CivE 602
Prestressed Concrete

Part 5:

Prestress Losses

Prof. J.S. West

INTRODUCTION

- **The stress in the prestressing tendon decreases over time → prestress losses.**
- **The rate of reduction in prestress force is initially rapid, and decreases gradually over time.**
- **Accurate estimation of prestress losses is important for serviceability in terms of prediction of cracking, deflection and camber.**
  - **Over-estimate losses: prestress is larger than expected**
    → excessive camber
    → excessive shortening
  - **Under-estimate losses: prestress is smaller than expected**
    → excessive deflections in service
    → allowable tension stress exceeded
    → excessive cracking (width and #)
- **Loss of prestress has only a small effect of the flexural resistance of prestressed elements with bonded tendons (effect may be larger for unbonded tendons).**
- **Prestress losses may be categorized as instantaneous or immediate losses (occurring during transfer and/or stressing), and long-term losses.**
- **The magnitude of prestress losses varies along the length of the member → evaluate at critical sections**
- **Gross-section properties may be used for an approximation of prestress losses.**
- **Transformed section properties are preferable for more accurate estimation of losses.**

- **Sources of *immediate* prestress losses:**

1. Elastic Shortening: Shortening of the concrete element when prestressing force is applied causes prestressing tendons to shorten, reducing prestress force.

2. Friction: Loss due to friction between the post-tensioning tendon and duct during stressing → prestressing force varies along length of tendon.

3. Anchorage Set: Wedge-type anchorages require some movement to engage the wedges and anchor the tendon → movement of the wedges causes tendon to shorten, reducing prestress force. Also known as seating loss.

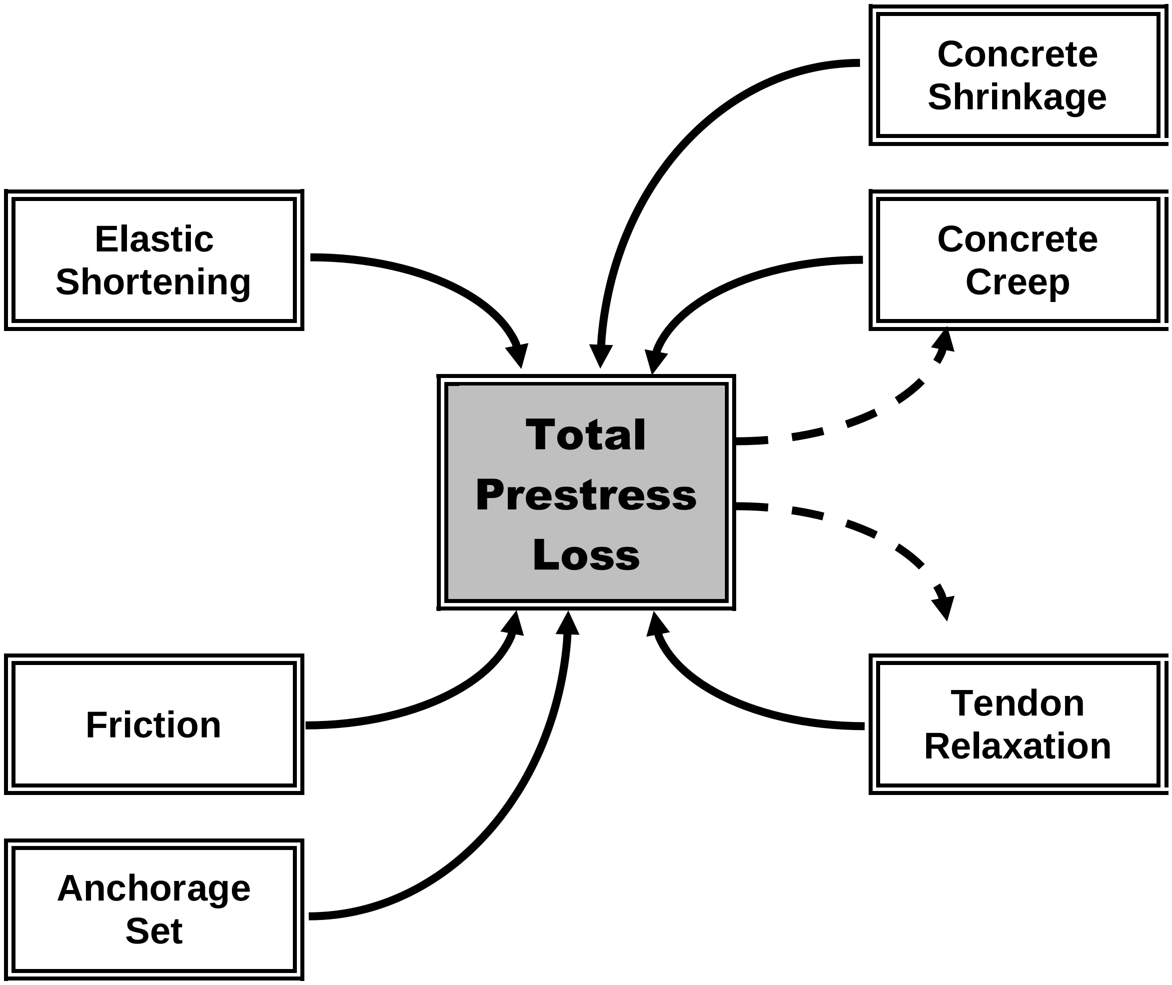
- **Sources of *long-term* prestress losses:**

4. Tendon Relaxation: Gradual reduction in tendon stress over time (tendon length and temperature is constant).

5. Concrete Creep: Gradual shortening of the concrete over time due to creep causes the tendon to shorten, reducing prestress force.

6. Concrete Shrinkage: Gradual shortening of the concrete over time due to shrinkage causes the tendon to shorten, reducing prestress force.

- Some of the prestress losses are not only time-dependent, but are also *inter-dependent*:



- Relaxation reduces the stress in the steel, reducing the stress in the concrete → creep will be reduced

- Creep and shrinkage of the concrete shortens the concrete, reducing the stress in the steel → relaxation is reduced

<u>Summary: sources of prestress loss and occurrence:</u>

Source	Stage of Occurrence		Notation (tendon stress loss)
	Pretensioned	Post-tensioned	
Elastic Shortening (ES)	**At transfer**	**At jacking**	Δf_{pES}
Anchorage Set (ANC)	**Before transfer***	**At transfer**	Δf_{pA}
Friction (FR)	**Not applicable**	**At jacking**	Δf_{pF}
Relaxation (REL)	**Before & after transfer**	**After transfer**	Δf_{pR}
Creep (CR)	**After transfer**	**After transfer**	Δf_{pCR}
Shrinkage (SH)	**After transfer**	**After transfer**	Δf_{pSH}
Total	**Lifetime**	**Lifetime**	Δf_{pT}

***Typically very small**

Losses in Pretensioned Elements

- **Prestressing Steps:**
  1. **Jacking:** strands are stressed and anchored to bulkhead before concrete is placed
  2. **Transfer:** strands are released and prestress force is transferred to concrete
  3. **Long-term:** prestress force gradually decreases due to time-dependent losses

- **Total prestress loss:**

$$\Delta f_{pT} = \left\{\Delta f_{pA} + \Delta f_{pR1}\right\} + \Delta f_{pES} + \left\{\Delta f_{pR2} + \Delta f_{pSH} + \Delta f_{pCR}\right\}$$

f_{pj} = stress in the tendon at jacking

f_{pi} = initial stress in the tendon after transfer

$$= f_{pj} - \left[\left\{\Delta f_{pA} + \Delta f_{pR1}\right\} + \Delta f_{pES}\right]$$

f_{pe} = effective stress in the tendons after all losses

$$= f_{pj} - \Delta f_{pT} \;=\; f_{pi} - \left\{\Delta f_{pR2} + \Delta f_{pSH} + \Delta f_{pCR}\right\}$$

Δf_{pA} = loss due to seating of anchorage chuck at bulkhead → normally very small → can be compensated for by slight overstressing or by using shims

Δf_{pR1} = loss due to relaxation of strands between jacking and transfer

Δf_{pES} = loss due to elastic shortening at transfer

Δf_{pR2} = loss due to relaxation of strands from time of transfer onwards (over life of structure)

Δf_{pSH} = loss due to concrete shrinkage from time of transfer onwards (over life of structure)

Δf_{pCR} = loss due to concrete creep from time of transfer onwards (over life of structure)

Losses in Post-tensioned Elements

- **Prestressing Steps:**
  1. **Jacking:** strands are anchored to the concrete element at one end and stressed from the other end
  2. **Transfer:** strands at jacking end are anchored and prestress force is transferred to concrete
  3. **Long-term:** prestress force gradually decreases due to time-dependent losses

- **Total prestress loss:**

$$\Delta f_{pT} = \left\{\Delta f_{pF} + \Delta f_{pES}\right\} + \Delta f_{pA} + \left\{\Delta f_{pR} + \Delta f_{pSH} + \Delta f_{pCR}\right\}$$

f_{pj} = stress in the tendon at jacking

f_{pi} = initial stress in the tendon after transfer

$$= f_{pj} - \left[\left\{\Delta f_{pF} + \Delta f_{pES}\right\} + \Delta f_{pA}\right]$$

f_{pe} = effective stress in the tendons after all losses

$$= f_{pj} - \Delta f_{pT} = f_{pi} - \left\{\Delta f_{pR} + \Delta f_{pSH} + \Delta f_{pCR}\right\}$$

Δf_{pF} = loss due to friction during stressing (varies long length of member)

Δf_{pES} = loss due to elastic shortening during stressing of multiple tendons (1st tendon experiences loss when 2nd is stressed)

Δf_{pA} = loss due to seating of anchorage at jacking end → may be significant → varies along length of tendon

Δf_{pR} = loss due to relaxation of strands from time of transfer onwards (over life of structure)

Δf_{pSH} = loss due to concrete shrinkage from time of transfer onwards (over life of structure)

Δf_{pCR} = loss due to concrete creep from time of transfer onwards (over life of structure)

Calculation Methods

- **Factors affecting prestress losses are complex → prediction is an approximation (± 10%) at best → consider a range of values.**
- **Approaches for *Time-dependent Losses*:**

  1. **Lump Sum Estimate (Simplified Method)**
  2. **Detailed Method (individual effects considered)**
  3. **Time Step Method**

- **Calculations for *Instantaneous Losses* are the same for each of the above methods**

- **Code Treatment:**
  - **CSA A23.3-04:** ***Building Code Requirements*** **Clause 18.5 indicates prestress losses should be considered, but no specific provisions or guidance are given. Commentary contains limited discussion for instantaneous losses.**
  - **CSA S6-06:** ***Canadian Highway Bridge Design Code*** **Clause 8.7.4 provides "detailed method" for loss estimation. Commentary provides further discussion, including a table of "lump sum losses" suitable for preliminary design**

- **The required "accuracy" of loss calculations depends on the structure.**

- **More accurate and detailed procedures require more input data:**
  - **pretensioned versus post-tensioned**
  - **material properties**
  - **shape of structural element**
  - **age at loading**
  - **environmental conditions**

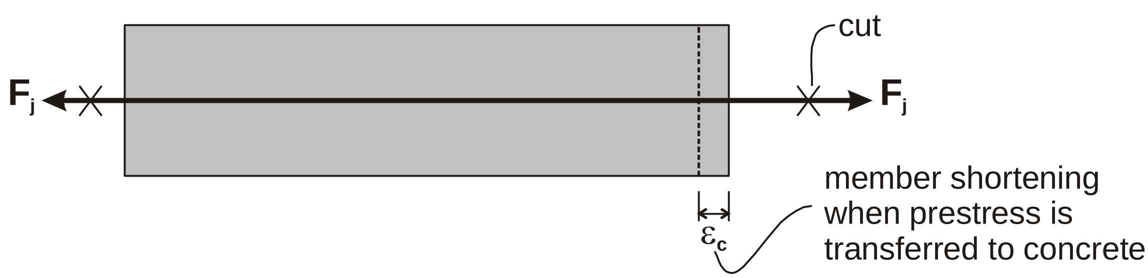
ESTIMATION OF INSTANTANEOUS LOSSES

- **Many methods have been proposed to estimate instantaneous prestress losses.**
- **Methods presented in the CAC Handbook,[1] CSA S6-06[2] and Naaman text[3] are presented for each of the three types of instantaneous prestress losses.**

ELASTIC SHORTENING (ES)

Pretensioned Elements:

- **A pretensioned concrete element shortens when the prestress force is transferred to the concrete → reduction of tendon length → elastic shortening loss**

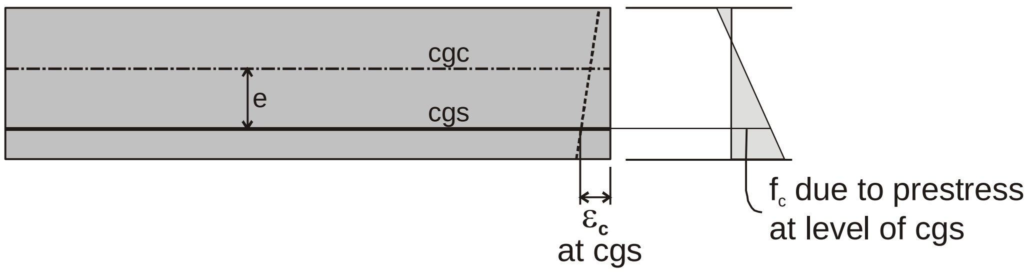


- **Prestress loss due to ES can be computed using basic mechanics of materials.**
- **Approximate solution:**

$$\Delta\varepsilon_{pES} = \varepsilon_c = \frac{F_j}{A_c E_{ci}}$$

$$\Delta f_{pES} = E_p\ \Delta\varepsilon_{pES} = \frac{E_p}{E_{ci}}\frac{F_j}{A_c} = \frac{E_p}{E_{ci}} f_c = n_{pi} f_c$$

Eccentric Tendons:



- **Prestress loss due to ES with eccentric tendon profile:**

e of tendon about which stresses are required

$$\Delta f_{pES} = n_{pi}\left[\frac{F_j}{A_{tr}} + \frac{(F_j e)e}{I_{tr}} - \frac{M_o e}{I_{tr}}\right]$$

where,

n_{pi} =modular ratio based on concrete modulus of elasticity at time of transfer

e =eccentricity of cgs based on transformed section

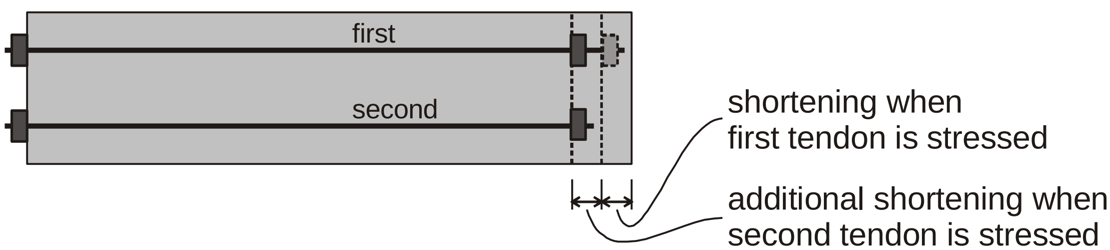
M_o =moment due to girder self-weight

- **In multiple strand configurations, the ES loss in each strand can be determined separately (strands farther from cgc will experience larger loss). However, the average loss for the prestressing steel will be similar to the value given by the expression above, since stress calculations are elastic.**
- **A more rigorous analysis is presented by Naaman[3] → differences between "approximate" and "accurate" methods are small**
- **Stresses in non-prestressed reinforcement can be computed in a similar manner**

$$f_{sES} = n_{si}\left[\frac{F_j}{A_{tr}} + \frac{(F_j e)y_s}{I_{tr}} - \frac{M_o y_s}{I_{tr}}\right]$$

Post-tensioned Elements:

- **A post-tensioned element with only one tendon does not experience prestress loss due to ES since the member shortens while the tendon is being stressed.**
- **If more than one tendon is used, the last tendon stressed will not have an ES loss, but the previously stressed tendons will experience an ES loss as each subsequent tendon is stressed.**



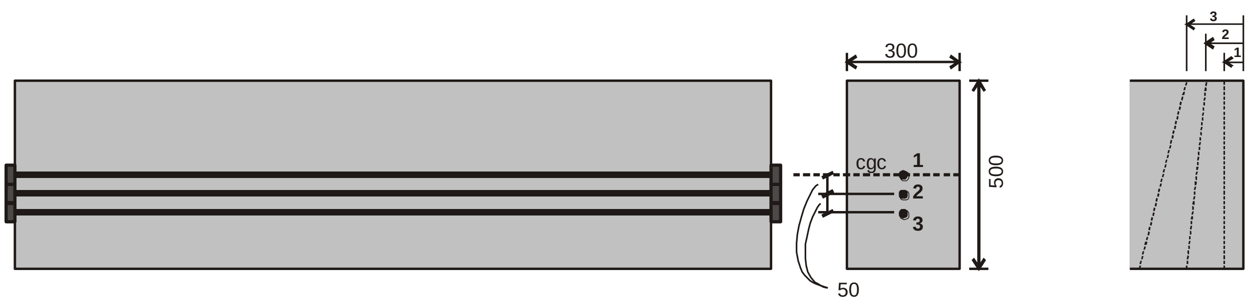
- **Calculations are similar to those for pretensioned elements, but with consideration of stressing sequence.**

Example – "Accurate" Method:

Post-tensioned beam with 3 tendons.

F_j = 500 kN in each tendon. A_p = 396 mm^2 (each tendon).

Find the final force in each tendon after ES losses.



Transformed section properties:

$$A_{tr} = 158{,}450\,\text{mm}^2 \qquad e_1 = -2.7\,\text{mm}$$

$$I_{tr} = 3.159 \times 10^9\,\text{mm}^4 \qquad e_2 = 47.3\,\text{mm}$$

$$(y_t)_{tr} = 252.7\,\text{mm} \qquad e_3 = 97.3\,\text{mm}$$

ES Calculations: Tendon 1 → Tendon 2 → Tendon 3

- **Calculate concrete stress at the level of each tendon due to stressing of each tendon.**

Cause of Stress		Concrete Stress at Level, y_i		
Tendon	**e_i**	**y_1 = -2.7 mm**	**y_2 = 47.3 mm**	**y_3 = 97.3 mm**
1	-2.7 mm	3.16	3.14	3.11
2	47.3 mm	3.14	3.51	3.88
3	97.3 mm	3.11	3.88	4.65
	Totals:	6.25 MPa	3.88 MPa	0 MPa

- **Elastic shortening loss is due to concrete stresses produced by stressing of subsequent tendons:**

Tendon 1: $\left(\Delta f_{pES}\right)_1 = n_{pi}\left[3.14\,\text{MPa} + 3.11\,\text{MPa}\right] = 50.7\,\text{MPa}$

Tendon 2: $\left(\Delta f_{pES}\right)_2 = n_{pi}\left[3.88\,\text{MPa}\right] = 31.5\,\text{MPa}$

Tendon 3: $\left(\Delta f_{pES}\right)_3 = \text{zero}$

Avg. ES: $\left(\Delta f_{pES}\right)_{Avg.} = 27.4\,\text{MPa}$ → $ES = \dfrac{27.4\,\text{MPa}}{f_{pj}} = 2.2\%$

General Form:[3]

$$\left(\Delta f_{pES}\right)_x = n_{pi}\left[\sum_{y=x+1}^{N}\left(\frac{F_{i,y}}{A_{tr}} + \frac{\left(F_{i,y}\cdot e_y\right)e_x}{I_{tr}}\right)\right]$$

where

N = number of tendons

$F_{i,y}$ = force in tendon "y" immediately after the last tendon has been stressed

e_x, e_y = eccentricities of tendon "x" and tendon "y" with respect to the centroid of the transformed concrete section

A_{tr} = transformed area of concrete section

I_{tr} = transformed moment of inertia of concrete section

- Note that tendon "N" does not experience elastic shortening loss → start with tendon "N" and proceed backwards to evaluate ES loss for each previous tendon in turn.

Approximate Method – CSA S6-06 and CAC Handbook

- Determine average loss due to ES:

$$\Delta f_{pES} = \left(\frac{N-1}{2N}\right)\frac{E_p}{E_{ci}} f_{cgp}$$

where,

N = number of tendons

f_{cgp} = sum of concrete stresses at the level of the cgs due to the prestress force at jacking and the self-weight of the member

$$= \frac{F_j}{A_{tr}} + \frac{\left(F_j\, e\right)e}{I_{tr}} - \frac{M_o\, e}{I_{tr}}$$

Redo Example:

$$f_{cgp} = 9.86\ \text{MPa} \qquad \text{(neglecting self-weight)}$$

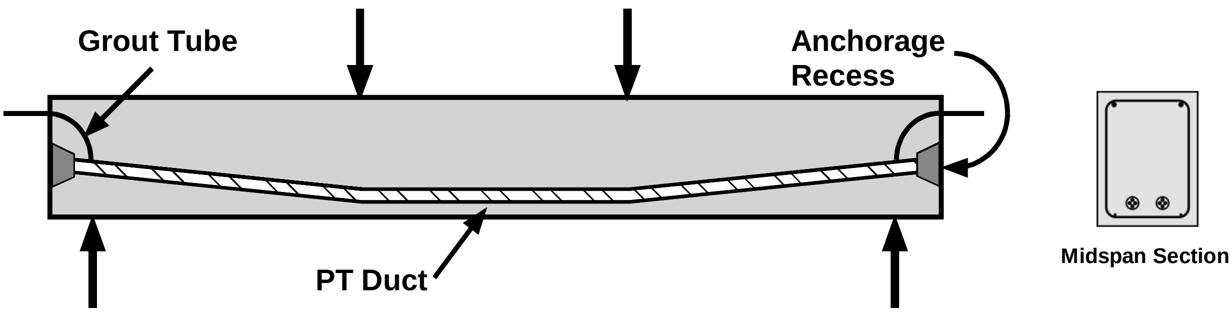
$$n_{pi} = 8.11$$

$$\Delta f_{pES} = \left(\frac{3-1}{2(3)}\right)(8.11)(9.86\ \text{MPa})$$

$$= 26.6\ \text{MPa}$$

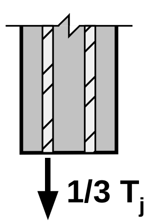
Use of Staged Post-tensioning to Minimize ES Losses

- **Consider beam with two parallel post-tensioned tendons:**

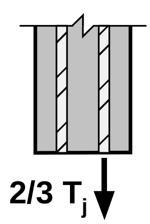


- **Prestress applied in 4 stages:**

Stage 1:
Tendon 1 to 1/3 T_j

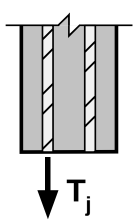


Stage 2:
Tendon 2 to 2/3 T_j

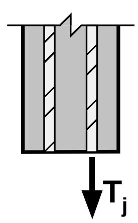


- **Tendon 2: no ES loss**
- **Tendon 1: ES loss only due to 1/3 T_j from Tendon 2**

Stage 3:
Tendon 1 from 1/3T_j to T_j



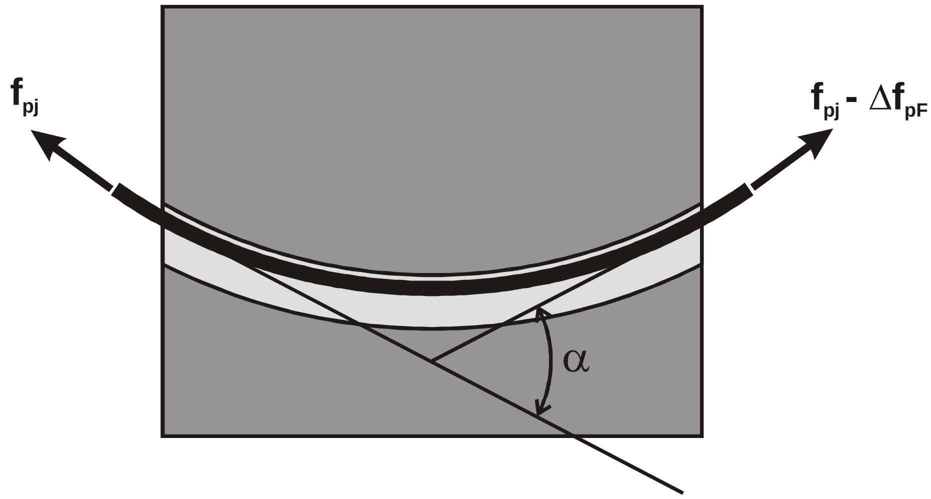
Stage 4:
Tendon 2 from 2/3T_j to T_j



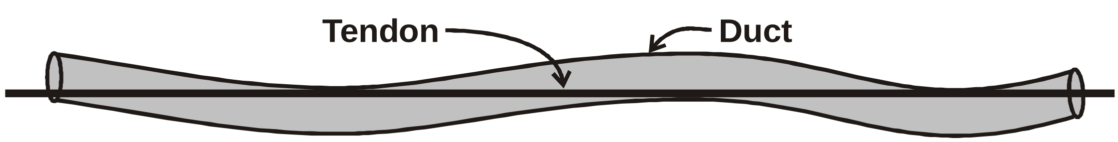
Plan View of Beam End

FRICTION LOSS (FR)

- **Friction between the post-tensioned tendon and the duct reduces the prestress force along the length of the tendon.**
- **Two components:**
    - **curvature friction: static angular friction produced by curved tendon profiles**



    - **wobble friction: linear friction due to unintended "wobble" in duct**



- **Tendon stress at a distance "x" from jacking end:**

$$f_{pj}(x) = f_{pj}\, e^{-(\mu\alpha + Kx)}$$

where,

f_{pj} = tendon stress at jacking end

x = tendon length from anchorage to section of interest

μ = coefficient of angular friction

α = angle change (in radians) between the force at the anchorage and the force at section of interest (distance x)

K = wobble coefficient, per unit length

- **Friction Loss over length, x:**

$$\Delta f_{pF} = f_{pj}\left[1 - e^{-(\mu\alpha + Kx)}\right]$$

- **Values of μ and K are determined by tests (supplied by Manufacturer). In the absence of test data, recommended values are available.[1,2,3,4,5]**

Friction Factors for 7-wire Strands (CSA S6-06)[2]

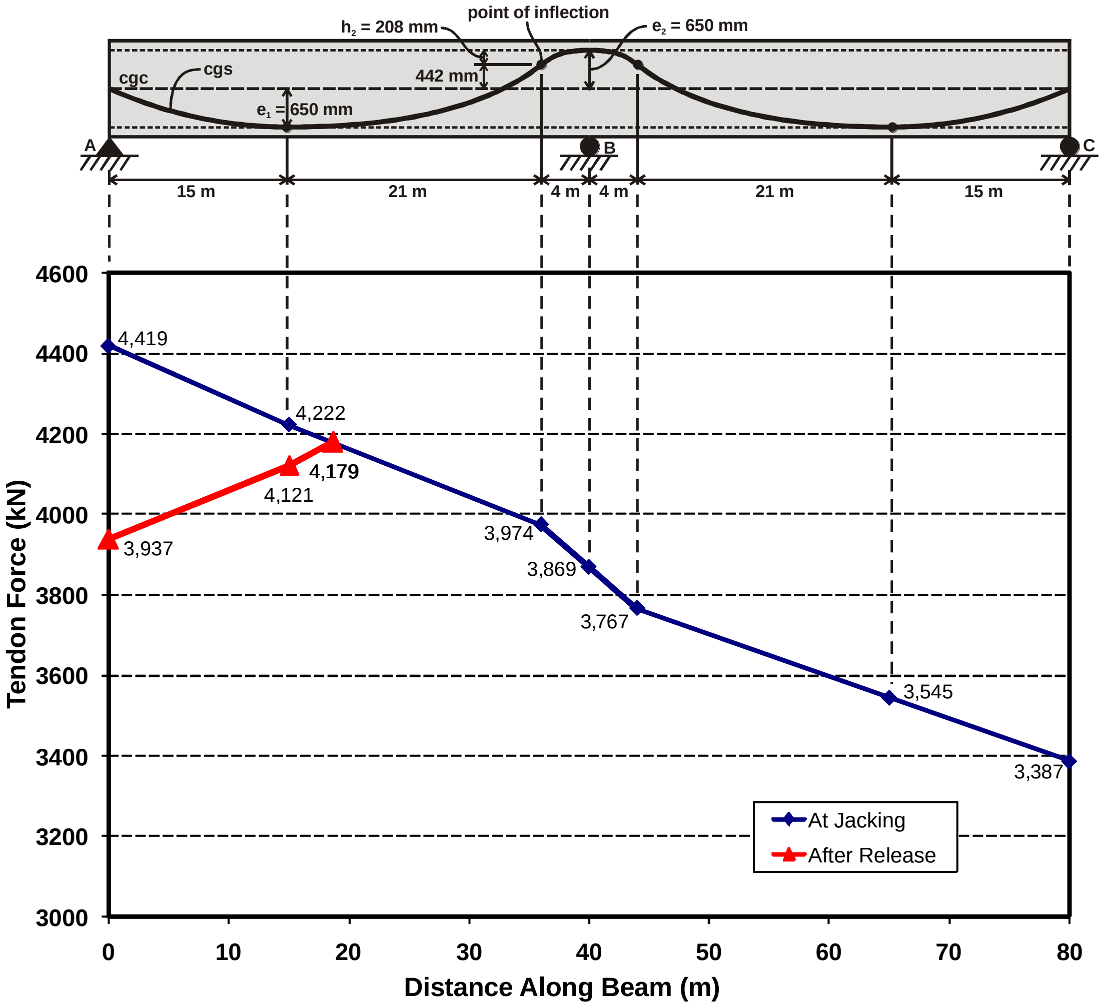
Duct Type	K	μ
Internal Ducts		
Rigid Steel	0.002	0.18
Semi-rigid Steel over 75 mm OD	0.003	0.20
Semi-rigid Steel up to 75 mm OD	0.005	0.20
Plastic	0.001	0.14
External Ducts		
Straight Plastic	0.000	--
Rigid Steel Pipe Deviators	0.002	0.25

Example Friction Loss Calculation:

Problem Data:

f_{pj} = 0.80 f_{pu} (f_{pu} = 1860 MPa)
A_{ps} = 2,970 mm^2
E_p = 190,000 MPa

μ = 0.20
K = 0.002/m
Δ_{set} = 8 mm

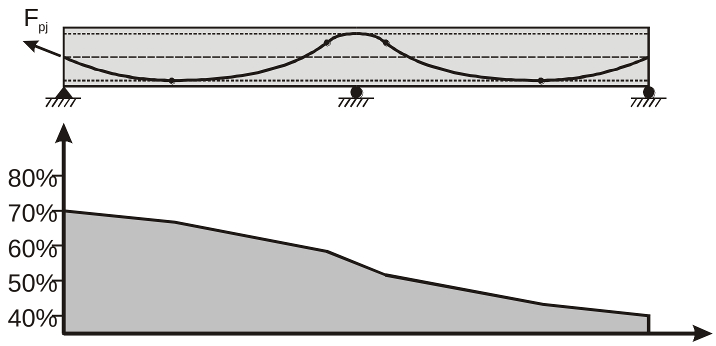


Calculations:

Parabola	Drape, f (m)	x_{start} (m)	x_{end} (m)	L_{seg} (m)	α (rad.)	$(\mu\alpha + Kx)$	$\Sigma(\mu\alpha + Kx)$	$e^{-\Sigma(\mu\alpha + Kx)}$	$e^{+\Sigma(\mu\alpha + Kx)}$	p_{seg} (N/mm)
1	**0.65**	**0**	**15**	15	0.087	0.046	0.046	0.955	1.047	13.133
2	**1.092**	**15**	**36**	21	0.104	0.061	0.106	0.899	1.112	11.845
3	**0.208**	**36**	**40**	4	0.104	0.027	0.133	0.875	1.142	26.192
4	**0.208**	**40**	**44**	4	0.104	0.027	0.160	0.852	1.173	25.502
5	**1.092**	**44**	**65**	21	0.104	0.061	0.220	0.802	1.247	10.567
6	**0.65**	**65**	**80**	15	0.087	0.046	0.266	0.766	1.305	10.535

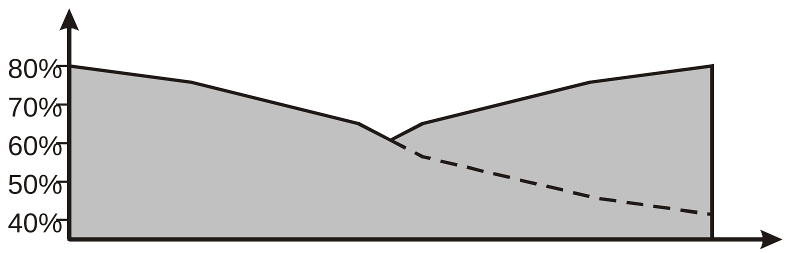
Friction Losses in Long Tendons – Stressing From Both Ends

- **For long tendons, the friction loss may be excessive if stressing is done only from one end.**

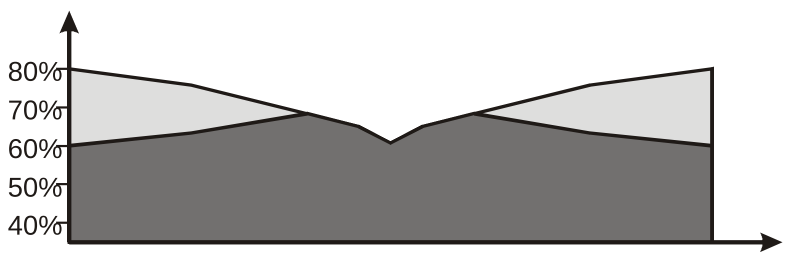


- **Friction losses may be reduced by stressing from both ends.**
- **Temporary over-stressing followed by release and re-stressing may also be used to minimize the effect of friction losses.**
- **Possible stressing sequence to minimize friction losses:[4]**

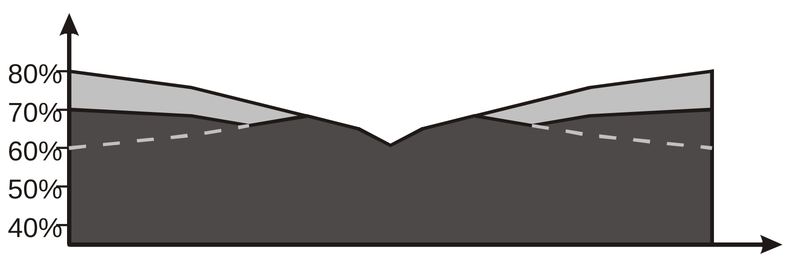
1. Tendon is stressed to 80% f_{pu} from both ends



2. Tendon is released to 60% of f_{pu} from both ends

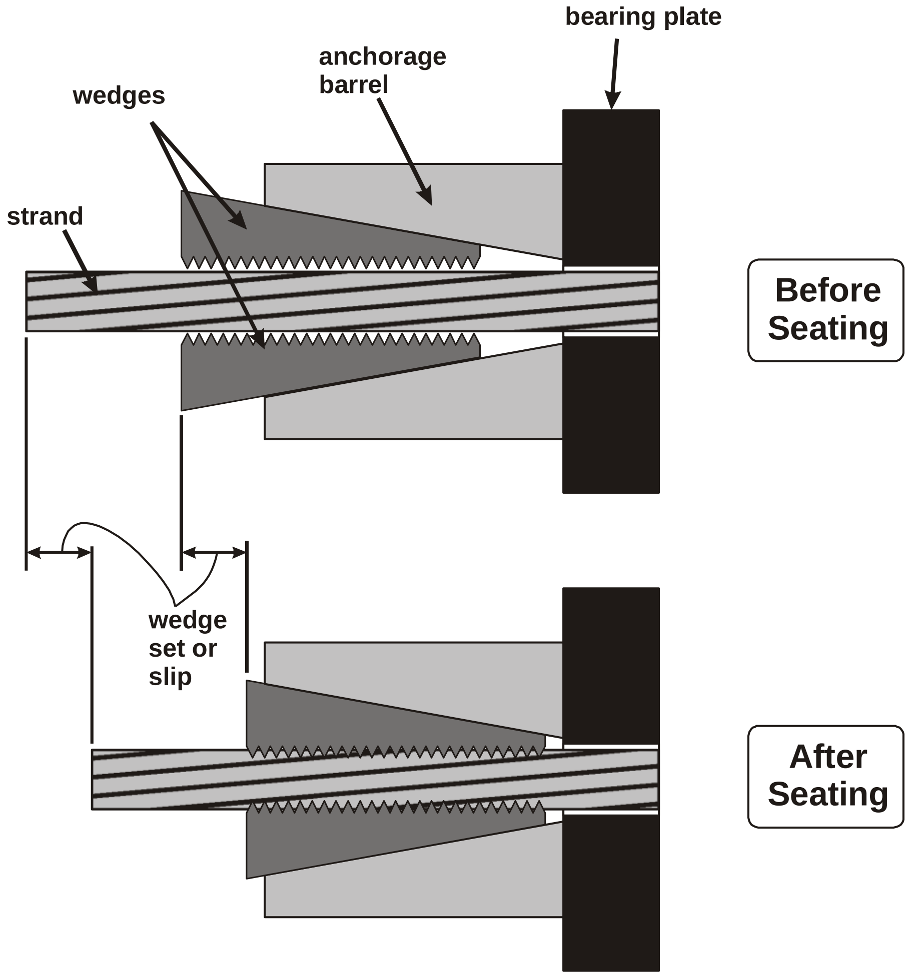


3. Tendon is re-stressed to 70% of f_{pu} from both ends



ANCHORAGE SET (ANC)

- Loss due to anchorage seating → amount of slip the wedges undergo inside the anchorage barrel before anchoring of the tendon is achieved.
- Slip of the wedges causes the tendon to shorten by the same amount → prestress is decreased.



- Values of slip may be up to 8 mm for a 13 mm dia. strand, and 12 mm for a 15 mm dia. strand. Recommended values are provided in the Commentary of CSA S6-06.[2]
- The actual value of anchorage slip should be provided by the manufacturer of the post-tensioning anchorage hardware or the post-tensioning contractor.
- "Power seating" of the wedges will reduce the amount of slip.
- A slip value of 6 mm is often assumed in design when no other information is available.

Calculation of Loss Due To Anchorage Set

- **In an unbonded tendon (frictionless):**

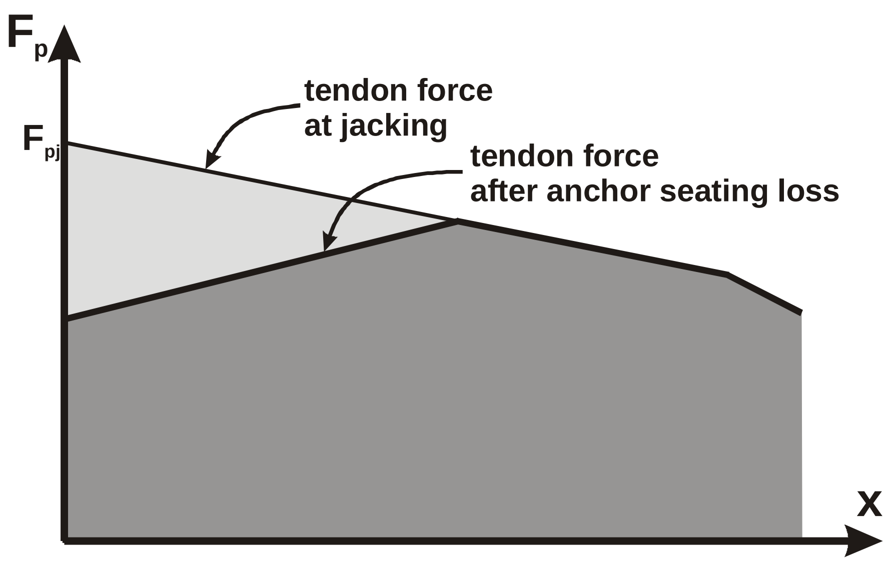
  $\Delta\varepsilon_{pA} =$ **constant over tendon length**

  $$= \frac{\Delta_{set}}{L_{tendon}}$$

  $$\Delta F_{pA} = A_p E_p \Delta\varepsilon_{pA}$$

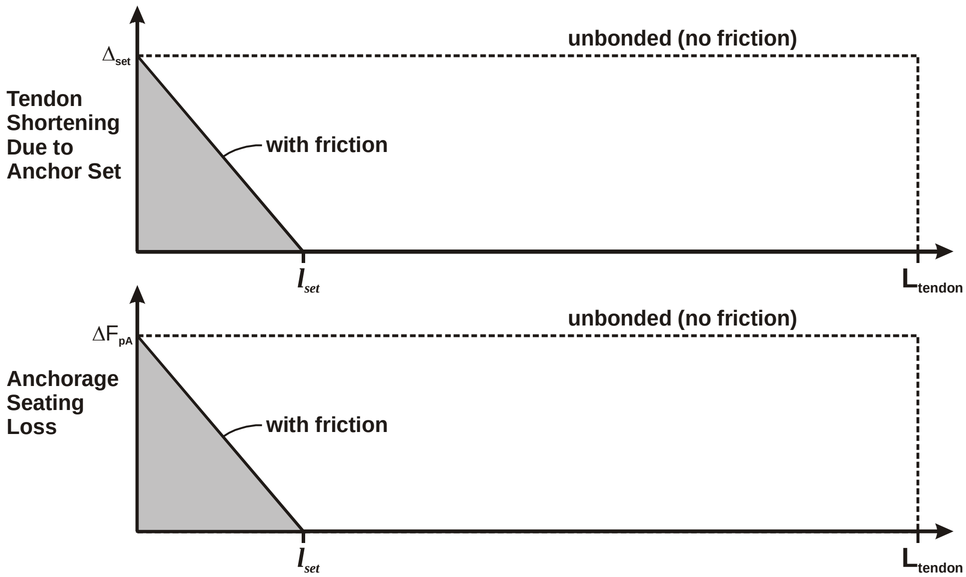
- **Where friction is present (i.e., most tendons), shortening of the tendon due to anchorage set is limited to a length, l_{set}, adjacent to the anchorage depending on the tendon frictional characteristics:**

  $\Delta\varepsilon_{pA} \neq \frac{\Delta_{set}}{L_{tendon}}$ → ΔF_{pA} **is <u>not</u> constant along length**



Define: p = **friction loss per unit length**

= **slope of $F_{pj}(x)$ curve**



$$\Delta_{set} = \int_0^{l_{set}} \frac{\Delta F_p(x)}{A_p E_p}\, dx$$

$$= \frac{1}{2} \frac{\Delta F_{pA}}{A_p E_p} l_{set}$$

Thus,

$$\Delta F_{pA} = 2 A_p E_p \frac{\Delta_{set}}{l_{set}}$$

From tendon force plot (p. 5.20)

$$\Delta F_{pA} = 2\, p\, l_{set}$$

Thus,

$$l_{set} = \sqrt{\frac{A_p E_p\, \Delta_{set}}{p}}$$

$$\Delta F_{pA} = 2\, p\, l_{set}$$

$$\Delta f_{pA} = \frac{\Delta F_{pA}}{A_p}$$

where,

Δ_{set} = anchorage set (assumed or specified by manufacturer)

ΔF_{pA} = prestress force loss due to anchorage set

Δf_{pA} = prestress (stress) loss due to anchorage set

l_{set} = length over which tendon force is affected by anchorage set

***p* = friction loss per unit length (for tendon conditions at anchorage)**

$$= \frac{\Delta F_{pj,x_1}}{x_1}$$

$\Delta F_{pj,x1}$ = friction loss (force) occurring over length x_1 measured from anchorage

A_p = total cross-sectional area of tendon

E_p = tendon modulus of elasticity

<u>Note:</u> In short beams, l_{set} may exceed the length of the beam
→ procedures shown above are no longer applicable
→ (See Naaman text,[3] Section 8.18).

<u>Power Seating of Wedges to Minimize Losses Due to Anchorage Set</u>

- **Most commercial hydraulic jacks for prestressing are equipped to "power seat" the wedges in the anchorage.**
- **Once the desired jacking force has been achieved, a secondary jack hydraulically seats (pushes in) the wedges with a predetermined force.**
- **Power seating reduces the reliance on friction to seat the wedges, and can significantly reduce the amount of slip required to fully anchor the tendon.**

Example – Anchorage Set

Consider tendon profile and friction loss calculations shown previously on p. 5.17.

Anchorage set: Δ_{set} ***= 8 mm***

Start by determining the friction loss per unit length near the anchorage. The previous calculations show:

$F_p = 4{,}419$ kN at $x = 0$

$F_p = 4{,}222$ kN at $x = 15$ m

Assume $l_{set} < 15$ **m**

$$p = \frac{4{,}419\,\text{kN} - 4{,}222\,\text{kN}}{15\,\text{m}} = 13.133\,\text{kN/m}$$

$$l_{set} = \sqrt{\frac{\left(2970\,\text{mm}^2\right)\left(190{,}000\,\text{MPa}\right)\left(8\,\text{mm}\right)}{13.133\,\text{N/mm}}} = 18{,}540\,\text{mm} = 18.54\,\text{m}$$

Since l_{set} **exceeds 15 m,** p **could be recalculated over a length of ~ 18.5 m if a more accurate estimation is desired. Looking at the plot of prestress force along the beam length, the slope of the curve does not change significantly up to 18.5 m. Thus, the change in** p **over 18.5 m compared to 15 m will be negligible.**

Compute prestress loss due to anchorage set:

$$\Delta F_{pA} = 2\left(13.133\,\text{kN/m}\right)\left(18.54\,\text{m}\right) = 487.0\,\text{kN}$$

$$\Delta f_{pA} = \frac{487 \times 10^3\,\text{N}}{2970\,\text{mm}^2} = 164.0\,\text{MPa}$$

$$\frac{\Delta f_{pA}}{f_{pj}} = \frac{164.0\,\text{MPa}}{0.80\left(1860\,\text{MPa}\right)} = 11.0\%$$

Prestress force at anchorage after set:

$$F_p = F_{pj} - \Delta F_{pA} = 4{,}419 - 487 = 3{,}932\,\text{kN}$$

TENDON ELONGATION DURING STRESSING

- **During the stressing operation, both the jacking force and corresponding tendon elongation are measured and recorded.**
- **Elongation measurements can be used to determine:**
  - **If the tendon is jammed/stuck in the duct, causing only a portion of the tendon to elongate → elongation will be less than expected.**
  - **If some of the strands in a multi-strand tendon are broken → elongation will be longer than expected since tendon area is reduced (stresses are higher).**
  - **If friction losses are greater than expected.**
- **The predicted tendon elongation is obtained by integrating the tendon strains along the length of the tendon:**

$$\Delta = \int_L \varepsilon(x)\,dx = \int_L \left(\frac{F_j(x)}{A_p E_p}\right) dx = \frac{1}{A_p E_p}\int_L F_j(x)\,dx$$

- **Since the distribution of prestress force at jacking is known along the tendon length from the friction calculations, the tendon elongation can be computed using the average force in the tendon:**

$$\Delta = \frac{F_{j,avg}\,L}{A_p E_p}$$

where,

$F_{j,avg}$ = average force in tendon along length at jacking

L = total length of tendon

A_p = total cross-sectional area of tendon

E_p = tendon modulus of elasticity

LUMP SUM ESTIMATION OF LONG-TERM LOSSES

- **Simplified approach to estimate long-term losses (REL, SH and CR)**
- **Instantaneous losses (FR, ANC and ES) computed separately**
- **Suitable for preliminary estimate of losses**

CPCI Manual[6]

- ***Simplified Method*** **proposed by** ***PCI Committee on Prestress Losses***[7]
- **Provides estimate of total loss due to ES, REL, SH and CR**
- **Applicable to precast pretensioned elements only**
- **Assumes $f_{pi} = 0.75\ f_{pu}$**

f_{co} = concrete compressive stress at level of tendon at critical section immediately after transfer

$$= \frac{F_i}{A} + \frac{F_i \cdot e^2}{I} - \frac{M_o \cdot e}{I}$$

f_{c1} = concrete stress at level of tendon at critical section caused by superimposed dead load, M_D (tension negative)

$$= \frac{M_D \cdot e}{I}$$

Δf_{pT} = total loss due to ES, REL, SH and CR

For normal density concrete:

$$\Delta f_{pT} = 137\,\text{MPa} + 16.3\,f_{co} + 5.4\,f_{c1}$$

For semi-low density concrete:

$$\Delta f_{pT} = 121\,\text{MPa} + 20.4\,f_{co} + 4.8\,f_{c1}$$

Adjustment for element volume-to-surface area ratio (V/S):

V/S Ratio (mm)	25	50	75	100
Adjustment	+3.2%	0	-3.8%	-7.6%

Canadian Highway Bridge Design Code (CSA S6-06)[2]

- **Commentary Table C8.2 provides estimated lump sum losses to be used for preliminary design purposes only.**
- **Conditions:**
  - **normal-density concrete**
  - **constructed and prestressed in one stage only**
  - **ratio of $A_s / A_{ps} \leq 1.0$**
  - **Grade 1860 low-relaxation strands, $f_{pi} = 0.74\ f_{pu}$**

Table C8.2 - Estimate of Lump Sum Losses (MPa)

Condition	Loss Type	Pretensioned	Post-tensioned
At transfer	REL_1	10	n.a.
	ES	110	20
	Subtotal: Δf_{s1}	120	20
After Transfer	CR	80	55
	SH	40	35
	REL_2	20	30
	Subtotal: Δf_{s2}	140	120
Total	$\Delta f_s = \Delta f_{s1} + \Delta f_{s2}$	260	140

- **For beams with high-strength concrete and with partial prestressing, the lump-sum losses proposed by Naaman and Hamza[8] are recommended by CSA S6-06.**

AASHTO LRFD Code[5]

- **Lump sum loss estimates based on Naaman and Hamza**[8]
- **Provides estimate of total loss due to REL, SH and CR**
- **Conditions:**
  - **post-tensioned, non-segmental elements with spans up to 50 m and stressed at 10 to 30 days**
  - **pretensioned members stressed with $f'_{ci} \geq 24\,\text{MPa}$**
  - **normal-weight or structural light-weight concrete**
  - **steam or moist-curing**
  - **average exposure conditions and temperature conditions**

Total Lump Sum Estimate of Time-Dependent Losses

Type of Beam Section	Level	Total Prestress Loss (Grade 1860 Strands)
Rectangular beam or solid slab	Upper Bound	$203 + 28 \cdot PPR$
	Average	$182 + 28 \cdot PPR$
Box Girder	Upper Bound	$174 + 28 \cdot PPR$
	Average	$133 + 28 \cdot PPR$
I-Girder		$231\left[1 - 0.15\left(\frac{f'_c - 42}{42}\right)\right] + 42 \cdot PPR$
Single-T, Double-T, Hollow-core and Voided Slabs	Upper Bound	$273\left[1 - 0.15\left(\frac{f'_c - 42}{42}\right)\right] + 42 \cdot PPR$
	Average	$231\left[1 - 0.15\left(\frac{f'_c - 42}{42}\right)\right] + 42 \cdot PPR$

- **Where PPR = partial prestressing ratio:**

$$PPR = \frac{A_{ps} f_{ps} \left(d_p - a/2 \right)}{A_{ps} f_{ps} \left(d_p - a/2 \right) + A_s f_y \left(d_s - a/2 \right)}$$

$= $ **1.0 for a fully-prestressed beam**

- **For low-relaxation strands, values in the table should be reduced by:**
  - **28 MPa for box girders**
  - **41 MPa for rectangular beams, solid slabs and I-girders**
  - **55 MPa for single-T, double-T, hollow core and voided slabs**
- **For members made with structural light-weight concrete, increase values in the table by 35 MPa.**
- **Upper bound and average values are the same for I-girders.**
- **Total lump-sum loss estimates are also provided for prestressing bars.[5]**

DETAILED ESTIMATION OF LONG-TERM LOSSES

- **Many methods have been proposed to estimate the separate contribution of each source of prestress loss (see Naaman text[3] for reference list).**

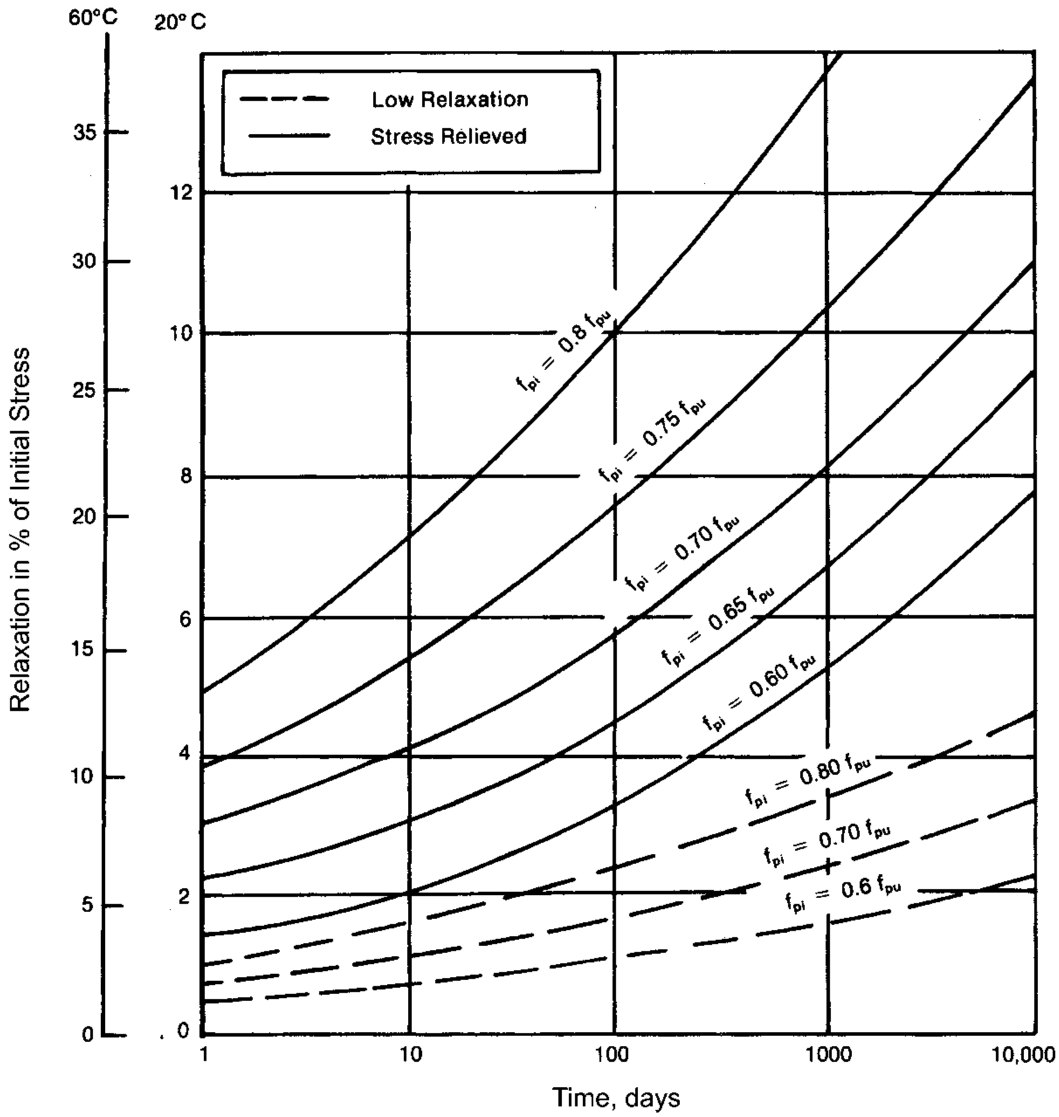
- **Methods presented in the CPCI Handbook,[6] CAC Handbook,[1] CSA S6-06[2] and Naaman text[3] are presented for the three types of long-term prestress losses.**

LOSSES DUE TO RELAXATION OF PRESTRESSING STEEL

- **Relaxation is the loss of stress in a material held at a constant length and temperature (i.e., constant strain).**
- **Relaxation of prestressing steels results in a loss of prestress.**
- **Relaxation of prestressing steel is a function of:**
  - **type of steel: normal (stress-relieved) versus low-relaxation**
  - **temperature during sustained stress: higher temperature results in greater relaxation**
  - **initial stress level: negligible for stresses below $0.5f_{pu}$, increases rapidly for higher stresses**
  - **time: stress loss continues over time but at a decreasing rate**

Low-Relaxation Strands

- **Standard stress-relieved prestressing strand has "normal" relaxation characteristics.**
- **Strands with "low-relaxation" are produced by subjecting stress-relieved strands to *stabilization:***
  - **Strands are subjected temperatures ranging from 20°C to 100°C for 1000 hours**
  - **stress level ~ 70% of f_{pu}**
  - **up to 75% of relaxation losses occur (in comparison to stress-relieved strands)**



Intrinsic Relaxation of Stress-Relieved and Low-Relaxation Strands[1]

➢ **The FIP Commission on Prestressing Steels[9] recommends the following typical relaxation losses:**

Strand Type	**Sustained Stress Level (1000 hrs)**		
	$f_{pi}/f_{pu} = 0.6$	**0.7**	**0.8**
Stress-relieved	**4.5%**	**8%**	**12%**
Low-relaxation	**1%**	**2%**	**4.5%**

Estimation of Relaxation Losses

- **For temperatures up to 20°C, the stress in the prestressing steel at any time, t, can be estimated as:**

$$f_p(t) = f_{pi}\left[1 - \frac{\log(t)}{K}\left(\frac{f_{pi}}{f_{py}} - 0.55\right)\right]$$

- **Stress loss due to relaxation, Δf_{pR}:**

$$\Delta f_{pR} = \frac{\log(t)}{K}\left(\frac{f_{pi}}{f_{py}} - 0.55\right)f_{pi}$$

where,

$f_p(t)$ = stress in prestressing steel at time "t" during which intrinsic or pure relaxation has occurred (under constant strain)

$\Delta f_{pR}(t)$ = prestressing loss at time "t" due to intrinsic or pure relaxation (under constant strain)

f_{pi} = initial prestress

f_{py} = yield stress of tendon

= 0.85 f_{pu} for stress-relieved strands

= 0.90 f_{pu} for low-relaxation strands

t = time since prestressing, <u>in hours</u> (or duration of sustained strain)

K = 10 for stress-relieved strands

= 45 for low-relaxation strands

Example:

Assume Gr. 1860 strand is stressed to 80% of f_{pu}. Compute relaxation loss after 1 day, 1 month, 1 year and 100 years.

Stress-relieved strands, t = 24 hours:

$$\Delta f_{pR} = \frac{\log(24\,hr)}{10}\left(\frac{0.80\,f_{pu}}{0.85\,f_{pu}} - 0.55\right)(0.80 \times 1860\,MPa)$$
$$= 80.3\ MPa$$

Low-relaxation strands, t = 24 hours:

$$\Delta f_{pR} = \frac{\log(24\,hr)}{45}\left(\frac{0.80\,f_{pu}}{0.90\,f_{pu}} - 0.55\right)(0.80 \times 1860\,MPa)$$
$$= 15.5\ MPa$$

Calculations Summary:

		Stress-Relieved		Low-Relaxation	
Time		Prestress Loss		Prestress Loss	
	(hours)	(MPa)	(%)	(MPa)	(%)
1 day	24	80.3	5.4%	15.5	1.0%
1 month	720	166.3	11.2%	32.0	2.2%
1 year	8760	229.5	15.4%	44.2	3.0%
100 years	876000	345.9	23.2%	66.6	4.5%
	f_{pi} =	1488 MPa		1488 MPa	
	f_{py} =	1581 MPa		1674 MPa	
	f_{pu} =	1860 MPa		1860 MPa	
	K =	10		45	

The reduction in prestress over time due to relaxation is computed more accurately by considering the loss occurring in discrete time steps.

Reduced Relaxation Due to Concrete Creep and Shrinkage

- **The "apparent" relaxation in a prestressed concrete element is less than the "intrinsic" or "pure" relaxation in a tendon stressed between two fixed supports since the concrete will creep over time, reducing the tendon length and stress level.**
- **Various approaches have been proposed to estimate the "apparent" reduced relaxation:**
  - **Ghali, A., and Trevino, J., (1985). "Relaxation of Steel in Prestressed Concrete," PCI Journal, Vol. 30, No. 5, Sept.-Oct., pp. 82-94.**
  - **Neville, A.M., Dilger, W.H., and Brooks, J.J.[10]**
    (to be discussed later)
  - **FIP Commission on Prestressing Steels and Systems:[9]**

$$\left(\Delta f_{pR}\right)_{app} = \left(\Delta f_{pR}\right)_{pure}\left[1 - 2\,\frac{\Delta f_{pSH} + \Delta f_{pCR}}{f_{pi}}\right]$$

CSA S6-06: Prestress Loss Due to Relaxation

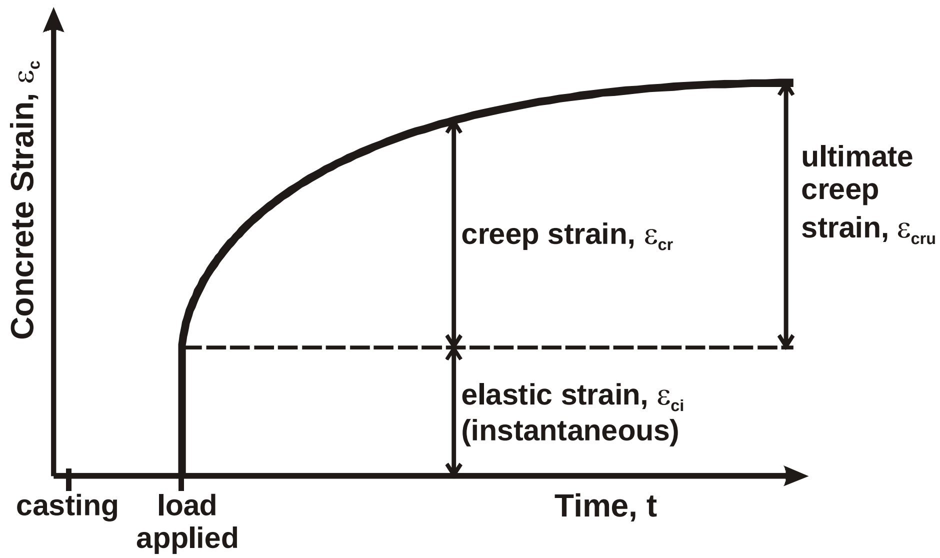
- **Prestress loss after transfer for low-relaxation strands:**

$$\Delta f_{pR2} = \left[\frac{f_{pi}}{f_{pu}} - 0.55\right]\left[0.34 - \frac{\Delta f_{pCR} + \Delta f_{pSH}}{1.25\,f_{pu}}\right]\frac{f_{pu}}{3} \quad \geq \quad 0.002\,f_{pu}$$

- **Accounts for "interaction" between losses due to creep, shrinkage and relaxation.**
- **Relaxation loss for high-strength bars should be obtained from manufacturer or assume to be 20 MPa.**

PRESTRESS LOSS DUE TO CREEP OF CONCRETE

- **Creep of concrete occurs as time-dependent strain/shortening under sustained compression stress.**
- **Creep of concrete gradually decreases over time, approaching a maximum or ultimate creep strain, ε_{cru}**



- **Creep depends on many factors:**
  - **duration of loading**
  - **age at loading**
  - **relative humidity**
  - **aggregate type**
  - **concrete mixture proportions**
  - **size and shape of concrete element**
  - **stress level (compression) in concrete**
- **The creep strain at time "t" can be related to the instantaneous or elastic strain using a creep coefficient, $C_c(t)$:**

$$\varepsilon_{cr}(t) = C_c(t)\ \varepsilon_{ci}$$

- **The ultimate creep strain is obtained considering an ultimate creep coefficient, C_{cu}:**

$$\varepsilon_{cru} = C_{cu}\ \varepsilon_{ci}$$

- **The creep coefficient at time "t" is often represented following the recommendations of ACI Committee 209[11]:**

$$C_c(t) = \left(\frac{t^{0.6}}{10 + t^{0.6}}\right) C_{cu}\, Q_{cr} \qquad \text{("t" in days)}$$

- **The above expression is valid for normal, semi-low and low density concretes subjected to sustained compressive stresses not exceeding 50% of f'_c.**
- **The ultimate creep coefficient varies between 1.3 and 4.15. An average value of C_{cu} = 2.35 is selected for standard conditions.[3,6,11]**
- **The modification factor, Q_{cr}, is used to account for non-standard conditions.**

$$Q_{cr} = Q_a Q_h Q_f Q_r Q_s Q_v$$

where,

Q_a = **modification factor for age at loading**

Q_h = **modification factor for relative humidity**

Q_f = **modification factor for ratio of fine to total aggregate**

Q_r = **modification factor for volume-to-surface area ratio**

Q_s = **modification factor for concrete slump**

Q_v = **modification factor for concrete air content**

- **Modification factors are described in detail by ACI 209.[11]**

- **The CAC Handbook (Table 1.2) and CPCI Handbook (Fig. 2.4.1) provide summaries of the ACI 209 factors in tabular form.**

Creep & Shrinkage Modification Factors for Non-Standard Conditions[1]

Age at Loading days	Q_a	
	moist cured	steam cured
1	1.25	1.00
7	1.00	0.94
20	0.87	0.85
60	0.77	0.76

Relative Humidity %	Q_h	P_h
40	1.00	1.00
60	0.87	0.80
80	0.73	0.60
100	0.60	0.00

Ratio of Fine to Total Aggr.	Q_f	P_f
0.30	0.95	0.72
0.40	0.98	0.86
0.50	1.00	1.00
0.70	1.05	1.04

Volume Surface Ratio, mm	Q_r	P_r
38	1.00	1.00
75	0.82	0.84
150	0.70	0.59
250	0.67	0.37

Slump. mm	Q_s	P_s
50	0.95	0.97
70	1.00	1.00
125	1.15	1.09

Air %	Q_v	P_v
≤ 6	1.00	1.00
8	1.18	1.01
10	1.36	1.03

	Cement Content, kg/m^3		
	225	300	410
P_c	0.89	0.93	1.00

Notes:
- Standard conditions produce modification factors of 1.0.
- Volume – surface ratio of a rectangular member having a X b cross-section is ab/(2a + 2b)
- Ratio of fine aggregate to total aggregate is expressed as the ratio of the weights
- For average ambient relative humidity see Fig. 1.6.

Estimation of Prestress Loss Due to Creep

- **The creep strain over an interval of time (t_i, t_j) can be estimated as follows:**

$$\varepsilon_{cr}(t_i,t_j)=\left[\left(\frac{t_j^{0.6}}{10+t_j^{0.6}}-\frac{t_i^{0.6}}{10+t_i^{0.6}}\right)C_{cu}\,Q_{cr}\right]\times\varepsilon_{ci}$$

- **Since the steel and concrete are assumed to bonded, the steel experiences the same strain (change) due to creep.**
- **The change in stress in the prestressing steel (prestress loss) over an interval of time (t_i, t_j) is given by:**

$$\begin{aligned}\Delta f_{pCR}(t_i,t_j)&=E_p\,\varepsilon_{cr}(t_i,t_j)\\&=E_p\left[\left(\frac{t_j^{0.6}}{10+t_j^{0.6}}-\frac{t_i^{0.6}}{10+t_i^{0.6}}\right)C_{cu}\,Q_{cr}\right]\times\varepsilon_{ci}(t_i)\\&=n_p\left[\left(\frac{t_j^{0.6}}{10+t_j^{0.6}}-\frac{t_i^{0.6}}{10+t_i^{0.6}}\right)C_{cu}\,Q_{cr}\right]\times f_{cgp}(t_i)\end{aligned}$$

where,

$\varepsilon_{ci}(t_i)$ = **instantaneous concrete strain at the level of the prestressing steel (c.g.s.) due to prestress and sustained loads at time t_i**

$f_{cgp}(t_i)$ = **stress in the concrete at the level of the prestressing steel (c.g.s.) due to prestress and sustained loads at time t_i**

$$=\frac{f_p(t_i)\times A_p}{A_c}\left(1+\frac{e^2}{r^2}\right)-\frac{(M_o+M_D)e}{I}$$

$f_p(t_i)$ = **stress in the prestressing tendon at time t_i**

e = **eccentricity of c.g.s.**

A_p = **area of prestressing tendon**

A_c = area of concrete cross-section

I = moment of inertia of concrete cross-section

r = radius of gyration = $\sqrt{I/A_c}$

n_p = modular ratio at time t_i

The prediction of prestress loss over time due to creep is computed more accurately by considering the loss occurring in discrete time steps.

CSA S6-06 – Prestress Loss Due to Creep

- **Prestress loss due to creep:**

$$\Delta f_{pCR} = \left[1.37 - 0.77\left(0.01RH\right)^2\right] K_{cr} \frac{E_p}{E_c}\left(f_{cir} - f_{cds}\right)$$

where,

RH = annual mean relative humidity

K_{cr} = 2.0 for pretensioned components

= 1.6 for post-tensioned components

f_{cir} = stress in concrete at level of prestressing tendon (cgs) due to prestress alone

f_{cds} = stress in concrete at level of prestressing tendon (cgs) due to sustained loads (self-weight + dead load + sustained LL).

PRESTRESS LOSS DUE TO CONCRETE SHRINKAGE

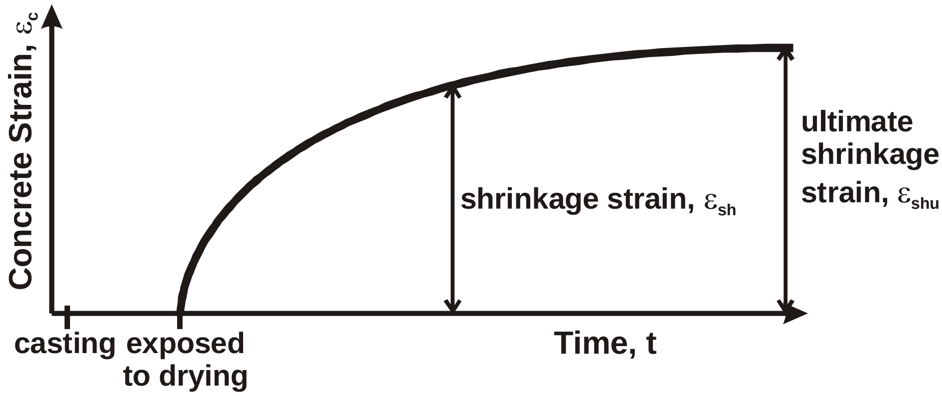
- **Hydraulic cement concretes experience volume changes with changes in moisture content.**
- **Changes in the moisture content of the cement paste causes the concrete to shrink or swell. The aggregate provides an internal restraint that significantly reduces the magnitude of the volume changes.**
- **Mechanism:**
  - **The hydration of cement produces calcium-silicate hydrate (C-S-H) gel → provides structure of hardened cement paste and binds aggregates.**
  - **Water that is not consumed in the hydration process resides in capillary and gel pores within the hydrated C-S-H gel:**

    <u>gel pores:</u> **interstitial voids between hydrated cement particles → diameter ~ 3nm (3×10^{-9} m)**

    <u>capillary pores:</u> **created by water unused by hydration → one to two orders of magnitude larger than gel pores**
  - **When the concrete is exposed to drying conditions, water evaporates from the capillary pores first → small amount of shrinkage**
  - **The evaporation of adsorbed water from the gel pores and inter-layer water from the gel can cause significant shrinkage of the cement paste.**
  - **If the concrete is subject to wetting after drying shrinkage occurs, the process is reversed causing an expansion of the concrete.**
- **Shrinkage is affected by a number of factors:[3,11]**
  - **amount of free water (related to w/c ratio)**
  - **type of aggregates**
  - **relative humidity of environment**
  - **ambient temperature**
  - **size and shape of structural element**
  - **cement content**

- **Shrinkage is independent of applied loading.**

- **The rate of shrinkage decreases over time, such that the shrinkage strain approaches an “ultimate” value:**



- **The shrinkage strain at time “t” can be represented in terms of the ultimate shrinkage strain and a time function:**[11]

$$\varepsilon_{sh}(t) = \left(\frac{t}{C_s + t}\right)\varepsilon_{shu} P_{sh}$$

where,

$\varepsilon_{sh}(t)$ = concrete shrinkage strain at time t (in days)

C_s = 35 for concrete moist-cured for 7 days

= 55 for concrete steam-cured for 1-3 days

ε_{shu} = ultimate shrinkage strain for standard conditions

= 780×10^{-6} mm/mm (Ref. 11)

P_{sh} = modification factor for non-standard conditions

= $P_c P_h P_f P_r P_s P_v$

with,

P_c = modification factor for cement content

P_h = modification factor for relative humidity

P_f = modification factor for ratio of fine to total aggregate

P_r = modification factor for volume-to-surface area ratio

P_s = modification factor for concrete slump

P_v = modification factor for concrete air content

- **Modification factors are described in detail by ACI 209.[11]**
- **The CAC Handbook (Table 1.2) and CPCI Handbook (Fig. 2.4.1) provide summaries of the ACI 209 factors in tabular form (see p. 5.36).**

Estimation of Prestress Loss Due to Concrete Shrinkage

- **The shrinkage strain over an interval of time (t_i, t_j) can be estimated as follows:**

$$\varepsilon_{sh}(t_i, t_j) = \left(\frac{t_j}{C_s + t_j} - \frac{t_i}{C_s + t_i} \right) \varepsilon_{shu} \, P_{sh}$$

- **Since the steel and concrete are assumed to bonded, the steel experiences the same strain (change) due to shrinkage.**
- **The change in stress in the prestressing steel (prestress loss) over an interval of time (t_i, t_j) is given by:**

$$\begin{aligned} \Delta f_{pSH}(t_i, t_j) &= E_p \, \varepsilon_{sh}(t_i, t_j) \\ &= E_p \left[\left(\frac{t_j}{C_s + t_j} - \frac{t_i}{C_s + t_i} \right) \varepsilon_{shu} \, P_{sh} \right] \end{aligned}$$

The prediction of prestress loss over time due to shrinkage is computed more accurately by considering the loss occurring in discrete time steps.

CSA S6-06 – Prestress Loss Due to Concrete Shrinkage

- **Prestress loss due to shrinkage:**

$$\Delta f_{pSH} = 117 - 1.05\,RH \qquad \text{for pretensioned components}$$

$$\Delta f_{pSH} = 94 - 0.85\,RH \qquad \text{for post-tensioned components}$$

Interaction Between Creep, Shrinkage and Relaxation – Total Long-Term Prestress Loss

- **The interaction between relaxation, creep and shrinkage and the resulting effect on prestress losses has been investigated by many researchers. The influence of non-prestressed reinforcement should also be included. Some approaches include:**
  - **Ghali, A., and Favre, R., (1994). "Concrete Structures: Stresses and Deformations," 2nd. Edition, E & FN Spon, London.**
  - **Ghali, A., (1989). "Stress and Strain Analysis in Prestressed Concrete: A Critical Review," PCI Journal, Vol. 34, No. 6, Nov.-Dec., pp. 80-97.**
  - **Neville, A.M., Dilger, W.H., and Brooks, J.J.[10] → included in CPCI Handbook and CAC Handbook.**

Neville, Dilger and Brooks Procedure

- **The total time dependent prestress loss, Δf_{pt}, for a member with one layer of tendons and non-prestressed steel at the same level:**

$$\Delta f_{pt} = \frac{n_o f_{cgp} C_c + \varepsilon_{sh} E_p + \alpha_r \Delta f_{pR}}{1 + n_o \left(\rho_s + \rho_p\right)\left(1 + e^2/r^2\right)\left(1 + 0.8 C_c\right)}$$

where,

n_o = **modular ratio at beginning of time step (time "t")**

= E_p / E_{co}

f_{cgp} = **stress in the concrete at the level of the prestressing steel (c.g.s.) due to prestress and sustained loads at time "t"**

C_c = **creep coefficient at time "t"**

ε_{sh} = **shrinkage strain at time "t"**

Δf_{pR} = **"pure" relaxation loss at time "t"**

α_r = relaxation reduction coefficient → accounts for reduced relaxation due to creep and shrinkage of concrete

ρ_s = $\left(A_s/A_c\right)\times\left(E_s/E_p\right)$

ρ_p = A_p/A_c

e = eccentricity of tendon

r^2 = I_c/A_c

- The relaxation reduction coefficient is a function of the initial prestress and the magnitude of prestress loss due to creep and shrinkage. It can be determined using Fig. 3.4.4. of the CPCI Handbook or Fig. 10.3 of the CAC Handbook with:

$$\Omega = \frac{\left(\Delta f_p\right)_{CR+SH}}{f_{pi}} \qquad \text{and} \qquad \beta = \frac{f_{pi}}{f_{pu}}$$

with $$\left(\Delta f_p\right)_{CR+SH} = \frac{n_o\, f_{cgp}\, C_c + \varepsilon_{sh}\, E_p}{1 + n_o\left(\rho_s + \rho_p\right)\left(1 + e^2/r^2\right)\left(1 + 0.8 C_c\right)}$$

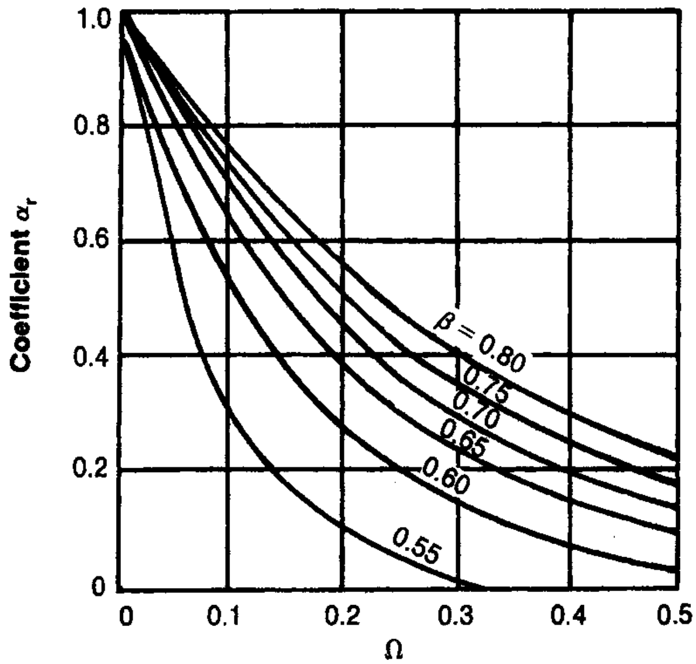
- The 0.8 factor applied to C_c in the denominator is an "aging coefficient" used to account for the reduction in creep over time due to shrinkage and relaxation.

- If the non-prestressed reinforcement is uniformly distributed over the depth of the section, the total time dependent prestress loss is given by:

$$\Delta f_{pt} = \frac{n f_{cgp}\, \alpha_c\, C_c + \varepsilon_{sh}\, E_p + \alpha_r \Delta f_{pR}}{1 + n\rho_p\left(1 + e^2/r^2\right)\left(1 + 0.8\,\alpha_c\, C_c\right)}$$

where,

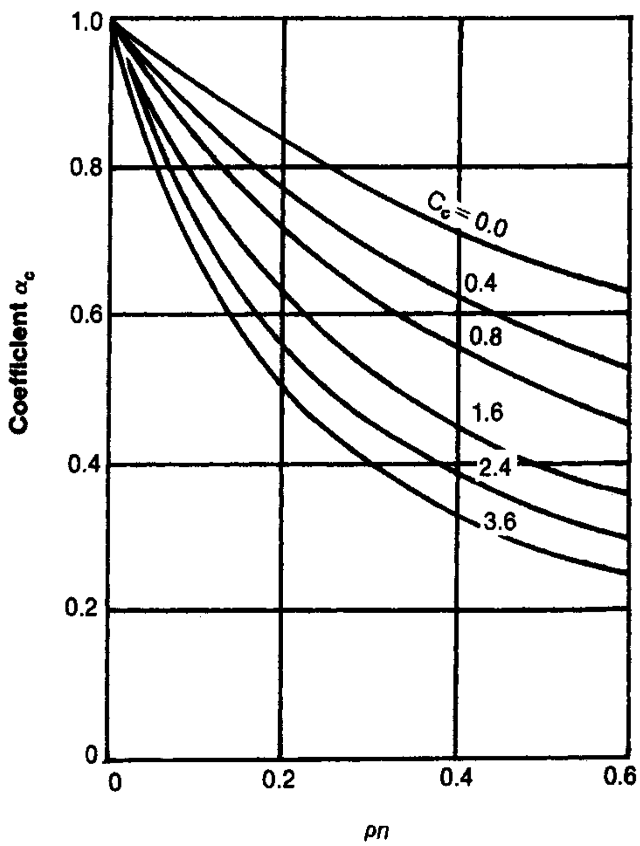
α_c = creep reduction coefficient (see Figure 10.4 from the CAC Handbook)



$$\Omega = \frac{(\Delta f_p)_{CR+SH}}{f_{pi}}$$

$$\beta = \frac{f_{pi}}{f_{pu}}$$

CAC Fig. 10.3 - Relaxation Reduction Coefficient, α_r [1]



C_c = creep coefficient at time "t"

$\rho = A_s/A_c$

n = modular ratio

CAC Fig. 10.4 - Creep Reduction Coefficient, α_c [1]

PRESTRESS LOSSES BY THE TIME-STEP METHOD

- **For situations where prestress losses are particularly critical, a more rigorous approach to loss prediction may be required**
  - **staged construction**
  - **staged prestressing**
  - **segmental construction**

- **The *time-step method* is an accurate approach to computing the losses due to creep, shrinkage and relaxation, accounting for interdependency of these time-dependent phenomena.**

- **The life of the structure is subdivided into time steps: short initial steps (time-dependent phenomena change more rapidly initially), considering construction stages, etc., or other milestones, followed by longer steps during service life.**

- **The losses during each step are computed, and a revised estimate of total prestress is obtained. This revised prestress level is then used to determine the losses in the next step.**

- **The time-step process allows changes in tendon characteristics (force and eccentricity) and section properties to be incorporated as construction progresses. In addition, time dependent changes in material properties (e.g., E_c) can be accounted for directly.**

- **A flow chart illustrating the process is shown on the following page.**

- **The *time-step method* may lead to prediction of smaller prestress losses than the lump-sum or detailed methods → more accurate representation of interdependence of time-dependent losses.**

- **The *time-step method* is well suited to computerization.**

- **See Naaman3 textbook and Neville et al.[10] for more information.**

REFERENCES

1). Cement Association of Canada, (2006). "Concrete Design Handbook," 3rd Ed. (Incl. CSA Standard: A23.3-04), Ottawa, ON.

2). Canadian Standards Association, (2006). "Canadian Highway Bridge Design Code," CAN/CSA-S6-06, and "Commentary on Canadian highway Bridge Design Code," CAN/CSA S6.1-06, Canadian Standards Association, Mississauga, Ontario, Canada

3). Naaman, A. E., (2004). "Prestressed Concrete Analysis and Design," 2nd Ed., Techno Press 3000, Ann Arbor, MI.

4). Collins, M.P., and Mitchell, D., (1991). "Prestressed Concrete Structures," Prentice Hall, Englewood Cliffs, NJ.

5). American Association of State Highway and Transportation Officials (AASHTO), (1998). "LRFD Bridge Design Specifications," 2nd Ed., Washington, DC.

6). Canadian Prestressed Concrete Institute, (2007). "Design Manual," 4th Ed., Ottawa, ON.

7). PCI Committee on Prestress Losses, (1975). "Recommendations for Estimating Prestress Losses," PCI Journal, Vol. 20, No. 4, pp. 43-75. Also see comments Vol. 21, No. 2, 1976.

8). Naaman, A.E., and Hamza, A.A., (1993). "Prestress Losses in Partially Prestressed High-Strength Concrete Beams," PCI Journal, Vol. 38, No. 3, pp. 98-114.

9). FIP Commission on Prestressing Steels and Systems, (1976). "Report on Prestressing Steels: Types and Properties," International Federation for Prestressed Concrete (FIP), Wexham Spring, UK. *(discussed in Naaman Text).*

10). Neville, A.M., Dilger, W.H., and Brooks, J.J., (1983). "Creep of Plain and Structural Concrete," Construction Press, London.

11). ACI Committee 209, (1992). "Prediction of Creep, Shrinkage and Temperature Effects in Concrete Structures" (ACI 209R-92), American Concrete Institute, Farmington Hills, MI.