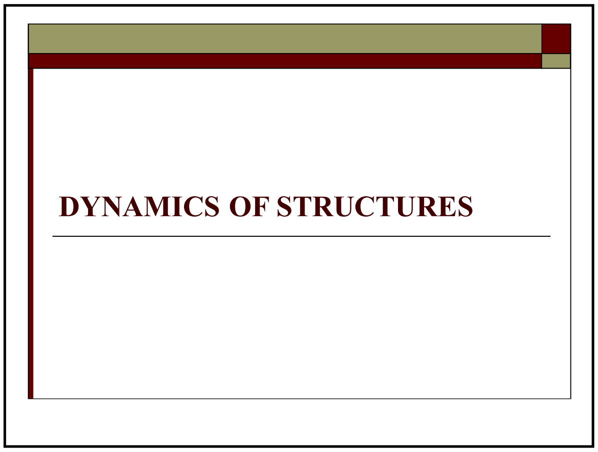
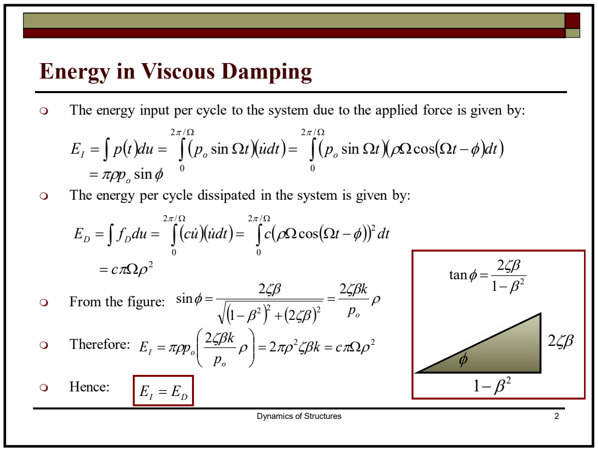


DYNAMICS OF STRUCTURES



Energy in Viscous Damping

- The energy input per cycle to the system due to the applied force is given by:

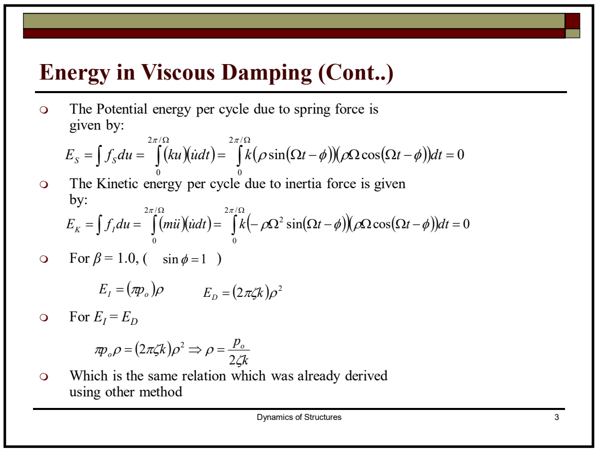
$$E_I = \int p(t)du = \int_0^{2\pi/\Omega} (p_o \sin \Omega t)(\dot{u}dt) = \int_0^{2\pi/\Omega} (p_o \sin \Omega t)(\rho\Omega\cos(\Omega t - \phi)dt)$$
$$= \pi\rho p_o \sin\phi$$

- The energy per cycle dissipated in the system is given by:

$$E_D = \int f_D du = \int_0^{2\pi/\Omega} (c\dot{u})(\dot{u}dt) = \int_0^{2\pi/\Omega} c(\rho\Omega\cos(\Omega t - \phi))^2 dt$$
$$= c\pi\Omega\rho^2$$

- From the figure: $\sin\phi = \frac{2\zeta\beta}{\sqrt{(1-\beta^2)^2 + (2\zeta\beta)^2}} = \frac{2\zeta\beta k}{p_o}\rho$
- Therefore: $E_I = \pi\rho p_o\left(\frac{2\zeta\beta k}{p_o}\rho\right) = 2\pi\rho^2\zeta\beta k = c\pi\Omega\rho^2$
- Hence: $\boxed{E_I = E_D}$

$$\tan\phi = \frac{2\zeta\beta}{1-\beta^2}$$



Energy in Viscous Damping (Cont..)

- The Potential energy per cycle due to spring force is given by:

$$E_S = \int f_S du = \int_0^{2\pi/\Omega} (ku)(\dot{u}dt) = \int_0^{2\pi/\Omega} k(\rho\sin(\Omega t - \phi))(\rho\Omega\cos(\Omega t - \phi))dt = 0$$

- The Kinetic energy per cycle due to inertia force is given by:

$$E_K = \int f_I du = \int_0^{2\pi/\Omega} (m\ddot{u})(\dot{u}dt) = \int_0^{2\pi/\Omega} k(-\rho\Omega^2\sin(\Omega t - \phi))(\rho\Omega\cos(\Omega t - \phi))dt = 0$$

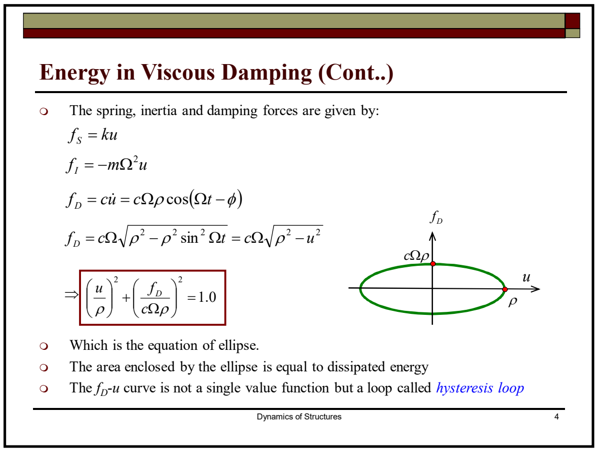
- For $\beta = 1.0$, ($\sin\phi = 1$)

$$E_I = (\pi p_o)\rho \qquad E_D = (2\pi\zeta k)\rho^2$$

- For $E_I = E_D$

$$\pi p_o\rho = (2\pi\zeta k)\rho^2 \Rightarrow \rho = \frac{p_o}{2\zeta k}$$

- Which is the same relation which was already derived using other method



Energy in Viscous Damping (Cont..)

- The spring, inertia and damping forces are given by:

$$f_S = ku$$

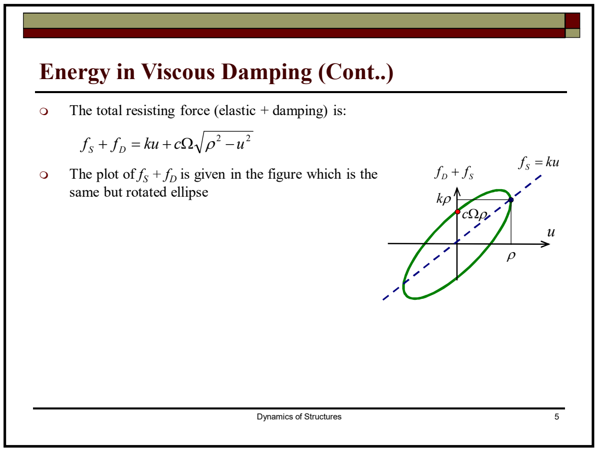
$$f_I = -m\Omega^2 u$$

$$f_D = c\dot{u} = c\Omega\rho\cos(\Omega t - \phi)$$

$$f_D = c\Omega\sqrt{\rho^2 - \rho^2\sin^2\Omega t} = c\Omega\sqrt{\rho^2 - u^2}$$

$$\Rightarrow \boxed{\left(\frac{u}{\rho}\right)^2 + \left(\frac{f_D}{c\Omega\rho}\right)^2 = 1.0}$$

- Which is the equation of ellipse.
- The area enclosed by the ellipse is equal to dissipated energy
- The f_D-u curve is not a single value function but a loop called *hysteresis loop*



Energy in Viscous Damping (Cont..)

- The total resisting force (elastic + damping) is:

$$f_S + f_D = ku + c\Omega\sqrt{\rho^2 - u^2}$$

- The plot of $f_S + f_D$ is given in the figure which is the same but rotated ellipse