DYNAMICS OF STRUCTURES

Course Contents

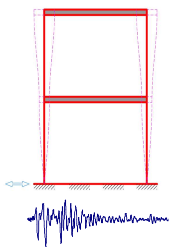
- Introduction to SDOF, MDOF and Continuous Systems
- Formulation of Equation of Motion for SDOF Systems
- Principles of Analytical Mechanics
- Free, Forced, Damped and Un-damped Response of SDOF and Continuous Systems
- Dynamic response to General Dynamic Loading and Transient Response
- Approximate and Numerical Methods for Analysis of SDOF and Continuous Systems
- Analysis of Response in Frequency Domain
- Wave Propagation Analysis

Recommended Books

- DYNAMICS OF STRUCTURES by *Jagmohan L. Humar*
- DYNAMICS OF STRUCTURES by *Ray W. Clough* and *Joseph Penzien*
- DYNAMICS OF STRUCTURES: THEOREY AND APPLICATION TO STRUCTURAL ENGINEERING by *Anil K. Chopra*
- VIBRATION PROBLEMS IN STRUCTURE: PRACTICAL GUIDELINES by *Hugo Vachmann* and *Lorrenz Steinbeisser*
- STRUCTURAL DYNAMICS: THEORY AND COMPUTATION by *Mario Paz*

Introduction

What is Dynamics of Structures?



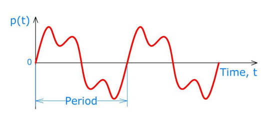
- A study related to the response of a structure subjected to dynamic loading
- Dynamic load is any load of which the magnitude, direction, or position varies with time
- The dynamic of structure may be:
  - Rigid Body or
  - Deformable
- Deformable-body dynamics is usually oscillatory in nature which induces displacement and stresses in the body.
- The analysis of these time-varying displacements and forces is the primary objective of structural dynamic.

Why to Study Dynamics of Structures?

- The response of structure to static load is different than its response to dynamic load
- Dynamic loading may cause large displacement and severe stresses, especially in cases where the frequency of loading is close to the natural frequency of structures
- Fluctuating stress, even of moderate intensity, may cause material failure through fatigue
- Oscillatory motion may at times cause wearing and malfunction of machinery
- The vibration from one machine may transferred to a delicate instrument through support structures
- Vibration cause discomfort to the occupants

Nature of Exciting Force

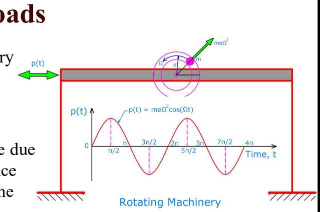
- Periodic and Nonperiodic (random)
  - Periodic Load repeats itself at regular interval of time called period. The reciprocal of period is called frequency of load
  - Nonperiodic load is random in nature and does not repeat itself

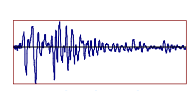


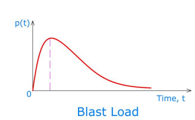
- Deterministic and Nondeterministic
  - Deterministic loads can be specified as definite function of time (continuous function or discrete numerical values at certain regular intervals of time)
  - Nondeterministic loads cannot be specified as definite function of time and are known in statistical sense, e.g. earthquake load, wind load

Sources of Dynamic Loads

- Rotating or reciprocating machinery (sinusoidal vibration)
- Wind (time-varying drag and lift forces)
- Bomb blast (The force on structure due to blast load depends on the distance from the center of explosion and the strength of explosive.
- Earthquakes

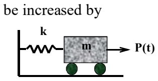




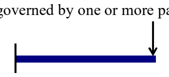


Mathematical Model of Dynamic System

- Discrete Parameter System (Discrete System):
  - A system with rigid-body masses and massless springs
  - Its response is completely defined by specifying displacements along the degree of freedom (DOF) that determine the position of masses in space.
  - The response is governed by a set of Ordinary Differential Equations (ODE) equal to DOF.
  - Simple and preferable approach and The accuracy can be increased by increasing DOF



- Distributed Parameter System (Continuous System)
  - Masses and the stiffness are distributed
  - The response is governed by one or more partial differential equations



Formulation of Equation of Motion

- A set of differential equations governing the motion of dynamic system, by defining the dynamic displacements, is known as Equation of Motion

$$m\ddot{u} + c\dot{u} + ku = p(t)$$

- Equation of Motion is formulated by different approaches:
  - d'Alembert's Principle or direct equilibrium
  - Principle of Virtual Displacement
  - Hamilton Principle

- **d'Alembert's Principle:**

  This principle converts a dynamic problem to an equivalent static problem

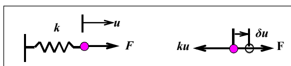
$$Q(t) = \frac{d}{dt}\left(m\frac{du}{dt}\right) = m\ddot{u}$$

Formulation of Equation of Motion (Cont..)

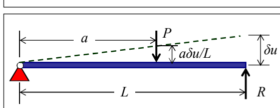
- **Principle of Virtual Displacement:**

  If a system which is in equilibrium under a set of forces is subjected to virtual displacements that are compatible with the constraint in the system, the total work done by the forces in going through the virtual displacements will be zero.

$$\delta W_e + \delta W_i = 0$$



$$\left(F\delta u - ku\delta u\right) = 0$$
$$\Rightarrow F = ku$$



$$\delta W_i = 0$$
$$\delta W_e = R(\delta u) - P\left(\frac{a\delta u}{L}\right)$$
$$\delta W = 0 \Rightarrow R = \frac{aP}{L}$$

Formulation of Equation of Motion (Cont..)

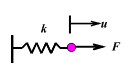
- **Hamilton's Principle:**

  The variation of kinetic and potential energy plus the variation of the work done by the non-conservative forces (which changes during the dynamic response) considered during any time interval t_1 to t_2 must equal to zero.

$$\int_{t_1}^{t_2} \delta\left(T - V\right)dt + \int_{t_1}^{t_2} \delta W_{nc}\,dt = 0$$

- *For static problem, the kinetic energy is zero and the expression converts to the well-known principle of minimum potential energy, i.e.*

$$\delta\left(V - W_{nc}\right) = 0$$



$$\frac{d}{du}\left(\frac{1}{2}ku^2 - Fu\right) = 0 \Rightarrow F = ku$$

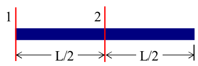
Mass Moment of Inertia

- Mass moment of inertial of a body about certain axis is the product of mass and square of radius of gyration.

$$I_x = \int_m (dm) y^2 = mr^2$$

- Mass moment of Inertia is a measure of the inertial resistance of a body to its rotation
- Rigid Prismatic Bar

  m = total mass of bar and L = Length of bar



$$I_1 = \frac{mL^2}{3}$$

$$I_2 = \frac{mL^2}{12}$$

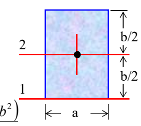
Mass Moment of Inertia (Cont..)

- Rigid Plate
  - Rectangular
  - Circular
  - Ellipse

$$I_1 = \frac{mb^2}{3}$$

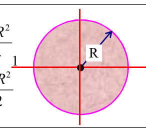
$$I_2 = \frac{mb^2}{12}$$

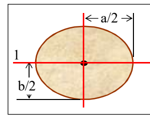
$$I_p = \frac{m(a^2 + b^2)}{12}$$



$$I_1 = \frac{mR^2}{4}$$

$$I_p = \frac{mR^2}{2}$$



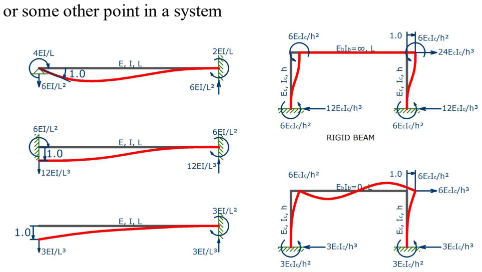


$$I_1 = \frac{mb^2}{16}$$

$$I_p = \frac{m(a^2 + b^2)}{16}$$

Stiffness: Linearly Elastic System

- Stiffness is defined as the action (force, moment, torque) at a certain point required to produce a unit deformation (translation, rotation, twist) at that or some other point in a system



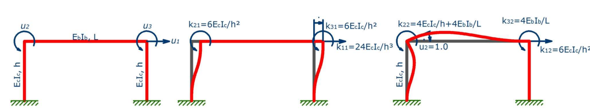
Stiffness: Linearly Elastic System

- Static Condensation

$$\begin{bmatrix} \frac{24EI_c}{h^3} & \frac{6EI_c}{h^2} & \frac{6EI_c}{h^2} \\ \frac{6EI_c}{h^2} & \frac{4EI_c}{h} + \frac{4EI_b}{L} & \frac{4EI_b}{L} \\ \frac{6EI_c}{h^2} & \frac{4EI_b}{L} & \frac{4EI_c}{h} + \frac{4EI_b}{L} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} f_s \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} F \\ 0 \end{bmatrix}$$

$$\Rightarrow K_{21}U_1 + K_{22}U_2 = 0 \Rightarrow U_2 = -[K_{22}]^{-1} K_{21} U_1$$

$$\Rightarrow F = K_{11}U_1 + K_{12}U_2 = \left[K_{11} + K_{12}\left\{-[K_{22}]^{-1} K_{21}\right\}\right] U_1$$



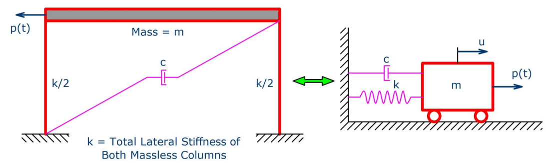
Module-2

Single Degree of Freedom System:

Formulation of Equation of Motion

Single Degree of Freedom System

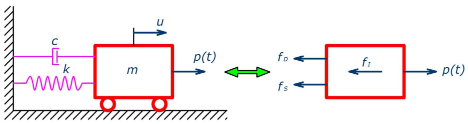
- A system whose dynamic response can be described by specifying displacement along only one coordinate.
- True Engineering Systems are Multi-degree System; SDOF Systems are Idealization
- For many Engineering Systems SDOF Idealization is Satisfactory
- Dynamic Analysis of SDOF Systems is Simple and can be extended to more complex MDOF Systems



Components of Dynamic System

- Components of a Dynamic System are:
  - Mass Resulting in Inertial Force (f_I)
  - Stiffness resulting in Spring Force (f_S)
  - Energy dissipating mechanism resulting in Damping Force (f_D)
  - External Exciting Force $p(t)$
- From Direct Equilibrium (d'Alembert's Principle):

$$f_I + f_D + f_S = p(t)$$
$$m\ddot{u} + c\dot{u} + ku = p(t)$$



Inertia and Spring Force

- Inertia Force:
  The force produced in a dynamic system due to:
  - Translational Acceleration of mass (Inertial force, $f_I = m\ddot{u}$)
  - Rotational Acceleration of mass (Inertial Moment, $M_I = I\ddot{\theta}$)
- Spring Force:
  Internal Forces induced in a body undergoing deformation.
  - Spring force exist both in static and dynamic system
  - For elastic system the spring force is directly proportional to the deformation produced and is equal to stiffness times the deformation ($f_S = ku$)
  - For inelastic system the spring force is dependent on both displacement and velocity.

Damping Force

- Damping Force:
  Damping is the process by which free vibration steadily diminishes in amplitude due to dissipation of energy and the corresponding force is called damping force. There are many energy dissipating mechanisms:
  - Heating of the material due to internal friction
  - Opening and closing of micro cracks
  - Friction between structural and non-structural components
  - Connections
  - External devices
- Equivalent Viscous Damping force $f_D = c\dot{u}$