Example 11.2

Design the beam shown in Fig. 11.23, without considering any pattern loading and correcting moments to face of supports. Locate the cut-off points for the proposed curtailment of bars. Use $f_c' = 20$ MPa, $f_y = 420$ MPa, clear cover = 40 mm, #13 stirrups and minimum transverse reinforcement throughout the length.

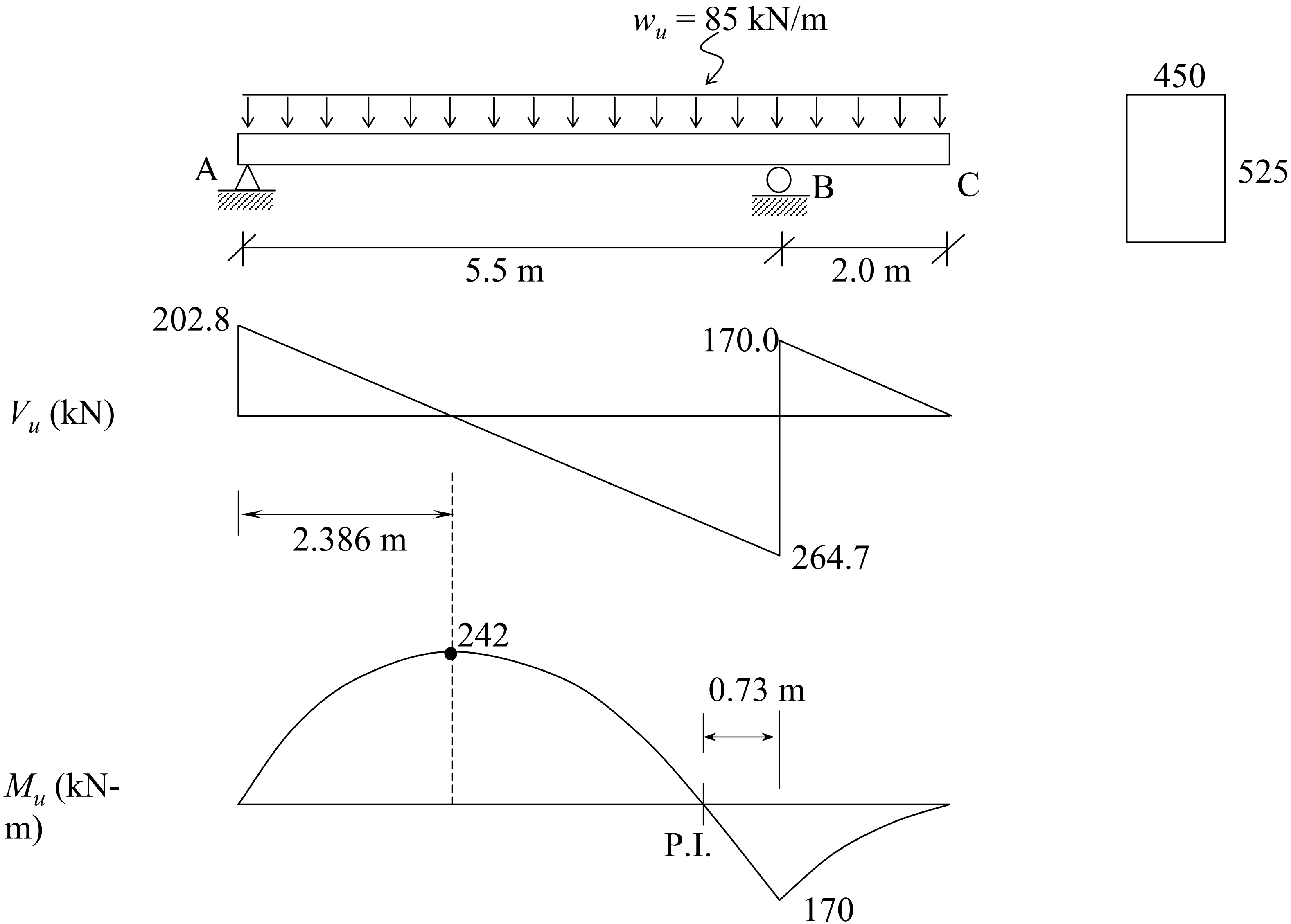


Fig. 11.23. Data for Beam of Example 11.2.

$$d \cong 525 - 75 = 450 \text{ mm}$$

$$d_{min} = \sqrt{\frac{M_u}{0.205 f_c' b}}$$

$$= \sqrt{\frac{242 \times 10^6}{0.205 \times 20 \times 450}} = 362 \text{ mm}$$

$$d > d_{min} \qquad OK$$

Positive Moment Steel

$$\frac{M_u}{bd^2} = \frac{242 \times 10^6}{450 \times 450^2} = 2.66 \text{ MPa}$$

ρ = 0.0078, *A^+_s = 1580 mm², 2 # 25 + 1 # 29*

1 # 29 may be curtailed.

Negative Moment Steel

$$\frac{M_u}{bd^2} = \frac{170 \times 10^6}{450 \times 450^2} = 1.87 \text{ MPa}$$

ρ = *0.0054*, $A^-_s = 1094\ mm^2$, *4 # 19*
(2 # 19 may be curtailed).

Development Lengths According To ACI 12.2.2

No. 19 top bars $\ell_d = \dfrac{(12)(420)}{25\sqrt{20}}(1.3)(1.0)(1.0)(19)$

$= 1113$ mm

No. 25 bottom bar $\ell_d = \dfrac{3}{5}\dfrac{(420)}{\sqrt{20}}(1.0)(1.0)(1.0)(25)$

$= 1409$ mm

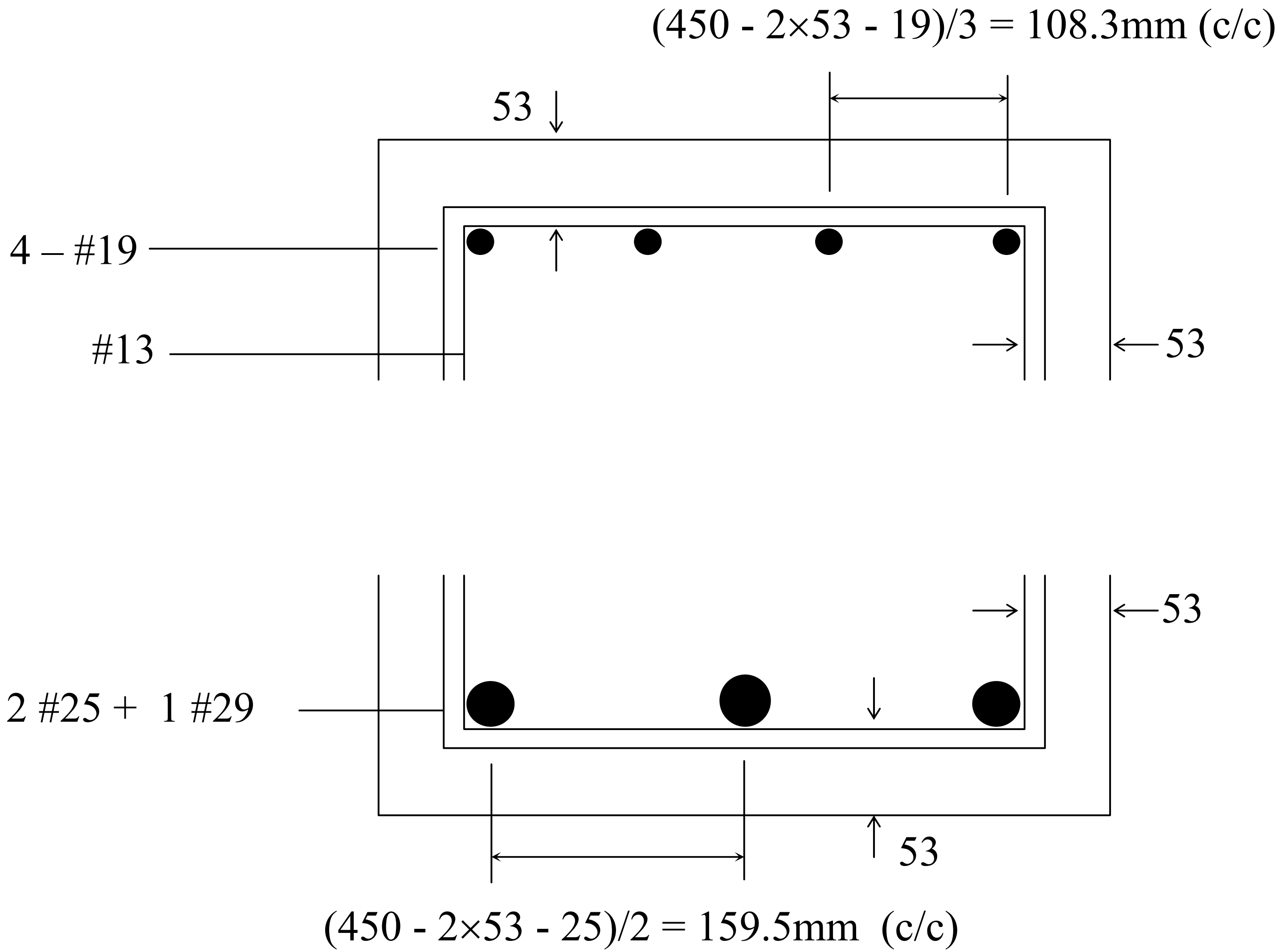


Fig. 11.24. Reinforcement Details For Positive And Negative Moment Sections.

No. 29 bottom bars, $\ell_d = \frac{3}{5}\frac{(420)}{\sqrt{20}}(1.0)(1.0)(1.0)(29)$

$= 1634$ mm

Development Lengths According To ACI 12.2.3 With $K_{tr} = 0$

For #19 bars, $c_e = 62.5$ mm, $c_s/2 = 54$ mm $\therefore$ $c_b = 54$ mm

$$\frac{c_b}{d_b} = 2.84 > 2.5; \quad \text{take} \quad \frac{c_b}{d_b} = 2.5$$

For #25 bars, $c_e = 65.5$ mm, $c_s/2 = 79.7$ mm,

$$\frac{c_b}{d_b} = 2.60 \text{ (take } \frac{c_b}{d_b} = 2.5)$$

For #29 bars, $c_e = 67.5$ mm, $c_s/2 = 79.7$ mm,

$$\frac{c_b}{d_b} = 2.31$$

$$\ell_d = \frac{9}{10}\frac{420}{\sqrt{20}}\frac{\psi_t\psi_s}{(c_b/d_b)}d_b = 84.52\frac{\psi_t\psi_s}{(c_b/d_b)}d_b$$

For #19 top bars

$$\ell_d = (84.52)(1.3)(0.8)(19)/2.5 = 668 \text{ mm}$$

For #25 bottom bars

$$\ell_d = (84.52)(1.0)(1.0)(25)/2.5 = 845 \text{ mm}$$

For #29 bottom bars

$$\ell_d = (84.52)(1.0)(1.0)(29)/2.33 = 1052 \text{ mm}$$

These values are considerably lesser than the approximate values and will be used here.

Examine Cutting 2 – #19 Bars In M⁻ Region

Capacity of Section With Continuing 2 – #19 Reinforcement

$$A_s(2-\#19) = 570 \text{ mm}^2$$

$$d = 525 - 53 - 19/2 = 462 \text{ mm}$$

$$a = \frac{(570)(420)}{(0.85)(20)(450)} = 31.3 \text{ mm}$$

$$\phi M_n\ (2-\#19) = (0.9)(570)(420)(462 - 31.3/2)/10^6$$

$$= 96.2\ kN\text{-}m$$

Distance From Maximum Moment To Theoretical Cutoff Point (T.C.P)

Left of Support B

Referring to Fig. 11.25, we get,

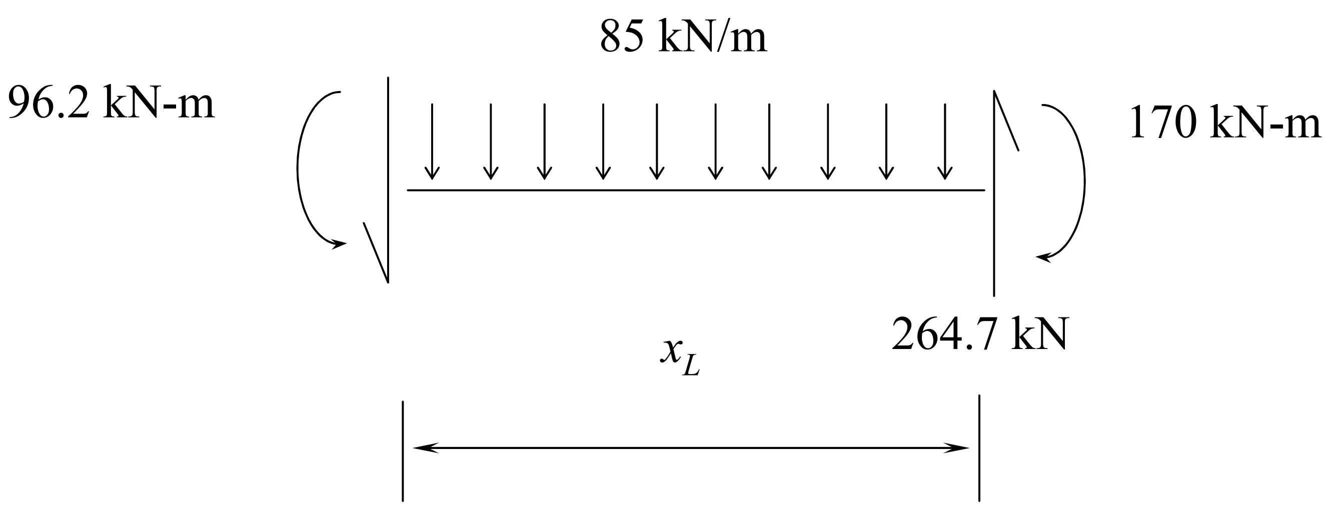


Fig. 11.25. Free Body Diagram of Span Between T.C.P. and Support B.

$$96.2 - 85\frac{x_L^2}{2} + 264.7\,x_L - 170 = 0 \quad \Rightarrow \quad x_L = 0.29 \text{ m}$$

- Two of the bars may be curtailed to the left of support while the other two may continue as hanger bars.
- Alternatively, all bars may be curtailed at the point of inflection and separate hanger bars may later be provided.

Right of Support B

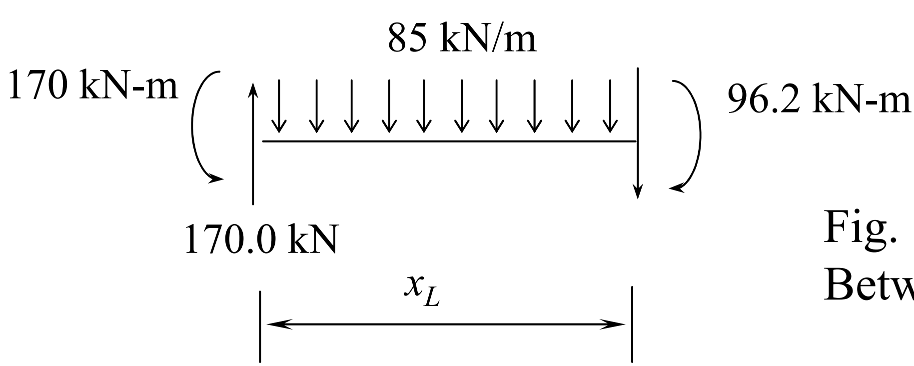


Fig. 11.26. Free Body Diagram of Span Between Support and Right PI.

Referring to Fig. 11.26, we get,

$$170 - 170\,x_R + \frac{85\,x_R^2}{2} - 96.2 = 0 \qquad \Rightarrow \qquad x_R = 0.50 \text{ m}$$

Cutoff Location For Right Side

The cut-off location may be found by considering Fig. 11.27.

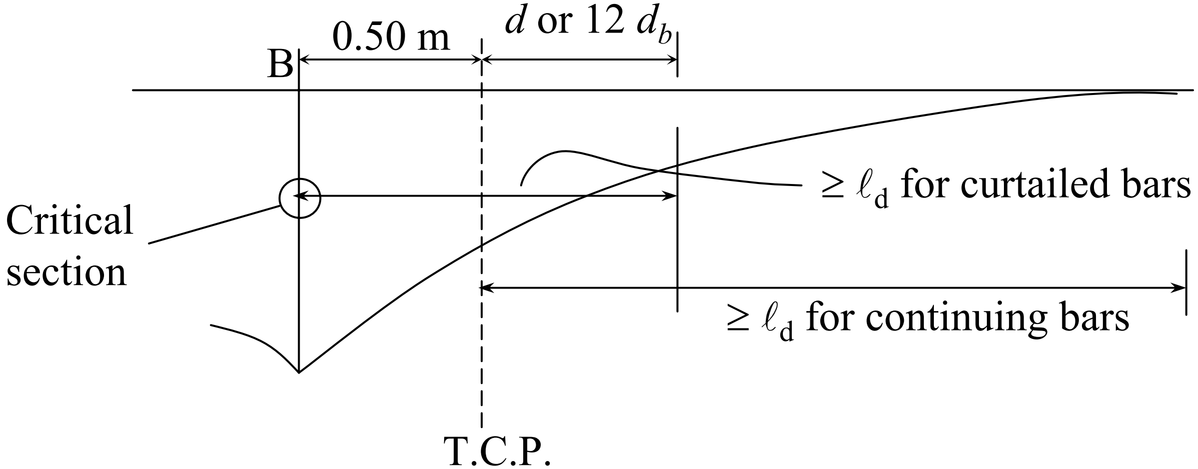


Fig. 11.27. Cutoff Location Right of Support B.

Distance of cut-off point from support B
$= x_R +$ greater of d & 12 d_b
$=$ 0.50 + greater of (450 & 228)mm $= 0.95$ m

$\ell_d = 0.723$ m for #19 bars
Say cutoff at 1.0m right of support B

Check That Continuing Reinforcement Is Developed

$\ell_{d\,\#\,19} = 0.723 \text{ m} < 2.0 - 0.50 = 1.5\text{m}$

$\therefore$ **OK**

In case sufficient space is not available, hooks are to be provided.

Cutoff Location For Left Side (Beyond P.I.)

Let us curtail all the four bars at the P.I.

Distance of cut-off from support B

$$= 0.73 + \text{larger of} \begin{cases} d = 0.450 \\ 12\, d_b = 0.228 \\ \ell_n/16 = \dfrac{5.5}{16} = 0.344 \end{cases}$$

$$= 0.73 + 0.45 = 1.18 \text{ m}$$

$$\textbf{\textit{vs.}} \quad \ell_{d\,\#\,19} = 0.723\text{m} \ \ (\boldsymbol{OK})$$

Say cutoff at 1.2 m left of support B

Examine Cutting 1 – #29 Bars In M^+ Region

Capacity Of Section With Continuing 2 – #25 Reinforcement

$$A_s(2 - \#25) = 1014 \text{ mm}^2$$

$$d = 525 - 53 - 25 / 2 = 459 \text{ mm}$$

$$a = \frac{(1014)(420)}{(0.85)(20)(450)} = 55.7 \text{ mm}$$

$$\phi M_n\,(2 - \#25) = (0.9)(1014)(420)(459 - 55.7/2)/10^6$$

$$= 165.3\ kN\text{-}m$$

Distance From M_{max} Point To T.C.P

- It is assumed that *x* is the distance of cut-off moment from the maximum moment point (Fig. 1.28).
- At maximum moment section, shear force is zero.
- In order to evaluate x, moment may be taken about the cut-off point because it will eliminate the shear force present there from the calculations.

$$\Sigma M = 0 \quad \Rightarrow \quad 165.3 + \frac{85x^2}{2} - 242 = 0$$

$x = 1.34$ m on either side of M^{+}_{max}

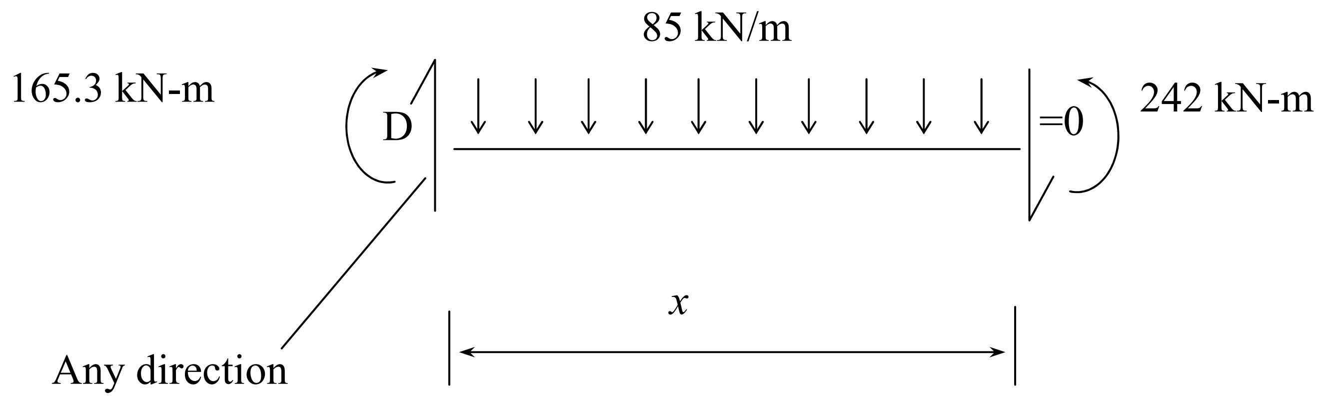


Fig. 11.28. Cut-Off Location on Either Side of Maximum Moment Section.

Now bars must extend up to larger of the following distances from the maximum moment section.

1) x + larger of $\begin{cases} 12d_b = 12(29) = 348\text{mm} \\ d = 459\text{mm} \end{cases}$

$$= 1.34 + \frac{459}{1000} = 1.80 \text{ m}$$

2) $\ell_{d\#29}$ = 1052mm = 1.052 m

Distance from left support = 2.386 – 1.80 = 0.584 m

<u>Say 0.5 m from left support</u>

Distance from right support
= (5.5 – 2.386) – 1.80 = 1.314 m

Say 1.3 m from support B

Check That Continuing Steel (2#25) Is Fully Developed At TCPs

Distance of theoretical cut-off point from left support
= 2.386 – 1.34 = 1.046 m

$$\ell_{d\,\#25} = 845 \text{ mm} = 0.845 \text{ m} < 1.046 \text{ m} \qquad \textbf{\textit{OK}}$$

On the right side, adequate space is available and there is no end of beam problem.

CHECK FOR M$^+$ STEEL DIAMETER

Support

Say $\ell_a = 150$ mm (embedment beyond center of support)

$$M_n\,(2 \text{ - } \#25) = \frac{165.3}{\phi} = 183.7 \text{ kN-m}$$

$$V_u = 202.8 \text{ kN}$$

$$1.3\frac{M_n}{V_u} + \ell_a = 1.3 \left(\frac{183.7\times1000}{202.8}\right) + 150 = 1327 \text{ mm}$$

$$> \ell_{d\#25} = 845 \text{ mm} \qquad \textbf{\textit{OK}}$$

Point Of Inflection

ℓ_a = 459 mm

M_n = 183.7 kN-m

V_u = 264.7 – (85)(0.73) = 202.65 kN

$$\frac{M_n}{V_u} + \ell_a = \frac{183.7\times1000}{202.65} + 459 = 1365 \text{ mm}$$

$$> \ell_{d\#25} = 845 \text{ mm} \qquad \textbf{\textit{OK}}$$

Note: In order to terminate bars in tension zone, ACI 12.10.5 must be satisfied.

Figure 11.29 shows the final detailing of reinforcement for the example beam. Hanger bars and transverse reinforcement are not shown for clarity of diagram.

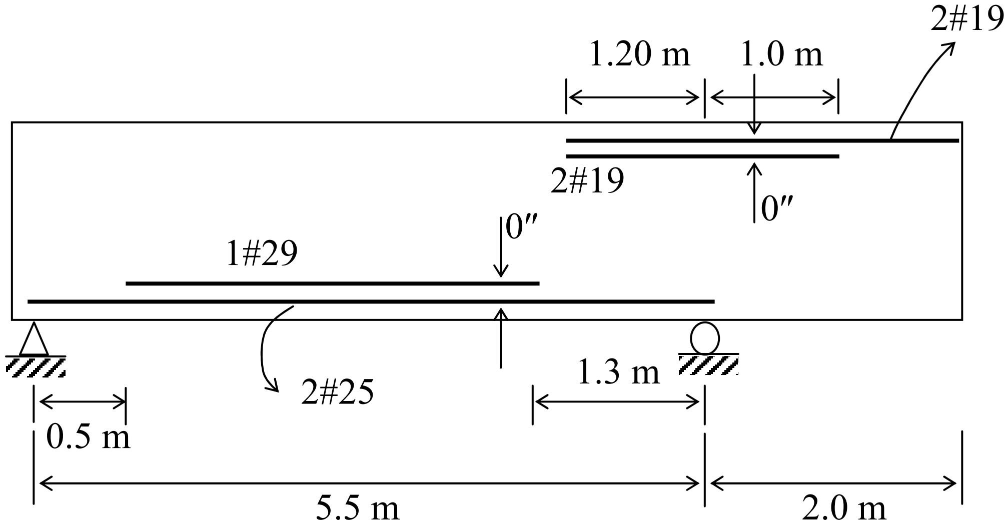
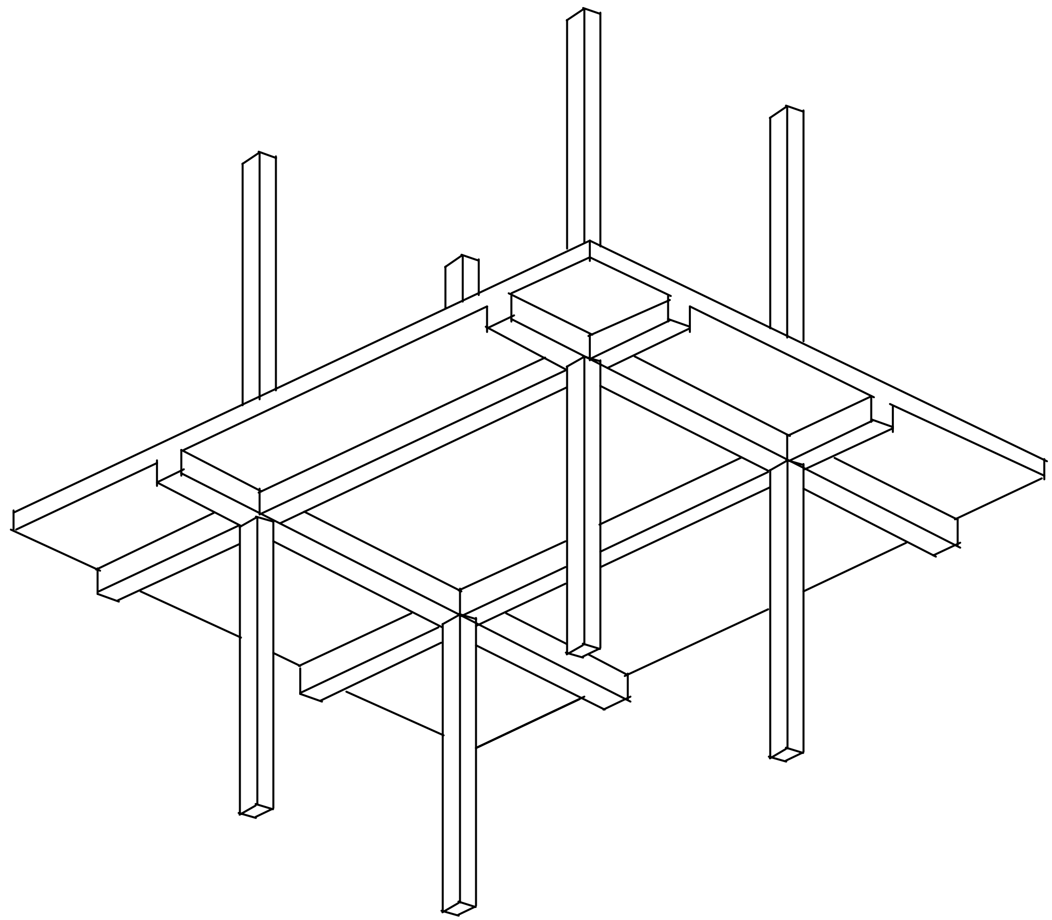


Fig. 11.29. Final Detailing of Reinforcement.

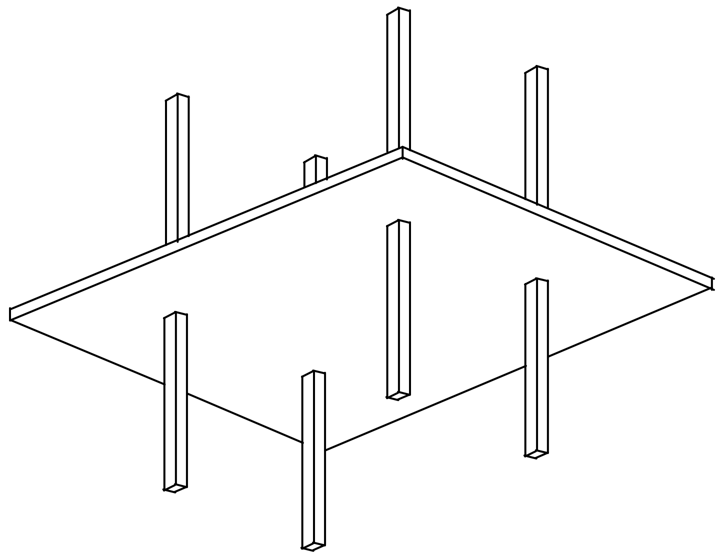
TWO-WAY COLUMN SUPPORTED SLABS

- The slab, which rests on supports on all the four sides and has longer to shorter span ratio lesser than 2.0 is called ***two-way slab***.
- The supports may be beams cast within the slab.
- If the supports are incorporated in both directions within the depth of the slab itself with or without projected beams, the resulting slab system is called ***two-way column supported slab system***.



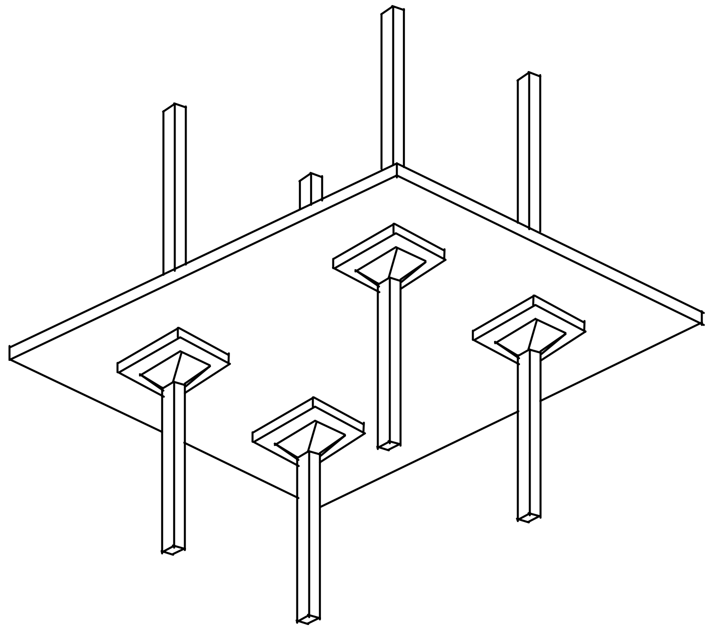
(d) Two-Way Slab With Beams

Fig. 2.1. Types of Two-Way Slabs.



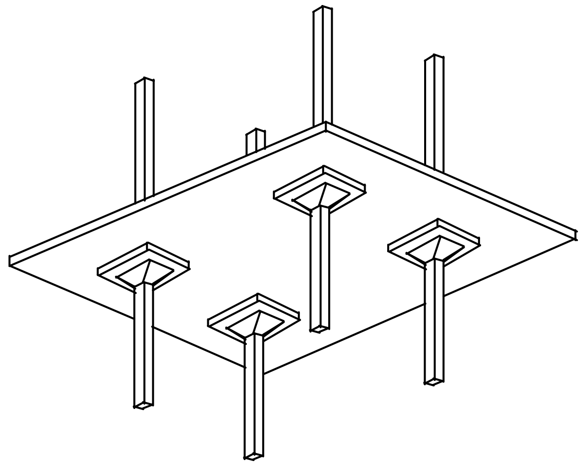
(a) Flat Plate

If there are no projected beams and the slab alone is directly resting on the columns, the resulting slab system is called a ***flat plate***.



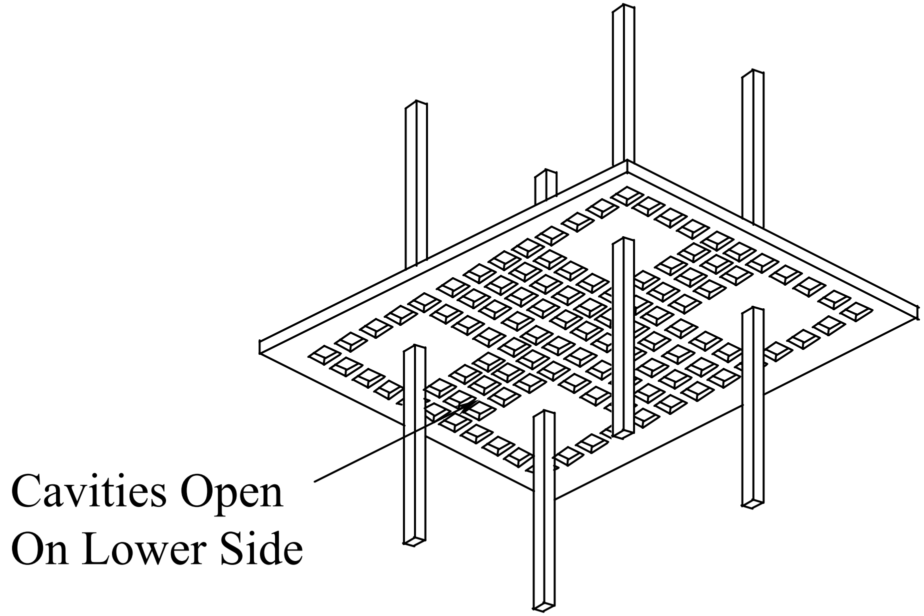
(c) Flat Slab

If some portion of the slab over the columns and surrounding area is constructed with greater but constant thickness or if the column is gradually enlarged like a cone or pyramid at the junction with the slab, the resulting system is called ***flat slab***.



(c) Flat Slab

- The thickened slab near the column is called ***drop panel***.
- Flaring up to the top of the column is called ***column capital***.
- The drop panel commonly extends about one-sixth of the span each way from the column, giving extra strength in the column region while minimizing the amount of concrete at mid-span.



(b) Waffle Slab

- To lighten the slab, reduce the slab moments and save material, the slab at mid-span can be replaced by intersecting ribs.
- However, near the columns, full depth is retained to transmit loads from the slab to the columns.
- This type of slab is known as a ***waffle slab*** or a ***two-way joist system***, where the cavities in the slab are made with fiberglass or metal dome forms.

- Using waffle slab, the cost of material is reduced but the labor cost and the cost of formwork is increased.
- The total cost of waffle slab will approximately become lesser when the total depth of solid slab required exceeds 180 mm.

$$\frac{\text{sum of slab panel sides(m)} \times 2 \times 1000}{180} \geq 190$$

$$\text{or} \quad \text{sum of slab panel sides (m)} \geq 17$$

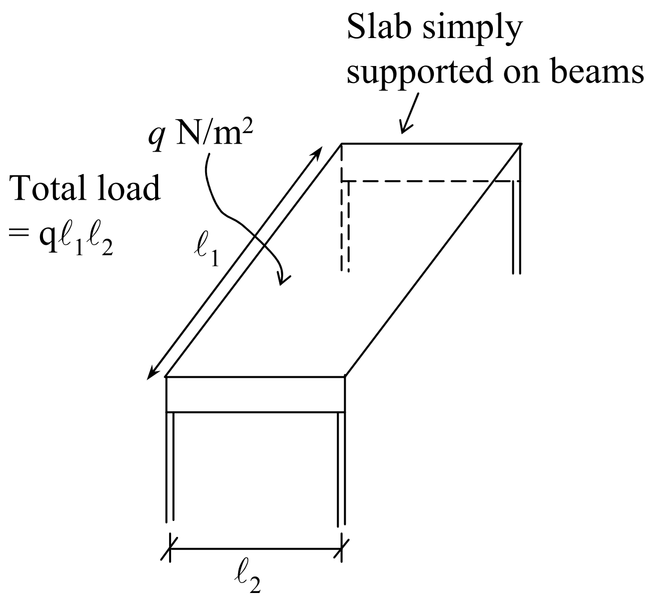
- This means that a square panel 9m × 9m or larger and rectangular panel 10m × 7m or larger may be designed as a waffle slab.

FRACTION OF LOAD TO BE CARRIED IN TWO MUTUALLY PERPENDICULAR DIRECTIONS

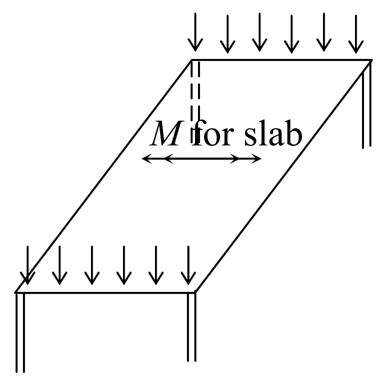
- The example of Fig. 12.3 deals with a rectangular slab simply supported on beams along its shorter edges, which in turn rest on columns.
- When the load is transferred to the edge beams (b-part of the figure), the moment produced in the slab at mid-span of its longer direction is as under:

$$M = \frac{q\ell_1^2}{8} \quad \text{(per unit width)}$$

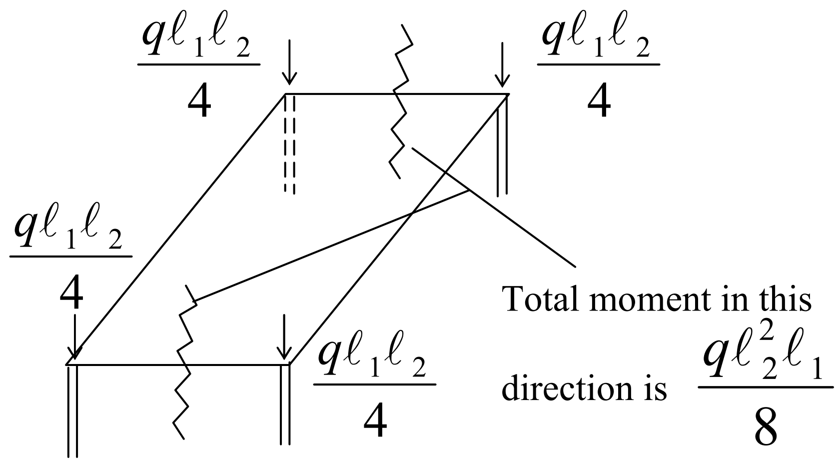
$$= \frac{q\ell_1^2\ell_2}{8} \quad \text{(total moment)}$$



a) Slab With Uniformly Distributed Load



b)Slab After Transferring Load Along the Longer Direction to Beams



c)Slab With Full Transfer of Load to Columns

Fig. 12.3. Example of Slab Load Distribution.

The load on each beam becomes $q\ell_1/2$ (N/m). In c-part of the figure, the beam load is transferred to columns and this produces the following moments along the shorter direction at the mid-span of each beam:

$$\text{Moment at the center} = \frac{q\ell_1}{2} \times \frac{\ell_2^2}{8} = \frac{q\ell_1\ell_2^2}{16}$$

Total moment in both the beams $= \dfrac{q\ell_1\ell_2^2}{8}$

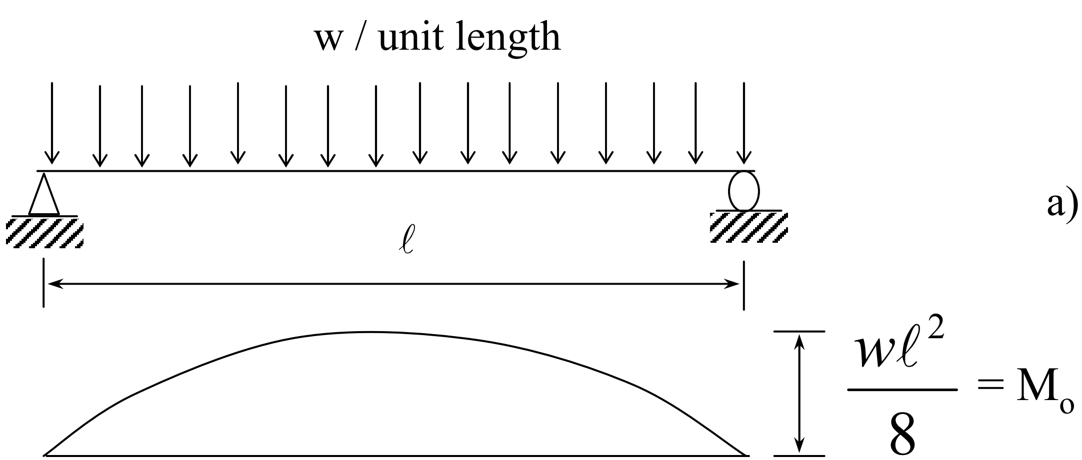
The reaction on each column is $q\ell_1/4$ and the total load transferred to the columns is $q\ell_1\ell_2$, which is equal to the total load of the slab. However, the bending moment in each direction is to be considered for full load. Hence, for every design strip of the slab in both the directions, the total moment in each direction is calculated as follows:

$$\text{Moment in each direction} = \frac{q(width\ of\ load)(span)^2}{8}$$

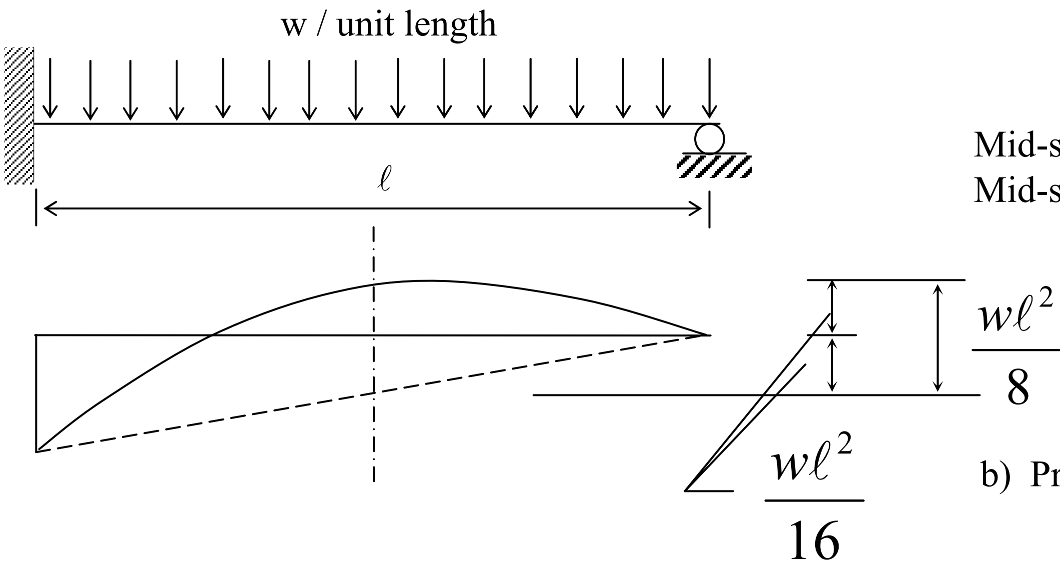
$$= \text{moment due to total load}$$

Following examples prove that the negative moment at the mid-span (or the average of the negative support moments) plus positive moment at the mid-span is always equal to the total static moment, given as under:

$$\text{Total static moment, } M_o = \frac{w\ell^2}{8}$$

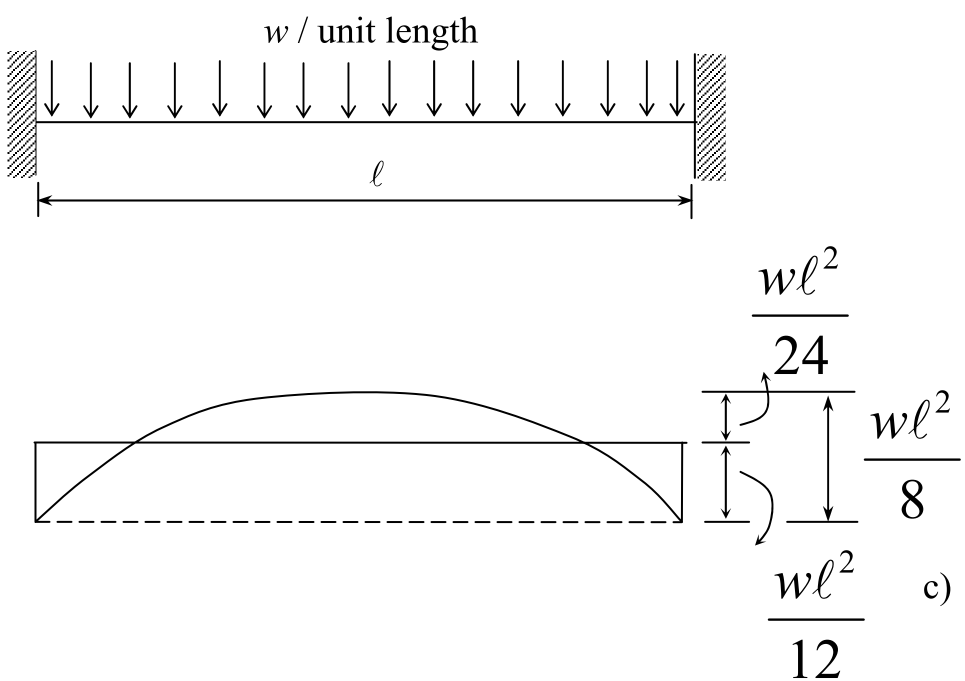


a) Simply Supported Beam



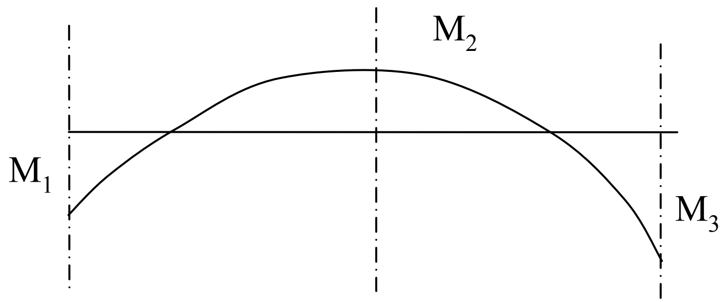
Mid-span +ve moment = $0.50\ M_o$
Mid-span –ve moment = $0.50\ M_o$

b) Propped Cantilever



Mid-span +ve moment = $^1/_3\ M_o$
Mid-span –ve moment = $^2/_3\ M_o$

c) Fixed Ended Beam



d) Bending Moment Diagram of Typical Span of a Continuous Beam

$$M_o = \left|\frac{M_1 + M_3}{2}\right| + \left|M_2\right| = \frac{w\ell^2}{8}$$

Fig. 12.4. Total Static Moment in Various Types of Beams.

VARIATION OF BENDING MOMENT IN A SLAB PANEL

Figure 12.5 shows variation of bending moments both across the longer column and panel centerlines.

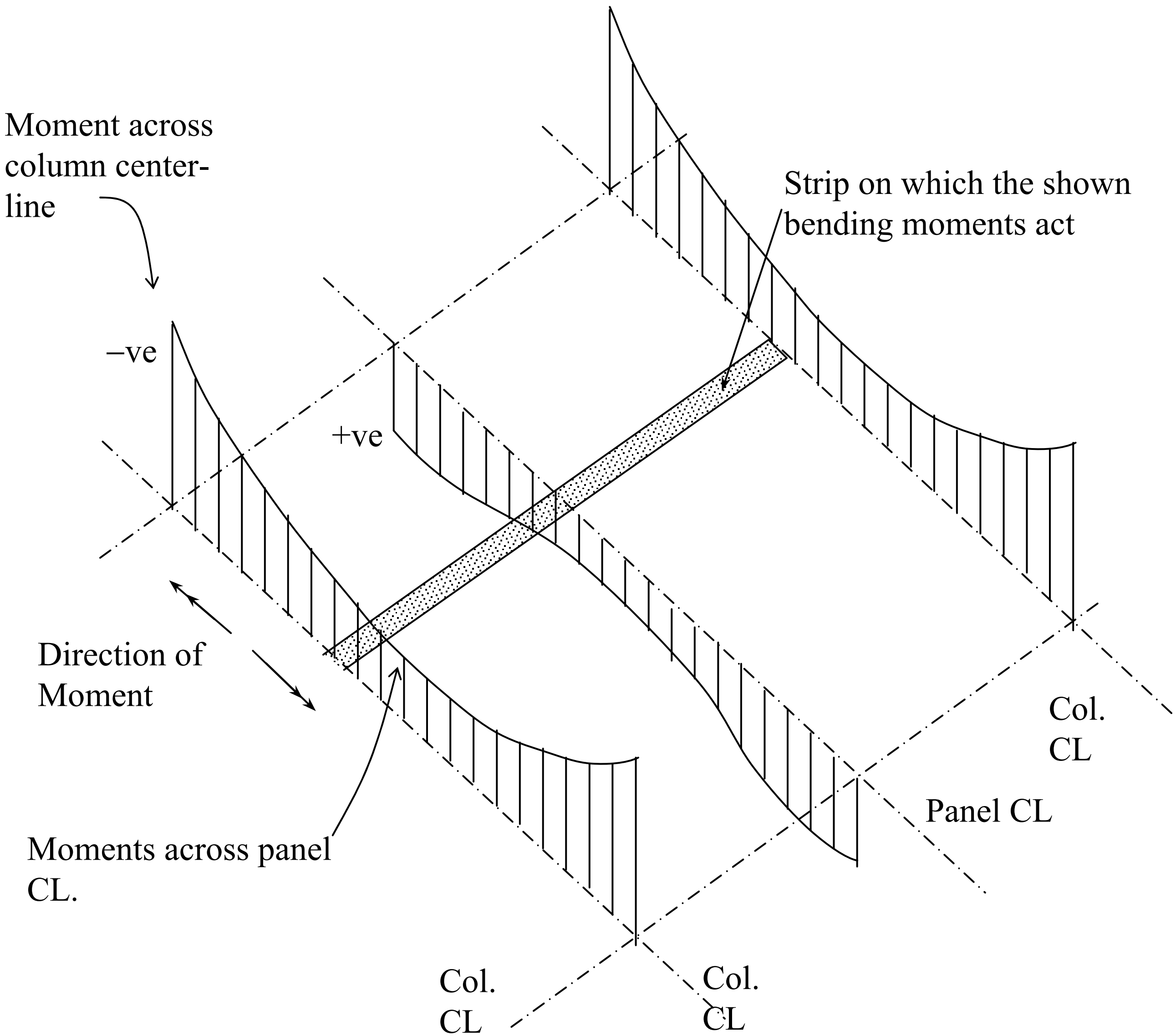


Fig. 12.5. Bending Moment Variation Across Shown Slab Strip.

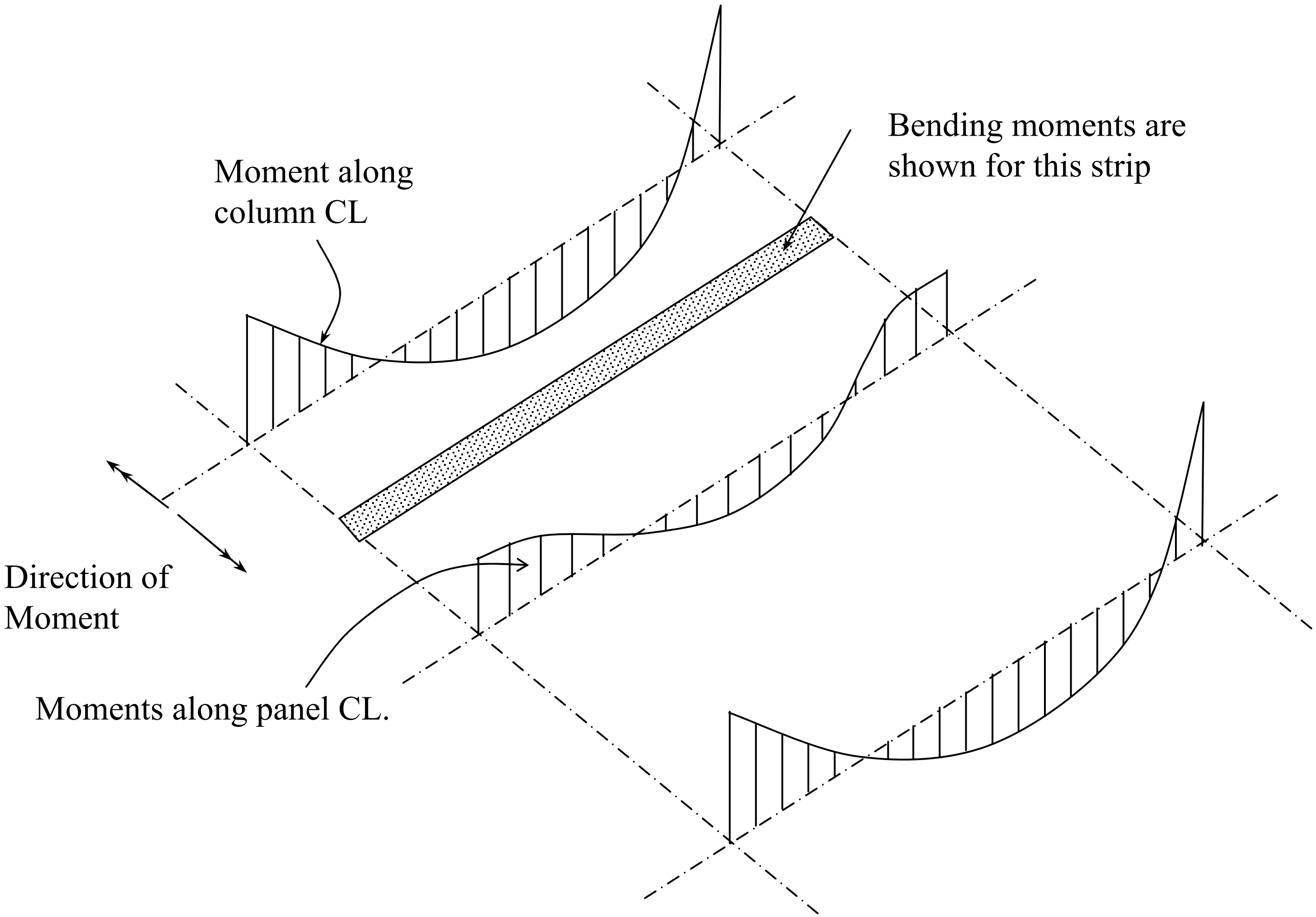


Fig. 12.6. Bending Moment Variation Along Shown Slab Strip.

DESIGN STRIP, COLUMN STRIP AND MIDDLE STRIP

- Each ***design strip / frame*** consists of one line of columns, beam running on these columns (if present) and portion of slab extending up to mid-span of adjacent panel or edge of slab.
- There are at least four design frames for a slab system, namely, exterior long frame, interior long frame, exterior short frame and interior short frame.

ℓ_1 = length of span in the direction in which moments are being determined, measured center-to-center of supports.

= length of a panel parallel to the design strip considered center-to center of supports.

ℓ_2 = length of span transverse to ℓ_1, measured center-to-center of supports.

= width of a panel perpendicular to the design strip considered between centerlines of adjacent panels.

Each design strip / frame is further sub-divided into ***column strips*** and ***middle strips*** (Fig. 12.8).

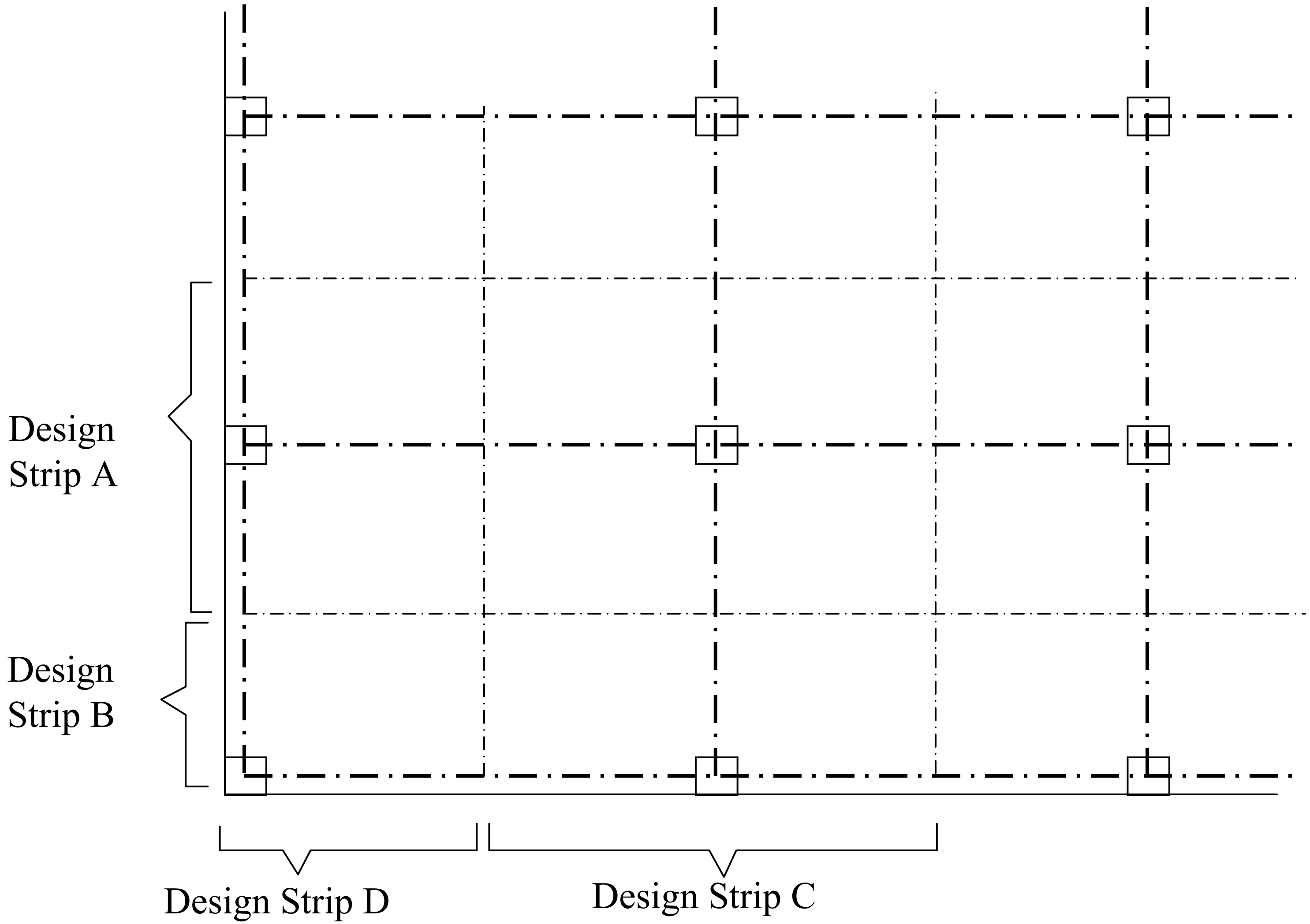


Fig. 12.7. Position of Design Strip in Plan.

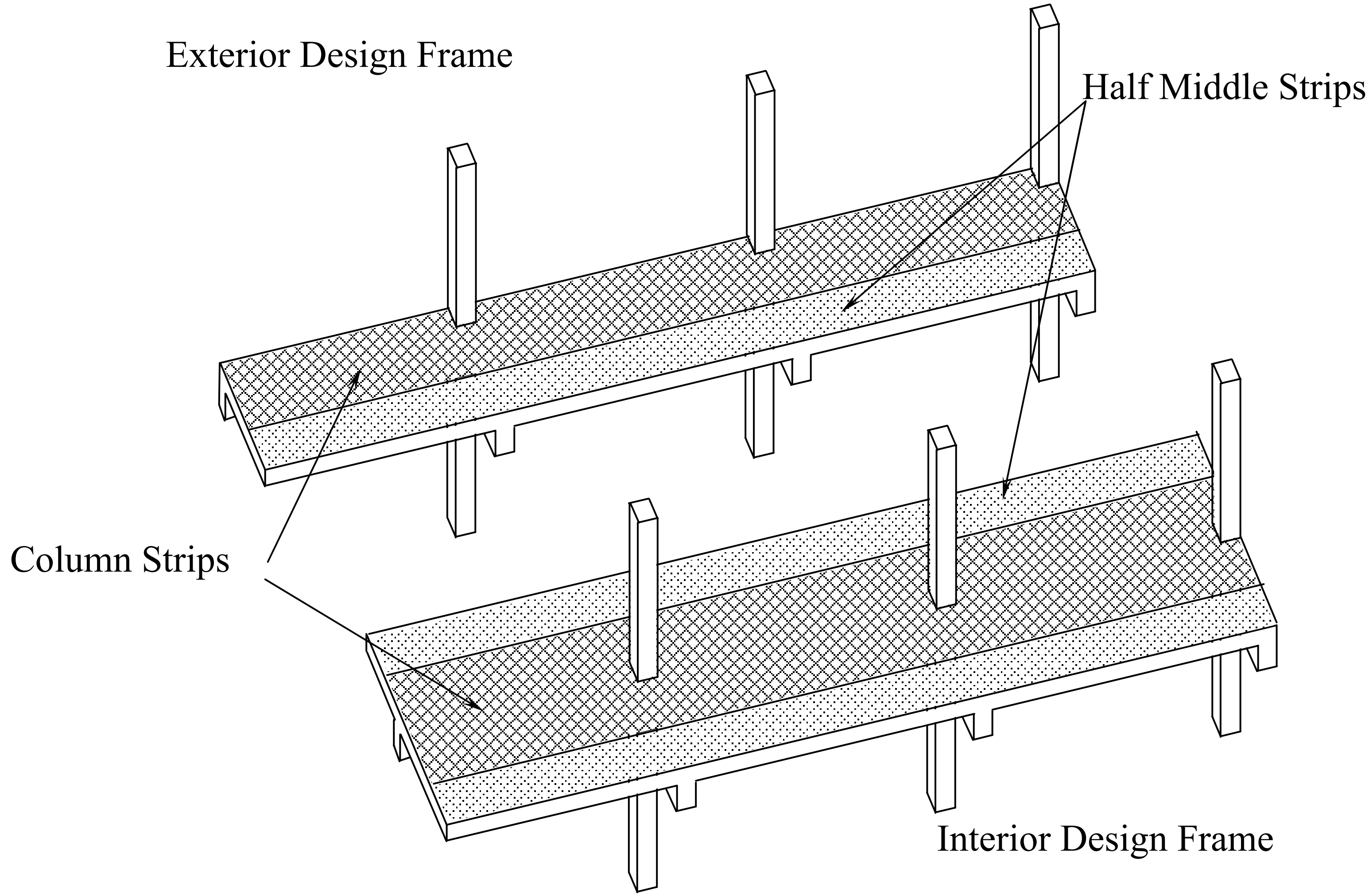


Fig. 12.8. Design, Column And Middle Strips.

- According to ACI 13.2.1, column strip is a design strip with width on each side of column centerline equal to $\ell_2/4$ or /4, whichever is less.
- Column strip includes beams, column capitals and drop panels, if any.
- Middle strip is a design strip bounded by two column strips (ACI 13.2.2).
- Half middle strip starts from column strip on one side and extends up to the panel mid-span on the other side.

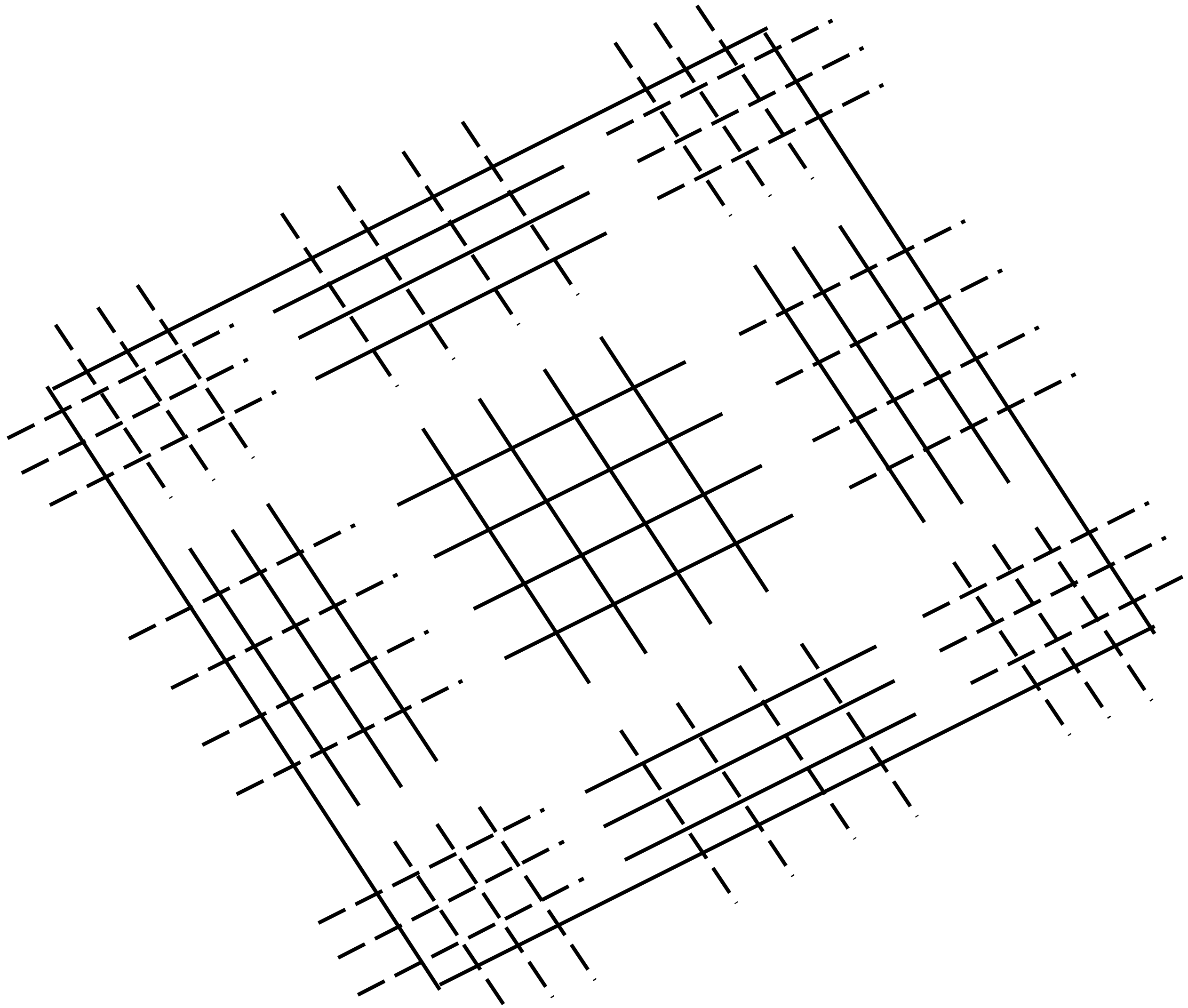


Fig. 12.9. Approximate Steel Placement For A Typical Slab Panel.