Steel Structures

M.Sc. Structural Engineering

SE-505

Plastic Analysis and Design of Structures
Lec #4

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Steel Structures

Effect of Shear Force on Moment Capacity

We can ignore the effect of shear force on plastic moment capacity as it is very less in magnitude. **AISC ALSO SUPPORTS THIS IDEA. British code considers this little effect.**

Shear stresses are maximum near N.A. and flexural stresses are maximum near periphery.

V_n = Nominal shear capacity of web

$$V_n = 0.6 F_{yw} A_w \qquad \phi_v = 0.9$$

$$V_u \leq \phi_v V_n$$

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Effect of Shear Force on Moment Capacity (contd…)

If $V_u \geq \phi_v V_n$, to compensate the capacity following are the options:

1. Select larger section. (may be uneconomical)
2. Provide web doublers.
   Web doublers is extra plate attached with web.
3. Provide diagonal stiffener to take some portion of shear strength.

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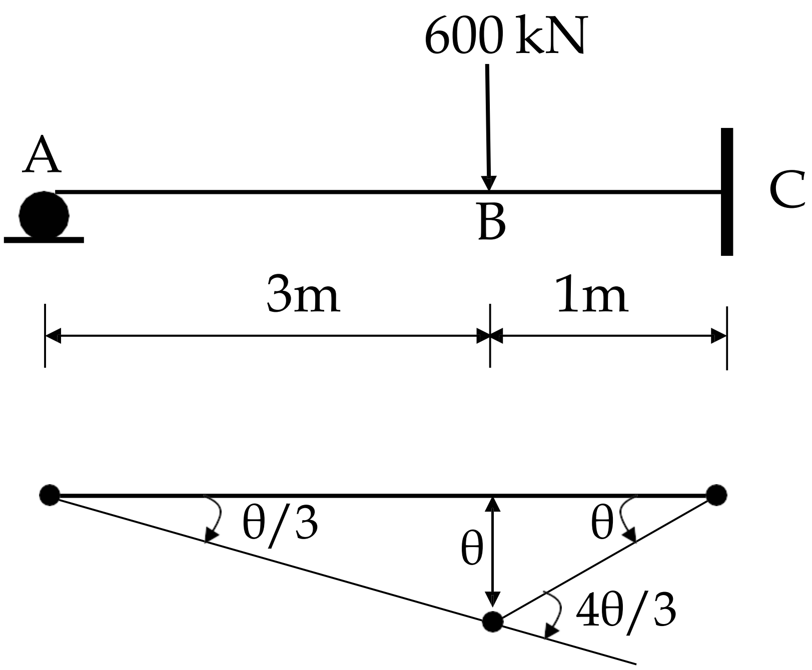
Example: Design the member assuming that significant bracing is available to prevent LTB. Rotation capacity up-to collapse is available.

Solution:



$$W_E = P \times \theta$$

$$W_I = M_P\left(\frac{4}{3}\theta + \theta\right) = \frac{7}{3}M_P\theta$$

$$P \times \theta = \frac{7}{3}M_P\theta$$

$$M_P = \frac{3}{7} \times P = \frac{3}{7} \times 600$$

$$M_P = 257\text{kN}-\text{m}$$

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Solution: (contd…)

For the selection of section

$$Z_t \geq Z_{req}$$

$$d_{min} = \frac{F_y \times L}{5500} = \frac{L}{22} \quad \text{For A36 Steel}$$

Should be minimum weight section

$$Z_{req} = \frac{257 \times 10^6}{0.9 \times 250} = 1142 \times 10^3 mm^3$$

$$d_{min} = \frac{4000}{22} = 182mm$$

From the beam selection table, Trial Section **W 410 x 60**

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Solution: (contd…)

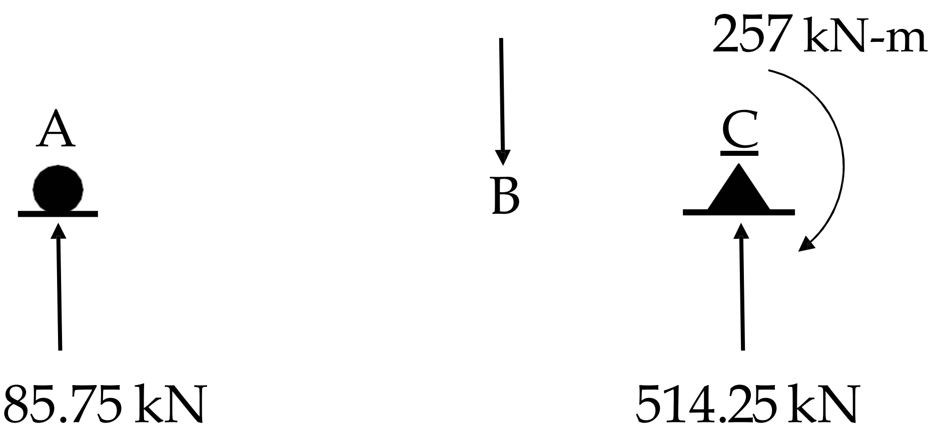
check the conditions for compact section:

web is continuously connected to flange

$$\frac{b_f}{2t_f} = 6.9 < \lambda_p 10.8$$

$$\frac{h}{t_w} = 46.6 < \lambda_p = 107$$

$L_p < L_b$ No LTB



<u>Shear Check</u>

$$V_{u\,max} = 514kN$$

$$\phi_v V_n = \phi_v 0.6 F_y d \times t_w$$

$$\phi_v V_n = 0.9 \times 0.6 \times 250 \times 407 \times 7.7 / 1000 = 423kN < V_{u\,max}$$

Portion AB is safe but BC needs to be strengthened.

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Solution: (contd…)

1. Provide web doubler

$$0.9 \times 0.6 \times 250 \times 407 \times t_w \Big/ 1000 = 514$$

$$t_w = 9.35\text{mm}$$

Extra thickness required = 9.35 – 7.70 = 1.65 mm

Provide 5mm thick (based on corrosion resistance) web doubler in portion BC of length 1m and height equal to 'd' of section. Connect at regular intervals with weld or bolts. Spacing of weld or bolt can be calculated by usual design procedure.

2. Use of diagonal stiffener

   Shear to be resisted by diagonal stiffener = 514 – 423 =91 kN

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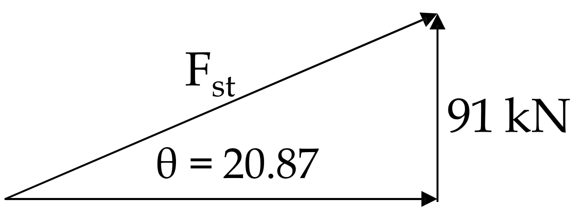
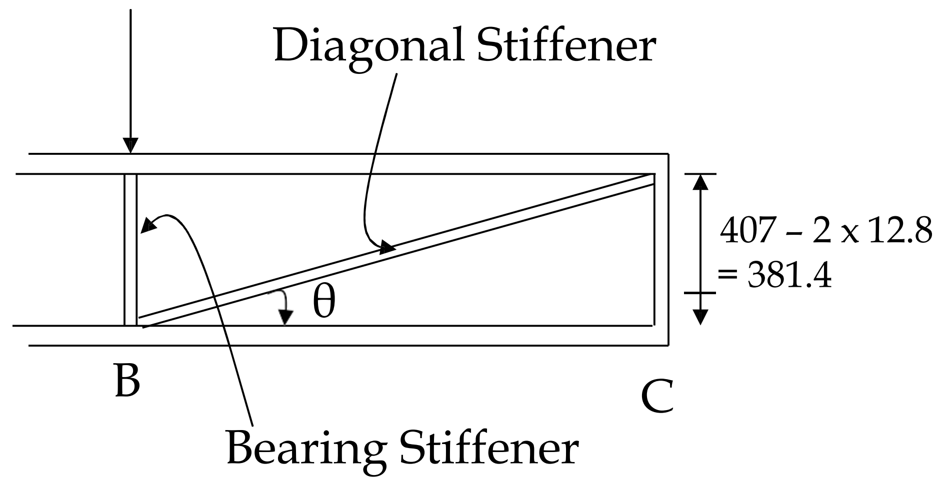
Solution: (contd…)

Length of diagonal stiffener = $\sqrt{(1000)^2 + (382)^2} = 1070\text{mm}$





Force resisted by diagonal stiffener, Fst = 91/Sinθ = 254.90 kN (tensile)

$$A_{st} = \frac{254.9 \times 1000}{0.9 \times 250} = 1133\text{mm}^2$$ For pair of stiffener on both sides

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Solution: (contd…)

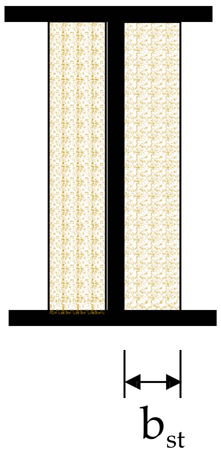
$$b_{st} = \sqrt{7.9A_{st}}$$

For b/t = 15.8

$$b_{st} = \sqrt{7.9 \times 1133} \cong 95\text{mm}$$

95 mm should not extend out of the section

$$b_{st} \leq \frac{b_f - t_w}{2} = \frac{178 - 7.7}{2} = 85\text{mm} < 95\text{mm}$$



So we use b_{st} = 85 mm

$$t_{st} = \frac{1133}{2 \times 85} = 6.7\text{mm}$$

Use 7 mm Thick Plate

If we use rivets or bolts angle section is better and for weld rectangular plate is better.

MOMENT CHECK AFTER PLASTIC ANALYSIS

When the final collapse load is obtained by considering all the possible failure mechanisms and by selecting the one corresponding to the minimum load, bending moment diagram is drawn to make sure that the plastic moment capacity is not exceeded at any point ($M \leq M_p$).

Following possibilities may exist for plotting the bending moment diagram:

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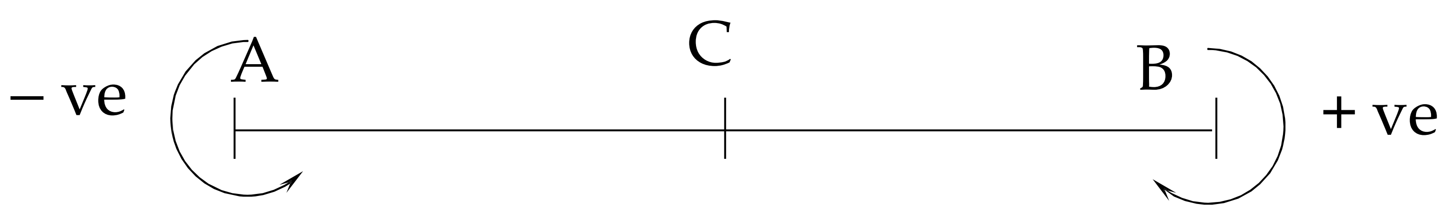
Situation – I: The structure, after formation of collapse mechanism, is fully determinate. The reactions and unknown moments may be calculated by the equations of static equilibrium.

Sign Convention For Moments

i) Clockwise end moments are considered positive.



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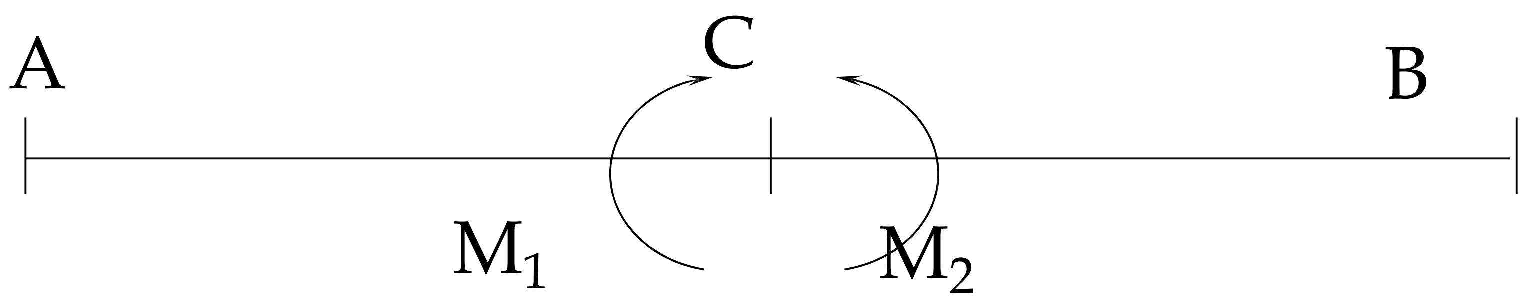
ii) Within the beam, positive moments are considered positive when tension is produced on the bottom fiber or when the frame tries to open.



$M_1 = M_2 = M$

M_1 = applied moment on segment CB

M_2 = applied moment on segment CA

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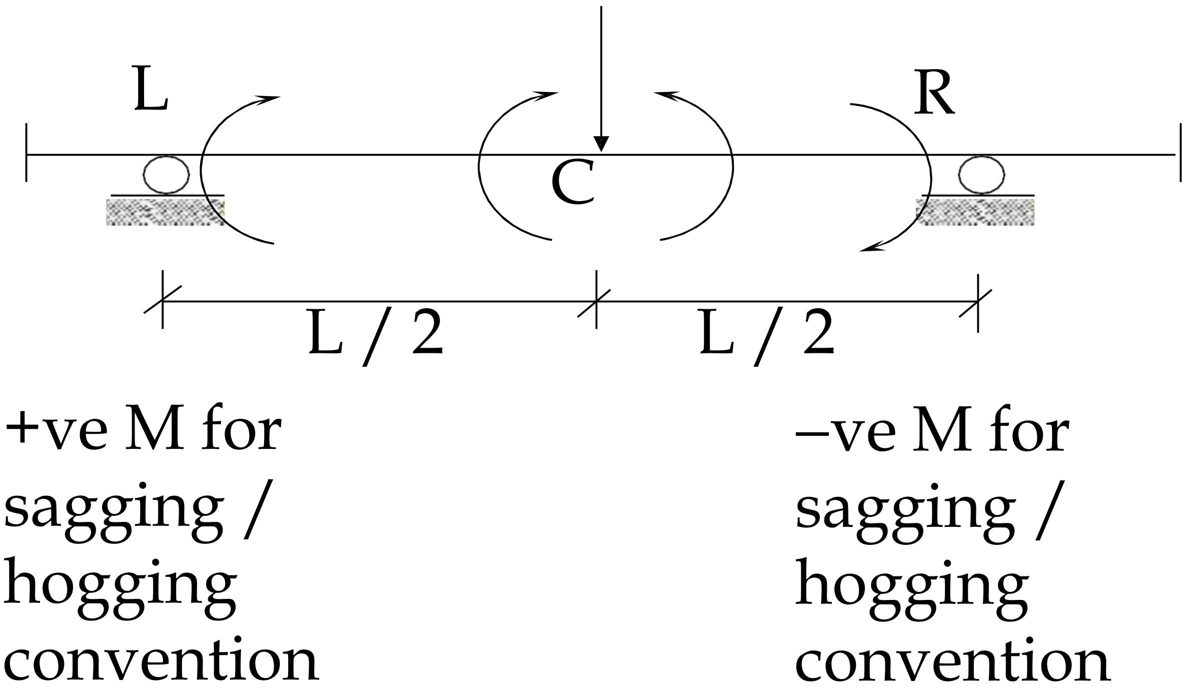
Common Equilibrium Equations

i) For beams, a typical example for which the central moment in terms of end moments and applied load can be written as:



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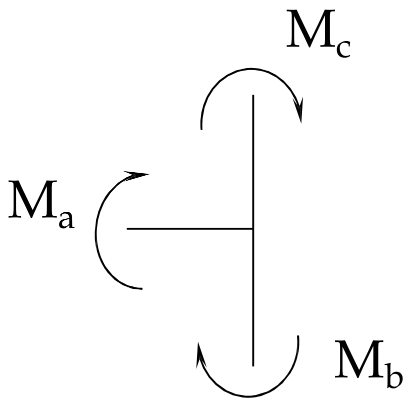
$$M_C = \frac{M_L}{2} - \frac{M_R}{2} + \frac{P_u L}{4}$$

or $$M_C = \frac{M_L}{2} - \frac{M_R}{2} + M$$

where M = simply supported positive moment at the center.

ii) For Joint Equilibrium:

$$M_a + M_b + M_c = 0$$



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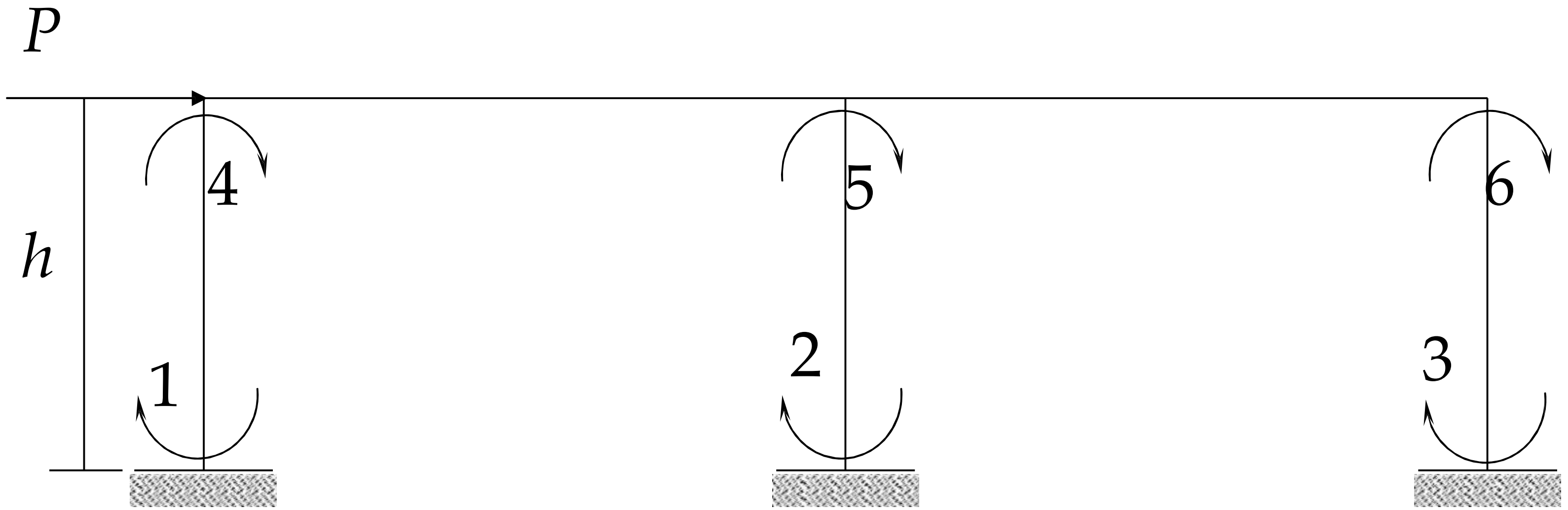
iii) For Sway Equilibrium:



$$M_1 + M_2 + M_3 + M_4 + M_5 + M_6 + Ph = 0$$

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2nd Method Of Applying Equilibrium Equations

In this method, applied loads, moments and reactions are worked out in terms of shear couples.

Couples due to shear forces and moments are always equal in magnitude but opposite in direction.

External loads and reactions are both applied shears on the beam, which act in the same direction.

Moments are resisting couples and are acting in the opposite directions.

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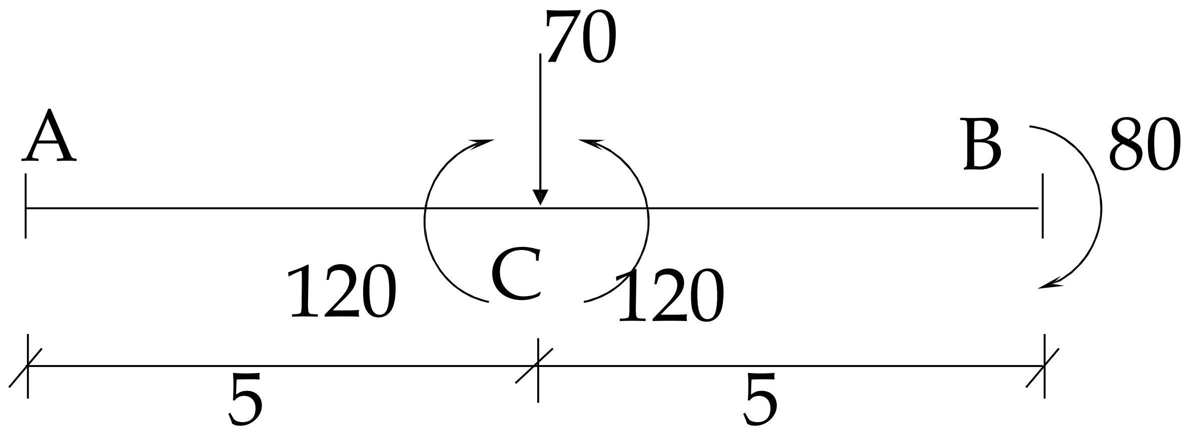
i) Calculating moment at A, if moments are known at B and C.



Total moment acting in portion CB = 120 + 80 = 200 clockwise

For equilibrium, the shear force in portion CB, having no load in between, must be that which produces equal and opposite couple.

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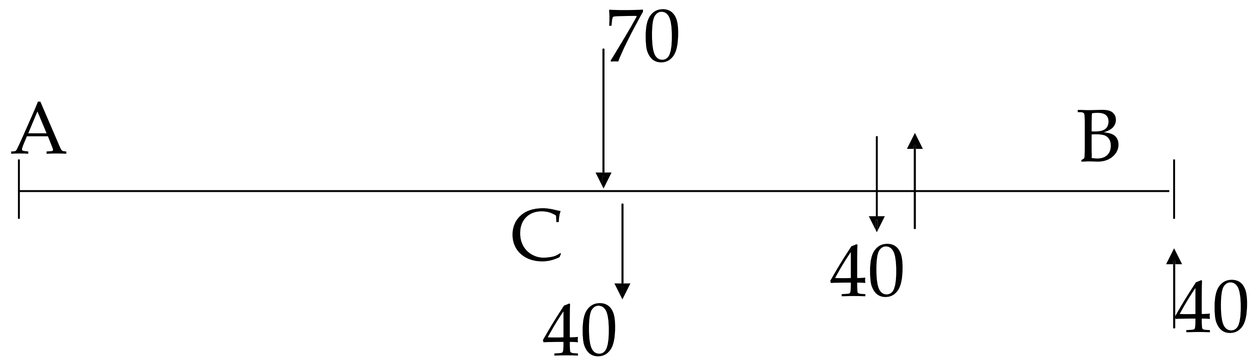
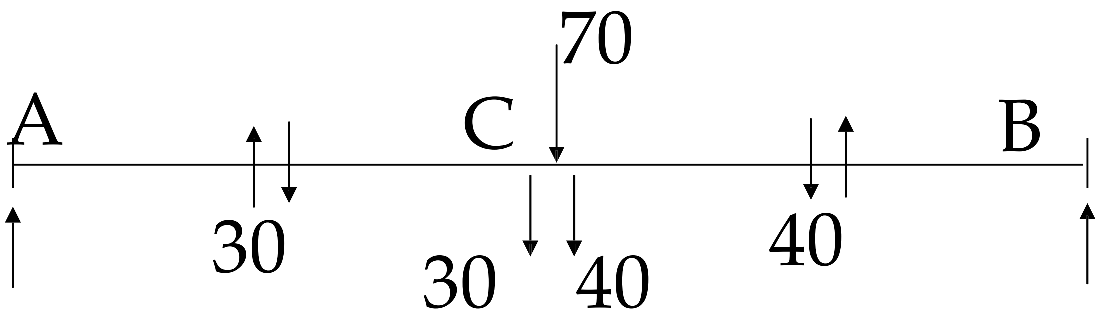
Shear force = 200 / 5 = 40



At point C, external load provides the required shear. For equilibrium of joint C, the balance shear force must be 30 units acting on portion CA.



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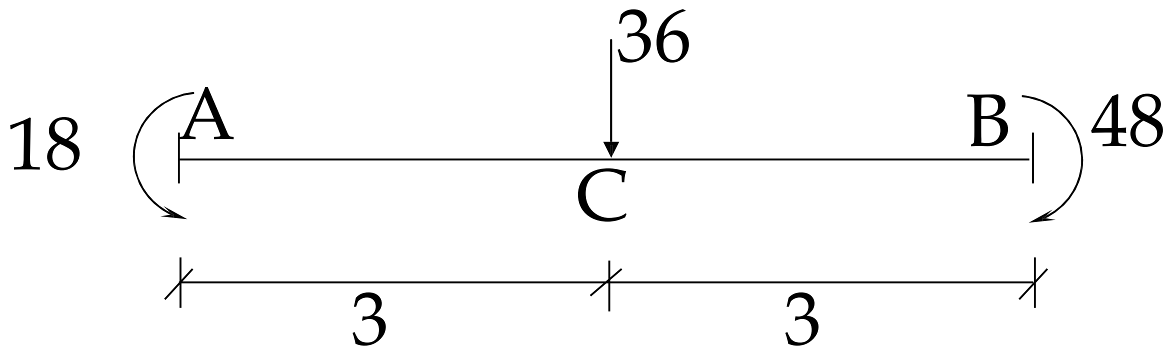
Assume that M_A is counterclockwise:

$$M_C + M_A = 30 \times 5$$

$$120 + M_A = 150$$

$$M_A = -30$$ or 30 units counterclockwise

ii) Calculating moment at C, if the end moments are known.



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In the absence of the central load, the shear in portion AB will be (48 – 18)/6 = 5 (counterclockwise).

Similarly, if only load is present in the absence of end moments, it produces a simply supported shear of 18 clockwise in portion AC and 18 counterclockwise in portion CB.

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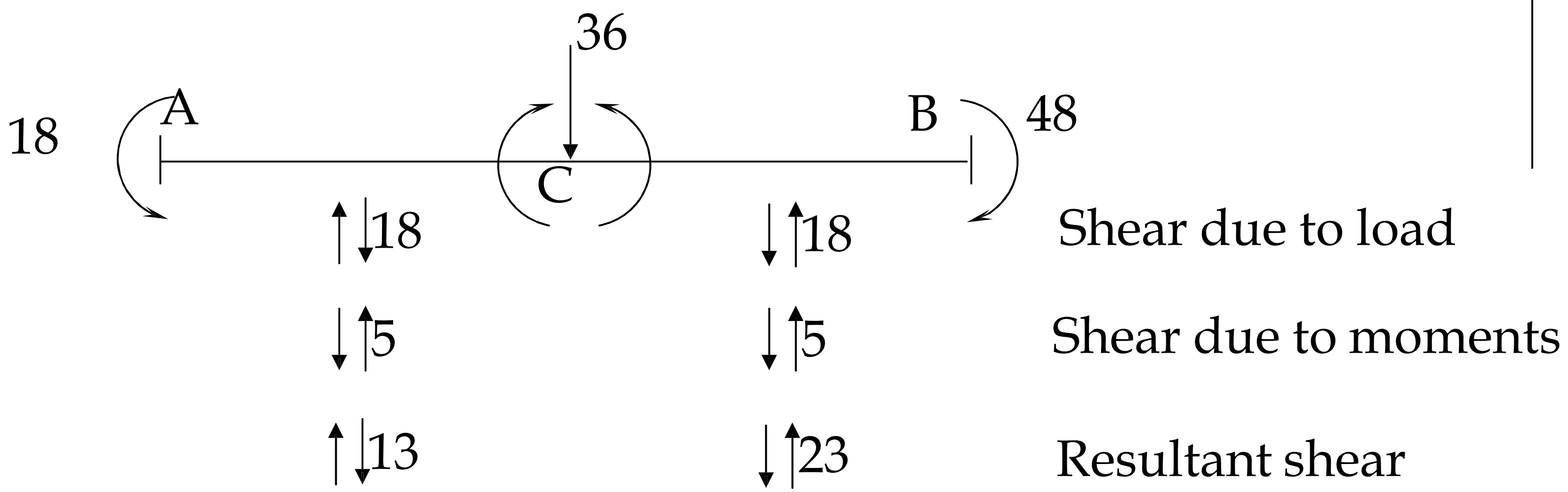

Moment at C may now be calculated from the left or the right side.

For portion AC:

$M_A \quad + \quad M_C \quad = \; 13 \times 3 \text{ (CW)}$

$M_C = \; 13x3 - 18 \; = \; 21$ (CCW or positive according to sagging / hogging concept)

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Situation – II: The structure, after formation of collapse mechanism, is partially redundant.

The objective of design is to utilize the strength of all members and all parts of the structure up to a maximum extent.

This means that a no. of plastic hinges in addition to the collapse mechanism are expected to be nearly formed, reducing the redundancy of the structure.

The condition having high degree of indeterminacy at the collapse load shows a design that is not the best.

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The member strengths in the indeterminate parts may be reduced to get economy without affecting the collapse strength.

There are two methods for analysis of the partially redundant structure.

a) Trial And Error Method

In this method, unknown moments equal in number to the remaining degree of indeterminacy are assumed reasonably and the remaining moments and shears are found by the equilibrium equations.

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Accurate values of moments at various sections are not generally needed.

The main objective is to make sure that the moment does not exceed M_p anywhere in the structure.

This method is suitable when the remaining redundancy is very less (like one or two).

Example

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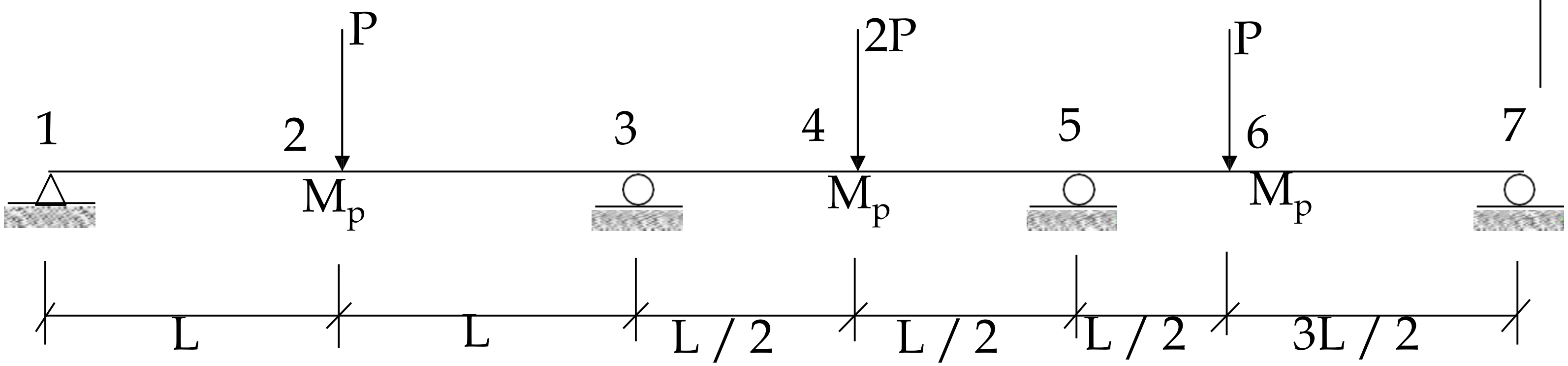

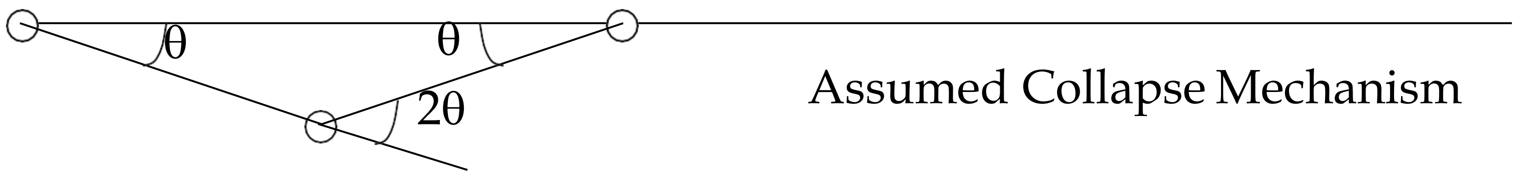

Assumed Collapse Mechanism

$$W_E = P\,\theta L$$

$$W_I = M_p(2\theta) + M_p(\theta) = 3M_p\,\theta$$

$$P = 3M_p / L \qquad or \qquad M_p = PL/3$$

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X = redundancy in the original structure = 2

M = no. of plastic hinges for the assumed mech.
= 2

I = remaining redundancy = $X - (M-1)$ = 1

Note that out of the **M** plastic hinges, one plastic hinge cause collapse but do not change the redundancy of the remaining structure.

Actually, moment M_5 is still redundant.

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Assume $M_5 = M_p$:

$$M_4 = -\frac{M_3 + M_5}{2} + \frac{2PL}{4}$$

$$= -M_p + \frac{PL}{3} \times \frac{6}{4}$$

$$= -M_p + 1.5\,M_p = 0.5\,M_p < M_p \qquad \text{(OK)}$$

$$M_6 = -\frac{3M_p}{4} + \frac{P(L/2)(3L/2)}{2L}$$

$$= -0.75M_p + \frac{PL}{3} \times \frac{9}{8} = -0.75M_p + 1.125\,M_p$$

$$= 0.875\,M_p < M_p \qquad \text{(OK)}$$

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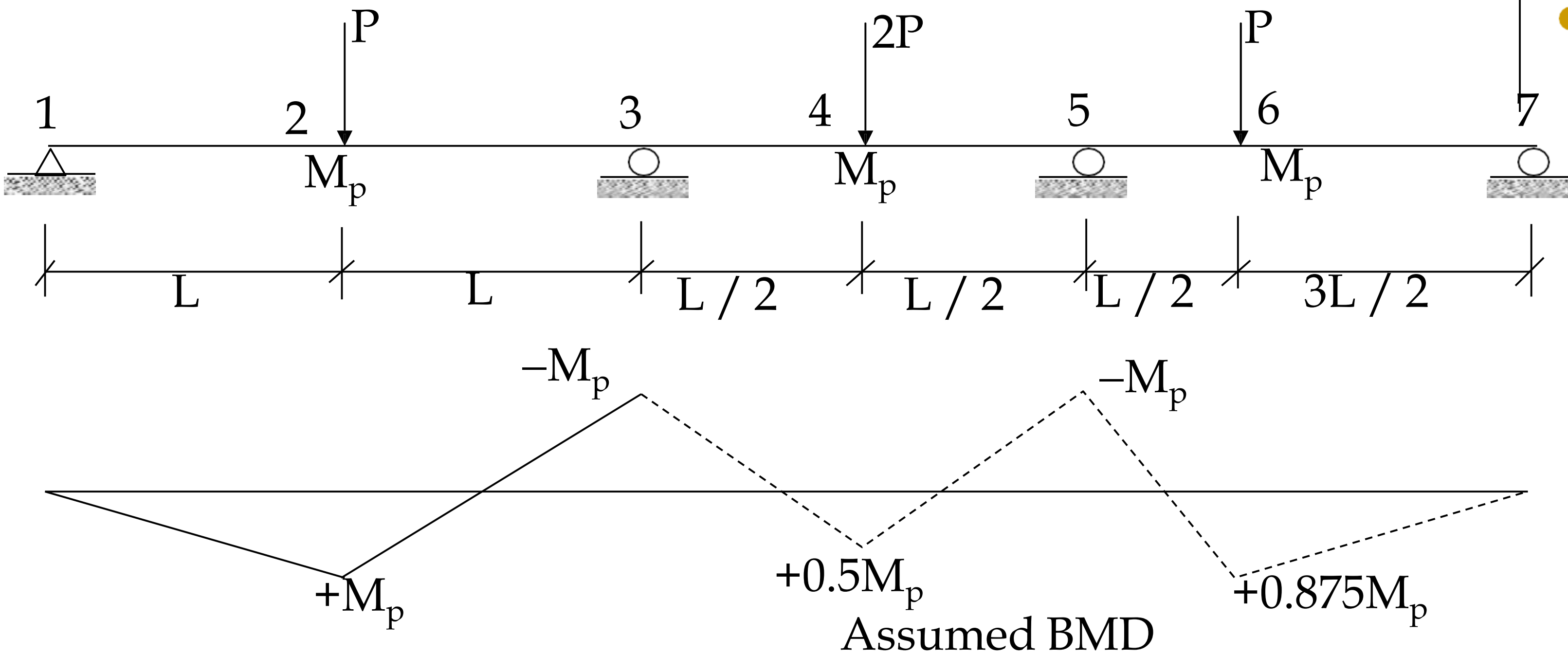

The condition that $M \leq -M_p$ throughout the beam indicates that the assumed mechanism may be the critical one.

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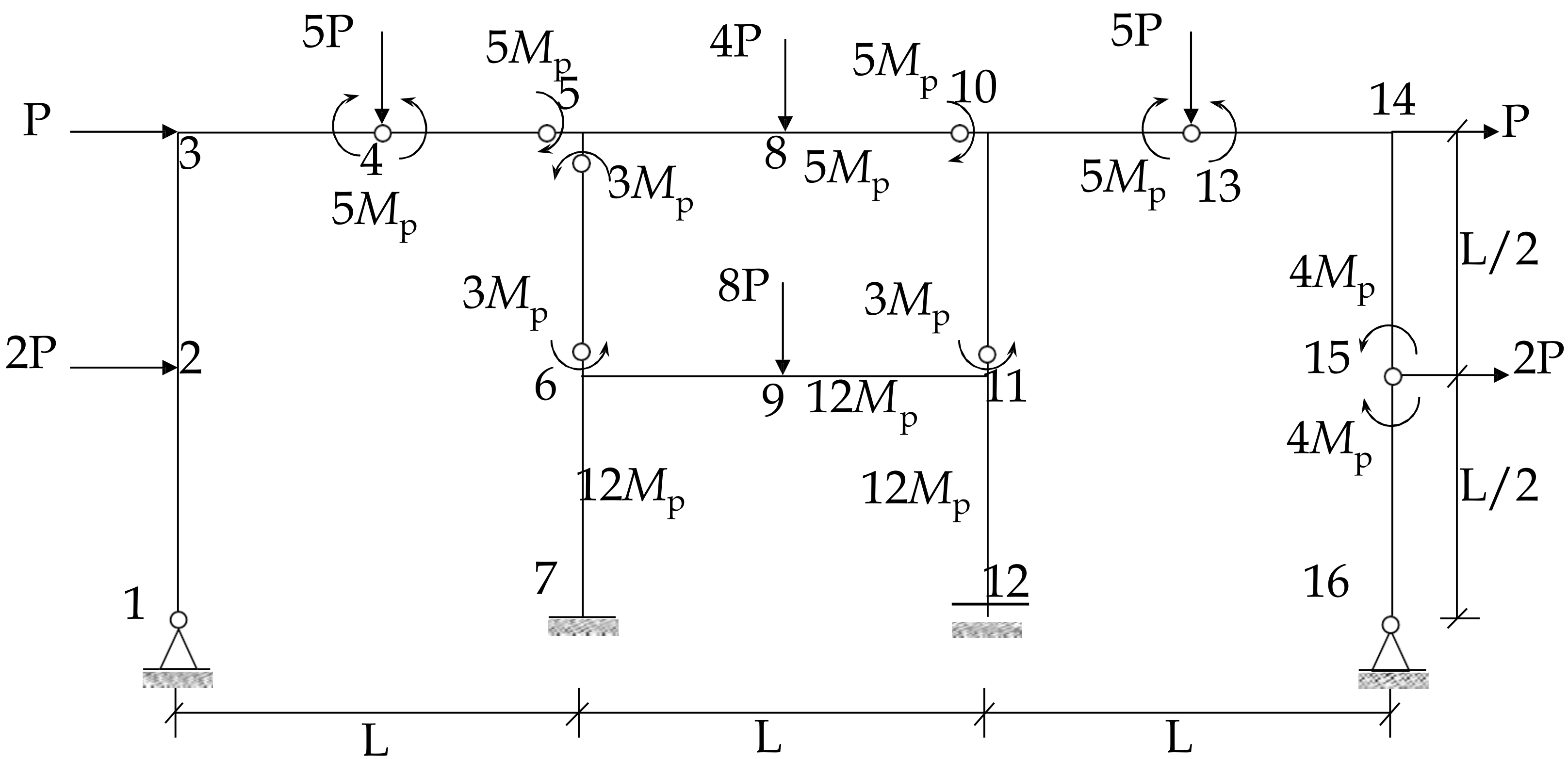
Example: $P_c = \qquad 6\, M_p / L$



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Portion 4-5: Shear $= \dfrac{5M_p + 5M_p}{L/2}$

$= 20\dfrac{M_p}{L}$ $\downarrow\uparrow$

Portion 3-4: Shear $= 5P - 20\dfrac{M_p}{L}$

$= 30\dfrac{M_p}{L} - 20\dfrac{M_p}{L}$

$= 10\dfrac{M_p}{L}$ $\uparrow\downarrow$

Assuming M_3 to be counterclockwise:

$M_3 + 5M_p = 10\,M_p/L \times L/2$ $\qquad M_3 = 0$

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Portion 1-2: Shear $= P = 6 \dfrac{M_p}{L}$

$$M_2 = 6\, M_p / L \times L / 2 = 3\, M_p$$

Portion 15-16: Shear $= \dfrac{4M_p}{L/2} = 8 \dfrac{M_p}{L}$

Portion 14-15: Shear $= 2P - 8 \dfrac{M_p}{L}$

$$= 12 \frac{M_p}{L} - 8 \frac{M_p}{L} = 4 \frac{M_p}{L}$$

Assuming M_{14} to be counterclockwise:

$$M_{14} - 4M_p = -4\, M_p / L \times L / 2$$

$$M_{14} = 2M_p$$

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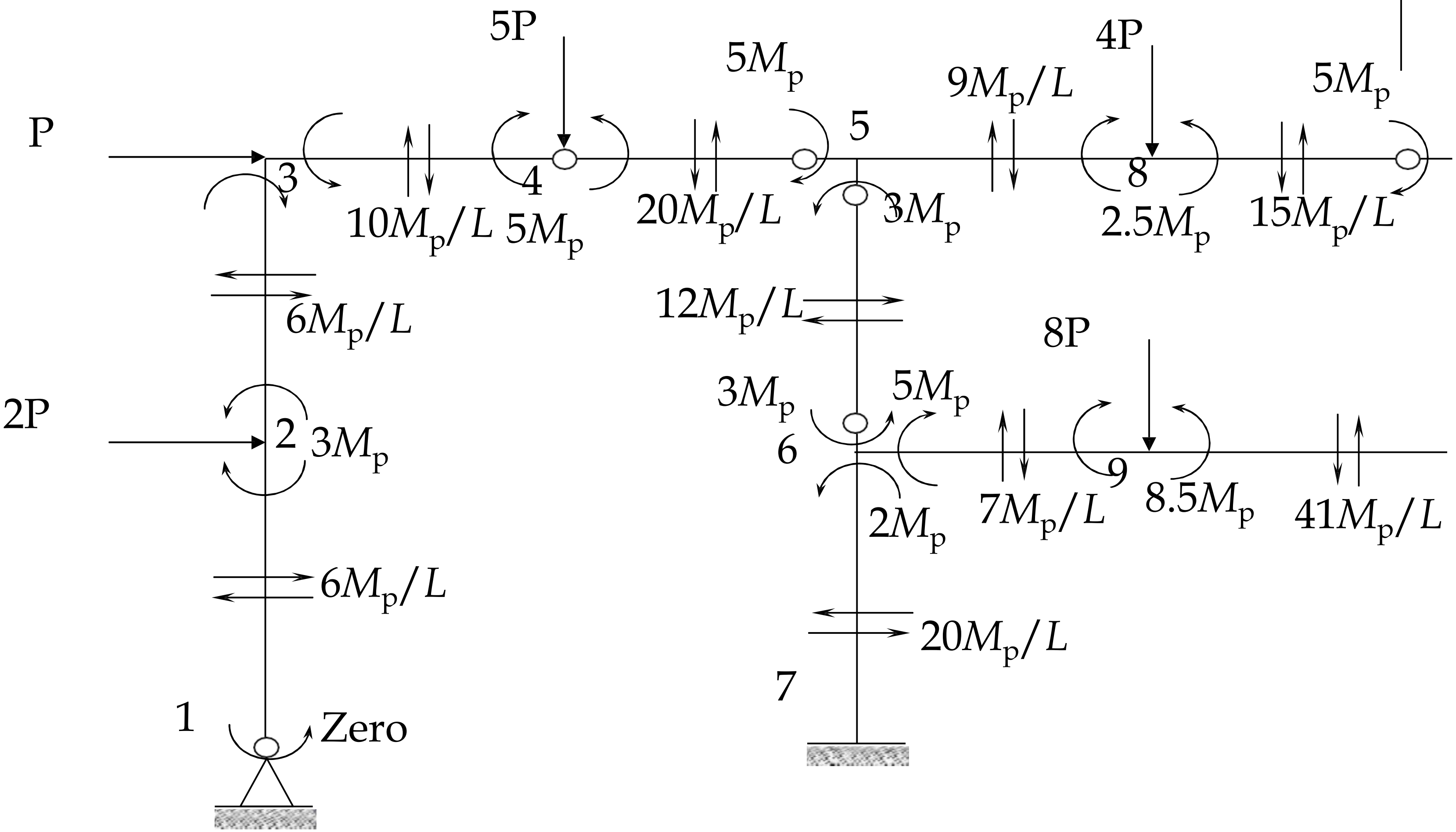

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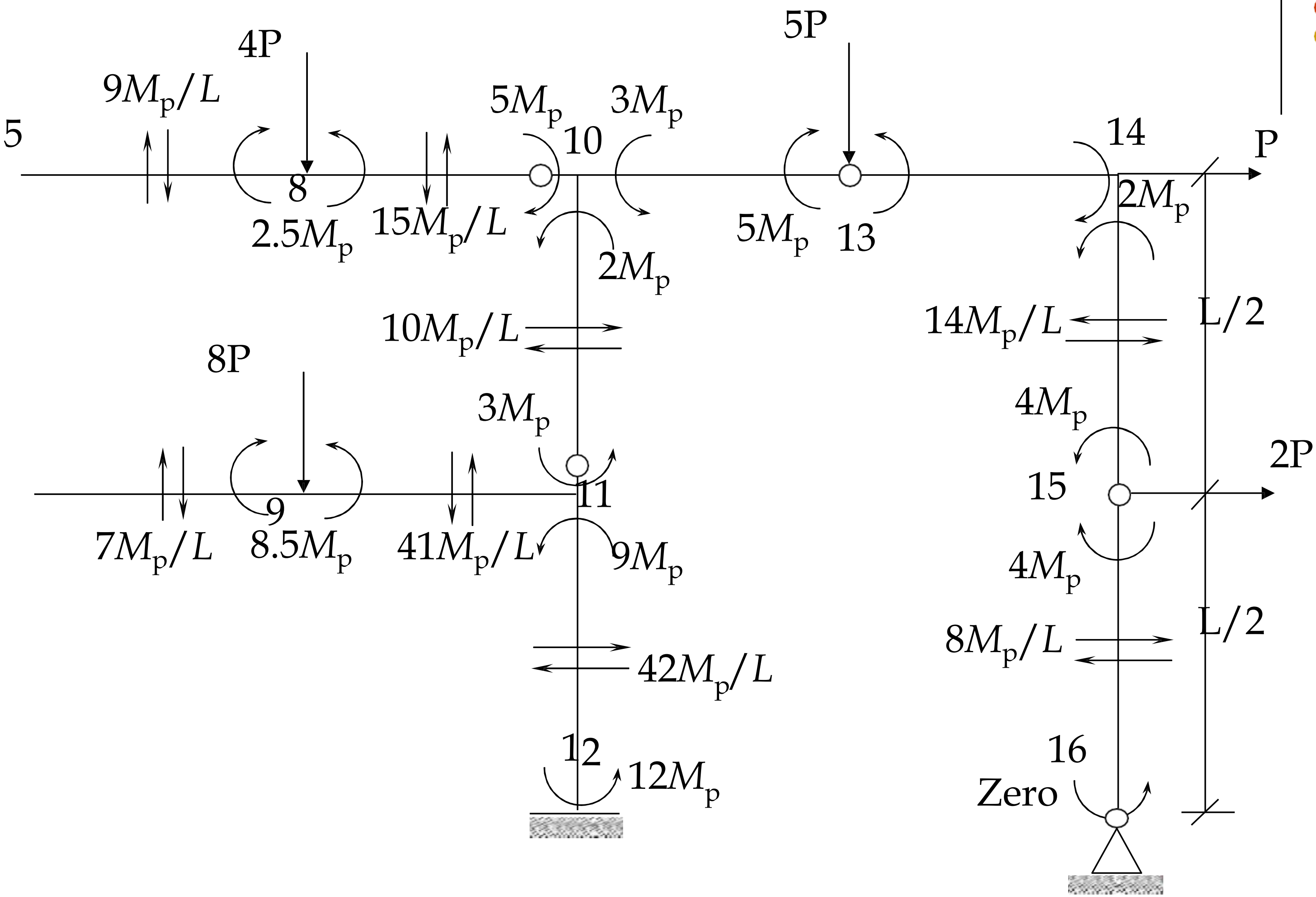

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Portion 13-14: Shear $= \dfrac{5M_p + 2M_p}{L/2}$

$= 14\dfrac{M_p}{L}$

Portion 10-13: Shear $= 5P - 14\dfrac{M_p}{L}$

$= 30\dfrac{M_p}{L} - 14\dfrac{M_p}{L}$

$= 16\dfrac{M_p}{L}$

Assuming M_{10} to be counterclockwise:

$M_{10} + 5M_p = 16\, M_p/L \times L/2$

$M_{10} = 3M_p$

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Portion 5-6: $$\text{Shear} = \frac{3M_p + 3M_p}{L/2}$$

$$= 12\,\frac{M_p}{L}$$

Portion 10-11: $$\text{Shear} = \frac{5M_p + 2M_p}{L/2} = 14\,\frac{M_p}{L}$$

Portion 5-10: $M_5 = 2M_p$ for equilibrium

$$\text{Shear without load} = \frac{5M_p - 2M_p}{L}$$

$$= 3\,\frac{M_p}{L}$$

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$$\text{Load} = 4P = 24\frac{M_p}{L}$$

$$\text{S.S. shear in portion 5-8} = 12\frac{M_p}{L}$$

$$\text{Total shear} = (12-3)\frac{M_p}{L} = 9\frac{M_p}{L}$$

$$\text{S.S. shear in portion 8-10} = 12\frac{M_p}{L}$$

$$\text{Total shear} = (12+3)\frac{M_p}{L} = 15\frac{M_p}{L}$$

Assuming M_8 to be counterclockwise:

$M_8 + 2M_p = 9\,M_p/L \times L/2$

$M_8 = 2.5M_p$

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Portion 7-6-9-11-12 is still indeterminate.

For its approximate analysis, we may assume additional hinges in this portion wherever they are likely to be produced in case of failure of this part.

Let us assume a hinge at point – 11 in the beam and a hinge at section – 12.

Similarly, a hinge may also be considered at section – 7.

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M_{11} for beam = $12M_p$ (clockwise)
then M_{11} for column = $9M_p$ (counterclockwise)
M_{12} for column = $12M_p$ (counterclockwise)

Portion 11-12: Shear = $\dfrac{12M_p + 9M_p}{L/2}$ = $42\dfrac{M_p}{L}$ $\rightleftarrows$

For Overall Lateral Equilibrium of the Frame

Leftward shear in member 7-6:

$$+42\frac{M_p}{L} + 8\frac{M_p}{L} + 6\frac{M_p}{L} = 2P + 2P + P + P$$

$$= 36\frac{M_p}{L}$$

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Shear in 7-6 $= (36 - 42 - 8 - 6)\ \dfrac{M_p}{L}$

$$= -20\ \frac{M_p}{L} \quad \text{or} \quad 20\ \frac{M_p}{L}$$

Assuming $M_7 = 12M_p$ (clockwise, opposite to shear in member 7-6) and assuming that M_6 is clockwise.

$$M_6 + M_7 = 20\ \frac{M_p}{L} \times L/2$$

M_6 in column $= 2M_p$ (counterclockwise)

M_6 in beam $= 5M_p$ (clockwise)

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Portion 6-11:

$$\text{Shear without load} = 17\,\frac{M_p}{L}\ \downarrow\uparrow$$

$$\text{S.S. shear in portion 6-9} = 4P = 24\,\frac{M_p}{L}\ \uparrow\downarrow$$

$$\text{Total shear in portion 6-9} = (24\text{-}17)\,\frac{M_p}{L}$$

$$= 7\,\frac{M_p}{L}\ \uparrow\downarrow$$

$$\text{S.S. shear in portion 9-11} = 24\,\frac{M_p}{L}$$

$$\text{Total shear in portion 9-11} = (24+17)\,\frac{M_p}{L}$$

$$= 41\,\frac{M_p}{L}\ \downarrow\uparrow$$

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Assuming M_9 to be clockwise:

$$M_9 + 5M_p = -7\frac{M_p}{L} \times L/2$$

$$M_9 = 8.5M_p \text{ (counterclockwise)}$$

Hence, moment is not exceeded anywhere in the frame and the assume mechanism is the critical mechanism.

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b) Moment Balancing Method

In this method, a pseudo-moment distribution process is used.

This method does not apply continuity conditions at the joints and hence is approximate and different from the actual moment distribution method.

The following steps are used in this method:

i)Sign convention for moments given earlier is used.

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ii) Carryover factors are used to restore the equilibrium of the beam spans between the joints when end moment change during joint balancing. These represent the effect of a unit change in moment at one point in a span on the moments at other points.

Carryover factors are only applied on beams in case of analysis of frames.

Case 1: A unit positive increase in the left end moment with no change at the right end produces the following increment at the center:

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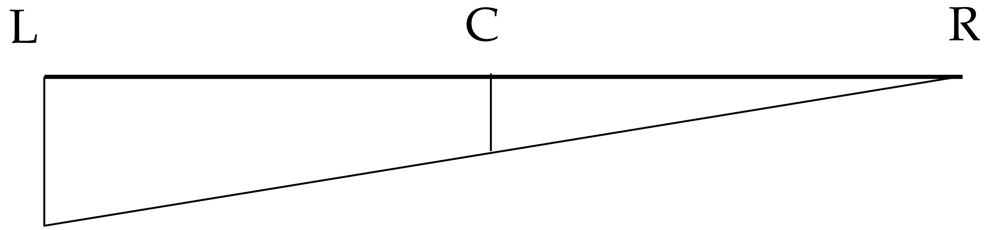
According to the already derived equation, we have,

$$M_C = M_L / 2 - M_R / 2 + M_s$$

$$\Delta M_C = \tfrac{1}{2}\, \Delta M_L$$

As $\Delta M_R = 0$ and $\Delta M_s = 0$

ΔM_L	ΔM_C	ΔM_R
+1	+1/2	0



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Case 2: Keeping the centerline moment constant, a unit change in moment at one end produces a unit change in moment at the opposite end of same sign.

According to the already derived equation, we have,

$$M_C = M_L / 2 - M_R / 2 + M_s$$

$\Delta M_L = \Delta M_R$ As $\Delta M_C = 0$ and $\Delta M_s = 0$

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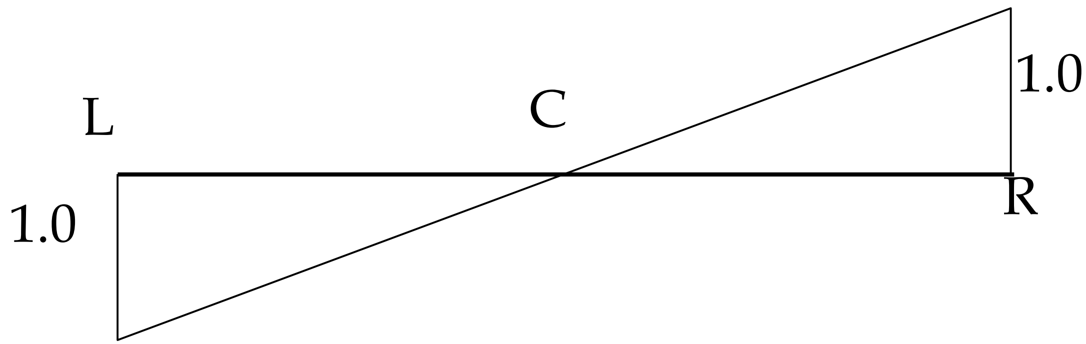

ΔM_L	ΔM_C	ΔM_R
+1	0	+1

Case 3: A unit positive increase in the right end moment with no change at the left end produces the following increment at the center:

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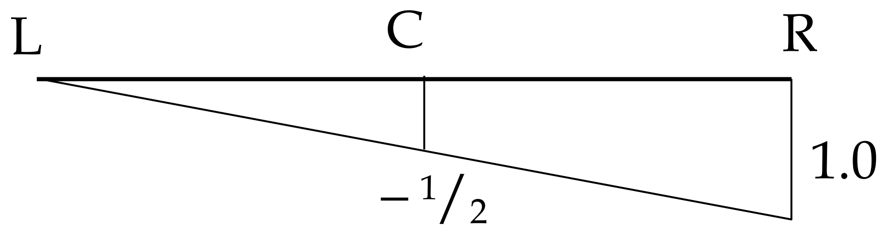
According to the already derived equation, we have,

$$M_C = M_L / 2 - M_R / 2 + M_s$$

$$\Delta M_C = -\tfrac{1}{2}\, \Delta M_R$$

As $\Delta M_L = 0$ and $\Delta M_s = 0$

ΔM_L	ΔM_C	ΔM_R
0	–1/2	+1



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iii) Selection of Starting Moments:

Moments at plastic hinge locations are taken equal to known Mp values. Moments at ends of a given indeterminate span may be selected equal in magnitude, keeping in mind the known end conditions.

If one end is hinge and other is any other type of support, moment at both sides is assumed equal to zero.

If both ends are continuous, a negative bending moment at each end equal to the simply supported moment of that span less than M_p may be assumed.

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The resulting central moment may be calculated as simply supported moment plus end moments.

iv) Moment Balancing:

The joint equilibrium is restored by applying opposite of unbalanced moment.

v) Carryover Operation:

The extra moment applied during moment balancing disturbs the equilibrium of the beam portions between the ends. This operation restores the equilibrium.

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vi) Column Adjustment:

Moment corrections are made to bring back the sway equilibrium.

More moment is given to that section which is already weak and near its full capacity.

Zero moment may be assigned to a section where hinge can not form.

Moment after adjustment at a column end must not be greater than moment at its other end.

This gives critical condition for the moment check.

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vii) Summation of Moments:

All the assumed, joint-balancing and carryover moments are added together to get the total moments.

viii) Drawing Bending Moment Diagram

Bending moment diagram is completed using the total moments obtained above.

Example: Solve the previous continuous beam example by moment distribution method.

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Simply supported moment at point-4

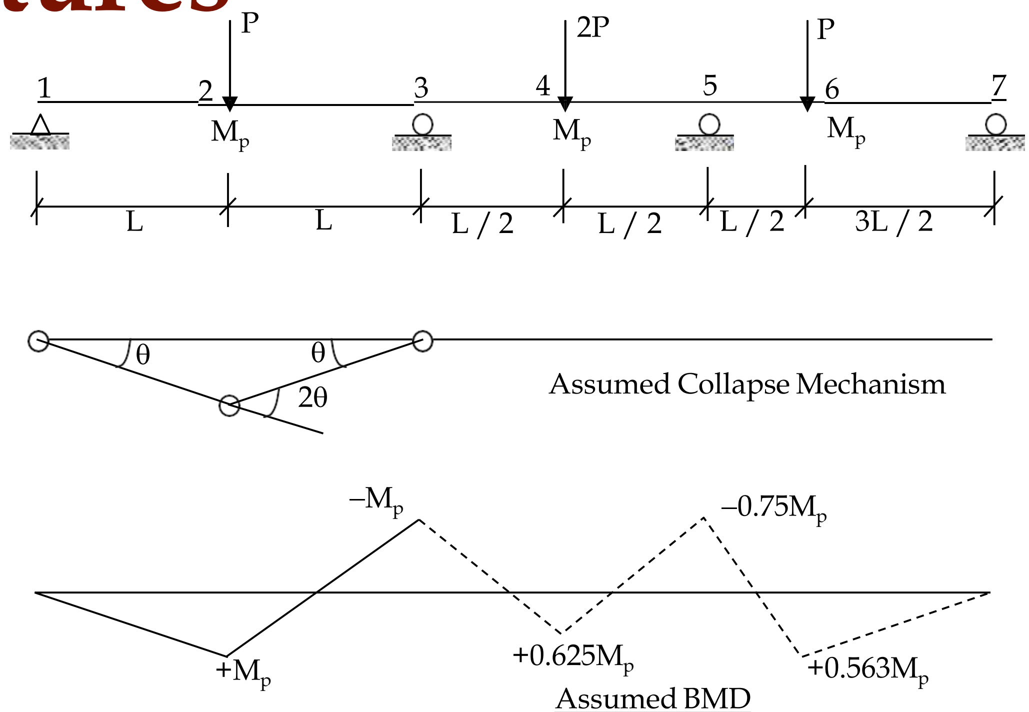
$$= \frac{2P \times L}{4} = \frac{2}{4} L \left(\frac{3M_p}{L} \right) = {}^3/_2\, M_p$$

Simply supported moment at point-4

$$= \frac{P \times \frac{L}{2} \times \frac{3L}{2}}{2L} = \frac{3}{8} LP = \frac{3}{8} L \left(\frac{3M_p}{L} \right)$$

$$= {}^9/_8\, M_p = 1.125\, M_p$$

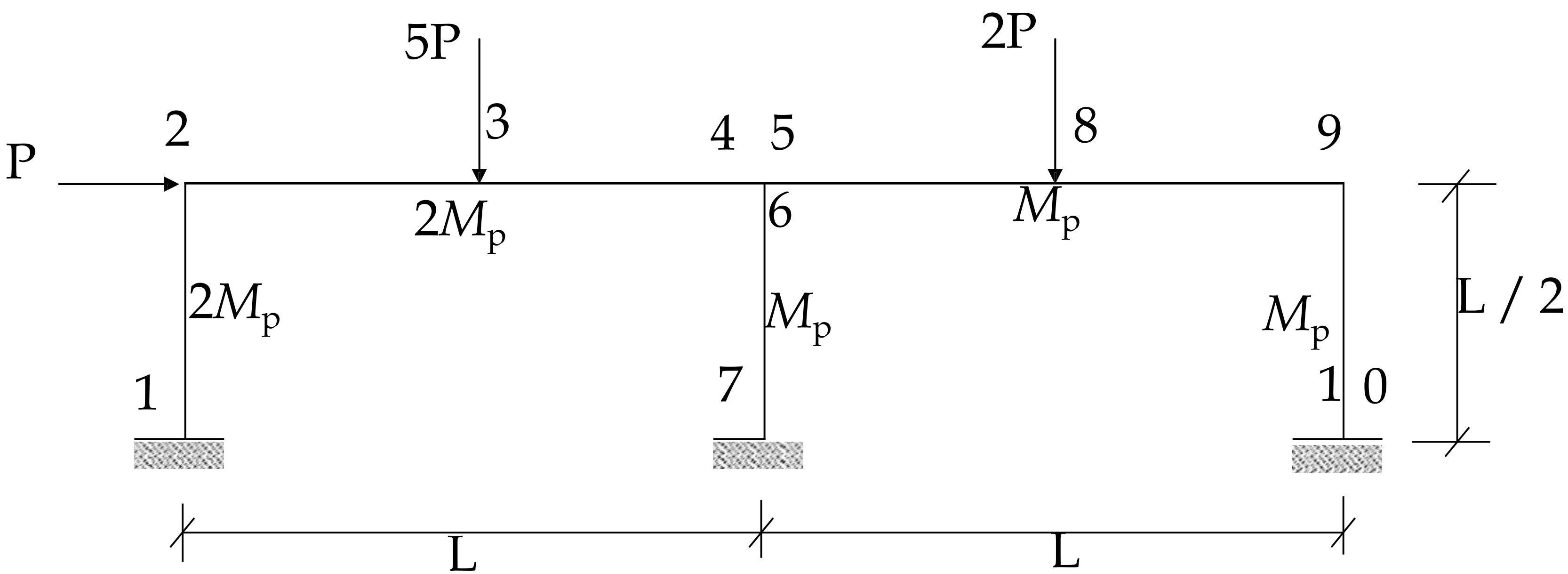
Steel Structures



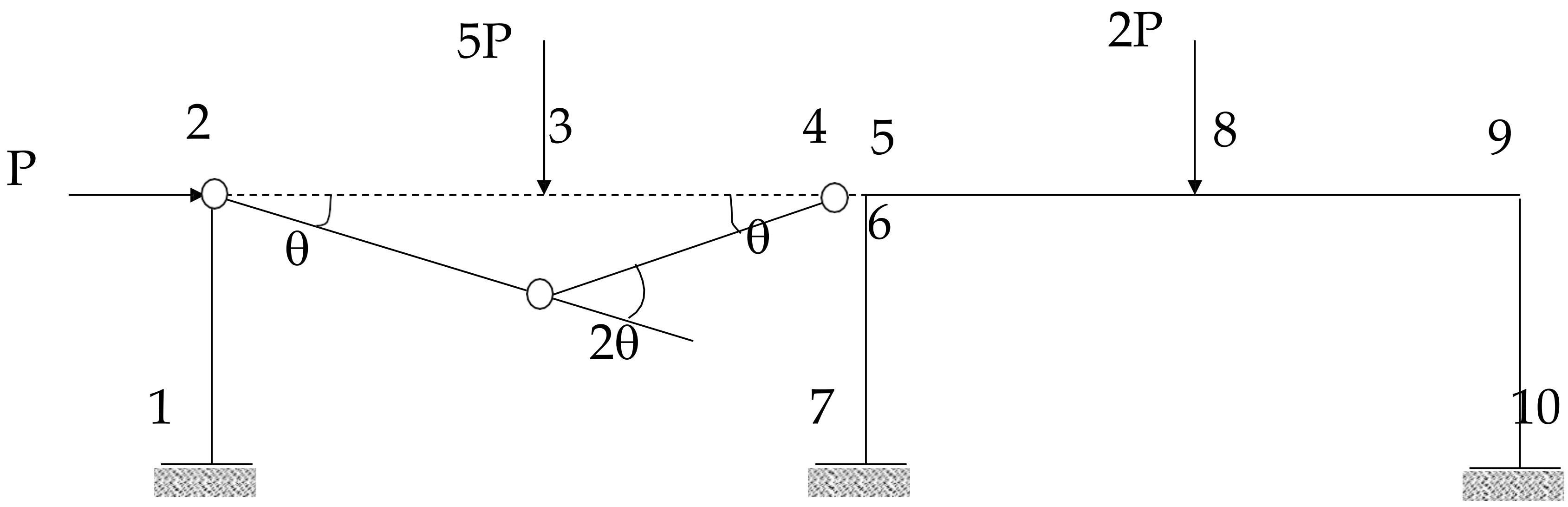
1. *Ms*	–			+1.5		1.125 0
2. Assumed moments	–	+1.0 +1.0	–0.75 +0.75	+0.75	0	1.125 0
3. Joint balancing	––	–– ––	–0.25 ––	0	–0.75	–– ––
4. Carryover	––	–– ––	0	–0.125 0	0	–0.563 ––
5. Total moments	––	+1.0 +1.0	–1.0 +0.625	+0.75	–0.75	+0.562 0

Steel Structures

Example



Steel Structures



Assumed Collapse Mechanism

Steel Structures

$$W_E = 5P \times \theta \times L / 2 = 2.5\, P\theta L$$

$$W_I = 2M_p(\theta + 2\theta + \theta) = 8\, M_p\theta$$

$$W_E = W_I \quad \Rightarrow \quad 2.5\, P\theta L = 8\, M_p\theta$$

$$P = 3.2\,\frac{M_p}{L} \qquad \text{or} \qquad M_p = \frac{PL}{3.2}$$

$$I = 6$$

$M = 3$ number of hinges in the assumed mechanism)

$$X = I - (M - 1)$$

$= 6 - (3 - 1) = 4$ (resulting structure is 4th degree indeterminate)

Steel Structures

M_s for portion 5-9 $= \dfrac{2P \times L}{4}$

$$= \frac{\left(\dfrac{3.2M_p}{L}\right) \times L}{2} = 1.6\,M_p$$

Assume $M_5 = M_{9\text{-}8} = 0.6\,M_p$

<u>Assumed Distribution of Sway Moments</u>

Clockwise moments are considered positive as before. As a first approximation, moment given to column 1-2 will be double of other columns because of heavy section.

Steel Structures

If $\quad M_1 = M_2 = M$

then $M_6 = M_7 = M_9 = M_{10} = M / 2$

The equation for sway becomes:

$$4M + PL / 2 = 0$$

$$\text{or} \quad M = -PL / 8$$

$$= -(3.2\, M_p / L) \times L / 8$$

$$= -0.4\, M_p$$

Steel Structures

Section	1	2		3	4	5	6	7	8	9		10
		2-1	2-3							9-8	9-10	
1. M_s									+1.6			
2. Assumed moments	–0.4	–0.4	–2.0	+2.0	+2.0	-0.6	–0.2	–0.2	+1.0	+0.6	–0.2	–0.2
3. Joint balancing		+2.4				–0.4	–0.8				–0.4	
4. Carryover									–0.2			
5. Column adjustment	0.0							–0.8				–0.4
6. Total / Final moments	–0.4	+2.0	–2.0	+2.0	+2.0	–1.0	–1.0	–1.0	+0.8	+0.6	–0.6	–0.6

Steel Structures

Joint Balancing

Joint 2: The moment 2-3 is fixed; the balancing moment of +2.4 can only be applied to the column.

Joints 4,5,6: The unbalanced moment of $-1.2M_p$ can only be given to joints 5 and 6. The moments for both the members become $-2M_p$ equal to their full capacity.

Joint 9: The unbalanced moment of $-0.4M_p$ may be given to column to get increase in moment.

Steel Structures

Carryover

Only one moment of $-0.4M_p$ is applied as balancing moment on the beam.

Changing moment 9-8 will create imbalance at joint 9. Hence, keeping moment at 9-8 constant, the carryover to the central moment is $-0.2M_p$.

Column Unbalance

Joint	2-1	6	9	
ΔM	+2.4	–0.8	–0.4	$\sum M = +1.2$

Steel Structures

The balancing moment of –1.2 is given to sections that are weaker to get critical conditions.

$\Delta M_1 = 0$ (very strong)

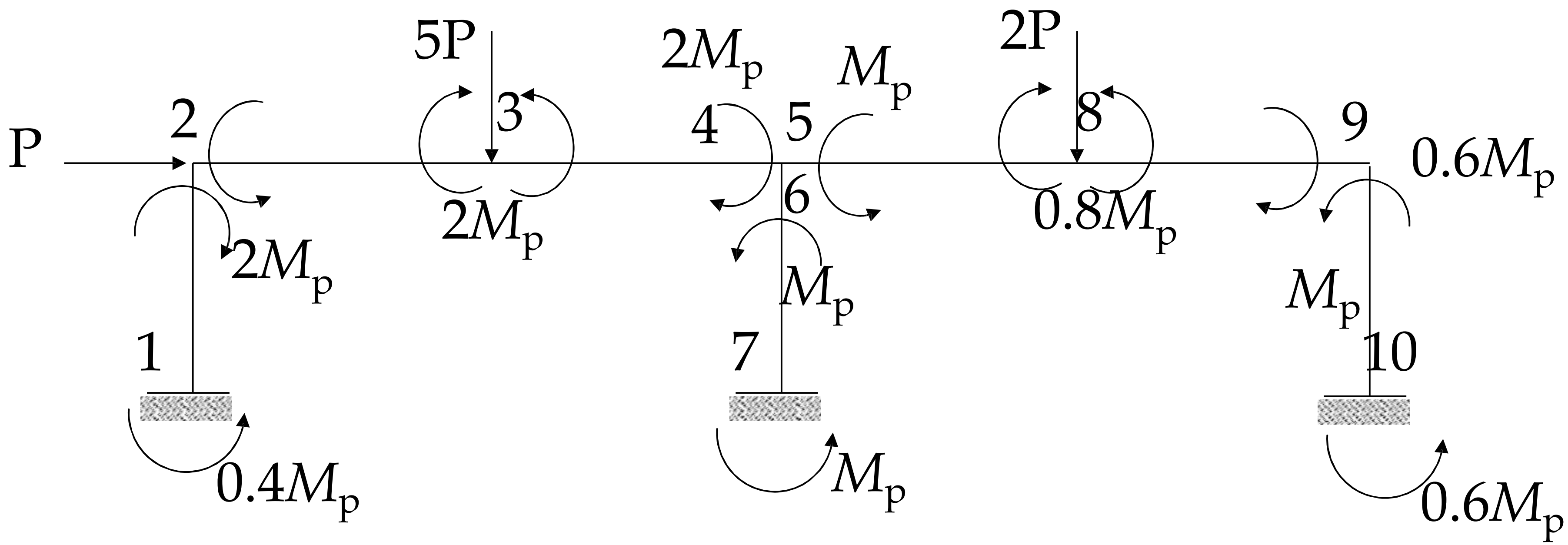
$\Delta M_7 = -0.8$ (hinge may be developed)

$\Delta M_{10} = -0.4$

$\sum \Delta M = -1.2$

Note: ΔM_7 and ΔM_{10} may be interchanged.

Steel Structures



Concluded