CivE 602
Prestressed Concrete

Part 7:

Design for Shear

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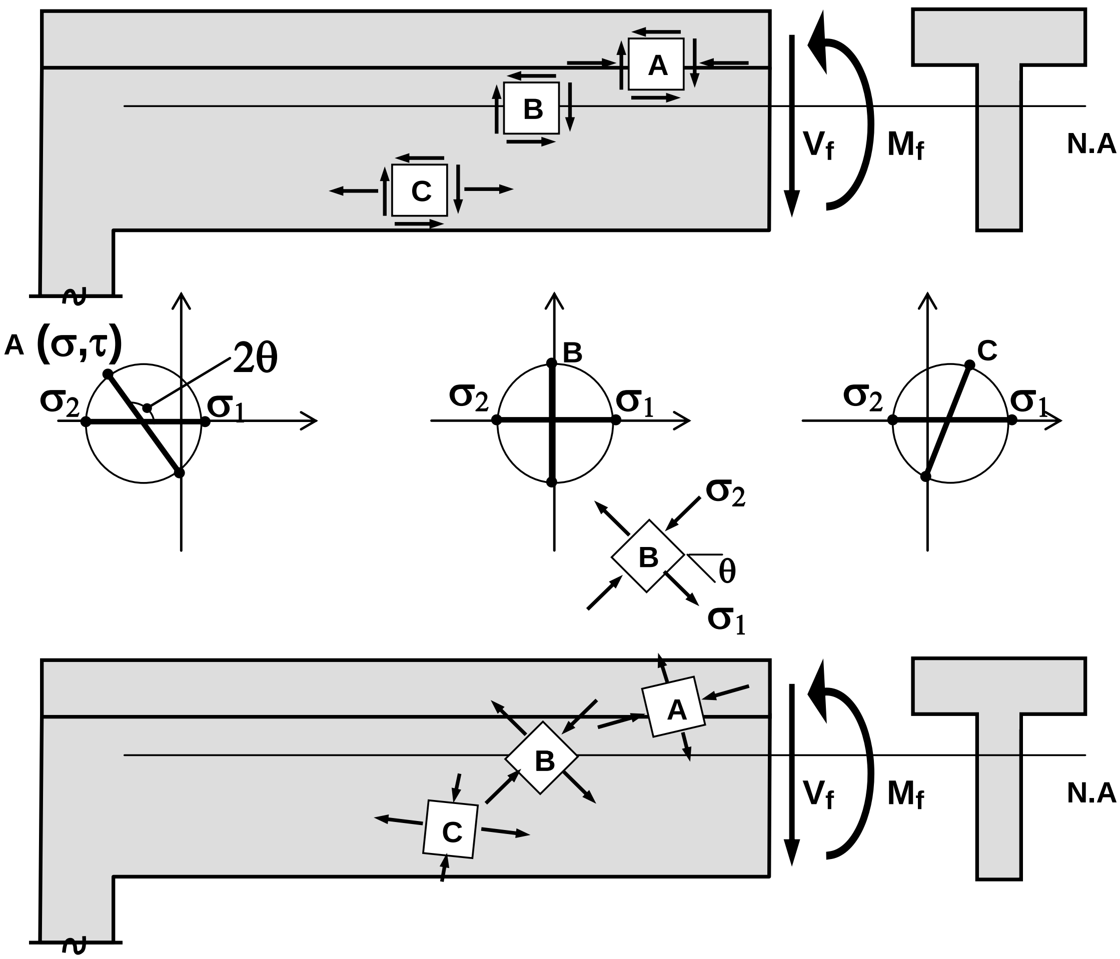
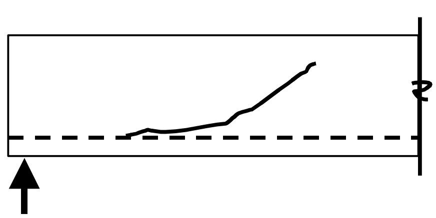
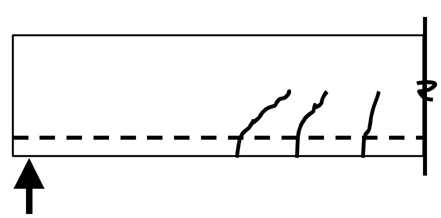
SHEAR IN REINFORCED CONCRETE BEAMS



- If σ_1 exceeds the tensile strength of the concrete, then cracking occurs
- Cracking is perpendicular to principal tension stress



Web shear crack



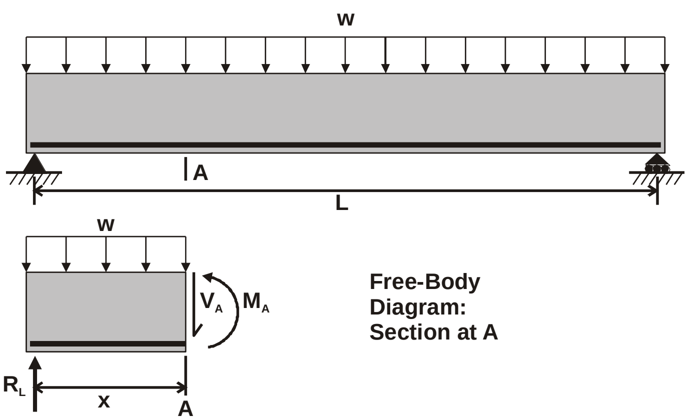
Flexure-shear crack

SHEAR IN PRESTRESSED CONCRETE BEAMS

- **Shear in prestressed concrete has two distinct differences in comparison to shear in reinforced concrete:**

1. For a given beam and loading, the shear force acting at a particular section is less in the prestressed concrete beam in comparison to the reinforced concrete beam. This is because the prestressing force typically has a vertical force component in the opposite direction to the applied (gravity) loads.

Reinforced Concrete Beam:

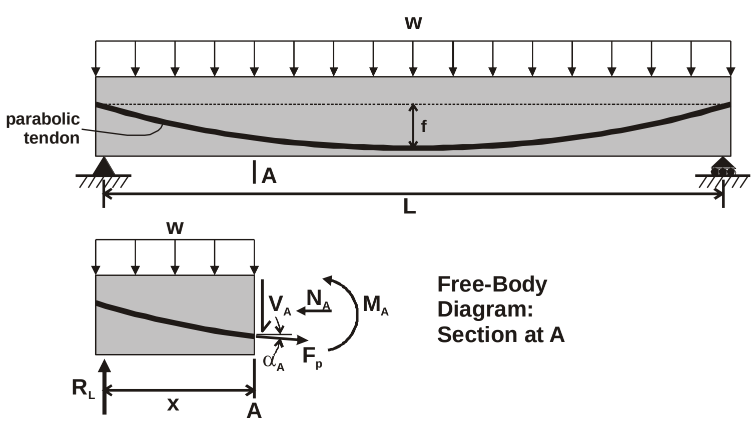


$$\sum F_y = 0 = R_L - w\,x - V_A = \frac{wL}{2} - w\,x - V_A$$

Thus,

$$V_A = \frac{wL}{2} - w\,x$$

Prestressed Concrete Beam:



$$\sum F_y = 0 = R_L - w\,x - V_A - F_p \sin\alpha_A$$

Thus,

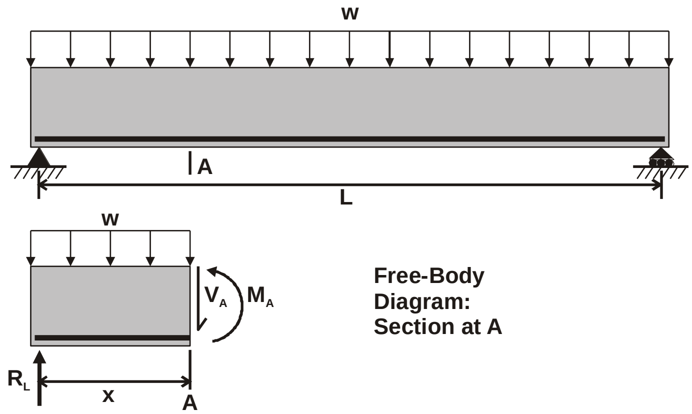
$$V_A = \frac{wL}{2} - w\,x - F_p \sin\alpha_A$$

$$\Rightarrow (V_A)_{PS} = (V_A)_{RC} - V_P$$ ~~sinα~~

- **The term $F_p \sin\alpha$ is the component of the effective prestress force in the direction of the applied shear, and is denoted as V_p. It is treated as a contribution to shear resistance for design purposes.**

Alternate derivation for V_p using load-balancing:

Reinforced Concrete Beam:

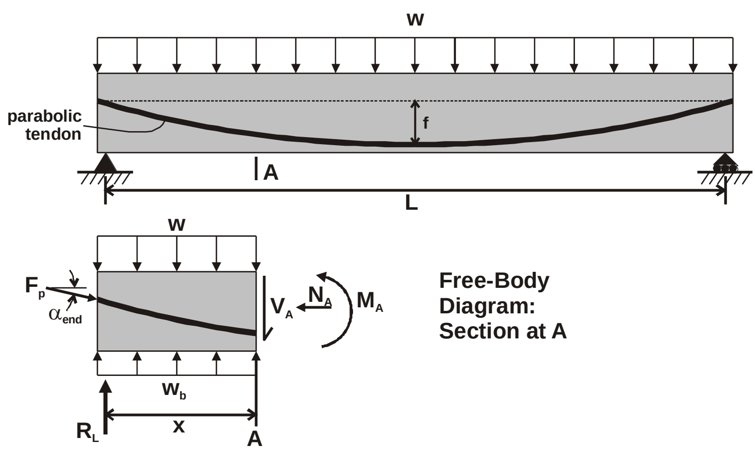


$$\sum F_y = 0$$
$$= R_L - w\,x - V_A$$
$$= \frac{wL}{2} - w\,x - V_A$$

Thus,

$$V_A = \frac{wL}{2} - w\,x$$

Prestressed Concrete Beam:



$$\sum F_y = 0$$
$$= R_L - w\,x - V_A$$
$$- F_p \sin\alpha_{end} + w_b x$$

Thus,

$$V_A = \frac{wL}{2} - w\,x - F_p \sin\alpha_{end} + w_b x$$

For small angles: $\sin\alpha_{end} \approx m_{end} = \frac{8f}{L^2}\left(\frac{L}{2}\right) = \frac{4f}{L}$

We know: $w_b = \frac{8fF_p}{L^2}$

Thus,

$$V_A = \frac{wL}{2} - w\,x - \frac{F_p f}{L}\left(4 - \frac{8x}{L}\right)$$

But,

$$\sin\alpha_A \approx m_A = \frac{8f}{L^2}\left(\frac{L}{2} - x\right) = \frac{f}{L}\left(4 - \frac{8x}{L}\right)$$

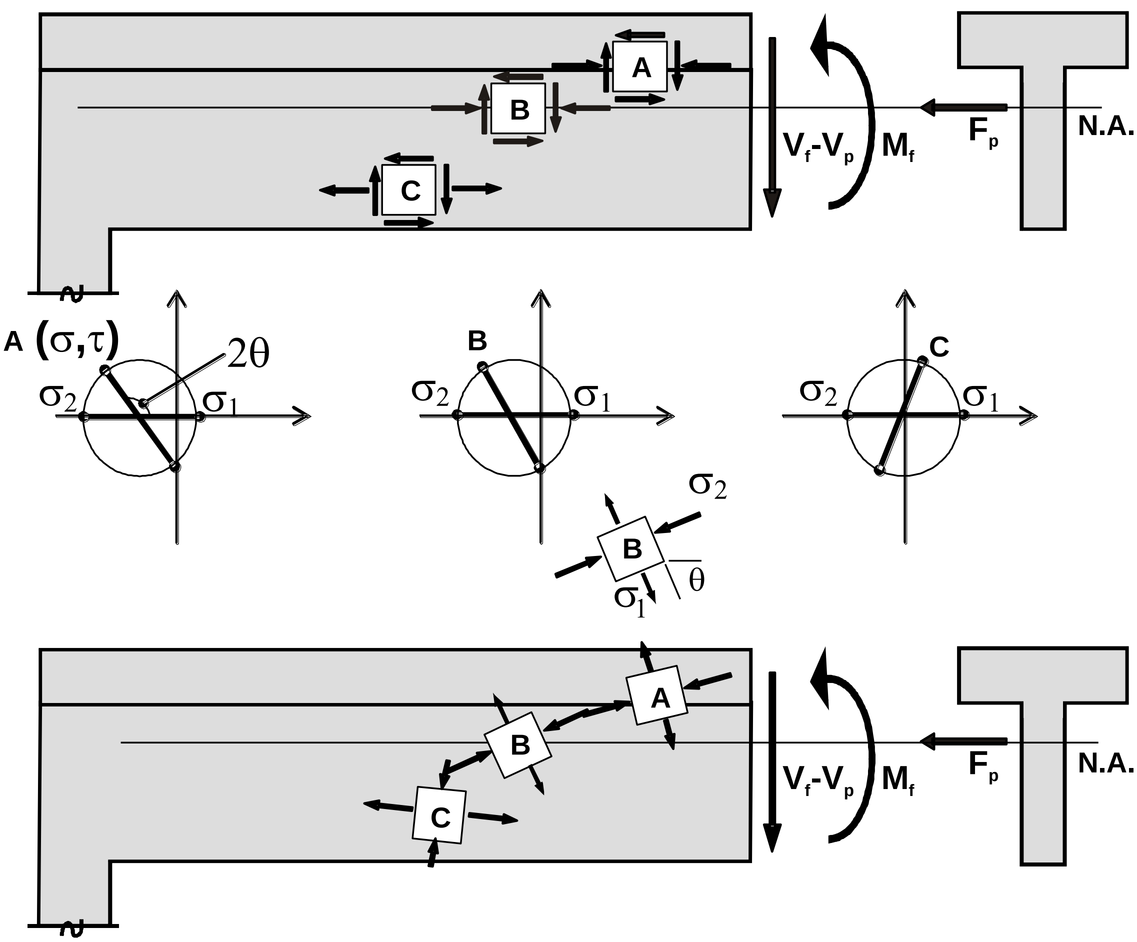
Thus,

$$V_A = \frac{wL}{2} - w\,x - F_p \sin\alpha_A$$

Define: $$\boxed{V_p = F_p \sin\alpha}$$

2. **The prestress force introduces a compressive force on the concrete section. This compression reduces the diagonal (principal) tension developed within the web of the section, increasing the shear capacity of the concrete.**

   In addition, the angle of inclination of the principal tension with respect to the beam axis is reduced. As such, the inclined cracking that develops will be "flatter," and stirrup spacing can be increased while still ensuring that at least one stirrup crosses the crack.



- **The positive effect of the compression force is accounted for in design by increasing the shear resistance provided by the concrete.**

DESIGN FOR SHEAR

- **Many different approaches have been proposed for design of reinforced and prestress concrete elements for shear:**
  - **truss model**
  - **softened truss model**
  - **modified compression field theory**
  - **strut and tie model**
- **See references for more information:**
  - **Collins and Mitchell text**[1]
  - **Naaman text**[2]
  - **MacGregor and Bartlett text**[3]

CSA A23.3 DESIGN FOR SHEAR

- **Shear design is considered at the Ultimate Limit State**
- **CSA A23.3-04 uses a *sectional design method* derived from the *modified compression field theory* for design of flexural elements for shear.**

  11.3 Design for Shear in Flexural Regions
  - ***General Method***
  - ***"Simplified"* Method is included for common conditions**
- **Application of the simplified case is similar in style to the previous versions of A23.3 and to the ACI 318 Code.**
- **The AASHTO LRFD provisions for shear design are similar to the CSA A23.3-04 provisions.**
- **Shear design in "disturbed regions" or for deep beams is addressed using the Strut and Tie method (Clause 11.4).**

CSA A23.3 SHEAR REQUIREMENTS (CHP. 11)

$$V_r \geq V_f$$

Additional provisions for:

- **Minimum Shear Reinforcement (location and amount)**
- **Maximum Spacing of Shear Reinforcement**
- **Maximum Shear Resistance**
- **Critical cross-section for shear design near supports**

FACTORED SHEAR RESISTANCE

CSA A23.3-04 *Clause 11.3.3*

$$V_r = V_c + V_s + V_p$$

$$V_{r,max} = 0.25\phi_c f_c' b_w d_v + V_p$$

Where,

ϕ_c = 0.65

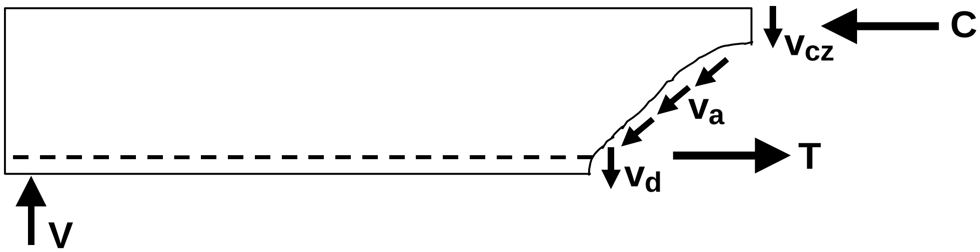
d_v = effective shear depth

= 0.9d or 0.72h, whichever is greater

b_w = web width

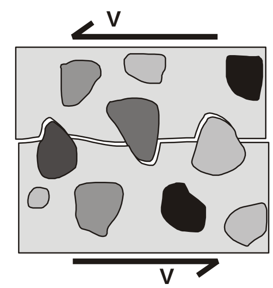
CONCRETE RESISTANCE IN SHEAR, V_C

CSA A23.3-04 *Clause 11.3.4*



Three contributions:

- **v_{cz} shear in compression zone**
- **v_a aggregate interlock** →
- **v_d dowel action**
  - → **v_a is the largest contribution to concrete shear strength**



$$V_c = \phi_c \lambda \beta \sqrt{f'_c}\, b_w d_v$$

Where,

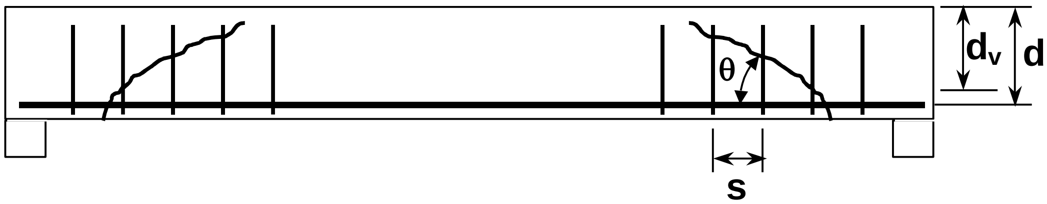
- **ϕ_c = 0.65 (A23.3-04)**
- **λ = factor to account for low-density concrete**
  = 1 for normal density concrete
- **β = factor accounting for shear resistance of cracked concrete, determined in *Clause 11.3.6***

Note that $\sqrt{f'_c} \leq 8\text{MPa}$ when computing V_c.

- **For f'_c > 64 MPa, use $\sqrt{f'_c} = 8\text{MPa}$**
- **Accounts for reduced aggregate interlock in high strength concrete**

STEEL RESISTANCE IN SHEAR, V_S

CSA A23.3-04 *Clause 11.3.5*



Force in one stirrup: $\phi_s A_v f_y$

number of stirrups crossing a crack: $n = \frac{d_v \cot\theta}{s}$

$$V_s = \frac{\phi_s A_v f_y d_v \cot\theta}{s}$$

Where,

- ϕ_s = 0.85
- A_v = area of shear reinforcement within distance "s"
  = A_b x no. of legs in stirrup
- s = stirrup spacing
- θ = angle of inclination of compression stresses, determined in *Clause 11.3.6*
  ≈ angle of inclined cracks due to shear

For design:

$$(V_s)_{req'd} = V_f - V_c - V_p = \frac{\phi_s A_v f_y d_v \cot\theta}{s}$$

Thus: $$s_{req'd} = \frac{\phi_s A_v f_y d_v \cot\theta}{V_f - V_c - V_p}$$

PRESTRESS RESISTANCE IN SHEAR, V_P

$$V_p = \phi_p F_{pe} \sin\alpha$$

Where,

ϕ_p = 0.90

F_{pe} = effective prestress force at section of interest

α = angle of inclination (from horizontal) of effective prestress force at section of interest

DETERMINATION OF β AND θ

CSA A23.3-04 *Clause 11.3.6*

Clause 11.3.6.2 - Special Member Types

β = 0.21
θ = 42 deg.

For

- slabs with thickness ≤ 350 mm
- beams with overall thickness ≤ 250 mm
- concrete joist construction (Clause 10.4)
- beams cast integrally with slabs where the depth of the beam below the slab is not greater than one-half of the web width or 350 mm

Clause 11.3.6.3 – Simplified Method

- **Applicable to cases other than Clause 11.3.6.2 and members not subject to significant axial tension**
- **May be used for prestressed concrete elements**
- **Limitations:** $f'_c \leq 60$ MPa, $f_y \leq 400$ MPa

θ = 35 deg.

β = 0.18 — for sections containing at least the minimum transverse reinforcement (Clause 11.2.8.2)

$$\beta = \frac{230}{1000 + d_v}$$

for sections containing no transverse reinforcement and having maximum C.A. size ≥ 20 mm

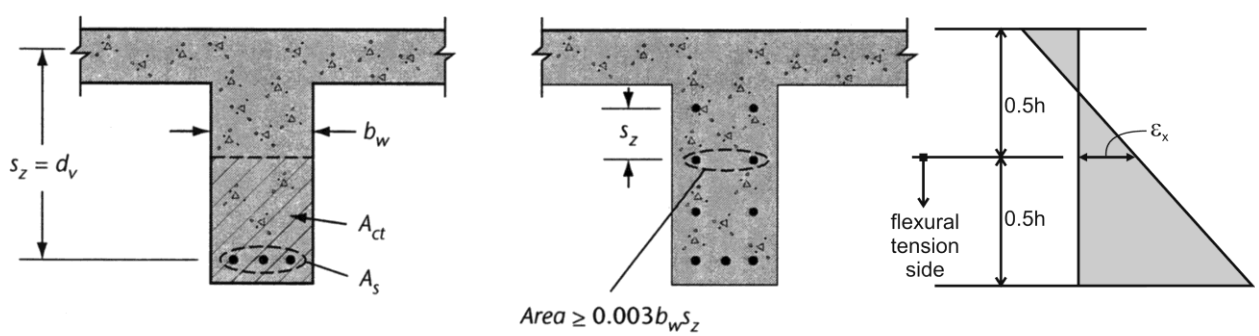
$$\beta = \frac{230}{1000 + s_{ze}}$$

for sections containing no transverse reinforcement and all aggregate sizes

s_{ze} = equivalent crack spacing parameter

$$= \frac{35 s_z}{15 + a_g} \geq 0.85 s_z$$

a_g = maximum C.A. size



(CSA A23.3-04)

Clause 11.3.6.4 – General Method

- **Based on Modified Compression Field Theory**
- **Use for:**
  - **$f'_c > 60$ MPa**
  - **Members subject to significant tension**
  - **Prestressed concrete elements**
  - **Situations where designer wants a more rigorous approach → non-typical members/structures**

$$\beta = \frac{0.40}{(1+1500\,\varepsilon_x)} \cdot \frac{1300}{(1000+s_{ze})}$$

$$\theta = 29 + 7000\,\varepsilon_x \qquad \text{(deg.)}$$

Where,

s_{ze} = equivalent crack spacing parameter

= 300 mm for members containing minimum transverse reinforcement *(Clause 11.2.8.2)*

$= \dfrac{35\,s_z}{15+a_g} \geq 0.85\,s_z$ for members without minimum transverse reinforcement

If $f'_c > 70\,\text{MPa}$, take $a_g = 0$

If $60\,\text{MPa} < f'_c \leq 70\,\text{MPa}$, a_g shall be linearly reduced to zero

ε_x = longitudinal concrete strain at mid-depth of the cross-section

$$= \frac{M_f/d_v + V_f - V_p + 0.5N_f - A_p f_{po}}{2\left(E_s A_s + E_p A_p\right)} \leq 3.0\times10^{-3}$$

- **If the value of ε_x is negative, it shall be taken as zero or recalculated as:**

$$\varepsilon_x = \frac{M_f/d_v + V_f - V_p + 0.5N_f - A_p f_{po}}{2\left(E_s A_s + E_p A_p + E_c A_{ct}\right)} \quad \geq \quad -2.0 \times 10^{-3}$$

f_{po} = stress in the prestressing tendons at decompression (i.e., when the stress in the surrounding concrete is zero)

= 0.70 f_{pu} for bonded tendons (outside of transfer length)

= f_{pe} for unbonded tendons

M_f = factored moment at section of interest (taken positive)

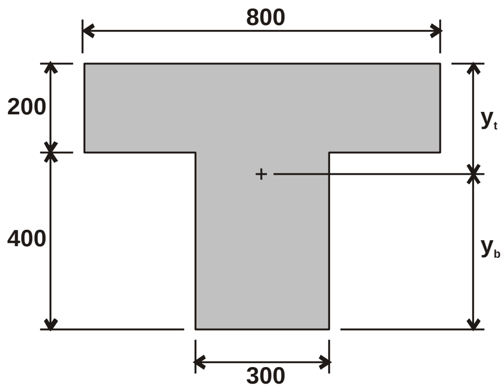
≥ $\left(V_f - V_p\right)d_v$

V_f = factored shear force at section of interest (taken positive)

N_f = factored axial force, acting simultaneously with V_f, at section of interest (taken positive for tension, negative for compression)

A_{ct} = area of concrete on flexural tension side of member (see figure on page 6.11)

Example – Comparison of β and θ for Simplified and General Methods



f'_c = 40 MPa
E_c = 28,460 MPa

Loading:
w_{self} = 6.58 kN/m
w_D = 8 kN/m
w_L = 16 kN/m

Beam span = 10 m (simply-supported)

Post-tensioned Tendon:
parabolic profile
e_{mid} = 240 mm
e_{ends} = 0
f = 240 mm (drape)

A_p = 891 mm^2
f_{pe} = 1116 MPa
F_{pe} = 994.4 kN

Determine effective shear depth, d_v

d_v = 0.9d or 0.72h, whichever is greater

Since e approaches zero at the ends of the member, d_v will be governed by 0.72h

d_v = 0.72(600 mm) = 432 mm

Determine V_f and M_f at the critical section for shear → distance d_v from support:

$$w_f = 1.25(6.58+8)+1.5(16) = 42.2 \text{ kN/m}$$

$$V_f(x) = \frac{w_f L}{2} - w_f x \qquad V_f(0.432) = \frac{(42.2)(10)}{2} - (42.2)(0.432) = 192.8\text{kN}$$

$$M_f(x) = \frac{w_f L x}{2} - \frac{w_f x^2}{2}$$

$$M_f(0.432) = \frac{(42.2)(10)(0.432)}{2} - \frac{(42.2)(0.432)^2}{2} = 83.3\text{kNm}$$

Determine V_p:

for parabolic profile, tendon slope, $m = \frac{8f}{L}x$

slope at critical section: $m(0.432) = \frac{8(.240)}{10}\left(\frac{10}{2} - 0.432\right) = 0.08771$

$$V_p = \phi_p F_{pe} \times \text{slope} = (0.90)(994.4)(0.08771) = 78.5\,\text{kN}$$

Calculate ε_x:

$$\varepsilon_x = \frac{M_f/d_v + V_f - V_p - A_p f_{po}}{2(E_p A_p)}$$
$$= \frac{(83.3 \times 10^6/432) + 192.8 \times 10^3 - 78.5 \times 10^3 - (891)(0.7 \times 1860)}{2[(200{,}000)(891)]}$$
$$= -0.00239$$

Since ε_x is negative, recalculate:

$$\varepsilon_x = \frac{M_f/d_v + V_f - V_p - A_p f_{po}}{2(E_p A_p + E_c A_{ct})}$$
$$= \frac{(83.3 \times 10^6/432) + 192.8 \times 10^3 - 78.5 \times 10^3 - (891)(0.7 \times 1860)}{2[(200{,}000)(891) + (28{,}460)(300 \times 300)]}$$
$$= -0.00015$$

Determine β and θ for General Method:

Assume minimum transverse reinforcement is provided → s_{ze} = 300 mm

$$\beta = \frac{0.40}{(1 + 1500\,\varepsilon_x)} \cdot \frac{1300}{(1000 + s_{ze})}$$
$$= \frac{0.40}{(1 + 1500(-0.00015))} \cdot \frac{1300}{(1000 + 300)} = 0.516$$

$$\theta = 29 + 7000\,\varepsilon_x$$
$$= 29 + 7000(-0.00015)$$
$$= 28 \text{ deg.}$$

Determine β and θ for Simplified Method:

Assume minimum transverse reinforcement is provided

$\theta = 35$ **deg.**

$\beta = 0.18$

Effect of β on V_C:

General Method:

$$V_c = \phi_c \lambda \beta \sqrt{f'_c}\, b_w d_v$$
$$= (0.65)(1.0)(0.516)\sqrt{40\,\text{MPa}}(300\,\text{mm})(432\,\text{mm}) \div 10^3 = 274.9\,\text{kN}$$

Simplified Method:

$$V_c = \phi_c \lambda \beta \sqrt{f'_c}\, b_w d_v$$
$$= (0.65)(1.0)(0.18)\sqrt{40\,\text{MPa}}(300\,\text{mm})(432\,\text{mm}) \div 10^3 = 95.9\,\text{kN}$$

As β increases, V_c increases → 187% higher for General Method

Effect of θ on V_S:

$$V_s = \frac{\phi_s A_v f_y d_v \cot\theta}{s}$$

$$s_{req'd} = \frac{\phi_s A_v f_y d_v \cot\theta}{V_f - V_c - V_p}$$

As θ decreases, cotθ increases:

- **V_S will be higher for General Method for a given stirrup spacing, s (~32%)**
- **Stirrup spacing will be greater for General Method for given A_V**

ADDITIONAL CODE REQUIREMENTS (CSA A23.3-04)

1. **Minimum shear reinforcement** *Clause 11.2.8.1*

   A minimum area of shear reinforcement is required in the following regions of flexural members:

   (a) where $V_f > V_c$

   (b) beams where h > 750 mm

   (c) where torsion, $T_f > 0.25\ T_{cr}$

2. **Minimum area of shear reinforcement** *Clause 11.2.8.2*

   Where shear reinforcement is required by Clause 11.2.8.1 or by calculation, a minimum area of shear reinforcement shall be provided:

$$A_{v,min} = 0.06\sqrt{f_c'}\,\frac{b_w\, s}{f_y}$$

3. **Maximum spacing of shear reinforcement** *Clause 11.3.8*

$$s \le \begin{cases} 0.7\, d_v \\ 600\,\text{mm} \end{cases} \quad \text{for } V_f \le 0.125\lambda\phi_c f_c' b_w d_v$$

$$s \le \begin{cases} 0.35\, d_v \\ 300\,\text{mm} \end{cases} \quad \text{for } V_f > 0.125\lambda\phi_c f_c' b_w d_v$$

4. Maximum shear resistance *Clause 11.3.3*

$$V_{r,max} = 0.25\phi_c f_c' b_w d_v + V_p$$

Thus,

$$V_{s,max} = 0.25\phi_c f_c' b_w d_v - V_c$$

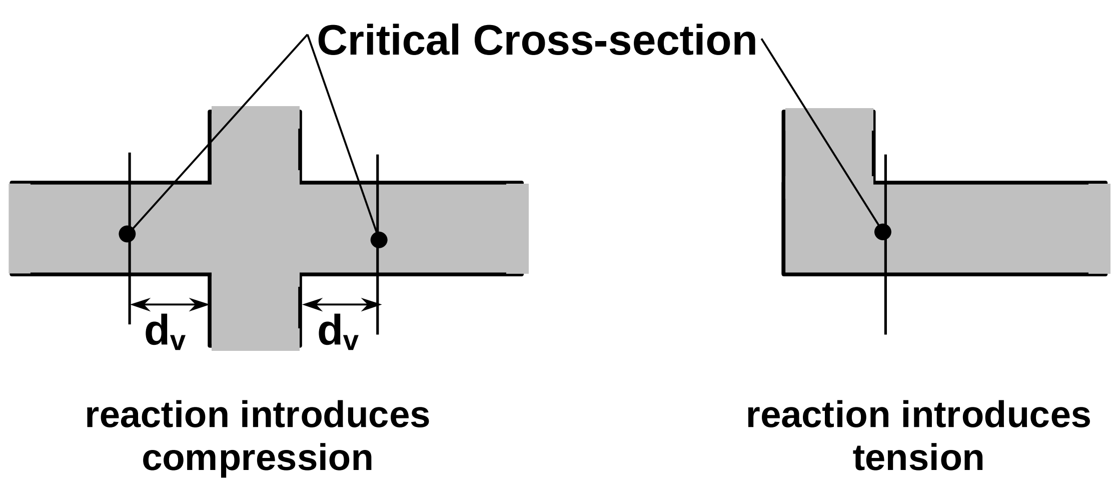
If too many stirrups are provided (V_s is too large), then the concrete web may crush before the stirrups yield.

If $V_{r,max} < V_f$, then the cross-section dimensions need to be increased.

5. Sections near supports *Clause 11.3.2*

<u>Critical section for shear design:</u>

- Compute V_f at a distance d_v from support where support reaction introduces compression
- Compute V_f at support where support reaction introduces tension



SHEAR DESIGN PROCEDURE

Must satisfy $V_r > V_f$ along length of member.

1. **Determine if size of cross-section is adequate:**

   Check $V_f \leq V_{r,max}$ If not, increase b_w and/or d

2. **Determine θ and β**

3. **Compute V_c (with and without stirrups)**

4. **Design stirrups for critical section, V_f, near support:**

   (a) If $V_f \leq V_c$, then no stirrups are required

   (b) If $V_f > V_c$, then:

   - **Choose A_v**
   - **Compute required spacing, s**
   - **Choose a reasonable value for s → round to nearest multiple of 10 mm or 25 mm less than or equal to calculated s**
   - **Check $A_v > A_{v,min}$**
   - **Check $s \leq s_{max}$**

5. **Design stirrups for selected other sections along length of beam following Step 4. procedures**

6. **Determine stirrup layout along beam length.
   Draw V_r diagram for beam and compare to V_f envelope**

SHEAR DESIGN EXAMPLES FOR PRESTRESSED CONCRETE

- **See CAC Handbook Chapter 10, Examples 10.7 and 10.8**

- **See CPCI Handbook Chapter 3, Examples 3-20 and 3-21.**

REFERENCES

1). Collins, M.P., and Mitchell, D., (1991). "Prestressed Concrete Structures," Prentice Hall, Englewood Cliffs, NJ.

2). Naaman, A. E., (2004). "Prestressed Concrete Analysis and Design," 2nd Ed., Techno Press 3000, Ann Arbor, MI.

3). MacGregor, J.G., and Bartlett, F.M., (2002). "Reinforced Concrete Mechanics and Design," 1st Canadian Ed., Prentice-Hall Canada, Scarborough, ON.