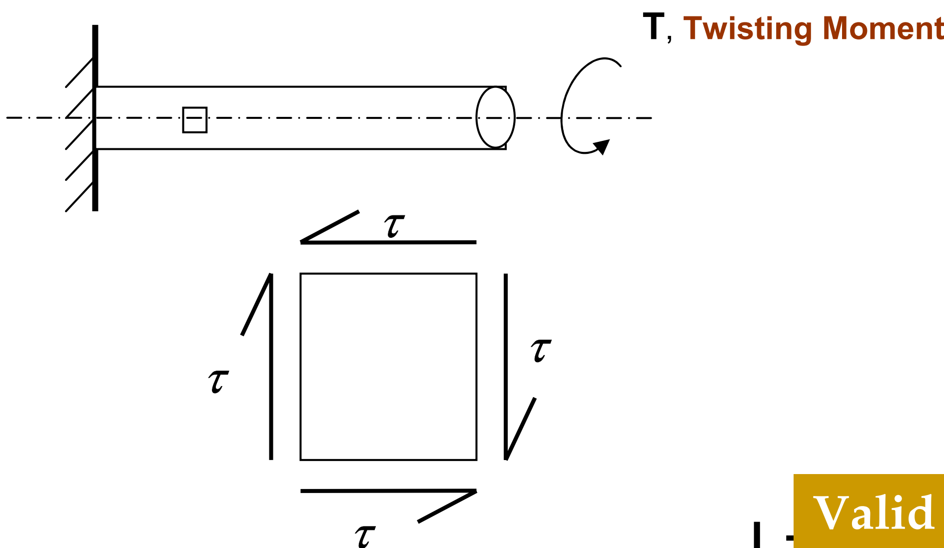
Torsion Design

Slides prepared by
Azhar
Lecturer of Civil Engineering

Torque:- (Twisting Moment)

- Moment about longitudinal axis.
- Corresponding deformation produced is twist or torsion.



$$\tau = \frac{Tr}{J}$$

J =

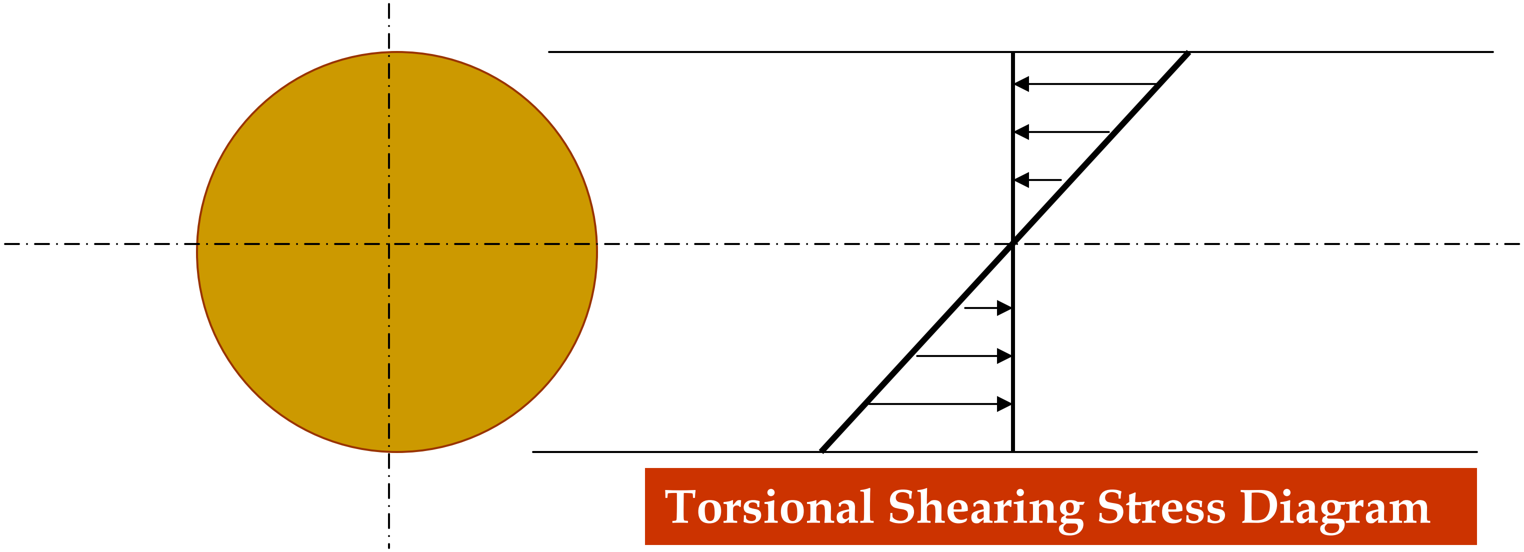
Valid only if no point on bar is stressed beyond proportional limit

τ = Torsional Shearing Stress

r = Radial distance from

$$\tau = \frac{Tr}{J}$$

- Assumption : Plane section remains plane

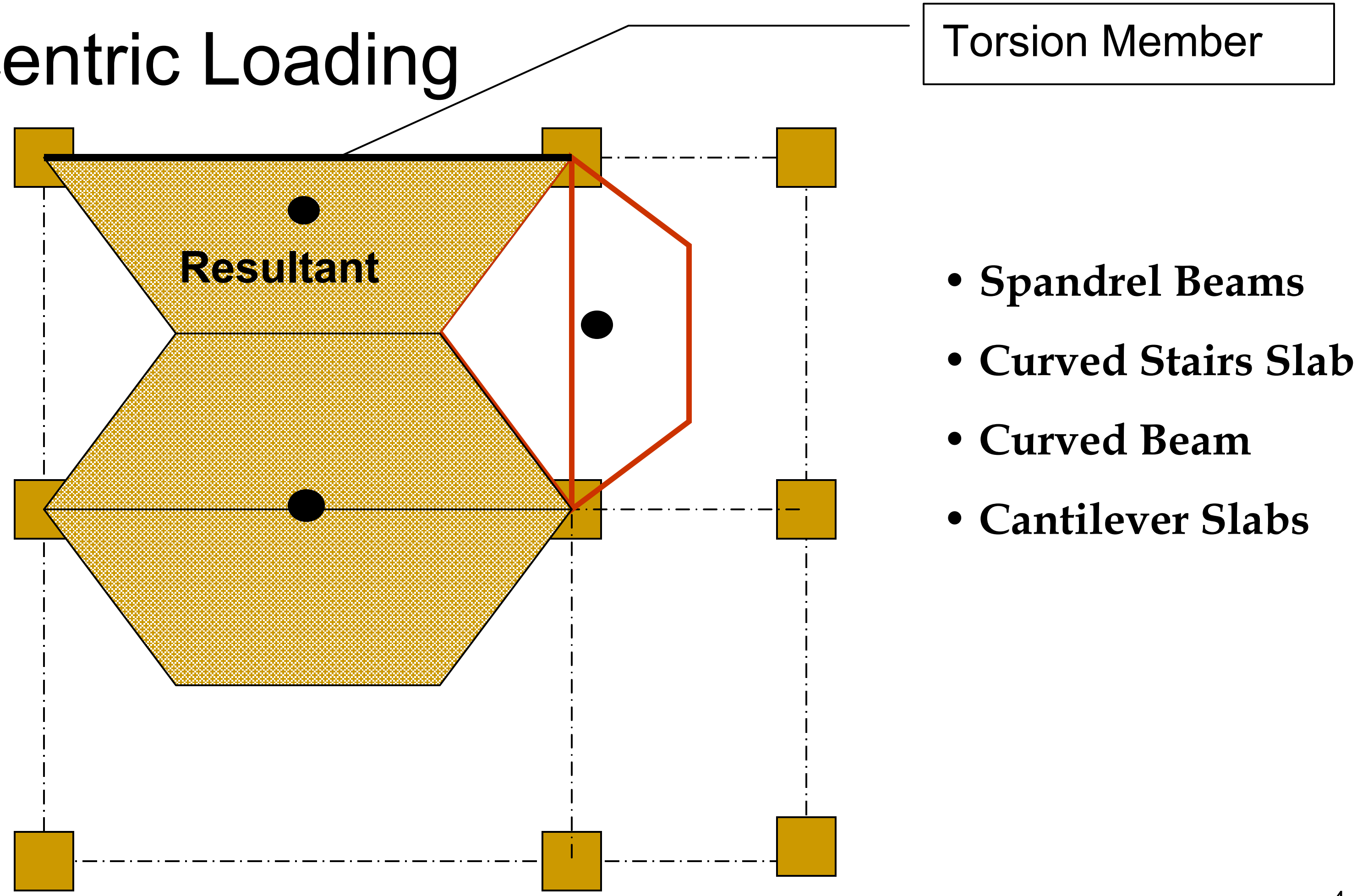


Torsional Shearing Stress Diagram

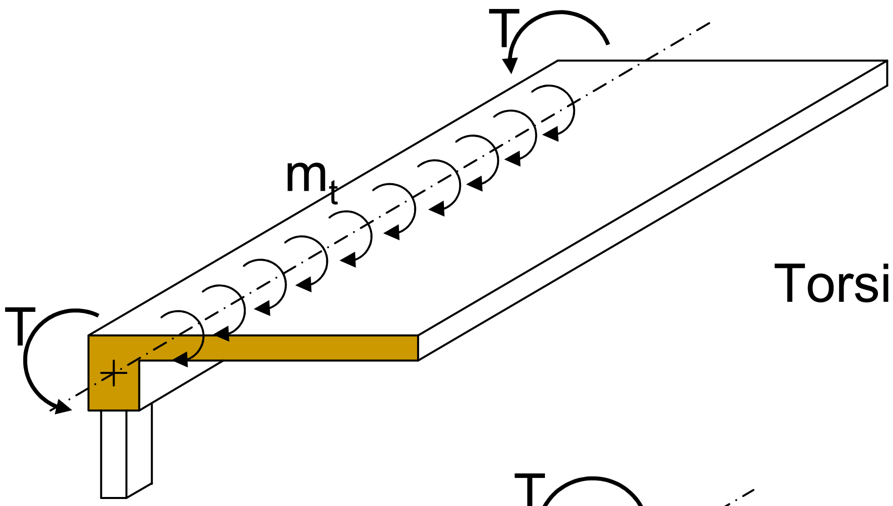
- **Here the emphasis will be on design**

Reasons of Torsion:

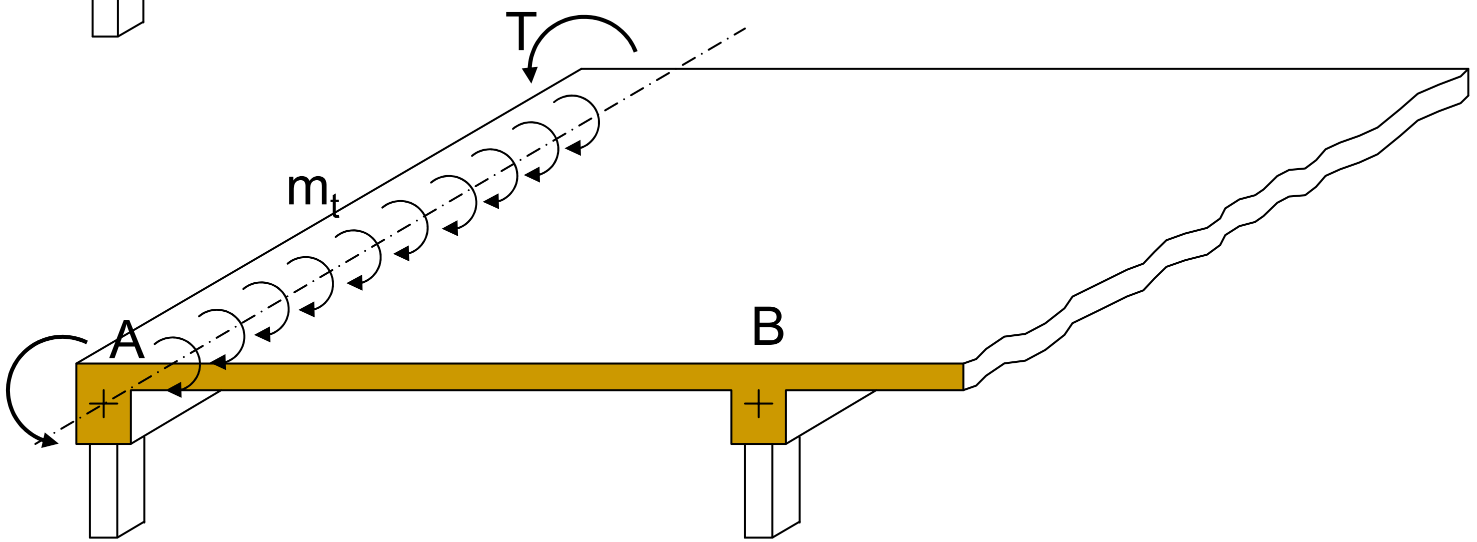
1. Eccentric Loading



- **Spandrel Beams**
- **Curved Stairs Slab**
- **Curved Beam**
- **Cantilever Slabs**



Torsion at a cantilever slab



Torsion at an edge beam

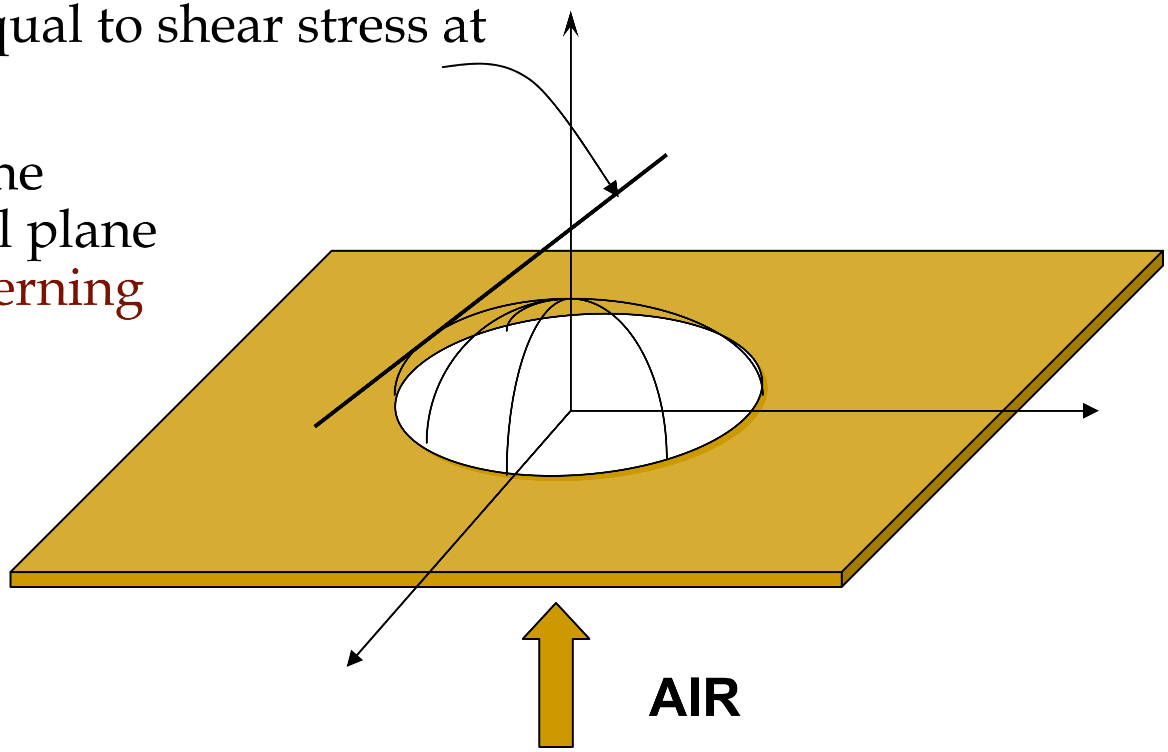
Reasons (contd…)

2. Different loadings and deformations of adjacent 2-D frames (connecting members shall be subjected to torque)

3. If members are meeting at an angle in space then part of bending moment will become torque for other member.

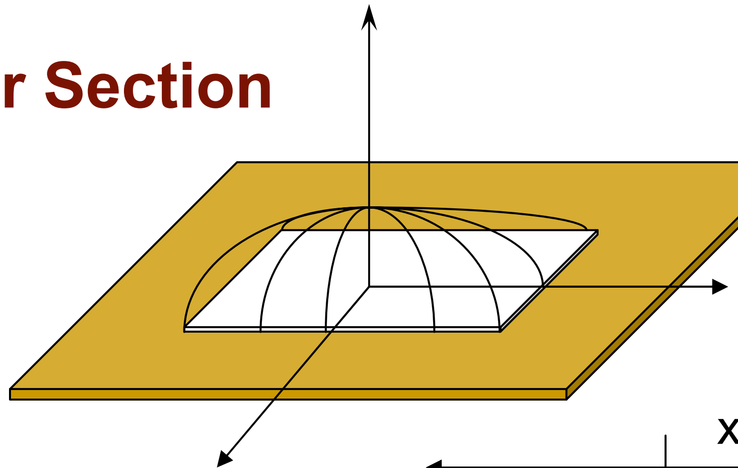
Soap Film Analogy

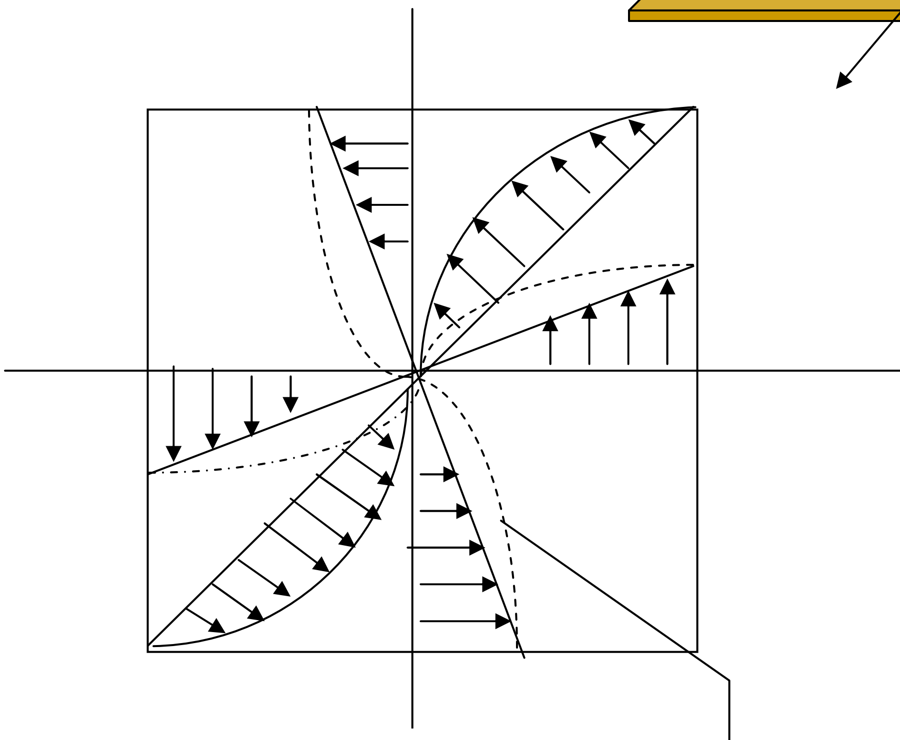
- Slope at any point is equal to shear stress at that point
- The volume between the bubble and the original plane (by the analogy of governing differential equation) is proportional to the total torque resistance (applied). Steeper the slope of tangent at any point greater will be the shear stress.



- SFA is more useful for noncircular and irregular section for which formulas are not available.

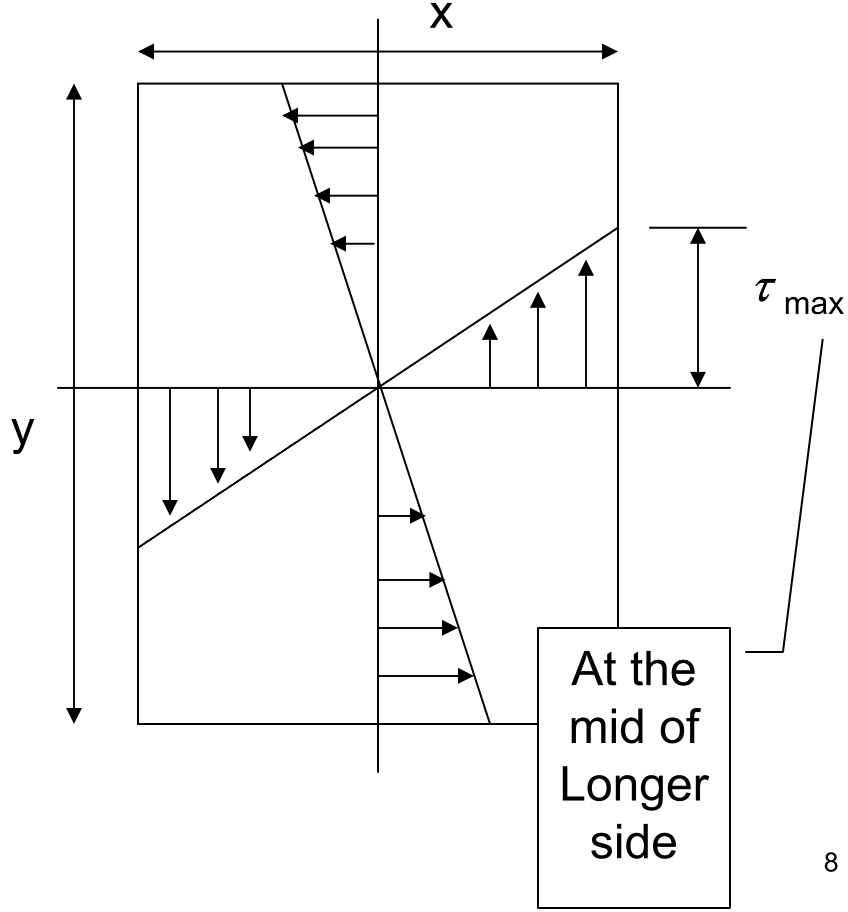
SFA For Rectangular Section



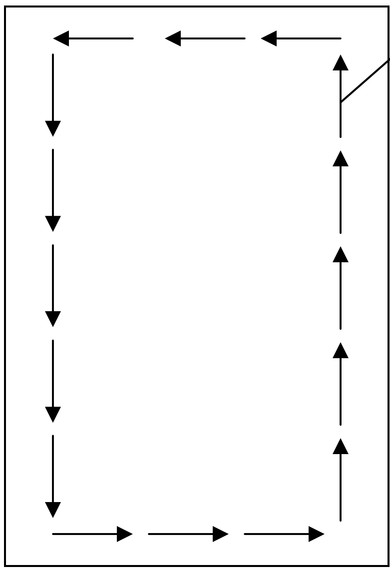


Square Cross Section

When material behaves inelastically



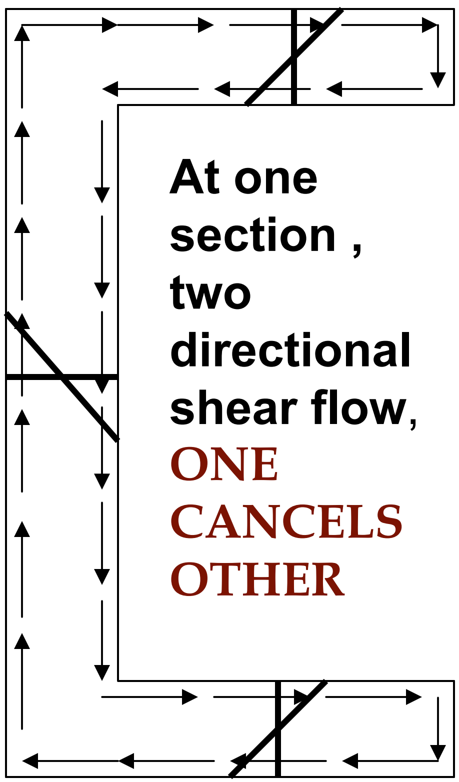
Shear Flow

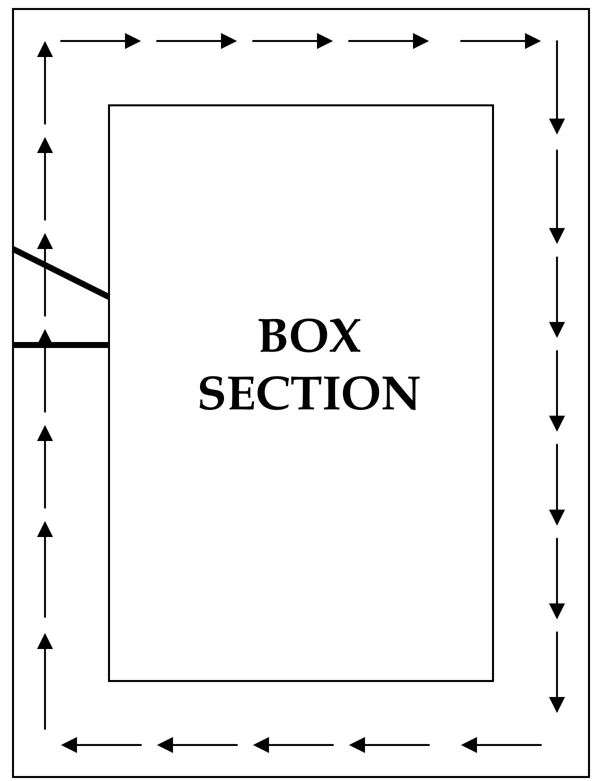


Shear Flow (Resultant shear at a point)

- Shear Flow = Shear Stress x Thickness
  = Shear force per unit length
- **Shear flow is opposite to applied shear**

Shear Flow (contd...)





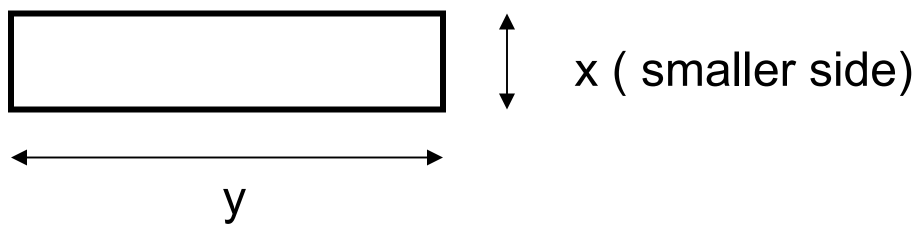
At one section only one directional flow. So closed sections are very efficient in resisting torque

Formula For Maximum Torsion

$$\tau_{max} = \frac{Tx}{\alpha x^3 y}$$

Valid for Rectangular Section only (PCC)

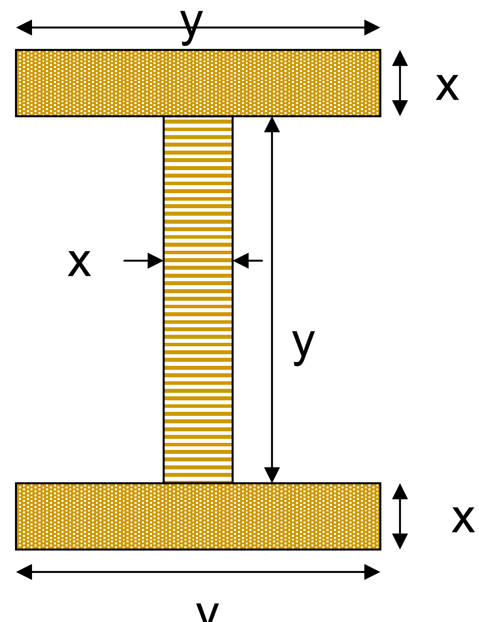
$$= \frac{Tx}{C}$$



- C, Torsion constant = $\alpha x^3 y$
- α depends on x / y ratio.

y/x	1.0	1.5	2.0	3.0	5.0	∞
α	.208	.219	.246	.267	.290	1/3

C For I-Section

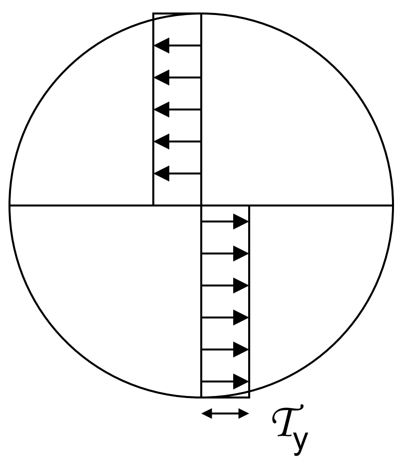


$$C = \sum \alpha x^3 y$$

Plastic Torsion

- Whole the section will yield in torsion, $\tau = \tau_y$

•Plastic analysis assumes **uniform shear intensity** all around the surface and all around the cross section.

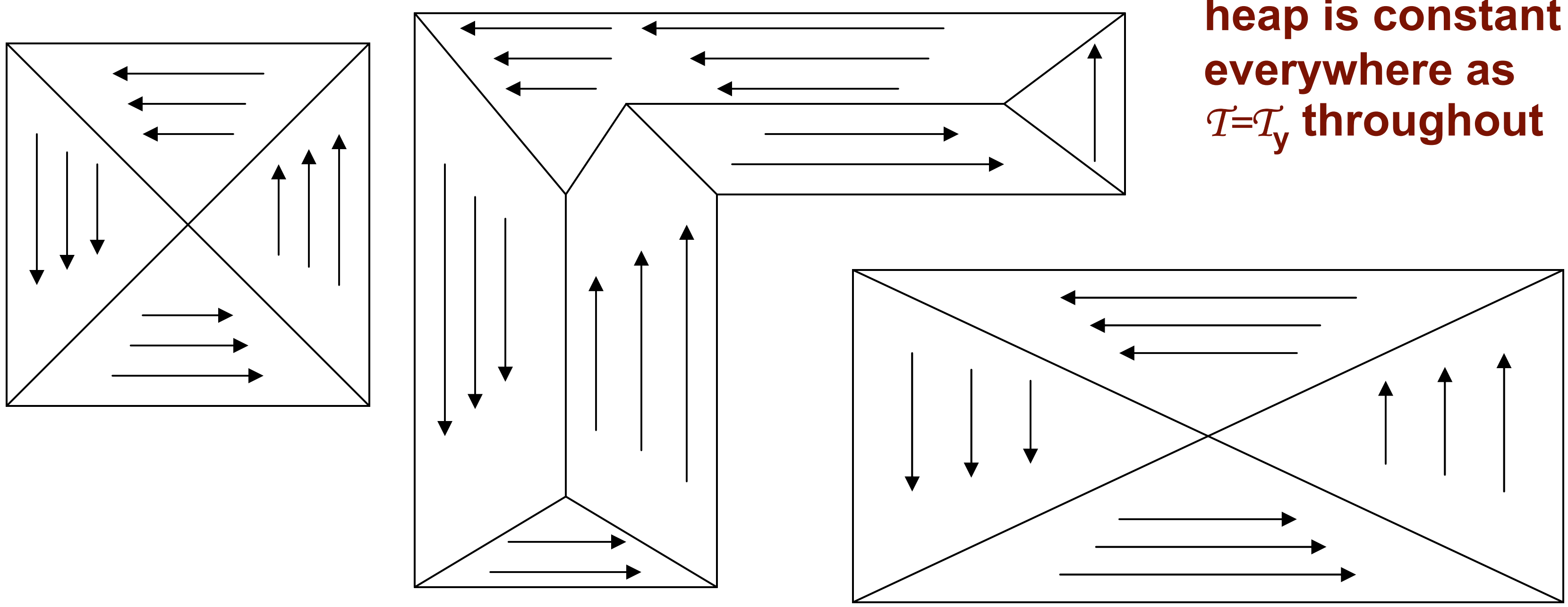


Plastic analysis can be envisioned in terms of **SAND HEAP ANALOGY**

Sand Heap Analogy

- Put sand on a plate having a shape same as that of cross section (Circular, Rectangular, Irregular)

Slope of sand heap is constant everywhere as $\tau=\tau_y$ throughout



Sand Heap Analogy (contd…)

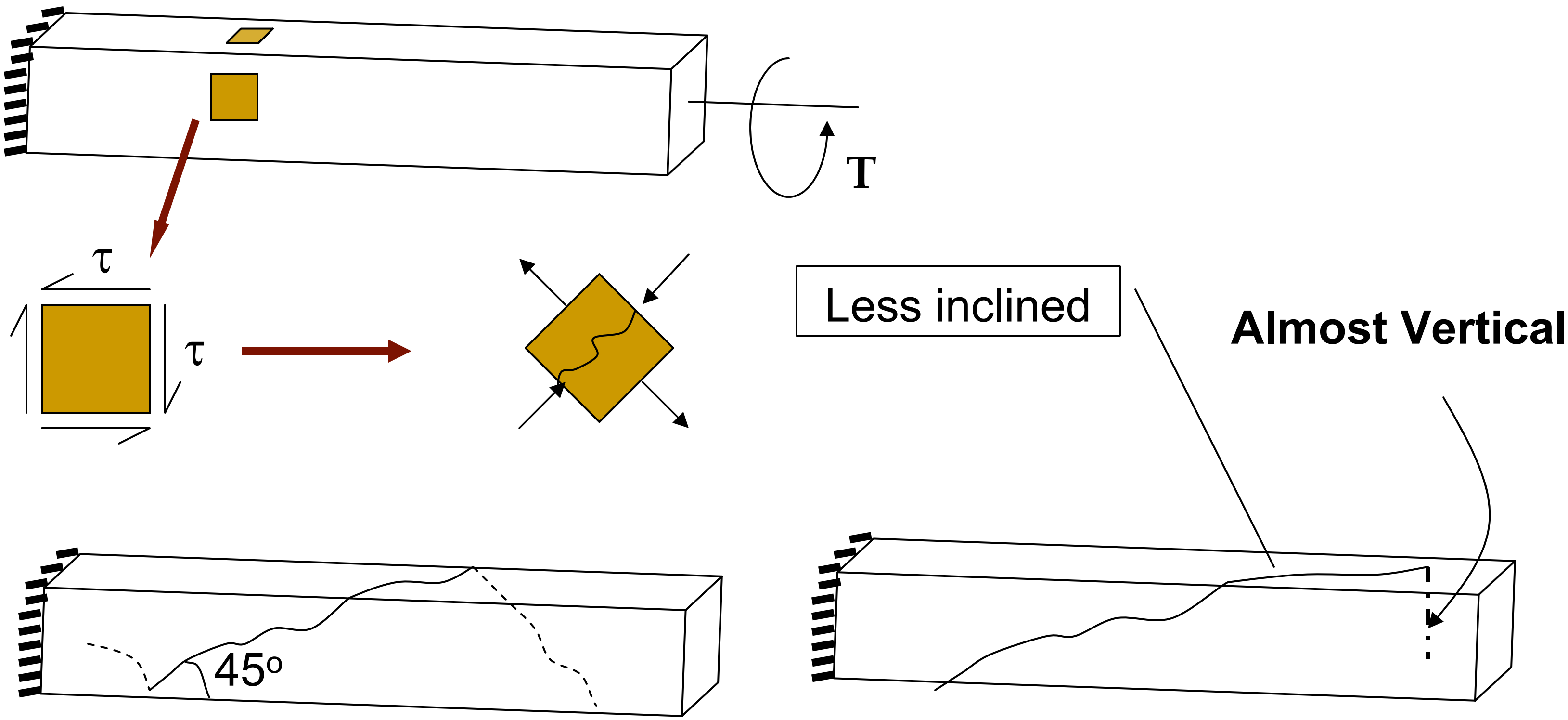
- Volume under the sand heap is proportional to the torque.

$$\tau_{max} = \frac{Tx}{\alpha_p x^3 y}$$

α_p = 0.33 for y/x = 1.0

= 0.5 for y/x = ∞

General Cracking Pattern

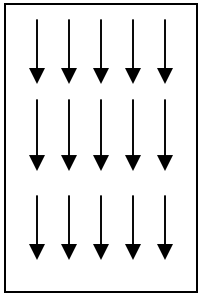


Inclined cracks due to torsion which spiral around the member

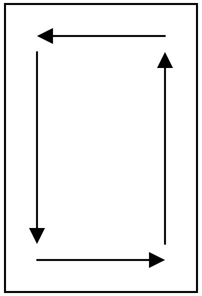
Cracks due to Vu, Mu, and Tu

General Cracking Pattern…

- On one web face the torque shear adds (& on the other face subtracts) relative to the vertical shear.



Section subjected to Shear



Section subjected to Torsion

- **Cracks due to shear are parallel on both sides of the member butt due to torsion cracks are just like spiral around the member.**

Types of Torsion

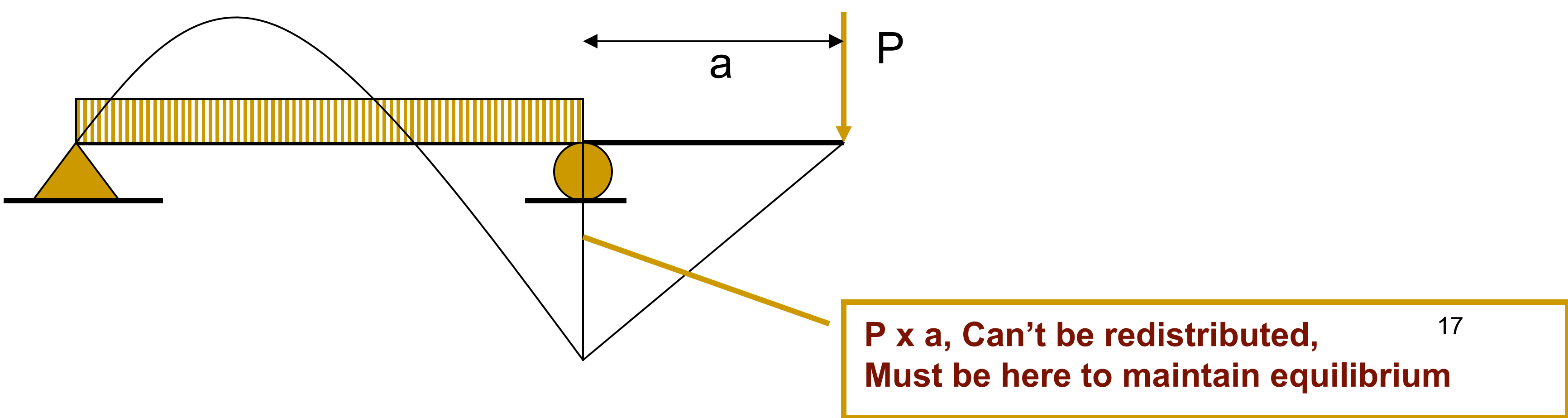
- There are two types of torsion depending upon its sources.

1- Equilibrium Torsion

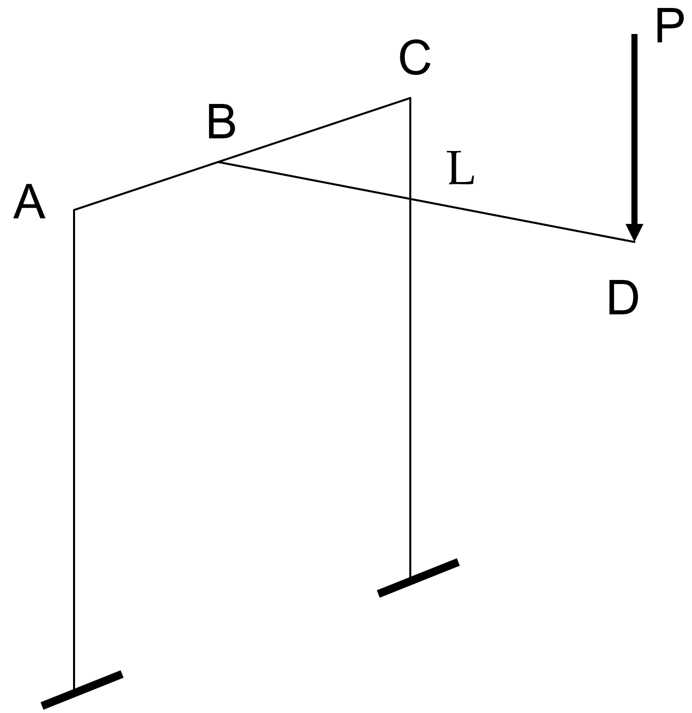
2- Compatibility Torsion

Equilibrium Torsion:

- Required for equilibrium & can't be redistributed.



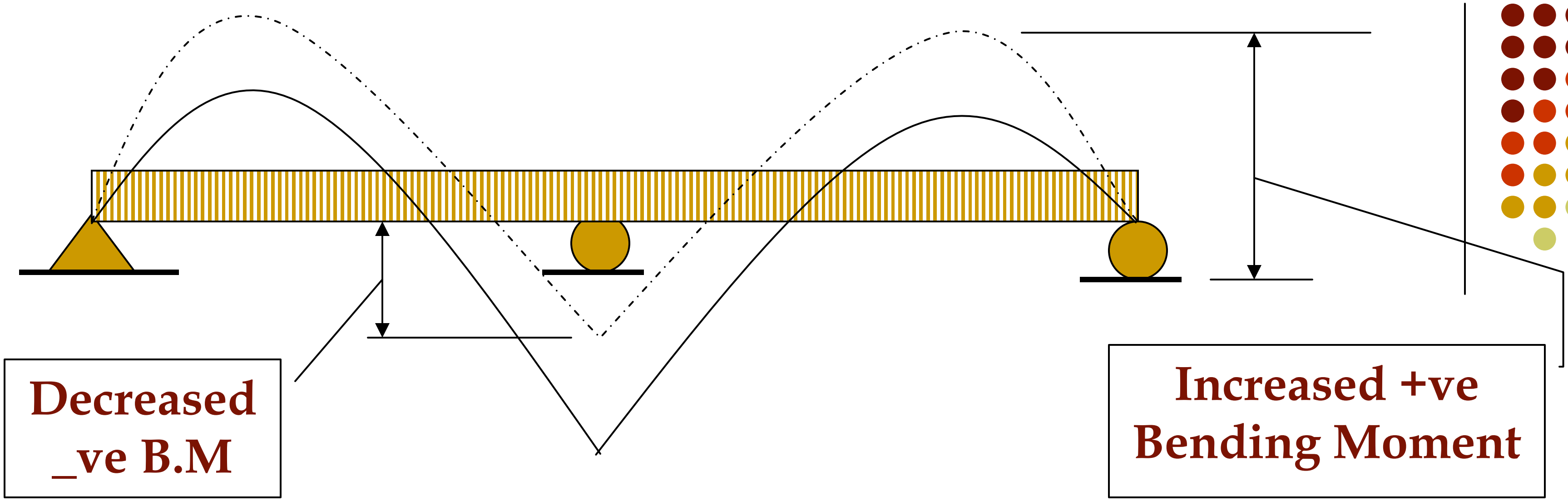
- Torque at B = P x L, it can't be redistributed
- If AC is not designed for torsion it will fail.



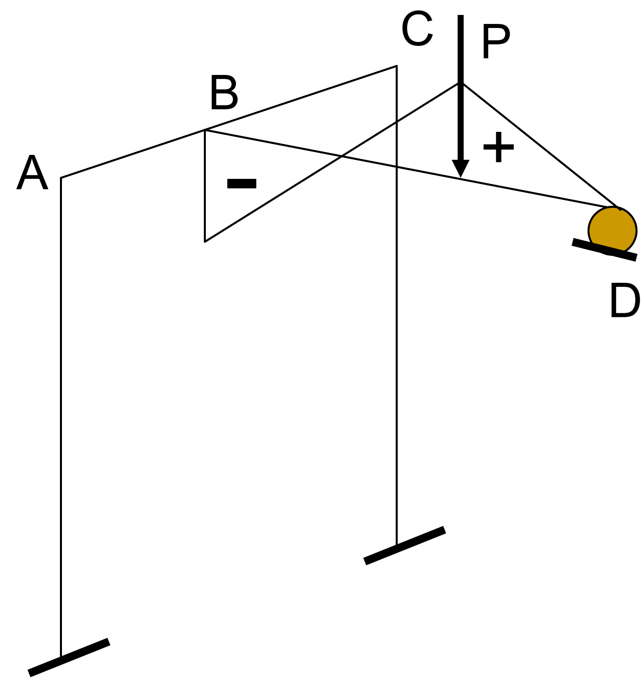
Compatibility Torsion

Compatibility means, compatibility of deformation b/w different members meeting together.

Redistribution/Adjustment is possible.



- Redistribution is allowed by the code



- **If AC is not designed for torque redistribution can be there, initially there will be some torque and if no resistance developed it will be redistributed and +ve BM in span BD will increase**

Design

ACI CODE

- Before 1995
  - Design of shear and torsion was combined
- 1995 & Onwards
  - Shear & Torsion design Separate. Reinforcement is combined in the end.

- **For shear design shear strength of concrete is considered.**
- **For torsion design shear strength of concrete is considered zero. Compressive strength is considered to an extent.**

Reinforcements

- Two types of reinforcements are required to resist torque.

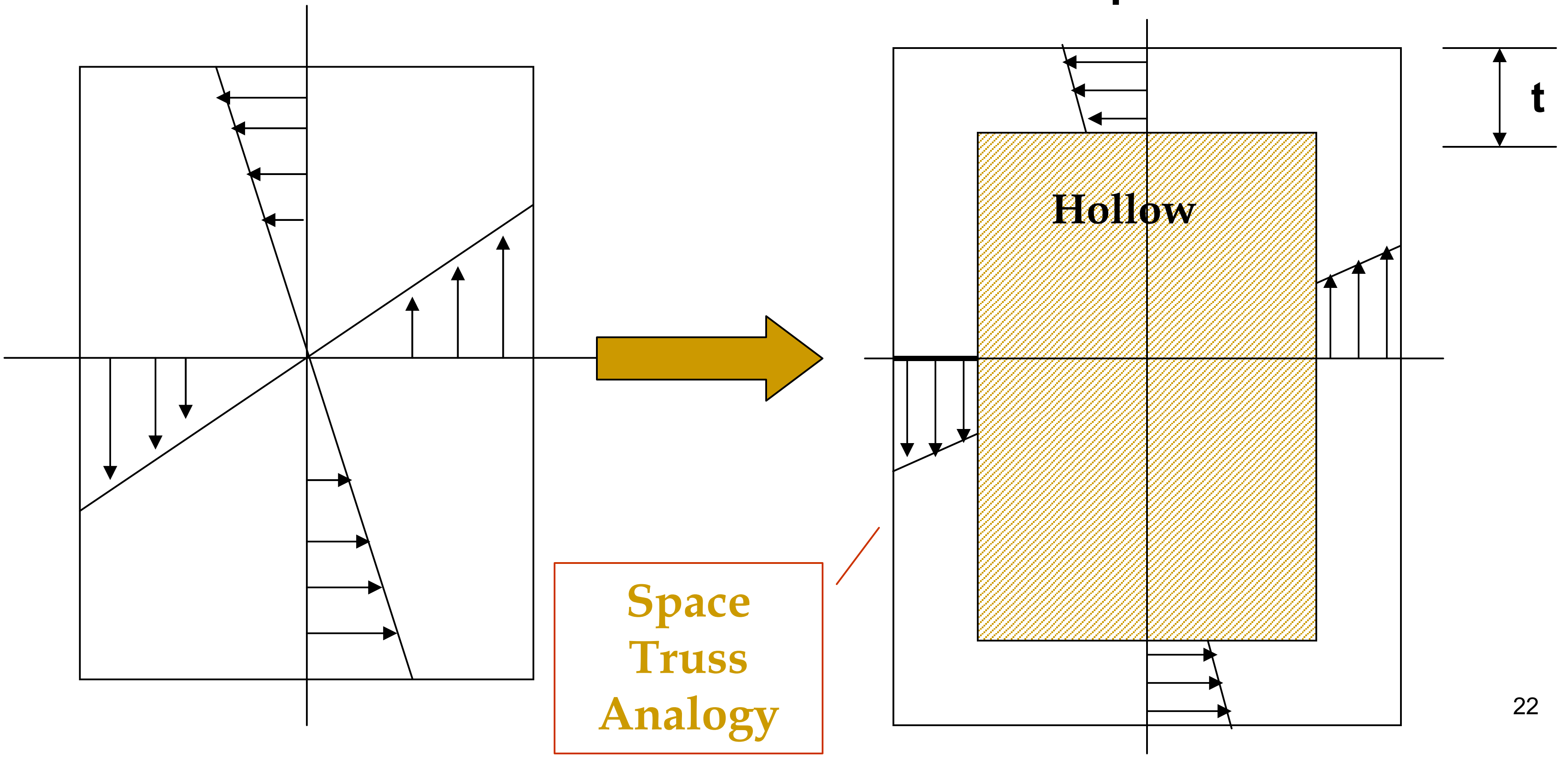
  i) Transverse reinforcement in the form of stirrups (closed loop)

  ii) Extra reinforcement in longitudinal directions specially in corners and around perimeter.

Open stirrups are for Shear not for Torsion

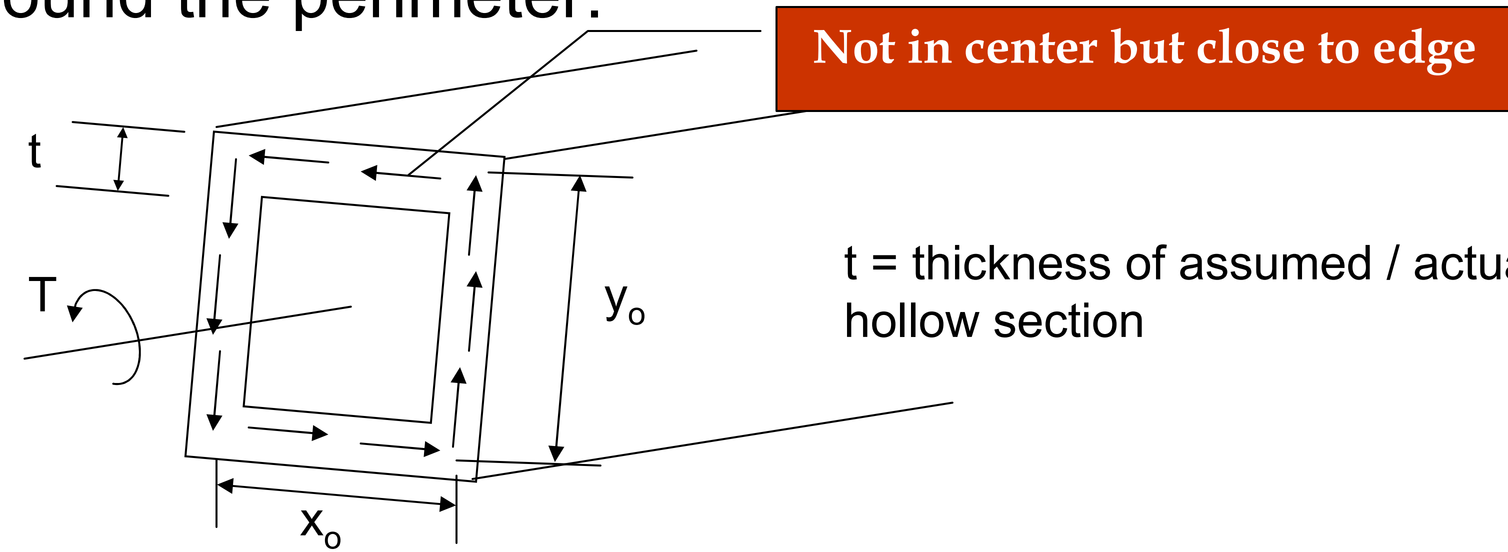
Design (contd…)

- If section is solid consider it hollow. Consider some outer surface to resist torque.



Space Truss Analogy

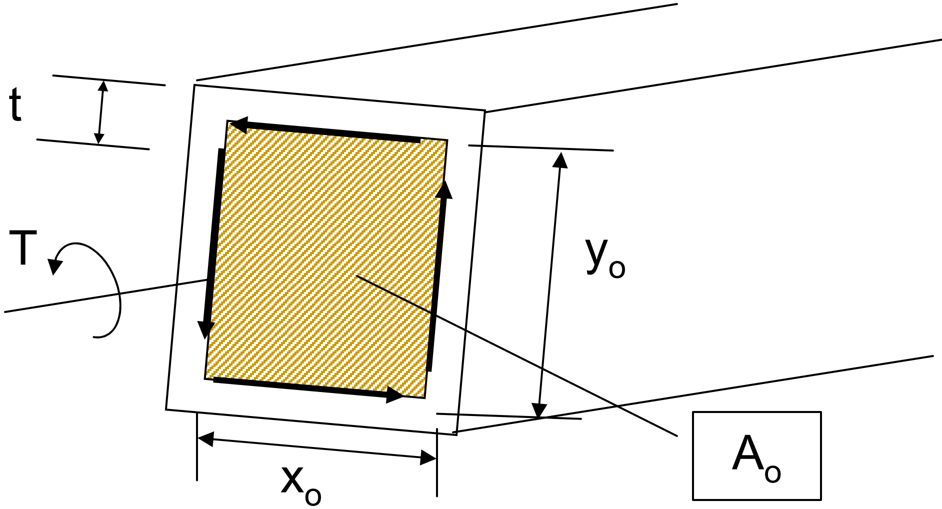
- Shear stresses are considered constant over a finite thickness t around the periphery of member, allowing the beam to be represented by an equivalent tube.
- Within the walls of the tube torque is resisted by the shear flow q. in the analogy q is treated as constant around the perimeter.



t = thickness of assumed / actual hollow section

Space Truss Analogy (contd…)

- x_o & y_o are measured from the center of wall.
- we are neglecting the internal area and using shear flow calculated from average shear stress, instead of maximum, so these two things are balancing each other. **MAKING THE PROBLEM SIMPLER.**



A_o = Area enclosed by the shear flow path,

$A_o = x_o y_o$

Space Truss Analogy (contd…)

- τ = Average shear stress in the wall thickness
- q = shear flow = τ x t

Moment about center due to shear flow must be equal to applied torque T, so

$$T = [\ q \times y_o \times \frac{x_o}{2} + \ q \times x_o \times \frac{y_o}{2}] \times 2$$

$$T = qx_oy_o + qx_oy_o$$

$$T = 2qx_oy_o$$

$$T = 2qA_o$$

$$q = \frac{T}{2A_o}$$

As, $q = \tau \times t \Rightarrow \tau = \frac{q}{t}$

$$\tau = \frac{T}{2A_o t}$$

ACI 11.6.1 Commentary

Prior to cracking due to torsion the thickness we consider is

$$t = 0.75\frac{A_{cp}}{P_{cp}}$$

Area enclosed by the wall center line, $A_o \approx \frac{2}{3}A_{cp}$

A_{cp} = Area enclosed by outside perimeter of concrete section or the gross area of the section

and p_{cp} = outside perimeter of the concrete cross-section.

Tensile strength of concrete is considered = $\frac{1}{3}\sqrt{fc'}$
(which is generally $\frac{1}{2}\sqrt{fc'}$)

Cracking Torque

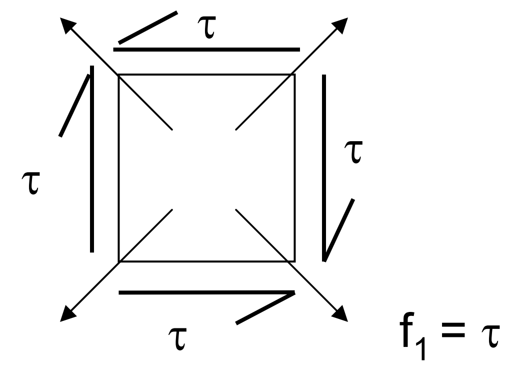
- Torque at which cracking starts.
- For pure shear case principal tensile stress is equal to Torsional shear stress.

$$\tau = \frac{T}{2A_o t} = \frac{1}{3}\sqrt{fc'}$$

$$T_{cr} = \frac{2}{3}\sqrt{fc'} \times A_o \times t$$

$$T_{cr} = \frac{2}{3}\sqrt{fc'} \times \frac{2}{3}A_{cp} \times \frac{3}{4}\frac{A_{cp}}{P_{cp}}$$

$$T_{cr} = \frac{1}{3}\sqrt{fc'}\frac{A_{cp}^{\ 2}}{p_{cp}}$$



T_{cr} = Torque at which cracking starts

ACI 11.6.1

Neglect torsion effect if

$$T_u <= \frac{\phi T_{cr}}{4}$$

$$\phi = 0.75$$

$$T_u <= \frac{\phi}{12}\sqrt{fc'}\frac{A_{cp}{}^2}{p_{cp}}$$

ACI 11.6.2.2

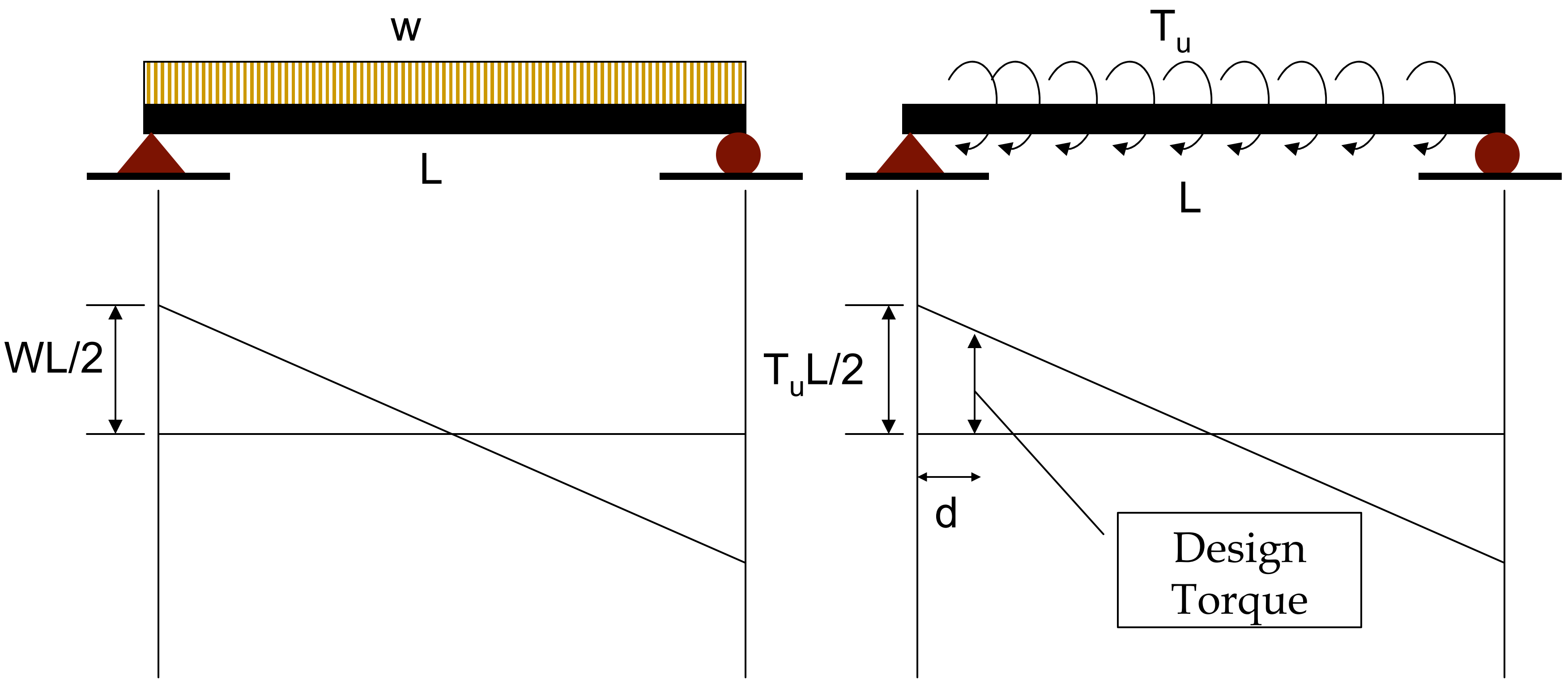
Compatibility torque is allowed to be reduced to

$$\frac{\phi}{3}\sqrt{fc'} \times \frac{A_{cp}{}^2}{p_{cp}}$$

Value of moment and shear in adjoining member must be consider

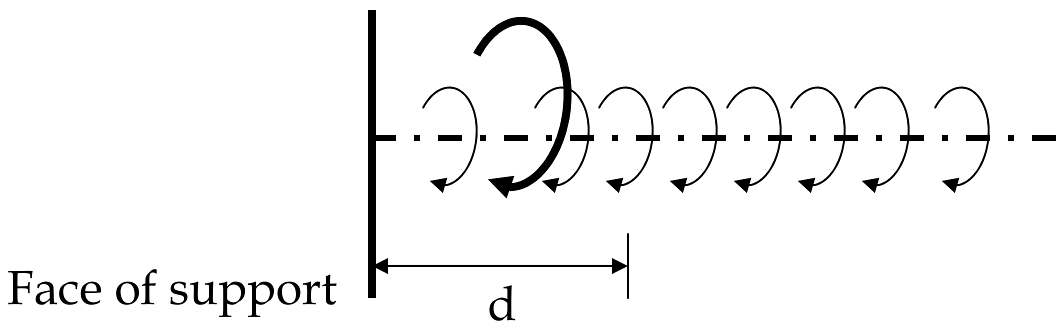
ACI 11.6.2.3

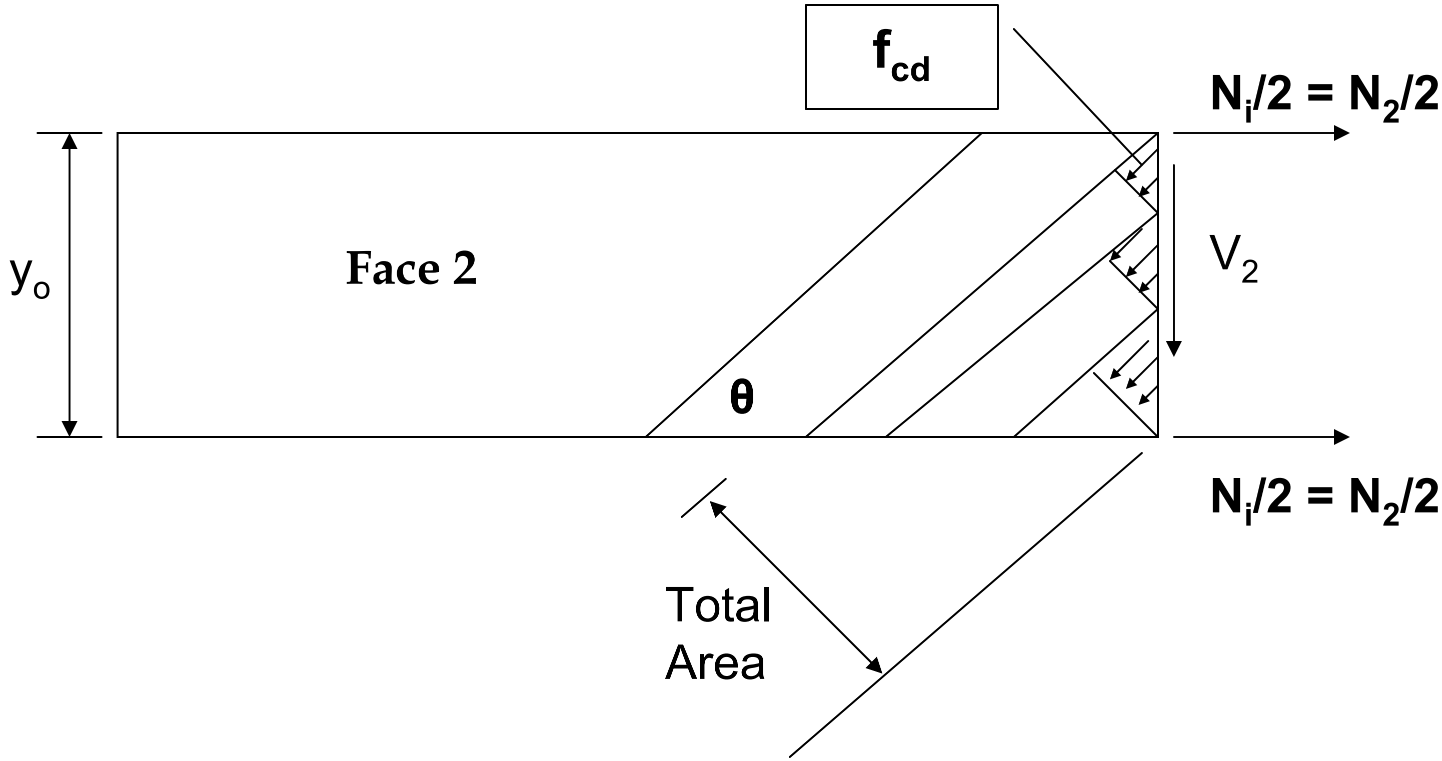
Unless determined by more exact analysis it shall be permitted to take the Torsional loading from slab as **uniformly distributed** along the member (beam).



ACI 11.6.2.4

Face of the support can be considered as critical section if there is some point moment within d distance from face of support.





f_{cd} = Stress acting over compression diagonal

N_2 = longitudinal force required for equilibrium at face 2

For Face - 2

$$V_2 = q \times y$$

$$V_2 = \frac{Ty}{2A_o}$$

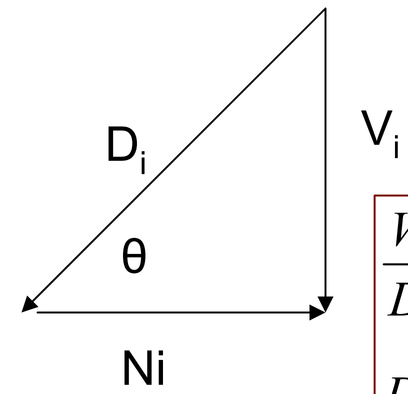
- V_i is resolved in D_i & N_i

$$f_{cd} = \frac{Di}{Area}$$

$$f_{cd} = \frac{V_i}{\sin\theta} \times \frac{1}{y_o sos\theta \times t}$$

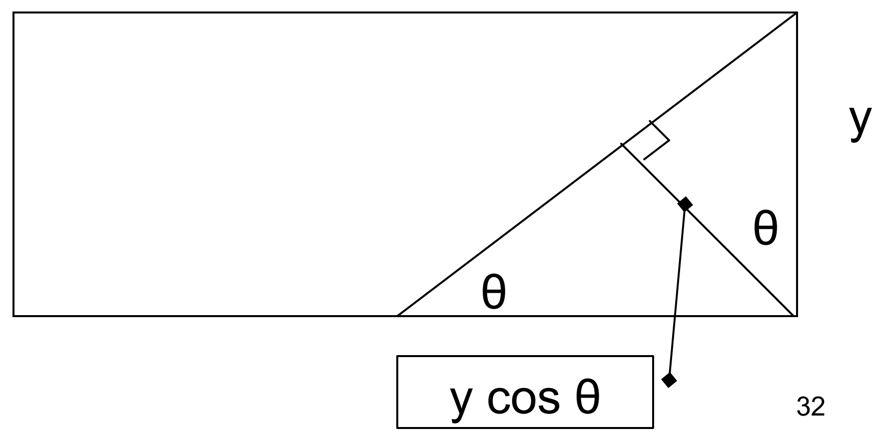
$$f_{cd} = \frac{T/2A_o \times y_o}{y_o t \sin\theta \cos\theta}$$

$$f_{cd} = \frac{T}{2A_o t \sin\theta \cos\theta}$$



$$\frac{Vi}{Di} = \sin\theta$$

$$Di = \frac{Vi}{\sin\theta}$$



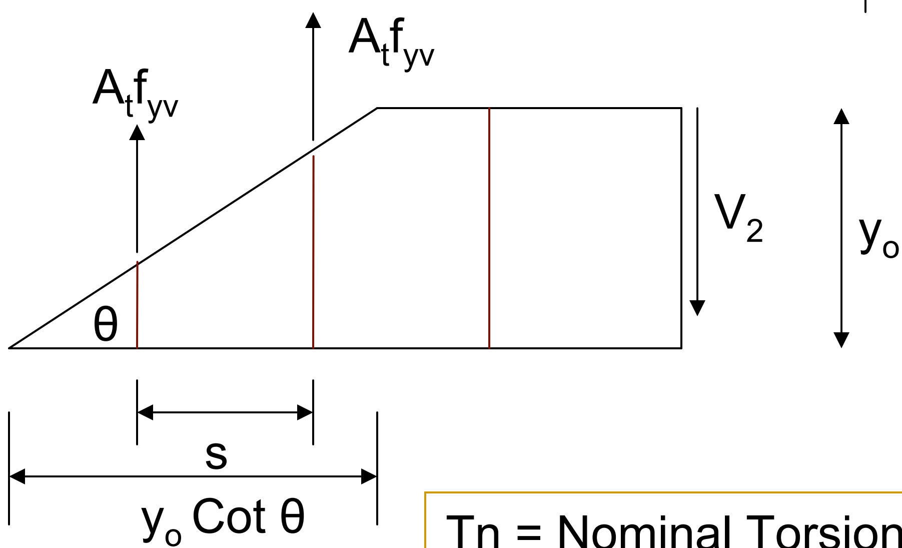
Area of this face is **(y Cos θ) t**

Face - 2

At = Area of one leg of closed stirrup.

n = number of stirrups intercepted by Torsional crack

$$n_2 = \frac{y_o \cot\theta}{s}$$



For vertical equilibrium

$$n_2 \times A_t f_{yv} = V_2$$

$$\frac{y_o \cot\theta}{s} \times A_t f_{yv} = \frac{T_n y_o}{2A_o}$$

$$T_n = \frac{2A_o A_t f_{yv}}{s} \times \cot\theta$$

ACI 11.6.3.6

Tn = Nominal Torsional moment strength

$A_o \approx 0.85A_{oh}$, **or get by exact analysis**

θ = 30° to 60°

= 45° better for non-prestressed members

Face – 2 (contd…)

$$N_2 = V_2 \cot\theta$$

$$N_2 = \frac{T_n}{2A_o} y_o \cot\theta$$

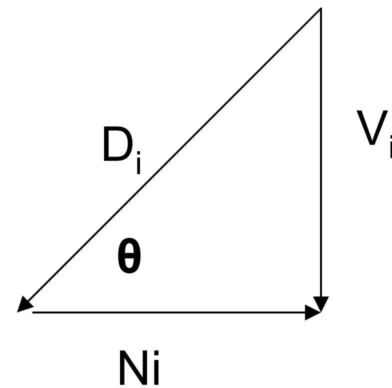
$$N_2 = \left[\frac{2A_o A_t f_{yv} \cot\theta}{s}\right] \frac{y_o \cot\theta}{2A_o}$$

$$N_2 = \frac{A_t f_{yv}}{s} \cot^2\theta \times y_o$$

$$\sum N_i = \frac{A_t f_{yv} \cot^2\theta}{s} \left(2x_o + 2y_o\right)$$

$$A_l \times f_{yl} = \frac{A_t f_{yv} \cot^2\theta}{s} \times p_h$$

$$A_l = \frac{A_t}{s} p_h \left(\frac{f_{yv}}{f_{yl}}\right) \cot^2\theta$$



p_h = perimeter of centerline of outermost closed transverse torsional reinforcement.

ACI 11.6.3.7

A_l = Total area of longitudinal steel to resist torsion

1

Get $\frac{A_t}{s} = \frac{T_n}{2A_o f_{yv} \cot\theta}$ and put in Eq-1

- Smaller θ value we use, lesser shall be the stirrups but more longitudinal reinforcement & vice versa.

θ ↓ Cot θ ↑

Total Reinforcement (Shear + Torsion)

$$\frac{A_{v+t}}{s} = \frac{A_v}{s} + \frac{2A_t}{s}$$

For closed loop stirrup

2 is not here, because we use twice the area of shear reinforcement

ACI 11.6.5.3

- Min longitudinal steel

$$A_{l\,\min} = \frac{5\sqrt{fc'}}{12} \times \frac{A_{cp}}{f_{yl}} - \frac{A_t}{s} p_h [\frac{f_{yv}}{f_{yl}}]$$

Where $\frac{A_t}{s} >= \frac{1}{6}\frac{b_w}{f_y}$

ACI 11.6.5.2

$$(\frac{A_{v+t}}{s})_{\min}$$ is larger of

$$i)\frac{3}{48}\sqrt{fc'}\frac{b_w}{f_y}$$

$$ii)\frac{1}{3}\frac{b_w}{f_y}$$

ACI 11.6.6 (Spacing requirement)

- Transverse stirrups spacing should not be more than

  i) p_h / 8

  ii) 300 mm

- Longitudinal bar spacing should not be more than 300 mm.

  •Distributed around the perimeter of closed stirrup.
  •At least one in each corner.

- Dia of longitudinal bar should not be less than

  i) s / 24

  ii) 10 mm

  s = spacing of shear reinforcement

ACI 11.6.6.3

- Torsional reinforcement shall be provided for a distance of at least **(b_t + d)** beyond the point theoretically required

  b_t = width of torsion section

ACI 11.6.3.1

- Check for the x-sectional dimensions for combined shear and torsion

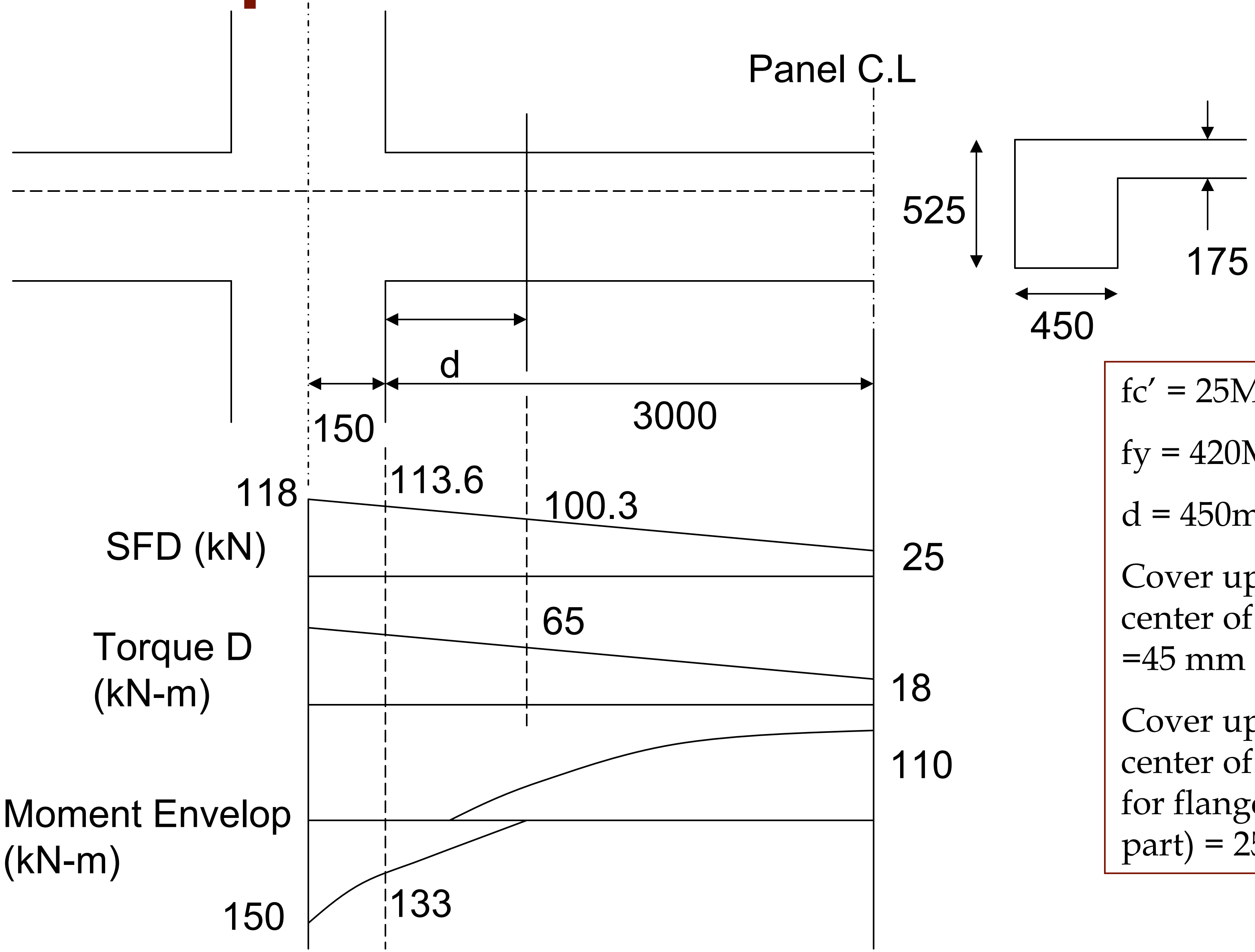
$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_h}{1.7 A_{oh}{}^2}\right)^2} <= \phi\left(\frac{V_c}{b_w d} + \frac{2}{3}\sqrt{fc'}\right)$$

If this equations does not satisfy increase x-sectional dimensions

Design Procedure

1. Plot SF & BM diagram, design for flexure, do partial detailing.
2. Draw factored torque diagram, get torque at different sections ad also get **critical torque**.
3. Neglect torsion if $$T_u <= \frac{\phi T_{cr}}{4}$$
4. If compatibility torsion it can be reduced to $$\frac{\phi}{3}\sqrt{fc'} \times \frac{A_{cp}^{2}}{p_{cp}}$$
5. Check the x-sectional dimensions for combined actions of shear and torque, **if not ok increase dimensions.**
6. Design for shear and calculate A_v/s.
7. Compute the torsion transverse reinforcement A_t/s.
8. Calculate total transverse reinforcement $$\frac{A_{v+t}}{s} = \frac{A_v}{s} + \frac{2A_t}{s}$$

Example



fc′ = 25Mpa

fy = 420Mpa

d = 450mm

Cover up to center of stirrup =45 mm

Cover up to center of stirrup for flange (slab part) = 25mm

<u>Statement</u> : **Design part of beam closer to junction and mid span section for combined action of shear, bending and torque.**

<u>Solution:</u>

M-ve = 133 kN-m

$$A_{smin} = \frac{1.4}{f_y} b_w d$$

$= 675.0\ mm^2$

$$s = 0.85 \frac{f_c'}{f_y} = 0.0506$$

$$\frac{R}{f_c'} = \frac{M_u}{bd^2 f_c'} = \frac{85 \times 10^6}{450 \times 450^2 \times 25}$$

$A_s = 815\ mm^2$

M +ve = 110 kN-m

$A_s = A_{smin} = 675.0\ mm^2$

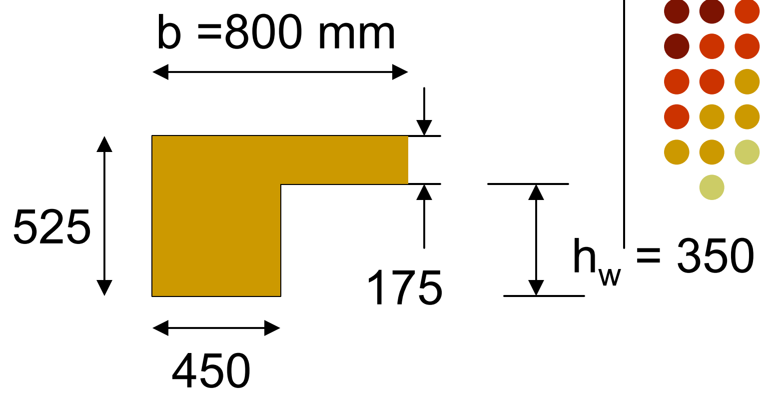
b is lesser of :

i) $b_w + 4h_f = 450 + 700$

ii) $b_w + h_w = 450 + 350 = 800$ mm

So b = 800 mm

Tu 65 kN-m



$$T_{cr}/4 = \frac{\phi}{12}\sqrt{fc'}\frac{A_{cp}^{\;2}}{p_{cp}}$$

$A_{cp} = 450 \times 525 + 175 \times 350 = 297500 \text{ mm}^2$

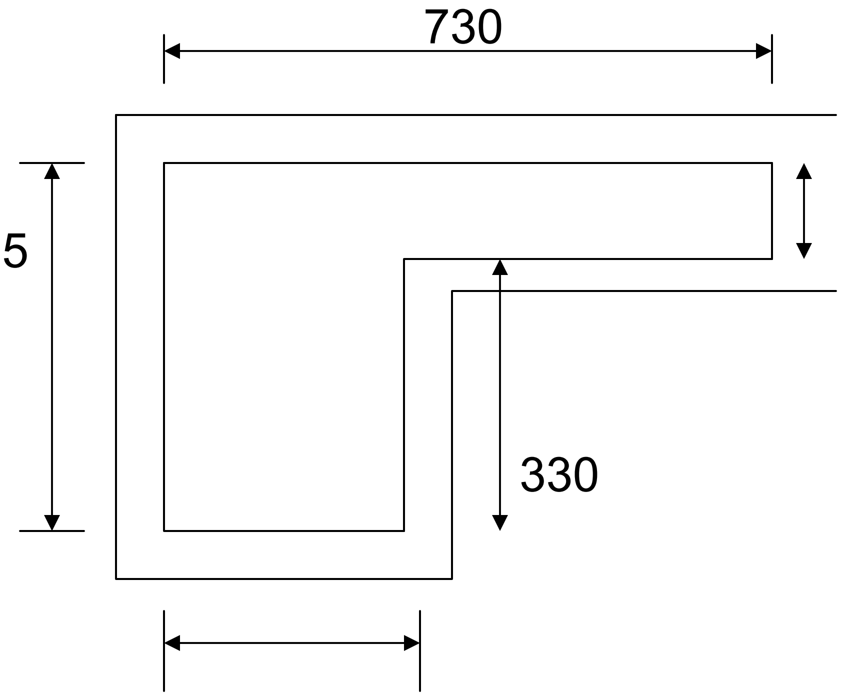
$P_{cp} = (800 + 525) \times 2 = 2650$ mm

$$T_{cr}/4 = \frac{\phi}{12}\sqrt{fc'}\frac{A_{cp}^{\;2}}{p_{cp}} = \frac{0.75}{12}\sqrt{25}\times\frac{297500^2}{2650}\times\frac{1}{10^6} = 10.44 kNm$$ **< Tu**

We cannot redistribute because we don't know about the other members. Their design will change if we redistribute.

A_{oh} = 210050 mm2

P_h = 2370 mm



Step # 6

Check for the x-sectional dimensions

$$LS = \sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_h}{1.7 A_{oh}{}^2}\right)2}$$

$$LS = \sqrt{\left(\frac{100.30x10^3}{450x450}\right)^2 + \left(\frac{65x10^6\,x2370}{1.7x210050^2}\right)^2} = 3.113MPa$$

For right hand side we need V_c

$$V_c = \frac{1}{6}\sqrt{fc'}b_w d$$

$$RS = \phi\left[\frac{1}{6}\sqrt{fc'} + \frac{2}{3}\sqrt{fc'}\right]$$

$$RS = \phi \times \frac{5}{6}\sqrt{fc'} = 0.75 \times \frac{5}{6}\sqrt{fc'} = 3.125MPa$$

$LS < RS$ **So dimensions are O.K.**

Step # 7 (independent design for shear)

$(V_u)_{max}$ = 100.3 kN

$$V_c = \frac{1}{6}\sqrt{f_c'}b_w d$$

$$V_c = \frac{1}{6}\sqrt{25}\times 450\times 450/1000$$

$$V_c = 168.75kN$$

$$\phi V_c = 0.75\times 168.75 = 126.56kN$$

$$\frac{\phi V_c}{2} = \frac{126.56}{2} = 63.3kN$$

$$\frac{\phi V_c}{2} < V_u < \phi V_c$$

$$V_s = 0$$

So $$\frac{A_v}{s} = 0$$

All shear is taken by concrete

For the actually applied shear force. However, min shear reinforcement is required for combined shear and torsion. To be calculated latter.

Step # 8 (Transverse reinforcement required for torque)

$$\theta = 45^o$$

$$A_o = 0.85 A_{oh}$$

$$\frac{A_t}{s} = \frac{Tu}{\phi \times 2 \times 0.85 A_{oh} \times f_{yv}}$$

$$\frac{A_t}{s} = \frac{65 \times 10^6}{0.75 \times 2 \times 0.85 \times 210050 \times 420} = 0.578 mm^2 / mm / leg$$

Step # 9 (Total Transverse reinforcement)

Area of #10 bar

$$\frac{A_{v+t}}{s} = \frac{A_v}{s} + \frac{2A_t}{s}$$

$$\frac{2 \times 71}{s} = 0 + 2 \times 0.578 = 1.156 mm^2 / mm / leg$$

$$s = 123mm$$

$$s = 120mm < \frac{d}{2} = \frac{450}{2} = 225mm$$

Check is required because $V_u > \phi V_c / 2$

Minimum shear + Torsional reinforcement

$$\left(\frac{A_{v+t}}{s}\right)_{\min} = \frac{1}{3}\frac{b_w}{f_{yv}}, f_c' < 28.5MPa$$

$$\left(\frac{A_{v+t}}{s}\right)\min = \frac{1}{3}\frac{450}{420} = 0.375 < 1.14$$ O.K.

Transverse stirrup spacing should be less than:

1. p_h / 8 = 2370 / 8 = 296 mm
2. 300 mm **O.K.**

Step # 10 (Longitudinal Reinforcement)

$$A_l = \frac{A_t}{s} p_h \left(\frac{f_{yv}}{f_{yl}} \right) \cot^2 \theta$$

$$A_l = \frac{A_t}{s} p_h \left(\frac{f_{yv}}{f_{yl}} \right) \cot^2 \theta = 0.578 \times 2370 \times 1 \times 1 = 1370 mm^2$$

$$A_{l\min} = \frac{5\sqrt{fc'}}{12} \times \frac{A_{cp}}{f_{yl}} - \frac{A_t}{s} p_h [\frac{f_{yv}}{f_{yl}}]$$

$$A_{l\min} = \frac{5\sqrt{f_c'}}{12} \times \frac{A_{cp}}{f_{yl}} - \frac{A_t}{s} p_h [\frac{f_{yv}}{f_{yl}}] = \frac{5}{12}\sqrt{25} \times \frac{297500}{420} \quad 0.157 \times 2370 \times \frac{450}{420} = 1043 mm2$$

Replaced by $b_w/(3f_y)$

$A_l = 1370\ mm^2$

(Longitudinal Reinforcement)

Maximum spacing = 300 mm

Minimum dia = 10 mm

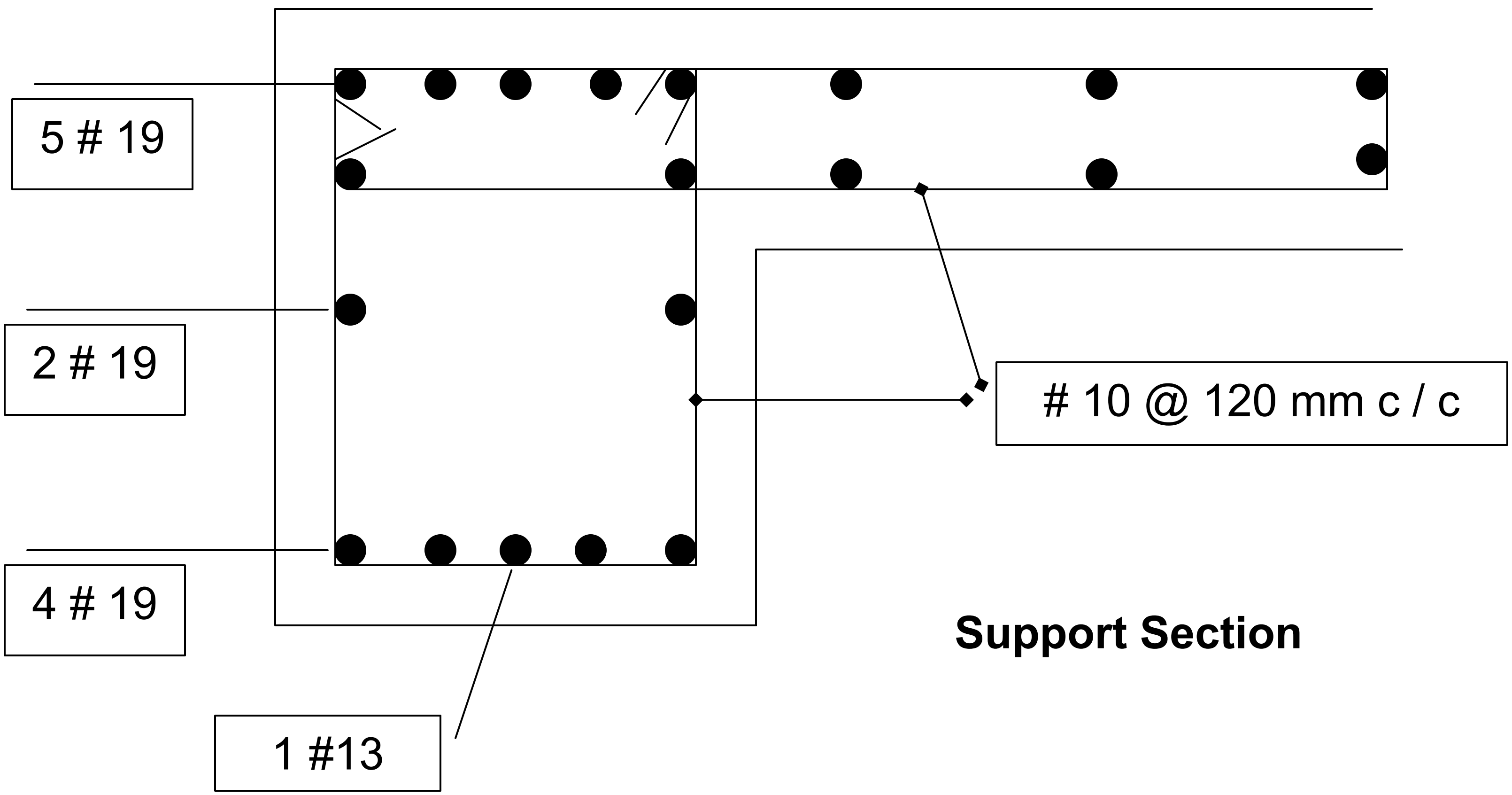
For three layers, A_l / 3 = 1370 / 3 = 457 mm^2

Top Flexural + Torsional Steel = 815 + 457 = 1272 mm^2

Bottom Flexural + Torsional Steel = 1/4A^+ + 457 = 626 mm^2

5 # 19 for top layer **4 # 19 + 1 # 13 for bottom layer**

For middle layer = 457 mm^2 **2 # 19**



Support Section

Concluded