Steel Structures

M.Sc. Structural Engineering

SE-505

Lecture # 01 (Contd.)

Plastic Analysis and Design of Structures

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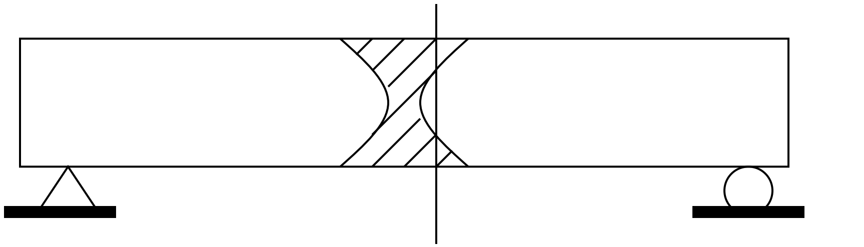
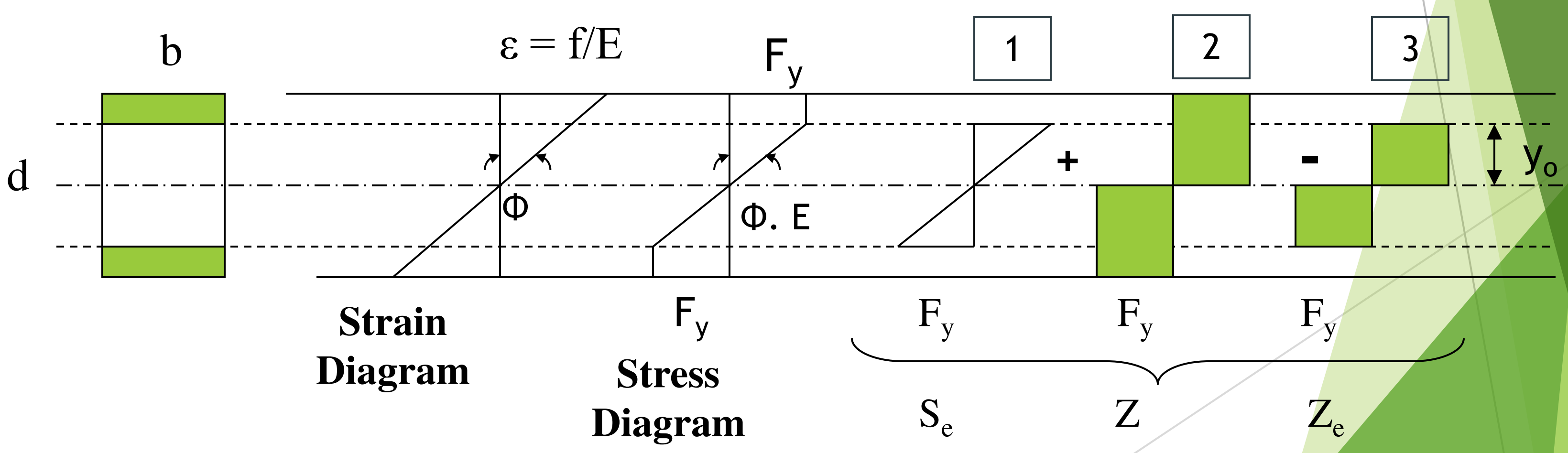
Steel Structures

Flexural Strength For Partially Plastic Section





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Flexural Strength For Partially Plastic Section (contd…)

Φ = Curvature (Rotation per unit length)

Original stress diagram = 1 + 2 + 3

Se = Elastic section modulus of the part which is still elastic

Ze = Plastic section modulus of the inner part that is still elastic.

Z = Plastic section modulus of the entire cross-section

From strain diagram

$$\tan\Phi = \frac{\varepsilon_y}{y_o} = \frac{F_y}{Ey_o}$$

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Flexural Strength For Partially Plastic Section (contd.)

From strain diagram

$$\left.\begin{aligned}
&\tan\Phi = \frac{\varepsilon_y}{y_o} = \frac{F_y}{Ey_o} \\
&\text{For smaller angle in radians} \\
&\Phi = \frac{F_y}{Ey_o} \\
&\text{For larger angle in radians} \\
&\Phi = \tan^{-1}\left(\frac{F_y}{Ey_o}\right)
\end{aligned}\right\} \text{———} \boxed{1}$$

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Flexural Strength For Partially Plastic Section (contd…)

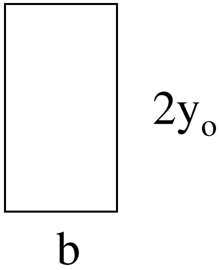
$$M = F_y\left(S_e + Z - Z_e\right) \qquad \boxed{2}$$

For rectangular section

$$S_e = \frac{bh^2}{6}$$

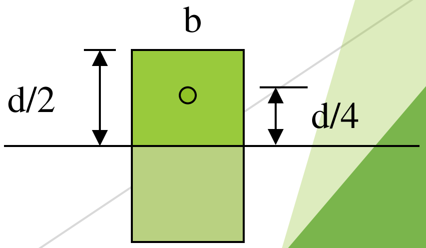
$$S_e = \frac{b\times(2y_o)^2}{6}$$

$$S_e = \frac{2}{3}by_o^{\,2}$$



Using Eq:1, 2 we can draw moment curvature relationship

$$Z = \left(b\times\frac{d}{2}\right)\frac{d}{4}\times 2 \Rightarrow \quad Z = \frac{bd^2}{4}$$



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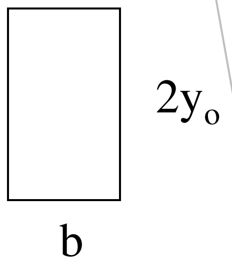
Flexural Strength For Partially Plastic Section (contd…)

$$z_e = \frac{b \times (2y_o)^2}{4} = by_o{}^2$$



$$M = F_y\left(\frac{2}{3}by_o{}^2 + \frac{bd^2}{4} - by_o{}^2\right)$$

$$M = F_y \times by_o{}^2\left(\frac{d^2/4}{y_o{}^2} - \frac{1}{3}\right)$$

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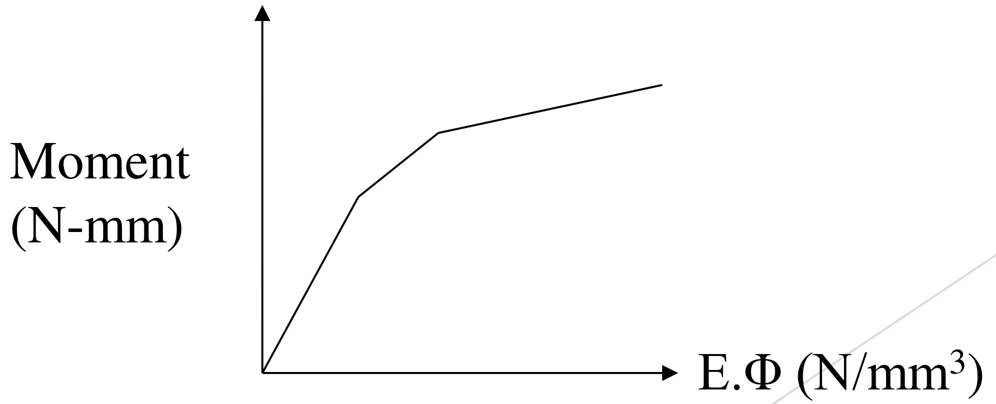
Moment Curvature Relationship For a Particular Section, **(M-Φ Curve)**

<u>Benefits of M-Φ Curve</u>

1. For any value of M we can calculate Φ and rotation capacity.
2. We can develop load-deflection curves to determine member ductility.
3. We can calculate section ductility.



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Example: Draw M-Φ relationship for W 250 x 70

$b_f = 254$ mm

$t_f = 14.2$ mm

$d = 253$ mm

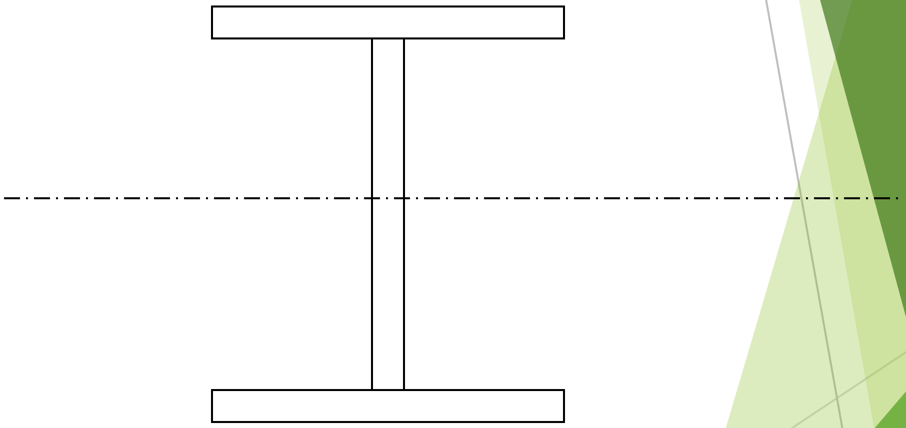
$t_w = 8.6$ mm

$I_x = 11{,}300 \times 10^4$ mm^4

$z_x = 990 \times 10^3$ mm^3

$A = 9290$ mm^2

$S_x = 895 \times 10^3$ mm^3



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Solution:

Let's Take y_o = d/2

$$\frac{d}{2} = 126.5mm$$

$$E\Phi = \frac{F_y}{y_o} = \frac{250}{126.5}$$

$$\boxed{E\Phi = 1.97 N/mm^3}$$

$$M = F_y\left(S_x + Z - Ze\right)$$

For y_o = d/2

$$S_e = S_x, Z_e = Z$$

$$\boxed{M = 250 \times 895 \times 10^3 / 10^6 = 223.75 kN - m}$$

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Solution:

Take y_o = **d/2 – t_f/2**

$$\frac{d}{2}-\frac{t_f}{2}=119.4\text{mm}$$

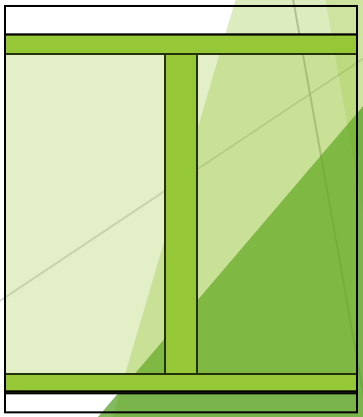
$$E\Phi=\frac{F_y}{y_o}=\frac{250}{119.4}=2.094\,\text{N}/\text{mm}^3$$

$$M=F_y\left(S_e+Z-Z_e\right)$$

Calculations for S_e

$$I_e=\frac{254\times 238.8^3}{12}-\frac{(254-8.6)\times(224.6)^3}{12}$$

$$I_e=56.54\times 10^6\,\text{mm}^4$$



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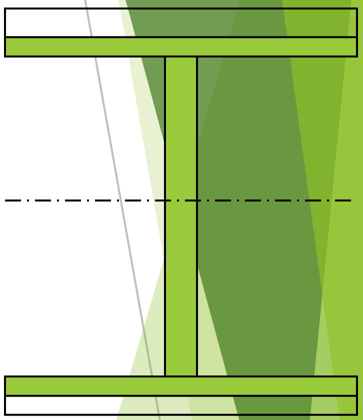
Solution: (contd…)

$$S_e = \frac{I_e}{y_o} = 473.6\times10^3\,mm^3$$

$$z_e = 2\left[254\times7.1\times\left(119.4-\frac{7.1}{2}\right)\right]+\frac{8.6\times224.6^2}{4}$$



$$z_e = 526.3\times10^3\,mm^3$$

$$M = 250\left(473.6\times10^3 + 990\times10^3 - 526.3\times10^3\right)/10^6$$

$$\boxed{M = 234.3kN - m}$$

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Solution:

Take $\mathbf{y_o = d/2 - t_f}$

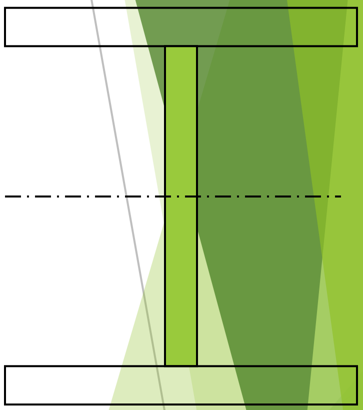
$$\frac{d}{2} - t_f = 112.3\text{mm}$$

$$E\Phi = \frac{F_y}{y_o} = \frac{250}{112.3} = 2.23\text{N}/\text{mm}^3$$

$$S_e = 72.3 \times 10^3 \text{mm}^3$$

$$z_e = 108.5 \times 10^3 \text{mm}^3$$

$$M = 238.5\text{kN} - \text{m}$$



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Solution:

Take **y_o = (d/2 – t_f)/2**

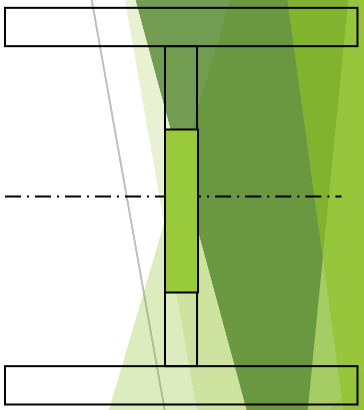
$$\frac{\left(\frac{d}{2}-t_f\right)}{2}=56.15\text{mm}$$

$$E\Phi=\frac{F_y}{y_o}=\frac{250}{56.15}=4.45\text{N/mm}^3$$

$$S_e=18.08\times10^3\text{mm}^3$$

$$z_e=27.11\times10^3\text{mm}^3$$

$$M=245.2\text{kN}-\text{m}$$



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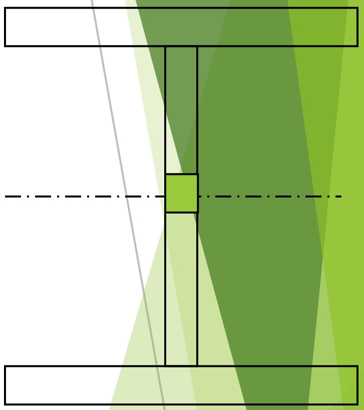
Solution:

Take **y_o = 20 mm**

$$E\Phi = \frac{F_y}{y_o} = \frac{250}{20} = 12.5 N/mm^3$$



$$S_e = 2.293 \times 10^3 mm^3$$

$$z_e = 3.44 \times 10^3 mm^3$$

$$M = 247.2 kN-m$$

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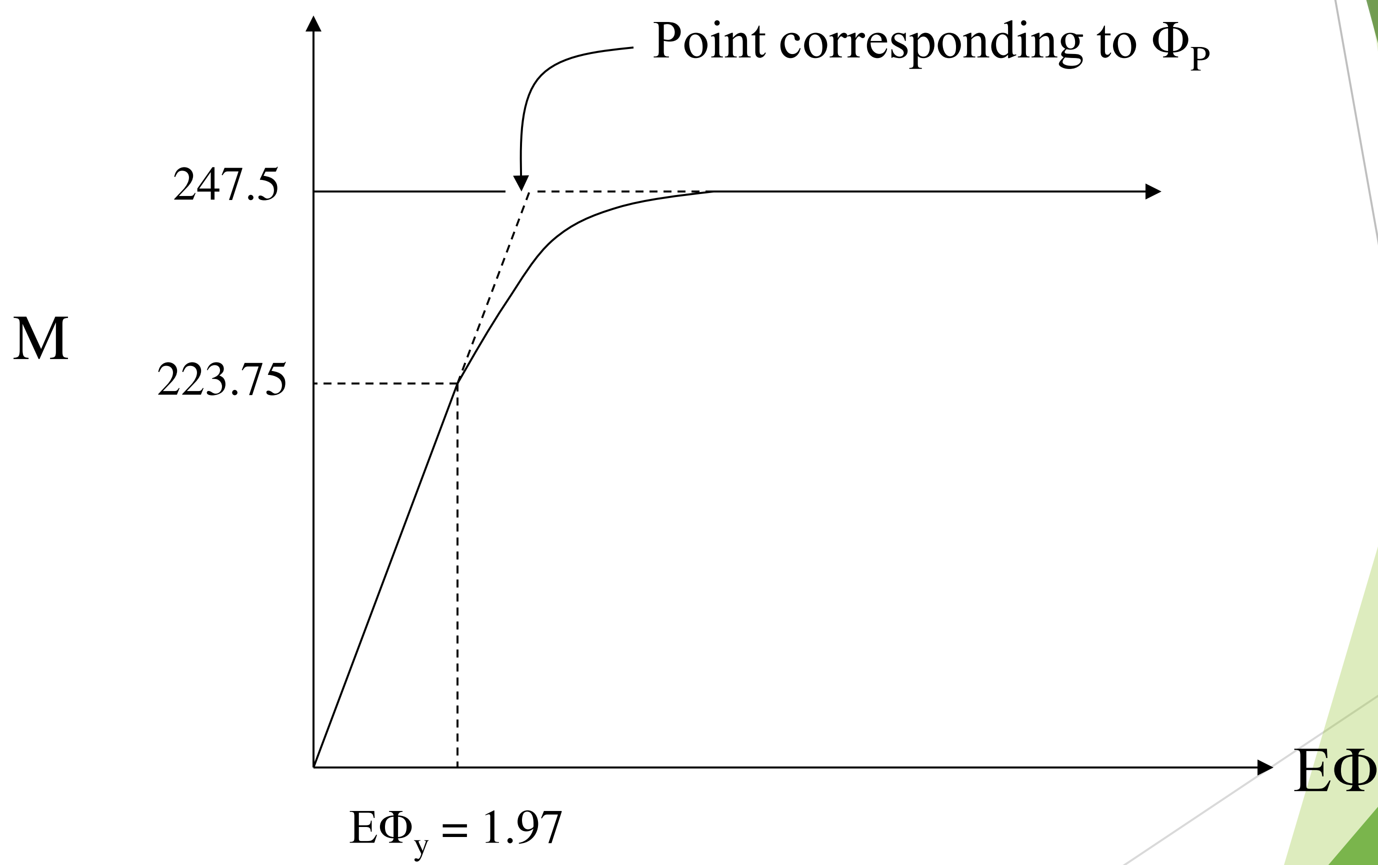
Solution:



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Solution:

From curve

$$\frac{E\Phi_P}{E\Phi_y} = \frac{M_P}{M_y}$$

$$\Phi_P = \Phi_y \times \frac{M_P}{M_y}$$

$$\Phi_P = \frac{1.97}{200,000} \times \frac{247.5}{223.75}$$

$$\Phi_P = 1.09 \times 10^{-5}\ \text{rad/mm}$$

Section Ductility $\mu = \Phi_u/\Phi_P = 3$ For ordinary structures and 22 for special earthquake resistant structures

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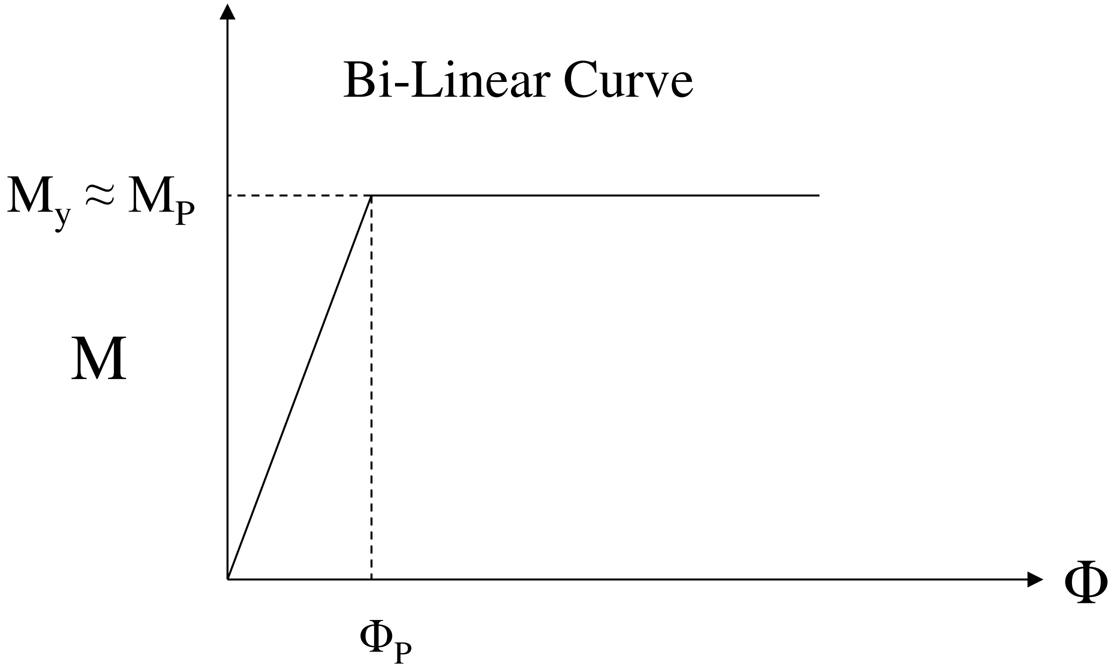
Simplification of M-Φ Curve



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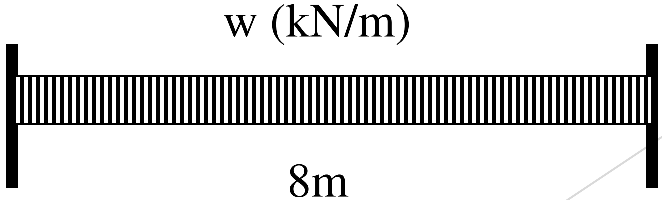
Load Deflection Curve

Example: Using the section of previous example and simplified M-Φ curve plot the load deflection curve for the beam shown and hence estimate the member ductility. Assume

1. Section ductility, $\mu = 3$
2. Length of plastic hinge is d/2 on each side of maximum moment section.
3. $M_y \approx M_P$



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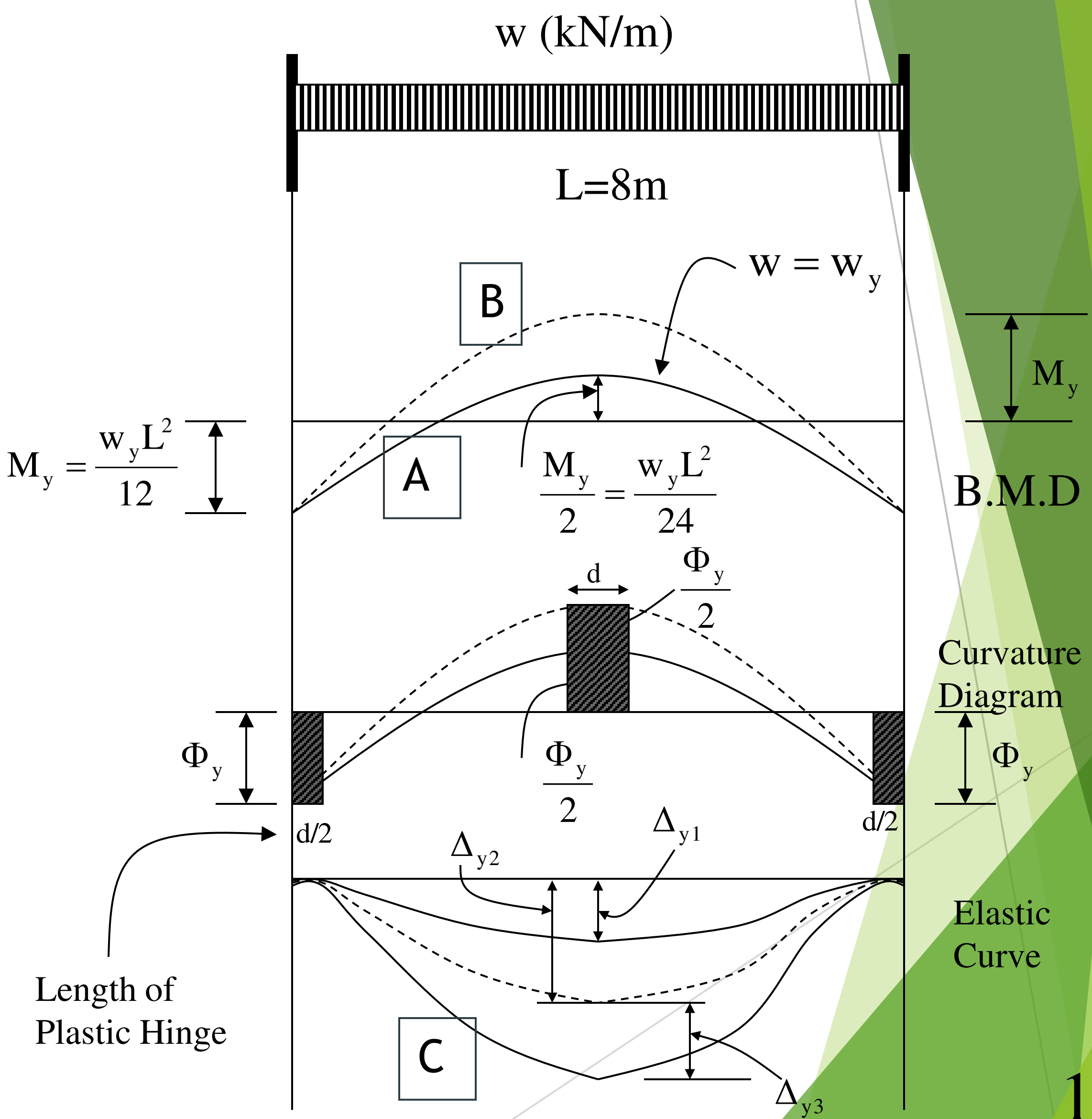
Solution:

w_y = Value of "w" that causes first yielding anywhere in the beam.

CASE-A : Before the development of end hinges or elastic range

CASE-B : Formation of central hinge.

CASE-C : Final failure



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Solution: (contd…)

Δ_{y1} = Deflection at the stage of yielding at the ends

Δ_{y2} = Deflection at the stage of yielding at the center

Δ_{y3} = Final Failure

Final failure is the stage when the rotation capacity at the ends or at the center exhausts.

<u>Load at the First Yield:</u> w_{y1}

$$M_y = M_P = \frac{w_{y1}L^2}{12}$$

$$w_{y1} = \frac{M_P \times 12}{L^2} = \frac{247.5 \times 12}{8^2} = 46.41 kN/m$$

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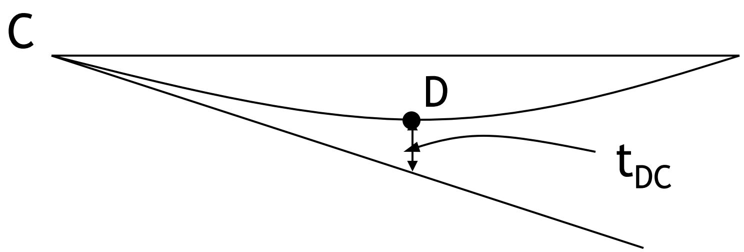
Solution: (contd…)

Deflection at the First Yield: Δ_{y1}



Rotation between two points C & D $= \int_{C}^{D} \Phi \times dx$

$$\int_{C}^{D} \Phi \times dx = \text{Area of curvature diagram between C \& D.}$$

t_{DC} = Tangential deviation of any point D on the elastic curve form tangent drawn on point C on the elastic curve.

$t_{CD} = \int_{D}^{C} \Phi.x.dx =$ First moment of area of curvature diagram between C & D about point D.

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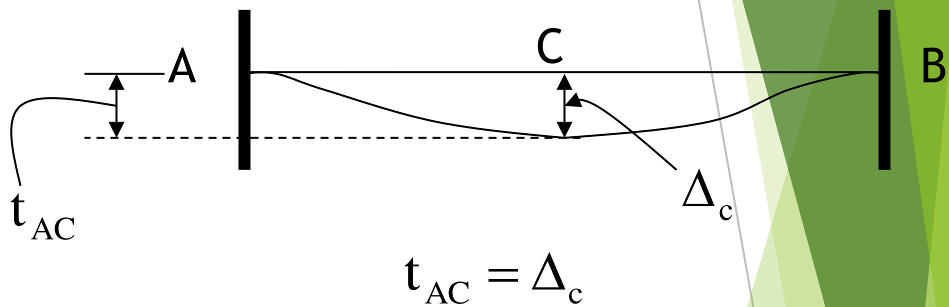
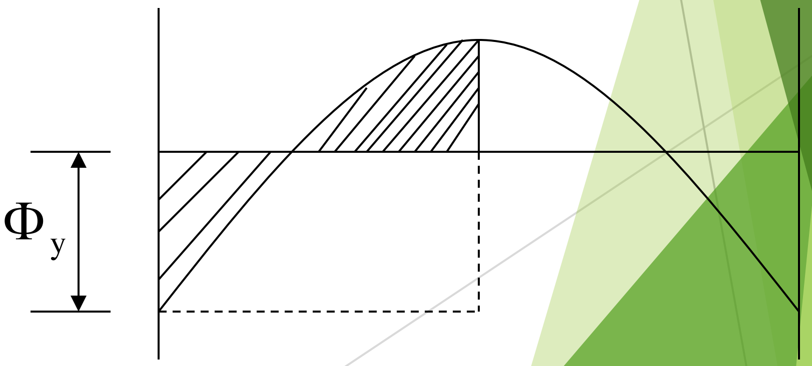
Solution: (contd…)

Deflection at the First Yield: Δ_{y1}

$$t_{AC} = \int_A^C \Phi . x . dx$$



Δ_{y1} = First moment of curvature diagram between A & C about A



$$\Delta_{y1} = A_1 x_1 - A_2 x_2$$

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Solution: (contd…)

Deflection at the First Yield: Δ_{y1}

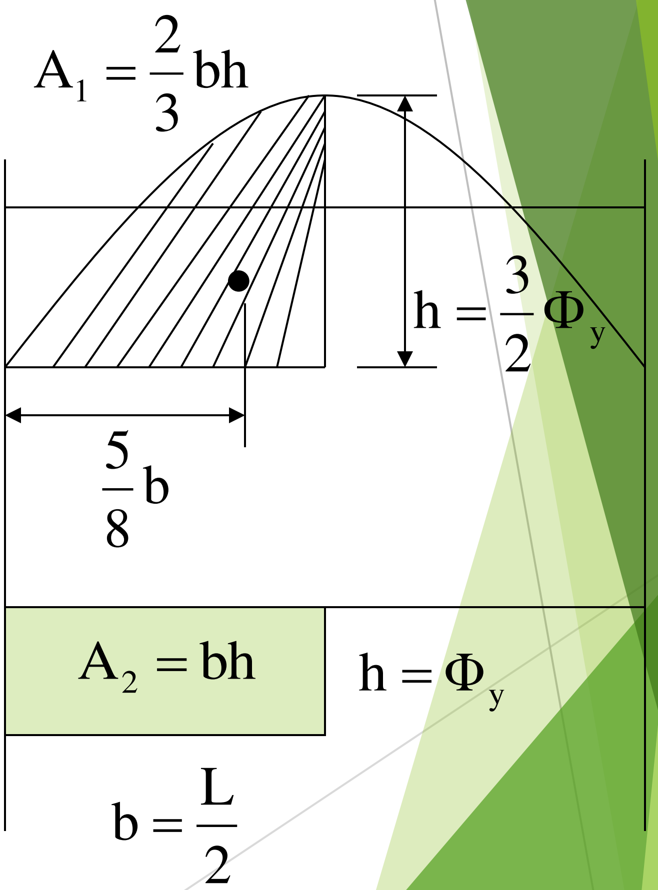
$$\Delta_{y1} = A_1 x_1 - A_2 x_2$$

$$\Delta_{y1} = \frac{2}{3}\left(\frac{3}{2}\Phi \times \frac{L}{2}\right)\left(\frac{5}{8} \times \frac{L}{2}\right) - \left[\Phi_y \times \frac{L}{2}\right] \times \frac{L}{4}$$

$$\Delta_{y1} = \Phi_y \frac{L^2}{32}$$

$$\Delta_{y1} = \frac{1.09 \times 10^{-5} \times 8000^2}{32}$$

$$\boxed{\Delta_{y1} = 21.8\text{mm}}$$



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Solution: (contd…)

Load at the Second Yield: w_{y2}

Assuming sufficient rotation capacity is available at the ends. Otherwise final failure will take place before the formation of second hinge

$$-M_P + \frac{w_{y2}L^2}{8} = M_P$$

Due to end moment, at the center

Due to end Load, at the center

Must be at the center at the to produce PH

$$2M_P = \frac{w_{y2}L^2}{8}$$

$$w_{y2} = \frac{16M_P}{L^2} = \frac{16 \times 247.5}{8^2} = 61.87\text{kN} - \text{m}$$

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Solution: (contd…)

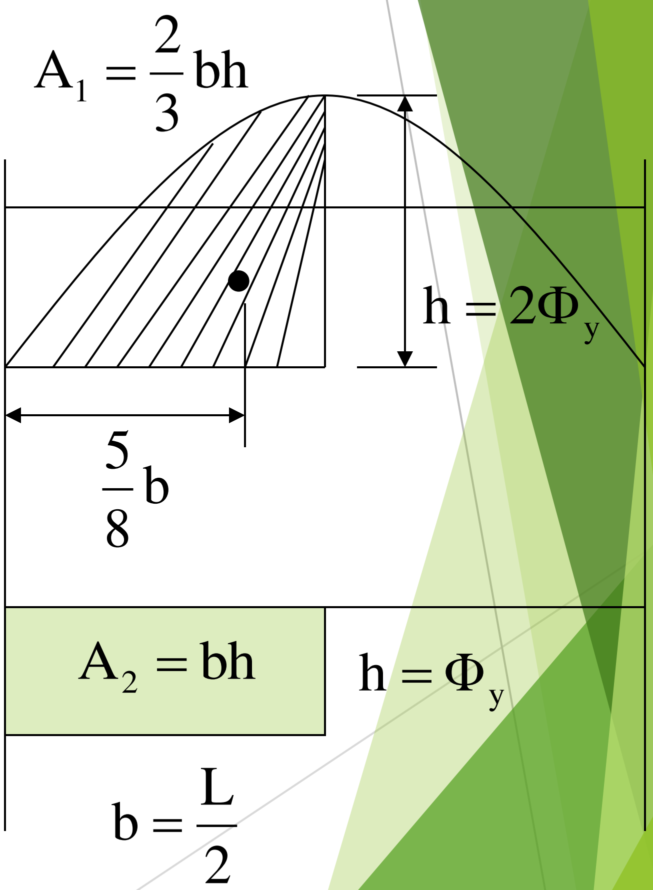
Deflection at the Second Yield: w_{y2}

$$\Delta_{y2} = \frac{2}{3}\left[2\Phi_y \times \frac{L}{2}\right]\left(\frac{5}{8}\times\frac{L}{2}\right) - \left(\Phi y \times \frac{L}{2}\times\frac{L}{4}\right)$$

$$\Delta_{y2} = \Phi_y \times \frac{L^2}{12}$$

$$\Delta_{y2} = 1.09\times 10^{-5} \times \frac{8000^2}{12}$$

$$\boxed{\Delta_{y2} = 58.13\text{mm}}$$



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Solution: (contd…)

Rotation Capacity At The End Hinges:

$$\Phi_{available} = 3 \times \Phi_P$$

$$\theta_{available} = 3 \times \Phi_P \times \frac{d}{2}$$

$$\theta_{available} = 3 \times 1.09 \times 10^{-5} \times \frac{253}{2}$$

$$\theta_{available} = 4.136 \times 10^{-3} \text{rad}$$

$$\boxed{\theta_{available} = 0.004136 \text{rad}}$$

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Solution: (contd…)

Rotation Capacity At The End Hinges:

After the formation of hinge, remaining rotation capacity

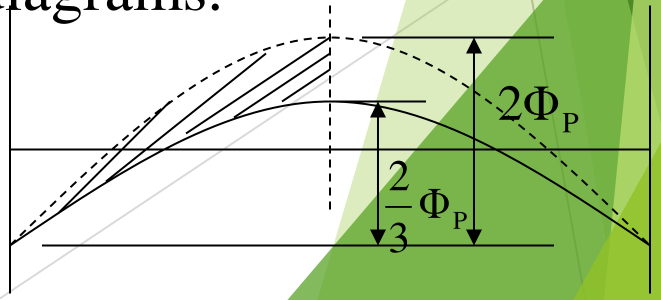
$$\theta_{balance} = 0.004136 - 1.09 \times 10^{-5} \times \frac{253}{2}$$

$$\boxed{\theta_{balance} = 0.00276 rad}$$

Rotation capacity used up-to the formation of central hinge = Difference of area between to curvature diagrams.

$$\theta = \frac{2}{3} \times 2\Phi_P \times \frac{L}{2} - \frac{2}{3} \times \frac{2}{3}\Phi_P \times \frac{L}{2} = \frac{\Phi_P L}{6}$$

$$\boxed{\theta = 0.0145}$$



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Solution: (contd…)

Rotation Capacity At The End Hinges:

$$\theta = 0.0145$$

This is the rotation capacity required for the formation of central hinge but the capacity available is only 0.00276rad. So before the formation of central hinge the rotation capacity at the ends will exhausts and failure will occur.

Concrete frames may have such situation.

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Example: Same as previous example but the specially designed end connection provides a total rotation capacity of 0.03rad.

Solution:

Calculations up-to w_{y1} and w_{y2} are the same.

Check For the Rotation Capacity

$$\theta_{available} = 0.03$$

After the formation of end hinge

$$\theta_{balance} = 0.03 - 1.09 \times 10^{-5} \times \frac{253}{2}$$

$$\boxed{\theta_{balance} = 0.0286 > 0.0145}$$

So central hinge will form

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Solution:

Rotation capacity after the formation of second hinge

$$\theta_{balance} = 0.0286 - 0.0145$$

$$\theta_{balance} = 0.0141 rad$$

$\theta_{balance}$ for central hinge

$$\theta_{balance} = (3-1)\Phi_P \times d$$

$$\theta_{balance} = 2 \times 1.09 \times 10^{-5} \times 253$$

$$\theta_{balance} = 0.0055 rad$$

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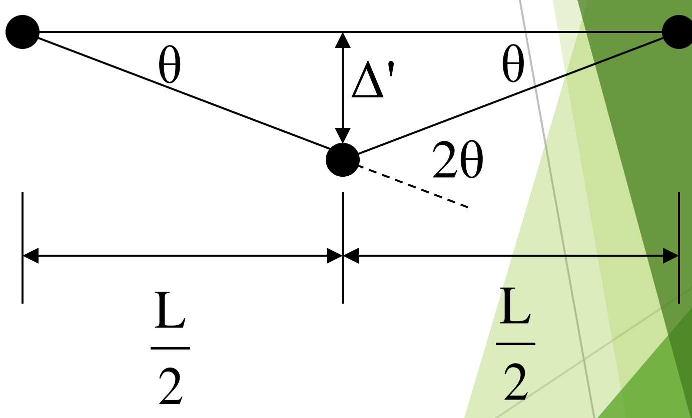
Solution:

<u>Failure Stage:</u>

$$w_{y3} = w_{y2} = 61.9 kN/m$$

As after the formation of second hinge beam can’t take more load because of the formation of mechanism

$$\Delta' = \theta' \times \frac{L}{2}$$



For the rotation capacity of central hinge

$$2\theta = 0.0055$$

$$\theta = \frac{0.0055}{2}$$

$$\Delta_{y3} = \frac{0.0055}{2} \times \frac{8000}{2} = 11.03mm$$

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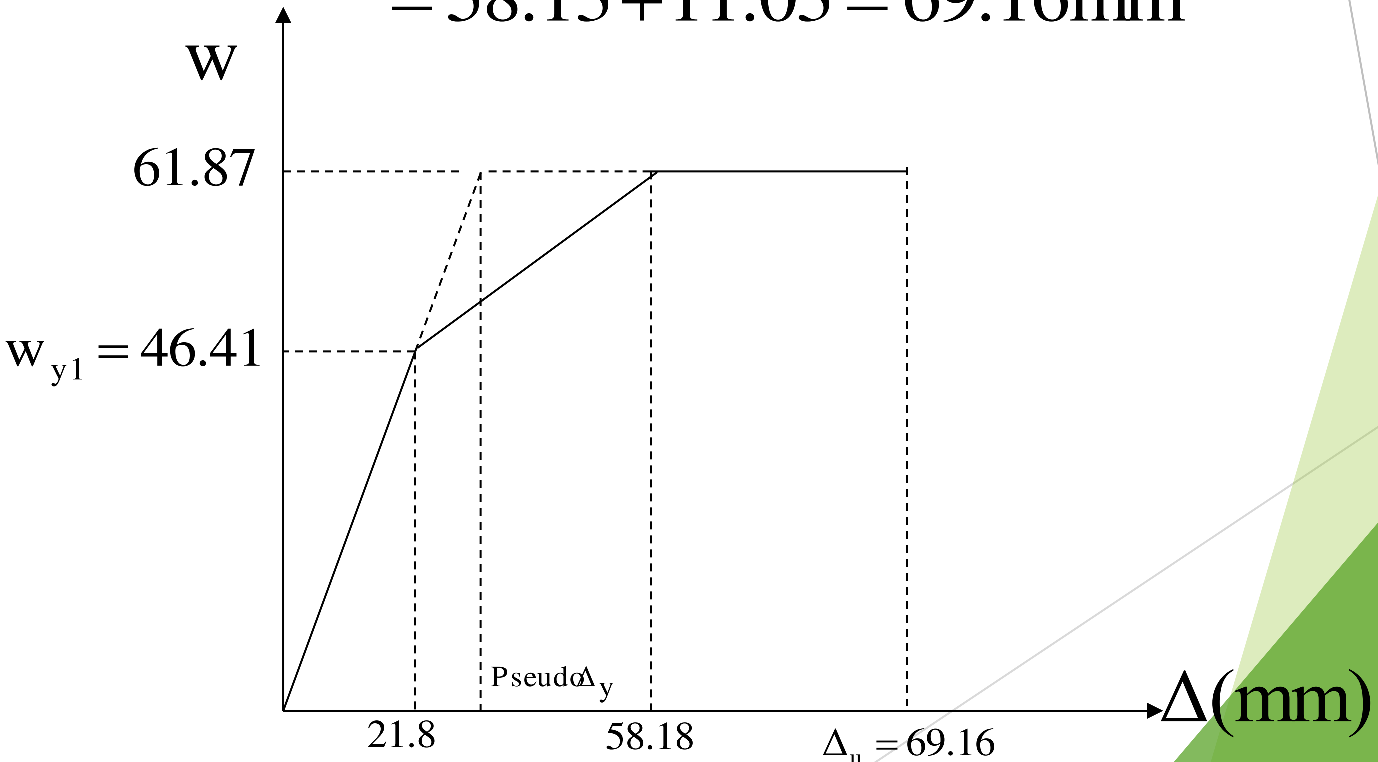
Solution:

Failure Stage:

Total deflection $= \Delta_{y2} + \Delta_{y3}$

$$= 58.13 + 11.03 = 69.16\text{mm}$$



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Solution:

$$\text{Member ductility} = \frac{\Delta}{\text{Pseudo}\Delta_y}$$

$$\text{Pseudo}\Delta_y = 21.8 \times \frac{61.87}{46.41} = 29.06\text{mm}$$

$$\mu = \frac{69.16}{29.06}$$

$$\boxed{\mu = 2.38}$$

Less then section ductility

Concluded