Steel Structures

M.Sc. Structural Engineering

SE-505

Plastic Analysis and Design of Structures

Steel Structures

Design of Frames

- We have to assume M_P value in terms of ratios of column M_P in the start.
- For single bay single story frame we may assume same M_P value for beams as well as columns.
- For multi storied and multi bay frame: Any one beam = M_P
  For other beams assign M_P in the ratio of simply supported moments.
- If at a joint one column is joined with one beam: M_P value of column = M_P value of beam
- If three or more members are meeting at a joint, M_P value of column is taken using joint equilibrium but not lesser than half of the larger beam M_P value.

Steel Structures

Design of Frames

Load Combinations

$$\begin{cases} 1.2D + 1.6L_r \\ 1.2D + 0.5L_r + 1.3W \end{cases} \longrightarrow \begin{cases} 1.2D + 1.6L + 0.5L_r \\ 1.2D + 1.6L_r + 0.5L \end{cases}$$

$$0.9D - 1.3W$$

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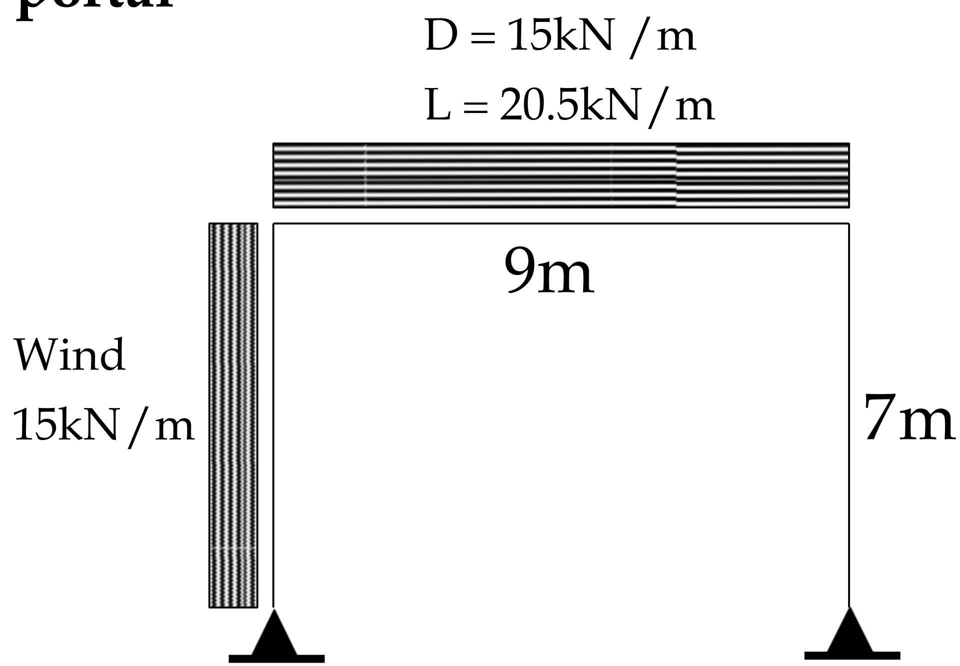
Design of Frames

Example: Design the given portal frame



$1.2D + 1.6L$

$1.2D + 0.5L_r + 1.3W$

Solution:

Let we consider three point load in place of UDL

$P = 0.3wL$

$P/6$ At ends

<u>First combinations</u>

$$w_u = 1.2 \times 15 + 1.6 \times 20.5 = 50.8kN/m$$

$$P = 0.3w_uL = 0.3(50.8) \times 9 = 137.2kN$$

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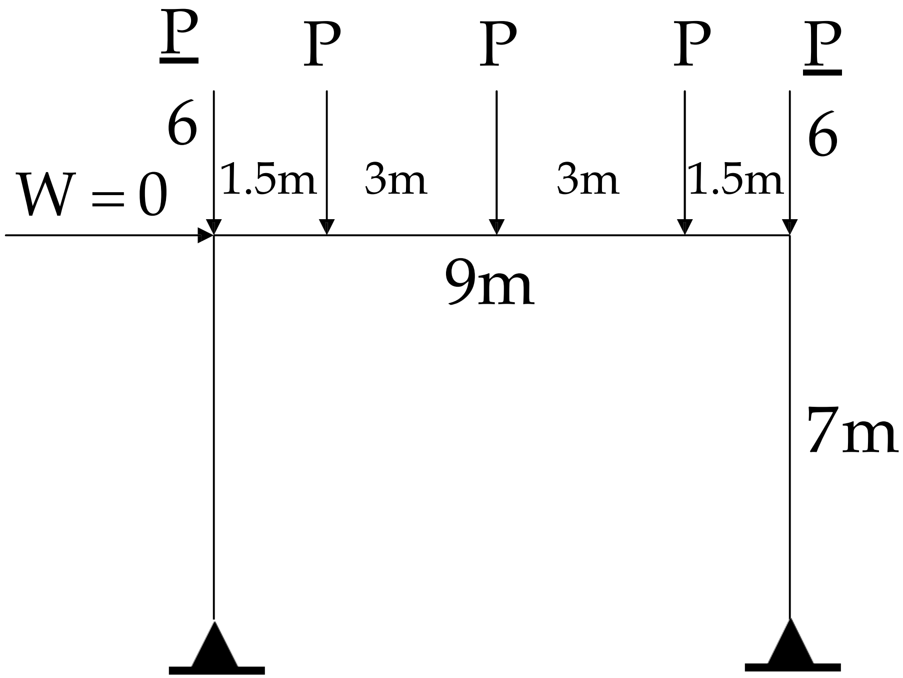
Design of Frames

Solution:



<u>Second combinations</u>

$$w_u = 1.2 \times 15 + 0.5 \times 20.5 = 28.25 kN/m$$

$$P = 0.3 w_u L = 76.3 kN$$

$$W = 1.3 \times 15 \times 7/2 = 68.25 kN \qquad \Rightarrow W = \frac{68.25}{76.3} P = 0.895P$$

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Design of Frames

Solution: (contd…)

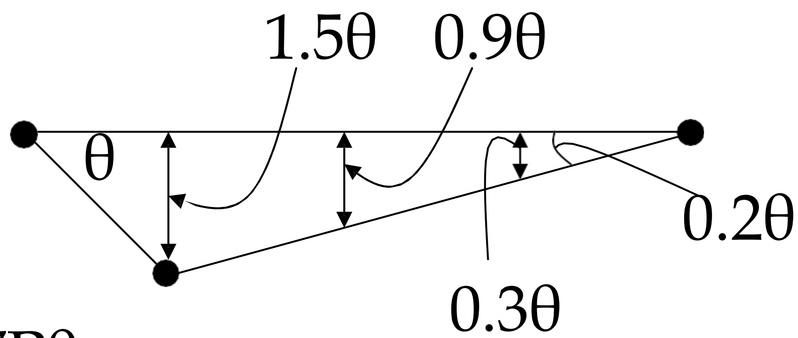
Plastic Moment Ratios

Use same M_P throughout

Analysis

1. 3 beam mechanisms
2. 1 sway mechanism

1- Beam Mech-1



$$W_E = P(1.5\theta + 0.9\theta + 0.3\theta) = 2.7P\theta$$

$$W_I = M_P(\theta + 0.2\theta + 1.2\theta) = 2.4M_P\theta$$

$$P = 0.889M_P$$

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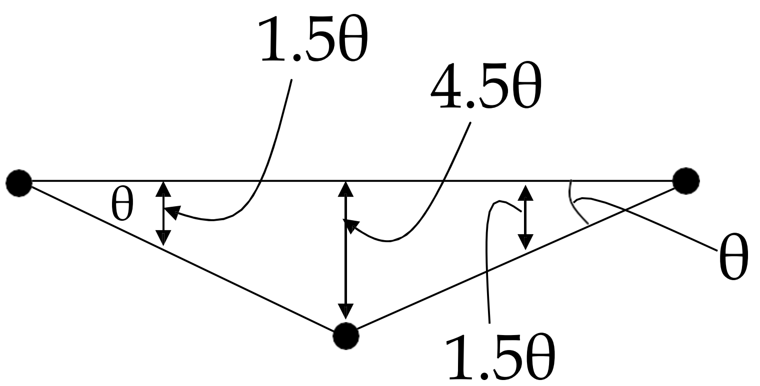
Design of Frames

Solution: (contd…)

2- Beam Mech-2

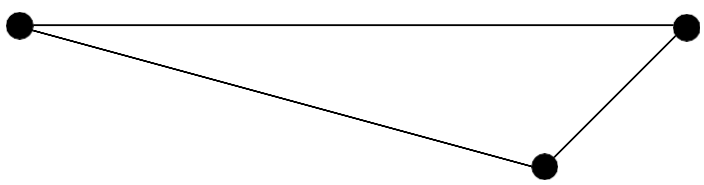


$$W_E = P(1.5\theta + 4.5\theta + 1.5\theta) = 7.5P\theta$$

$$W_I = 4M_P\theta$$

$$P = 0.533M_P$$

3- Beam Mech-3



$$P = 0.889M_P$$

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Design of Frames

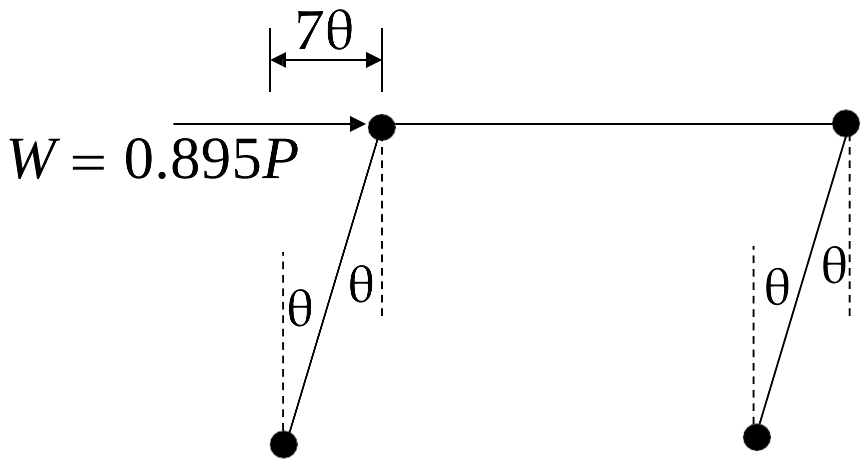
Solution: (contd…)

4- Panel or Sway Mechanism

$$W_E = 0.895P \times 7\theta$$

$$W_I = 2M_P\theta$$

$$P = 0.319M_P$$



5- Combined Mech- 1 + 4

$$W_E = 2.7P\theta + 0.895P \times 7\theta = 8.965P\theta$$

$$W_I = 2.4M_P\theta + 2M_P\theta - 2M_P\theta = 2.4M_P\theta$$

$$P = 0.268M_P$$

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Design of Frames

Solution: (contd…)

6- Combined Mech- 2 + 4

$$P = 0.291M_P$$

Factored Critical M_P Values:

First Combination:

Second mechanism is critical. None of 4 – 6 because there is no wind load in first combination.

$$M_P = \frac{137.2}{0.533} = 257.41kN - m$$

Second Combination:

$$M_P = \frac{76.3}{0.268} = 284.7kN - m$$

5th mechanism is critical

$$\boxed{M_P = 284.7kN - m}$$

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Design of Frames

Solution: (contd…)

Reaction For Critical Mechanisms:

<u>Combination 1: Second Mechanism</u>

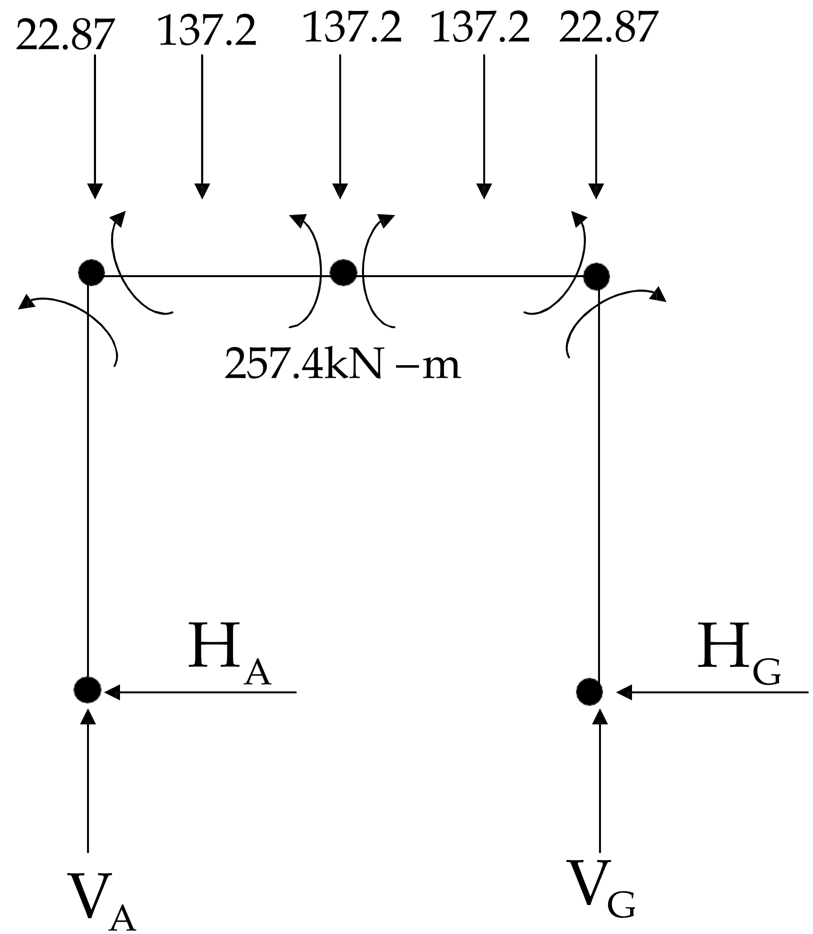
Find reactions also for the less critical mechanism also, because sometimes some of their forces may be more critical.

$$V_A = 228.6\text{kN}$$

$$V_G = 228.6\text{kN}$$

$$H_G = 36.7\text{kN}$$

$$H_A = -36.77\text{kN}$$



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Design of Frames

Solution: (contd…)

Reaction For Critical Mechanism:

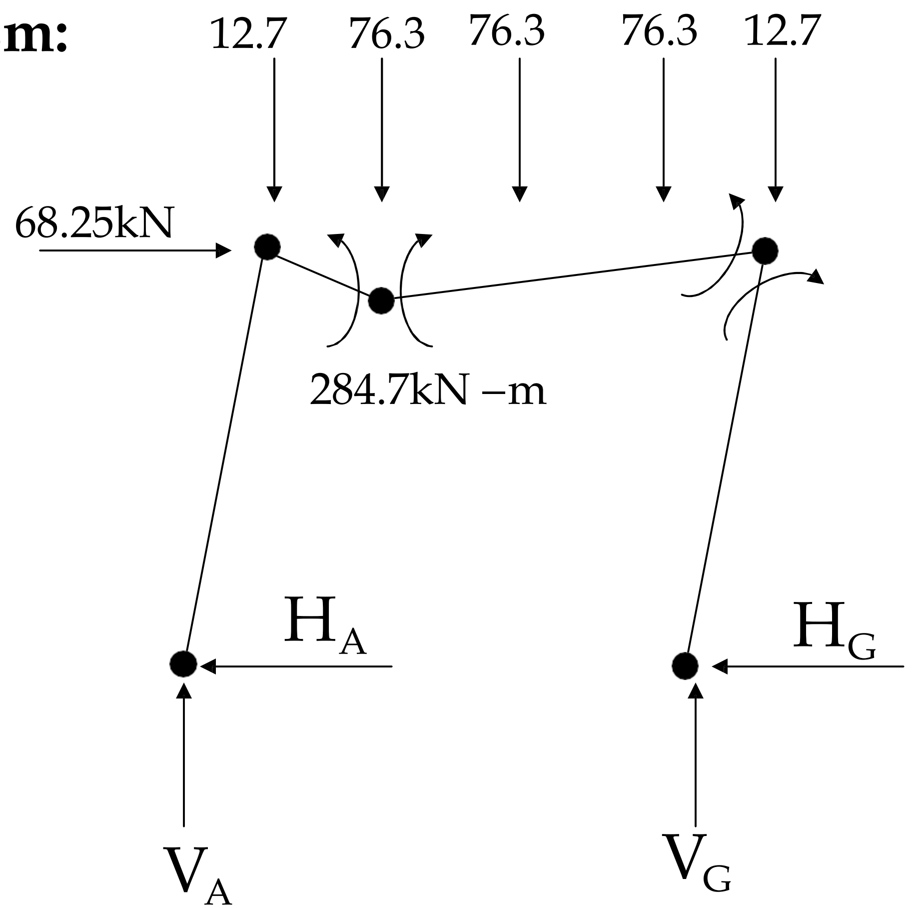
Load comb 2. Fifth Mechanism

$V_A = 73.79kN$

$V_G = 180.51kN$

$H_G = 27.58kN$

$H_A = 40.67kN$



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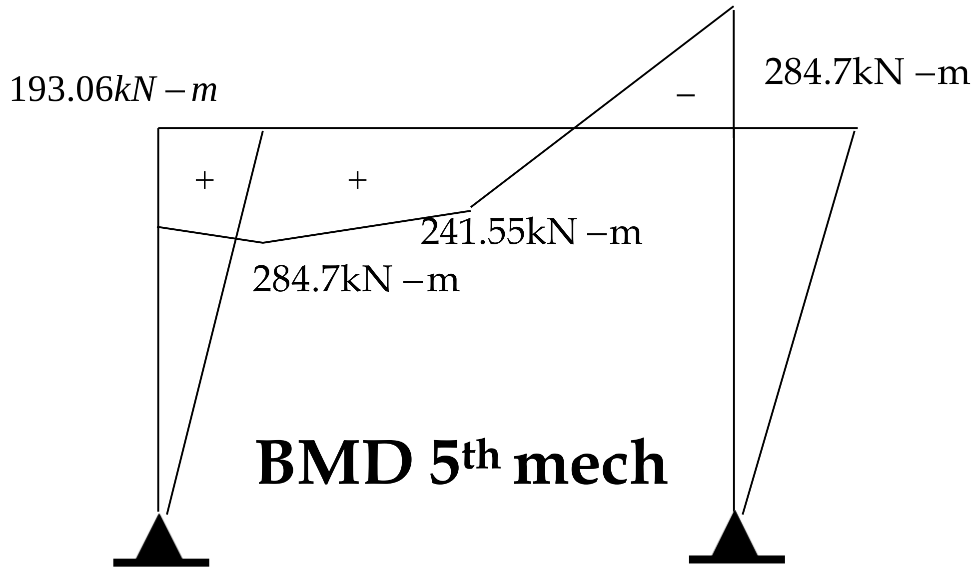
Design of Frames

Solution: (contd…)



BMD 5th mech

Nowhere moment can be more than M_P

Steel Structures

Selection of Trial Section

For beam $d_{\min} = \frac{9000}{22} = 409mm$

$$(Z_x)_{req} = \frac{M_P}{0.9F_y} = \frac{284.7\times10^6}{0.9\times250} = 1265\times10^3 mm^3$$

Trial Section

Beam: W460 x 60 $\phi_b M_p$ = 289 kN-m

Column: W 360 x 72 $\phi_b M_p$ = 289 kN-m

Heavier section is selected for column because it has axial for unlike beam. Greater weight is good for out of plane stability.

Beam:

$\frac{b_f}{2t_f} = 5.7 < 10.8$ **OK**

$\frac{h}{t_w} = 51 < 70$ **OK**

70 Because generally in this case $P_u/\phi P_y < 0.125$

Steel Structures

Design of Frames

Solution: (contd…)

Column:

$$\frac{b_f}{2t_f} = 6.7 < 10.8 \quad \textbf{OK} \qquad \frac{h}{t_w} = 33.5 < 70 \quad \textbf{OK}$$

Design of Beam Column

$$P_u = 180.51\text{kN} = V_G$$

Most critical value

From Page # 70 of LRFD manual

$$G_{top} = \frac{20{,}200 \times 10^4 / 7000}{25{,}500 \times 10^4 / 9000} = 1.02$$

$$G_{bottom} = 10(\text{braced})$$

$$G_{bottom} = 20(\text{unbraced})$$

$$K_b = 1.00 \qquad K_u = 2.15$$

Steel Structures

Design of Frames

Solution: (contd…)

$$A = 9100 mm^2$$

$$r_x = 149 mm$$

$$\frac{k_b L_u}{r_x} = \frac{1.00 \times 7000}{149} = 47 < 200$$ **OK**

$$\frac{k_u L_u}{r_x} = \frac{2.15 \times 7000}{149} = 101 < 200$$ **OK**

Not checking for y-direction it will braced later on.

One requirement specially for plastic analysis..

$$\frac{L_u}{r_x} = \frac{7000}{149} = 47 < 4.71\sqrt{\frac{E}{F_y}} = 133 \text{ for A36 steel}$$ **OK**

$$R = 101 \Rightarrow \phi_c F_{cr} = 131.02 MPa$$

Steel Structures

Design of Frames

Solution: (contd…)

$$\phi_c P_n = \phi_c F_{cr} A = 1192 kN$$

$$\frac{P_u}{\phi_c P_n} = \frac{180.51}{1192} = 0.15 < 0.2$$

Moment effect is dominating and axial effect is less

$$P_{e1} = \frac{\pi^2 EA}{(K_b L_u / r)^2} = 8132 kN$$

$$P_{e2} = \frac{\pi^2 EA}{(K_u L_u / r)^2} = 1760 kN$$

From page # 104 of LRFD manual

$$C_m = 1.0$$

As lateral load is acting within the column

Steel Structures

Design of Frames

Solution: **(contd…)**

$$B_1 = \frac{C_m}{1 - \frac{P_u}{P_{e1}}} = \frac{1.0}{1 - \frac{180.51}{8132}} = 1.023$$

$$B_2 = 1 + 2.5\frac{\left(\sum P_u\right)\left(\sum H\right)L\theta}{\left(\sum P_{e2}\right)\left(\sum M_P\phi\right)}$$

$\sum \mathbf{H} = \mathbf{68.25kN}$ $\sum P_u = 254.3kN$ Sum of all gravity loads.

$\sum P_{e2} = 2 \times 1760 = 3520kN$

Different when support conditions for both columns are different.

ϕ = rotation above the column under consideration

Steel Structures

Design of Frames

Solution: (contd…)

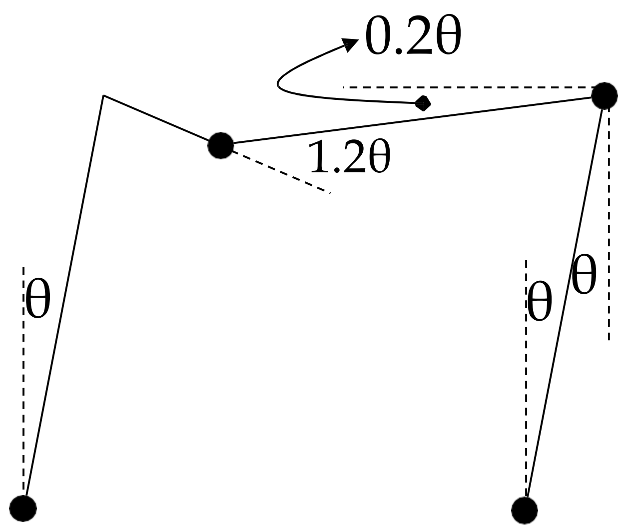
$$\phi = 1.2\theta + 0.2\theta + 1.0\theta$$

$$\phi = 2.4\theta$$

Putting the values and solving

$$B_2 = 1.124$$

Revised value of Z for column



$$Z = Z_t\left(0.5\frac{P_u}{\phi_c P_n} + B_1 B_2\right)$$

$$= 1265\times10^3\left(0.5\times0.15 + 1.023\times1.124\right)$$

$$= 1550\times10^3$$

Steel Structures

Design of Frame

Solution: (contd…)

Revised value of Z for beam. B_2 is applied to both beam and column.

$$Z = Z_t \times B_2 = 1265 \times 10^3 \times 1.121$$

$$= 1418 \times 10^3 mm^3 < 1285 \times 10^3 mm^3$$ **Revise**

Next Trial:

Column: W 360 x 91, r_x = 152mm, A = 11500mm^{2,} ϕ_b M_P=376kN-m

Beam: W 530 x 66

Steel Structures

Design of Frames

Solution: (contd…)

Magnified moment

We can revise B_2 for more accuracy as P_{e2} has changed.

$$\text{Magnified moment} = M_P \times B_1 \times B_2 = 284.7 \times 1.023 \times 1.124$$

$$= 327.4kN - m$$

$$\frac{k_b L_u}{r_x} = \frac{1.00 \times 7000}{152} = 46.05 < 200$$

$$\frac{k_u L_u}{r_x} = \frac{2.15 \times 7000}{152} = 99 < 200$$

$$R = 99 \Rightarrow \phi_c F_{cr} = 133.83MPa$$

Steel Structures

Design of Frames

Solution: (contd…)

$$\phi_c P_n = 1539 kN$$

$$\frac{P_u}{\phi_c P_n} = 0.117 < 0.2$$

$$\frac{P_u}{2\phi_c P_n} + \frac{M_{ux}}{\phi_b M_{Px}} = \frac{0.117}{2} + \frac{327.4}{376}$$

$$= 0.929 < 1.01$$ **OK**

So column section W 360 x 91 is OK for wind load combination

Steel Structures

Design of Frames

Solution: (contd…)

Check The Column For Gravity Load Combination.

$$\mathrm{P_u} = 228.67\mathrm{kN} \qquad M_u = 257.4 \times B_1 \cong 263.3\, kN - m$$

No sway, so no B_2

$$R = 46 \Rightarrow \phi_c F_{cr} = 201.13\, MPa$$

$$\phi_c P_n = 2313 kN$$

$$\frac{P_u}{\phi_c P_n} = 0.099 < 0.2$$

$$\frac{P_u}{2\phi_c P_n} + \frac{M_{ux}}{\phi_b M_{Px}} = \frac{0.099}{2} + \frac{263.3}{376}$$

$$= 0.75 < 1.01$$

OK

Steel Structures

Beam: W530 x 66

$$\frac{b_f}{2t_f} = 7.2 < 10.8$$ **OK**

$$\frac{h}{t_w} = 53.6 < 70$$ **OK**

70 Because generally in this case $P_u/\phi P_y < 0.125$

Column: W360 x 91

$$\frac{b_f}{2t_f} = 7.7 < 10.8$$ **OK**

$$\frac{h}{t_w} = 30.4 < 70$$ **OK**

Steel Structures

Design of Frames

Solution: (contd…)

Shear Check

Column:

$$(\mathrm{Vu})_{\max} = 40.67\mathrm{kN}$$

$$\phi_v V_n = 0.9 \times 0.6 \times 250 \times 353 \times 9.5 / 1000$$

$$= 452.7kN > 40.67kN \quad \textbf{OK}$$

Beam:

$$(Vu)_{\max} = 228.67kN$$

$$\phi_v V_n = 0.9 \times 0.6 \times 250 \times 525 \times 8.9 / 1000$$

$$= 630.8kN > 228.67kN \quad \textbf{OK}$$

Steel Structures

Design of Frames

Solution: (contd…)

Lateral Bracing

L_P for W 360 x 91 = 3.10 m
L_P for W 530 x 66 = 1.59 m

L_P is for elastic analysis not for plastic analysis. For plastic analysis we need to compute L_{Pd}

$$L_{Pd} = \left[0.096 + 0.061 \frac{M_1}{M_2} \right] r_y$$ For A36 steel.

M_1/M_2 is positive for reverse curvature.

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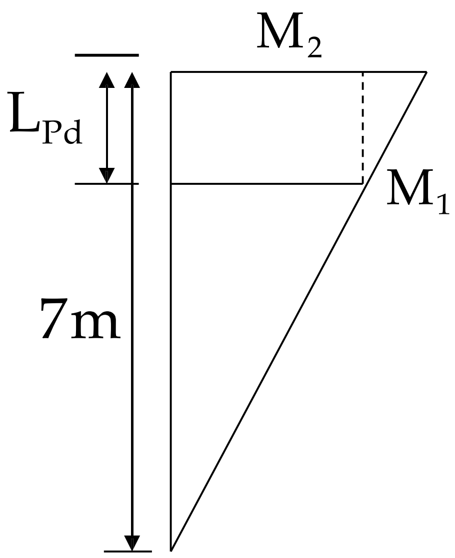
Design of Frames

Solution: (contd…)

W 360 x 91 (Column)

$M_2 = 284.7 \times B_2 = 320$ kN-m



$$M_1 = \frac{320}{7}\left(7 - L_{pd}\right) = 320 - \frac{320}{7} L_{pd}$$

$$\frac{M_1}{M_2} = -\frac{320 - \frac{320}{7} L_{pd}}{320}$$

$$\frac{M_1}{M_2} = \left(-1 + \frac{1}{7} L_{pd}\right)$$

Steel Structures

Design of Frames

Solution: **(contd…)**

$$L_{Pd} = \left[0.097 + 0.061\left(-1 + \frac{1}{7} L_{Pd}\right)\right] 62.2$$

$$= (0.097 + 0.061) 62.2 + \frac{0.061}{7} \times 62.2 L_{Pd}$$

$$= 4.89m$$

<u>W 530 x 66</u> (Beam)

M_1/M_2 = +ve because of reverse curvature

Assume $M_1/M_2 = 0$, it is more critical

$$\Rightarrow L_{Pd} = 3.104\text{m}$$

Steel Structures

Design of Frames

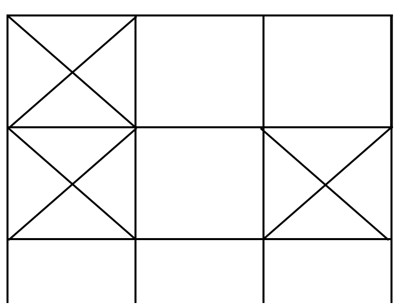
Solution: (contd…)

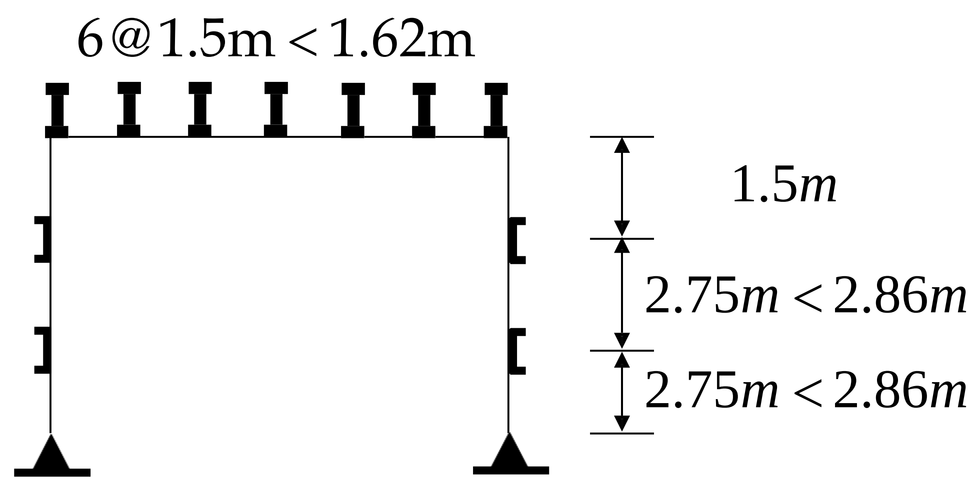
For Column $\dfrac{K_y L_y}{r_y} = \dfrac{K_x L_x}{r_x}$ Considering $K_x = K_y = 1.0$

$$L_y = \frac{1.00 \times 7 \times 62.2}{152} = 2.86m$$

To avoid buckling in y-direction

Cross bracing can also be provided in some portions for stability.



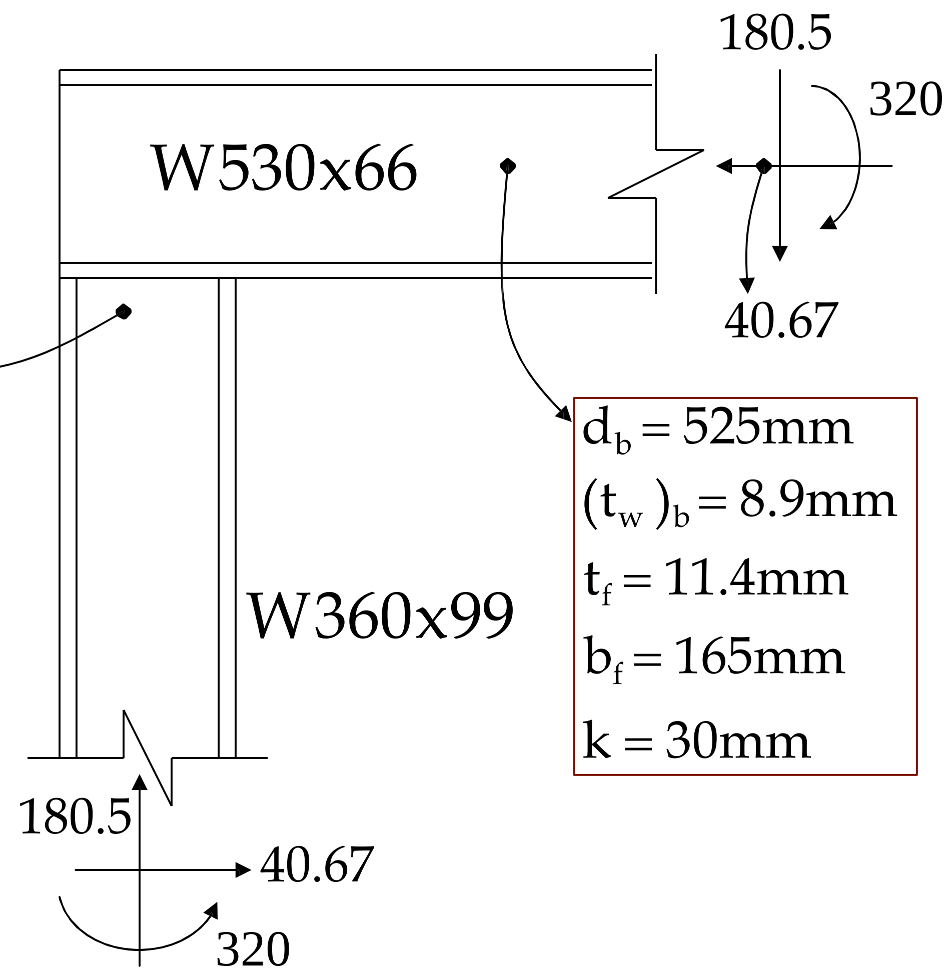


Steel Structures

Design of Frames

Solution: (contd…)

Design of Connection



Magnified moment

$$= B_2 \times 284.7$$

$$= 1.124 \times 284.7$$

$$= 320kN - m$$

Steel Structures

Design of Frames

Solution: (contd…)

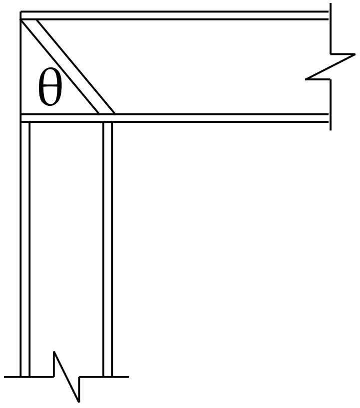
Check Web (beam) Without Diagonal Stiffener

$$\left(t_w\right)_{req} = \frac{1.95 M_u}{F_y d_b d_c}$$

$$= \frac{1.95 \times 320 \times 10^6}{250 \times 525 \times 353} = 13.49 > t_w = 8.9$$

So diagonal stiffener is required

$$\tan\theta = \frac{d_b}{d_c}$$



$$\theta = 56^o$$

$$\cos\theta = 0.558$$

Steel Structures

Design of Frames

Solution: (contd…)

$$\left(A_{st}\right)_{req} = \frac{1}{\phi_c F_{cr}\cos\theta}\left[\frac{M_u}{0.95 d_b} - \phi_v \left(0.6 F_y\right) t_w d_c\right]$$

Assume that

$$F_{cr} = 0.95 F_y = 238 MPa$$ Check later

$$\left(A_{st}\right)_{req} = \frac{1}{0.90\times 238\times 0.558}\left[\frac{320\times 10^6}{0.95\times 525} - 0.9\left(0.6\times 250\right)8.9\times 353\right]$$

$$\left(A_{st}\right)_{req} = 1820 mm^2$$ (910 mm^2 on one side)

$$\frac{b_f - t_w}{2} = 78mm$$ For beam section

Steel Structures

Design of Frames

Solution: (contd…)

Let

$$t_{st} = 15mm$$

$$b_{st} = \frac{910}{15} = 61 \cong 65mm$$

$$\lambda = \frac{65}{15} = 4.33 < \lambda_P = 10.8$$ **O.K**

So Diagonal Stiffener is

15x65mm

Over all width of stiffener = $b = 2b_{st} + t_w = 138.9mm$

Radius of gyration, $r = \frac{b}{\sqrt{12}} = 40.14mm$

Steel Structures

Design of Frames

Solution: **(contd…)**

Let

$$\frac{KL}{r} = \frac{1.0 \times d_c / \cos\theta}{40.14} \cong 16$$

K = 1.0 for stiffener, AISC

$$\Rightarrow \phi_c F_{cr} = 221.97 MPa$$

$$\Rightarrow F_{cr} = 221.97 / 0.9 = 246.63 > 0.95 F_y \, MPa$$

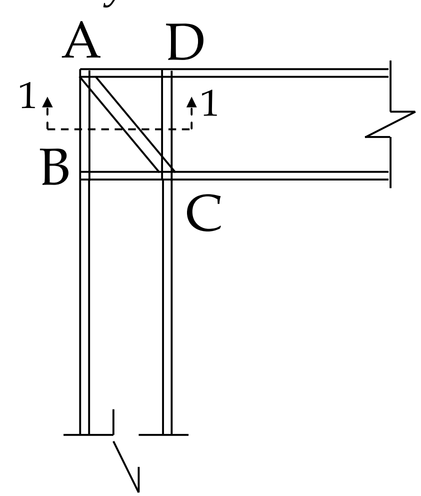
O.K

Stiffener Along AB

Same size as that of the column flange

$$b_f \times t_f = 254 \times 16.4$$

$$= 4165.6 mm^2$$



Steel Structures

Design of Frames

Solution: (contd…)

$b_{st} = 78mm$ Same as beam $t_{st} = 28mm$

$$\text{Maximum flange force for column} = 0.9F_yA_f$$

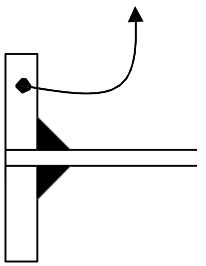
$$= 0.9 \times 250 \times 254 \times 16.4 = 937.3kN$$

E-70 electrode is used for welding, 495 MPa

$$\phi R_{n_w} = 2\phi_v \times 0.6F_y \times 0.707t_w$$

$$= 2 \times 0.75 \times 0.6 \times 495 \times 0.707t_w / 1000$$

$$= 0.315t_w kN / mm$$



l_w = available length of weld = 525 – 2 x 11.4 = 502 mm

Steel Structures

Design of Frames

Solution: (contd…)

Total strength of weld available $= 502 \times 0.315 t_w$

$$= 502 \times 0.315 t_w = 937.4$$

$$t_w = 6mm$$

Check for $t_{w\,min}$ and $t_{w\,max}$
From LRFD Manual

Weld at BC

Conservative estimate of the flexural component of force is the amount of tension in the beam web per unit length.

Tensile flexural component $= \phi_t F_y (t_w)_b$ Per unit length

$$= 0.9 \times 250 \times 8.9 / 1000 = 2.0 kN / mm$$

Steel Structures

Design of Frames

Solution: (contd…)

$$\text{Shear Component} = \frac{V_u}{d_c - 2t_f} = \frac{\square 40.67}{353 - 2\times 16.4} = 0.127\text{kN}/\text{mm}$$

$$\text{Resultant Force} = \sqrt{(2)^2 + (0.127)^2} = 2.004\text{kN}/\text{mm}$$

$$0.315t_w = 2.004\text{kN}/\text{mm}$$

$$t_w = 6.36\text{mm}$$

Use $\boxed{t_w = 8\text{mm}}$

<u>Weld Along Diagonal Stiffener</u>

$$\text{Force on one side of web} = \phi F_y \frac{A_{st}}{2} = 0.9\times 250\times \frac{15\times 65}{1000} = 219.37kN$$

Steel Structures

Design of Frames

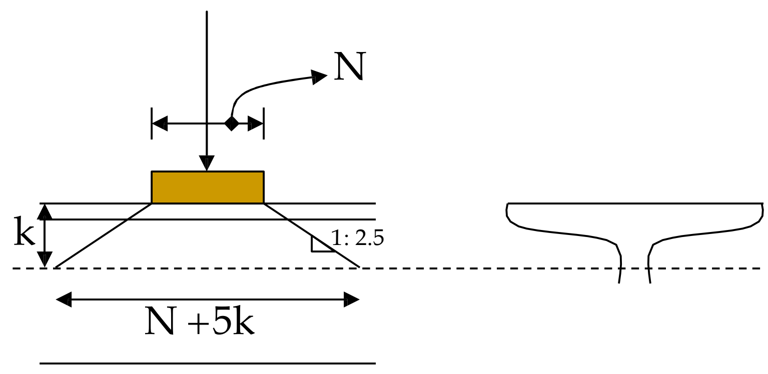
Solution: (contd…)

$$0.315t_w \times \frac{d_c}{\cos\theta} = 219.4$$

$$t_w = 1.11\text{mm}$$

Use min thickness, or say

$$\boxed{t_w = 6\text{mm}}$$



Stiffener Along CD

Inner flange force = 937.4 kN

Without stiffener CD

$$P_{bf} = \phi(5k + t_f)F_y t_w$$

$$P_{bf} = 1.0(5\times30 + 16.4)250\times8.9/1000 = 370.1\text{kN} < 937.4\text{kN}$$

Stiffener is required

Steel Structures

Design of Frames

Solution: (contd…)

A_{st} on one side $$= \frac{937.4 - 370.1}{\phi_c F_{cr}} \times \frac{1}{2} \times 1000 \quad \phi_c F_{cr} = 0.9 \times 0.95 \times 250$$

$$= 1327 \quad mm^2$$

If we keep b_{st} = 78 mm or say 75 mm

$$t_{st} = 18mm$$

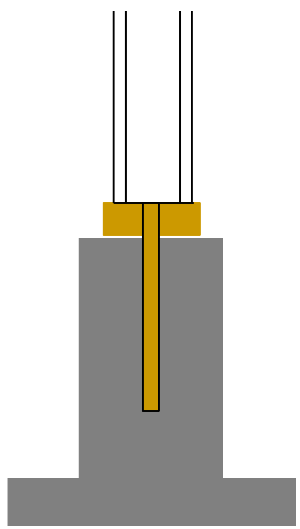
$$\lambda = \frac{b_{st}}{t_{st}} = \frac{75}{18} = 4.17 < \lambda_P \quad \textbf{O.K}$$

Steel Structures

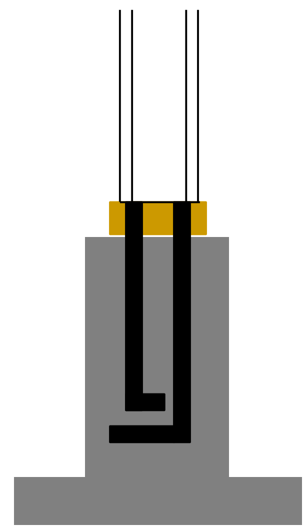
Design of Frames

<u>Design of Base Plate</u>

Design of base plate is very simple in this case as there is no bending moment. Concrete pedestal is provided up-to plinth level to avoid rusting.



Hinged connection



Fixed connection

See P # 367 to 370 of steel book

Steel Structures

Design of Frames

Example: Design of Base Plate

P_u = 228.67kN, W 360 x 91, d = 353mm, b_f = 254 mm

Solution:

Let f_c' = 18MPa

A_1 = Area of Base plate

A_2 = Area of Supporting Concrete

If we keep A_2 equal to or larger than $2A_1$ sufficient bearing strength is available. Let we Assume

$$A_2 = 2A_1$$

Steel Structures

Design of Frames

Solution:

A_1 is larger of

$$= \frac{1}{\sqrt{A_2 / A1}} \left[\frac{P_u}{0.6 \times 0.85 f_c'} \right] \qquad \text{When } A_2 < 2A_1$$

$$= \frac{P_u}{0.6 \times 1.7 f_c'}$$

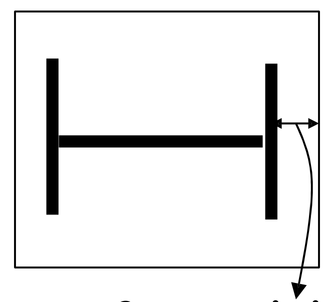
$$= (d + 16) x (b_f + 16)$$ Minimum size

A_1 is larger of

$$= \frac{1}{\sqrt{2}} \left[\frac{228.67 \times 1000}{0.6 \times 0.85 \times 18} \right] = 17614 \text{mm}^2$$

$$= \frac{228.67 \times 1000}{0.6 \times 1.7 \times 18} = 12455 \text{mm}^2$$

$$= (d + 16) x (b_f + 16) = 275 \times 375 \text{mm} = 99630 \text{mm}^2$$



8 mm minimum for welding

Use $275 \times 375\text{mm}$ Plate

Steel Structures

Design of Frames

Solution:

Use size of pedestal as

$$\boxed{400 \times 525\text{mm}}$$

Double the area of base plate

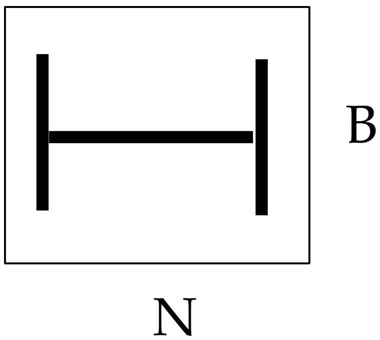
Cantilever length, ℓ is larger, of

$$n = \frac{(B - 0.8b_f)}{2} = \frac{275 - 0.8 \times 254}{2} = 35.9\text{mm}$$

$$m = \frac{(N - 0.95d)}{2} = \frac{375 - 0.95 \times 353}{2} = 19.8\text{mm}$$

$$n' = \frac{1}{4}\sqrt{db_f} = \frac{1}{4}\sqrt{353 \times 254} = 74.9\text{mm}$$

$$\ell = 74.9mm$$



Steel Structures

Design of Frames

Solution:

$$t_P = l\sqrt{\frac{2P_u}{0.9F_y B \times N}}$$

$$= 74.9\sqrt{\frac{2 \times 228.67 \times 1000}{0.9 \times 250 \times 275 \times 375}} \qquad = 10.5mm$$

Say $\quad t_P = 12\text{mm}$

Final Base Plate Size

$$\boxed{12 \times 275 \times 375\text{mm}}$$

Use two anchor bolts of minimum diameter.

Concluded