COLUMN BEHAVIOR

- A column is a structural member transmitting axial compressive loads, with or without moments.
- The cross-sectional dimensions of a column are generally considerably less than its height distinguishing it from a wall.
- Columns support vertical loads from the floors and roof and transfer these loads to the foundations.

- ***Eccentricity*** of load '*P*' is defined as the distance between the line of action of the load and the plastic centroidal axis (which coincides with the geometric centroid for symmetric cross-sections) of the member.
- A ***concentrically loaded column*** is that ideally straight and parallel-to-load member where the load is acting exactly at the plastic centroid.
- There is only axial force in the member and no bending moment.
- In case of an ***eccentrically loaded column***, load acts at an eccentricity from the centroidal axis and hence both axial force and moment act on the member.
- A member having simultaneous action of axial load and moment may be converted into an equivalent eccentrically loaded column.

Short Column

- The columns where the 2nd order effects, such as buckling and P–Δ effects, are very small and may be neglected are termed short columns.
- When the slenderness ratio is relatively low, the strength of a column is governed by the strength of the material and the geometry of the cross-section.
- Load acts at little or no eccentricity.

In the past, for nearly axially loaded members, a column was considered to be short when its effective height was lesser than 15 times its least dimension.

However, the more accurate distinction of a short column from a slender column may be made according to the ACI approach (ACI 10.12.2) as explained below:

1. For compression members braced against side-sway, the effects of slenderness may be neglected when

$$\frac{k\ell_u}{r} \leq 34 - 12\,\frac{M_1}{M_2}$$

where k = effective length factor

M_1 = magnitude of smaller factored end moment due to loads that result in no appreciable side-sway,

M_2 = magnitude of larger factored end moment due to loads that result in no appreciable side-sway,

$\frac{M_1}{M_2}$ = considered positive if the member is bent in single curvature and negative in case the member is bent in reverse curvature, needs not to be considered less than – 0.5,

ℓ_u = unsupported length of the column.

According to ACI 10.11.3, the unsupported length of a column is taken equal to the clear distance between floor slabs, beams or other lateral supports in the considered direction.

In case column capitals are present, the unsupported length is outside the capitals.

r = radius of gyration in the direction of possible bending / buckling or more generally the minimum radius of gyration

For rectangular columns $r \approx 0.30\,h$

For circular members $r \approx 0.25\,D$

h = overall cross-sectional dimension in the direction stability is being considered. Smaller lateral dimension is taken as the value of h to get the maximum slenderness ratio.

and D = diameter of the circular

Appropriate values of effective length factor 'k' for identification of a short column may be used as under:

$k = 1.0$ for columns hinged at both ends,
$k = 1.0$ for stocky columns restrained by flat slab floor, and
$k = 0.90$ for columns in beam-column frames.

2. For compression members not braced against side-sway, the effect of slenderness may be neglected when kl_u/r is less than 22 (ACI 10.13.2).

$$\frac{k\ell_u}{r} \leq 22$$

Slender Column

- If the moments induced by slenderness effects weaken a column appreciably, it is referred to as a slender or long column.
- Strength of a column may be significantly reduced by lateral deflections.
- Moment magnification takes place in columns having lateral deflections due to P-Δ effects and due to reduction of flexural rigidity caused by cracking and time dependent effects.

- The elastic analysis carried out to calculate deflections and member forces for the given loads is called 1st order and analysis.
- The high axial load present in the column combined with this elastic deflection produces extra bending moment in the column, as is clear from Fig.14.1.
- The analysis of structure including this extra moment is called 2nd order analysis.
- Similarly, other higher order analysis may also be performed.
- In practice, usually 2nd order analysis is sufficiently accurate with the high order results of much lesser numerical value.

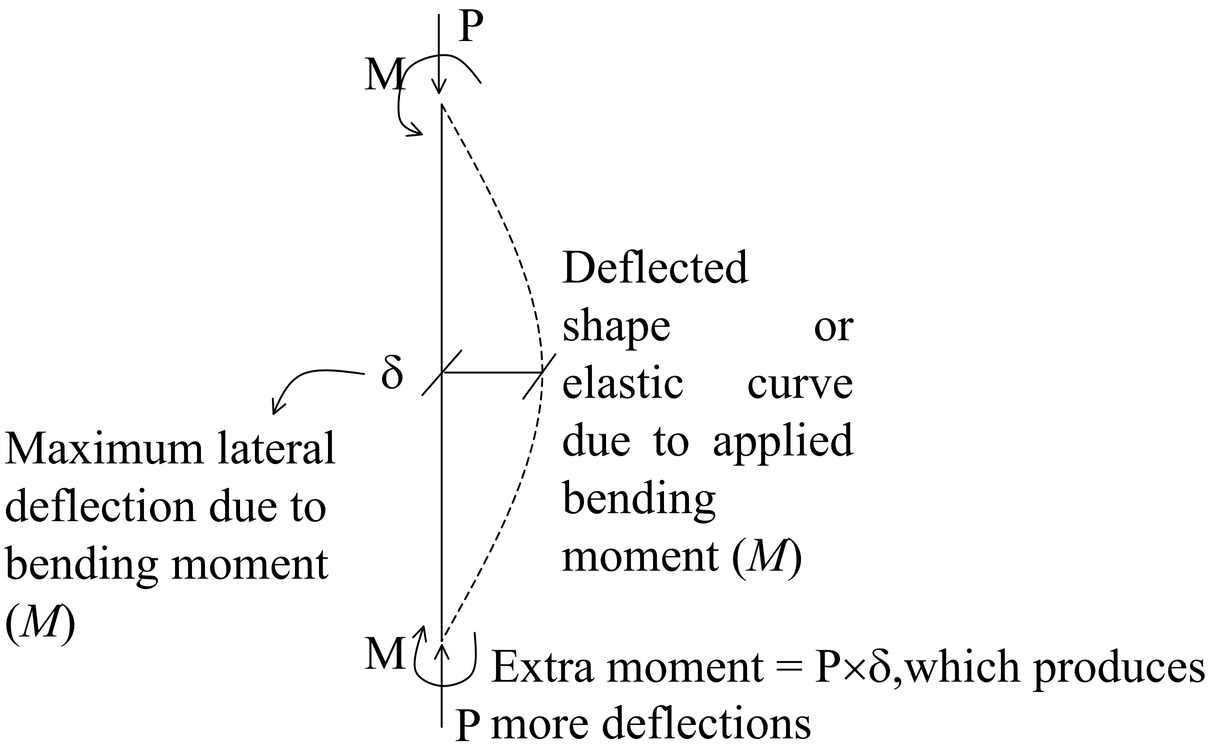


Fig. 14.1. Eccentricity Due to First Order Deflections.

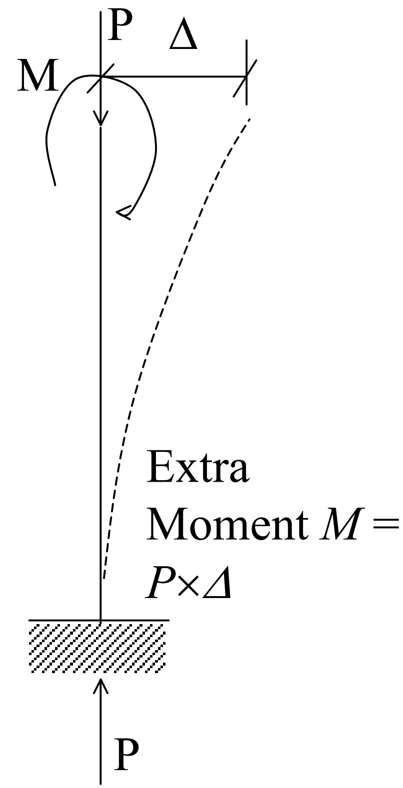


Figure 14.2. A Deflected Beam-Column.

- The phenomenon in which the moments are automatically increased in a column beyond the usual analysis for loads is called ***moment magnification*** or ***2nd order effects***.
- The moment magnification depends on many factors but, in some cases, it may be higher enough to double the 1st order moments or even more.
- In majority of practical cases, this magnification is appreciable and must always be considered for a safe design.
- 1st order deflection produced within a member (δ) usually has a smaller 2nd order effect called P-δ effect, whereas magnification due to sides-way (Δ) is much larger denoted by P-Δ effect (refer to Fig. 14.2).

- P-Delta effect is defined as the secondary effect of column axial loads and lateral deflections on the moments in members.
- Although the deflections are usually very small but their product with the load (*P*) may be considerable due to very high axial loads in columns.
- 2nd order effects are more pronounced when loads closer to buckling loads are applied and hence the empirical moment magnification formula contains a ratio of applied load to elastic buckling load.

According to ACI 10.3.6, an additional reduction factor is also applied on nominal strength of concentrically loaded columns to take care of accidental eccentricities, which is 0.80 for tied columns and 0.85 for spirally reinforced columns.

These values approximate the axial load strengths at e / h ratios of 0.05 and 0.10, specified in the earlier codes.

ULTIMATE STRENGTH OF CONCENTRICALLY LOADED SHORT COLUMNS

- **Axially Loaded Spiral Short Columns**

$$\phi_c P_{no} = 0.85 \times 0.70\,[0.85\, f'_c\,(A_g - A_{st}) + f_y\, A_{st}]$$

where A_g = gross area of cross-section of the column

and A_{st} = total area of longitudinal steel.

- **Axially Loaded Tied Short Columns**

$$\phi_c P_{no} = 0.80 \times 0.65\,[0.85\, f'_c\,(A_g - A_{st}) + f_y\, A_{st}]$$

TRIAL COLUMN SIZE

For a spirally reinforced column, the required trial column area is as under:

$$A_g\ (\text{trial}) \geq \frac{P_u + 2M_{ux} + 2M_{uy}}{0.5 f_c' + 0.01 f_y}$$

For a tied column:

$$A_g\ (\text{trial}) \geq \frac{P_u + 2M_{ux} + 2M_{uy}}{0.43 f_c' + 0.008 f_y}$$

INTERACTION BETWEEN COMPRESSIVE ACTION AND BENDING

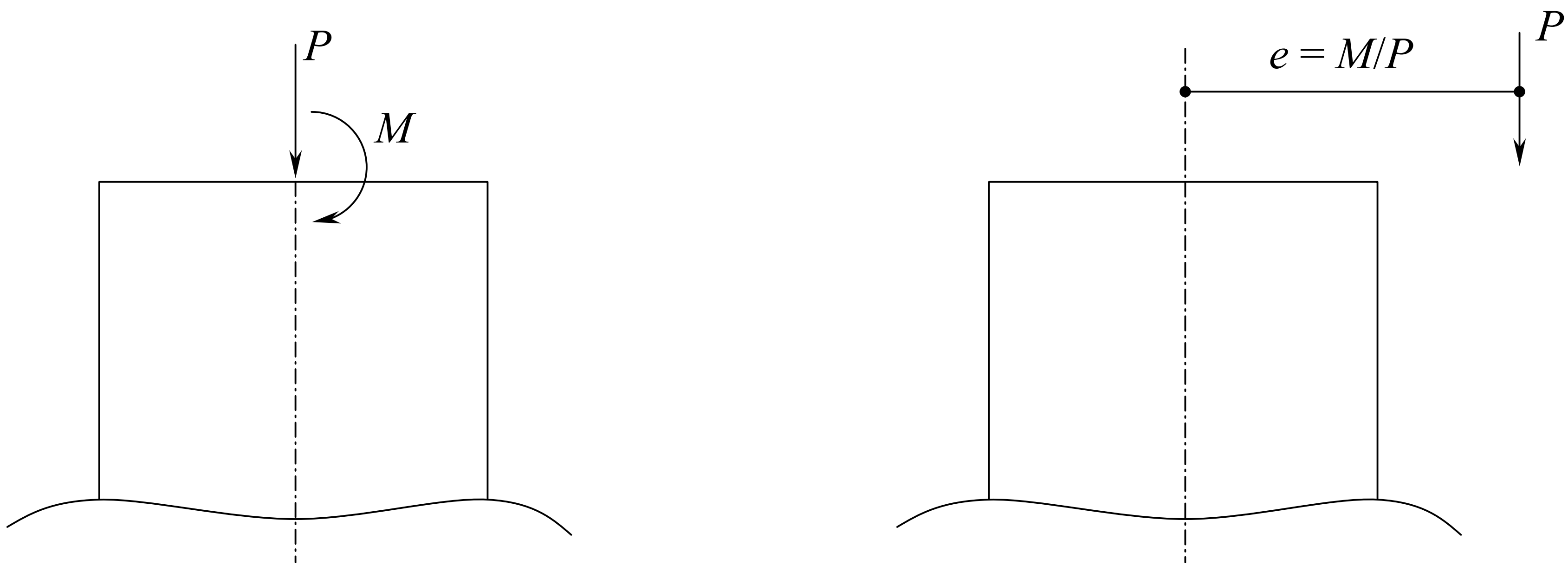


Fig. 14.3. Beam-Column And Eccentrically Loaded Column.

Columns with relatively small e have compressive stresses almost uniformly distributed over the entire cross-section. If overloaded, such columns will fail by crushing of the concrete simultaneously with yielding of the steel in compression on the more heavily loaded side.

Columns with large eccentricity develop tension over at least a part of the section. If overloaded, these columns fail due to tensile yielding of the steel on the tension side away from the load.

For an idealized homogeneous and elastic column with an equal compressive and tensile strengths, f_u, the following is true:

Axial compressive stress + Extreme fiber flexural stress

$$\leq \quad f_u$$

$$\frac{P}{A} + \frac{M\,y}{I} \leq f_u$$

$$\frac{P}{A\,f_u} + \frac{M}{S\,f_u} \leq 1$$

$$\frac{P}{P_{max}} + \frac{M}{M_{max}} \leq 1$$

The above equation is known as an ***interaction equation*** because it relates *P* and *M* at failure.

If the same equation is plotted as the line AB, as in Fig. 14.4, the resulting curve is called an ***interaction diagram***.

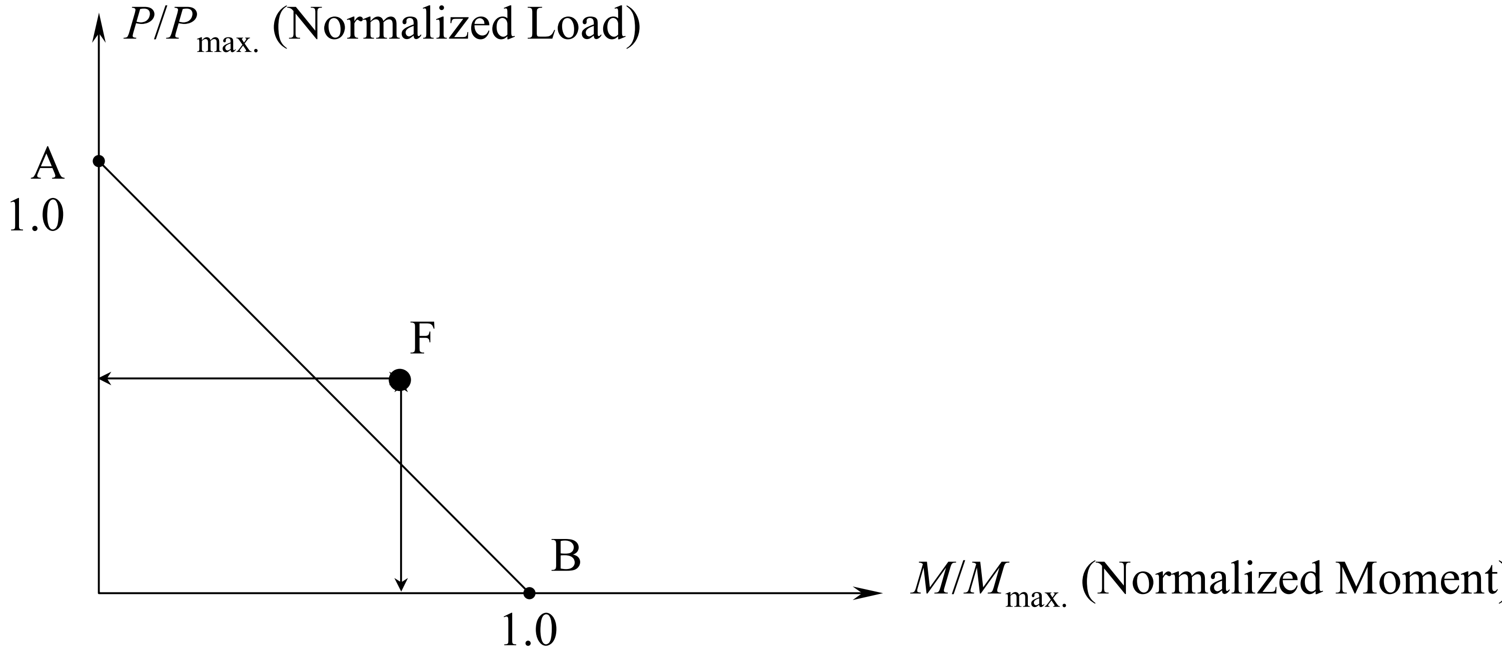


Fig. 14.4. Basic Linear Interaction Curve.

Typical Interaction Diagram

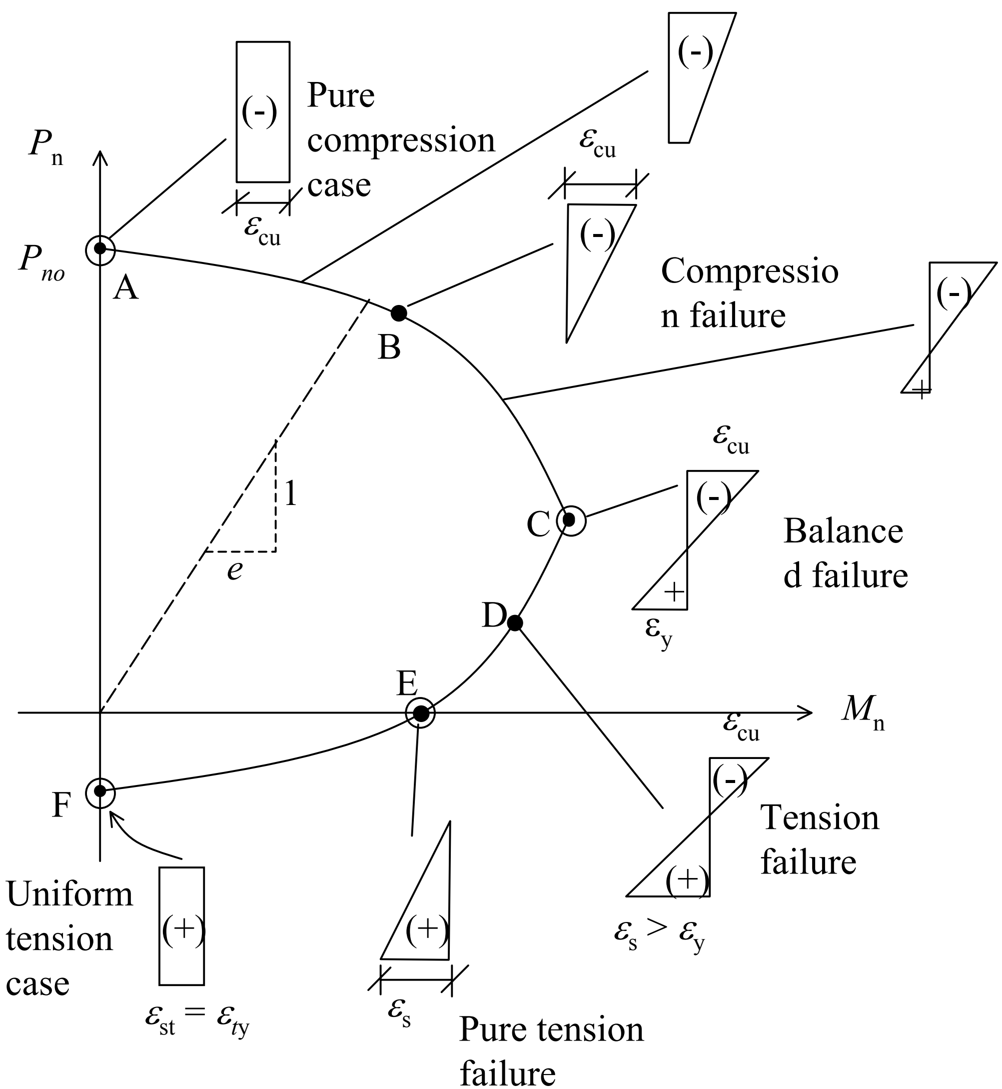


Fig. 14.5. A Typical Column Interaction Curve.

Slope of any line like dotted one shown in Fig. 14.5

$$= \frac{P}{M} = \frac{P}{P \times e} = \frac{1}{e}$$

PLASTIC CENTROID

The plastic centroid is location of the compression resultant for the section under pure axial load without any bending moment at the ultimate stage.

In other words, plastic centroid is the centroid of resistance of a section when all the concrete is compressed to a maximum stress of $0.85\, f_c'$ and the steel is stressed to yield stress, with uniform strain over whole of the section.

For column calculations, all moment calculations are performed with respect to the plastic centroid. That is, all the eccentricities are measured from the plastic centroid.

The plastic centroid may be located by taking the moments of all internal forces, shown in Fig. 14.6, about any edge and then dividing the result by the magnitude of all the resultant forces, as under:

$$\begin{aligned}\text{Resultant} &= \Sigma F_c + \Sigma F_s \\ &= C_c + C_{s1} + C_{s2} + C_{s3}\end{aligned}$$

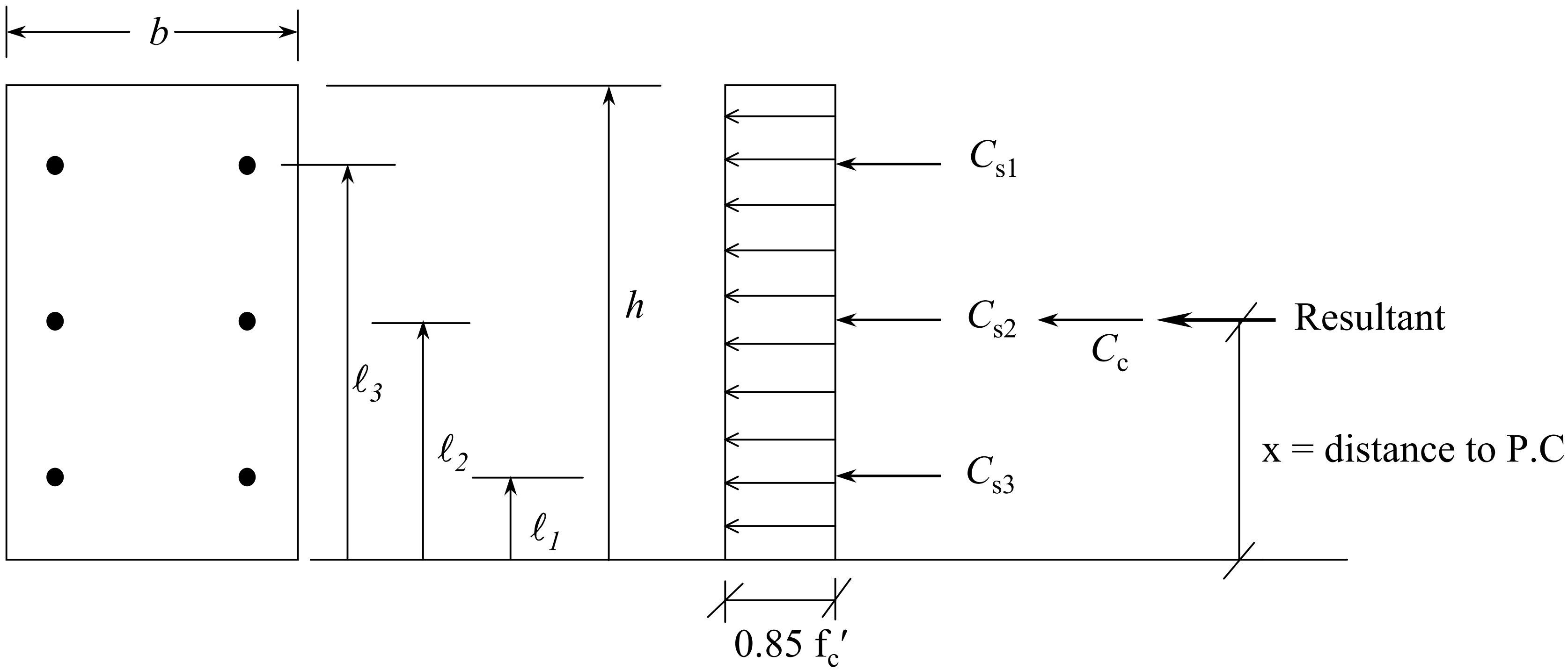


Fig. 14.6. Location of Plastic Centroid.

Distance of plastic centroid from compression face is:

$$x = \frac{\left(\sum M_{conc} + \sum M_{steel}\right)\text{about bottom edge}}{\text{Resultant}}$$

$$= \frac{A_{s1} f_y \ell_1 + A_{s2} f_y \ell_2 + A_{s3} f_y \ell_3 + 0.85 f_c' bh \, h/2}{A_{s1} f_y + A_{s2} f_y + A_{s3} f_y + 0.85 f_c' bh}$$

For symmetric sections, the plastic centroid is located at the geometric centroid.

DESIGN INTERACTION CURVE

Design interaction curve is obtained by applying the strength reduction factors (ϕ and additional reduction factors for pure axial load case) to the nominal interaction diagram.

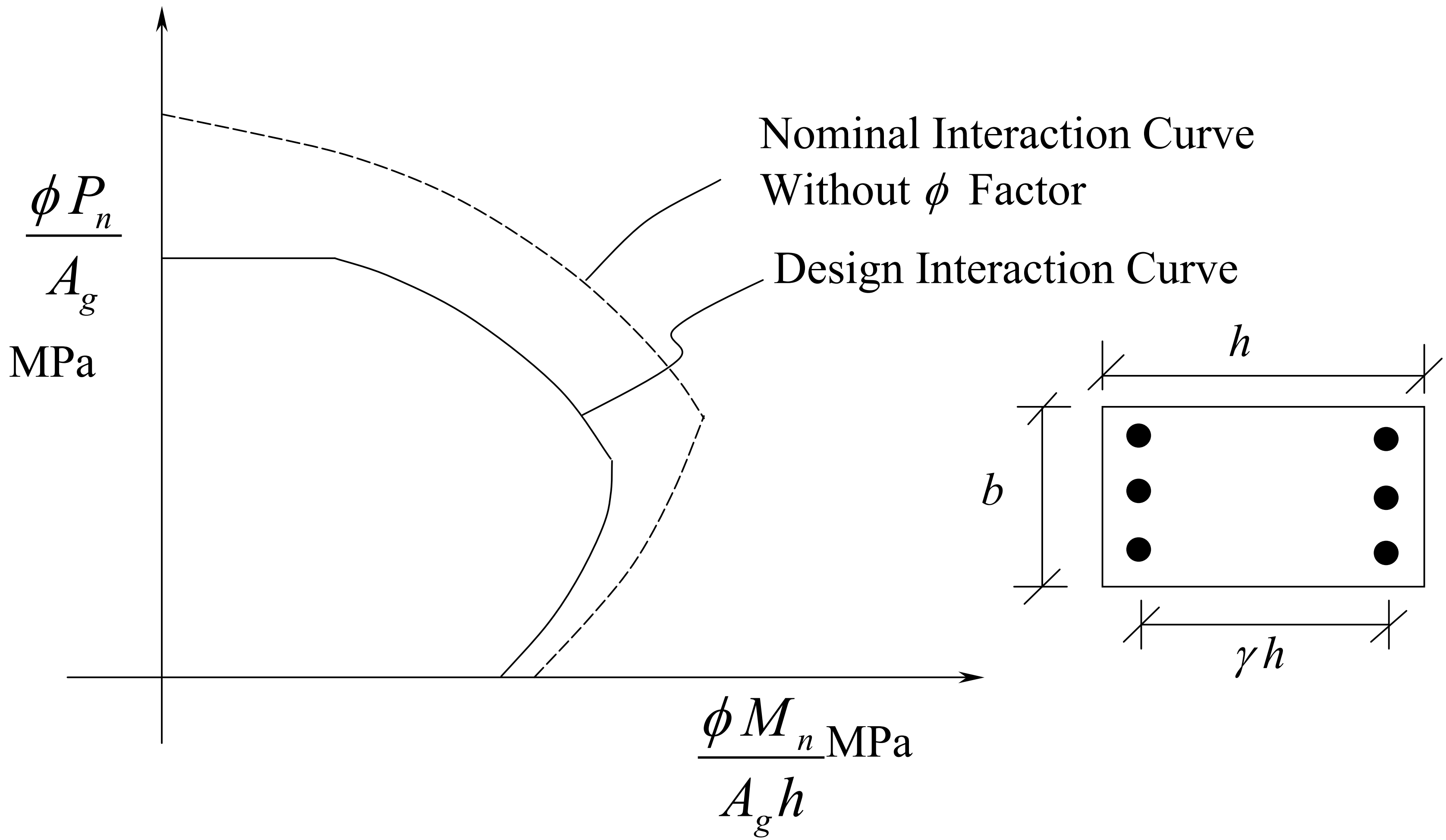


Fig. 14.7. Design Capacity Interaction Curve For Columns.

- A simplified $P - M$ interaction curve may be obtained by plotting only three points and joining them by straight lines.
- These three points correspond to uniaxial compression, pure flexure and balanced conditions (Fig. 14.8).
- This diagram is a safe approximation as compared with the actual diagram.

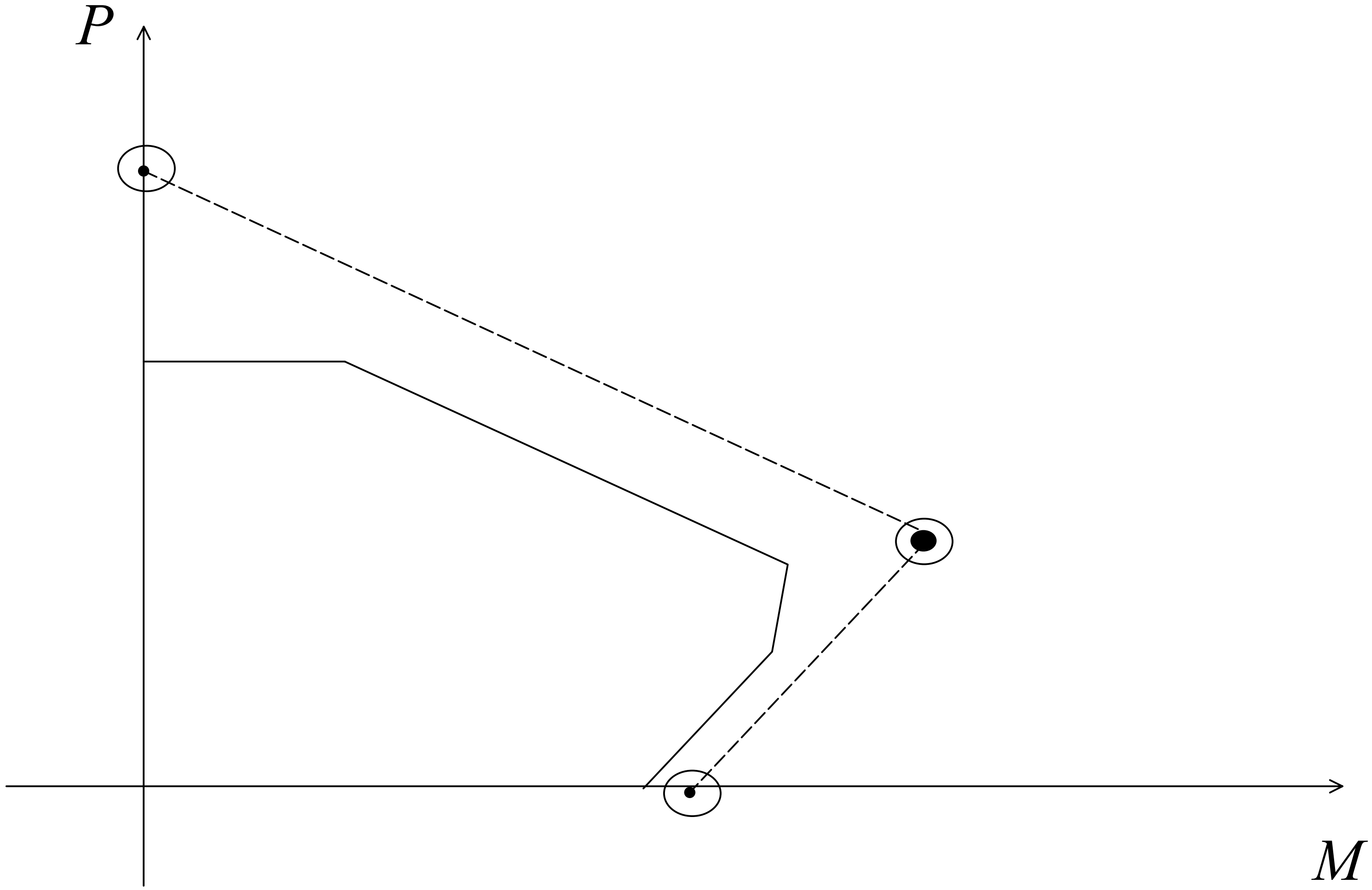


Fig. 14.8. Typical Simplified Interaction Curve.

ANALYSIS OF RECTANGULAR COLUMNS HAVING UNIAXIAL ECCENTRICITY

e	=	eccentricity of load from plastic centroid,
e′	=	distance of load from the centroid of tension steel or eccentricity of load with respect to the tension steel,
d	=	distance of extreme tension steel from the compression face,
d'	=	distance of centroid of compression steel from the compression face,
d''	=	distance of tension steel from plastic centroid,
b	=	width of the column,

h	=	size of column in the direction of bending,
c	=	depth of the neutral axis from the compression face,
ε_s	=	strain in tension steel,
ε_s'	=	strain in compression steel,
ε_c	=	strain in concrete at extreme compression face,
f_s	=	stress in tension steel,
f_s'	=	stress in compression steel,
T	=	force in tension steel,
C_s'	=	force in compression steel,
C_c	=	compressive force in concrete,
a	=	depth of equivalent rectangular stress block,

A_s = area of tension steel,

A_s' = area of compression steel,

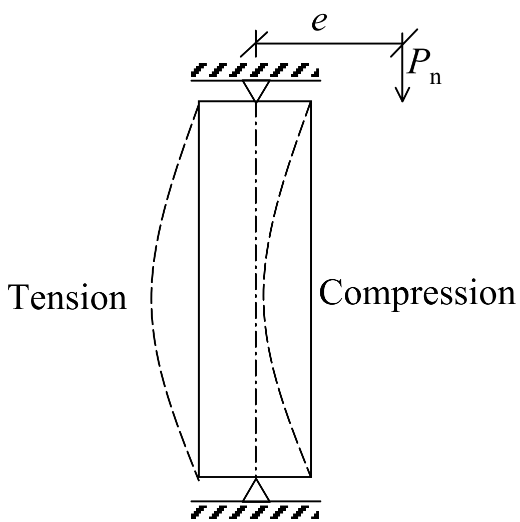
and

A_{st} = total area of steel.

Two basic equations are formulated for the analysis of columns loaded at uniaxial eccentricity by considering the equilibrium of forces in the force diagram of Fig. 14.9.

<u>Load Equation</u>

$$\begin{matrix}\text{Applied axial} \\ \text{compressive load}\end{matrix} = \begin{matrix}\text{Sum of internal} \\ \text{longitudinal forces}\end{matrix}$$



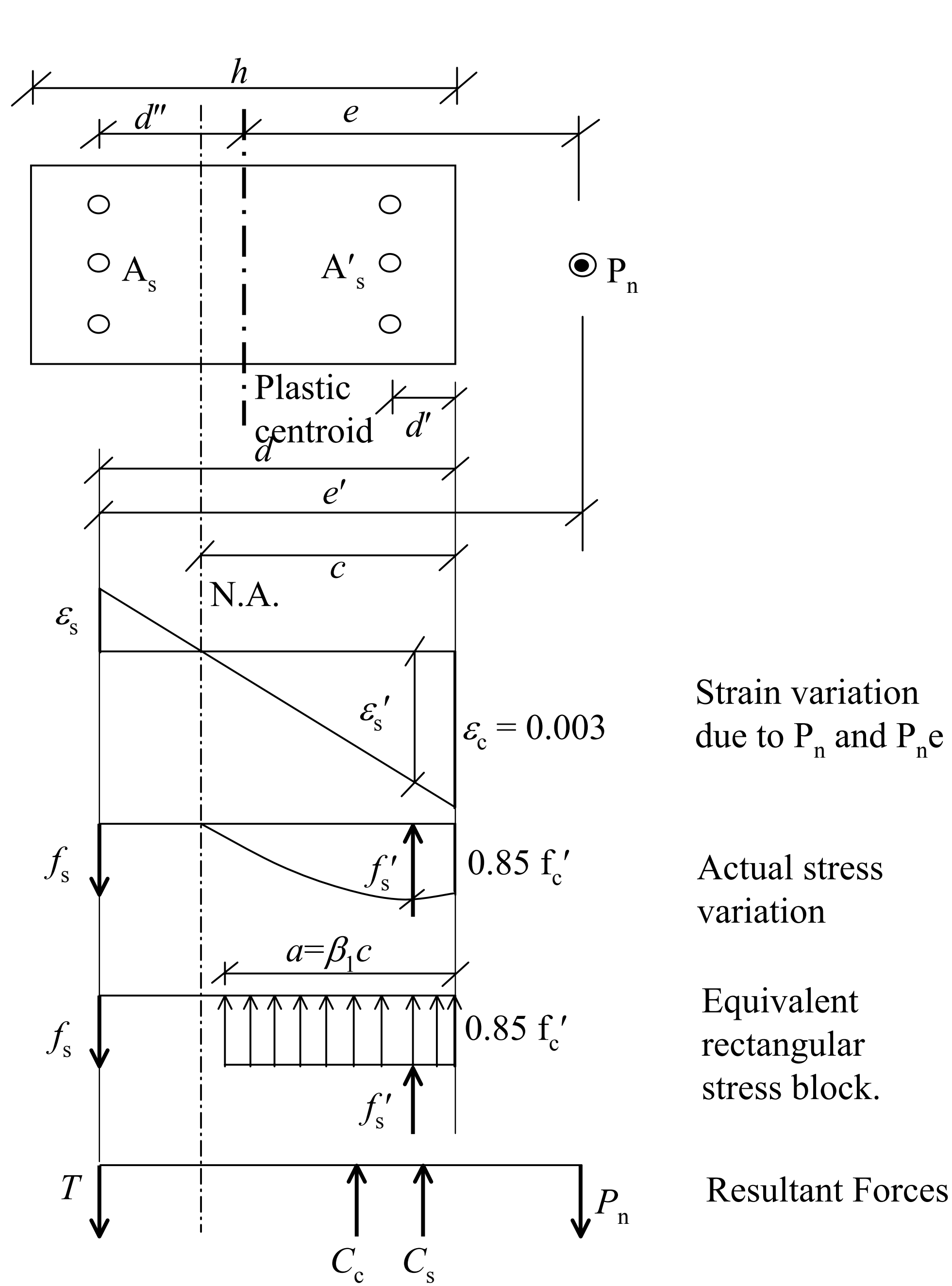


Fig. 14.9. Rectangular Column Subjected to Uniaxial Bending.

Moment Equation

Moment due to external load about either the plastic centroid or tension steel = Sum of moments due to all the internal forces about either the plastic centroid or tension steel (corresponding to left side of the equation)

$$C_c = 0.85 f_c' \, b \, a$$

$$C_s = A_s' f_s'$$

$$= A_s' f_y \quad \text{if the compression steel is yielding}$$

$$T_s = A_s \, f_s$$

$$= A_s \, f_y \quad \text{if the tension steel is yielding}$$

The load equation becomes:

Total load = Total compressive forces – Total tensile forces

$$P_n = C_c + C_s - T$$

$$= 0.85 f_c' \, ba + A_s' f_s' - A_s \, f_s$$

Assuming that both the compression and the tension steels to have yielded, the above equation becomes:

$$P_n \quad = \quad 0.85 f_c' \, ba \; + \; A_s' f_y - A_s f_y$$

<u>Load equation</u> (I)

Taking moments about the tension steel, the moment equation may be written as:

$$P_n \times e' \; = \; 0.85 f_c' ba \,(d - a/2) + A_s' f_y \,(d - d')$$

<u>Moment equation</u> (II)

Sometimes this moment may also be taken about the plastic centroid. The moment equation about plastic centroid is:

$$\begin{aligned} P_n \times e \quad &= 0.85\, f_c{}' ba\, (d - d'' - a/2) \\ &+ A_s{}' f_s{}' (d - d'' - d') + A_s f_s\, d'' \end{aligned}$$

<u>Moment equation</u> (III)

Nominal Column Strength

<u>**Case 1** – Axial Compression</u>

In this case, only the axial load P_{no} is to be calculated. The strength reduction factor (ϕ) is 0.65 with an extra reduction factor of 0.8 for a tied column.

P_{no} = strength provided by concrete + strength provided by steel

$$= 0.85 f_c' bh + A_s' f_y + A_s f_y$$

Case 2 – Balanced Failure

Here, the balanced load P_{nb} and balanced eccentricity e_b are to be calculated. For balanced failure, $f_s = f_y$. The strength reduction factor (ϕ) is 0.65 for a tied column.

The following steps are performed to analyze the column:

Step 1: Calculate c_b and a_b from the strain diagram of Fig. 14.10.

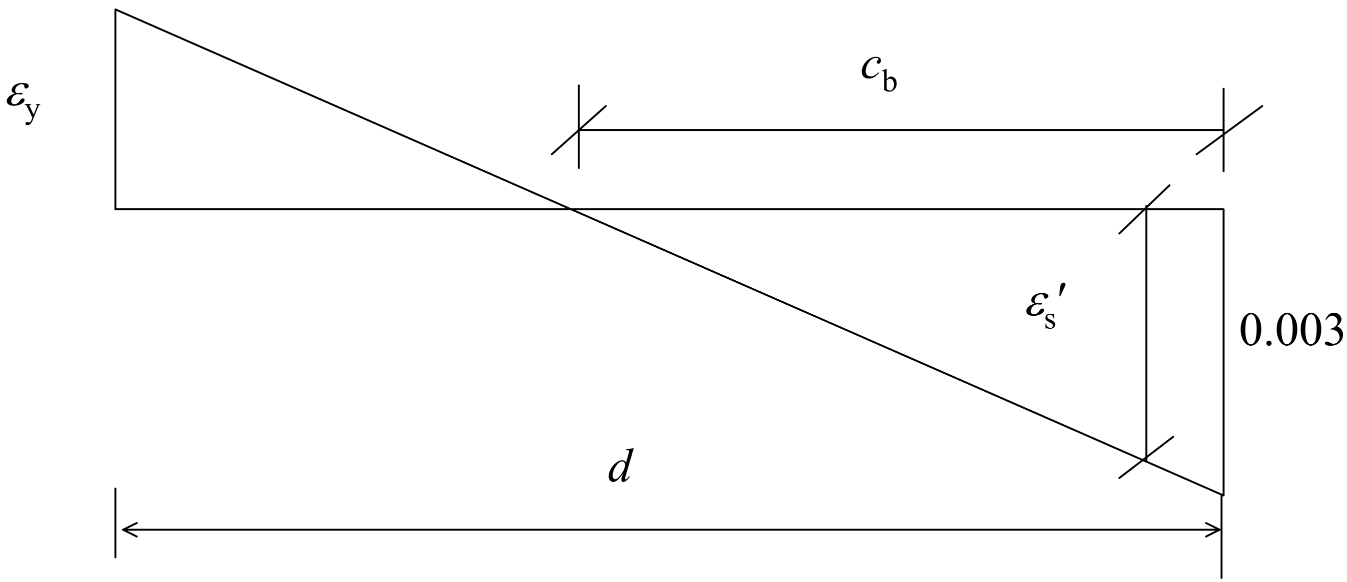


Fig. 14.10.Strain Diagram at Balanced Condition.

$$\frac{c_b}{d - c_b} = \frac{0.003}{\varepsilon_y}$$

$$c_b \, \varepsilon_y = 0.003\,d - 0.003\,c_b$$

$$c_b\,(0.003 + \varepsilon_y) = 0.003\,d$$

$$\frac{c_b}{d} = \frac{0.003}{0.003+\varepsilon_y} \times \frac{E_s}{E_s}$$

$$\text{or} \quad c_b = \frac{600}{600+f_y}\,d$$

$$a_b = \beta_1\,c_b = \beta_1\,\frac{600}{600+f_y}\,d$$

The strain in compression steel may then be checked by the equation:

$$\varepsilon_s' = 0.003\;\frac{a_b - \beta_1 d'}{a_b}$$

If this strain is greater than the yield strain, $f_s' = f_y$; and if this strain is lesser than the yield strain, $f_s' = \varepsilon_s' \times E$.

Step 2: Calculate load P_{nb} from the load equation.

$$P_{nb} = 0.85\, f_c' b\, a_b + A_s' f_s' - A_s f_y$$

If $A_s = A_s'$ and $f_s' = f_y$, the expression for P_{nb} is simplified as under:

$$P_{nb} = 0.85\, f_c' b\, a_b$$

Step 3: Calculate moment M_{nb} using the moment equation and Fig. 14.11.

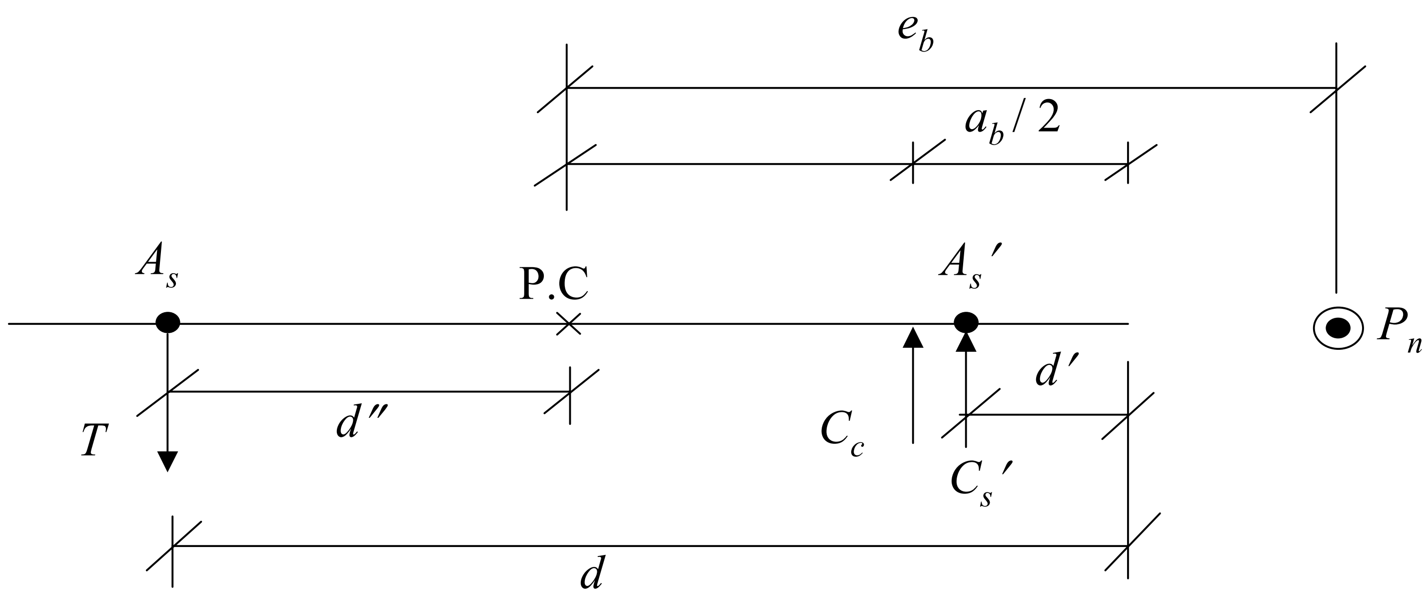


Fig. 14.11. Force Diagram for Calculation of Moment Capacity of a Column.

<u>*First Method*:</u> Taking moments about the plastic centroid.

$$
\begin{aligned}
P_{nb}\, e_b &= M_{nb} = C_c\,(d - d'' - a/2) \\
&\quad + C_s'(d - d'' - d') + T\ d'' \\
&= 0.85 f_c' b\, a_b\,(d - d'' - a_{\mathrm{b}}/2) \\
&\quad + A_s' f_s'(d - d'' - d') + A_s f_y\ d''
\end{aligned}
$$

Second Method: Taking moments about the tension steel.

$$P_{nb}\, e_b' = C_c\,(d - a_b/2) + C_s'(d - d')$$

$$= 0.85\, f_c'\, b\, a_b\,(d - a_b/2) + A_s' f_y\,(d - d')$$

Where e_b' = balanced eccentricity from the tension steel.

After calculation of e_b' from the above equation, e_b and M_{nb} are calculated as follows:

$$e_b = e_b' - d''$$

$$M_{nb} = P_{nb} \times e_b$$

Case 3 – Pure Bending Without Axial Force

When $P_n = 0$, we cannot consider load acting at some eccentricity.

The moment M_n is to be considered as such and not in the form $P_n \times e'$ or $P_n \times e$.

Further, in case of columns where $A_s = A_s'$, f_s' cannot be equal to f_y because this will mean that concrete does not resist any compression (not the actual case).

The strength reduction factor (ϕ) is 0.90 for such a case.

$$f_s = f_y \; and \; P_n = 0$$

Step 1: Find f_s' *from the strain diagram in terms of* '*a*', using Fig. 14.12.

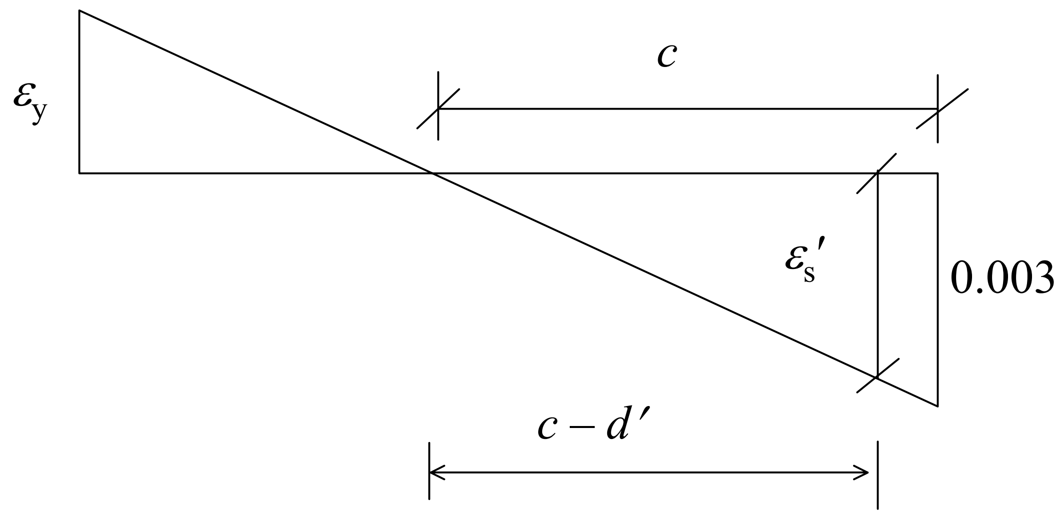


Fig. 14.12. Strain Diagram For Pure Bending.

$$\varepsilon_s' = 0.003 \, \frac{c - d'}{c}$$

$$f_s' = 600 \, \frac{a - \beta_1 d'}{a} \quad \le f_y \qquad \text{(I)}$$

Step 2: Write the equation for the load and equate it to zero to calculate 'a'.

$$P_n = 0 = 0.85 f_c' \, b \, a + A_s' f_s' - A_s f_y$$

If $f_s' = f_y$

$$P_n = 0.85 f_c' \, b \, a + A_s' f_s' - A_s f_y = 0$$

Otherwise,

$$P_n = 0.85 f_c' \, b \, a + A_s' (600) \left(\frac{a - \beta_1 d'}{a} \right) - A_s f_y \quad \text{(II)}$$

The value of a is calculated from one of the above equations.

Step 3: Put the value of 'a' from Eq. II into Eq. I to find f_s'. Also the yielding of tension steel may also be verified.

$$\frac{\varepsilon_s}{0.003} = \frac{d-c}{c}$$

$$\text{or} \quad f_s = 600 \, \frac{\beta_1 d - a}{a} \qquad \text{(III)}$$

Step 4: Take moments about the plastic centroid.

$$M_n = 0.85 f_c' \, b \, a \, (d - d'' - a/2) + A_s' f_s' (d - d'' - d') + A_s f_y d'' \qquad (IV)$$

The strength reduction factor (ϕ) is 0.90 because it will be a tension controlled section. *The following quantities are known:*

$$\varepsilon_s = 0.005$$
$$\varepsilon_{cu} = 0.003$$
$$f_s = f_y$$

Step 1: *Calculate depth of neutral axis c* and depth of equivalent rectangular stress block *a* from the load equation.

$$\frac{c}{d} = \frac{0.003}{0.008} = \frac{3}{8}$$

$$c = 3/8\, d \quad \text{and} \quad a = \beta_1\, 3/8\, d$$

Step 2: *Check for yielding of compression steel.*

$$\varepsilon_s' = 0.003\, \frac{a - \beta_1 d'}{a}$$

If $\varepsilon_s' \geq \varepsilon_y, f_s' = f_y$; otherwise, $f_s' = \varepsilon_s' E_s$

Step 3: Calculate load P_n from the load equation.

$$P_n = 0.85\, f_c'\, b\, a + A_s' f_s' - A_s f_y$$

Step 4: Calculate load M_{n} from the moment equation.

$$M_n = 0.85\, f_c'\, b\, a\, (d - d'' - d') + A_s f_y\, d''$$

<u>**Case 5** – Given Load Is Lesser Than P_{nb}</u>

For load lesser than P_{nb}, the tensile yielding of steel will cause the failure. P_n is known and either e or M_n is to be calculated. The strength reduction factor (ϕ) is 0.90 if the tensile steel strain is larger than or equal to 0.005 or between 0.65 and 0.9 for a tied column.

$$P_n < P_{nb} \Rightarrow \text{tension failure will occur}$$

$$\therefore\ \boldsymbol{f_s} = \boldsymbol{f_y}$$

Assume compression steel to be yielding in the start ($f_s' = f_y$).

Step 1: Calculate '*a*' from the load equation.

$$P_n = 0.85 f_c' \, b \, a + A_s' f_y - A_s f_y$$

Step 2: Check for yielding of compression steel.

$$\varepsilon_s' = 0.003 \frac{a - \beta_1 d'}{a}$$

If $\varepsilon_s' \geq \varepsilon_y$, $f_s' = f_y$.
Otherwise, use $f_s' = \varepsilon_s' E$ in terms of unknown '*a*' and repeat *Step 1* to find the correct value of '*a*'.

Step 3: From the moment equation about the plastic centroid, calculate e or M_n.

$$\begin{aligned} M_n \quad &= P_n \times e = 0.85 f_c' \, b \, a \, (d - d'' - a/2) \\ &\quad + A_s' f_s' (d - d'' - d') + A_s f_y d'' \end{aligned}$$

Step 4: *Calculate the tension steel strain from the following equation and then determine the* ϕ factor.

$$\varepsilon_s \quad = \quad 0.003 \; \frac{\beta_1 d - a}{a}$$

Case 6 – Given Load Greater Than P_{nb}

For load greater than P_n, the compression failure of concrete will cause the failure. P_n is known and either e or M_n is to be calculated. The strength reduction factor (ϕ) is 0.65 for a tied column.

$$
\begin{aligned}
& P_n > P_{nb} \quad \Rightarrow \textit{compression failure} \\
& \therefore f_s < f_y \\
& \textit{Assume}\ f_s' = f_y
\end{aligned}
$$

Step 1: Calculate f_s from strain diagram in terms of 'a'.

$$f_s = 600\ \frac{\beta_1 d - a}{a}$$

Step 2: Calculate '*a*' from the load equation by using the value of f_s from Step-1, which will be a quadratic equation in terms of '*a*'.

$$P_n = 0.85 f_c' \, b \, a + A_s' f_y - A_s f_s$$

$$\text{Calculate c} = a / \beta_1$$

Step 3: If the calculated depth of neutral axis (c) is greater than the column depth (h), the above equation does not strictly apply because neutral axis lies outside the section and shape of stress block changes. In such cases, compression will be present all along the section. For this portion of the interaction diagram, a straight line from the pure axial case to the point where the N.A. lies within the section may be used.

If N.A. lies within the section, the compression steel is checked for yielding.

$$f_s' \quad = \quad 600\,\frac{a - \beta_1 d'}{a} \leq \quad f_y$$

If $f_s' < f_y$, the steps 1 to 3 are to be repeated by using the value of f_s' in terms of '*a*' replacing a stress of f_y in compression steel.

Step 4: Calculate f_s from the equation given in Step-1.

Step 5: Use moment equation to calculate the moment or the eccentricity.

$$M_n = 0.85 f_c' \, b \, a \, (d - d'' - a/2) + A_s' f_y \, (d - d'' - d') + A_s f_s \, (d'')$$

<u>**Case 7** – Determination Of Nominal Strength (P_n) For Given Eccentricity ($e > e_b$)</u>

In this case, eccentricity (e) is known and the corresponding failure load (P_n) is to be calculated.

For larger eccentricities, the failure will be by yielding of tension steel in tension and the strength reduction factor (ϕ) will depend upon the strain in tension steel at the instant when the concrete strain reaches a value of 0.003.

$e > e_b$ means $P_n < P_{nb}$

$\Rightarrow$ tension failure

$\therefore \quad f_s = f_y$

Assume $f_s' = f_y$ (to be checked later)

Step 1: The nominal load capacity P_n is calculated in terms of 'a' as follows:

$$P_n = 0.85 f_c' \text{ ba} + A_s' f_y - A_s f_y$$

Step 2: The value of P_n is used in the expression for moment and the value of 'a' is calculated by solving the resulting equation in terms of 'a'.

$$P_n \times e = 0.85 f_c' ba\,(d - d'' - a/2) + A_s' f_y\,(d - d'' - d') + A_s f_y\, d''$$

$a \quad = \quad ?$ (quadratic or higher order equation)

Step 3: The yielding of compression steel is checked as under:

$$\varepsilon_s' = 0.003\ \frac{a - \beta_1 d'}{a}$$

$$f_s' = 600\ \frac{a - \beta_1 d'}{a} \leq f_y$$

If the compression steel is not yielding, an expression for f_s' is formulated in terms of 'a' and this value of f_s' is used in Steps 1 and 2 to get a 3rd order equation in terms of 'a'. This equation is solved to get the correct value of 'a'.

Step 4: Calculate load from the load equation.

$$P_n = 0.85 f_c' ba + A_s' f_y - A_s f_y$$

Step 5: Calculate the tension steel strain from the following equation and then determine the ϕ factor.

$$\varepsilon_s = 0.003 \frac{\beta_1 d - a}{a}$$