Steel Structures

M.Sc. Structural Engineering

SE-505

Lecture # 2

Design of Locally Unstable Compression Members

LOCALLY UNSTABLE MEMBERS IN COMPRESSION

INTRODUCTION

The design of locally unstable sections, at least with respect to overall buckling (local instability does not occur before the chances of overall buckling), is discussed earlier.

Sometimes, thin / slender elements are used in the compression members, which may carry substantial loads even after this local instability.

In fact, the thin plate cold-formed sections are always made such that the individual elements are not locally stable.

ELASTIC STABILITY OF PLATES

The buckling of a plate section, having a size of $b \times t$, depends on the equivalent slenderness ratio.

This equivalent slenderness ratio is equal to the width / thickness ratio denoted by λ, equal to b / t.

DIFFERENTIAL EQUATION FOR BENDING OF HOMOGENEOUS PLATES

The general forces acting a 3-d element are shown in Figure 11.1.

Following nomenclature is used for various force effects:

M_x = Bending moment per unit length on x-face.

M_y = Bending moment per unit length on y-face.

M_{xy} = Twisting moment per unit length on x-face.

M_x = Bending moment per unit length on x-face.

M_y = Bending moment per unit length on y-face.

M_{xy} = Twisting moment per unit length on x-face.

M_{yx} = Twisting moment per unit length on y-face.

Q_x = Shearing force per unit length in z-direction acting on x-face.

Q_y = Shearing force per unit length in z-direction acting on y-face.

q = Intensity of continuously distributed load in z-direction.

N_x = Normal force per unit length on x-face.

N_y = Normal force per unit length on y-face.

w = Deformation in z-direction load “q”.

$\frac{\partial w}{\partial x}$ = Slope in x-direction.

$\frac{\partial w}{\partial y}$ = Slope in y-direction.

$$\frac{\partial^2 w}{\partial x^2}$$ = Curvature in x-direction, proportional to moment M_x.

$$\frac{\partial^2 w}{\partial y^2}$$ = Curvature in x-direction, proportional to moment M_y.

$$\frac{\partial^2 w}{\partial x \partial y}$$ = Change of x-direction slope measured in y-direction or vice versa, showing torsional shear curvature proportional to torsional moments M_{xy} and M_{yx}.

$$\frac{\partial^3 w}{\partial x^3}$$ = First derivative of x-direction curvature with respect to x-axis (indicating rate of change of moment in x-direction), proportional to the shear force Q_x.

$$\frac{\partial^3 w}{\partial y^3}$$ = First derivative of y-direction curvature with respect to x-axis (indicating rate of change of moment in y-direction), proportional to the shear force Q_y.

$$\frac{\partial^4 w}{\partial x^4}$$ = Second derivative of x-direction curvature with respect to x-axis (indicating rate of change of shear force in x-direction).

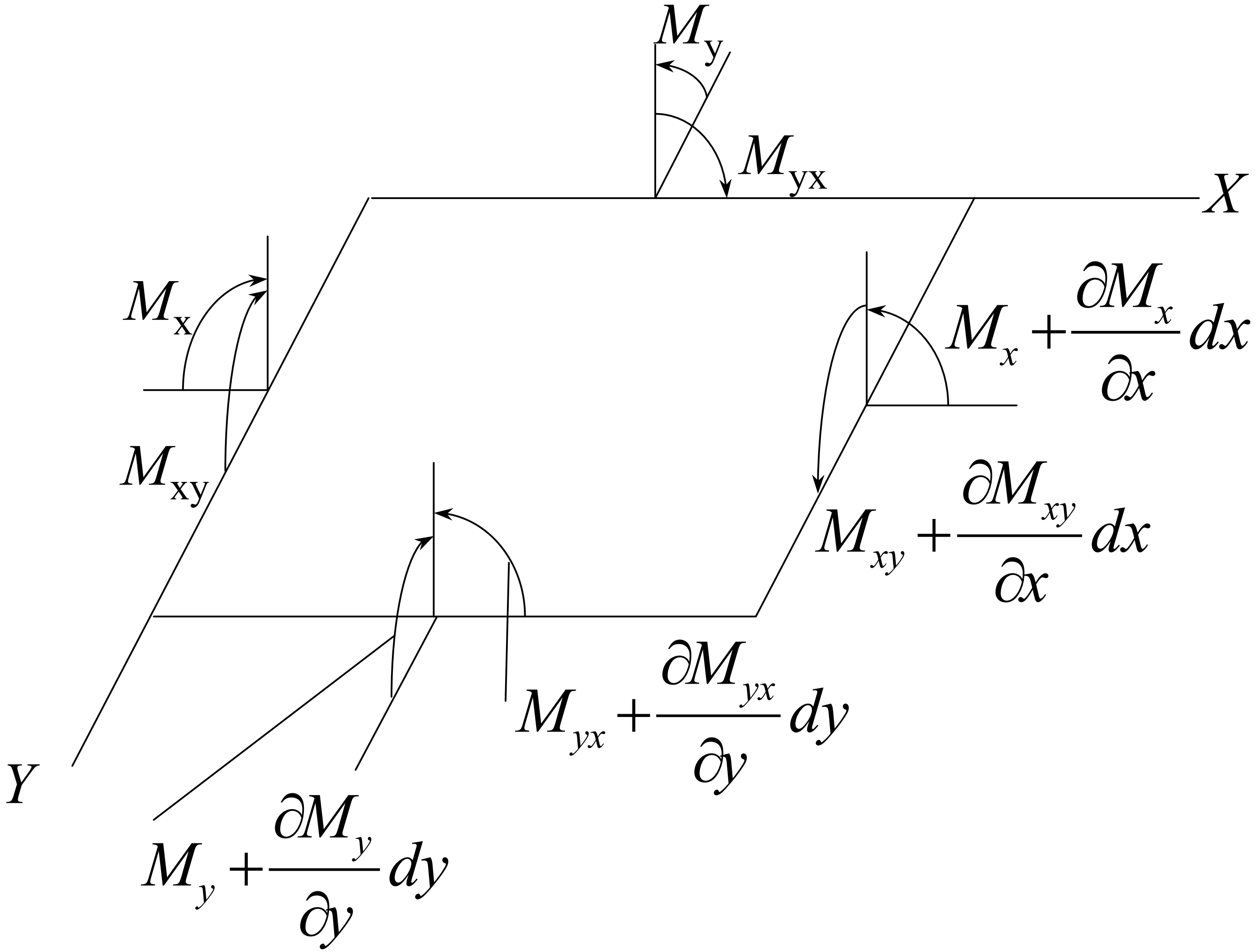
$$\frac{\partial^4 w}{\partial x^4}$$ = Load change along x-axis.

$$\frac{\partial^4 w}{\partial y^4}$$ = Second derivative of y-direction curvature with respect to y-axis (indicating rate of change of shear force in y-direction).

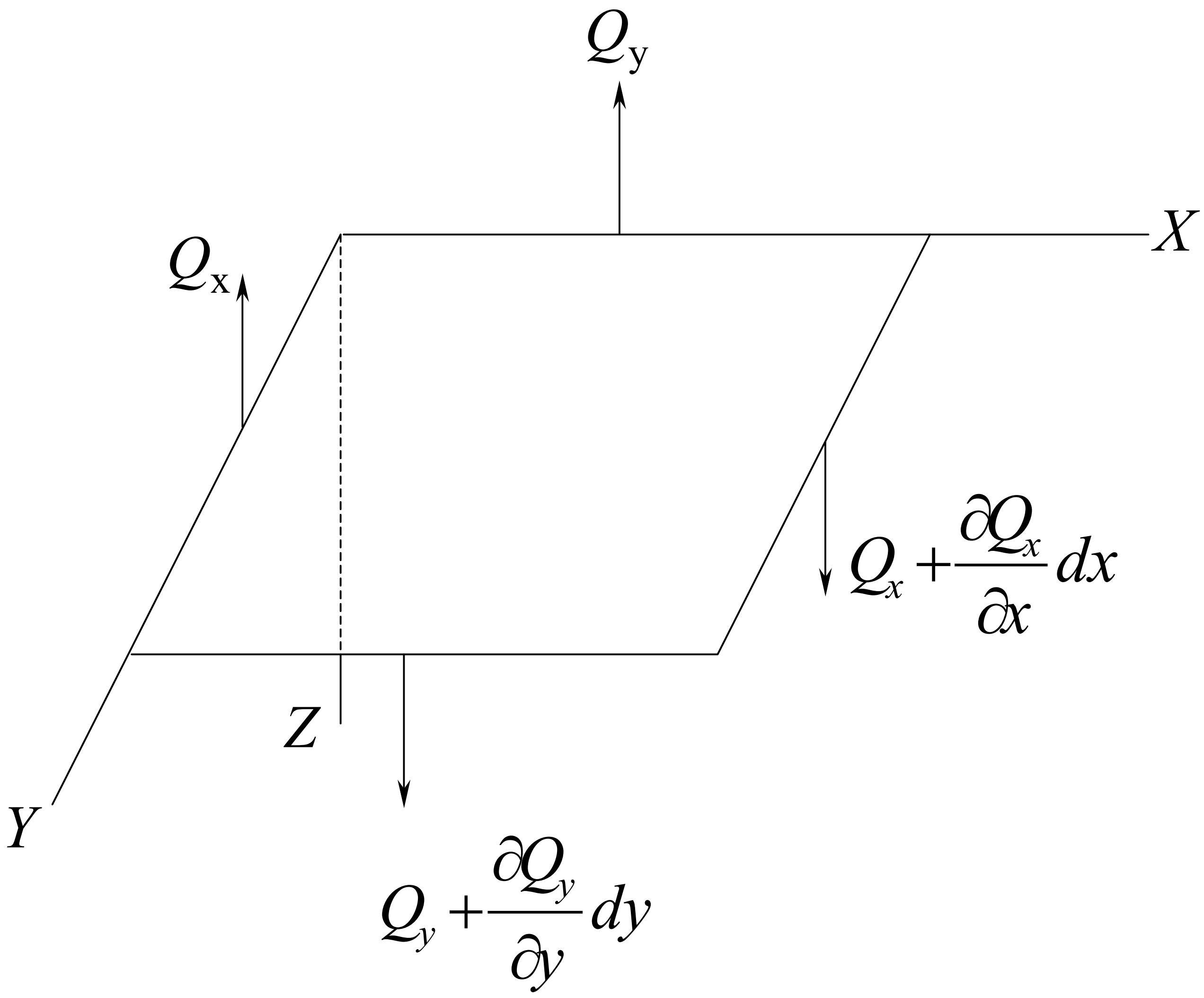
= Load change along y-axis.

$$\frac{\partial^4 w}{\partial x^2 \partial y^2}$$ = Second derivative of y-direction curvature with respect to y-axis (indicating rate of change of shear force in y-direction).

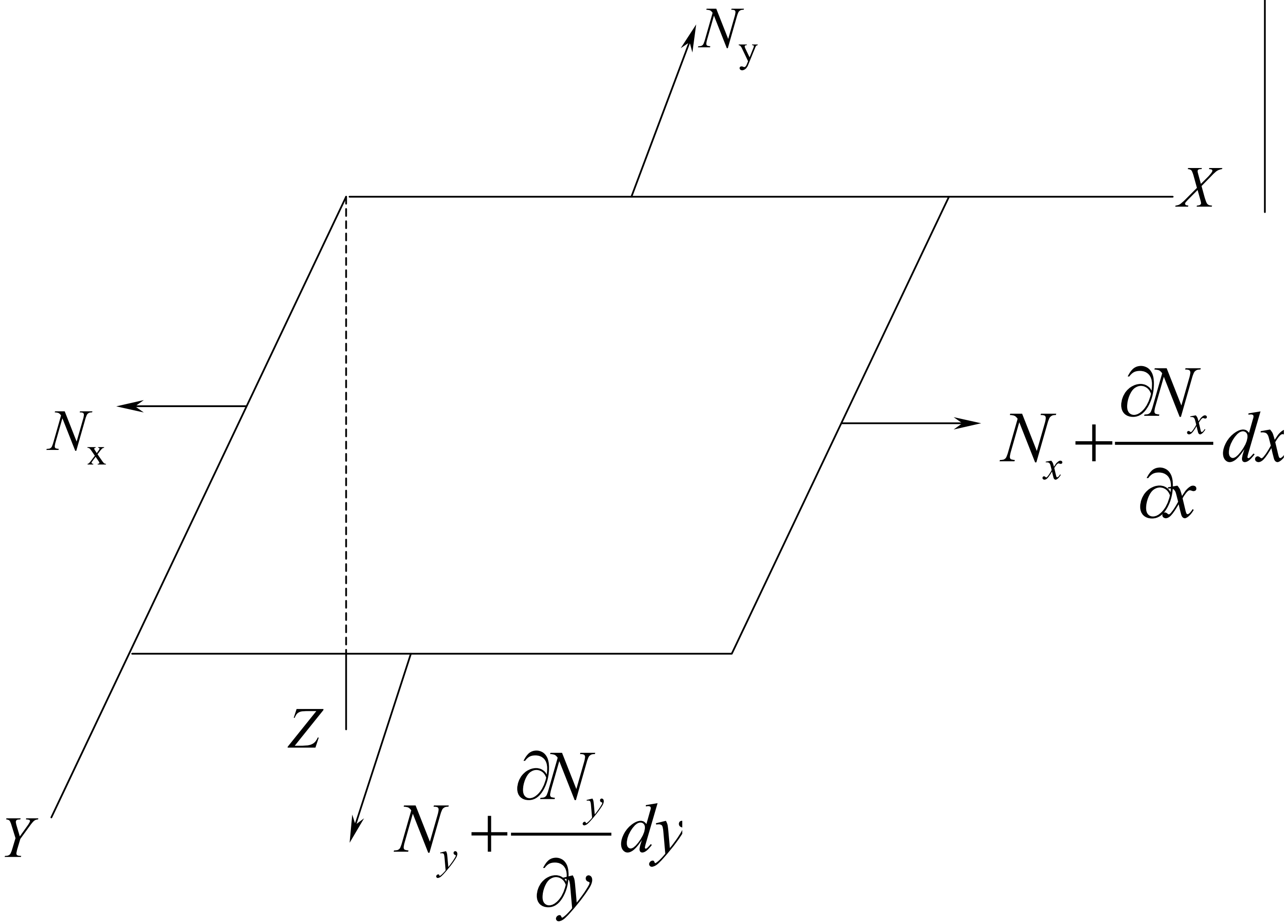
t = Thickness of plate.



M_y
M_{yx}
X
M_x
$M_x + \frac{\partial M_x}{\partial x} dx$
M_{xy}
$M_{xy} + \frac{\partial M_{xy}}{\partial x} dx$
$M_{yx} + \frac{\partial M_{yx}}{\partial y} dy$
Y
$M_y + \frac{\partial M_y}{\partial y} dy$



Q_y
X
Q_x
$Q_x + \frac{\partial Q_x}{\partial x} dx$
Z
Y
$Q_y + \frac{\partial Q_y}{\partial y} dy$



N_y
X
N_x
$N_x + \frac{\partial N_x}{\partial x} dx$
Z
Y
$N_y + \frac{\partial N_y}{\partial y} dy$

D = Flexural rigidity of the plate.

$$= \frac{Et^3}{12(1-\nu^2)}$$

The differential equation for bending of a plate element may be written by adding the load resistance by flexure and shear in the two directions (the related derivatives along with the constant of proportionality equal to the flexural rigidity of the plate, *D*) and equating it to the applied load.

The D-term may be taken on the right hand of the equation.

$$\frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q}{D}$$

After solving this equation for the deflection function *w*(*x*), analytically for some simple cases or numerically, the corresponding load effects may be calculated by using the following expressions:

$$M_x = -D\left(\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2}\right)$$

$$M_y = -D\left(\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2}\right)$$

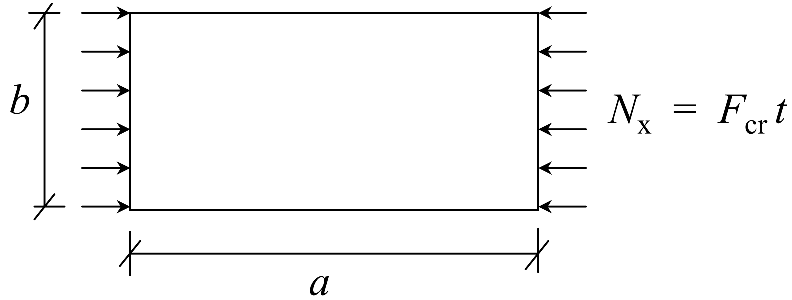
$$M_{xy} = M_{yx} = -D(1-\nu)\frac{\partial^2 w}{\partial x \partial y}$$

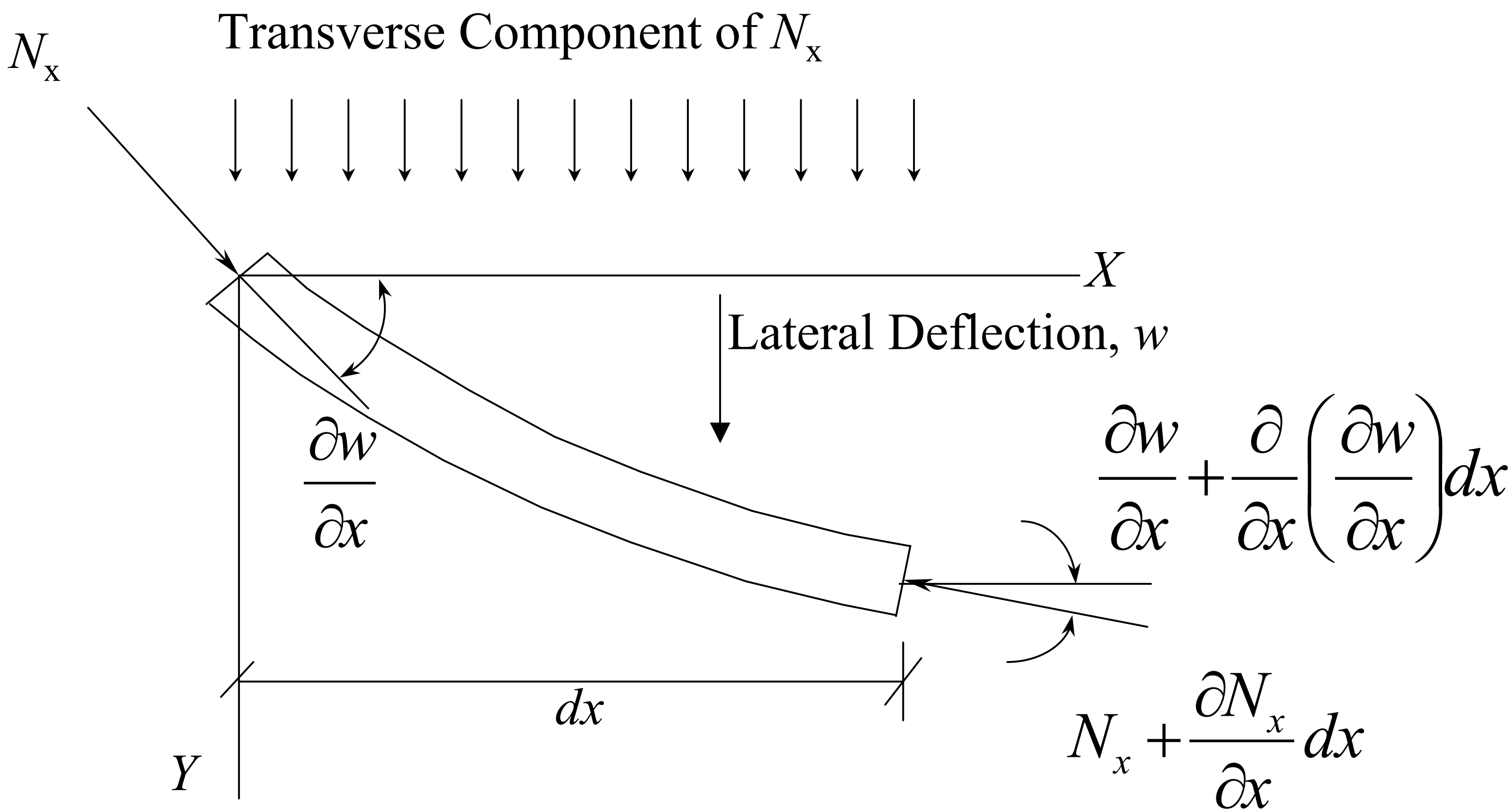
$$Q_x = \frac{\partial M_{yx}}{\partial y} + \frac{\partial M_x}{\partial x} = -D\frac{\partial}{\partial x}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)$$

$$Q_y = \frac{\partial M_y}{\partial y} - \frac{\partial M_{xy}}{\partial x} = -D \frac{\partial}{\partial y}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)$$

BUCKLING OF UNIFORMLY COMPRESSED PLATE

The buckling of a uniformly compressed plate may be studied by considering a thin plate, just at the stage of buckling, with free to rotate edges and subjected to compressive force uniformly distributed at the edges.


b
a
$N_x = F_{cr} t$


Transverse Component of N_x
N_x
X
Lateral Deflection, w
$\frac{\partial w}{\partial x}$
$\frac{\partial w}{\partial x} + \frac{\partial}{\partial x}\left(\frac{\partial w}{\partial x}\right)dx$
$N_x + \frac{\partial N_x}{\partial x} dx$
dx
Y

Considering a thin plate element of size $a \times b$, subjected to a critical buckling stress on the edges denoted by F_{cr}, the applied axial force per unit length on the edges will become $F_{cr}\, t = N_x$ in our general nomenclature for the plate element.

Now considering a differential element of size $dx \times dy$, a component of the force N_x after buckling acts as the transverse load q on the element.

The magnitude of this load may be estimated by considering the equilibrium of the element in the z-direction after buckling as follows:

$$N_x dy \frac{\partial w}{\partial x} - \left(N_x + \frac{\partial N_x}{\partial x} dx \right) dy \left(\frac{\partial w}{\partial x} + \frac{\partial^2 w}{\partial x^2} dx \right) = q\,dx\,dy$$

Where $N_x dy$ is the total inclined force, $\partial w/\partial x$ is the slope or tangent of the angle and $N_x dy \, \partial w/\partial x$ is the z-direction component of this load.

Remember that for small angles in radians, the angle itself, its sine and tangent are almost equal.

The second term in the equation is the same expression developed for the right end of the element.

After opening the brackets, the following is obtained:

$$N_x dy \frac{\partial w}{\partial x} - N_x \frac{\partial w}{\partial x} dy - N_x \frac{\partial^2 w}{\partial x^2} dxdy - \frac{\partial N_x}{\partial x} \frac{\partial w}{\partial x} dxdy - \frac{\partial N_x}{\partial x} \frac{\partial^2 w}{\partial x^2} dx^2 dy \quad = \quad qdxdy$$

$$-\left(N_x \frac{\partial^2 w}{\partial x^2} - \frac{\partial N_x}{\partial x} \frac{\partial w}{\partial x} - \frac{\partial N_x}{\partial x} dx \frac{\partial^2 w}{\partial x^2} \right) dxdy \quad = \quad qdxdy$$

Neglecting the product of infinitesimal terms, the expression simplifies to:

$$q \quad = \quad -N_x \frac{\partial^2 w}{\partial x^2}$$

Using his load, the D.E. of plate bending may provide all the required results. The form of this equation will become as under:

$$\frac{\partial^4 w}{\partial x^4} + 2\,\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = -\frac{N_x}{D}\frac{\partial^2 w}{\partial x^2}$$

This is a partial differential equation involving two variables. For its solution, a deflection function to be fitted may be assumed to be a product of a *x*-function $F_1(x)$ and a *y*-function $F_2(y)$.

$$w = F_1(x)\,F_2(y)$$

Further assuming that the buckling will yield a sinusoidal variation along the x-axis, following function may be tried:

$$F_1(x) = \sin(m \pi x / a),$$

where a is the length of the plate and m is an integer number.

This function satisfies the boundary conditions as shown below:

$$x = 0 \Rightarrow F_1(x) = 0 \Rightarrow w = 0 \quad \text{(BC \#1)}$$

$$\frac{\partial^2 w}{\partial x^2} = F_2(y)\left(-\sin\frac{m\pi x}{a}\right)\frac{m^2\pi^2}{a^2}$$

$$x = 0 \Rightarrow \quad \frac{\partial^2 w}{\partial x^2} = 0 \quad \text{(BC \#2)}$$

(Moment at edge is zero)

$$x = a \Rightarrow F_1(x) = m\pi \Rightarrow w = 0 \quad \text{(BC \#3)}$$

$$x = a \Rightarrow \frac{\partial^2 w}{\partial x^2} = 0 \text{ (as above)} \quad \text{(BC \#4)}$$

Substituting the trial function into the governing differential equation, the following result is obtained:

$$\left(\frac{m\pi}{a}\right)^4 \sin\frac{m\pi x}{a} F_2(y) - 2\left(\frac{m\pi}{a}\right)^2 \sin\frac{m\pi x}{a}\frac{d^2}{dy^2}F_2(y) + \sin\frac{m\pi x}{a}\frac{d^4}{dy^4}F_2(y)$$

$$= \frac{N_x}{D}\left(\frac{m\pi}{a}\right)^2 F_2(y)\sin\frac{m\pi x}{a}$$

$$\frac{d^4}{dy^4}F_2(y) - 2\left(\frac{m\pi}{a}\right)^2 \frac{d^2}{dy^2}F_2(y) + \left[\left(\frac{m\pi}{a}\right)^4 - \frac{N_x}{D}\left(\frac{m\pi}{a}\right)^2\right]F_2(y) = 0$$

This is an ordinary fourth order homogeneous differential equation in terms of only one variable, that is, *y*. The solution of this equation is:

$$F_2(y) = C_1 \sinh \alpha y + C_2 \cosh \alpha y + C_3 \sinh \beta y + C_4 \cosh \beta y$$

where

$$\alpha = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \sqrt{\frac{N_x}{D}\left(\frac{m\pi}{a}\right)^2}}$$

and

$$\beta = \sqrt{-\left(\frac{m\pi}{a}\right)^2 + \sqrt{\frac{N_x}{D}\left(\frac{m\pi}{a}\right)^2}}$$

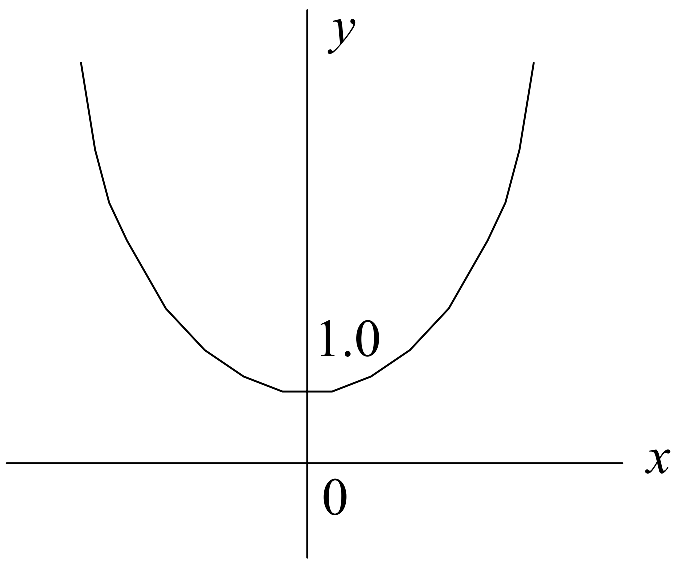
The complete solution for the plate deflection function is:

$$w = [\sin(m\pi x / a)](C_1 \sinh\alpha y + C_2 \cosh\alpha y + C_3 \sinh\beta y + C_4 \cosh\beta y)$$

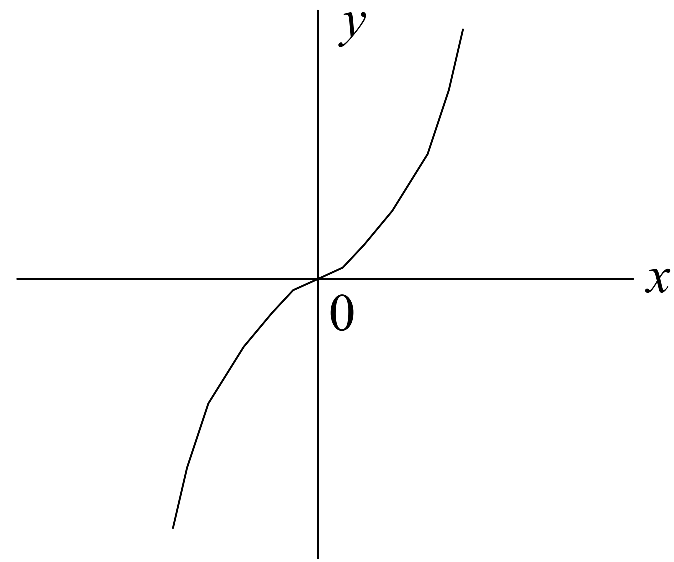
This function must satisfy all the boundary conditions.

However, sine and sinh functions are not symmetrical about x = 0 line.

For identical support conditions along the two edges parallel to the direction of loading ($y = \pm b / 2$), the odd function coefficients must be zero.



$$y = \cosh x = (e^{x} + e^{-x}) / 2$$



$$y = \sinh x = (e^{x} - e^{-x}) / 2$$

$C_1 = C_3 = 0$

$w = (C_2 \cosh \alpha y + C_4 \cosh \beta y)$
$\quad \times \sin (m \pi x / a)$

$$\frac{\partial w}{\partial y} = \left[C_2 \alpha \sinh \alpha y + C_4 \beta \sin \beta y \right] \sin \frac{m \pi x}{a}$$

$$\frac{\partial^2 w}{\partial y^2} = \left[C_2 \alpha^2 \cosh \alpha y - C_4 \beta^2 \cos \beta y \right] \sin \frac{m \pi x}{a}$$

The boundary conditions are that the edges, $y = \pm b / 2$, are simply supported.

$w = 0$ and $\dfrac{\partial^2 w}{\partial y^2} = 0$

Which give the following results:

$$C_2 \cosh \alpha\frac{b}{2} + C_4 \cosh\beta\frac{b}{2} = 0$$

$$C_2\alpha^2 \cosh\alpha\frac{b}{2} - C_4\beta^2 \cos\beta\frac{b}{2} = 0$$

$$\begin{bmatrix} \cosh\alpha\frac{b}{2} & \cos\beta\frac{b}{2} \\ \alpha^2\cosh\alpha\frac{b}{2} & -\beta^2\cos\beta\frac{b}{2} \end{bmatrix} \begin{bmatrix} C_2 \\ C_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

For a non-trivial solution, C2 and C4 must be non-zero and the determinant of the coefficients matrix must be zero:

$$-\left(\cosh\alpha\frac{b}{2}\right)\left(\beta^2\cos\beta\frac{b}{2}\right)-\left(\alpha^2\cosh\alpha\frac{b}{2}\right)\left(\cos\beta\frac{b}{2}\right) = 0$$

$$\left(\alpha^2+\beta^2\right)\cosh\alpha\frac{b}{2}\cos\beta\frac{b}{2} = 0$$

The condition that $\alpha_2 = -\beta_2$ *represents a trivial solution giving* N_x *= 0 and cosh*($\alpha b/2$) can not be zero (it is always greater than or equal to 1.0).

The only way in which the above equation may be satisfied for a real problem is the following:

$$\cos \beta \frac{b}{2} = 0$$

or $$\beta \frac{b}{2} = \pi / 2,\ 3\pi / 2,\ 5\pi / 2, \text{ etc.}$$

The first mode of buckling along the width (quarter wave in *b* / 2 distance) is usually the most critical, which is represented by the least values out of the above solutions.

$$\beta \frac{b}{2} = \pi / 2$$

$$\frac{b}{2}\sqrt{-\left(\frac{m\pi}{a}\right)^2 + \sqrt{\frac{N_x}{D}\left(\frac{m\pi}{a}\right)^2}} = \pi / 2$$

$$-\left(\frac{m\pi}{a}\right)^2 + \sqrt{\frac{N_x}{D}\left(\frac{m\pi}{a}\right)^2} = \pi^2 / b^2$$

$$\frac{N_x}{D}\left(\frac{m\pi}{a}\right)^2 = \left[\frac{\pi^2}{b^2} + \left(\frac{m\pi}{a}\right)^2\right]^2$$

$$\frac{N_x}{t} = \frac{D}{t}\left[\frac{\pi^2 a}{b^2 m\pi} + \frac{m\pi}{a}\right]^2$$

$$\text{or} \quad F_{\mathrm{cr}} = \frac{D\pi^2}{b^2 t}\left[\frac{1}{m}\frac{a}{b} + m\frac{b}{a}\right]^2$$

The term in the brackets is called the plate-buckling coefficient (*k*), *m* is the number of half sine waves in the buckled shape along the x-axis (or along the length) and *D* is the plate rigidity defined earlier.

$$k = \left[\frac{1}{m}\frac{a}{b} + m\frac{b}{a}\right]^2 \qquad D = \frac{Et^3}{12(1-\nu^2)}$$

The expression for F_{cr} becomes:

$$F_{cr} = \frac{\pi^2 E}{12(1-\nu^2)(b/t)^2}k$$

The buckling coefficient depends on the type of stress (uniform compression or otherwise), edge conditions (value of *m* will be different), and the aspect ratio *a* / *b*.

For a larger value of *a* / *b* and if *m* also becomes larger, the *k*-curve becomes flatter and approaches a constant value of 4.0. For example for *a* / *b* of 15, the value of *k* becomes:

$$k = \left[\frac{15}{m} + \frac{m}{15}\right]^2$$

$m = 1 \quad \Rightarrow \quad k = 227$

$m = 4 \quad \Rightarrow \quad k = 16.1$

$m = 1 \quad \Rightarrow \quad k = 5.8$

$m = 1 \quad \Rightarrow \quad k = 4.02$

It is to be noted that the less value of k gives less buckling strength and is more critical.

Hence for simply supported ends, a value of 4.0 is taken.

For the other end conditions, the critical values are listed below:

k_{min} value for two edges simply supported	=	4.00
k_{min} value for two edges fixed	=	6.97
k_{min} value for one edge fixed and other simply supported	=	5.42
k_{min} value for one edge fixed and other free	=	1.277
k_{min} value for one edge free and other simply supported	=	0.425

The form of equation for F_{cr} may be modified as under in order to study the factors affecting the buckling strength:

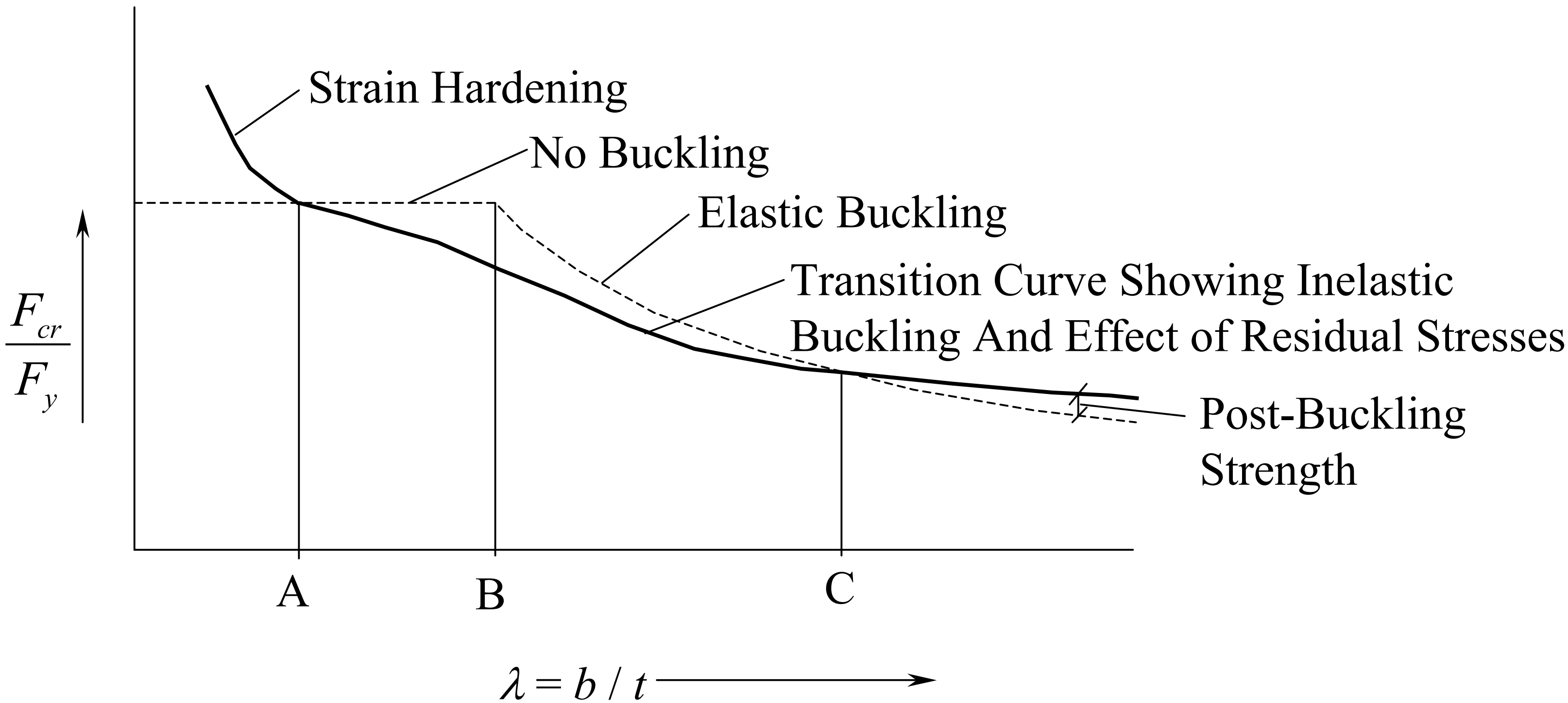
$$\frac{F_{cr}}{F_y} = \frac{\pi^2 Ek}{12(1-\nu^2)(b/t)^2} = \frac{1}{\lambda_c^2}$$

The square root of the ratio of yield strength to the elastic critical buckling strength may be denoted by the parameter λ_c, *while the b / t* ratio may be denoted by the parameter λ.

The Poisson's ratio for steel may be taken equal to 0.3 to simplify the above equation to the following:

$$\lambda_c = 1.052\sqrt{\frac{F_y}{Ek}} \times \lambda$$

A graph between the parameter λ (b / t ratio) and the ratio F_{cr} / F_y is shown in Figure. This graph shows distinct phases of strength under the action of edge compression as described below:



a. For very low values of λ, the strength becomes higher than F_y *due to strain hardening, without any buckling.*

In such cases, post-buckling strength is absent but the entire plate reaches strain hardening after undergoing all the yielding.

Hence, F_{cr} / F_y may become greater than unity.

The value λ_o is an equivalent elastic value of λ_c at which chances of inelastic buckling just start corresponding to the yielding stress.

Some typical values of λ_o are as under:

λ_o = 0.455 for long hinged flanges.
λ_o = 0.461 for fixed flanges.
λ_o = 0.588 for hinged webs.
λ_o = 0.579 for fixed webs.

An average value of 0.5 may be considered for the general discussion.

b. The value of λ_c is equal to one at the point where no elastic buckling occurs up to F_y stress.

c. Inelastic buckling occurs for values of λ_c less than approximately1.45.

d. The point B indicates a situation where the elastic buckling formula gives strength equal to F_y.

e. Elastic buckling according to the derived formula when the value of λ_c is greater than or equal to 1.45.

f. Post buckling strength with stress redistribution and large deformations results after λ_c equal to 1.45.

Point A

F_{cr} / F_y = 1.0 according to the inelastic buckling formula

Corresponding λ_c *for elastic formula* $\approx$ *0.5*

λ for elastic λ_c *of 0.5* $\approx 0.475\sqrt{\frac{Ek}{F_y}}$

Point C

$\lambda_c \approx 1.45$

$F_{cr} / F_y = 0.476$ according to the elastic buckling formula

$\lambda \approx 1.378\sqrt{\frac{Ek}{F_y}}$ using the elastic formula

Slope of straight line between A and C = $0.58\sqrt{\frac{F_y}{Ek}}$

Point B

λ_c *on the elastic curve = 1.00*

$F_{cr} / F_y = 1.00$ on the elastic curve

$\lambda \approx 0.951\sqrt{\frac{Ek}{F_y}}$ using the elastic formula

Corresponding value of F_{cr} / F_y calculated using the inelastic straight line = 0.724

The values of the factor λ for the three points are listed below for different critical values of *k*-factor:

For k = 0.425 For Overhanging Flanges

$$0.475\sqrt{\frac{Ek}{F_y}} = 0.310\sqrt{\frac{E}{F_y}} \quad : \quad 0.951\sqrt{\frac{Ek}{F_y}} = 0.620\sqrt{\frac{E}{F_y}} \quad :$$

$$1.378\sqrt{\frac{Ek}{F_y}} = 0.898\sqrt{\frac{E}{F_y}}$$

AISC Table B4.1:

Flexure in flanges of rolled I-shaped sections:

$$\lambda_p = 0.38\sqrt{\frac{E}{F_y}}$$

Uniform compression in flanges of rolled I-shaped sections:

$$\lambda_r = 0.56\sqrt{\frac{E}{F_y}}$$

For k = 4.0 For Stiffened Webs

$$0.475\sqrt{\frac{Ek}{F_y}} = 0.95\sqrt{\frac{E}{F_y}} \; : \; 0.951\sqrt{\frac{Ek}{F_y}} = 1.902\sqrt{\frac{E}{F_y}} \; :$$

$$1.378\sqrt{\frac{Ek}{F_y}} = 2.756\sqrt{\frac{E}{F_y}}$$

AISC Table B4.1:

Flexure in webs of doubly symmetric I-shaped sections:

$$\lambda_p = 3.76\sqrt{\frac{E}{F_y}}$$

(Half of the web is in compression and that compression also varies along the member.)

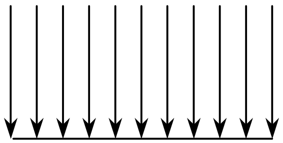
Uniform compression in webs of doubly symmetric I-shaped sections:

$$\lambda_r = 1.49\sqrt{\frac{E}{F_y}}$$

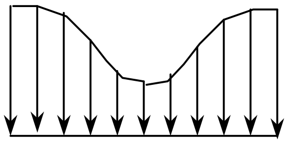
Any plate with no residual stresses develops uniform axial stresses up to the stage when the portions away from the side supports buckles in out-of-plane direction (Figure).

After buckling, the stresses become non-uniform and post-buckling strength is available near the relatively stable ends.

The post-buckling strength becomes larger as the width-to-thickness ratio is increased.



Stress Distribution
Before Buckling



Stress Distribution
After Buckling

The AISC values are in general excessively conservative because of the presence of residual stresses and imperfections.

For design, the local buckling of a column component must be prevented if it occurs before achieving full strength of the column based on its overall slenderness ratio KL / r.

This means that the acceptable b / t ratios vary depending on the overall slenderness ratio of the column.

$$F_{\text{cr component}} \geq F_{\text{cr overall column}}$$

Steel Structures

Design of Compression Members With Some Parts Locally Unstable

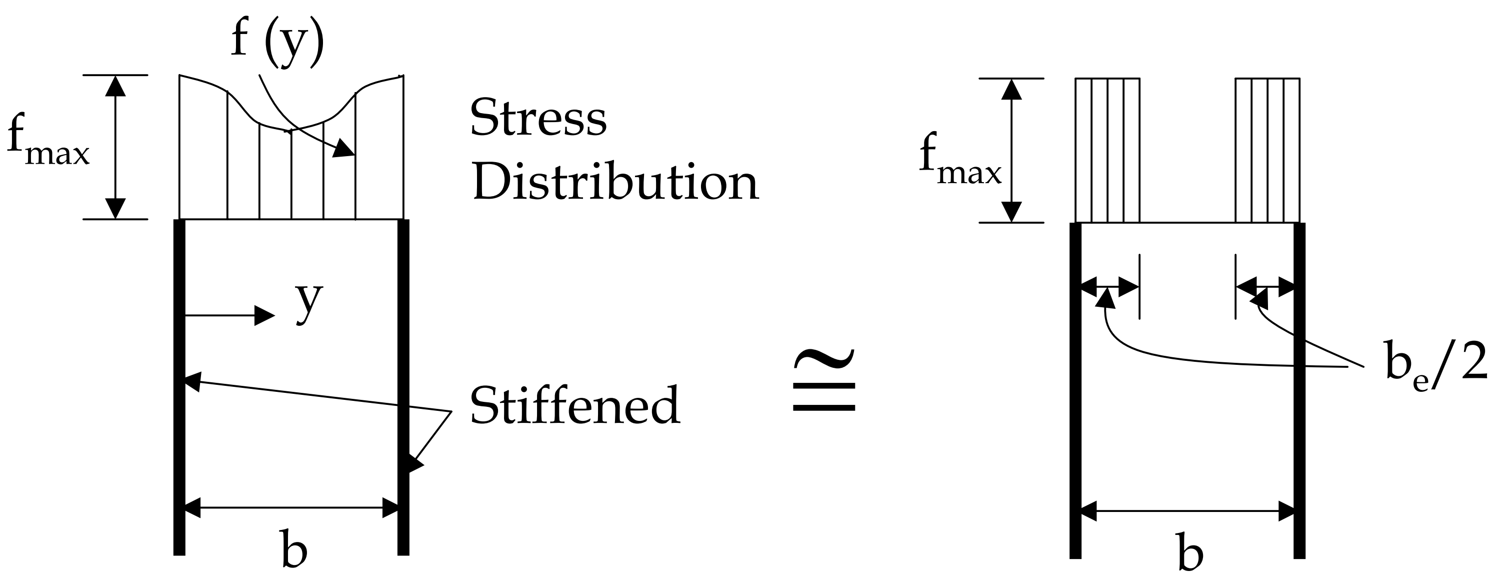
AISC 2005 – E7

- Plate elements in compression, either "stiffened" or "unstiffened" have post buckling strength.

- Stiffened elements have large post buckling strength while unstiffened elements have little.

Steel Structures

Post Buckling Strength



Stiffened Elements

Plate elements under axial compression, showing actual stress distribution and an equivalent system

Steel Structures

Post Buckling Strength (contd…)

Nominal strength of a stiffened elements

$$P_n = t\int_0^b f(y)dy$$

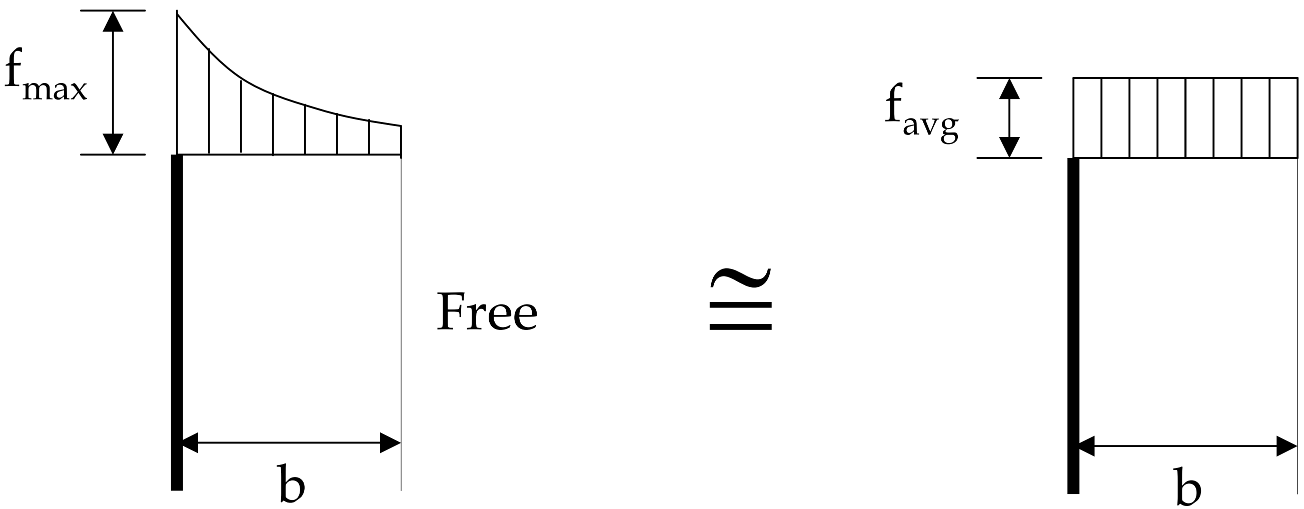
$$P_n = t \times b_e \times f_{\max}$$

$$P_n = A_{eff} \times f_{\max}$$

b_e = Effective width over which the maximum stress may be considered uniform and still gives almost correct answer.

Steel Structures

Post Buckling Strength (contd…)



Unstiffened Elements

Plate elements under axial compression, showing actual stress distribution and an equivalent system

Steel Structures

Post Buckling Strength (contd…)

Nominal strength of an unstiffened element

$$P_n = t \times b \times f_{avg}$$

$$\boxed{P_n = A_g \times f_{avg}}$$

Unstiffened elements have less post-buckling strength. They may be idealized as not buckled but subjected to a reduced equivalent stress.

Steel Structures

Post Buckling Strength (contd…)

Nominal strength of the member having both stiffened and unstiffened elements (W-section)

$$P_n = f_{avg} \times A_{eff}$$

$$P_n = \frac{f_{avg}}{f_{\max}} \times f_{\max} \times \frac{A_{eff}}{A_g} \times A_g$$

$$P_n = Q_s \times Q_a \times f_{\max} \times A_g$$

$$P_n = Qf_{\max} A_g = F_{cr} A_g$$

A compression member consisting of both stiffened and unstiffened elements may be treated as unstiffened for establishing the stress f_{avg}, while effective area is determined after deducting the ineffective area out of the stiffened elements.

Steel Structures

Post Buckling Strength (contd…)

Q = Full reduction factor for slender compression elements, 1.0 for members with compact and non-compact elements

Q_s = Reduction factor for slender unstiffened elements, 1.0 if no slender unstiffened element is present

Q_a = Reduction factor for slender stiffened elements , 1.0 if no slender stiffened element is present

Steel Structures

$$F_e = \frac{\pi^2 E}{(KL/r)^2}$$

Critical/Ultimate Compressive Strength, $\phi_c F_{cr}$

For $\frac{KL}{r} \le 4.71\sqrt{\frac{E}{QF_y}}$ or $F_e \ge 0.44QF_y$

$$F_{cr} = \left(0.658^{\frac{QF_y}{F_e}}\right) QF_y$$

For $\frac{KL}{r} > 4.71\sqrt{\frac{E}{QF_y}}$ or $F_e < 0.44QF_y$

$$F_{cr} = 0.877Fe$$

Steel Structures

Reduction Factor Q_s for Unstiffened Elements

For columns

$$Q_s = \frac{F_{cr,plate}}{F_{cr,column}} \geq \frac{F_{cr,plate}}{F_y}$$

For compression flanges of beams

$$Q_s = \frac{F_{cr,plate}}{F_{cr,beam..flange}} \geq \frac{F_{cr,plate}}{F_y}$$

Steel Structures

Reduction Factor Q_s for Unstiffened Elements (contd…)

a) For single angles and double angles with separators

For $\frac{b}{t} \leq 0.45\sqrt{\frac{E}{F_y}}$ $\quad Q_s = 1.0$

For $0.45\sqrt{\frac{E}{F_y}} < \frac{b}{t} < 0.91\sqrt{\frac{E}{F_y}}$ $\quad Q_s = 1.340 - 0.76\left(\frac{b}{t}\right)\sqrt{\frac{F_y}{E}}$

$12.8 < b/t < 25.8$ for A-36 steel

For $\frac{b}{t} \geq 0.91\sqrt{\frac{E}{F_y}}$ $\quad Q_s = \frac{0.53E}{F_y\left(b/t\right)^2}$

$b/t \geq 25.8$ for A-36 steel

Steel Structures

Reduction Factor Q_s for Unstiffened Elements (contd…)

b) For flanges, angles and plates projecting from rolled beams or columns

For $\frac{b}{t} \leq 0.56\sqrt{\frac{E}{F_y}}$ $\quad Q_s = 1.0$

For $0.56\sqrt{\frac{E}{F_y}} < \frac{b}{t} < 1.03\sqrt{\frac{E}{F_y}}$ $\quad Q_s = 1.415 - 0.74\left(\frac{b}{t}\right)\sqrt{\frac{F_y}{E}}$

$15.8 < b\,/\,t < 29.1$ for A-36 steel

For $\frac{b}{t} \geq 1.03\sqrt{\frac{E}{F_y}}$ $\quad Q_s = \frac{0.69E}{F_y\left(b/t\right)^2}$

$b\,/\,t \geq 29.1$ for A-36 steel

Steel Structures

Reduction Factor Q_s for Unstiffened Elements (contd…)

c) For flanges, angles and plates projecting from built-up columns or other compression members

For $\frac{b}{t} \le 0.45\sqrt{\frac{Ek_c}{F_y}}$ $\quad Q_s = 1.0$

For $0.64\sqrt{\frac{Ek_c}{F_y}} < \frac{b}{t} < 1.17\sqrt{\frac{Ek_c}{F_y}}$ $\quad Q_s = 1.415 - 0.65\left(\frac{b}{t}\right)\sqrt{\frac{F_y}{Ek_c}}$

For $\frac{b}{t} \ge 1.17\sqrt{\frac{Ek_c}{F_y}}$ $\quad Q_s = 0.90\frac{Ek_c}{F_y\left(b/t\right)^2}$

$$k_c = \frac{4}{\sqrt{h/t_w}} \qquad 0.35 \le k_c \le 0.76$$

Steel Structures

Reduction Factor Q_s for Unstiffened Elements (contd…)

d) For Stem of Tees

For $\frac{d}{t} \leq 0.75\sqrt{\frac{E}{F_y}}$ $\quad Q_s = 1.0$

For $0.75\sqrt{\frac{E}{F_y}} < \frac{d}{t} < 1.03\sqrt{\frac{E}{F_y}} \Rightarrow$

$$Q_s = 1.908 - 1.22\left(\frac{d}{t}\right)\sqrt{\frac{F_y}{E}}$$

For $\frac{d}{t} \geq 1.03\sqrt{\frac{E}{F_y}} \Rightarrow$

$$Q_s = \frac{0.69E}{F_y\left(\frac{d}{t}\right)^2}$$

d = the full nominal depth of tee

Steel Structures

Reduction Factor Q_a for Stiffened Elements (contd…)

$$Q_a = \frac{A_{eff}}{A_g}$$

a) For flanges of square and rectangular sections of uniform thickness

For $\frac{b}{t} \geq 1.04\sqrt{\frac{E}{f}} \Rightarrow$ $\boxed{b_e = 1.92\,t\sqrt{E/f}\left[1 - \frac{0.38}{b/t}\sqrt{\frac{E}{f}}\right]}$

Otherwise $\boxed{b_e = b}$

f = Computed elastic compressive stress in the stiffened element

= P_n/A_{eff} (may conservatively be taken equal to F_y.

Steel Structures

Reduction Factor Q_a for Stiffened Elements (contd...)

b) For other uniformly compressed elements.

For
$$\frac{b}{t} \geq 1.49\sqrt{\frac{E}{f}}$$

$$b_e = 1.92\,t\sqrt{\frac{E}{f}}\left[1 - \frac{0.34}{b/t}\sqrt{\frac{E}{f}}\right]$$

Otherwise
$$b_e = b$$

f is taken as F_{cr} with F_{cr} calculated based on $Q = 1.0$.

Steel Structures

Reduction Factor Q_a for Stiffened Elements (contd…)

c) For axially loaded circular sections

For $$0.11 \frac{E}{F_y} < \frac{D}{t} < 0.45 \frac{E}{F_y}$$

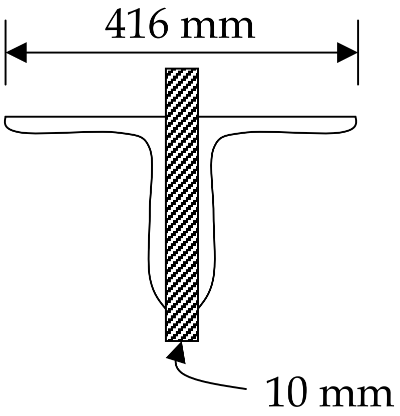
$$Q = Q_a = \frac{0.038\, E}{F_y \left(D/t \right)} + \frac{2}{3}$$

D = Outside diameter , mm

t = Wall thickness, mm

Steel Structures

Example: Design a double equal leg angle compression member of width 416 mm connected by 10 mm gusset plate and stay plates. Steel with F_y = 420 MPa is to be used



$$P_u = 1700\ kN$$

$$KL = 6\,m$$

$$F_y = 420\ MPa$$

Solution

Assume Slenderness ratio R = 90

$$F_e \geq 0.44 F_y \Rightarrow$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2 \times 200000}{(90)^2} = 243.69\ MPa$$

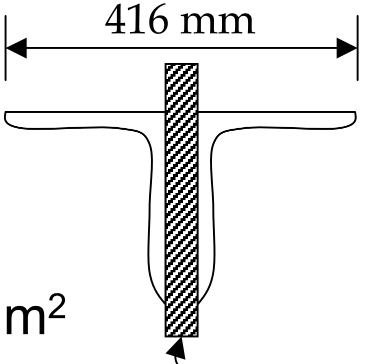
Steel Structures

Solution (contd…)

$$F_e \geq 0.44 F_y \Rightarrow$$

$$\phi_c F_{cr} = 0.9 \times \left(0.658^{\frac{F_y}{F_e}} \right) F_y$$

$$= 0.9 \times \left(0.658^{\frac{420}{243.69}} \right) \times 420 \quad = \ 183.74 \text{ mm}^2$$



$$A_{req} = \frac{P_u}{\phi_c F_{cr}} = \frac{17000 \times 1000}{183.74}$$

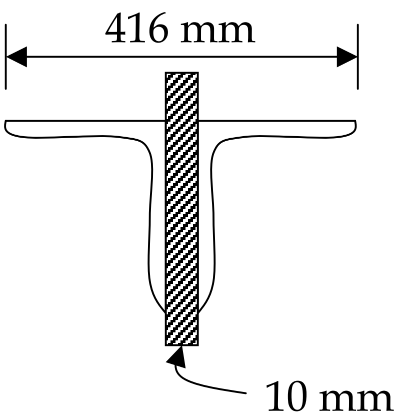
$$= 9252 \ mm^2$$ For 2Ls

$$= \frac{9252}{2} = 4626 \ mm^2$$ For single angle

Steel Structures

Solution (contd…)

Trial Section - 1: **$2L_s$ 203 x 203 x 12.7**



$$A = 5000 \; mm^2$$ For single angle

$$r_{\min} = r_{x,\text{ of single angle}} = 63.5\,mm$$

$$R = \frac{KL}{r_{\min}} = \frac{6000}{63.5} \cong 95$$

$$F_e = \frac{\pi^2 \times 200000}{95^2} = 218.72 \text{ MPa}$$

$$\phi_c F_{cr} = 0.9 \times \left(0.658^{420/218.72}\right) \times 420 = 169.21 \text{ MPa}$$

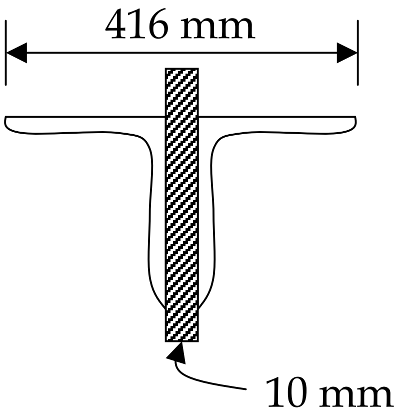
Steel Structures

Solution (contd…)

$$\phi_c P_n = 169.21 \times 2 \times 5000 \ / 1000$$

$$= 1692.1 kN$$

Based on assumption that member is locally stable



$\phi_c P_n < P_u$ Revise the section

Trial Section - 2: **$2L_s$ 203 x 203 x 14.3**

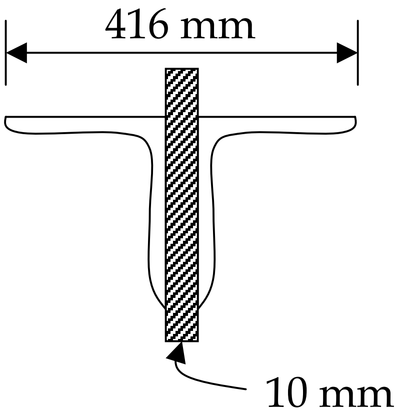
$$A = 5600 \ mm^2$$

For single angle

$$r_{min} = r_{x, \text{of single angle}} = 63.5 mm$$

Steel Structures

Solution (contd…)



$$R = \frac{KL}{r_{\min}} = \frac{6000}{63.5} \cong 95$$

$$F_e = \frac{\pi^2 \times 200000}{95^2} = 218.72 \text{ MPa}$$

$$\phi_c F_{cr} = 0.9 \times \left(0.658^{420/218.72}\right) \times 420 = 169.21 \text{ MPa}$$

$$\phi_c P_n = 169.21 \times 2 \times 5600 / 1000$$

$$\phi_c P_n = 1895 \text{ kN}$$

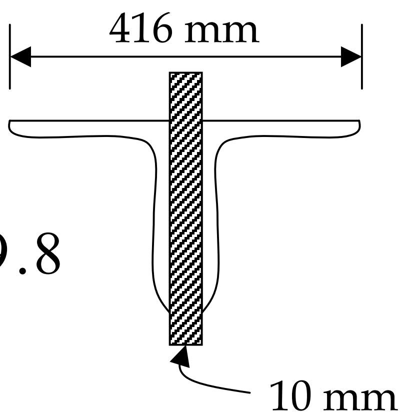
Based on assumption that member is locally stable

Steel Structures

Solution (contd…)

Check Local Stability

$$\lambda = \frac{b}{t} = \frac{203}{14.3} = 14.2 \quad > \quad \lambda_r = 0.45\sqrt{\frac{E}{Fy}} = 9.8$$



$\lambda > \lambda_r$ section is locally unstable

$$\lambda_r = 0.91\sqrt{\frac{E}{F_y}} = 19.9$$

For unstiffened elements, if $0.45\sqrt{\frac{E}{F_y}} < \frac{b}{t} < 0.91\sqrt{\frac{E}{F_y}}$

Steel Structures

Solution (contd…)

$$Q = Q_s = 1.340 - 0.76\left(\frac{b}{t}\right)\sqrt{\frac{F_y}{E}}$$

$$= 1.340 - 0.76(14.2)\sqrt{\frac{420}{200{,}000}}$$

$$= 0.846$$ 15.4 % reduction

So $$\phi_c F_{cr} = 0.90\left(0.658^{\frac{QFy}{Fe}}\right)QF_y$$

$$= 0.90\left(0.658^{\frac{0.845 \times 420}{218.72}}\right)0.845 \times 420$$

$$= 161.96 \text{ MPa}$$

Steel Structures

Solution (contd…)

$$\phi_c P_n = \phi_c F_{cr} A_g = 161.96 \times 5600 \times 2 / 1000$$

$$\boxed{\phi_c P_n = 1814 \ \text{kN} > P_u}$$

O.K.

Steel Structures

Example: Calculate the factored axial load capacity of the shown 300 mm x 300 mm non-standard structural tube having a thickness of 5mm and an effective length of 5.5 m. Use F_y = 345 MPa.

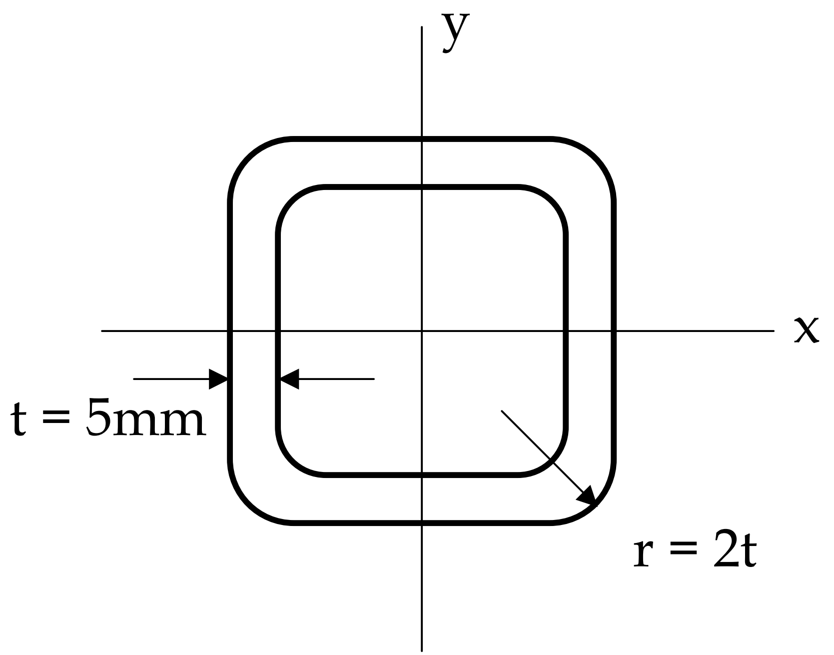
<u>Solution:</u>

$$KL = 5.5m$$

$$F_y = 345MPa$$

$$A \cong 300^2 - 290^2 = 5900mm^2$$

Neglecting chamfer



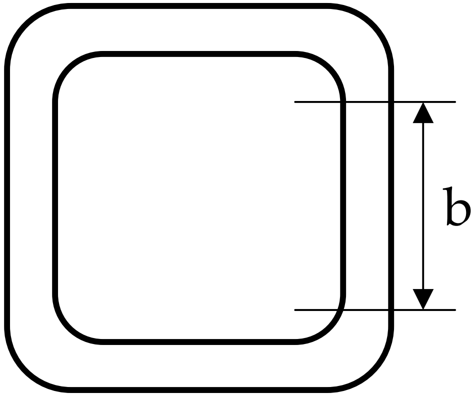
$$I_x = I_y \cong \frac{300^4}{12} - \frac{290^4}{12} \cong 8560 \times 10^4 mm^4$$

Steel Structures

<u>Solution:</u> (contd…)

$$r_x = r_y = \sqrt{\frac{I}{A}} \cong 120mm$$

Straight width $b = 300 - 2\times(2\times5) = 280mm$



$$\lambda = \frac{\text{b}}{\text{t}} = \frac{280}{5} = 56$$

$$\lambda_\text{r} = 1.40\sqrt{\frac{E}{F_y}} = 33.7$$

$\lambda > \lambda_r$ **Stiffened, locally unstable element**

The section does not have unstiffened elements

Steel Structures

Solution: (contd…)

$$Q = Q_s \times Q_a$$ For stiffened elements $Q_s = 1.0$

$$\Rightarrow Q = Q_a$$

Overall Stability

$$R = \frac{KL}{r_{min}} = \frac{5.5 \times 1000}{120} = 45.83$$

Assume $f = F_y = 345$ MPa

$$b_e = 1.92\, t \sqrt{\frac{E}{f}} \left[1 - \frac{0.38}{b/t} \sqrt{E/f} \right]$$

Steel Structures

Solution: (contd…)

$$b_e = 1.92 \times 5\sqrt{\frac{200,000}{345}}\left[1 - \frac{0.38}{56}\sqrt{\frac{200,000}{345}}\right]$$

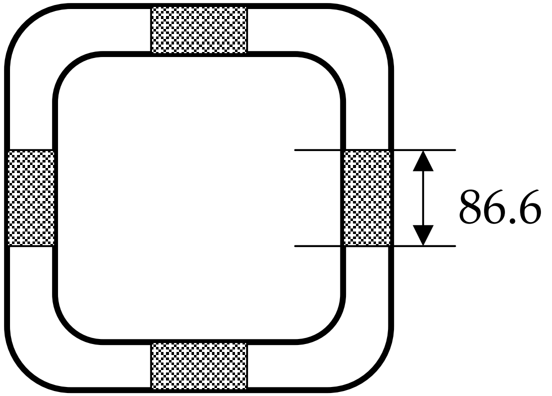
$$b_e = 193.4mm$$

Ineffective width

$$= 280 - 193.4 = 86.6mm$$

$$A_{eff} = 5900 - 86.6 \times 4 \times 5$$

$$= 4168mm^2$$



Steel Structures

<u>Solution:</u> (contd…)

$$Q_a = \frac{A_{eff}}{A_g} = \frac{4168}{5900} = 0.70$$

$$F_e = \frac{\pi^2 \times 200000}{45.83^2} = 939.79 \text{ MPa}$$

$$\phi_c F_{cr} = 0.9 \times \left(0.658^{0.7 \times 345 / 939.79}\right) \times 0.7 \times 345 = 195.19 \text{ MPa}$$

$$\phi_c P_n = 195.19 \times 5900 / 1000$$

$$= 1151.6 \text{ kN}$$

Steel Structures

Solution: (contd…)

If we ignore local buckling

$$\phi_c F_{cr} = 0.9 \times \left(0.658^{345/939.79}\right) \times 345 = 266.27 \text{ MPa}$$

$$\phi_c P_n = 266.27 \times 5900 / 1000$$

$$\Phi_c P_n = 1571 \text{ kN}$$

Steel Structures

Example: Determine the compression capacity of the given built-up I-section for an effective length of 2.5m. F_y = 345 MPa. Ignore the residual stresses.

Solution

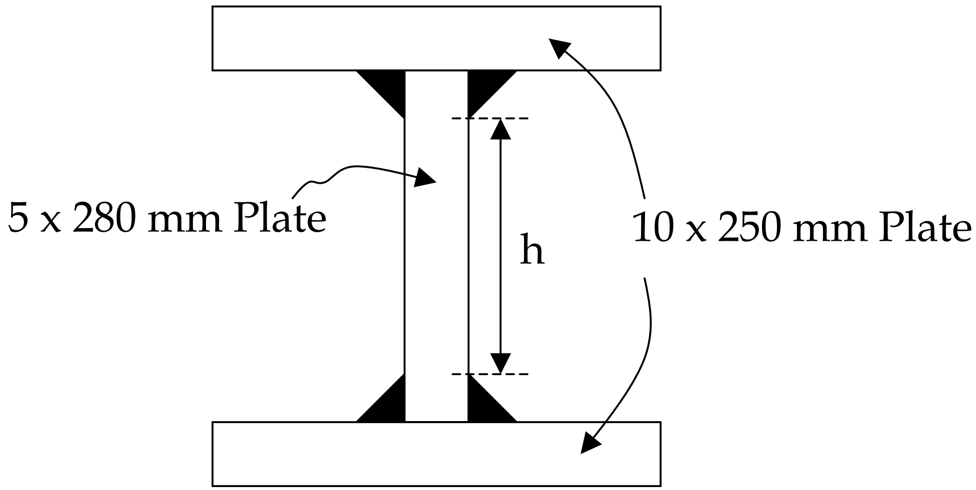
$$I_y = 2 \times \frac{10 \times 250^3}{12}$$

$$= 2604 \times 10^4 mm^4$$

$$A = 2 \times 10 \times 250 + 5 \times 280$$

$$= 6400 mm^2$$

$$r_y = \sqrt{I_y / A} = 63.8 mm$$



Steel Structures

Solution

Local Stability Check

For flange $$\lambda = \frac{b_f}{2t_f} = \frac{250}{2\times 10} = 12.5$$

$$k_c = \frac{4}{\sqrt{h/t_w}}$$ 0.35 to 0.76

$$= \frac{4}{\sqrt{280/5}} = 0.534$$

$$\lambda_r = 0.64\sqrt{\frac{Ek_c}{F_y}} = 0.64\sqrt{\frac{200{,}000\times 0.534}{345}} = 11.26$$

Steel Structures

<u>Solution</u> $\lambda > \lambda_r$ Flange is locally unstable

Now for the built-up sections

$$0.64\sqrt{\frac{Ek_c}{F_y}} = 11.26 \quad \text{and} \quad 1.17\sqrt{\frac{Ek_c}{F_y}} = 20.6$$

As $$11.26 < \lambda < 20.6$$

So $$Q_s = 1.415 - 0.65\left(b/t\right)\sqrt{F_y/k_cE} = 0.953$$

<u>Local Stability Check For Web</u>

$$\lambda = \frac{h}{t_w} = \frac{280}{5} = 56$$

Steel Structures

Solution

$$\lambda_r = 1.49\sqrt{\frac{E}{F_y}} = 35.9$$

$\lambda > \lambda_r$ Web is locally unstable

Assume $Q = 1.0$

$$\frac{KL}{r_{min}} = \frac{2500}{63.8} = 39.2$$

$$F_e = \frac{\pi^2 \times 200000}{39.2^2} = 1284.6 \text{ MPa}$$

$$f = F_{cr} = \left(0.658^{345/1284.6}\right) \times 345 = 308.3 \text{ MPa}$$

Steel Structures

Solution

$$b_e = 1.92t\sqrt{\frac{E}{f}}\left[1 - \frac{0.34}{b/t}\sqrt{\frac{E}{f}}\right]$$

$$= 1.92 \times 5\sqrt{\frac{200{,}000}{308.3}}\left[1 - \frac{0.34}{56}\sqrt{\frac{200{,}000}{308.3}}\right]$$

$$= 206.7mm$$

Ineffective width = $280 - 206.7 = 73.3mm$

$$A_{eff} = 6400 - 73.3 \times 5 = 6033mm^2$$

Steel Structures

Solution

$$Q_a = \frac{6033}{6400} = 0.943$$

$$Q = Q_a \times Q_s = 0.943 \times 0.953 = 0.898$$

$$\phi_c F_{cr} = 0.9 \times \left(0.658^{\,0.898 \times 345 / 1284.6}\right) \times 0.898 \times 345 = 252.06 \text{ MPa}$$

$$\phi_c P_n = 252.06 \times 6400 / 1000$$

$$= 1613 \text{ kN}$$

Concluded