MOMENT CURVATURE RELATIONSHIPS AND DUCTILITY

* Moment curvature relationships are very important to find out ductility of the structure and the amount of possible redistribution of stresses.

* It is also very important for earthquake resistance and resistance against blasts.

* Ductility is the deformation capacity of a member / structure after the first yield.

* Ductility gives a measure of the energy dissipation capacity of a member / structure.

* Work done on a structure after yield cannot be stored in the material; it is converted into heat energy and is dissipated to environment.

* Energy dissipation is desirable for structures because the heavy energy imparted by the ground motions, etc., are to be somehow released.

* The expected earthquake forces for the most heavy motions are reduced for design according to the ductility and energy dissipation capacity of the member, the reduction may be zero to more than 12 times.

* The design for strength is carried out for these forces.

* For the remaining loads, the design is carried out for ductility and the detailing is modified to improve the rotation and deformation capacities of the members prior to final collapse.

* Ductility may be categorized as material ductility, section ductility, member ductility and structure ductility.

* In general, the magnitudes of the ductility for each type in case of a single structure are not the same and there is no direct relationship between these; structure ductility is usually much more complex.

* Material ductility is the largest as all the fibers are assumed to be stressed equally.

* The measure of ductility in case of mild steel specimen in tension is the percentage elongation, which may be 0.12 to more than 0.25.
* Section ductility in flexure may be less than the material ductility because all the layers of the material are not equally stressed.
* The fibers away from the N.A. are strained more than the inner fibers and these do not contribute fully in providing the energy absorption.

* Member ductility is lesser than the section ductility because after first yield at the critical sections, most of the curvature only occurs at the yielded sections with other elastic sections preventing the rotation.

* In this way the overall curvature up to fracture is always less.

* In other words, the sections having plastic hinges dissipating energy have a limited length and the other sections do not contribute in energy dissipation.

* The structure ductility is even lesser than the member ductility as all the members are not equally stresses and some of the members may not have plastic hinges in them near collapse.

* The general order of magnitude of ductility in various categories is as under:

Material ductility > Section ductility > Member ductility > Structure ductility

SECTION DUCTILITY

* The section ductility mainly depends on whether the section is under-reinforced or over-reinforced.

* The balanced condition has to be investigated to get insight into the behavior of a flexural member.

* Balanced condition is achieved when the concrete stresses reach some limiting condition and the steel starts yielding at the same time, as shown in Fig. 17.1.

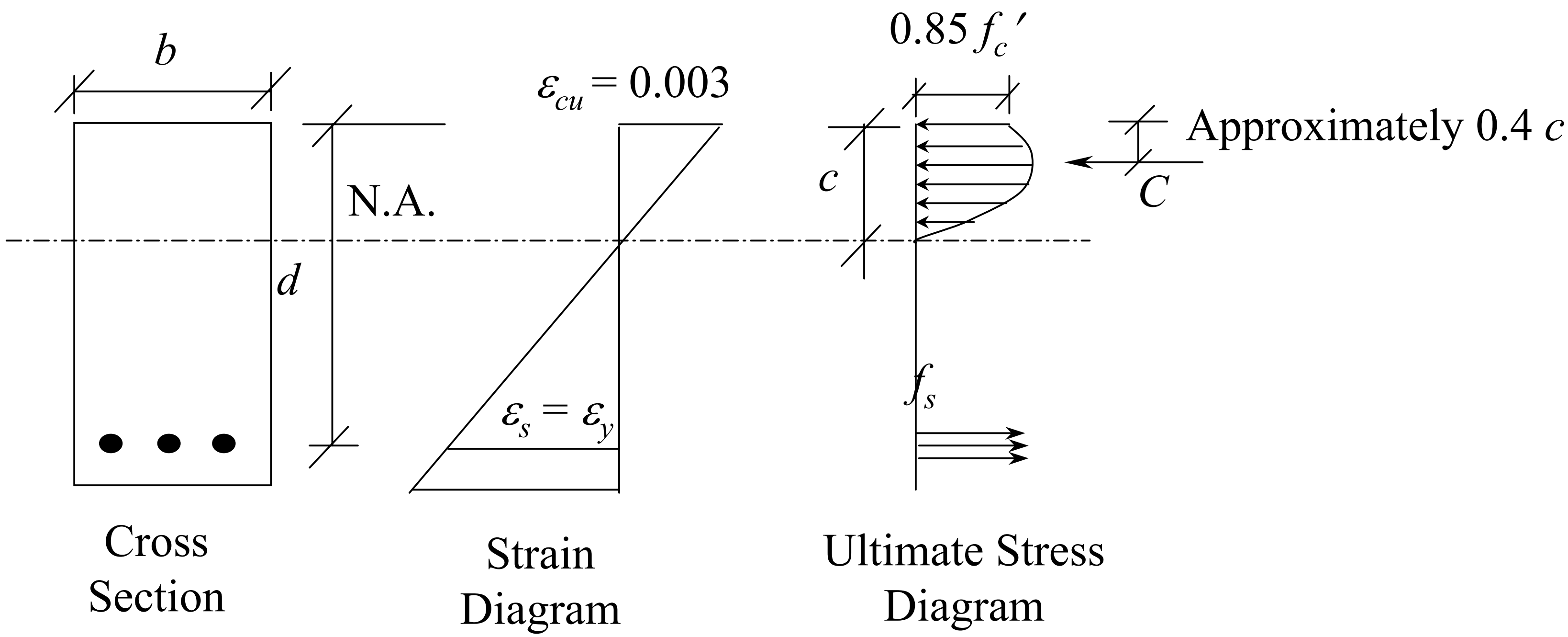

Fig. 17.1. Strains and Stresses at Balanced Condition.

A typical stress-strain curve for concrete is shown in Fig. 17.2.

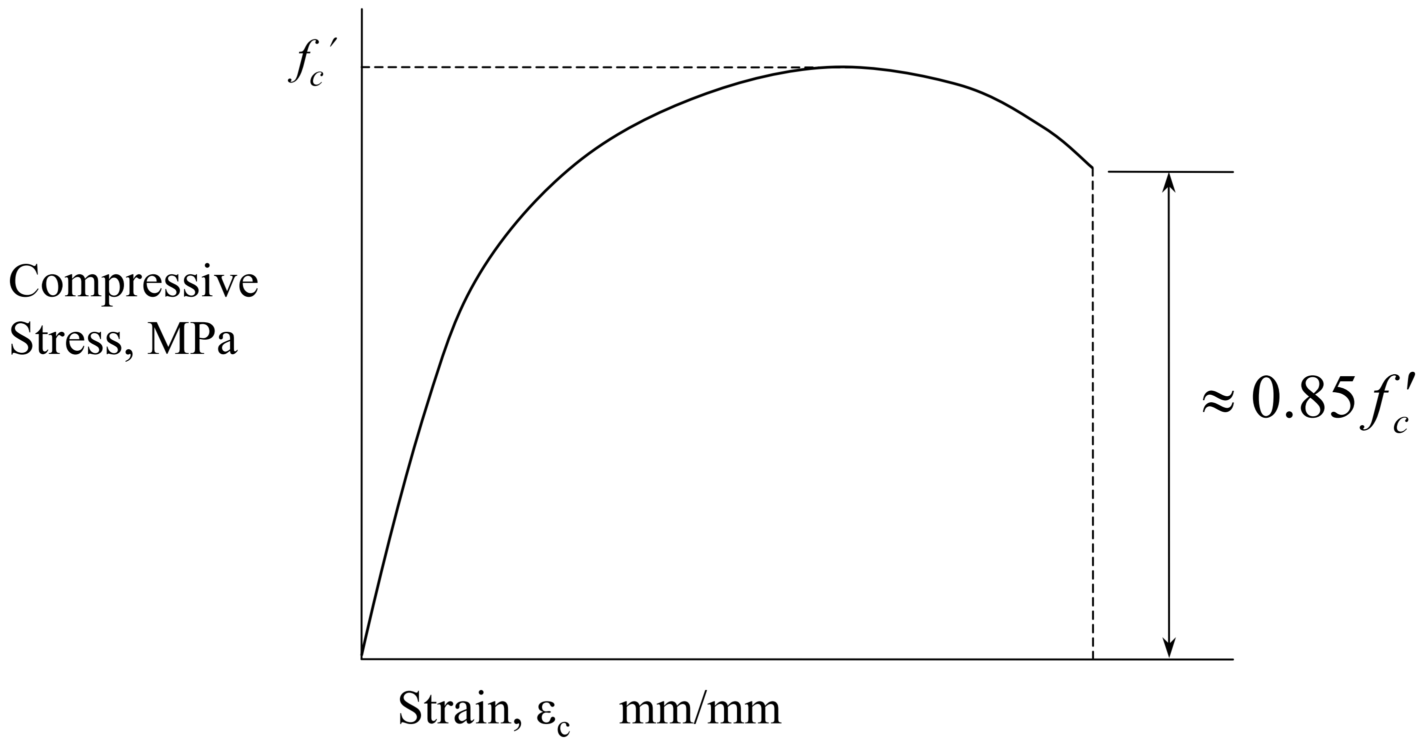

Fig. 17.2. Typical Concrete Stress-Strain Curve.

A typical stress-strain curve for steel is shown in Fig. 17.3.

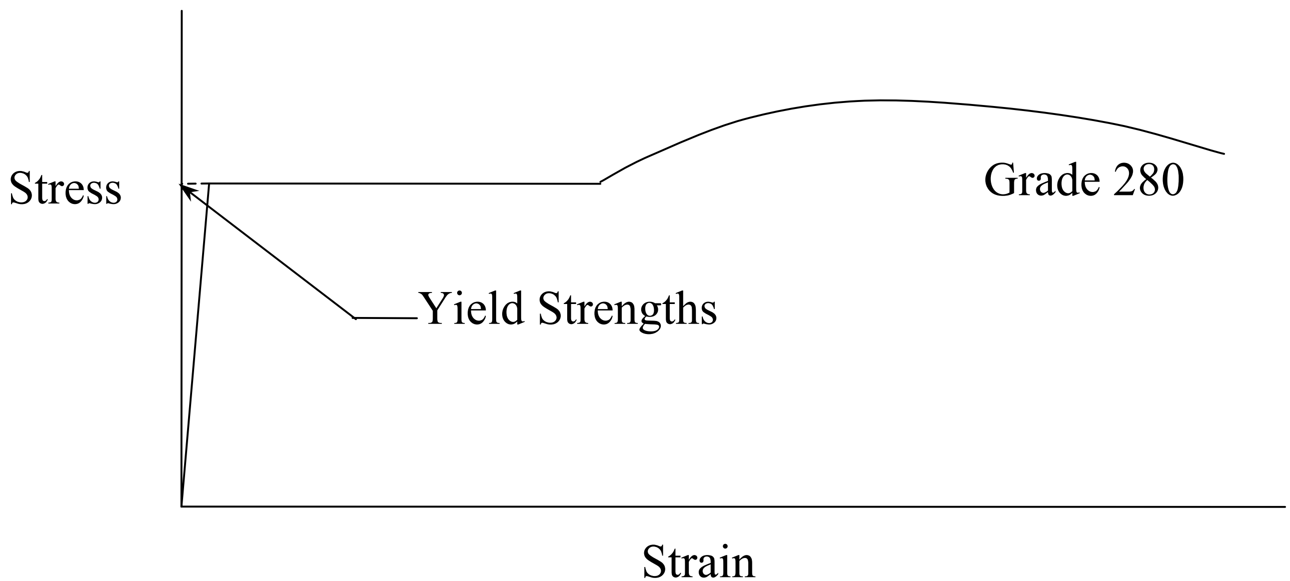

Fig. 17.3. Typical Stress-Strain Curve for Steel.

Knowing the size of the member (b and d) and the material strengths (f_c' and f_y), the steel ratio for the balanced condition may be determined as follows:

$$\rho_b = 0.85\,\beta_1 \frac{f_c'}{f_y} \frac{600}{600 + f_y}$$

If the steel is provided more than that given by the balanced steel, the section will be over-reinforced having insignificant ductility. However, if the steel is provided less than that given by the balanced steel, the section will be under-reinforced and its ductility is discussed here.

Stage 1: Before Cracking

$$M_{\mathrm{cr}} = \frac{f_{t,\max} I_{trans}}{y_t}$$

$$\phi_{\mathrm{cr}} = \frac{\varepsilon_{ct}}{y_t} = \frac{f_{t,\max}}{E_c y_t}$$

or $$\phi_{\mathrm{cr}} = \frac{M_{cr}}{E_c I_{trans}}$$

Moment-curvature relationship is shown by a graph between the curvature (ϕ) taken on the x-axis and moment taken on the y-axis.

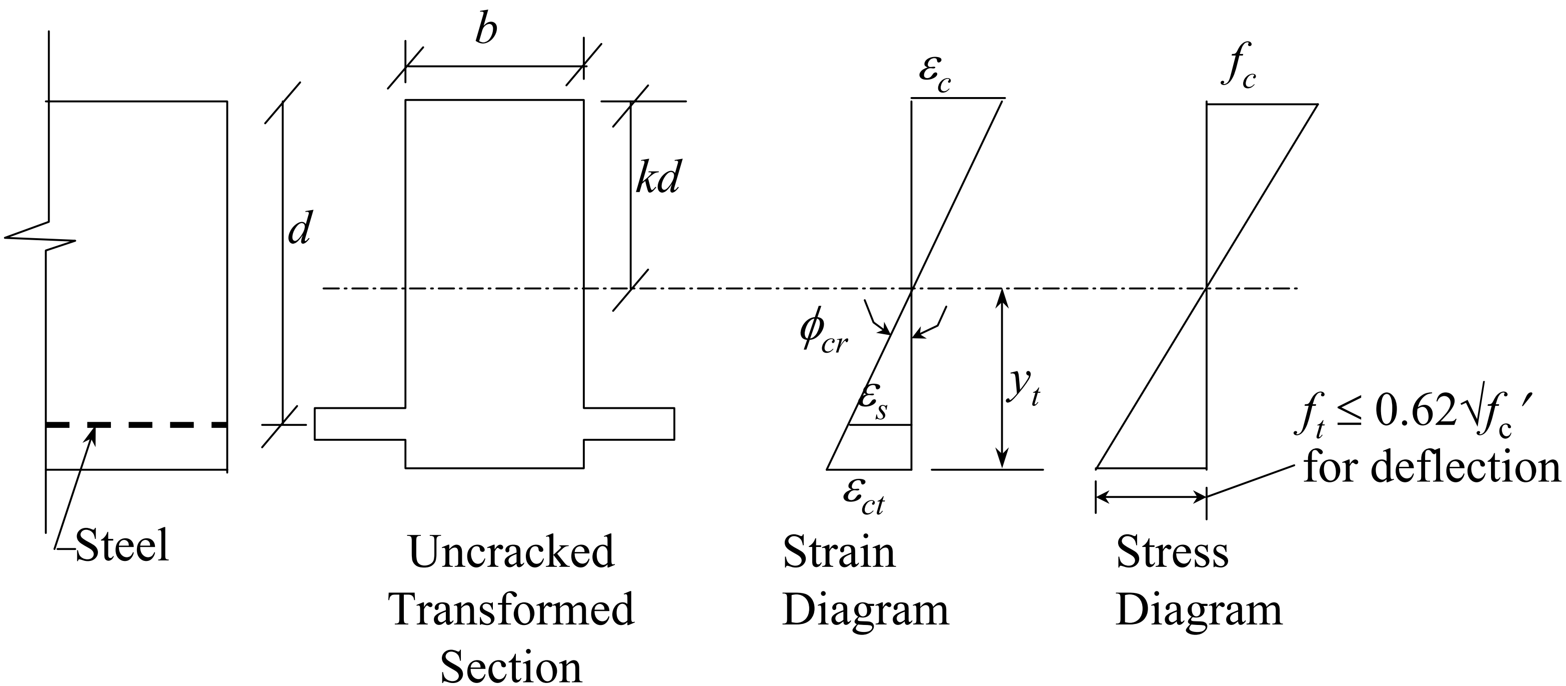

Fig. 17.4. Curvature Capacity of Uncracked Section.

Stage 2: After Cracking To Steel Yielding

* In this stage, we neglect the contribution of concrete in tension.

* The concrete in compression can still be assumed to behave elastically as the concrete stress is less than $0.7 f_c'$.

* The strain diagram, stress diagram and the transformed section are shown in Fig. 17.5.

From the strain diagram, we get:

$$\frac{\varepsilon_c}{\varepsilon_y} = \frac{kd}{d-kd} = \frac{k}{1-k}$$

$$\varepsilon_{\mathrm{c}} = \frac{k}{1-k}\times\varepsilon_y = \frac{k}{1-k}\times\frac{f_y}{E_s}$$

$$f_{\mathrm{c}} = E_{\mathrm{c}} \times \varepsilon_{\mathrm{c}} = \frac{E_c}{E_s}f_y\times\frac{k}{1-k} = \frac{f_y}{n}\times\frac{k}{1-k}$$

$$C_c = T \qquad \tfrac{1}{2} f_c\, b\, kd = A_s f_y$$

$$\tfrac{1}{2}\,\frac{f_y}{n}\times\frac{k}{1-k}\, b\, kd = \rho bd\times f_y$$

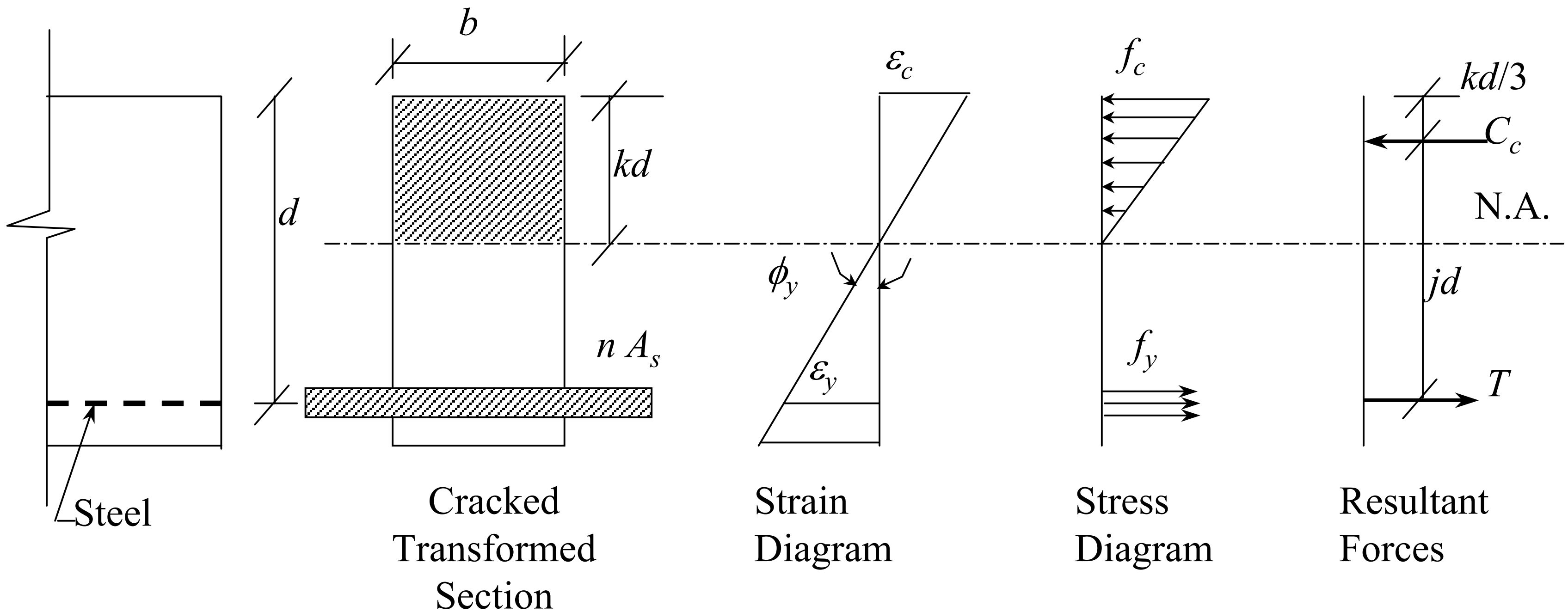

Fig. 17.5. Curvature Capacity of Cracked Elastic Section.

$$\frac{k^2}{1-k} = 2\rho n$$

$$k^2 + 2n\rho k - 2n\rho = 0$$

$$k = \sqrt{(\rho n)^2 + 2\rho n} - \rho n$$

$$j = 1 - k/3$$

$$M_{\mathrm{y}} = A_{\mathrm{s}} f_{\mathrm{y}} jd$$

$$\phi_{\mathrm{y}} = \frac{\varepsilon_y}{d - kd} \quad \text{or} \quad \phi_{\mathrm{y}} = \frac{M_y}{E_c (I_{cr})_{trans}}$$

Stage 3: Steel Yielding To Ultimate

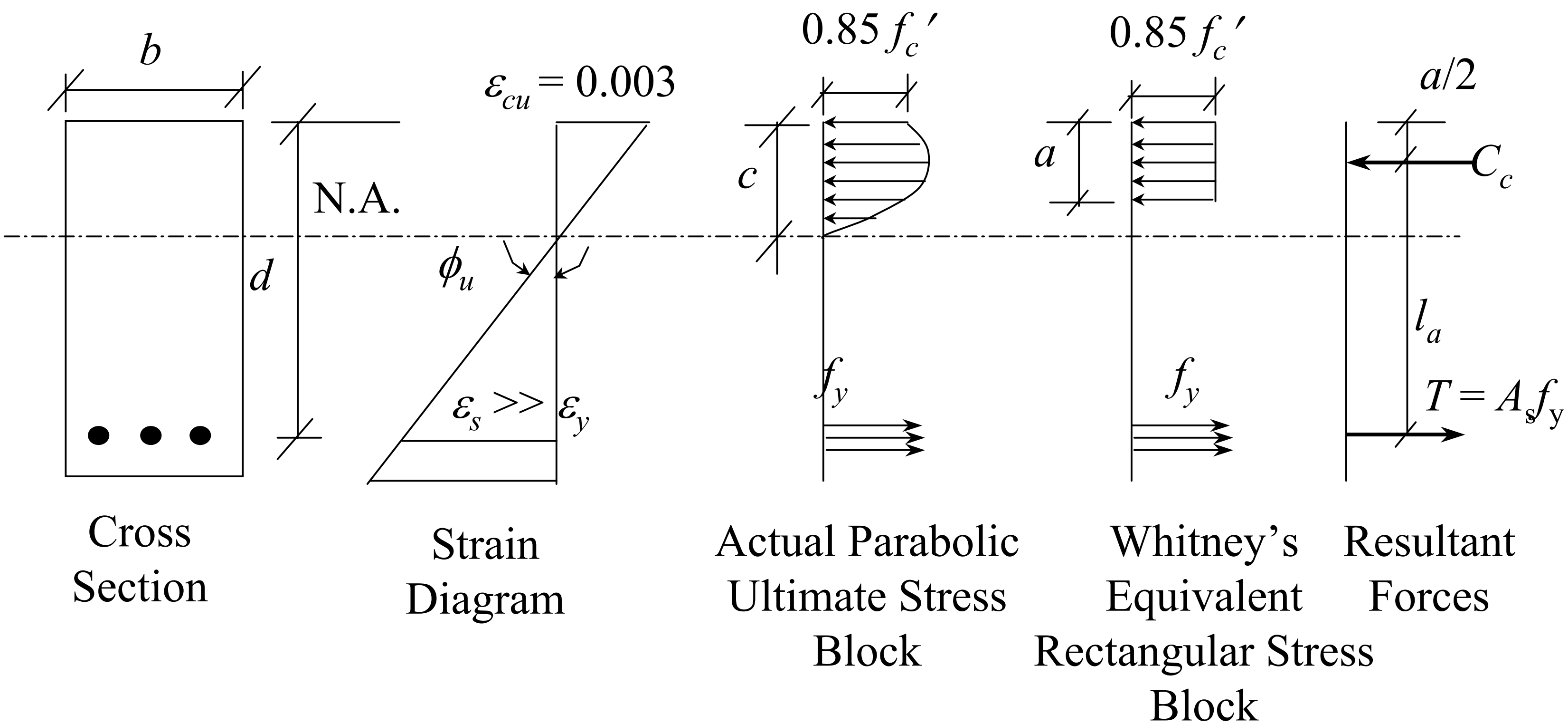

Fig. 17.6. Ultimate Curvature Capacity of Section.

$$T = C_c \quad \Rightarrow \quad A_s f_y = 0.85 f_c' b\, a$$

$$a = \frac{A_s\, f_y}{0.85 f_c' b} \qquad \text{and} \qquad c = a / \beta_1$$

$$M_u = T\, l_a = A_s f_y\, (d - a/2)$$

$$\phi_{\mathrm{u}} = \frac{\varepsilon_{cu}}{c}$$

M – ϕ **Curve**

Moment – curvature diagram ($M - \phi$ diagram) for a particular cross-section is plotted with x-axis representing the curvature and the y-axis representing the moment, as shown in Fig. 17.7.

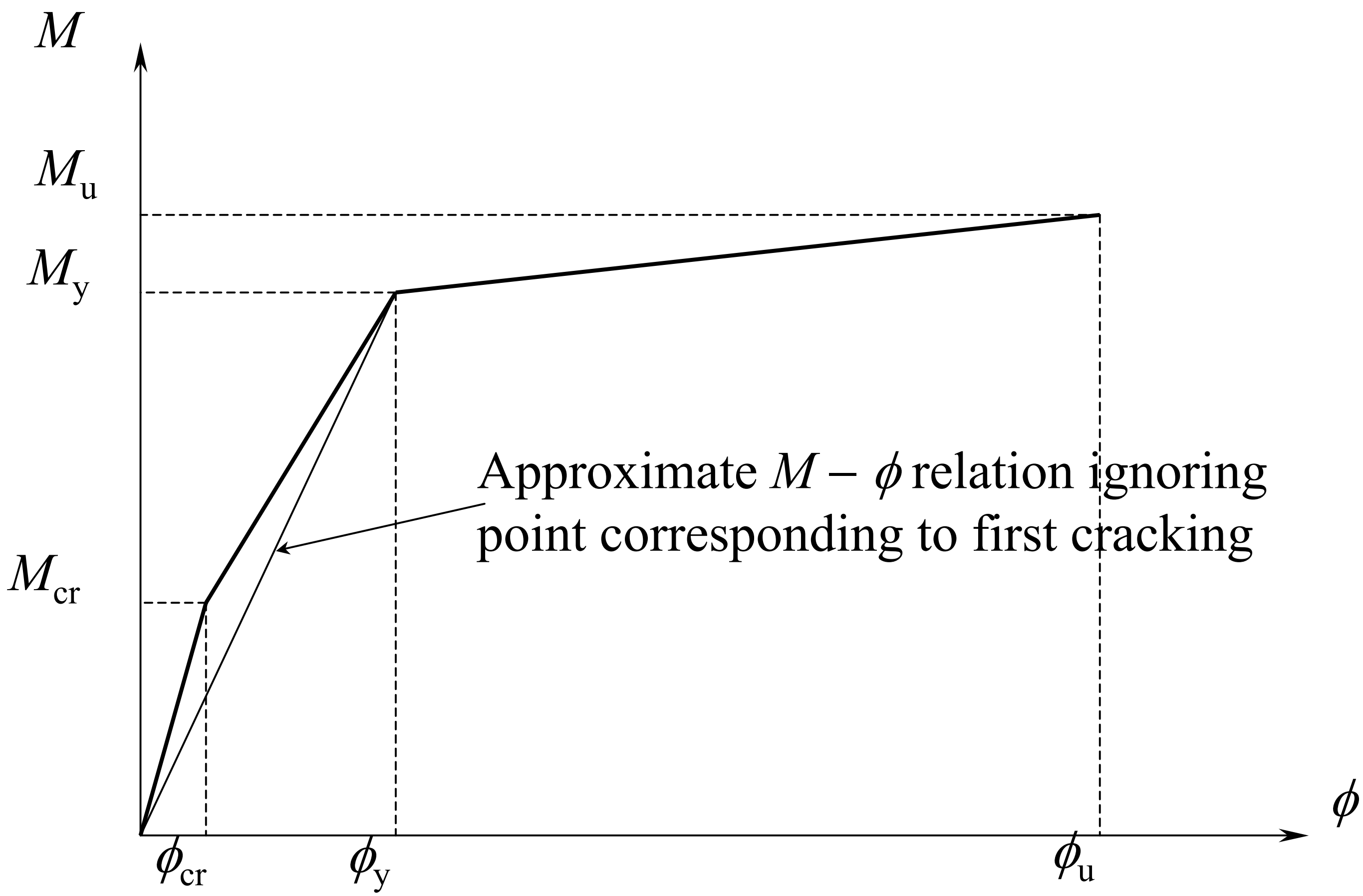

Fig. 17.7. A Typical Moment – Curvature Curve.

Section Ductility

Section ductility is defined as the curvature at ultimate (ϕ_u) divided by the curvature at the first yield (ϕ_y).

$$\text{Section ductility, } \mu = \frac{\phi_u}{\phi_y}$$

Effect Of f_c' On Section Ductility

The effect of varying f_c' keeping all other parameters constant will be studied here.

By increasing f_c', E_c will increase, modular ratio will reduce, factor $n\rho$ will reduce and factor k will also reduce.

This will result in less value of ϕ at the first yield.

At the ultimate, larger value of f_c' will give less depth of neutral axis, c, as shown below:

Effect Of f_c' On Section Ductility

$$c = \frac{1}{\beta_1} \frac{A_s f_y}{0.85 f_c' b} \qquad\qquad c \propto \frac{1}{f_c'}$$

The result will be a larger value of $\phi_u = \varepsilon_{cu} / c$. Due to larger ϕ_u and smaller ϕ_y, the value of section ductility will be increased.

Effect Of f_y On Section Ductility

By increasing f_y, ε_y will increase, $d - kd$ will remain constant and the value of ϕ at the first yield will be more.

At the ultimate condition, larger value of f_y will give larger depth of neutral axis, c, as shown below:

$$c = \frac{1}{\beta_1} \frac{A_s f_y}{0.85 f_c' b} \qquad c \propto f_y$$

The result will be a lesser value of $\phi_u = \varepsilon_{cu} / c$. Due to lesser ϕ_u and larger ϕ_y, the value of section ductility will be reduced.

Effect Of Steel Ratio On Section Ductility

When the steel ratio (ρ) is increased keeping all other parameters constant, $n\rho$ is increased, the value of k becomes more, $d - kd$ reduces and the value of ϕ at the first yield is increased.

At the ultimate condition, more steel ratio gives larger depth of neutral axis, c, as shown below:

$$\boldsymbol{c} = \frac{1}{\beta_1}\frac{A_s f_y}{0.85 f_c' b} = \frac{1}{\beta_1}\frac{\rho d f_y}{0.85 f_c'} \qquad c \propto \rho$$

The result is be a lesser value of $\phi_u = \varepsilon_{cu} / c$.

Due to lesser ϕ_u and larger ϕ_y, the value of section ductility will be reduced.

Effect Of Section Depth On Section Ductility

By increasing the section effective depth (d) and keeping all other parameters including the amount of steel A_s (not the steel ratio) constant, ρ will reduce, k will be less, $d - kd$ will increase and the value of ϕ at the first yield will be less.

At the ultimate condition, varying d will have no significant effect on the depth of neutral axis, c, as shown below:

$$c = \frac{1}{\beta_1} \frac{A_s f_y}{0.85 f_c' b}$$

The value of ϕ_u will remain nearly constant and hence the value of section ductility will be slightly increased.

MEMBER DUCTILITY

* The member ductility may be evaluated by plotting the load – deflection curve and determining the values of maximum deflection at first yield, Δ_y, and the ultimate maximum deflection, Δ_u.
* The member ductility is defined as the ultimate maximum deflection, Δ_u divided by the maximum deflection at first yield, Δ_y.
* It depends upon the section properties, the loading and the boundary conditions.

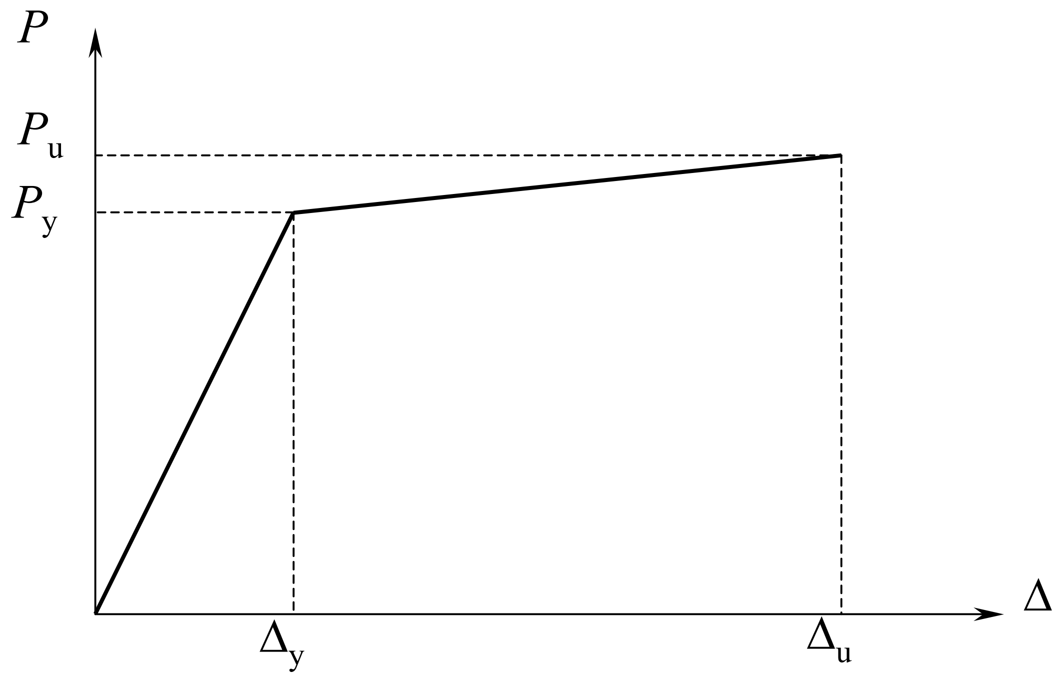

Fig. 17.8. A Typical Load – Deflection Curve.

$$\text{Member ductility, } \mu = \frac{\Delta_u}{\Delta_y}$$

A typical load – deflection curve is shown in Fig. 17.8. The area under this curve shows the energy absorbing capacity of the member.

A simplified $M - \phi$ curve may be used for plotting the load – deflection curve in order to reduce the calculation efforts.

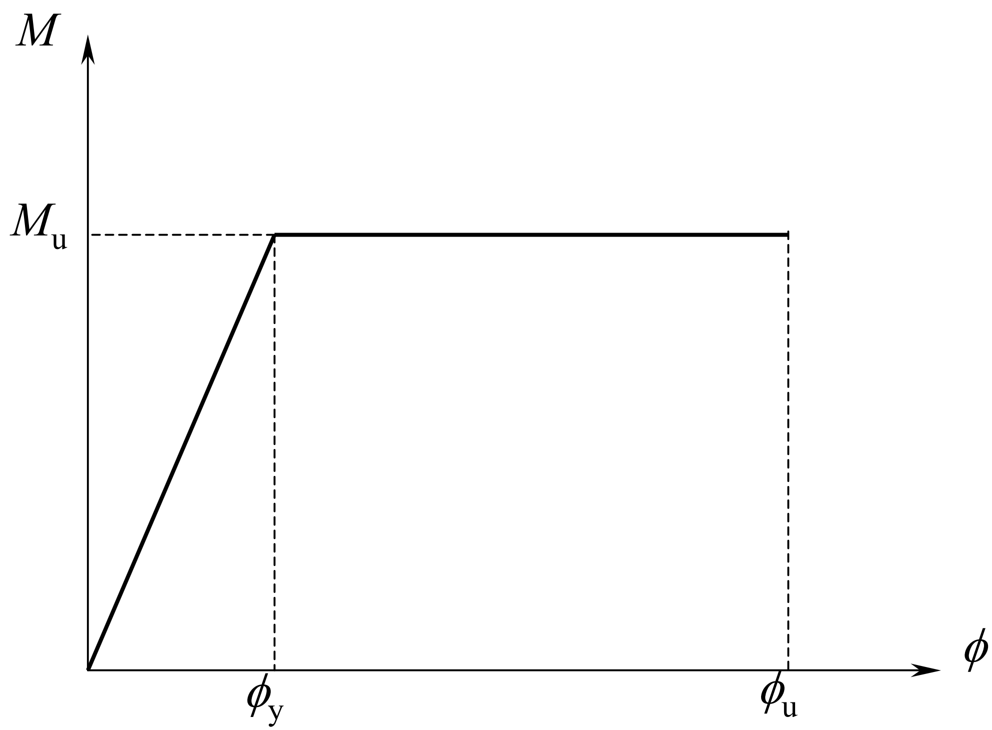

Fig. 17.9. A Typical Simplified Moment – Curvature Curve.

$P - \Delta$ CHARACTERISTICS

* To explain the procedure of showing the force – deformation characteristics, a beam of known cross section present throughout the length is considered in Fig. 17.10.

* Two loads are applied at the third points and are gradually increased in magnitude to observe the flexural characteristics.

* A simplified $M - \phi$ curve is used to find out curvatures from bending moments.

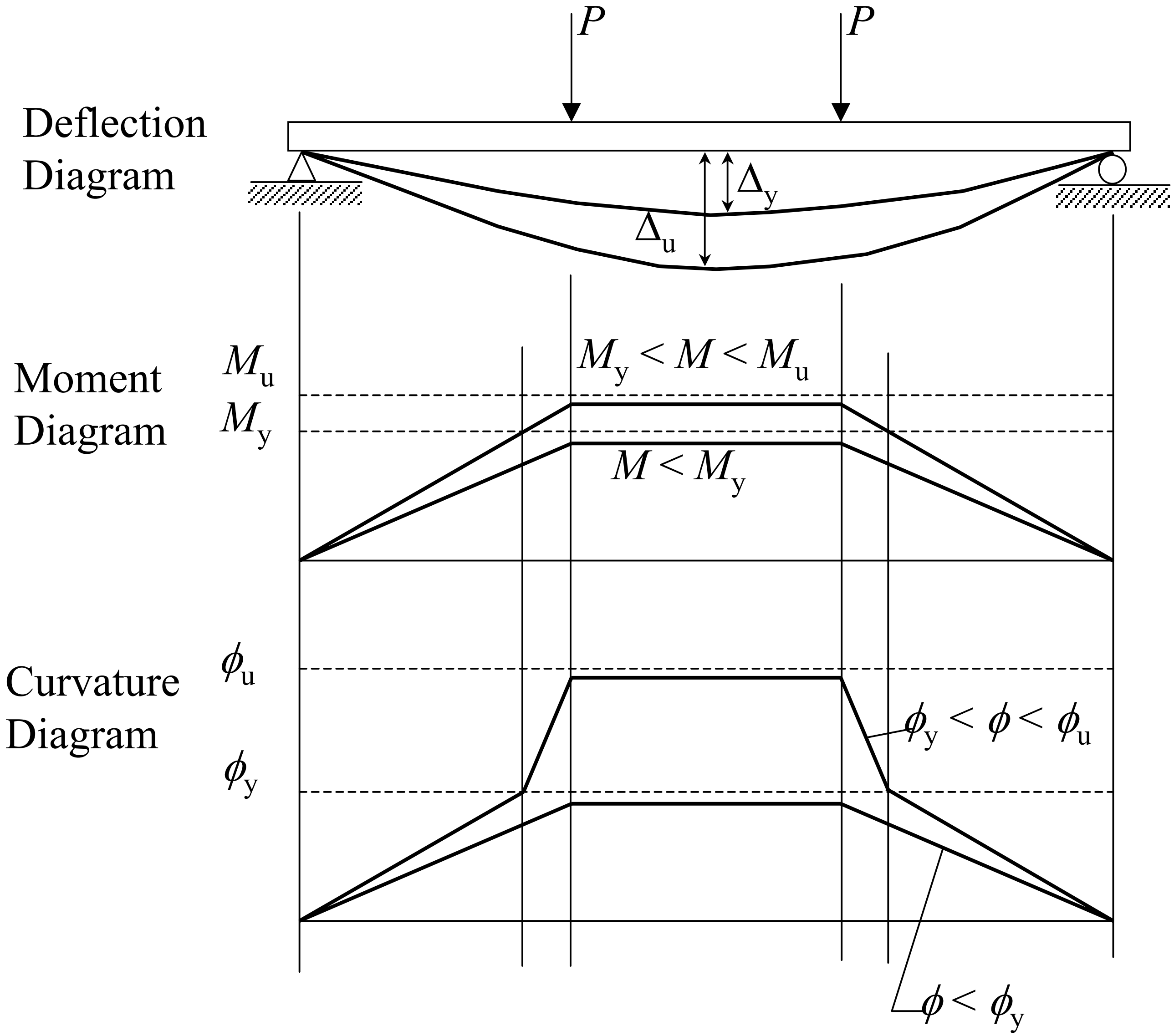

Fig. 17.10. Flexural Behavior of Beam Beyond First Yielding.

Deflections From Curvatures

The curvature is defined as the rotation per unit length of member for infinitely small length.

The total rotation between any two points A and B of a member is given by:

$$\theta_{\mathrm{AB}} = \int_A^B \phi \, dx = \text{area of curvature diagram}$$

between the points A and B.

According to one of the moment area theorem, the tangential deviation of a point from tangent through another point B, also on the elastic curve, ($t_{A/B}$) is equal to the first statical moment of the area of curvature diagram ($\phi = M/EI$) between points A and B taken around a vertical line through point A (see Fig. 17.11).

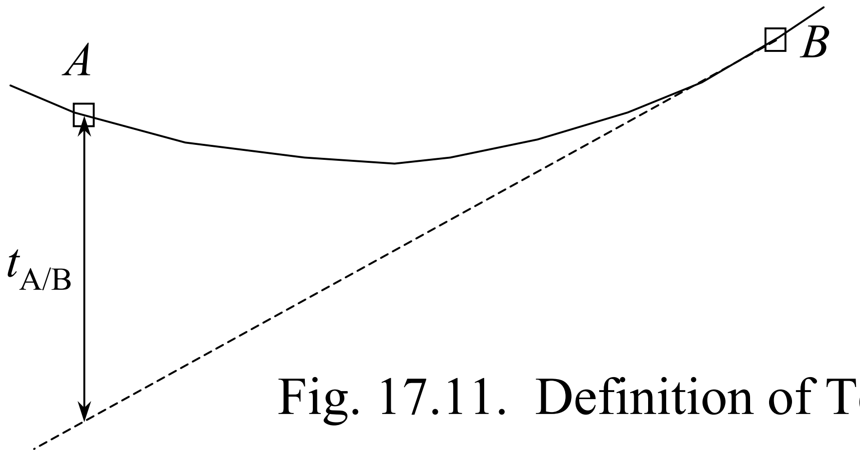

Fig. 17.11. Definition of Term $t_{A/B}$.

In case of Fig. 17.12b, the central deflection may be found as follows:

$$v_c = \frac{t_{A/B}}{2} - t_{C/B}$$

For fixed ended beam: $v_c = t_{A/C}$

$t_{A/C}$ = **vertical distance of A on elastic curve from tangent to elastic curve at C**

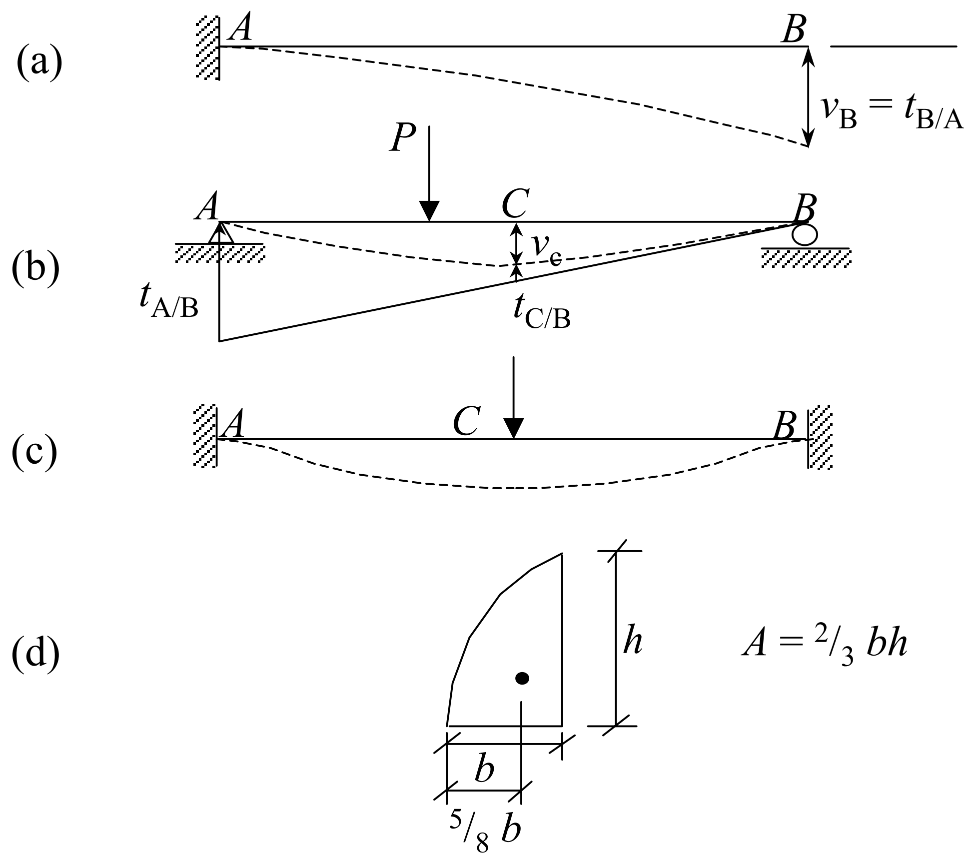

Fig. 17.12. Examples for Calculation of Deflections.

Plastic Hinge Length

* The zone of yielding of a plastic hinge in case of reinforced concrete member is spread out from the point of maximum moment.

* It can be assumed that this zone spreads out by distance lesser of $d / 2$ and $\ell / 28$ on either side of the location of maximum moment.

Member Ductility When More Than One Yields Occur Before Ultimate

The first and last portions of the curves may be extended backwards and there point of intersection is taken as the point of fictitious single yield denoted by pseudo Δ_y.

The member ductility is then defined as follows:

$$\text{Member ductility, } \mu = \frac{\Delta_u}{pseudo\ \Delta_y}$$

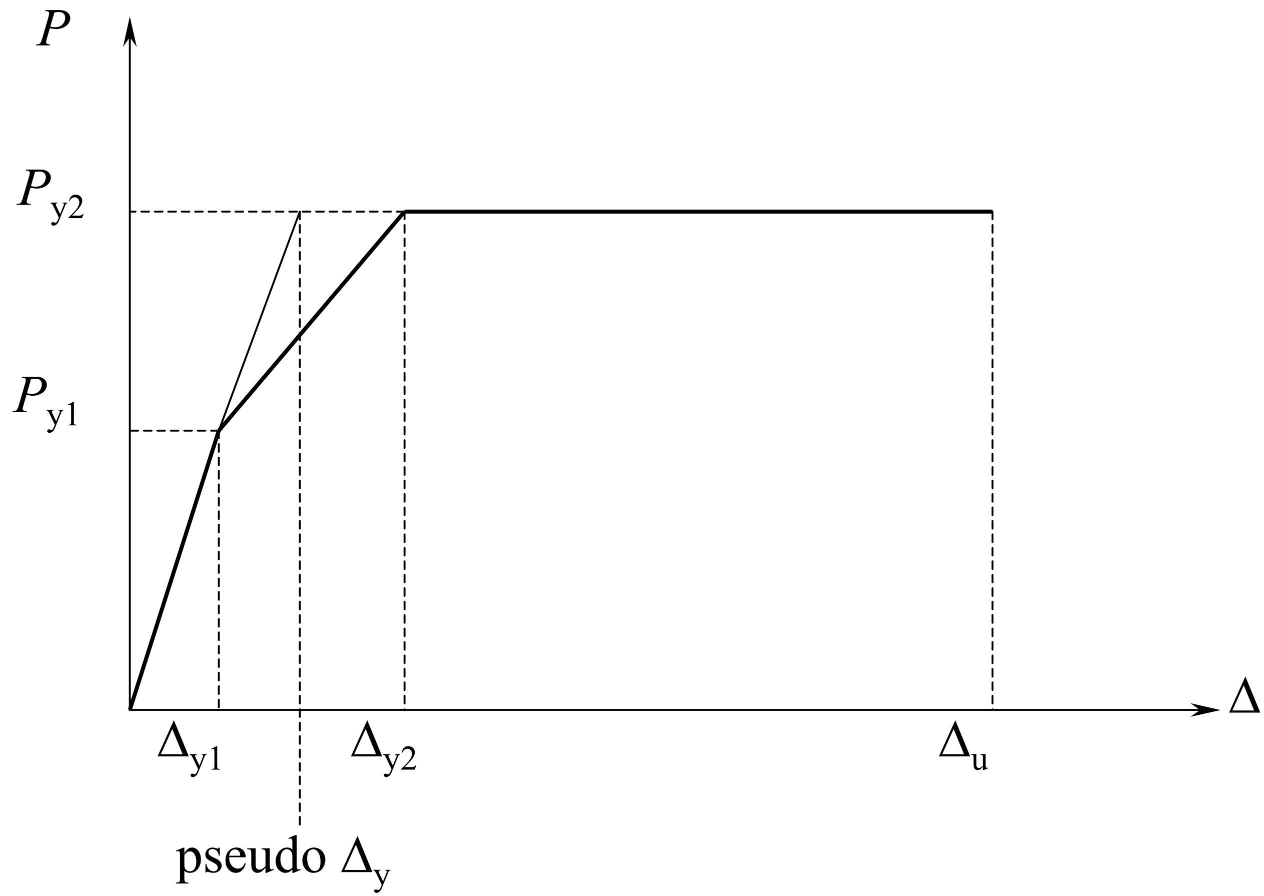

Fig. 17.13. Load Deflection Curve in Case of Two Yields.

Example 17.1: Plot moment – curvature diagram for the section shown in Fig. 17.14, ignoring the cracking. Also calculate the section ductility. $f_c' = 25$ MPa and $f_y = 420$ MPa.

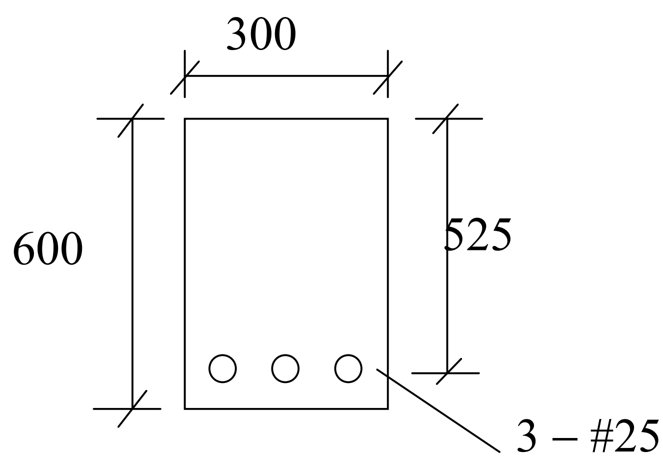

Fig. 17.14. Section Details for Example 17.1.

Solution:

$$f_c' = 25\ MPa$$

$$f_y = 420\ MPa$$

$$E_c = 4700\sqrt{25} = 23{,}500 \text{ MPa}$$

$$n = E_s / E_c = 8.51 \ \ (say\ 9)$$

$$A_s = 1530\ mm^2$$

$$\rho = A_s / bd = \frac{1530}{300 \times 525} = 0.0097$$

$$\rho_b = 0.85\,\beta_1 \frac{f_c'}{f_y} \frac{600}{600 + f_y}$$

$$= 0.85^2 \frac{25}{420} \frac{600}{1020} = 0.0253$$

$\rho < \rho_b$, the beam is tension controlled or under-reinforced.

1. Yield Stage

$$n\rho = 9 \times 0.0097 = 0.0873$$

$$k = \sqrt{(\rho n)^2 + 2\rho n} - \rho n$$

$$= 0.4269 - 0.0873 = 0.34$$

$$j = 1 - \frac{k}{3} = 0.887$$

$$M_y = A_s f_y jd = 1530 \times 420 \times 0.887 \times 525 / 10^6 = 299.2 \text{ kN-m}$$

$$\phi_{\mathrm{y}} = \frac{\varepsilon_y}{(1-k)d}$$

$$= \frac{0.0021}{0.66 \times 525} = 6.0 \times 10^{-6} \ \text{rad / mm}$$

2. *Ultimate Stage*

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{1530 \times 420}{0.85 \times 25 \times 300} = 100.8 \text{ mm}$$

$$c = a / \beta_1 = 100.8 / 0.85 = 118.6 \ mm$$

$$M_u = A_s f_y (d - a / 2)$$
$$= 1530 \times 420 \times (525 - 50.4) / 10^{6}$$
$$= 305.0 \text{ kN-m}$$

$$\phi_u = \frac{\varepsilon_{cu}}{c}$$

$$= 0.003 / 118.6 = 25.2 \times 10^{-6}\ rad / mm$$

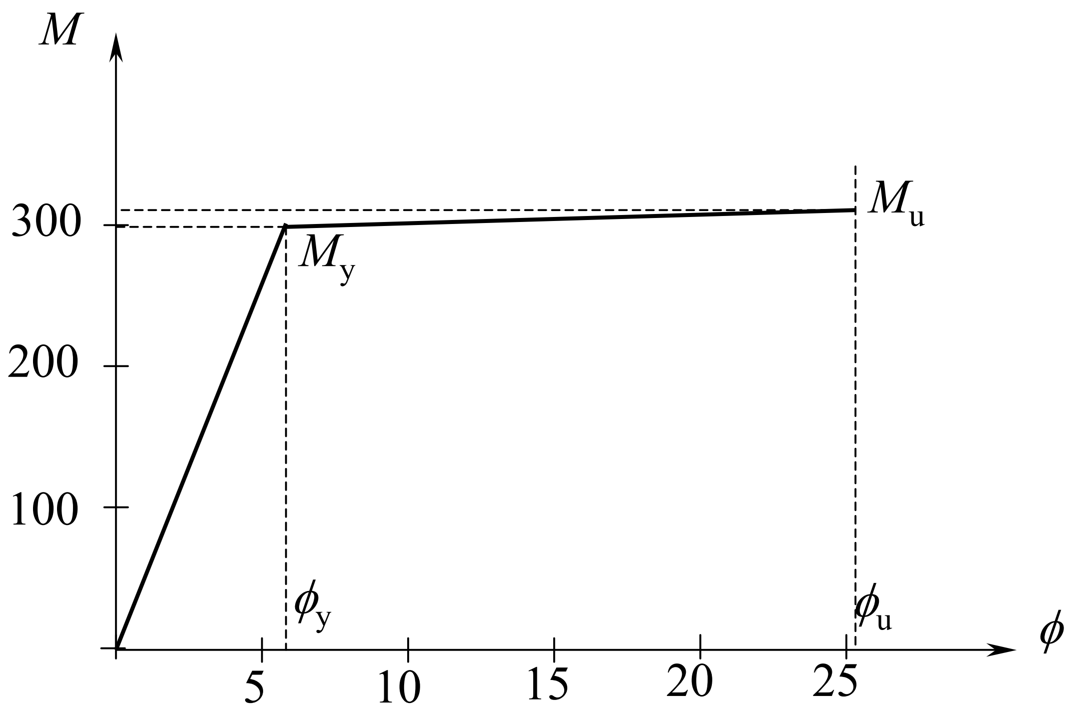

Fig. 17.15. Moment – Curvature Diagram for Example 17.1.

$$\text{Section ductility, } \mu = \frac{\phi_u}{\phi_y}$$

$$= \frac{25.2 \times 10^{-6}}{6.0 \times 10^{-6}} = 4.2$$

Example 17.2: Plot load – deflection curve and calculate member ductility for a fixed ended beam of span 8 m, subjected to a uniformly distributed load of w (kN/m). $f_c' = 25$ MPa and $f_y = 420$ MPa. Use the simplified $M - \phi$ curve shown in Fig. 17.16 for both positive and negative moments. The effective depth of the section is 525 mm.

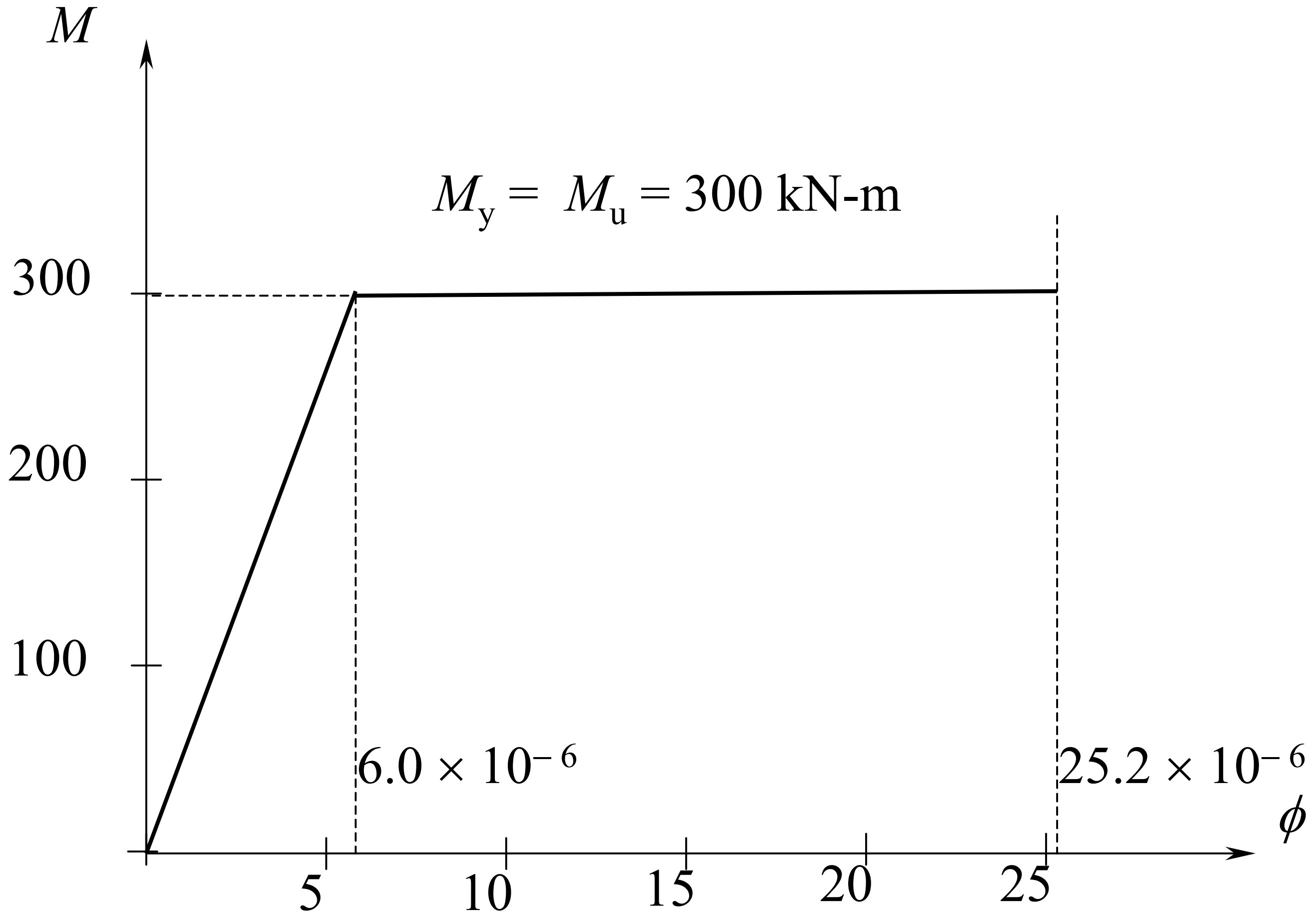

Fig. 17.16. Moment – Curvature Diagram for Example 17.2.

Solution:

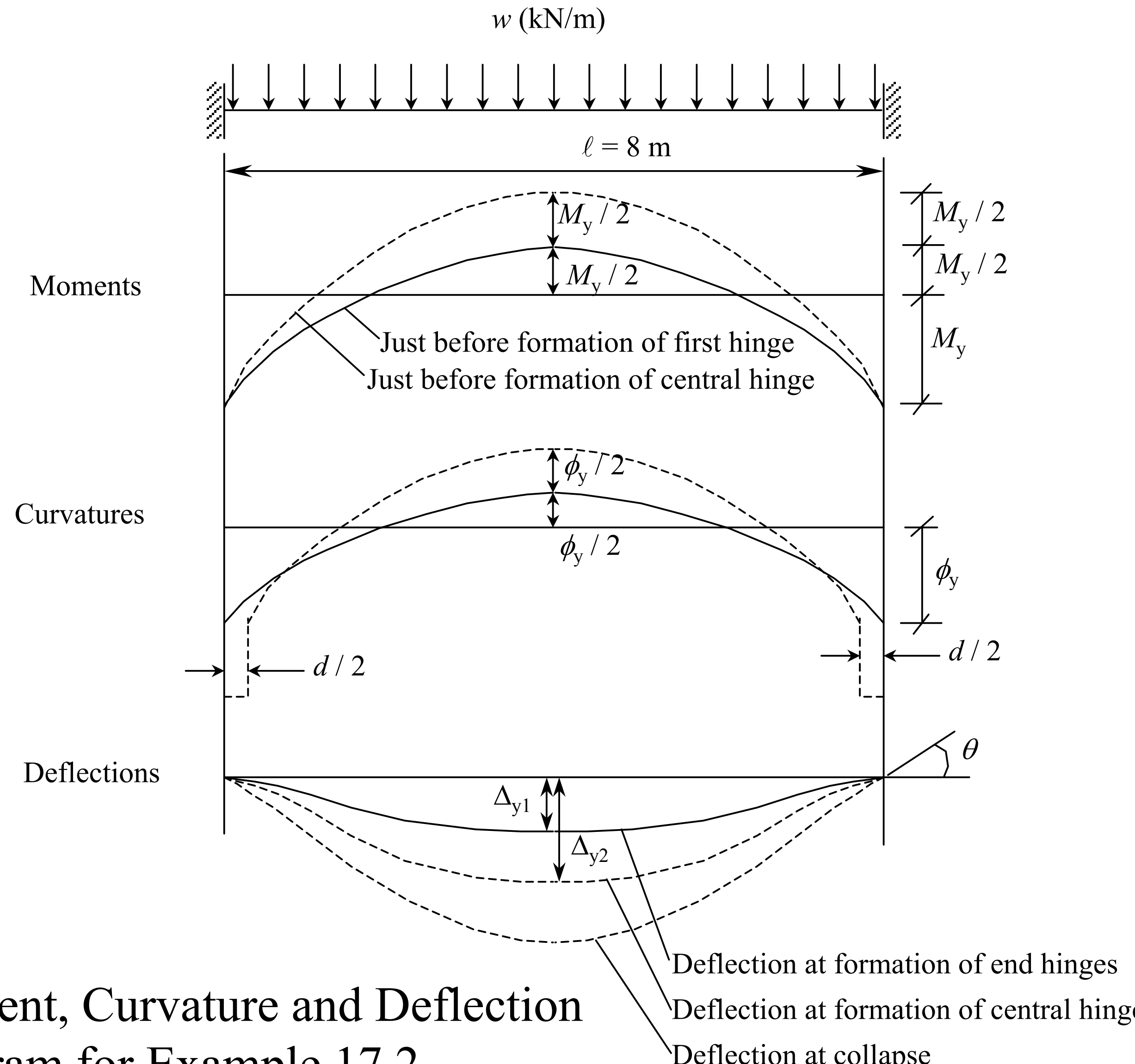

Fig. 17.17. Moment, Curvature and Deflection Diagram for Example 17.2.

1. First Yield – Formation Of End Hinges

$$M_y = w_{y1} \frac{\ell^2}{12}$$

$$w_{y1} = \frac{12 \times 300}{8^2} = 56.25 \text{ kN/m}$$

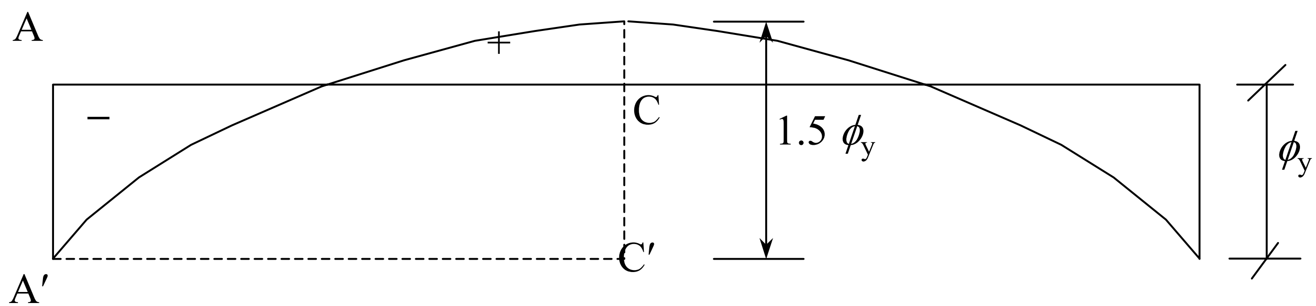

Fig. 17.18. Curvature Diagram Taken From Fig. 17.17.

$$\Delta_{y1} = t_{C/A}$$

$=$ moment of parabola between A and C about C – moment of rectangle AA′C′C about C

$$= \tfrac{2}{3}\,(1.5\,\phi_y)\,\tfrac{\ell}{2}\left(\tfrac{3}{8}\,\tfrac{\ell}{2}\right) - (\phi_y)\,\tfrac{\ell}{2}\left(\tfrac{\ell}{4}\right)$$

$$= -\tfrac{1}{32}\,\phi_y\,\ell^2$$

(negative sign shows that point C deflects below the tangent)

$$= \tfrac{1}{32} \times 6.0 \times 10^{-6} \times 8000^2 = 12 \text{ mm}$$

2. Second Yield – Formation Of Central Hinge

For the formation of the central hinge, it is necessary that the end hinges have sufficient rotation capacity and this condition must be checked later.

When this central hinge is formed, a positive moment of M_y must be present at the center as a resultant of moment of $-M_y$ at the support and simply supported moment of $w\ell^2 / 8$.

$$M_y = -M_y + w_{y2}\ell^2 / 8$$

$$w_{y2} = \frac{16M_y}{\ell^2} = \frac{16 \times 300}{8^2} = 75.00 \text{ kN/m}$$

Using first method

$$\begin{aligned}
\Delta_{y2} &= t_{A/C} \\
&= {}^2/_3\,(2\,\phi_y)\,{}^{\ell}/_2\,({}^5/_8\,{}^{\ell}/_2) - (\phi_y)\,{}^{\ell}/_2\,({}^{\ell}/_4) \\
&= {}^1/_{12}\,\phi_y\,\ell^2 \\
&= {}^1/_{12} \times 6.0 \times 10^{-6} \times 8000^2 = 32 \text{ mm}
\end{aligned}$$

Using second method

$$\Delta_{y2} = \Delta_{y1} + {}^{2}/_{3}\,(\phi_y / 2)\,({}^{5}/_{8}\,\ell/_{2})$$
$$= 12 + {}^{5}/_{96}\,\phi_y\,\ell^{2}$$
$$= 12 + {}^{5}/_{96} \times 6.0 \times 10^{-6} \times 8000^{2} = 32 \text{ mm}$$

3. Check For Rotation Capacity Of Support Hinges

$$\theta_{avail} = (\phi_u - \phi_y) \times d / 2$$
$$= (25.2 - 6.0) \times 10^{6} \times 525 / 2$$
$$= 0.00504 \text{ rad.}$$

θ_{req} = change in area of curvature diagrams between points A and C from formation of end hinges to formation of central hinge

$$= \,^{2}/_{3}\,(\phi_y / 2)\,(\ell/_{2}) \;=\; ^{1}/_{6}\,\phi_y\,\ell$$

$$= \frac{6.0\times 10^{-6}\times 8000}{6} = 0.008 \text{ rad.}$$

$\theta_{avail} < \theta_{req}$, central hinge will not form.

If the curvature at the center due to θ_{avail} is denoted by ϕ_{av}, its value may be determined as follows:

$$
\begin{aligned}
{}^{2}/_{3}\,(\phi_{av})\,(\ell/_{2}) &= 0.00504 \\
\phi_{av} &= 1.8 \times 10^{-6} \text{ rad/mm} \\
&= 0.315\ \phi_{y}
\end{aligned}
$$

The moment equation then becomes the following:

$$(0.5 + 0.315)\, M_{y} = -M_{y} + w_{y2}\ell^{2} / 8$$

$$w_{y2} = \frac{1.815 \times 8 M_{y}}{\ell^{2}} = \frac{14.52 \times 300}{8^{2}} = 68.06 \text{ kN/m}$$

$$\Delta_{y2} = \Delta_{y1} + {}^{2}/_{3}\,(0.315\phi_y)\,(\ell/_2)\,({}^{5}/_{8}\,\ell/_2)$$

$$= \Delta_{y1} + {}^{5}/_{96}\,\phi_y\,\ell^2$$

$$= 12 + \frac{6\times10^{-6}\times8000^2}{30.5} = 24.6 \text{ mm}$$

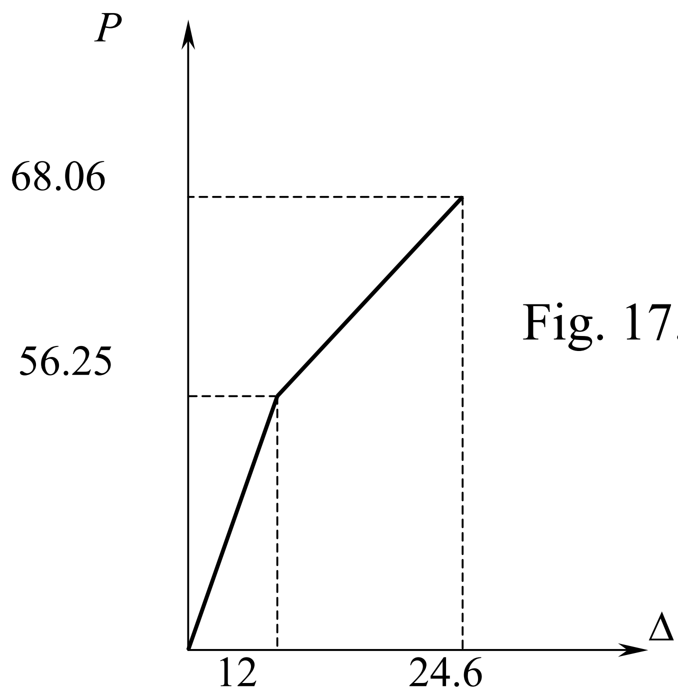

Fig. 17.19. Load Deflection For Example 17.2.

$$\text{Member ductility, } \mu = \frac{\Delta_u}{\Delta_y}$$

$$= 24.6 / 12 = 2.05$$

Example 17.3: Repeat Example 17.2 if Grade 300 steel is used in place of Grade 420 steel.

Solution:

$$f_c' = 25 \text{ MPa}$$
$$f_y = 300 \text{ MPa}$$

$$E_c = 4700\sqrt{25} = 23{,}500 \text{ MPa}$$

$$n = E_s / E_c = 8.51 \text{ (say 9)}$$

$$A_s = 1530 \text{ mm}^2$$

$$\rho = A_s / bd = \frac{1530}{300 \times 525} = 0.0097$$

$$\rho_b = 0.85\,\beta_1 \frac{f_c'}{f_y} \frac{600}{600 + f_y}$$

$$= 0.85^2 \frac{25}{300} \frac{600}{1020} = 0.0401$$

$\rho < \rho_b$, the beam is tension controlled or under-reinforced.

1. Yield Stage

$$n\rho = 9 \times 0.0097 = 0.0873$$

$$k = \sqrt{(\rho n)^2 + 2\rho n} - \rho n$$

$$= 0.4269 - 0.0873 = 0.34$$

$$j = 1 - \frac{k}{3} = 0.887$$

$$M_{\mathrm{y}} = A_{\mathrm{s}} f_{\mathrm{y}} jd = 1530 \times 300 \times 0.887 \times 525 / 10^{6}$$
$$= 213.7 \text{ kN-m}$$

$$\phi_{\mathrm{y}} = \frac{\varepsilon_y}{(1-k)d} = \frac{0.0015}{0.66 \times 525} = 4.3 \times 10^{-6} \text{ rad / mm}$$

2. Ultimate Stage

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{1530 \times 300}{0.85 \times 25 \times 300} = 72 \text{ mm}$$

$$c = a / \beta_1 = 72 / 0.85 = 84.7 \text{ mm}$$

$$M_u = A_s f_y (d - a/2)$$

$$= 1530 \times 300 \times (525 - 72/2) / 10^6$$

$$= 224.5 \text{ kN-m}$$

$$\phi_u = \frac{\varepsilon_{cu}}{c} = 0.003 / 84.7$$

$$= 35.4 \times 10^{-6} \text{ rad / mm}$$

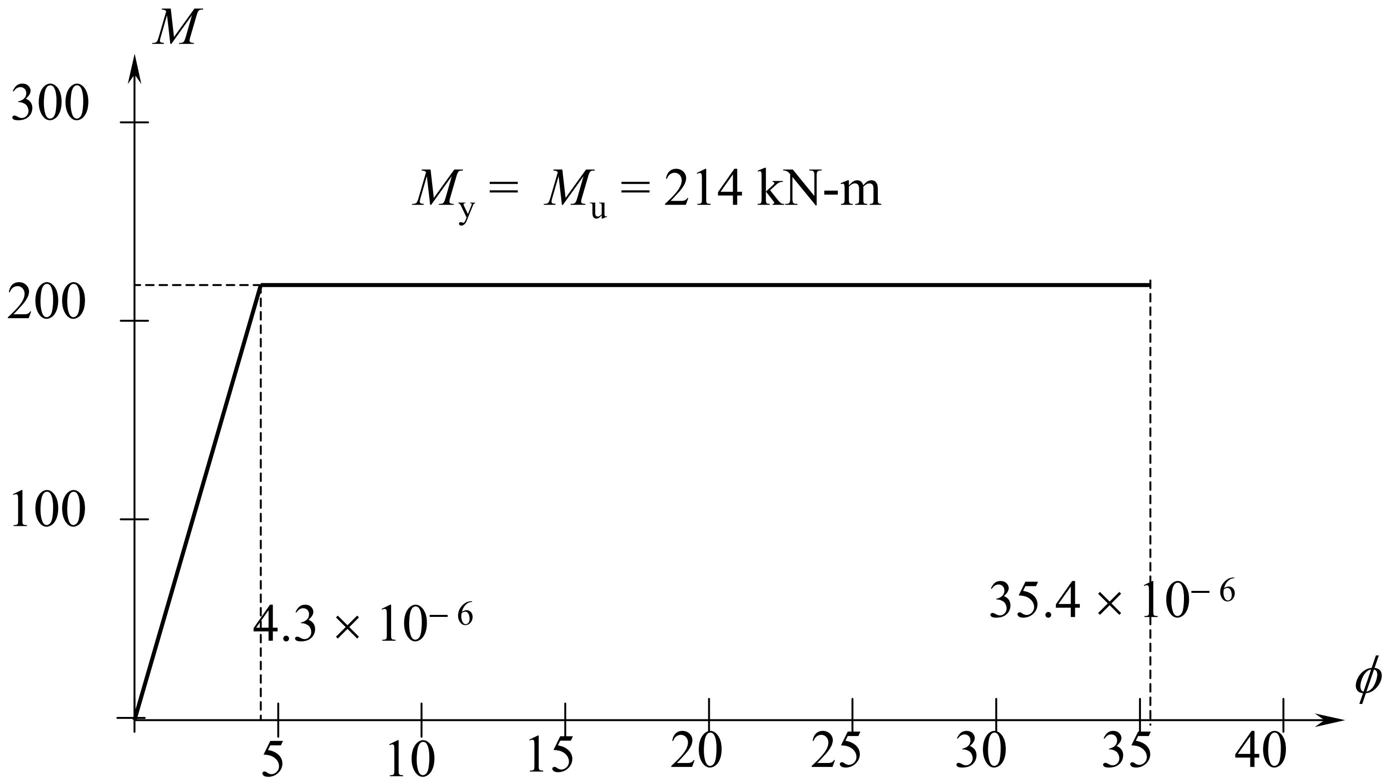

Fig. 17.20. Moment – Curvature Diagram for Example 17.3.

1. First Yield – Formation Of End Hinges

$$M_y = w_{y1}\,\frac{\ell^2}{12}$$

$$w_{y1} = \frac{12 \times 214}{8^2} = 40.13\ \text{kN/m}$$

$$\begin{aligned}\Delta_{y1} &= t_{C/A} \\ &= {}^{1}/_{32}\ \phi_y\ \ell^{2} \\ &= {}^{1}/_{32} \times 4.3 \times 10^{-6} \times 8000^{2} \\ &= 8.6\ \text{mm}\end{aligned}$$

2. Second Yield – Formation Of Central Hinge

$$M_y = -M_y + w_{y2}\ell^2 / 8$$

$$w_{y2} = \frac{16M_y}{\ell^2} = \frac{16 \times 214}{8^2} = 53.5 \text{ kN/m}$$

$$\begin{aligned}
\Delta_{y2} &= \Delta_{y1} + {}^2/_3\,(\phi_y / 2)\,({}^5/_8\,\ell/_2) \\
&= 8.6 + {}^5/_{96}\,\phi_y\,\ell^2 \\
&= 8.6 + {}^5/_{96} \times 4.3 \times 10^{-6} \times 8000^2 \\
&= 22.93 \text{ mm}
\end{aligned}$$

3. *Check For Rotation Capacity Of Support Hinges*

$$\theta_{avail} = (\phi_u - \phi_y) \times d / 2$$
$$= (35.4 - 4.3) \times 10^6 \times 525 / 2$$
$$= 0.00816 \text{ rad.}$$

θ_{req} = change in area of curvature diagrams between points A and C from formation of end hinges to formation of central hinge

$$= {}^2/_3\,(\phi_y / 2)\,(\ell/_2) = {}^1/_6\,\phi_y\,\ell$$

$$\theta_{req} = \frac{4.3 \times 10^{-6} \times 8000}{6} = 0.00573 \text{ rad.}$$

$\theta_{avail} > \theta_{req}$, central hinge will form.

3. Additional Central Deflection After Formation Of Mechanism

The load remains constant after the second yield, $w_3 = w_{y2}$.

$$\theta' = \theta_{avail} - \theta_{req}$$

$$= 0.00816 - 0.00573 = 0.00243 \text{ rad.}$$

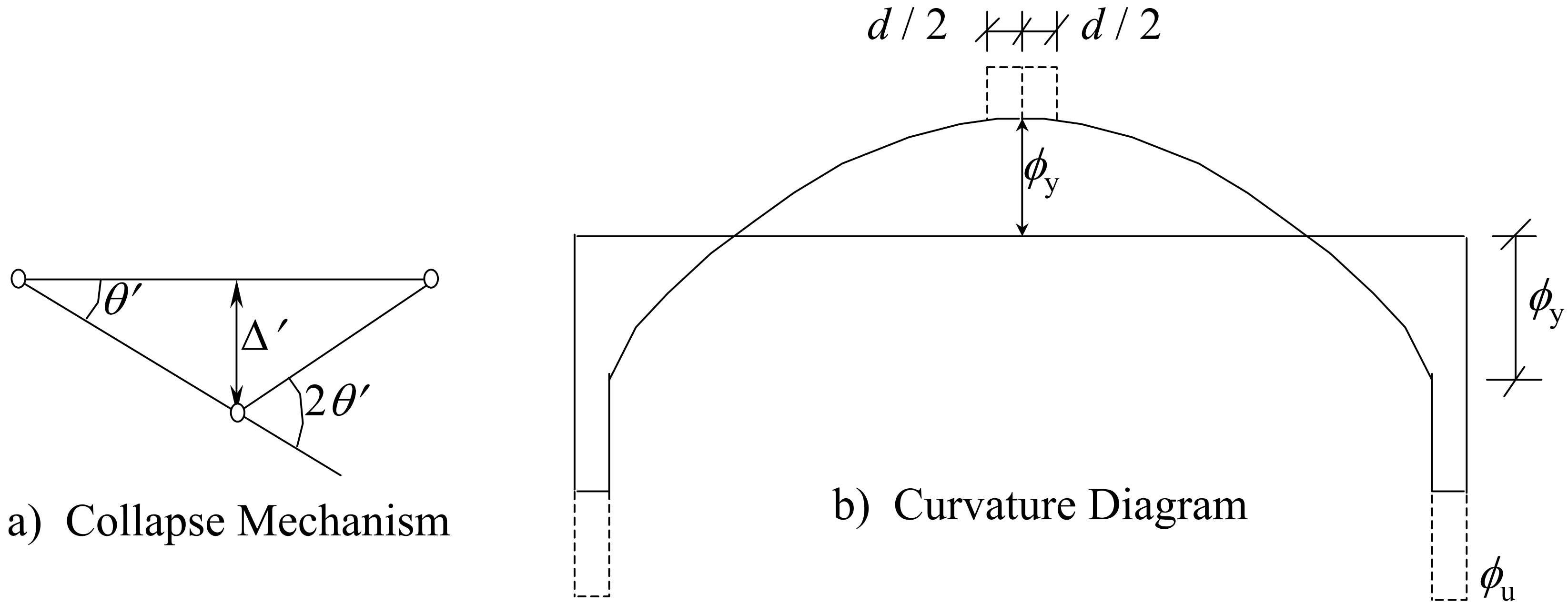

Fig. 17.21. Collapse Behavior for Example 17.3.

$$\Delta' = \theta' \times \ell / 2$$

$$= 0.00243 \times 8000 / 2 = 9.72 \text{ mm}$$

$$\mathit{\Delta}_u = \Delta_{y2} + \Delta'$$

$$= 22.93 + 9.72 = 32.65 \text{ mm}$$

The resulting load-deflection curve is shown in Fig. 17.22.

The first and third lines of the curve are extended backwards and the point of intersection gives the pseudo yield deflection.

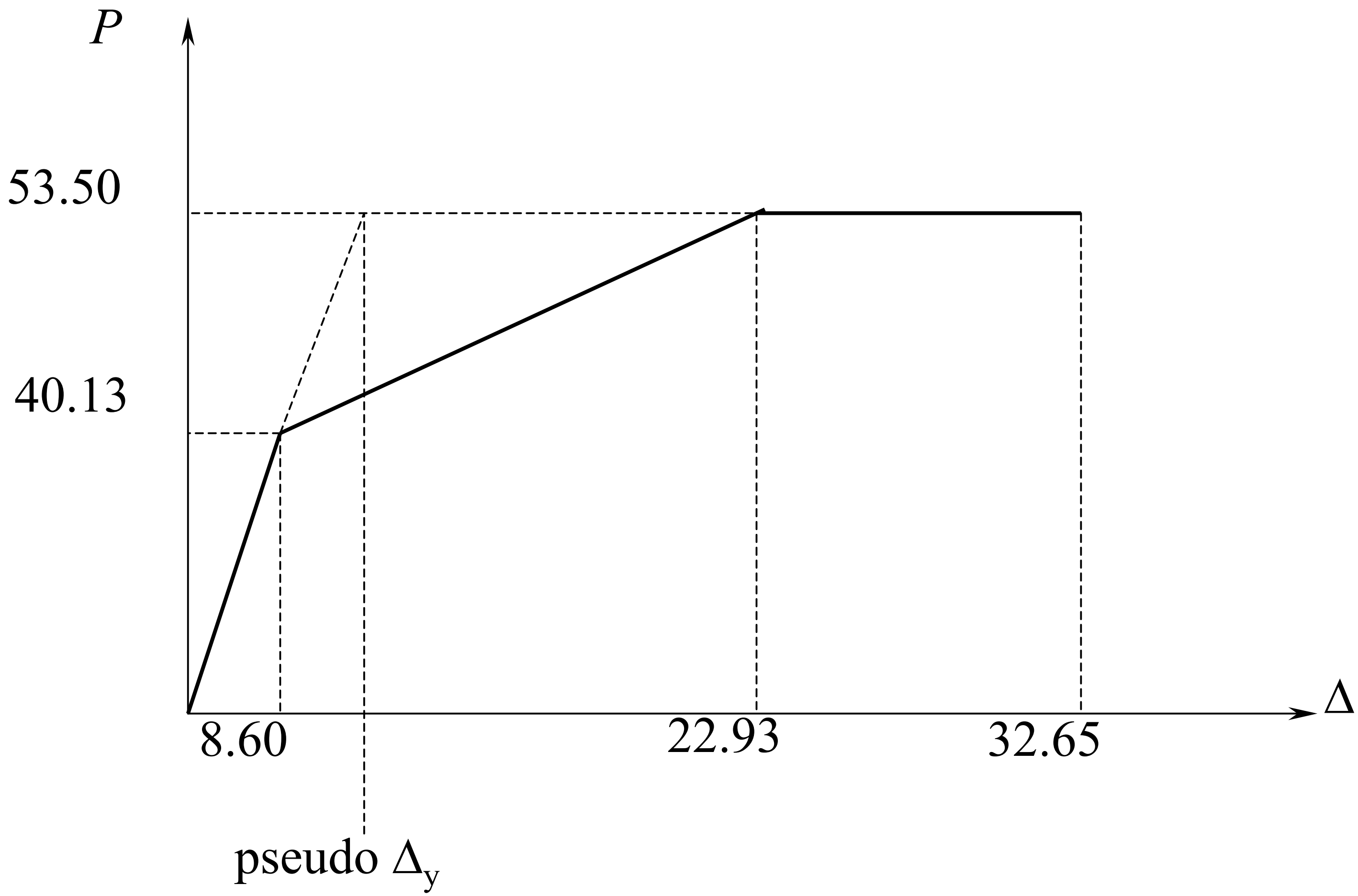

Fig. 17.22. Load – Deflection Curve for Example 17.3.

$$\text{Pseudo } \Delta_y = \frac{8.60}{40.13} \times 53.6 = 11.49 \text{ mm}$$

$$\text{Member ductility, } \mu = \frac{\Delta_u}{Pseudo\ \Delta_y}$$

$$= 32.65 / 11.49 = 2.84$$

REDISTRIBUTION OF MOMENTS

Redistribution of moments involving reduction of negative moments associated with the corresponding increase in the positive moments is usually desired for design in order to get better detailing and economical designs.

The redistribution is actually decided based on the plastic analysis, rotation capacities at the hinges and the material, section, and member ductility.

This redistribution is only possible for compatibility moments and not for the equilibrium moments.

ACI 8.4 gives guidelines for such redistribution of negative moment at continuous supports as under:

(a) Elastic negative moments may be increased or decreased for any loading arrangement by a maximum of the smaller of the following two:

   (i) 1000 ε_t per cent for ε_t up to 0.02
   (ii) 20 per cent.

(b) Redistribution is only allowed if extreme tension steel strain, ε_t, is greater than or equal to 0.0075 at the section where the moment is changed.

(c) The positive moments must be calculated using the modified end moments.

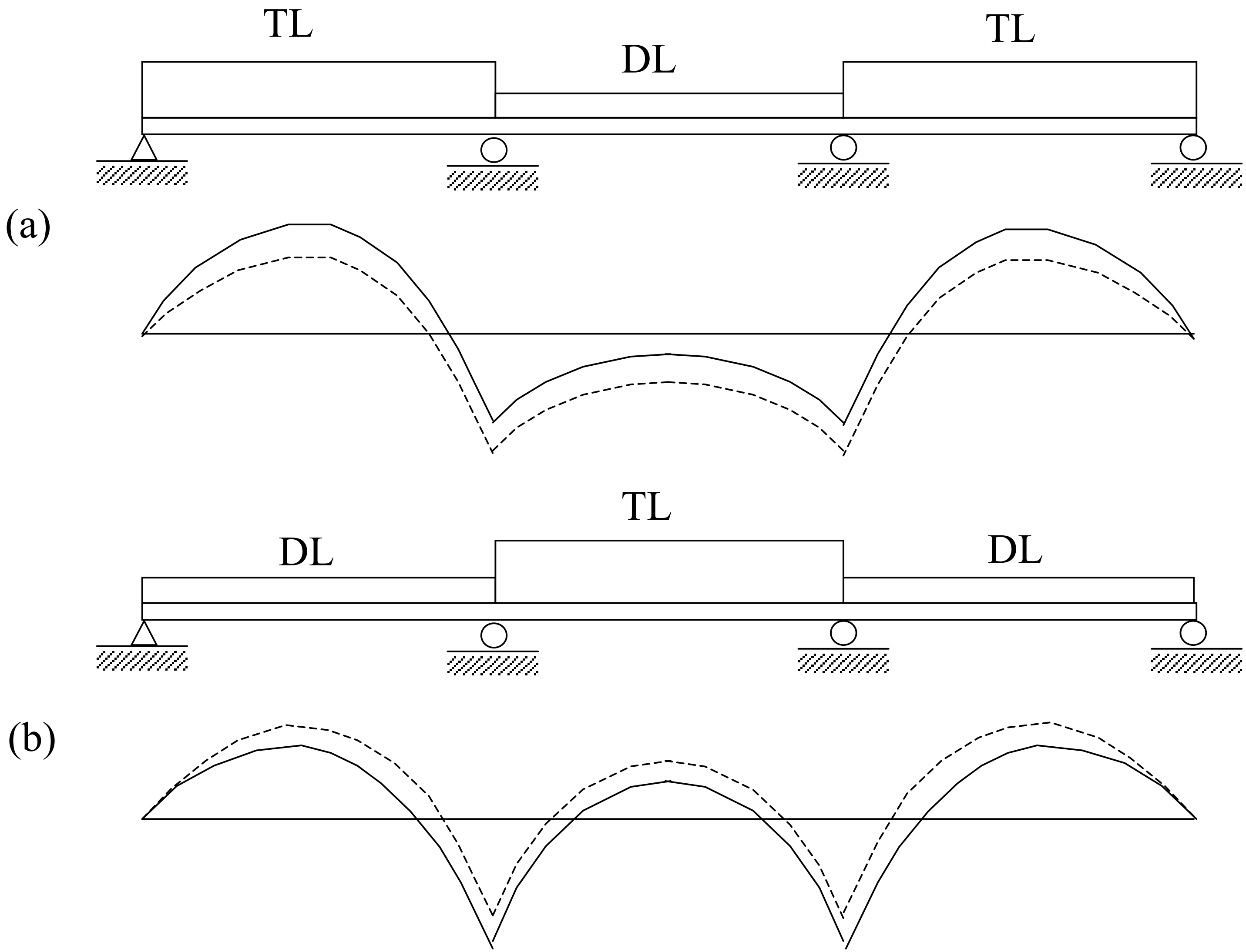

Fig. 17.23. Moment Redistribution in Two Different Cases.