First Approximate Method Of Design For Compression Failure

Step 3: Calculate e_b from the moment equation in terms of A_{st}.

$$P_{\mathrm{nb}} \times e_{\mathrm{b}} = 0.85 f_{\mathrm{c}}' b a_{\mathrm{b}} \left(\frac{h}{2} - \frac{a}{2} \right) + \frac{A_{st}}{2} f_y \left(\frac{h}{2} - d' \right) + \frac{A_{st}}{2} f_y (d'')$$

For symmetric sections, $h/2 - d' = d''$

$$P_{\mathrm{nb}} \times e_{\mathrm{b}} = 0.85 f_{\mathrm{c}}' b a_{\mathrm{b}} \left(\frac{h}{2} - \frac{a}{2} \right) + A_{\mathrm{st}} f_{\mathrm{y}} \left(\frac{h}{2} - d' \right) \qquad \text{(XVI)}$$

Step 4: Calculate P_{o} in terms of A_{st}.

$$P_{\mathrm{o}} = \phi [0.85 f_{\mathrm{c}}' (\mathrm{A_g} - \mathrm{A_{st}}) + \mathrm{A_{st}}\, \mathrm{f_y}] \qquad \text{(XV)}$$

Step 5: The interaction diagram in the compression region is assumed to be a straight line from the safe purely axial load point (P_o) to safe balanced point (P_b).

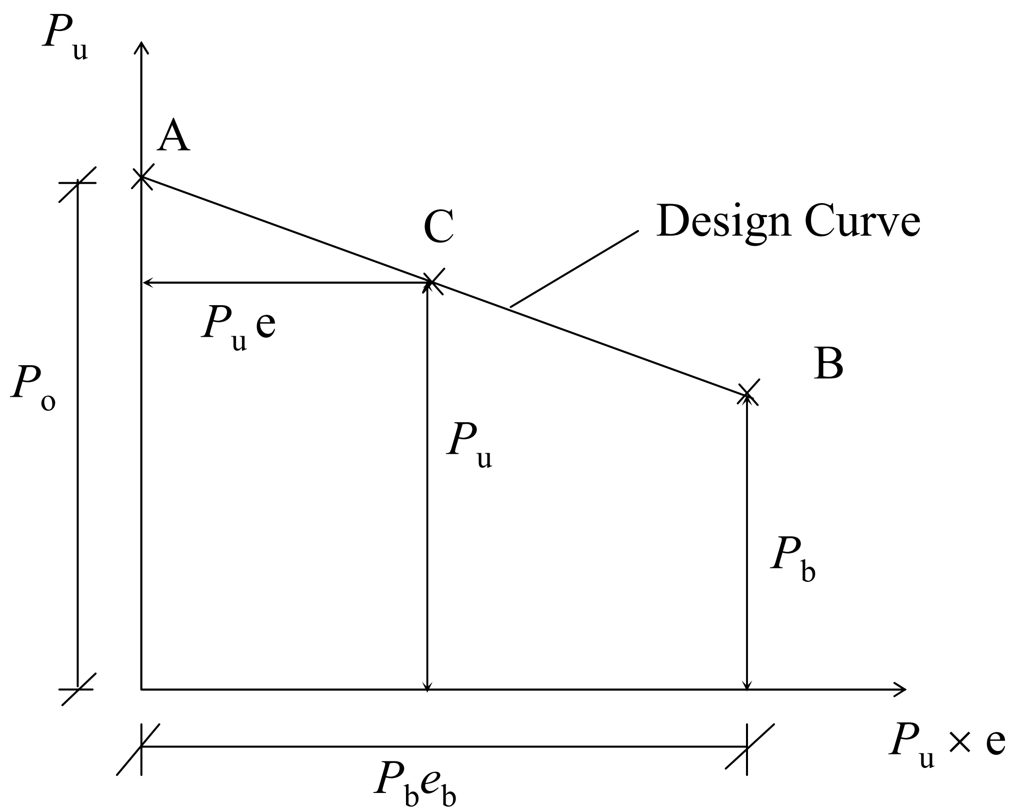


Fig. 14.19. Approximate Design Interaction Curve For Compression Region.

Slope of line $= \dfrac{P_o - P_b}{P_b e_b}$

Hence for any point '*C*', $P_\mathrm{u} = P_\mathrm{o} - \dfrac{P_o - P_b}{P_b e_b}(P_\mathrm{u}\, e)$

$$P_\mathrm{u} = \frac{P_o}{1 + \left(\dfrac{P_o}{P_b} - 1\right)\dfrac{e}{e_b}} \qquad \text{(XVI)}$$

The values of P_o and P_b may be put in terms of A_st to form a quadratic equation in terms of A_st.

The resulting equation may be solved for the unknown steel area.

However, the solution is still lengthy and difficult to solve for design.

Whitney's Empirical Equation

Assumptions:

The column has a rectangular cross-section with reinforcement placed in two layers parallel to the axis of bending at equal distances from this axis.

The compression reinforcement has yielded. This is true for small eccentricities.

The area of concrete displaced by the compression steel may be neglected.

A straight line can represent the interaction diagram for compression failures starting from the point corresponding to the pure axial load capacity P_{no} to the point corresponding to a balanced failure.

The depth of the compression stress block for a balanced failure is $a_b = \beta_1 c_b$.

From strain compatibility, assuming that $f_c' \leq 28$ MPa and $f_y = 420$ MPa, we get,

$$a_b = \beta_1 \frac{600}{600 + f_y} d$$

$$= 0.85 \frac{600}{600 + 420} d \cong 0.50\, d$$

Summing the moments about the tension reinforcement at balanced stage gives the following:

$$P_{\mathrm{nb}}\left(e_b + d - \frac{h}{2}\right) = C_{\mathrm{s}}'\,(d - d') + C_{\mathrm{c}}\,(d - a_b/2)$$

$$d - h/2 = (d - d')\,/\,2,\quad C_s' = A_s' f_y,\quad C_c = 0.85\,f_c'\,b\,a_b$$

$$P_{\mathrm{nb}}\left(e_b + \frac{d - d'}{2}\right)$$

$$= A_s' f_y\,(d - d') + 0.85\,f_c'\,b\,(0.50d)\left(d - \frac{0.50d}{2}\right)$$

$$P_{\mathrm{nb}} = \frac{0.319\,f_c' b d^2 \times \dfrac{h}{d^2}}{e_b + \dfrac{d - d'}{2} \times \dfrac{h}{d^2}} + \frac{A_s' f_y}{\dfrac{e}{d - d'} + 0.5}$$

$$P_{\mathrm{nb}} = \frac{0.319 f_c' b h}{\dfrac{e_b h}{d^2} + \dfrac{(d - d')h}{2d^2}} + \frac{A_s' f_y}{\dfrac{e}{d - d'} + 0.5} \qquad \text{(XVII)}$$

This equation is basically valid for balanced failure.

To make it valid for smaller eccentricities, the above equation is forced to pass through the value of P_{no} at $e = 0$.

Also to make it a general equation, e is used in place of e_b and P_{n} is used in place of P_{nb}.

For $e = 0$, Eq. XVII gives:

$$P_{\mathrm{no}} = \frac{0.319 f_c' b h}{\dfrac{(d - d')h}{2d^2}} + 2A_{\mathrm{s}}' f_{\mathrm{y}} \qquad \text{(XVIII)}$$

Actually, $P_{\mathrm{no}} = 0.85 f_{\mathrm{c}}' \, b \, h + 2 A_{\mathrm{s}}' f_{\mathrm{y}}$ (XVIV)

Comparing Eqs. XVIII and XVIV,

$$\frac{0.319 f_c' b h}{\frac{(d-d')h}{2d^2}} = 0.85 f_{\mathrm{c}}' \, b \, h$$

$$\frac{(d-d')h}{2d^2} = 0.375 \qquad \text{(XVV)}$$

From XVII: $P_{\mathrm{n}} = \dfrac{0.319 f_c' b h}{\dfrac{eh}{d^2} + 0.375} + \dfrac{A_{st} f_y}{1 + \dfrac{2e}{d-d'}}$

$P_{\mathrm{u}} = \phi P_{\mathrm{n}}$ *for* $f_{\mathrm{y}} = 420 \; MPa$

In general, the formula for all grades of steel becomes:

$$P_n = \frac{\alpha f_c' b h}{\frac{eh}{d^2} + \frac{\alpha}{0.85}} + \frac{A_{st} f_y}{1 + \frac{2e}{d - d'}}$$

where $\alpha = 0.408 - 0.00021 f_y$

Example 14.3: A 375×450 mm tied column is to be reinforced symmetrically by bars placed in two opposite faces of the section. $f_c' = 20$ MPa, $f_y = 420$ MPa, $d' = 65$ mm and $d = 450 - 65 = 385$ *mm*. Determine the steel areas required for the column to support the following ultimate loads:

1. 675 kN at $e = 450$ mm, **2.** 1180 kN at $e = 200$ mm.

Solution:

$$a_{\mathrm{b}} \quad = \quad \beta_1 \frac{600}{600+f_y} d = \quad 192.5\ mm$$

$$\varepsilon_{\mathrm{s}}' \text{ (balanced condition)} \quad = \quad (0.003)\ \frac{a_b - \beta_1 d'}{a_b}$$

$$= \quad 0.00214 \quad > \quad \frac{f_y}{E_s} = 0.0021$$

$\therefore$ Compression steel is yielding at balanced stage,

$$f_{\mathrm{s}}' = f_{\mathrm{y}}$$

$$P_{\mathrm{nb}} \quad \approx \quad 0.85 \times 200 \times 375 \times 192.5 + A_{\mathrm{s}}' f_{\mathrm{y}} - A_{\mathrm{s}} f_{\mathrm{y}}$$

$$= \quad 1227.2 \text{ kN}$$

$$P_b = 0.65\, P_{nb} = 797.7 \text{ kN}$$

1. $\underline{P_u = 675 \text{ kN}, e = 450 \text{ mm}}$

$$P_u < P_b \quad \Rightarrow \quad \text{tension failure} \quad \Rightarrow \quad f_s = f_y$$

$$\phi_{appr.} = 0.9 - \frac{P_u}{4\, P_{nb}}$$

$$= 0.9 - \frac{675}{4 \times 1227.2} = 0.76$$

Assuming $f_s' = f_y$, the load equation becomes:

$$\frac{675000}{0.76} = 0.85 \times 20 \times 375\, a \quad \therefore \quad a = 139.3 \text{ mm}$$

$$\varepsilon_s' = 0.003 \frac{a - \beta_1 d'}{a} = 0.00181 < \varepsilon_y$$

$\therefore$ Compression steel is not yielding.

Let $f_s' = (\varepsilon_s' E_s + \text{Previous } f_s') / 2$
$= (0.00181 \times 200{,}000 + 420) / 2 = 391$ MPa

From the moment equation about the plastic centroid,

$$\frac{675000 \times 450}{0.76} = 0.85\,(20)(375)(139.3)(225 - 139.3/2)$$

$$+ \frac{A_{st}}{2}(391)(225 - 65) + \frac{A_{st}}{2}(420)(225 - 65)$$

$A_{st} = 4034 \text{ mm}^2$

2nd Trial

$$\frac{675000}{0.76} = 0.85 \times 20 \times 375\, a + \frac{4034}{2}(391) - \frac{4034}{2}(420)$$

$$\therefore \quad a = 148.5 \text{ mm}$$

$$\varepsilon_s' = 0.003\ \frac{a - \beta_1 d'}{a} = 0.00188 < \varepsilon_y$$

$\therefore$ Compression steel is not yielding.

$$\text{Let} \quad f_s' = (\varepsilon_s' E_s + Previous\, f_s')\, /\, 2$$

$$= (0.00188 \times 200{,}000 + 391)\, /\, 2 = 384\, MPa$$

From the moment equation about the plastic centroid, we get:

$$\frac{675000 \times 450}{0.76} = 0.85\,(20)(375)(148.5)(225 - 148.5/2)$$

$$+ \frac{A_{st}}{2}(384)(225 - 65) + \frac{A_{st}}{2}(420)(225 - 65)$$

$$A_{\mathrm{st}} = 3995 \text{ mm}^2$$

<u>3rd Trial</u>

$$\frac{675000}{0.76} = 0.85 \times 20 \times 375\,a + \frac{3995}{2}(384) - \frac{3995}{2}(420)$$

$$\therefore \quad \boldsymbol{a = 150.6 \text{ mm}}$$

$$\varepsilon_{\mathrm{s}}' = 0.003\,\frac{a - \beta_1 d'}{a} = 0.00190 < \varepsilon_{\mathrm{y}}$$

$\therefore$ Compression steel is not yielding.

Let $f_s' = (\varepsilon_s' E_s + Previous\ f_s')\ /\ 2$

$$= (0.00190 \times 200{,}000 + 384)\ /\ 2 = 382\ MPa$$

From the moment equation about the plastic centroid,

$$\frac{675000 \times 450}{0.76} = 0.85\ (20)(375)(150.6)(225 - 150.6/2)$$

$$+ \frac{A_{st}}{2}(382)(225 - 65) + \frac{A_{st}}{2}(420)(225 - 65)$$

$$A_{\mathrm{st}} = 3989\ \mathrm{mm}^2$$

<u>4th Trial</u>

$$A_{\mathrm{st}} = 3990\ \mathrm{mm}^2$$

As an alternate, exact solution may be carried out following almost the same procedure as that for the compression failure.

$$\varepsilon_s = 0.003 \; \frac{\beta_1 d - a}{a} = 0.00349$$

$$\phi = 0.65 + \frac{0.25}{0.005 - \varepsilon_y}(\varepsilon_t - \varepsilon_y) = 0.77$$

This value is sufficiently close to the assumed value, and is on the safe side.

2. <u>P_u = 1180 kN, e = 200 mm</u>

$$P_u > P_b \quad \Rightarrow \quad \phi = 0.65$$

A. EXACT SOLUTION

Assuming the compression to be yielding, the load equation may be written as follows:

$$\frac{1180,000}{0.65} = 0.85(20)(375)(a) + \frac{A_{st}}{2}(420) - \frac{A_{st}}{2}(600)\left(\frac{0.85(385) - a}{a}\right)$$

$$A_{st}\,(210 - 300\ \frac{327.25 - a}{a}) = 1,815,385 - 6375\ a$$

$$\therefore \qquad \frac{A_{st}}{a} = \frac{1,815,385 - 6375\,a}{510\,a - 98175} \qquad \text{(I)}$$

From moment equation, we get,

$$\frac{1180{,}000 \times 200}{0.65} = 0.85(20)(375)(a)(225 - 0.5a)$$

$$+ \frac{A_{st}}{2}(420)(160) + \frac{A_{st}}{2}(600)\left(\frac{0.85(385) - a}{a}\right)(160)$$

$$\therefore \quad \frac{A_{st}}{a} = \frac{3.1875\,a^2 - 1434.4\,a + 363{,}077}{15{,}708 - 14.4\,a} \qquad \text{(II)}$$

From Eqs. I and II:

$$F(a) = 0.00163\,a^3 - 1.1363\,a^2 + 452.27\,\mathrm{a} - 64161 = 0$$
$$F'(a) = 0.00489\,a^2 - 2.2726\,a + 452.27$$

Assume $a_{\mathrm{o}} = h / 2 = 225$ mm

$$a_1 = a_o - \frac{F(a_o)}{F'(a_o)}$$

$$= 225 - \frac{(-1358.7)}{188.49} = 232.2 \text{ mm}$$

$$a_2 = a_1 - \frac{F(a_1)}{F'(a_1)}$$

$$= 232.2 - \frac{(-1.28)}{188.22} = 232.2 \text{ mm}$$

$$\varepsilon_s' = 0.003\,\frac{a - \beta_1 d'}{a} = 0.00229 > \varepsilon_y$$

(Compression steel is yielding)

From Eq. I: $A_{st} = 3843 \text{ mm}^2$

B. FIRST APPROXIMATE METHOD

Try at home. Results like those given below will be obtained:

$$1180 = \frac{1864.7 + 0.273\,A_{st}}{1 + \left(\dfrac{1864.7 + 0.273\,A_{st}}{797.7} - 1\right)\dfrac{200}{128.75 + 0.05476\,A_{st}}}$$

$$A_{st}^2 - 543.2\mathrm{A}_{\mathrm{st}} - 15{,}218{,}662 = 0$$

$A_{\mathrm{st}} = 4182\ \mathrm{mm}^2$ (8.83% greater than actual result)

C. USING WHITNEY'S EQUATION

$$\begin{aligned} \alpha &= 0.408 - 0.00021\,f_{\mathrm{y}} \\ &= 0.408 - 0.00021 \times 420 = 0.32 \end{aligned}$$

$$P_n = \frac{\alpha f_c' b h}{\frac{eh}{d^2} + \frac{\alpha}{0.85}} + \frac{A_{st} f_y}{1 + \frac{2e}{d - d'}}$$

$$\frac{1180,000}{0.65} = \frac{(0.32)(20)(375)(450)}{\frac{(200)(450)}{(385)^2} + \frac{0.32}{0.85}} + \frac{A_{st}(420)}{1 + \frac{(2)(200)}{320}}$$

$$\therefore \quad A_{st} = 3844 \text{ mm}^2$$

(can be on unsafe side in some cases)

ANALYSIS OF RECTANGULAR COLUMNS HAVING BARS AT FOUR FACES

For a general bar 'i' in the section, from the strain diagram of Fig. 14.20,

$$\varepsilon_{si} = 0.003 \frac{d_i - c}{c} \;;\; d_i < c \Rightarrow$$ Compressive strains denoted by negative sign

$d_i > c \Rightarrow$ Tensile strains denoted by positive sign

The stresses can be calculated according to the following three possibilities:

1. If, $\varepsilon_{si} \geq \dfrac{f_y}{E_s}$, $f_{si} = f_y$ (tensile)

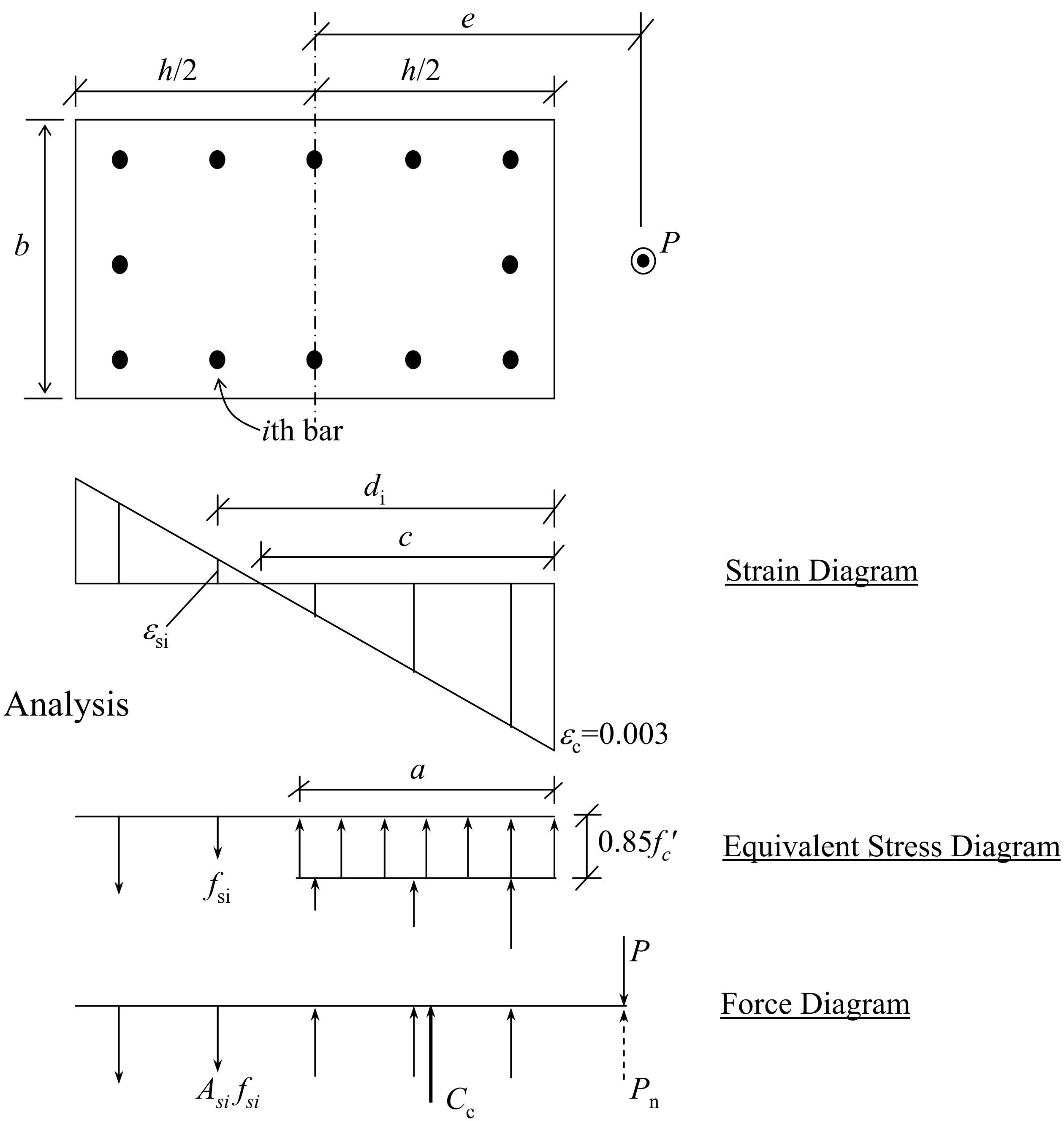


Fig. 14.20. General Analysis of a Short Column.

2. If, $\varepsilon_{\mathrm{si}} \leq -\dfrac{f_y}{E_s}$, $f_{\mathrm{si}} = -(f_{\mathrm{y}} - 0.85\, f_{\mathrm{c}}')$ (compressive, exact)

$f_{\mathrm{si}} = -f_{\mathrm{y}}$ (compressive, approximate)

3. Otherwise, $f_{\mathrm{si}} = \varepsilon_{\mathrm{si}}\, E_s + 0.85\, f_{\mathrm{c}}'$ (compressive, exact)

$f_{\mathrm{si}} = \varepsilon_{\mathrm{si}}\, E_s$ (compressive, approximate)

$$C_{\mathrm{c}} = -0.85\, f_{\mathrm{c}}'\, a\, b$$

Load equation $P_{\mathrm{n}} = -C_{\mathrm{c}} - \sum_{i=1}^{n} f_{si} A_{si}$

Moment equation $P_{\text{n}} \times e$

$$= -C_{\text{c}}\left(\frac{h}{2}-\frac{a}{2}\right) - \sum_{i=1}^{n} f_{si} A_{si}\,(h/2 - d_{\text{i}})$$

$$= C_{\text{c}}\left(\frac{a}{2}-\frac{h}{2}\right) + \sum_{i=1}^{n} f_{si} A_{si}\,(d_{\text{i}} - h/2)$$

To find out the load carrying capacity of such a column at a known eccentricity, trial method is suitable. The general procedure is summarized below:

1. Select 'c' and 'a' arbitrarily.
2. Find f_{s} in all the steel bars.
3. Calculate P_{n} from the load equation.

4. Calculate P_n from the moment equation knowing the eccentricity.

5. Repeat steps 1,2,3 and 4 until the values of P_n in steps 3 and 4 become the same.

Example 14.4: The column shown in Fig. 14.21 is reinforced with 10 # 25 (US Customary) bars. Find P_n and M_n, if the neutral axis is known to be at 500 mm from the right face. $f_c' = 25$ MPa, $f_y = 520$ MPa, $d' = 67.5$ mm and $d = 675 - 67.5 = 607.5$ *mm*.

Solution:

Using strain diagram of Fig. 14.21, $E_s = 200{,}000$ MPa, and $c = 500$ mm:

$$\varepsilon_{si} = 0.003 \frac{d_i - c}{c} \quad ; \quad \varepsilon_y = 0.0026;$$

$$0.85 f_c' = 21.25 \, MPa$$

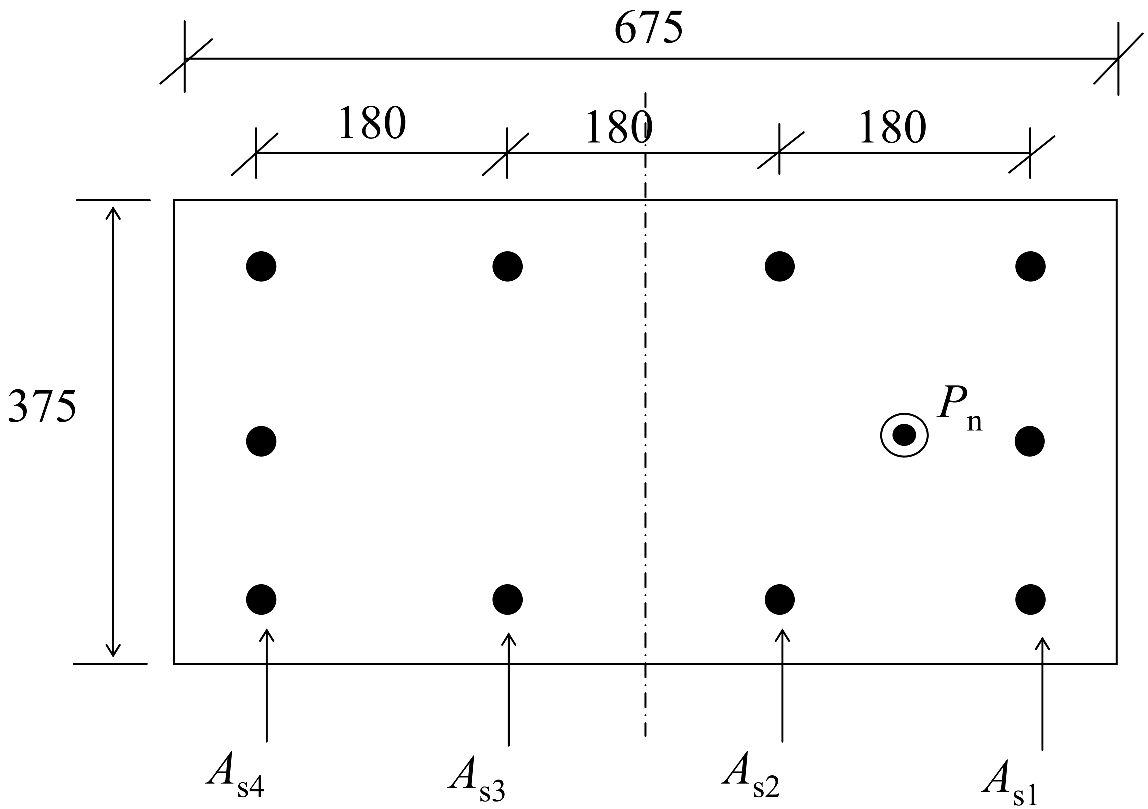


Fig. 14.21. Column Cross-Section For Example 14.4.

$$d_1 = 67.5 \text{ mm} \qquad A_{s1} = 1530 \text{ mm}^2$$

$$d_2 = 247.5 \text{ mm} \qquad A_{s2} = 1020 \text{ mm}^2$$

$$d_3 = 427.5 \text{ mm} \qquad A_{s3} = 1020 \text{ mm}^2$$

$$d_4 = 607.5 \text{ mm} \qquad A_{s4} = 1530 \text{ mm}^2$$

$$\varepsilon_{s1} = -0.002595 \qquad f_{s1} = -519 \text{ MPa (compressive)}$$

$$\varepsilon_{s2} = -0.001515 \qquad f_{s2} = -303 \text{ MPa (compressive)}$$

$$\varepsilon_{s3} = -0.000435 \qquad f_{s3} = -87 \text{ MPa (compressive)}$$

$$\varepsilon_{s4} = +0.000645 \qquad f_{s4} = +129 \text{ MPa (tensile)}$$

$$a = \beta_1 c = 425 \text{ mm}$$

$$C_c = 0.85 \times (-25) \times 375 \times 425 / 1000 = -3386.7\ kN$$

$$F_{si} = f_{si} \times A_{si}$$

$$F_{s1} = (1530) \times (-519) / 1000 = -794.1\ kN$$

$$F_{s2} = (1020) \times (-303) / 1000 = -309.1\ kN$$

$$F_{s3} = (1020) \times (-87) / 1000 = -88.7\ kN$$

$$F_{s4} = (1530) \times (+129) / 1000 = +197.4\ kN$$

$$P_n = -C_c - \sum_{i=1}^{n} f_{si} A_{si}$$

$$= 3386.7 + 794.1 + 309.1 + 88.7 - 197.4$$

$$\approx 4381 \text{ kN}$$

$$M_n = P_n \times e$$

$$= C_c \left(\frac{a}{2} - \frac{h}{2}\right) + \sum_{i=1}^{n} f_{si} A_{si} \ (d_i - h/2)$$

$$
\begin{aligned}
M_n = {} & [(-3386.7)(212.5 - 337.5) \\
& + (-794.1)(67.5 - 337.5) + (-309.1)(247.5 - 337.5) \\
& + (-88.7)(427.5 - 337.\ 5) \\
& + (197.4)(607.5 - 337.5)] / 1000 \\
= {} & 710.9 \text{ kN-m}
\end{aligned}
$$

BIAXIAL BENDING

In practice many columns are subjected to bending about both principal axes simultaneously.

A typical example is the corner columns of a building.

A failure surface for axial load (P_n) plus biaxial bending ($M_{nx} = P_n\, e_y$ and $M_{ny} = P_n\, e_x$) is plotted on the three mutually perpendicular axes, exactly analogous to the failure line for axial load plus uniaxial bending.

Any combination of P_u, M_{ux} and M_{uy} falling inside the design surface can be applied safely, but any point falling outside the surface would represent failure.

e_x = eccentricity of load from the plastic centroid parallel to the x-axis

and e_y = eccentricity of load from the plastic centroid parallel to the y-axis

Three Types of Interaction Surfaces

1. Surface plotted for parameters P_n, e_x and e_y (Diagram – A of Fig. 14.22).

2. Surface plotted for parameters $1 / P_n$, e_x and e_y (Diagram – B of Fig. 14.22).

3. Surface plotted for parameters P_n, M_{nx} and M_{ny} (Diagram – C of Fig. 14.22).

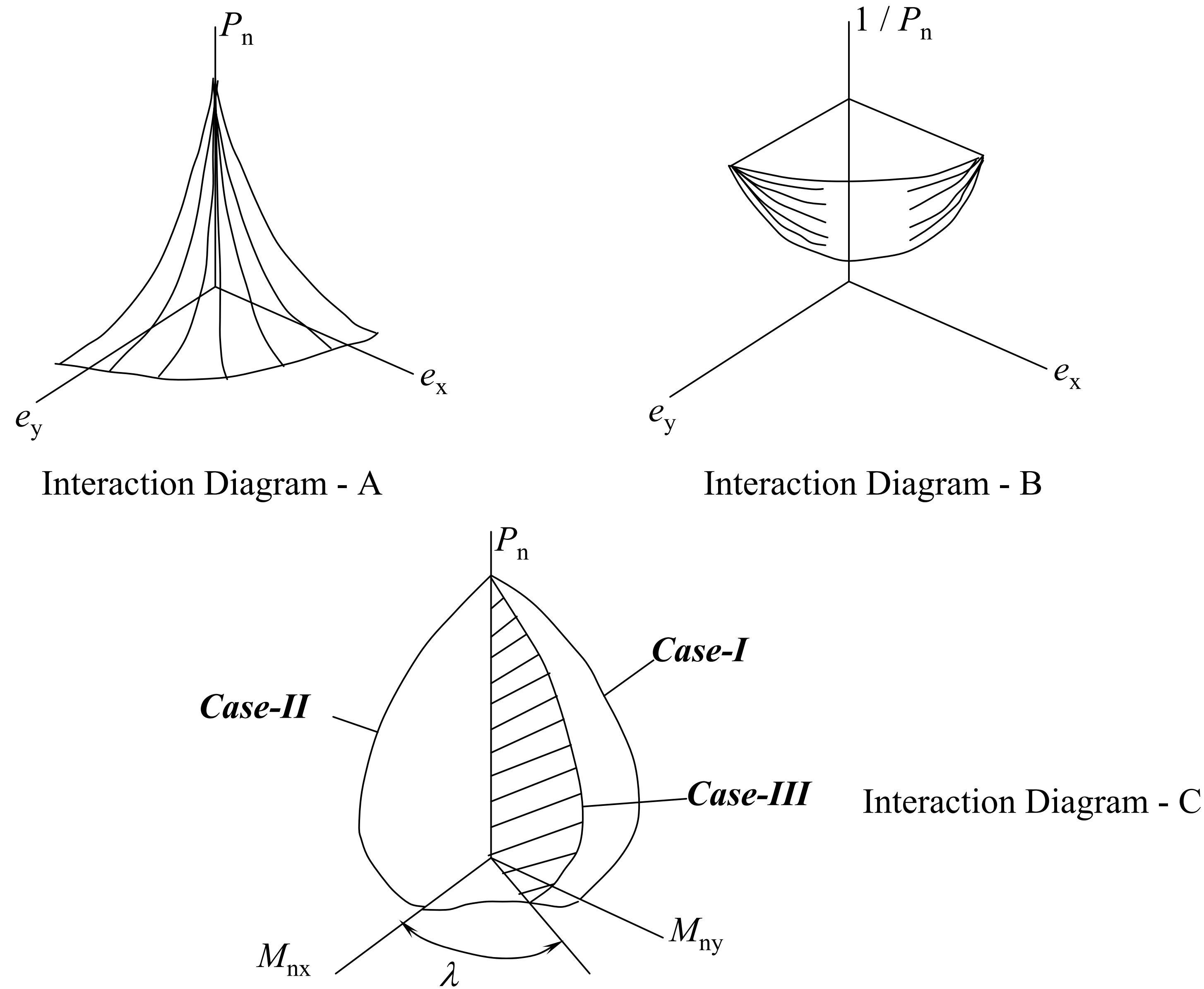


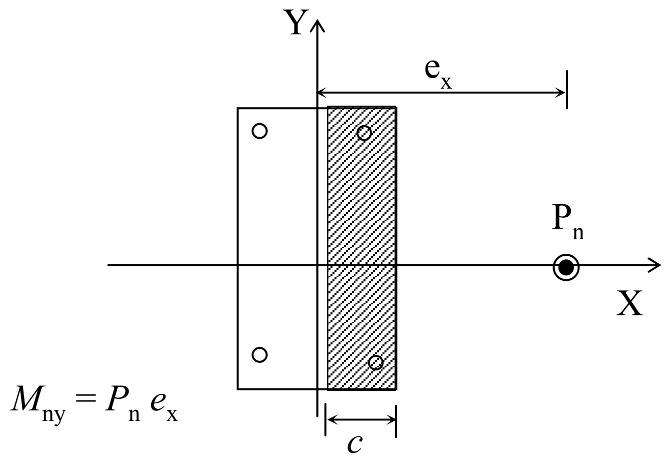
Fig. 14.22. Typical Interaction Surfaces For Biaxial Bending of Columns.

λ = eccentricity angle, defined as the angular distance of load from the y-axis.

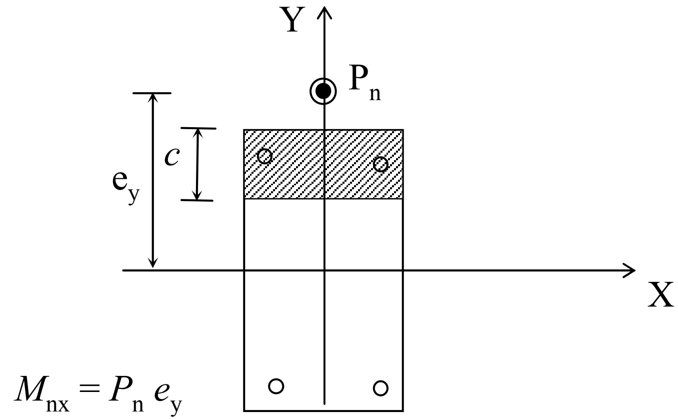
$$= \tan^{-1}\frac{e}{e_y} = \tan^{-1}\frac{M_{ny}}{M_{nx}}$$

θ = counter-clockwise angle of the neutral axis with respect to the y-axis.

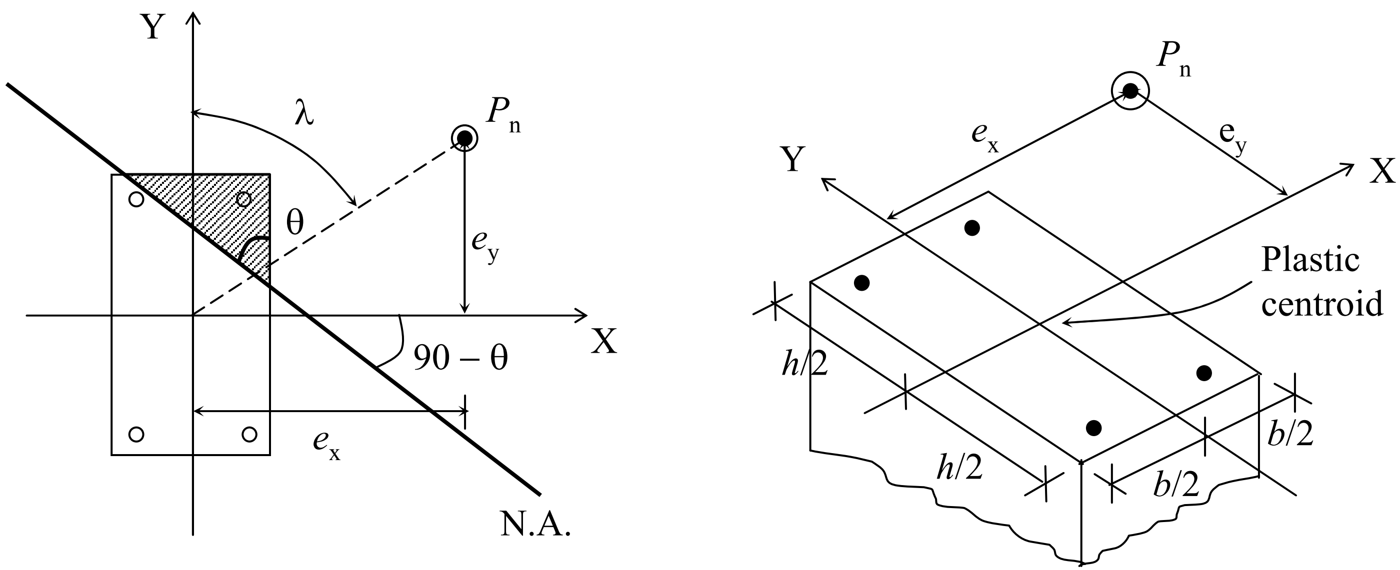
Analysis and design of column sections having biaxial bending are lengthy compared to uniaxial bending cases because a trial and adjustment procedure is required to find the inclination of the neutral axis, θ, and the depth of neutral axis, c, satisfying the equilibrium equations. *There are two unknowns in this analysis, namely,* θ and c, as compared to only one unknown in the uniaxial analysis.



Case-I



Case-II



Case-III

The four possible shapes of the equivalent compressive stress block for Case III, are shown in Fig. 14.24.

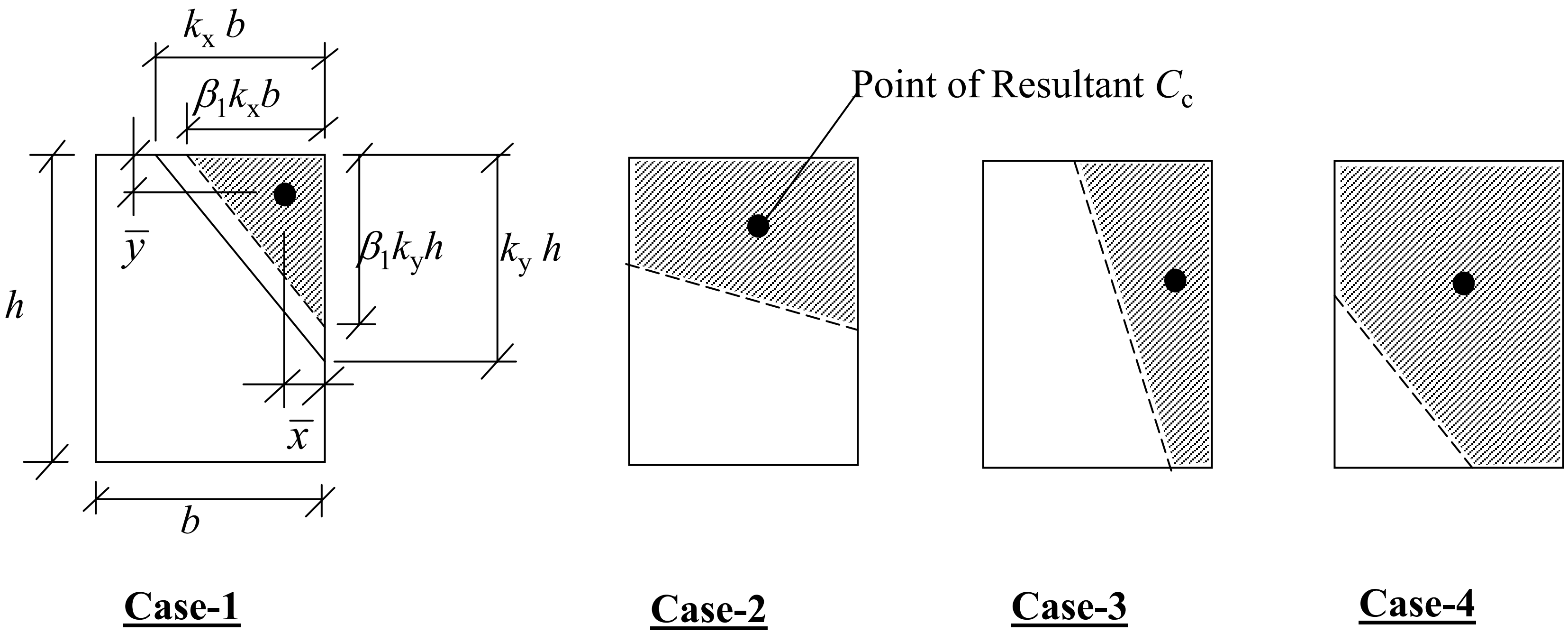


Fig. 14.24. Various Concrete Stress Blocks For Biaxial Bending of Columns.

For First Stress Block, we have,

$$C_{\mathrm{c}} = \frac{(0.85 f_c')(\beta_1 k_y h)(\beta_1 k_x b)}{2} = 0.425 f_c' \beta_1^2 k_x k_y bh$$

$$\bar{x} = 0.333\, \beta_1 k_{\mathrm{x}} b \quad : \quad \bar{y} = 0.333\, \beta_1 k_{\mathrm{y}} h$$

The strain, stress and force diagrams for any one of these cases can be drawn perpendicular to the neutral axis, as shown in Fig. 14.25.

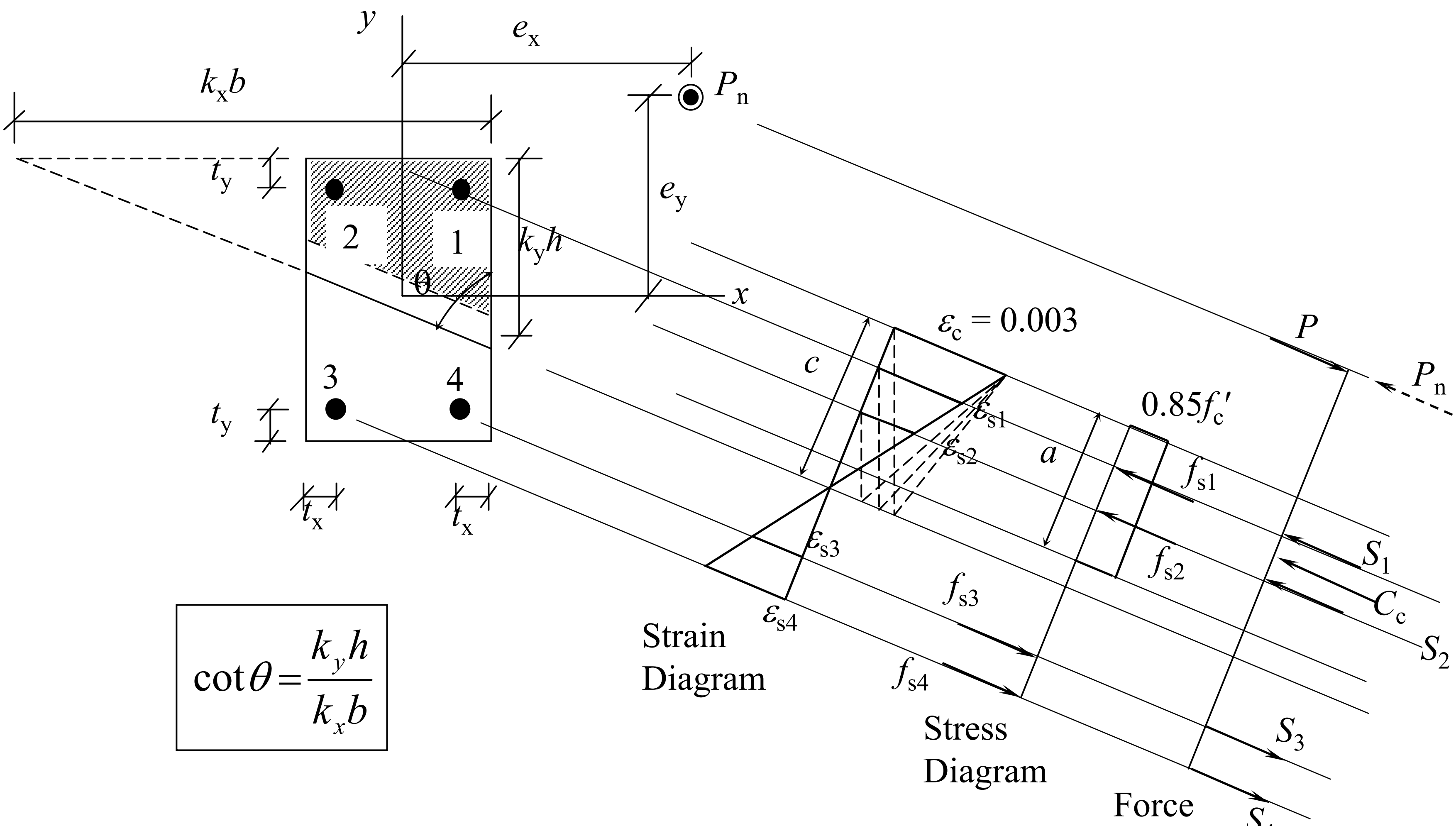


Fig. 14.25. Strain, Stress and Force Diagrams For Biaxial Bending of a Column Section.

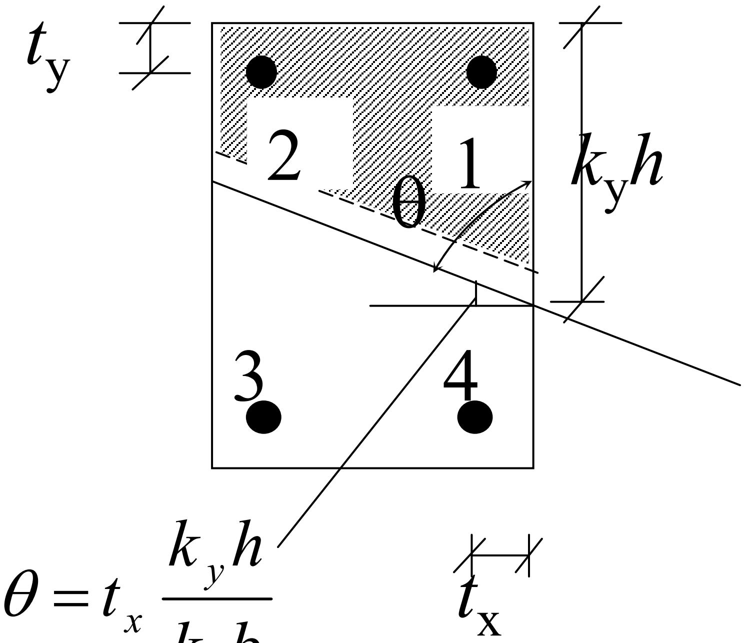
The strain, stress and force diagrams for any one of these cases can be drawn perpendicular to the neutral axis, as shown in Fig. 14.25.

From similar diagrams of the strain diagram, considering compressive strains and stresses to be positive, the following is obtained:

$$\frac{\varepsilon_{s1}}{0.003} = \frac{\text{distance of } \varepsilon_{s1} \text{ from N.A. along } c\text{ - dimension}}{c}$$

$$= \text{ratio of vertical corresponding distances}$$

The above is true because lines perpendicular to the neutral axis and the vertical lines are both intersecting a set of parallel lines and hence the ratio between the corresponding intercepts must be equal.



$$t_x \cot\theta = t_x \frac{k_y h}{k_x b}$$

Fig.14.26.Calculation of Vertical Dimensions.

$$\frac{\varepsilon_{s1}}{0.003} = \frac{k_y h - t_x \cot\theta - t_y}{k_y h}$$

$$\varepsilon_{s1} = 0.003\left(1-\frac{t_x}{k_x b}-\frac{t_y}{k_y h}\right)$$

Let, d_{xi} = horizontal distance of steel from the compression face

d_{yi} = vertical distance of steel from the compression face

= t_y for steels 1 and 2

= $h - t_y$ for steels 3 and 4

Then, $\varepsilon_{si} = 0.003\left(1-\frac{d_{xi}}{k_x b}-\frac{d_{yi}}{k_y h}\right)$

For example,

$$\varepsilon_{s2} = 0.003\left(1 - \frac{b - t_x}{k_x b} - \frac{t_y}{k_y h}\right)$$

$$\varepsilon_{s3} = 0.003\left(1 - \frac{t_x}{k_x b} - \frac{h - t_y}{k_y h}\right)$$

$$\varepsilon_{s4} = 0.003\left(1 - \frac{b - t_x}{k_x b} - \frac{h - t_y}{k_y h}\right)$$

It is to be noted that the positive strains indicate compression. The stresses in the steel bars are calculated as follows:

$$\text{If} \quad \varepsilon_{si} \geq f_y / E_s, \quad f_{si} = f_y$$
$$\text{or if} \quad \varepsilon_{si} \leq -f_y / E_s, \quad f_{si} = -f_y$$
$$\text{else} \quad f_{s1} = \varepsilon_{si} E_s$$

The forces in steel bars are calculated as follows:

$$S_i = A_{si} f_{si}$$

The equilibrium equations are then written as follows to calculate the load and the moment capacities:

$$P_n = C_c + \Sigma\, S_i$$

$$M_{nx} = P_n e_y = C_c \left(\frac{h}{2} - \bar{y} \right) + \Sigma\, S_i \left(\frac{h}{2} - d_{yi} \right)$$

$$M_{\text{ny}} = P_{\text{n}}\, e_{\text{x}} = C_{\text{c}} \left(\frac{b}{2} - \bar{x}\right) + \Sigma\, S_{\text{i}} \left(\frac{b}{2} - d_{xi}\right)$$

The appropriate signs should always be substituted into the above equations to get the correct answers.

The procedure to plot the strength interaction curve for a particular value of the eccentricity angle, λ, is as follows:

1. Neutral axis distance from the most heavily compressed corner is selected.

2. Some value of angle θ is selected.

3. The concrete stress resultant and its point of application are determined by using Eqs. I to III.

4. Strains are calculated in all the steel bars by employing Eq. IV.

5. Stresses are determined for the conditions of Eq. V.

6. The values of P_n, M_{nx} and M_{ny} are evaluated.

7. The value of λ for this trial is determined as $\tan^{-1}(M_{ny} / M_{nx})$.

8. Steps 2 to 7 are repeated until $\lambda = \lambda_t$, with some pre-defined tolerance, which gives one point on the curve.

Calculate $M_{n\lambda} = \sqrt{M_{nx}^2 + M_{ny}^2}$ and plot the point P_n, $M_{n\lambda}$.

9. The ϕ-factor is calculated for the maximum tensile strain in any steel bar. If no steel bar is in tension, the value of 0.65 is used.
10. A new value of the neutral axis depth is selected and steps 2 to 9 are repeated until full curve is plotted.

APPROXIMATE MEHTODS OF ANALYSIS AND DESIGN FOR BIAXIAL BENDING

Method Of Superposition

In this method, the reinforcement required for the two uniaxial bending cases, (P_u, M_{ux}) and (P_u, M_{uy}), is calculated separately and is then added to get the design for the biaxial bending.

Equivalent Uniaxial Eccentricity Method

The biaxial eccentricities, e_{x} and e_{y}, are replaced by an equivalent uniaxial eccentricity, e_{ox}, and the column is designed for the uniaxial bending case, $(P_{\mathrm{n}}, P_{\mathrm{n}} \times e_{\mathrm{ox}})$.

If $\quad e_{\mathrm{x}} / b \geq e_{\mathrm{y}} / h, \qquad e_{\mathrm{ox}} = e_{\mathrm{x}} + \alpha \dfrac{e_y}{h} b$

For $\dfrac{P_u}{f_c' A_g} \leq 0.4 \qquad \alpha = \left(0.5 + \dfrac{P_u}{f_c' A_g} \right) \dfrac{f_y + 300}{720} \geq 0.6$

For $\dfrac{P_u}{f_c' A_g} > 0.4 \qquad \alpha = \left(1.3 - \dfrac{P_u}{f_c' A_g} \right) \dfrac{f_y + 300}{720} \geq 0.5$

This method has certain restrictions.

Firstly, it is applicable only for columns symmetrical about both the axes and the ratio of their sides (b / h) lying between 0.5 and 2.0.

Secondly, the resulting reinforcement is to be placed in all the four faces of the column.

For the cases when the condition $e_x / b \geq e_y / h$ is not satisfied, either the axes may be interchanged (e_x becomes e_y and vice versa) or the equation may be written for e_{oy}.

Bresler Reciprocal Load Method

It is derived from a plane segment inside the interaction surface defined by $1/P_n$, e_x and e_y.

If P_{ni} is the approximate value of the ultimate load in biaxial bending case having eccentricities e_x and e_y, P_{nx} is the nominal load strength when only e_x is present ($e_y = 0$), P_{ny} is the nominal load strength when only e_y is present ($e_x = 0$) and P_o is the nominal load strength for concentrically loaded column, the following expression is used to find the strength corresponding to biaxial bending:

$$\frac{1}{P_{ni}} = \frac{1}{P_{nx}} + \frac{1}{P_{ny}} - \frac{1}{P_o}$$

The same equation may be modified for the interaction curve with ϕ-factor as follow:

$$\frac{1}{\phi P_{ni}} = \frac{1}{\phi P_{nx}} + \frac{1}{\phi P_{ny}} - \frac{1}{\phi P_o}$$

The value of ϕP_{n} obtained in this way should not exceed $0.80\phi P_{\mathrm{no}}$ for tied columns and $0.85\phi P_{\mathrm{no}}$ for spirally reinforced columns. In the design problems, the trial size and reinforcement of the column is selected which is then checked to see if $P_{\mathrm{u}} \leq \phi P_{\mathrm{n}}$.