Exercise 8.1

Q=1

$1+5+9+\dots+(4n-3) = n(2n-1)$ → Target

For $n=1$

$1 = 1[2(1)-1]$

$1 = 1(2-1)$

$1 = 1$

The result is true for $n=1$

Target:

$k+1[2(k+1)-1]$

$(k+1)(2k+2-1)$

$(k+1)(2k+1)$

$\therefore$ Condition-I is satisfied

Suppose that the result is true for $n=K$; $K \geq 1$

i.e $1+5+9+\dots+(4K-3) = K(2K-1)$

Adding next term i.e $4K+1$ on B.S

$1+5+9+\dots(4K-3)+(4K+1) = K(2K-1)+(4K+1)$

$= 2K^2-K+4K+1$

$= 2K^2+3K+1$

$= 2K^2+2K+K+1$

$= 2K(K+1)+1(K+1)$

$= (K+1)(2K+1)$

Next term

$= 4n-3$

$= 4(K+1)-3$

$= 4K+4-3$

$= 4K+1$

$\Longrightarrow (K+1)(2K+2-1)$

$(K+1)[2(K+1)-1] \quad \because n(2n-1)$

$\therefore$ The result is true for $n=K$

$\Longrightarrow$ it is also true for $n=K+1$

$\therefore$ condition II is satisfied

Hence the result is true for all +ve integers 'n'

Q=2

$$1+3+5+\dots+(2n-1)=n^2$$

For $n=1$

$1=(1)^2$

$1=1$

The result is true for $n=1$

∴ Condition I is satisfied

(Target)

$n^2=(K+1)^2$

$=K^2+2K+1$

Suppose that result is true for $n=K$, $K\geq 1$

i.e $1+3+5+\dots+(2K-1)=K^2$

Adding next term i.e $(2K+1)$ on b.S

$1+3+5+\dots(2K-1)+(2K+1)=K^2+2K+1$

$=(K+1)^2$

(Next term)

$2(K+1)-1$

$2K+2-1$

$2K+1$

∴ Result is true for $n=K$

⟹ it is also true for $n=K+1$

∴ C-II is satisfied

Hence result is true for all +ve integers 'n'

Q=3

$$1+4+7+\dots+(3n-2)=\frac{n(3n-1)}{2}$$

For $n=1$

$1=\frac{1(3(1)-1)}{2}$

$1=1$

The result is true for $n=1$

∴ C-I is satisfied

Target

$\frac{(K+1)[3(K+1)-1]}{2}$

$\frac{(K+1)[3K+3-1]}{2}$

$\frac{(K+1)(3K+2)}{2}$

Suppose that, result is true for $n = K$, then

$$1 + 4 + 7 + \dots (3K-2) = \frac{K(3K-1)}{2}$$

Adding next term i.e $(3K+1)$ on b.s

$$1 + 4 + 7 + \dots (3K-2) + (3K+1) = \frac{K(3K-1)}{2} + (3K+1)$$

Next term
$3(K+1)-2$
$= 3K+3-2$
$= 3K+1$

$$= \frac{3K^2 - K + 3K + 1}{2}$$

$$= \frac{3K^2 - K + 6K + 2}{2}$$

$$= \frac{3K^2 + 5K + 2}{2}$$

$$= \frac{3K^2 + 3K + 2K + 2}{2}$$

$$= \frac{3K(K+1) + 2(K+1)}{2}$$

$$= \frac{(3K+2)(K+1)}{2}$$

$$\Longrightarrow \frac{(K+1)(3K+3-1)}{2}$$

$$= \frac{(K+1)[3(K+1)-1]}{2}$$

$\therefore$ The result is true for $n = K$

$\Longrightarrow$ it is also true for $n = K+1$

$\therefore$ C-II is satisfied

Hence result is true for all positive integers n?

Q=4

$$1+2+4+\dots+2^{n-1}=2^n-1$$

For $n=1$

$1 = 2^1-1$

$1=1$

Target: $2^{k+1}-1$

The result is true for $n=1$

$\therefore$ C-I is satisfied

Suppose the result is true for $n=k$

i.e $1+2+4+\dots+2^{k-1}=2^k-1$

Adding next term i.e 2^k on b.s

$1+2+4+\dots+2^{k-1}+2^k = 2^k-1+2^k$

$= 2^k[1+1]-1$

$= 2\cdot 2^k-1$

$= 2^{k+1}-1$

Next term: $2^{(k+1)}-1$, $2^{k+1}-1$, 2^k

$\therefore$ The result is true for $n=k$

$\Rightarrow$ it is also true for $n=k+1$

$\therefore$ C-II is satisfied

Hence result is true for all +ve integers 'n'

Q=5

$$1+\frac{1}{2}+\frac{1}{4}+\dots+\frac{1}{2^{n-1}}=2\left[1-\frac{1}{2^n}\right]$$

For $n=1$

$1 = 2\left[1-\frac{1}{2^1}\right]$

$1 = 2\left[\frac{2-1}{2}\right]$

$1=1$

Target

$2\left[1-\frac{1}{2^{k+1}}\right]$

$2\left[\frac{2^{k+1}-1}{2^{k+1}}\right]$

Suppose the result is true for $n = k$

i.e $1 + \frac{1}{2} + \frac{1}{4} + \cdots + \frac{1}{2^{k-1}} = 2\left[1 - \frac{1}{2^k}\right]$

Adding next term i.e $\frac{1}{2^k}$ on b.S

Next term $= \frac{1}{2^{n-1}} = \frac{1}{2^{k+1-1}} = \frac{1}{2^k}$

$1 + \frac{1}{2} + \frac{1}{4} + \cdots + \frac{1}{2^{k-1}} + \frac{1}{2^k} = 2\left(1 - \frac{1}{2^k}\right) + \frac{1}{2^k}$

$= 2\left[1 - \frac{1}{2^k} + \frac{1}{2 \cdot 2^k}\right]$

$= 2\left[1 - \frac{1}{2^k} + \frac{1}{2^{k+1}}\right]$

$= 2\left[\frac{2^{k+1} - 2 + 1}{2^{k+1}}\right]$ $\because 2^{k+1} = 2^k \cdot 2$

$= 2\left[\frac{2^{k+1} - 1}{2^{k+1}}\right]$

$\Longrightarrow 2\left[1 - \frac{1}{2^{k+1}}\right]$

The result is true for $n = k$

$\rightarrow$ it is also true for $n = k+1$

C-II is satisfied

Hence the result is true for all +ve integers n

Q-6

$2+4+6+\dots+2n = n(n+1)$

For $n=1$

$2 = 1(1+1)$

$2 = 2$

The result is true for $n=1$

C-I is satisfied

Target
$k+1(k+1+1)$
$k+1(k+2)$
$k^2+2k+k+2$
k^2+3k+2

Suppose the result is true for $n=k$

i.e $2+4+6+\dots 2k = k(k+1)$

Adding next term i.e $2k+2$ on b.S

Next term
$2(k+1)$
$2k+2$

$2+4+6+\dots 2k+2k+2 = k(k+1)+2k+2$

$= k^2+k+2k+2$

$= k^2+3k+2$

$\Rightarrow k^2+2k++k+2$

$= k+1(k+2)$

$= k+1(k+1+1)$

The result is true for $n=k$

$\Rightarrow$ it is also true for $n=k+1$

C-II is satisfied

Hence result is true for all +ve integers 'n'

Q-7

$$2+6+18+\dots+2\times3^{n-1}=3^n-1$$

For $n=1$

$2 = 3^1-1$

$2=2$

The result is true for $n=1$

$\therefore$ C-I is satisfied

Target
$3^{k+1}-1$

Suppose that result is true for $n=K$

i.e $2+6+18+\dots+2\times3^{K-1}=3^K-1$

Adding next term i-e 6^K on b.S

$2+6+18+\dots+2\times3^{K-1}+6^K=3^K-1+6^K$

$=3^K+6^K-1$

$=3^K(1+2)-1$

$=3^K\cdot3-1$

$=3^{K+1}-1$

Next term
$=2\times3^{K+1-1}$
$=2\times3^K$
$=6^K$

The result is true for $n=K$

$\therefore$ it is also true for $n=K+1$

$\therefore$ C-II is satisfied

Hence the result is true for all +ve integers 'n'

Q-8

$$1\times3+2\times5+3\times7+\dots+n\times(2n+1)=\frac{n(n+1)(4n+5)}{6}$$

For $n=1$

$1\times3=\dfrac{1(1+1)(4(1)+5)}{6}$

$3=\dfrac{2(9)}{6}=\dfrac{18}{6}=3$

$3=3$

Target
$\dfrac{K+1(K+1+1)(4(K+1)+5)}{6}$
$\dfrac{(K+1)(K+2)(4K+4+5)}{6}$
$\dfrac{(K+1)(K+2)(4K+9)}{6}$

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The result is true for n=1

C I is satisfied

Suppose that result is true for n=K

i.e $1\times 3 + 2\times 5 + 3\times 7 + \dots + K(2K+1) = \frac{K(K+1)(4K+5)}{6}$

Adding next term i.e $(2K+3)(K+1)$ on b.s

$1\times 3 + 2\times 5 + 3\times 7 + \dots K(2K+1) + (2K+3)(K+1)$

$= \frac{K(K+1)(4K+5)}{6} + (2K+3)(K+1)$

Next term

$= K+1\,[2(K+1)+1]$

$= K+1\,(2K+2+1)$

$= K+1\,(2K+3)$

$= 2K^2+3K+2K+3$

$= K(2K+3)+1(2K+3)$

$= (2K+3)(K+1)$

$= K+1\left[\frac{K(4K+5)}{6} + 2K+3\right]$

$= K+1\left[\frac{4K^2+5K+12K+18}{6}\right] = K+1\left[\frac{4K^2+17K+18}{6}\right]$

$= (K+1)\left[\frac{4K^2+9K+8K+18}{6}\right] = K+1\left[\frac{K(4K+9)+2(4K+9)}{6}\right]$

$= \frac{(K+1)(4K+9)(K+2)}{6}$

$\Rightarrow \frac{(K+1)(K+2)(4K+4+5)}{6}$

$= \frac{(K+1)(K+1+1)[4(K+1)+5]}{6}$

The result is true for n=K

$\Rightarrow$ it is also true for n=K+1

C-II is satisfied

Hence the result is true for all +ve integers 'n'

Q=9

$$1\times 2 + 2\times 3 + 3\times 4 + \dots + n(n+1) = \frac{n(n+1)(n+2)}{3}$$

For n=1

$$1\times 2 = \frac{1(1+1)(1+2)}{3}$$

$$2 = \frac{2\times \cancel{3}}{\cancel{3}}$$

$$2 = 2$$

Target

$$\frac{(K+1)(K+1+1)(K+1+2)}{3}$$

$$\frac{(K+1)(K+2)(K+3)}{3}$$

The result is true for n=1

∴ C-I is satisfied

Suppose the result is true for n=K

i.e $1\times 2 + 2\times 3 + 3\times 4 + \dots K(K+1) = \frac{K(K+1)(K+2)}{3}$

Adding next term i.e $(K+1)(K+2)$ on b.S

$$1\times 2 + 2\times 3 + 3\times 4 + \dots K(K+1) + (K+1)(K+2)$$

$$= \frac{K(K+1)(K+2)}{3} + (K+1)(K+2)$$

$$= (K+1)(K+2)\left[\frac{K}{3} + 1\right]$$

$$= (K+1)(K+2)\left(\frac{K+3}{3}\right)$$

$$= \frac{(K+1)(K+2)(K+3)}{3} \Rightarrow \frac{(K+1)(K+1+1)(K+1+2)}{3}$$

Next term

$K+1\,(K+1+1)$

$(K+1)(K+2)$

$K^2+2K+K+2$

$K(K+2)+1(K+2)$

$(K+2)(K+1)$

The result is true for n=K

⇒ Result is also true for n=K+1

∴ C-II is satisfied

Hence result is true for all +ve integers 'n'

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Q=10

$$1\times2+3\times4+5\times6+\dots+(2n-1)\times2n=\frac{n(n+1)(4n-1)}{3}$$

For n=1

$$1\times2=\frac{1(1+1)(4(1)-1)}{3}$$

$$2=\frac{2\times\cancel{3}}{\cancel{3}}$$

$$2=2$$

Target

$$\frac{k+1(k+1+1)(4(k+1)-1)}{3}$$

$$\frac{(k+1)(k+2)(4k+4-1)}{3}$$

$$\frac{(k+1)(k+2)(4k+3)}{3}$$

The result is true for n=k

∴ C-I is satisfied

Suppose result is true for n=k

i.e $1\times2+3\times4+5\times6+\dots+(2k-1)\times2k=\frac{k(k+1)(4k-1)}{3}$

Adding next term i.e $(2k+1)(2k+2)$ on b.s

$$1\times2+3\times4+5\times6+\dots+(2k+1)\times2k+(2k+1)(2k+2)$$

$$=\frac{k(k+1)(4k-1)}{3}+(2k+1)(2k+2)$$

Next term

$$[2(k+1)-1]\times2(k+1)$$

$$\cancel{(2k+2-1)(2k+2)}$$

$$(2k+1)(2k+2)$$

$$=\frac{k(k+1)(4k-1)}{3}+(2k+1)2(k+1)$$

$$=k+1\left[\frac{k(4k-1)}{3}+2(2k+1)\right]$$

$$=(k+1)\left[\frac{4k^2-k}{3}+4k+2\right]$$

$$=(k+1)\left(\frac{4k^2-k+12k+6}{3}\right)$$

$$=(k+1)\left(\frac{4k^2+11k+6}{3}\right)$$

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$$= (k+1)\left(\frac{4k^2+8k+3k+6}{3}\right)$$

$$= (k+1)\left(\frac{4k(k+2)+3(k+2)}{3}\right)$$

$$= (k+1)\left(\frac{(4k+3)(k+2)}{3}\right)$$

$$= \frac{(k+1)(4k+3)(k+2)}{3}$$

$$\Rightarrow = \frac{(k+1)(k+2)(4k+4-1)}{3}$$

$$= \frac{(k+1)(k+1+1)[4(k+1)-1]}{3}$$

The result is true for $n = k$

$\Rightarrow$ it is also true for $n = k+1$

$\therefore$ C-II is satisfied

Hence result is true for all +ve integers 'n'

Ex 8.1

Q=11

$$\frac{1}{1\times2}+\frac{1}{2\times3}+\frac{1}{3\times4}+\dots+\frac{1}{n(n+1)}=1-\frac{1}{n+1}$$

For $n=1$

$$\frac{1}{1\times2}=1-\frac{1}{1+1}$$

$$\frac{1}{2}=\frac{2-1}{2}$$

Target

$$1-\frac{1}{(K+1)+1}$$

$$1-\frac{1}{K+2}$$

$$\frac{K+2-1}{K+2}$$

$$\frac{K+1}{K+2}$$

$$\frac{1}{2}=\frac{1}{2}$$

The result is true for $n=1$

$\therefore$ C-I is satisfied.

Suppose the result is true for $n=K$

i.e $\frac{1}{1\times2}+\frac{1}{2\times3}+\frac{1}{3\times4}+\dots+\frac{1}{K(K+1)}=1-\frac{1}{K+1}$

Adding next term i.e $\frac{1}{(K+1)(K+2)}$ on b.s

Next term

$$\frac{1}{(K+1)(K+1+1)}$$

$$\frac{1}{(K+1)(K+2)}$$

$$\frac{1}{1\times2}+\frac{1}{2\times3}+\frac{1}{3\times4}+\dots\frac{1}{K(K+1)}+\frac{1}{(K+1)(K+2)}$$

$$=1-\frac{1}{K+1}+\frac{1}{(K+1)(K+2)}$$

$$=\frac{(K+1)(K+2)-(K+2)+1}{(K+1)(K+2)}$$

$$=\frac{K^2+2K+\cancel{K}+\cancel{2}-\cancel{K}-\cancel{2}+1}{(K+1)(K+2)}$$

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$$= \frac{k^2 + 2k + 1}{(k+1)(k+2)}$$

$$= \frac{k^2 + k + k + 1}{(k+1)(k+2)}$$

$$= \frac{k(k+1) + 1(k+1)}{(k+1)(k+2)} = \frac{\cancel{(k+1)}(k+1)}{\cancel{(k+1)}(k+2)}$$

$$\Longrightarrow \frac{k+2-1}{k+2}$$

$$= 1 - \frac{1}{k+2}$$

$$= 1 - \frac{1}{(k+1)+1}$$

The result is true for $n = k$

$\Rightarrow$ the result is also true for $n = k+1$

$\therefore$ C-II is satisfied

The result is true for all values of +ve integers 'n'

$$Q=12$$

$$\frac{1}{1\times3}+\frac{1}{3\times5}+\frac{1}{5\times7}+\dots+\frac{1}{(2n-1)(2n+1)}=\frac{n}{2n+1}$$

Target

$$=\frac{k+1}{2(k+1)+1}=\frac{k+1}{2k+2+1}=\frac{k+1}{2k+3}$$

For $n=1$

$$\frac{1}{1\times3}=\frac{1}{2(1)+1}$$

$$\frac{1}{3}=\frac{1}{3}$$

The result is true for $n=1$

$\therefore$ C-I is satisfied

Suppose result is true for $n=k$

i.e $\frac{1}{1\times3}+\frac{1}{3\times5}+\frac{1}{5\times7}+\dots+\frac{1}{(2k-1)(2k+1)}=\frac{k}{2k+1}$

Adding next term i.e $\frac{1}{(2k+1)(2k+3)}$ on b.s

Next term

$$\frac{1}{[2(k+1)-1][2(k+1)+1]}$$

$$\frac{1}{(2k+2-1)(2k+2+1)}$$

$$\frac{1}{(2k+1)(2k+3)}$$

$$\frac{1}{1\times3}+\frac{1}{3\times5}+\frac{1}{5\times7}+\dots+\frac{1}{(2k-1)(2k+1)}+\frac{1}{(2k+1)(2k+3)}$$

$$=\frac{k}{(2k+1)}+\frac{1}{(2k+1)(2k+3)}$$

$$=\frac{k(2k+3)+1}{(2k+1)(2k+3)}$$

$$=\frac{2k^2+3k+1}{(2k+1)(2k+3)}$$

$$=\frac{2k^2+2k+k+1}{(2k+1)(2k+3)}=\frac{2k(k+1)+1(k+1)}{(2k+3)(2k+1)}$$

$$=\frac{\cancel{(2k+1)}(k+1)}{(2k+3)\cancel{(2k+1)}}=\frac{k+1}{2k+3}$$

$$\Rightarrow \frac{K+1}{K+2+1}$$

$$= \frac{K+1}{2(K+1)+1}$$

The result is true for $n=K$

Q=13

$$\frac{1}{2\times5}+\frac{1}{5\times8}+\frac{1}{8\times11}+\dots+\frac{1}{(3n-1)(3n+2)}=\frac{n}{2(3n+2)}$$

For $n=1$

$$\frac{1}{2\times5}=\frac{1}{2(3(1)+2)}$$

$$\frac{1}{10}=\frac{1}{10}$$

Target

$$\frac{(K+1)}{2(3(K+1)+2)}$$

$$\frac{(K+1)}{2[3K+3+2]}$$

$$\frac{(K+1)}{6K+10}$$

The result is true for $n=1$

$\therefore$ C-I is satisfied

Suppose the result is true for $n=K$

$$\frac{1}{2\times5}+\frac{1}{5\times8}+\frac{1}{8\times11}+\dots+\frac{1}{(3K-1)(3K+2)}=\frac{K}{2(3K+2)}$$

Adding next term ie $\frac{1}{(3K+2)(3K+5)}$ on b.s

Next term

$$=\frac{1}{[3(K+1)-1][3(K+1)+2]}$$

$$=\frac{1}{(3K+3-1)(3K+3+2)}$$

$$=\frac{1}{(3K+2)(3K+5)}$$

$$\frac{1}{2\times5}+\frac{1}{5\times8}+\frac{1}{8\times11}+\cdots+\frac{1}{(3K-1)(3K+2)}+\frac{1}{(3K+2)(3K+5)}$$

$$=\frac{K}{2(3K+2)}+\frac{1}{(3K+2)(3K+5)}$$

$$=\frac{1}{(3K+2)}\left\{\frac{K}{2}+\frac{1}{3K+5}\right\}$$

$$=\frac{1}{3K+2}\left\{\frac{K(3K+5)+2}{2(3K+5)}\right\}$$

$$=\frac{1}{3K+2}\left\{\frac{3K^2+5K+2}{2(3K+5)}\right\}$$

$$=\frac{3K^2+3K+2K+2}{2(3K+5)(3K+2)}$$

$$=\frac{3K(K+1)+2(K+1)}{2(3K+2)(3K+5)}$$

$$=\frac{\cancel{(3K+2)}(K+1)}{2\cancel{(3K+2)}(3K+5)}$$

$$=\frac{K+1}{2(3K+5)}=\frac{K+1}{6K+10}$$

$$\rightarrow\frac{K+1}{2(3K+3+2)}=\frac{K+1}{2[3(K+1)+2]}$$

The result is true for $n=K$

It is also true for $n=K+1$

Q=14

$$r + r^2 + r^3 + \dots + r^n = \frac{r(1-r^n)}{1-r}, \quad r \neq 1$$

For $n=1$

$$r = \frac{r(1-r^1)}{1-r}$$

$$r = r$$

Target

$$= \frac{r(1-r^{k+1})}{1-r}$$

The result is true for $n=1$

$\therefore$ C-I is satisfied

Suppose result is also true for $n=k$

$$r + r^2 + r^3 + \dots + r^k = \frac{r(1-r^k)}{1-r}$$

Adding next term i.e r^{k+1} on b.s

$$r + r^2 + r^3 + \dots + r^k + r^{k+1} = \frac{r(1-r^k)}{1-r} + r^{k+1}$$

$$= \frac{r(1-r^k) + 1-r(r^{k+1})}{1-r}$$

$$= \frac{r(1-r^k) + (1-r)(r^k \cdot r^1)}{1-r}$$

$$= \frac{r[1-r^k + (1-r)r^k]}{1-r}$$

$$= \frac{r[1-r^k + r^k - rr^k]}{1-r}$$

$$= \frac{r[1 - r^k(1 - 1 + r)]}{1-r}$$

$$= \frac{r\,[1 - r^{k}(r)]}{r}$$

$$= \frac{r\,(1 - r^{k+1})}{r}$$

The result is true for $n = K$

The result is also true for $n = K+1$

Hence C-II is satisfied

The result is true for all values of +ve integers 'n'

Q=15

$$a + (a+d) + (a+2d) + \dots + [a + (n-1)d] = \frac{n}{2}[2a + (n-1)d]$$

For $n = 1$

$$a = \frac{1}{2}[2a + (1-1)d]$$

$$a = \frac{1}{\cancel{2}}\cancel{2}a$$

$$a = a$$

Target

$$= \frac{K+1}{2}[2a + (K+1-1)d]$$

$$= \frac{K+1}{2}[2a + Kd]$$

$$= \frac{(K+1)(2a+Kd)}{2}$$

The result is true for $n = 1$

C-I is satisfied

Suppose result is also true for $n = K$

$$a + (a+d) + (a+2d) + \dots + [a + (K-1)d] = \frac{K}{2}(2a + (K-1)d]$$

Next term

$$[a + (K+1-1)d]$$

$$= a + Kd$$

Adding next term i.e $a+kd$ on b.s

$$a+(a+d)+(a+2d)+\dots[a+(k-1)d]+a+kd$$

$$=\frac{k}{2}[2a+(k-1)d]+a+kd$$

$$=\frac{k[2a+(k-1)d]+2[a+kd]}{2}$$

$$=\frac{k(2a+kd-d)+2(a+kd)}{2}$$

$$=\frac{2ak+k^2d-dk+2a+2kd}{2}$$

$$=\frac{k^2d+kd+2ak+2a}{2}$$

$$=\frac{kd(k+1)+2a(k+1)}{2}$$

$$=\frac{(k+1)(kd+2a)}{2}$$

$$\Rightarrow \frac{k+1}{2}[2a+(k+1-1)d]$$

Truth for $n=k$ $\Rightarrow$ truth for $n=k+1$

Hence statement is true for all +ve integeral values of 'n'

Q-16

$1\cdot 1! + 2\cdot 2! + 3\cdot 3! + \dots + n\cdot n! = (n+1)! - 1$

For $n=1$

$1\cdot 1! = (1+1)! - 1$

$1\cdot 1! = 2! - 1$

$1 = 2\times 1 - 1$

$1 = 1$

Target

$= (K+1+1)! - 1$

$= (K+2)! - 1$

Result is true for $n=1$ ∴ C-I is satisfied

Suppose statement is true for $n=K$

$1\cdot 1! + 2\cdot 2! + 3\cdot 3! + \dots + K\cdot K! = (K+1)! - 1$

Now we want to prove for $n=K+1$

For this Adding next term i.e $K+1\,(K+1)!$ on b.s

Next term

$= K+1\,(K+1)!$

$1\cdot 1! + 2\cdot 2! + 3\cdot 3! + \dots + K\cdot K! + (K+1)(K+1)!$

$= (K+1)! - 1 + (K+1)(K+1)!$

$= (K+1)!\,[1 + K+1] - 1$

$= (K+1)!\,(K+2) - 1$

$= K+2\,(K+1)! - 1$

$= (K+2)(K+1)! - 1$

$= (K+2)! - 1$

$= (K+1+1)! - 1$

Truth for $n=K \Rightarrow$ truth for $n=K+1$

Hence statement is true for all +ve integral values of 'n'

Q=17

$a_n = a_1 + (n-1)d$ when a_1, a_1+d, a_1+2d form A.P.

For $n=1$

$a_1 = a_1 + (1-1)d$

$a_1 = a_1$

Result is true for $n=1$

C-I is satisfied

Suppose result is also true for $n=K$

$a_K = a_1 + (K-1)d$

Now we want to prove for $n=K+1$

Adding next term i.e 'd' on b.S

$a_{K+1} = a_1 + (K+1-1)d + d$ | Next term

$a_{K+1} = a_1 + (K+1-1)d$

$a_{K+1} = a_1 + (K+1-1)d$

The result is true for $n=K$

$\Rightarrow$ it is also true for $n=K+1$

Hence C-II is satisfied

Result will be true for all +ve integral values of 'n'

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Q=18

$a_n = a_1 r^{n-1}$ when $a_1, a_1 r, a_1 r^2 \ldots$ form a G.P

For $n=1$

$a_1 = a_1 r^{1-1}$

$a_1 = a_1 r^0$

$a_1 = a_1$

Target

$= a_1 r^{n-1}$

$= a_1 r^{k+1-1}$

$= a_1 r^{k}$

Result is true for $n=1$

$\therefore$ C-I is satisfied.

Suppose result is also true for $n=k$

$a_k = a_1 r^{k-1}$

Now multiply b.s by r

$a_k \cdot r = a_1 r^{k-1} r$

$= a_1 r^{k-1+1}$

$= a_1 r^{\underline{k+1}-1}$

The result is true for $n=k$

$\Rightarrow$ it is also true for $n=k+1$

Hence C-II is satisfied.

Result will be true for any +ve integral number. 'n'

Q-19

$$1^2+3^2+5^2+\cdots+(2n-1)^2=\frac{n(4n^2-1)}{3}$$

For $n=1$

$$1^2=\frac{1(4(1)^2-1)}{3}$$

$$1^2=\frac{3}{3}$$

$$1=1$$

Target

$$=\frac{(K+1)(4(K+1)^2-1)}{3}$$

$$=\frac{(K+1)[4(K^2+2K+1)-1]}{3}$$

$$=\frac{(K+1)(4K^2+8K+4-1)}{3}$$

The result is true for $n=1$

C–I is satified

Suppose result is also true for $n=K$

i.e $1^2+3^2+5^2+\cdots+(2K-1)^2=\frac{K(4K^2-1)}{3}$

Adding next term i.e $(2K+1)^2$ on b.S

Next term

$$=(2(K+1)-1)^2$$

$$=(2K+2-1)^2$$

$$=(2K+1)^2$$

$$1^2+3^2+5^2+\cdots+(2K-1)^2+(2K+1)^2$$

$$=\frac{K(4K^2-1)}{3}+(2K+1)^2$$

$$=\frac{K(4K^2-1)+3(2K+1)^2}{3}$$

$$=\frac{K(2K+1)(2K-1)+3(2K+1)^2}{3}$$

$$=\frac{(2K+1)[K(2K-1)+3(2K+1)]}{3}$$

$$=\frac{(2K+1)(2K^2-K+6K+3)}{3}$$

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$$= \frac{(k+1)(4k^2+8k+3)}{3}$$

$$= \frac{(k+1)(4k^2+6k+2k+3)}{3}$$

$$= \frac{(k+1)[2k(2k+3)+1(2k+3)]}{3}$$

$$= \frac{(k+1)(2k+3)(2k+1)}{3}$$

$$= \frac{(2k+1)[2k^2+5k+3]}{3}$$

$$= \frac{(2k+1)(2k^2+3k+2k+3)}{3}$$

$$= \frac{(2k+1)[k(2k+3)+1(2k+3)]}{3}$$

$$= \frac{(2k+1)(2k+3)(k+1)}{3}$$

$$\longrightarrow \frac{(k+1)[2k(2k+3)+1(2k+3)]}{3}$$

$$= \frac{(k+1)(4k^2+6k+2k+3)}{3}$$

$$= \frac{(k+1)(4k^2+8k+3)}{3}$$

$$= \frac{(k+1)(4k^2+8k+4-1)}{3}$$

$$= \frac{(K+1)\left[4(K^2+2K+1)-1\right]}{3}$$

$$= \frac{(K+1)\left[4(K+1)^2-1\right]}{3}$$

Hence result is true for $n=K$

It is also true for $n=K+1$

C-II is satisfied

The result is true for all +ve integers 'n'

Q=20

$$\binom{3}{3}+\binom{4}{3}+\binom{5}{3}+\dots+\binom{n+2}{3}=\binom{n+3}{4}$$

For $n=1$

$$\binom{3}{3}=\binom{1+3}{4}$$

$$\binom{3}{3}=\binom{4}{4}$$

Target

$$=\binom{K+1+3}{4}$$

$$=\binom{K+4}{4}$$

Result is true for $n=1$

C-I is satisfied

Suppose result is also true for $n=K$

i.e $\binom{3}{3}+\binom{4}{3}+\binom{5}{3}+\dots+\binom{K+2}{3}=\binom{K+3}{4}$

Adding next term i.e $\binom{k+3}{3}$ on b.S

$$\binom{3}{3}+\binom{4}{3}+\binom{5}{3}+\dots\binom{k+2}{3}+\binom{k+3}{3}$$

$$=\binom{k+1+2}{3}$$

$$=\binom{k+3}{3}$$

$$=\binom{k+3}{4}+\binom{k+3}{3}$$

$$=\binom{k+3+1}{3+1}$$

$$=\binom{k+4}{4}$$

$$\Longrightarrow \binom{\underline{k+1}+3}{4}$$

Q-21

i) n^2+n is divisible by 2

For $n=1$

$(1)^2+1 = 1+1=2$

Clearly 2 is divisible by '2'

Statement is true for $n=1$

C-I is satisfied

Suppose statement is true for $n=K$

i.e K^2+K is divisible by 2

$K^2+K = 2(q)$ where 'q' is some quotient — ①

Now we want to prove for $n=K+1$

i.e $(K+1)^2+(K+1)$ is divisible by 2

or $(K+1)^2+(K+1)=2$

consider $(K+1)^2+(K+1)$

$= K^2+2K+1+K+1$

$= (K^2+K)+2K+2$

$= 2(q)+2(K+1)$ — from ①

$= 2(q+K+1)$; where $q+K+1$ is some quotient

$= 2$(some quotient)

$\Rightarrow (K+1)^2+(K+1)$ is divisible by 2

Truth for $n=K$ implies truth for $n=K+1$

C-II is satisfied.

Result is true for all +ve integral values of 'n'

ii) $5^n - 2^n$ is divisible by 3

For $n=1$

$5^1 - 2^1 = 5-2=3$

is divisible by '3'

Statement is true for $n=1$

C-I is satisfied

Suppose it is true for $n=k$

$5^k - 2^k$ is divisible by 3 —— ①

$5^k - 2^k = 3(q)$ $\quad q \in Z$ —— ②

Now we want to prove for $n=k+1$

i.e $5^{k+1} - 2^{k+1}$ is divisible by 3

or $5^{k+1} - 2^{k+1} = 3$

consider $5 \cdot 5^k - 2 \cdot 2^k$

$= (3+2)5^k - 2 \cdot 2^k$

$= 3 \cdot 5^k + 2 \cdot 5^k - 2 \cdot 2^k$

$= 3 \cdot 5^k + 2(5^k - 2^k)$

$= 3 \cdot 5^k + 2 \cdot 3q$ —— from ②

$= 3(5^k + 2q)$

$= 3$ (some quotient)

$\Rightarrow 5^{k+1} - 2^{k+1}$ is divisible by 3

C-II is satisfied

Truth for $n=k \Rightarrow$ truth for $n=k+1$

Hence statement is true for all +ve integral values of 'n'

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(iii)

$5^n - 1$ is divisible by 4

For $n=1$

$5^1 - 1 = 4$

4 is divisible by '4'

Thus statement is true for $n=1$

C-I is satisfied

Suppose statement is true for $n=k$

i.e $5^k - 1$ is divisible by 4 —①

$5^k - 1 = 4(q) \quad q \in Z$ —②

Now we want to prove for $n=k+1$

i.e $5^{k+1} - 1$ is divisible by 4

or $5^{k+1} - 1 = 4(q)$

Consider $5^{k+1} - 1$

$= 5.5^k - 1$

$= (4+1)5^k - 1 \quad = 4 \times 5^k + 1 \times 5^k - 1$

$= 4.5^k + (5^k - 1)$

$= 4.5^k + 4(q)$ from eq ②

$= 4(5^k + q)$

$= 4$ (some quotient)

$5^{k+1} - 1$ is divisible by 4 C-II is satisfied

Truth for $n=k$ implies truth for $n=k+1$

Hence statement is true for all +ve integral values of 'n'

(iv)

$8 \times 10^n - 2$ is divisible by 6

For $n = 1$

$8 \times 10^1 - 2 = 80 - 2 = 78 \div 6 = 13$

78 is divisible by 6

Thus statement is true for $n = 1$ C-I is satisfied

Suppose statement is true for $n = k$

i.e $8 \times 10^k - 2$ is divisible by 6 — ①

or $8 \times 10^k - 2 = 6(q)$ where q is some quotient ②

Now we want to prove for $n = k+1$

i.e $8 \times 10^{k+1} - 2$ is divisible by 6

or $8 \times 10^{k+1} - 2 = 6$ (quotient)

Consider

$$8 \times 10^{k+1} - 2$$

$$= 8 \times 10^k \cdot 10 - 2$$

$$= (\{8 \times 10^k \cdot 10\} - 10 \cdot 2) + 10 \cdot 2 - 2$$

$$= 10(8 \times 10^k - 2) + 2(10 - 1)$$

$$= 10(6q) + 18 \quad \text{from ②}$$

$$= 6(10q) + 18$$

$$= 6[10q + 3]$$

$$= 6 \text{ (some quotient)}$$

v) $n^3 - n$ is divisible by 6

For $n = 1$

$n^3 - n = (1)^3 - 1 = 0$

0 is divisible by 6

Statement is true for $n = 1$

C-I is satisfied

Suppose statement is true for $n = k$

i.e $k^3 - k$ is divisible by 6 — ①

$\Rightarrow$ $k^3 - k = 6q \quad q \in Z$ — ②

Now we want to prove for $n = k+1$

i.e $(k+1)^3 - (k+1)$ is divisible by 6

Consider $(k+1)^3 - (k+1)$

$= k^3 + 1 + 3k(k+1) - k - 1$

$= k^3 + 1 + 3k^2 + 3k - k - 1$

$= (k^3 - k) + (3k^2 + 3k)$

$= (k^3 - k) + 3k(k+1)$

$= (6q) + 3(2p) \quad \because k(k+1) = 2p = \text{even integer}$

$= 6q + 6p$

$= 6(p+q)$ where $(p+q)$ is an integer

$\Rightarrow (k+1)^3 - (k+1)$ is divisible by 6

Truth for $n = k$ implies truth for $n = k+1$

C-II is satisfied.

Hence it is true for all +ve integral values of 'n'

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Q=22

$$\frac{1}{3}+\frac{1}{3^2}+\cdots+\frac{1}{3^n}=\frac{1}{2}\left[1-\frac{1}{3^n}\right]$$

Target

$$=\frac{1}{2}\left[1-\frac{1}{3^{k+1}}\right]$$

For $n=1$

$$\frac{1}{3}=\frac{1}{2}\left[1-\frac{1}{3^1}\right]$$

$$\frac{1}{3}=\frac{1}{2}\left(\frac{3-1}{3}\right)$$

$$\frac{1}{3}=\frac{1}{\cancel{2}}\left(\frac{\cancel{2}}{3}\right)$$

$$\frac{1}{3}=\frac{1}{3}$$

Result is true for $n=1$

C-I is satisfied.

Suppose result is true for $n=k$

i.e $\frac{1}{3}+\frac{1}{3^2}+\cdots+\frac{1}{3^k}=\frac{1}{2}\left[1-\frac{1}{3^k}\right]$

Adding next term i.e $\frac{1}{3^{k+1}}$ on b.S

Next term $=\frac{1}{3^{k+1}}$

$$\frac{1}{3}+\frac{1}{3^2}+\cdots+\frac{1}{3^k}+\frac{1}{3^{k+1}}=\frac{1}{2}\left[1-\frac{1}{3^k}\right]+\frac{1}{3^{k+1}}$$

$$=\frac{1}{2}\left(\frac{3^k-1}{3^k}\right)+\frac{1}{3^{k+1}}$$

$$=\frac{3^k-1}{2\cdot 3^k}+\frac{1}{3^k\cdot 3}$$

$$= \frac{3(3^k-1)+2}{3^k.6}$$

$$= \frac{3(3^k-1)+2}{3^k.3\times 2}$$

$$= \frac{3(3^k-1)+2}{3^{k+1}.2}$$

$$= \frac{1}{2}\left(\frac{3.3^k-3+2}{3^{k+1}}\right)$$

$$= \frac{1}{2}\left(\frac{3^{k+1}-1}{3^{k+1}}\right)$$

$$\Rightarrow \frac{1}{2}\left(1-\frac{1}{3^{k+1}}\right)$$

The result is true for $n=k$

$\Rightarrow$ result is also true for $n=k+1$ C-II satisfied

Hence it is true for all +ve integral values of 'n'

Q=23

$$1^2 - 2^2 + 3^2 - 4^2 + \dots + (-1)^{n-1} \cdot n^2 = \frac{(-1)^{n-1} \cdot n(n+1)}{2}.$$

For $n=1$

$$1^2 = \frac{(-1)^{1-1} \cdot 1(1+1)}{2}$$

$$1^2 = \frac{2}{2}$$

$$1 = 1$$

Target

$$= \frac{(-1)^{k+1-1}(k+1)(k+1+1)}{2}$$

$$= \frac{(-1)^{k}(k+1)(k+2)}{2}$$

The result is true for $n=1$

C-I is satisfied

Suppose result is also true for $n=k$

$$1^2 - 2^2 + 3^2 - 4^2 + \dots + (-1)^{k-1} \cdot k^2 = \frac{(-1)^{k-1} k(k+1)}{2}$$

Adding next term i.e $(-1)^k (k+1)^2$ on b.s

$$1^2 - 2^2 + 3^2 - 4^2 + \dots + (-1)^{k-1} \cdot k^2 + (-1)^k (k+1)^2$$

$$= \frac{(-1)^{k-1} k(k+1)}{2} + (-1)^k (k+1)^2$$

Next term

$$= (-1)^{k+1-1}(k+1)^2$$

$$= (-1)^k (k+1)^2$$

$$= \frac{(-1)^k (-1)^{-1} k(k+1)}{2} + (-1)^k (k+1)^2$$

$$= (-1)^k (k+1) \left\{ -\frac{k}{2} + (k+1) \right\}$$

$$= (-1)^k (k+1) \left(\frac{-k + 2k+2}{2} \right)$$

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$$= (-1)^k (k+1) \left\{ \frac{k+2}{2} \right\}$$

$$= \frac{(-1)^k (k+1)(k+1+1)}{2}$$

Result is true for $n = k$

$\Longrightarrow$ it is also true for $n = k+1$

C-II is satisfied

Hence result is true for all value of +ve integers 'n'

Q=24

$$1^3 + 3^3 + 5^3 + \dots + (2n-1)^3 = n^2\left[2n^2-1\right]$$

For $n=1$

$$1^3 = (1)^2\left[2(1)^2-1\right]$$

$$1 = 1$$

Result is true for $n=1$

C-I is satisfied

Target

$$= (k+1)^2\left[2(k+1)^2-1\right]$$

$$= (k+1)^2\left[2(k^2+2k+1)-1\right]$$

$$= (k+1)^2\left[2k^2+4k+2-1\right]$$

$$= (k+1)^2(2k^2+4k+1)$$

Suppose result is also true for $n=k$

i.e $1^3+3^3+5^3+\dots+(2k-1)^3 = k^2\left[2k^2-1\right]$

Adding next term i.e $(2k+1)^3$ on b.s

Next term

$$(2(k+1)-1)^3$$

$$= (2k+2-1)^3$$

$$= (2k+1)^3$$

$$1^3+3^3+5^3+\dots+(2k-1)^3+(2k+1)^3$$

$$= k^2(2k^2-1)+(2k+1)^3$$

$$= k^2(2k^2-1)+8k^3+(1)^3+3(2k)(1)(2k+1)$$

$$= 2k^4-k^2+8k^3+1+12k^2+6k$$

$$= 2k^4+8k^3+11k^2+6k+1$$

$$= 2k^4+2k^3+6k^3+6k^2+5k^2+5k+k+1$$

$$= 2k^3(k+1)+6k^2(k+1)+5k(k+1)+1(k+1)$$

$$= (k+1)(2k^3+6k^2+5k+1)$$

$$= (k+1)(2k^3+2k^2+4k^2+4k+k+1)$$

$$= (k+1)\left[2k^2(k+1)+4k(k+1)+(k+1)\right]$$

$$= (k+1)^2(2k^2+4k+1)$$

$$\Rightarrow (K+1)^2\left[2K^2+4K+2-1\right]$$
$$= (K+1)^2\left[2(K^2+2K+1)-1\right]$$
$$= (K+1)^2\left[2(K+1)^2-1\right]$$

Truth for $n=K$ implies truth for $n=K+1$

C-II is satisfied

Result is true for all +ve integral values of 'n'

Q=25

$x+1$ is a factor of $x^{2n}-1$; $(x \neq -1)$

For $n=1$

$x^{2n}-1 = x^2-1 = (x+1)(x-1)$

$(x+1)$ is a factor of (x^2-1)

Result is true for $n=1$

C-I is satisfied

Suppose statement is true for $n=k$

i.e $x+1$ is a factor of $x^{2k}-1$ —— ①

Now we want to prove for $n=k+1$

i.e $x+1$ is a factor of $x^{2(k+1)}-1$

Consider $x^{2(k+1)}-1$

$= x^{2k+2}-1$

$= x^{2k}\cdot x^2 - 1$

$= (x^2-1+1)\cdot x^{2k} - 1$

$= \{(x^2-1)+1\}\cdot x^{2k} - 1$

$= (x^2-1)x^{2k} + x^{2k} - 1$

$= \{(x+1)(x-1)x^{2k}\} + \{x^{2k}-1\}$

Now, $x+1$ is a factor of $\{(x+1)(x-1)\cdot x^{2k}\}$

and $x+1$ is a factor of $\{x^{2k}-1\}$

$\Rightarrow$ $x+1$ is a factor of $\{(x+1)(x-1)\cdot x^{2k}\}+\{x^{2k}-1\}$

$\Rightarrow$ $x+1$ is a factor of $x^{2k+2}-1$

C-II is satisfied.

Truth for $n=k \Rightarrow$ truth for $n=k+1$

Hence statement is true for all +ve integral values of 'n'

Q-26

~~$x-y$ is a factor of x^n-y^n~~

Let $S(n)$ be a given statement, i.e

$S(n)$: $x-y$ is a factor of x^n-y^n

Condition 1

when $n=1$

$S(1) = x-y$ is a factor of x^1-y^1

clearly $x-y$ " " " " $x-y$

Thus $S(1)$ is true i.e condition (1) is satisfied.

Condition 2

Suppose $S(n)$ is true for $n=k \in N$

$S(k)$: $x-y$ is a factor of x^k-y^k; $(x \neq y)$ —— ①

Now we will prove it for $n=k+1$

$S(k+1)$: $x-y$ is a factor of $x^{k+1}-y^{k+1}$; $(x \neq y)$

$$S(k+1) : x^{k+1}-y^{k+1}$$

$$= x^k \cdot x - y^k \cdot y$$

$$= x \cdot x^k - \underbrace{x \cdot y^k + x \cdot y^k} - y \cdot y^k$$

$$= x(x^k-y^k) + y^k(x-y)$$

Here $x(x^k-y^k)$ has a factor $x-y$ from ①

And $y^k(x-y)$ has also a factor $x-y$

Thus $S(k+1)$ is true if $S(k)$ is true So condition (2) is satisfied.

Since both condition are satisfied, therefore $S(n)$ is true for all +ve integers

Q=29

$$AB = BA \quad \text{--- } (1)$$

and $AB^n = B^nA$

For $n=1$

$$AB' = B'A$$

$$AB = BA$$

Statement is true for $n=1$

C-I is satisfied

Suppose statement is true for $n=K$

i.e $\quad AB^K = B^KA \quad \text{--- } (2)$

Now we want to prove for $n=K+1$

i.e $\quad AB^{K+1} = B^{K+1}A$

Post multiply b.S by B on (2)

$$AB^K \cdot B = B^K A B$$

$$AB^{K+1} = B^K(AB)$$

$$AB^{K+1} = B^K(BA) \quad \text{from } (1)$$

$$\underline{AB^{K+1} = B^{K+1}A}$$

The result is true for $n=K$

$\Rightarrow$ it is also true for $n=K+1$

C-II is satisfied

Hence statement is true for all values of +ve integers 'n'

Ex 8.2

Expand the following

Q=1

i) $(a+2b)^5$

$$= \binom{5}{0}(a)^{5-0}(2b)^0 + \binom{5}{1}a^{5-1}(2b)^1 + \binom{5}{2}a^{5-2}(2b)^2$$

$$\binom{5}{3}a^{5-3}(2b)^3 + \binom{5}{4}a^{5-4}(2b)^4 + \binom{5}{5}a^{5-5}(2b)^5$$

$$= 1\cdot a^5\cdot 1 + 5a^4\cdot 2b + \frac{5\times 4}{2\times 1}a^3\cdot 4b^2 + \frac{5\times\cancel{4}^2\times\cancel{3}}{\cancel{3}\times\cancel{2}\times 1}a^2\, 8b^3$$

$$+ \frac{5\times\cancel{4}}{\cancel{4}}a^1\, 16b^4 + 1\cdot a^0\, 32b^5$$

$$= a^5 + 10a^4b + 40a^3b^2 + 80a^2b^3 + 80ab^4 + 32b^5$$

(ii)

$$\left(\frac{x}{2} - \frac{2}{x^2}\right)^6$$

$$= \binom{6}{0}\left(\frac{x}{2}\right)^{6-0}\left(\frac{-2}{x^2}\right)^0 + \binom{6}{1}\left(\frac{x}{2}\right)^{6-1}\left(\frac{-2}{x^2}\right)^1$$

$$+ \binom{6}{2}\left(\frac{x}{2}\right)^{6-2}\left(\frac{-2}{x^2}\right)^2 + \binom{6}{3}\left(\frac{x}{2}\right)^{6-3}\left(\frac{-2}{x^2}\right)^3$$

$$+ \binom{6}{4}\left(\frac{x}{2}\right)^{6-4}\left(\frac{-2}{x^2}\right)^4 + \binom{6}{5}\left(\frac{x}{2}\right)^{6-5}\left(\frac{-2}{x^2}\right)^5$$

$$+ \binom{6}{6}\left(\frac{x}{2}\right)^{6-6}\left(\frac{2}{x^2}\right)^6$$

$$= \left(1 \times \frac{x^6}{64} \times 1\right) - \left(\cancel{6}^{3} \times \frac{x^5}{\cancel{32}_{16\,8}} \times \frac{2}{x^2}\right) + \left(\frac{\cancel{6}^{3}\times 5}{\cancel{2}}\left(\frac{x^4}{\cancel{16}_4}\right)\right.$$

$$\left.\left(\frac{4}{x^4}\right)\right) - \left(\frac{6 \times 5 \times 4}{3 \times 2 \times 1} \times \frac{x^3}{\cancel{8}} \times \frac{8}{x^6}\right)$$

$$+ \left(\frac{\cancel{6}^{3} \times 5 \times 4 \times 3}{4 \times 3 \times 2 \times 1} \times \frac{x^2}{\cancel{4}} \times \frac{\cancel{16}^{4}}{x^8}\right) - \left(\frac{\cancel{6 \times 5 \times 4 \times 3 \times 2 \times 1}}{\cancel{5 \times 4 \times 3 \times 2 \times 1}} \times \frac{x}{\cancel{2}} \times \frac{32}{x^{10}}\right)$$

$$+ \; 1 \times 1 \times \frac{64}{x^{12}}$$

$$\Longrightarrow \frac{x^6}{64} - \frac{3x^3}{8} + \frac{15}{4} - \frac{20}{x^3} + \frac{60}{x^6} - \frac{96}{x^9} + \frac{64}{x^{12}}$$

(iii)

$$\left(3a - \frac{x}{3a}\right)^4$$

$$= \binom{4}{0}(3a)^{4-0}\left(\frac{-x}{3a}\right)^0 + \binom{4}{1}(3a)^{4-1}\left(\frac{-x}{3a}\right)^1$$

$$+ \binom{4}{2}(3a)^{4-2}\left(\frac{-x}{3a}\right)^2 + \binom{4}{3}(3a)^{4-3}\left(\frac{-x}{3a}\right)^3$$

$$+ \binom{4}{4}(3a)^{4-4}\left(\frac{-x}{3a}\right)^4$$

$$= (1 \times 81a^4) + \left(4 \times \cancel{27a^3}^{9a^2} \times \frac{-x}{\cancel{3a}}\right) + \left(\frac{\cancel{4}^{2} \times 3}{\cancel{2} \times 1} \times \cancel{9a^2} \times \frac{x^2}{\cancel{9a^2}}\right)$$

$$+ \left(\frac{4 \times \cancel{3} \times \cancel{2}}{\cancel{3} \times \cancel{2} \times 1} \times \cancel{3}a \times \frac{-x^3}{\cancel{27}^{9} a^3}\right) + \left(1 \times 1 \times \frac{x^4}{81a^4}\right)$$

$$= 81a^4 - 36a^2x + 6x^2 - \frac{4x^3}{9a^2} + \frac{x^4}{81a^4}$$

(iv)

$$\left(2a - \frac{x}{a}\right)^7$$

$$= \binom{7}{0}(2a)^{7-0}\left(\frac{-x}{a}\right)^0 + \binom{7}{1}(2a)^{7-1}\left(\frac{-x}{a}\right)^1 + \binom{7}{2}(2a)^{7-2}\left(\frac{-x}{a}\right)^2$$

$$+ \binom{7}{3}(2a)^{7-3}\left(\frac{-x}{a}\right)^3 + \binom{7}{4}(2a)^{7-4}\left(\frac{-x}{a}\right)^4$$

$$+ \binom{7}{5}(2a)^{7-5}\left(\frac{-x}{a}\right)^5$$

(iii)

$$\left(3a - \frac{x}{3a}\right)^4$$

$$= \binom{4}{0}(3a)^{4-0}\left(\frac{-x}{3a}\right)^0 + \binom{4}{1}(3a)^{4-1}\left(\frac{-x}{3a}\right)^1$$

$$+ \binom{4}{2}(3a)^{4-2}\left(\frac{-x}{3a}\right)^2 + \binom{4}{3}(3a)^{4-3}\left(\frac{-x}{3a}\right)^3$$

$$+ \binom{4}{4}(3a)^{4-4}\left(\frac{-x}{3a}\right)^4$$

$$= (1 \times 81a^4) + \left(4 \times \cancel{27a^3}^{\,9a^2} \times \frac{-x}{\cancel{3a}}\right) + \left(\frac{\cancel{4}^{\,2} \times 3}{\cancel{2} \times 1} \times \cancel{9a^2} \times \frac{x^2}{\cancel{9a^2}}\right)$$

$$+ \left(\frac{4 \times \cancel{3} \times \cancel{2}}{\cancel{3} \times \cancel{2} \times 1} \times \cancel{3}a \times \frac{-x^3}{\cancel{27}_{9}a^{\cancel{3}}}\right) + \left(1 \times 1 \times \frac{x^4}{81a^4}\right)$$

$$= 81a^4 - 36a^2x + 6x^2 - \frac{4x^3}{9a^2} + \frac{x^4}{81a^4}$$

(iv)

$$\left(2a - \frac{x}{a}\right)^7$$

$$= \binom{7}{0}(2a)^{7-0}\left(\frac{-x}{a}\right)^0 + \binom{7}{1}(2a)^{7-1}\left(\frac{-x}{a}\right)^1 + \binom{7}{2}(2a)^{7-2}\left(\frac{-x}{a}\right)^2$$

$$+ \binom{7}{3}(2a)^{7-3}\left(\frac{-x}{a}\right)^3 + \binom{7}{4}(2a)^{7-4}\left(\frac{-x}{a}\right)^4$$

$$+ \binom{7}{5}(2a)^{7-5}\left(\frac{-x}{a}\right)^5$$

$$+\binom{7}{6}(2a)^{7-6}\left(\frac{-x}{a}\right)^{6}+\binom{7}{7}(2a)^{7-7}\left(\frac{-x}{a}\right)^{7}$$

$$=\left(1\times 128a^{7}\times 1\right)+\left(7\times 64a^{6}\times \frac{-x}{a}\right)+\left(\frac{7\times 6}{2\times 1}\times 32a^{5}\times \frac{x^{2}}{a^{2}}\right)$$

$$+\left(\frac{7\times 6\times 5}{3\times 2\times 1}\times 16a^{4}\times \frac{-x^{3}}{a^{3}}\right)+\left(\frac{7\times 6\times 5\times 4}{4\times 3\times 2\times 1}\times 8a^{3}\times \frac{x^{4}}{a^{4}}\right)$$

$$+\left(\frac{7\times 6\times 5\times 4\times 3}{5\times 4\times 3\times 2\times 1}\times 4a^{2}\times \frac{-x^{5}}{a^{5}}\right)$$

$$+\left(\frac{7\times 6\times 5\times 4\times 3\times 2}{6\times 5\times 4\times 3\times 2\times 1}\times \frac{x^{6}}{a^{6}}\times 2a\right)+\left(1\times 1\times \frac{-x^{7}}{a^{7}}\right)$$

$$\Rightarrow 128a^{7}-448a^{5}x+672x^{2}a^{3}-560ax^{3}$$

$$+280\frac{x^{4}}{a}-84\frac{x^{5}}{a^{3}}+14\frac{x^{6}}{a^{5}}-\frac{x^{7}}{a^{7}}$$

Q=2

(i)

$(0.97)^3$

$= (1-0.03)^3$

$= \binom{3}{0}(1)^{3-0}(-0.03)^0 + \binom{3}{1}(1)^{3-1}(-0.03)^1$

$+ \binom{3}{2}(1)^{3-2}(-0.03)^2 + \binom{3}{3}(1)^0(-0.03)^3$

$= (1)(1)(1) + 3(1)(-0.03) + \left(\frac{3\times 2}{2\times 1}\right)(1)(0.0009)$

$+ (1)(1)(-0.000027)$

$= 1 - 0.09 + 0.0027 - 0.000027$

$= 0.912673$

(ii)

$(2.02)^4$

$= (2+0.02)^4$

$= \binom{4}{0}(2)^{4-0}(0.02)^0 + \binom{4}{1}(2)^{4-1}(0.02)^1$

$+ \binom{4}{2}(2)^{4-2}(0.02)^2 + \binom{4}{3}(2)^{4-3}(0.02)^3$

$+ \binom{4}{4}(2)^{4-4}(0.02)^4$

$= (1\times 16\times 1) + (4\times 8\times 0.02) + \left(\frac{4\times 3}{2\times 1} \times 4\times 0.0004\right)$

$$+\left(\frac{4\times3\times2}{3\times2\times1}\times 2\times 0.000008\right)+\left(1\times1\times0.00000016\right)$$

$$=16+0.64+0.0096+0.000064+0.00000016$$

$$=16.64966416$$

(iii)

$$(9.98)^4$$

$$=(9+0.98)^4$$

$$=\binom{4}{0}(9)^{4-0}(0.98)^0+\binom{4}{1}(9)^{4-1}(0.98)^1$$

$$+\binom{4}{2}(9)^{4-2}(0.98)^2+\binom{4}{3}(9)^{4-3}(0.98)^3$$

$$+\binom{4}{4}(9)^{4-4}(0.98)^4$$

$$=(1\times6561\times1)+(4\times729\times0.98)+\left(\frac{\cancel{4}^{2}\times3}{\cancel{2\times1}}\times81\times0.960\right)$$

$$+\left(\frac{4\times\cancel{3\times2}}{\cancel{3\times2\times1}}\times9\times0.941192\right)+\left(1\times1\times0.92236816\right)$$

$$=6561+2857.68+466.7544+33.882912$$

$$+0.92236816$$

$$=9920.23968$$

(iv)

$(21)^5$

$= (20+1)^5$

$= \binom{5}{0}(20)^{5-0}(1)^0 + \binom{5}{1}(20)^{5-1}(1)^1 + \binom{5}{2}(20)^{5-2}(1)^2$

$+ \binom{5}{3}(20)^{5-3}(1)^3 + \binom{5}{4}(20)^{5-4}(1)^4 + \binom{5}{5}(20)^{5-5}(1)^5$

$= (1 \times 3200000 \times 1) + (5 \times 160000 \times 1) + \left(\frac{5 \times \cancel{4}^{2}}{\cancel{2} \times 1} \times 8000 \times 1\right)$

$+ \left(\frac{5 \times \cancel{4}^{2} \times \cancel{3}}{\cancel{3} \times \cancel{2} \times 1} \times 400 \times 1\right) + \left(\frac{\cancel{5 \times 4 \times 3 \times 2}}{\cancel{4 \times 3 \times 2 \times 1}} \times 20 \times 1\right)$

$+ (1 \times 1 \times 1)$

$= 3200000 + 800000 + 80000 + 4000 + 100 + 1$

$= 4084101$

Q=3

(i)

$(a+\sqrt{2}x)^4 + (a-\sqrt{2}x)^4$

$(a+\sqrt{2}x)^4$

$= \binom{4}{0}(a)^{4-0}(\sqrt{2}x)^0 + \binom{4}{1}(a)^{4-1}(\sqrt{2}x)^1 + \binom{4}{2}(a)^{4-2}(\sqrt{2}x)^2$

$+ \binom{4}{3}(a)^{4-3}(\sqrt{2}x)^3 + \binom{4}{4}(a)^{4-4}(\sqrt{2}x)^4$

$(1 \times a^4 \times 1) + (4 \times a^3 \times \sqrt{2}x) + (6 \times a^2 \times 2x^2)$