

# Module 2

## The Science of Surface and Ground Water

# Lesson

# 8

# Flow Dynamics in Open Channels and Rivers

## Instructional Objectives

On completion of this lesson, the student shall be able to learn:

1. The physical dynamics of water movement in open channels and rivers
2. The mathematical description of flow processes in the above cases
3. Different types of free surface flows: uniform, non uniform, etc.
4. Different channel shapes and cross sections and their representations
5. Computation steps for gradually varied water surface profiles

### 2.8.0 Introduction

It is common for water resources engineers to design a water system involving flow of water from one place to another, usually passing a variety of structures on the way some of them meant for controlling the flow quantity. Rivers and artificial channels, like canals, convey water with a free surface, that is, the surface of water being exposed to air as opposed to flow of water in pipes. It is easy to visualize that for any such open channel flow, as they are called; the presence or absence of a hydraulic structure controls the position of the free surface of water. Knowing the mathematical description of flowing water, it is possible to compute the water surface profile, which is important for example in designing the height of the channel walls of the water conveying system.

Another example, the case of river flow obstruction by the presence dam may be mentioned. The water level of the river increases on construction of the dam and it is essential to know the maximum possible rise, perhaps during the maximum flood, in order to know the degree of submergence of the land behind the dam. Barrages are low height structures, and hence, the rise of water will not be occurring uniformly across the river, again due to the difference of gate operation.

In this lesson, the behavior and corresponding mathematical description of flow in open channels are reviewed in order to utilize them in designing water resources systems.

## 2.8.1 Flow in natural rivers

Figure1 shows a river carrying a low discharge.

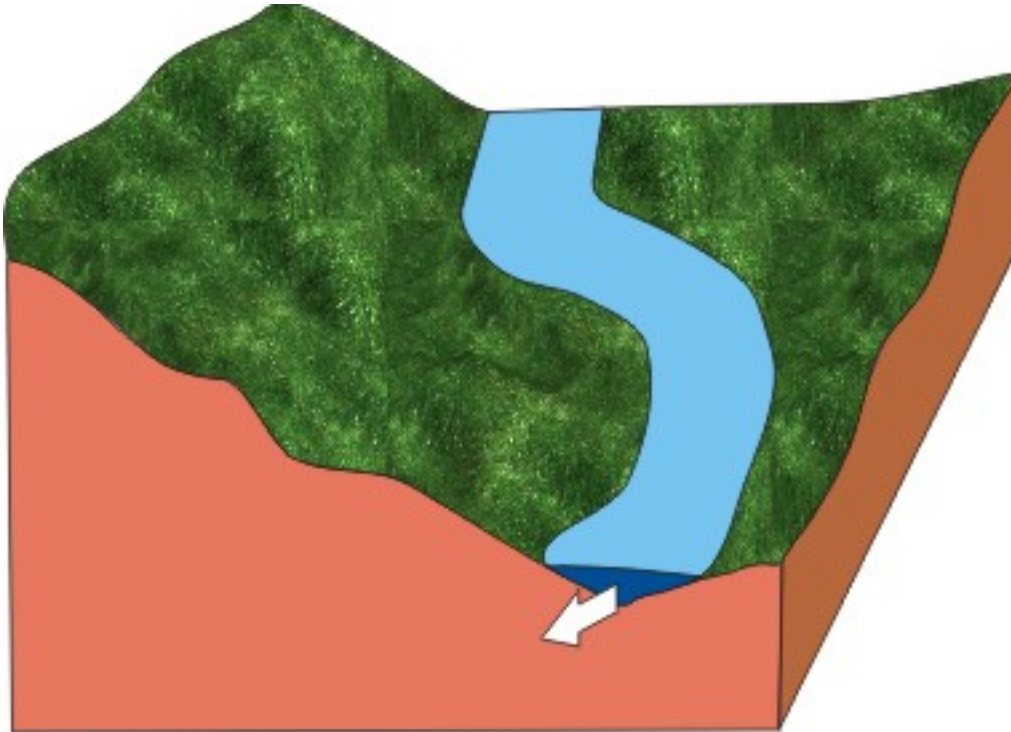


FIGURE. 1 A river flowing within its banks .

When the water surface of the river just touches its banks, the discharge flowing through the river at this stage is called the “**bank full discharge**”. It is also sometimes called the “**dominant discharge**”. If the discharge in the river increases, the water will overflow the banks and would spill over to the adjacent land, called the **flood plains** (Figure 2).

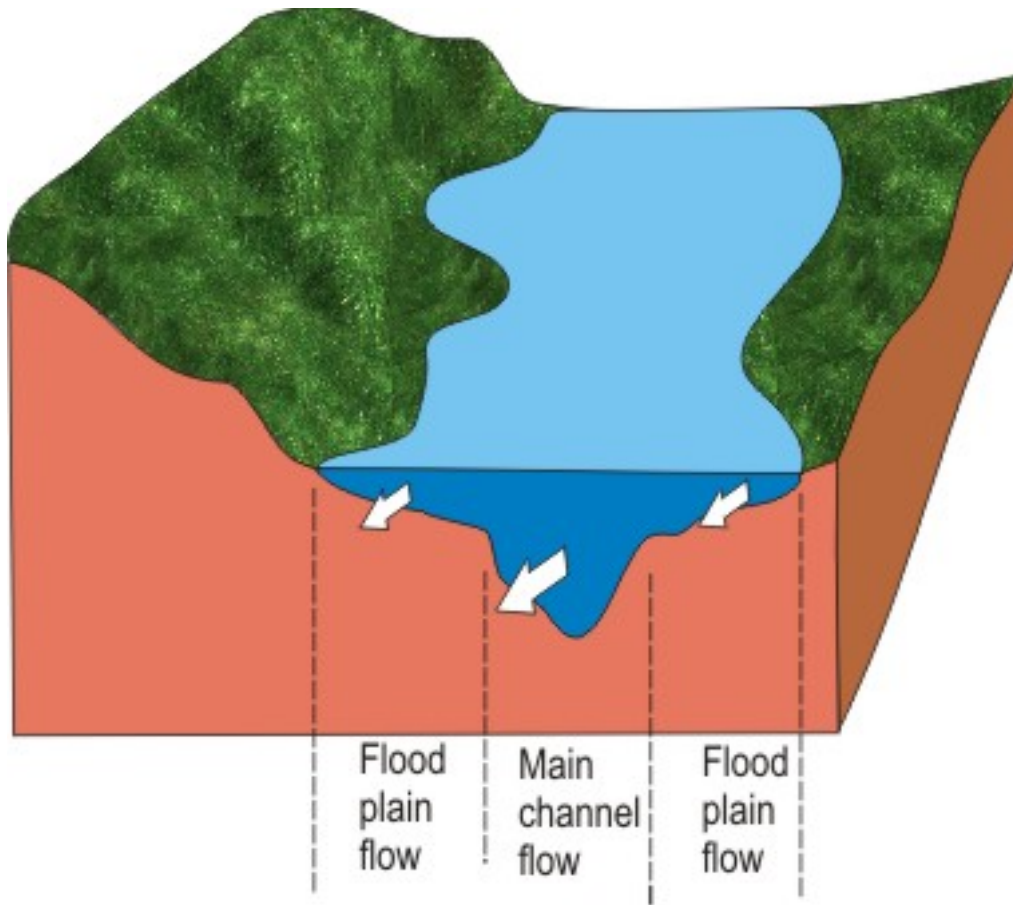


FIGURE .2 A river flowing its banks during flood

Though the amount of discharge flowing through the river is of interest to the water resources engineer it cannot be measured directly by any instruments. Rather, an indirect method is used which requires knowledge of the velocity distribution in a river or an open channel.

If we plot the velocity profile across a river, as shown in Figure 1, it would actually vary in three dimensions. Figure 3 shows the variation of velocity at the water surface.

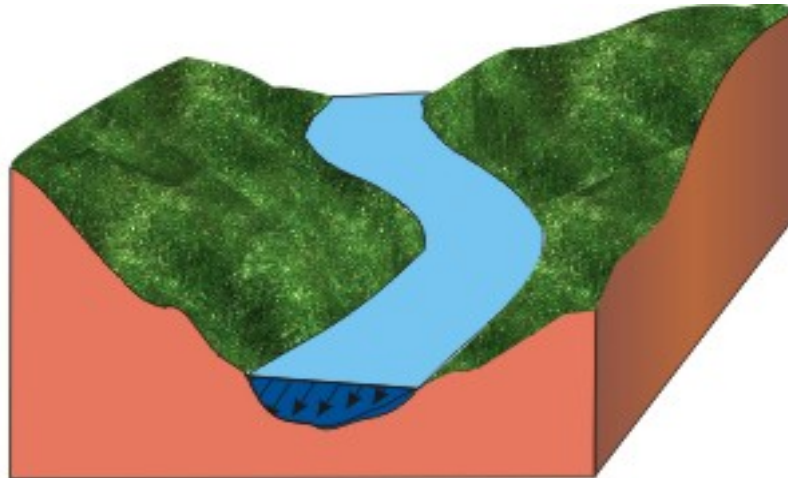


FIGURE 3 . Variation of surface velocity across a river section

It may be observed that velocity is highest at the center of the river but is zero at the banks. If a velocity profile were plotted on another horizontal plane at certain depth of the river, there too the velocity profile would be found to be similar in shape, but smaller in magnitude (Figure 4).

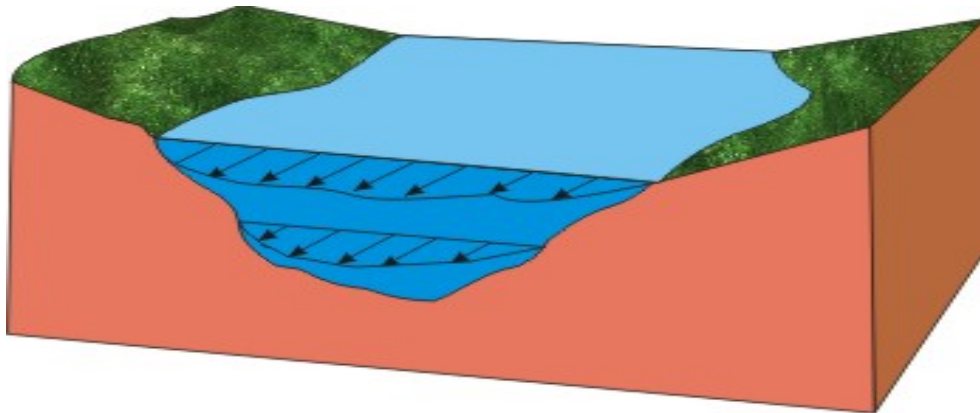


FIGURE 4 . Velocity variation of a river section at two levels.

Similarly the velocity profile of the river flowing in flood would be as shown in Figure 5, showing that the velocities over the flood plains is smaller compared to the main stream flow.

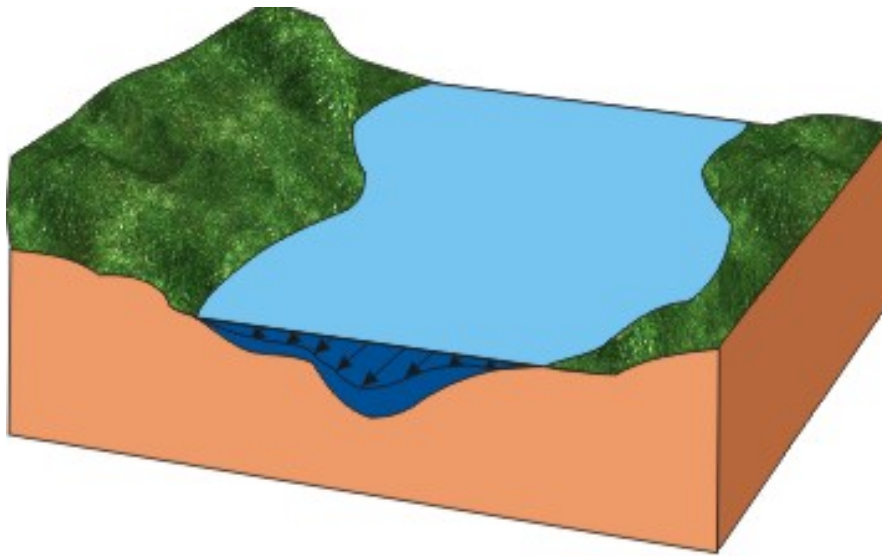
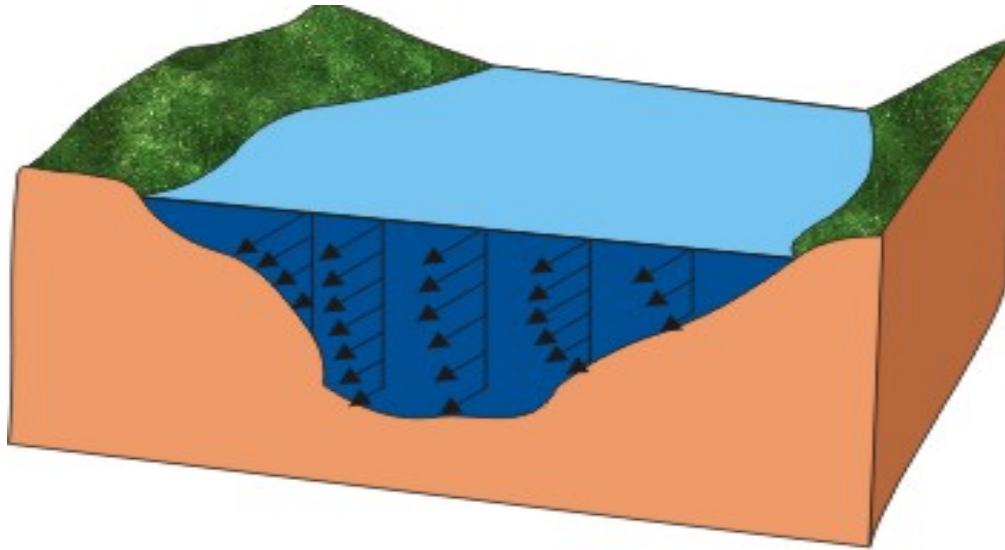


FIGURE 5. Surface velocity profile across a river section for a river flowing in flood

If we now take a look at the variation of velocity in a vertical plane within a river, and we plot them along different vertical lines across the river, then we may find the velocity profiles similar to those shown in Figure 6.



**FIGURE 6. Vertical velocity profiles in a river section**

In order to measure the discharge being conveyed in a river, the velocity profile or the average velocity at a number of equally spaced sections are measured, as in Figure 6. The total discharge is then approximately taken equal to the sum of the discharges passing through each segment.

Another way of depicting the velocity variation across a river cross-section is to plot "Isovels", which are actually the locus of points having equal velocity (Figure 7).



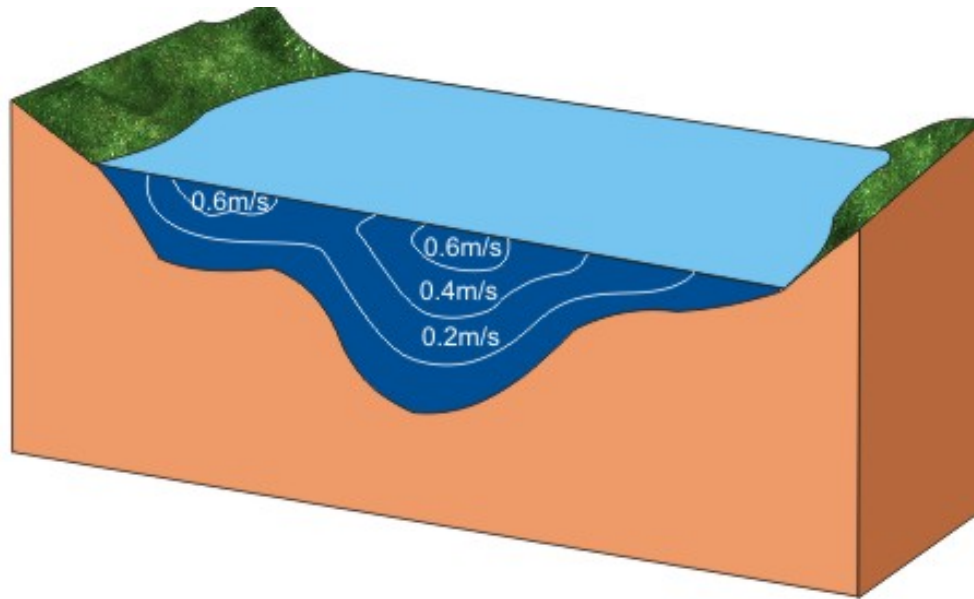


FIGURE 7. Isovells showing contours of typical equal velocities across a typical river section

It has been observed through experiments that a plot of velocity in the vertical plane would show that the maximum velocity occurs slightly below the surface (Figure 8) for a typical river flow.

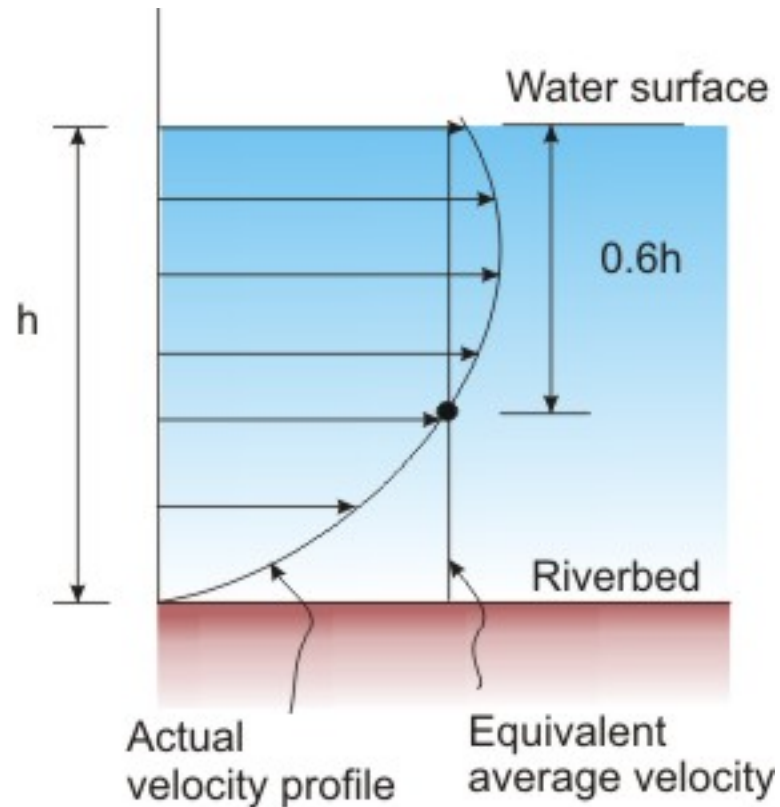


FIGURE 8. Vertical velocity distribution and average velocity for flow in a river

It has further been observed that an equivalent average velocity is almost equal to the actual velocity measured at 0.6 depth.

### 2.8.2 Variation of discharge with river stage

The water level in a river is sometimes called the “**stage**” and as this varies, there is a proportional change in the total discharge conveyed. For each point of a river, the relation between stage and discharge is unique but a general form is found to be as shown in Figure 9.

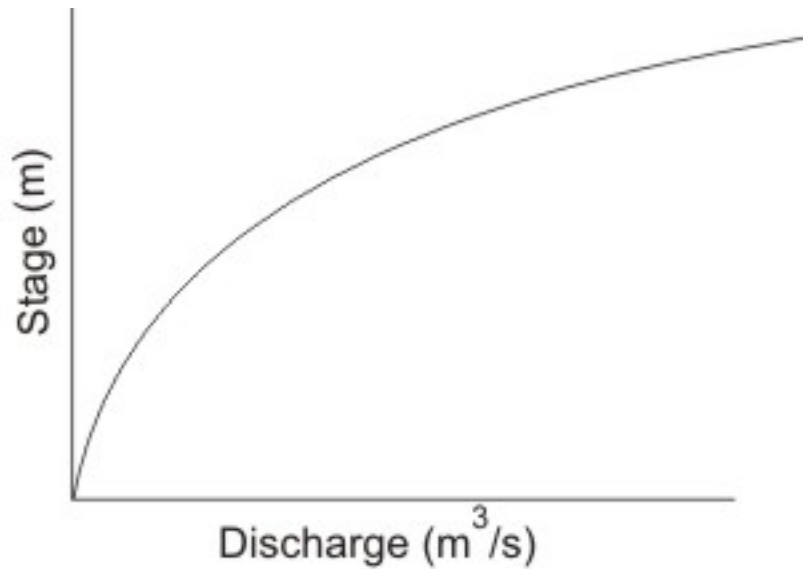


FIGURE 9. Stage-discharge curve for a river

The general mathematical description for the stage-discharge relation is given as:

$$Q = k (h - h_0)^m \quad (1)$$

Where  $h$  is the gauge corresponding to a discharge  $Q$  and  $h_0$  is the corresponding to zero discharge  $k$  and  $m$  are constants. If the variables ( $Q$  and  $H$ ) are plotted on a log-log graph, then it generally plots in a straight line as:

$$\log Q = m \log (h - h_0) + \log k \quad (2)$$

### 2.8.3 Flow variation along river length

It may be interpreted from Figures 4 or 6 that the velocity in a river cross section actually varies from bank to bank and from riverbed to free water surface and hence, can be called a two dimensional variation in a vertical plane. However, for engineering purposes it is, sufficient, generally, to use an equivalent velocity in the direction of river motion (perpendicular to river cross section) which may be

obtained by dividing the total discharge by the cross sectional area. In a natural river, therefore, these flow velocities may vary from section to section (Figure 10).

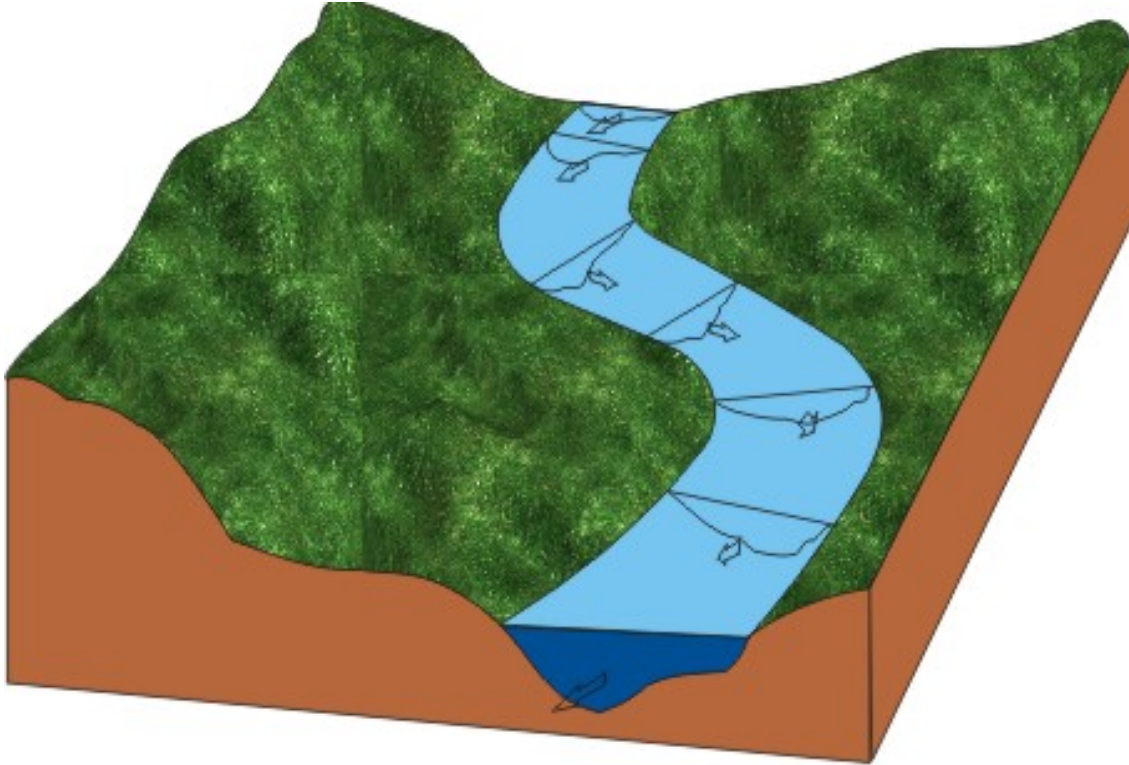


FIGURE. 10:- Variation of cross section and velocity along the length of a river showing vertical section at each point

If we now consider an axis along the length of the river, the total energy ( $H$ ) is given as:

$$H = Z + h + \frac{V^2}{2g} \quad (3)$$

We may plot the total energy as shown in Figure 11, where the variables are as follows:

- **Z**: Height of riverbed above a datum
- **h**: Depth of water
- **V**: Average velocity at a section

- $\frac{V^2}{2g}$  : Kinetic energy head

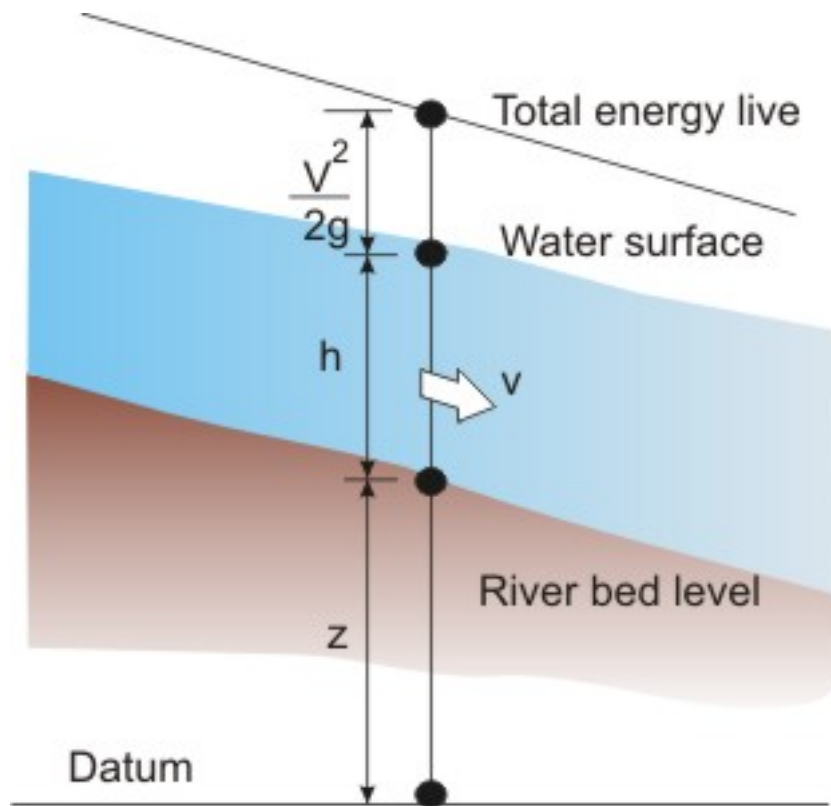


FIGURE 11. Variation of total energy along a river

Since the cross section, bed slope and flow resistance vary along a river length, the depth and velocity would vary correspondingly. However, if a short stretch of a river section is taken, then the variations in riverbed, water surface and the total energy may be considered as linear (Figure 12).

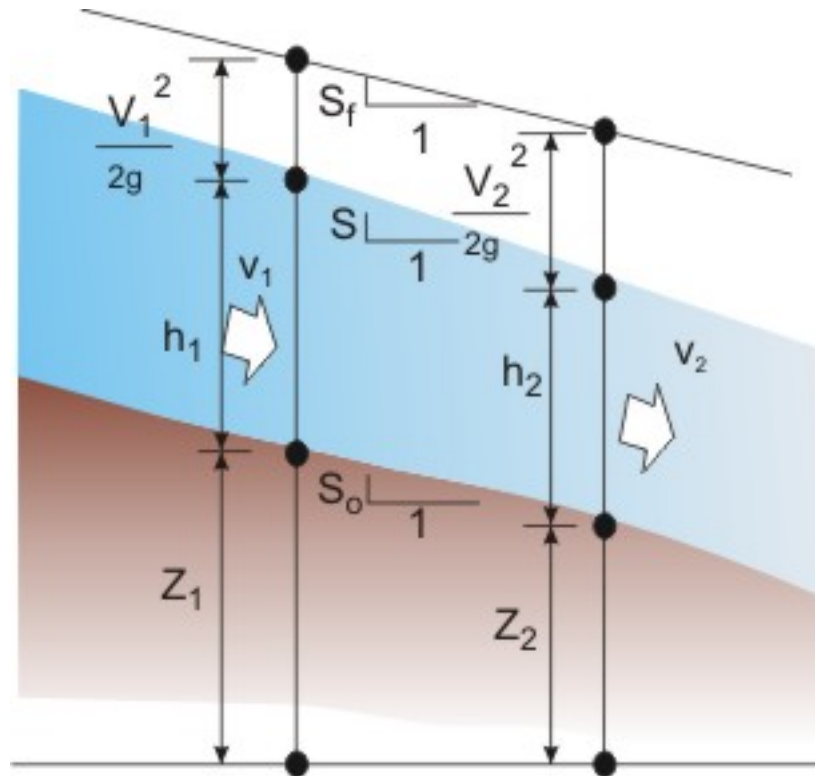


FIGURE 12. Short section between two points along a river

In Figure 12, three slopes have been marked, which are:

- $S_0$ : Riverbed slope
- $S$ : Water surface slope
- $S_f$ : Energy surface slope

Since the total energy of flowing water reduces along the river length due to friction the “energy surface slope” is generally termed as the “friction slope”. The energy loss in a river or an open channel occurs mostly due to the resistance at the channel sides, as the turbulent characteristics of the flowing water implies a smaller loss internally within the water body itself.

It has been nearly 200 years when scientists first attempted to mathematically express (or “model”) the friction slope in terms of known variables like average velocity, cross section properties and riverbed slope. One of the earliest models for friction slope  $S_f$  or, in effect, the channel resistance was derived from the considerations of “uniform flow” (Figure 13) where the flow variables and cross section are supposed to remain constant over a short reach.

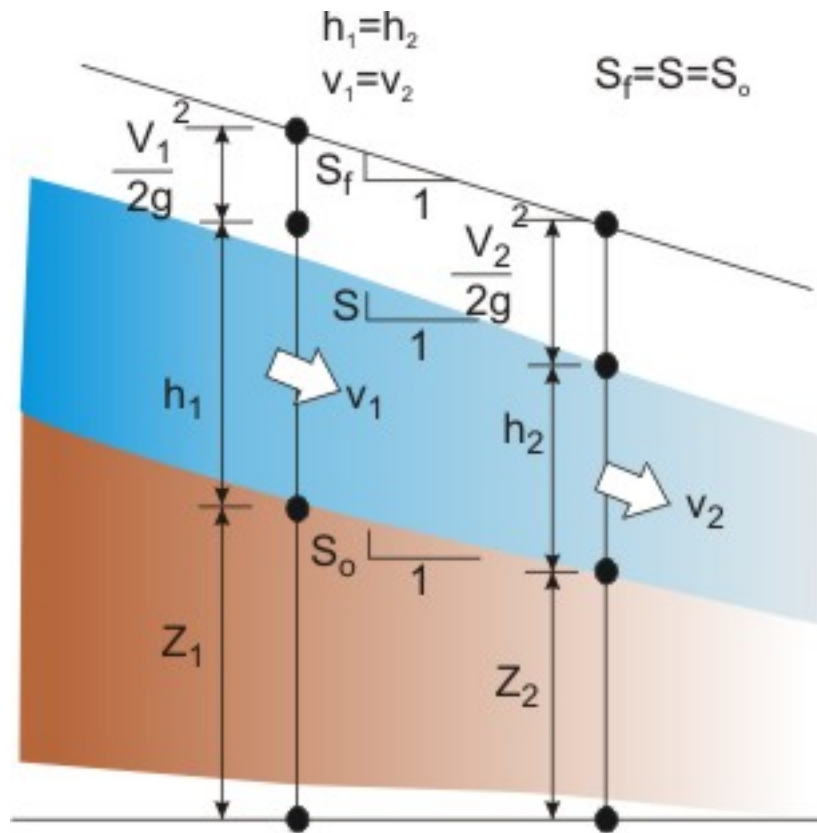


FIGURE 13. Uniform flow along a channel reach

If we take small volume of fluid from these two sections we may make a free body diagram of the forces acting on it (Figure 14).

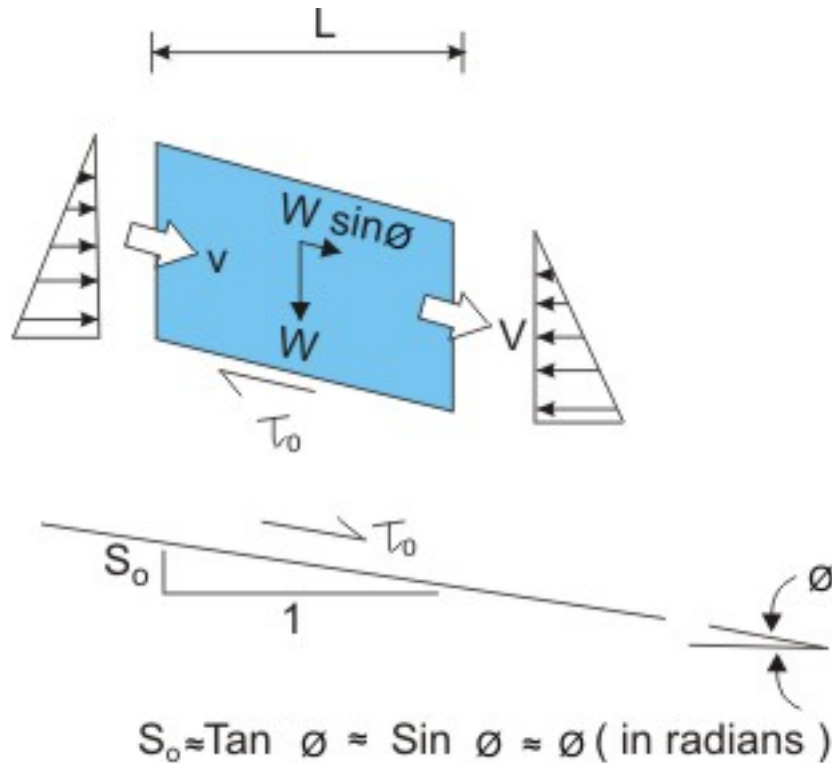


FIGURE 14. Free body diagram of forces on a control volume in uniform flow

The variables represented in the figure are as follows

- **W**: Weight of water contained in the control volume
- **V**: Inflow velocity, which is the same as the outflow velocities
- $\theta$ : Angle of slope river bed, which is also equal to that water surface and friction slopes
- $\tau_0$ : Shear stress due to friction acting on the control volume of fluid from the river bed and all along the periphery, though in Figure 14 only the resistance due to the riverbed is shown.

Equating the forces and noting that the inflowing and out flowing momenta are equal as well as the pressure forces at either end of the control volume one obtains:

$$\tau_0 P L = W \sin \theta = \rho g A L \sin \theta \quad (4)$$



Where the remaining variables are:

- **P**: wetted perimeter
- **A**: Cross section of flow area
- **L**: Length of control volume

Assuming  $\theta$  to be very small and nearly equal to bed slope, we have

$$\tau_0 = \rho g R S_0 \quad (5)$$

Assuming a state of rough turbulent flow, as is the case for natural rivers and channels, one may write

$$\tau_0 \propto V^2 \quad \text{or} \quad \tau_0 = kV^2 \quad (6)$$

Substituting into (4),

$$V = \sqrt{\frac{\rho g}{k} R S_0} \quad (7)$$

This may be written as

$$V = C \sqrt{RS} \quad (8)$$

This is known as Chezy equation after the French hydraulic engineer. Antoine Chezy who first proposed the formula around 1768 while designing a canal for Paris water supply. The constant **C** in equation (8) actually varies depending on Reynolds number and boundary roughness.

In 1869, Swiss engineers, Ganguillet and Kutter proposed an elaborate formula for Chezy's **C** which they derived from actual discharge data from the river Mississippi and a wide range of natural and artificial channels in Europe. The formula, in metric units, is given as

$$C = 0.552 \left( \frac{41.6 + \frac{1.811}{n} + \frac{0.00281}{S_0}}{1 + \left[ 41.65 + \left( \frac{0.000281}{S_0} \right) \right] \frac{n}{\sqrt{R}}} \right) \quad (9)$$

Where **n** is a coefficient known as Kutter's **n**, and is dependent solely on the boundary roughness.

In 1889, Robert Manning's, an Irish engineer proposed another formula for the evaluation of the Chezy coefficient, which was later simplified to:

$$C = \frac{R^{1/6}}{n} \quad (10)$$

From Equation (8), the Manning equation may be written as:

$$V = \frac{1}{n} R^{2/3} S_0^{1/2} \quad (11)$$

Where the Manning  $n$  is numerically equivalent to Kutter's  $n$ .

Many research workers have experimentally found the value of  $n$ , and for natural rivers, the following books may be consulted:

1. Chow, V T (1959) "Open Channel Hydraulics", McGraw Hill.
2. Chaudhry, M H (1994) "Open Channel Flow", Prentice Hall of India.

## 2.8.4 Uniform flow in channels of simple cross section

For problems concerning the steady uniform flow in rivers and open channels, the Manning's equation is commonly used in India. The depth of water corresponding to a discharge in a channel or river under uniform flow conditions is called "normal depth". By combining the continuity equation with that of Mannings, one obtains

$$Q = \frac{1}{n} A R^{2/3} S^{1/2} \quad (12)$$

Where the variables have been defined in the earlier sections.

One may also write equation (12) as follows

$$Q = K \sqrt{S} \quad (13)$$

Where  $K = \frac{A R^{2/3}}{n}$ , also called Conveyance, is often necessary to find out the normal depth of flow corresponding to a discharge  $Q$ , flowing in a channel for which equation (11) may be rearranged as

$$A R^{2/3} = \frac{n Q}{S^{1/2}} \quad (14)$$

In equation (14), the right hand side terms are known where as those in left hand are unknown and are functions of water depth. For a few commonly encountered sections the parameters A and R are given in the table below.

	Rectangle	Trapezoid	Circle
Flow Area, A	$b.h$	$(b+my).y$	$\frac{1}{8}(\phi - \sin \phi)$
Wetted Perimeter, P	$b + 2h$	$b + 2h.\sqrt{1+m^2}$	$\frac{1}{2}\phi D$
Hydraulic Radius, R	$\frac{bh}{b+2h}$	$\frac{(b+my).y}{b+2h\sqrt{1+m^2}}$	$\frac{1}{4}(1 - \frac{\sin \phi}{\phi})D$
Free surface width, B	$b$	$b + 2mh$	$(\sin \frac{\phi}{2})D$

In the table,  $m$  stands for the side slope of a trapezoidal channel and  $\phi$  stands for the angle subtended at the centre by the water surface chord line.

As seen from the above table except for the very simple rectangular section it is not possible directly to evaluate  $h$ , corresponding to  $Q$  as the left hand side of equation 13 is nonlinear in terms of  $h$ . One way of solving is by Newton's method, where equation (14) is written as

$$f(h) = AR^{\frac{2}{3}} - \frac{nQ}{S^{\frac{1}{2}}} = 0 \quad (15)$$

For using Newton's method the derivative of the function is required

$$f'(h) = \frac{d}{dh} \left( A \frac{A^{\frac{2}{3}}}{P^{\frac{2}{3}}} - \frac{nQ}{S^{\frac{1}{2}}} \right) = 0 \quad (16)$$

$$f'(h) = \frac{5}{3} P^{-\frac{2}{3}} A^{\frac{2}{3}} \frac{dA}{dh} - \frac{2}{3} P^{-\frac{5}{3}} A^{\frac{5}{3}} \frac{dP}{dh} \quad (17)$$

$$f'(h) = \frac{5}{3} BR^{\frac{2}{3}} - \frac{2}{3} R^{\frac{2}{3}} \frac{dP}{dh} \quad (18)$$

Where we have used  $\frac{dA}{dh} = B$ . Similarly the expression  $\frac{dP}{dh}$  may be evaluated for any section.

Starting with a realistic value  $h_i$  the iteration may be carried out as given below:

$$h^{i+1} = h^i - \frac{f(h^i)}{f'(h^i)} \quad (19)$$

Where  $h^{i+1}$  is the value of  $h$  at next iteration, which is an improvement of initial guess  $h^i$ . The iteration may be continued till a desired accuracy is achieved.

### 2.8.5 Uniform flow in channels of compound cross section

A compound section may be defined as a section in which various portions of the cross-section have different flow properties, like surface roughness or channel depth. (Figure 15)

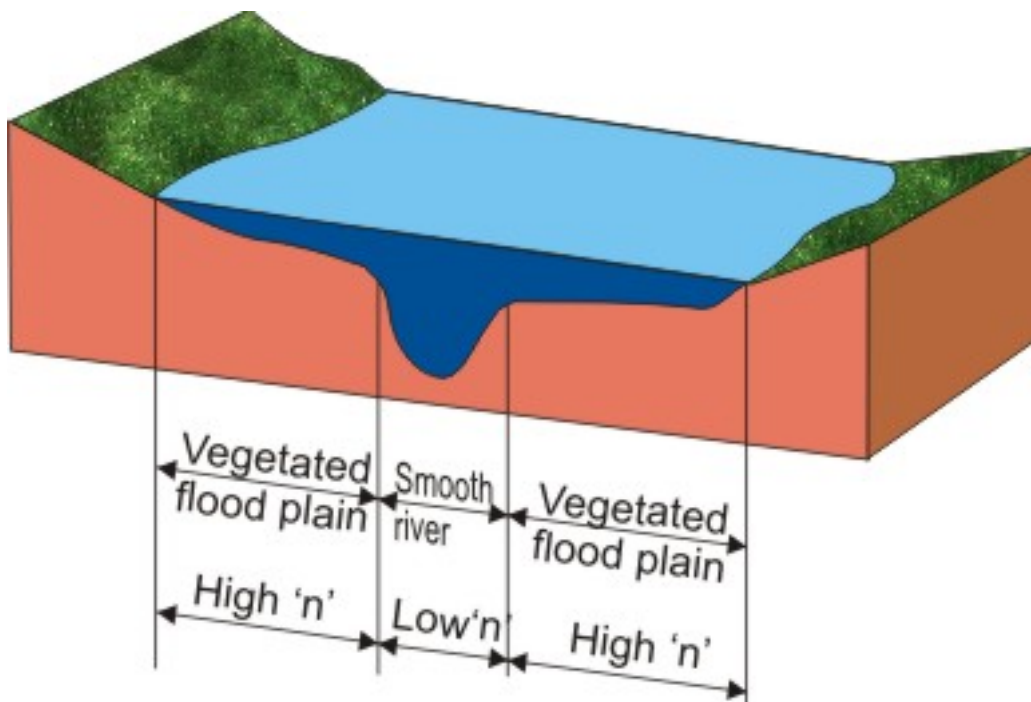


FIGURE 15. A river in flood  
example of a compound  
flow section

In order to use the uniform flow formula in compound channels one way may be to divide the flow section into sub areas (Figure 16) and treat the flow in each area separately.

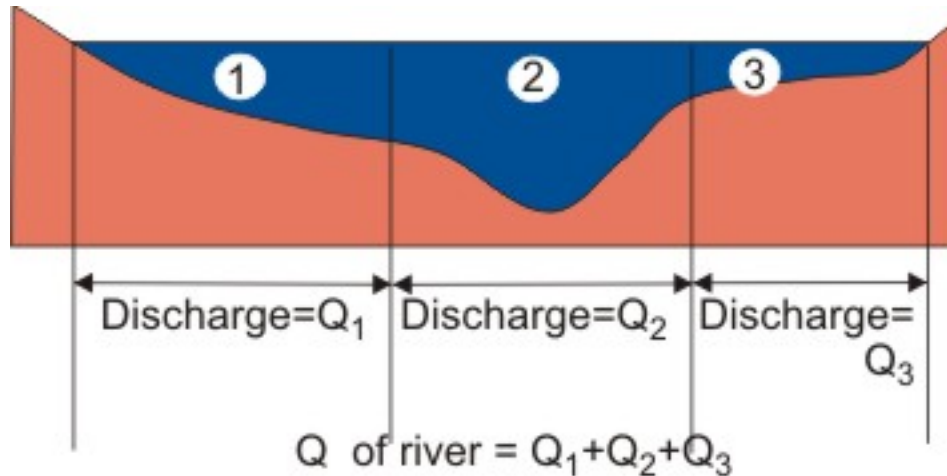


FIGURE 16. Simple flow analysis method for compound channels

However, it has been found that this method may lead to errors by as much as  $\pm 20\%$  or even more (Chadwick et al 2004). The error is largely due to the neglecting of mass and momentum interchange between adjacent sub-areas. The current solution would however be more complex by using a two or even three-dimensional model.

In another method, the **energy coefficient** ( $\alpha$ ) and **friction slope**  $S_f$  are evaluated in terms of conveyance  $K$  of the sub areas. With these expressions, the flow in compound section may be computed without knowing the individual flows in each sub area. For a compound channel divided into  $N$  sections. (For example  $N = 3$  in Figure 15). The energy coefficient,  $\alpha$ , is found out as:

$$\alpha = \frac{\sum_{i=1}^N V_i^3 A_i}{V_m^3 \sum_{i=1}^N A_i} \quad (20)$$

Where  $V_m$  is the mean flow velocity in the entire section and is given as follows

$$V_m = \frac{\sum V_i A_i}{\sum A_i} \quad (21)$$

Where  $V_i = Q_i / A_i$  and  $A_i$  is the area of its  $i^{\text{th}}$  sub-area. Equation (18) now can be written as

$$\alpha = \frac{\left( \sum Q_i^3 / A_i^2 \right) (\sum A_i)^2}{(\sum Q_i)^3} \quad (22)$$

Now, the flow in sub-areas  $i$  may be written as

$$Q_i = K_i S_{fi}^{1/2} \quad (23)$$

$$S_{fi}^{1/2} = \frac{Q_i}{K_i} \text{-----} \quad (21)$$

Here, an assumption has been made that  $S_f$  has the same value for all sub-areas, which is not quite correct since the velocities of each of these areas being different, would not give equal velocity heads. Where as, the water surface is almost level over the entire cross section.

$$\frac{Q_1}{K_1} = \frac{Q_2}{K_2} = \text{-----} = \frac{Q_n}{K_n} = \text{Constant} = S_f^{1/2} \quad (24)$$

It follows from equation (23) that

$$Q_1 = K_1 \frac{Q_n}{K_n} \quad (25)$$

$$Q_2 = K_2 \frac{Q_n}{K_n}$$

-  
-  
-

$$Q_n = K_n \frac{Q_n}{K_n} \quad (26)$$

Adding all the above equation yields

$$Q = \sum_{i=1}^n Q_i = \frac{Q_n}{K_n} \sum_{i=1}^n K_i \quad (27)$$

By substituting this expression for  $Q_i = K_i \left( \frac{Q_n}{K_n} \right)$  into equation (27) and simplifying the equation, one obtains

$$\alpha = \frac{\left( \sum_{i=1}^n A_i \right)^2}{\left( \sum_{i=1}^n K_i \right)^3}, \sum_{i=1}^n \left( \frac{K_i^3}{A_i^2} \right) \quad (28)$$

Elimination of  $\frac{Q_n}{K_n}$  from equations (24) and (26) and squaring both sides give

$$S_f = \left( \frac{\sum Q_i}{\sum K_i} \right)^2 \quad (29)$$

$$S_f = \frac{Q^2}{\sum K_i^2} \quad (30)$$

Thus, expressions for  $\alpha$  and  $S_f$  have been evaluated for any given stage without explicitly determining the flow in each sub areas,  $Q_i$ . In addition, equation (30) may be used in the procedure for determining varied flow profiles as discussed in Section 2.8.6.

## 2.8.6 Non uniform in channels

There are quite a few examples of non-uniform flow in rivers or open channels that may be encountered by a water resources engineer. Some of these have been illustrated in Figure 17.

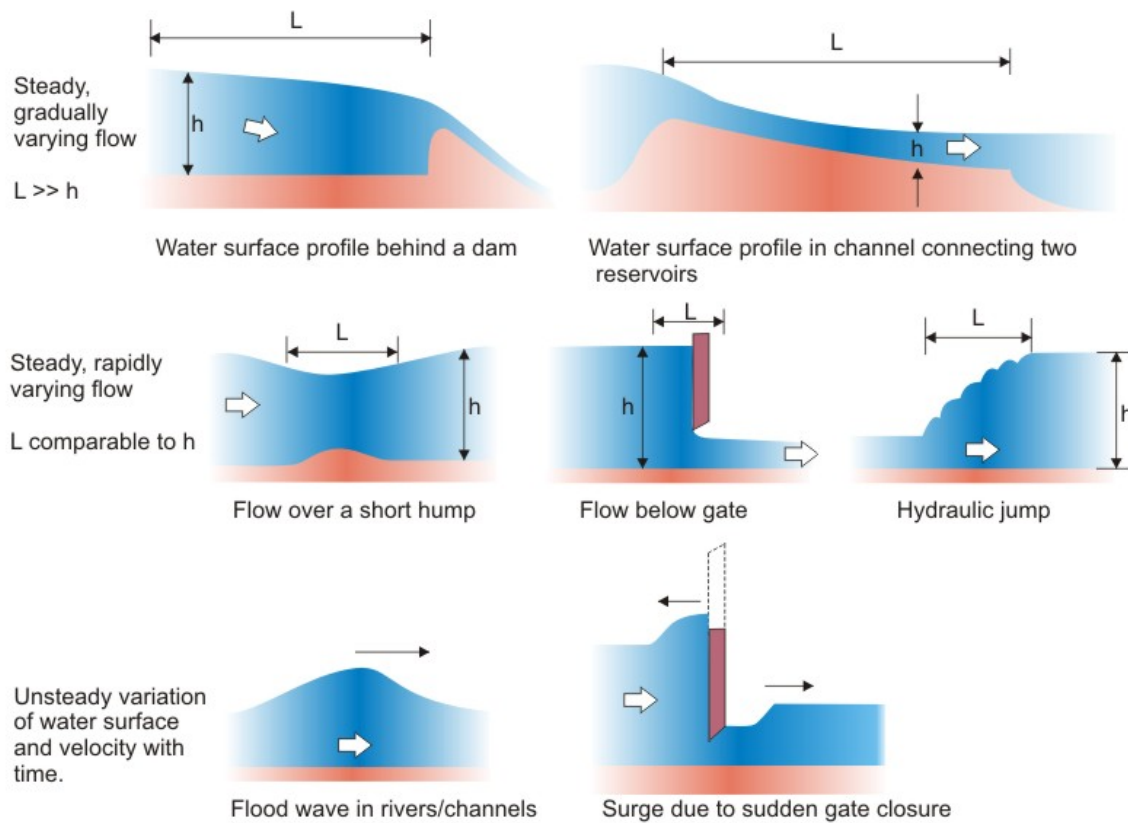


FIGURE 17. Some example of flow situation encountered in practice.

In this lesson we shall discuss the procedure to evaluate water surface profiles for steady, gradually varying flow situations. For steady, rapidly varying and unsteady flow situations, reference may be made to following or similar texts on hydraulics of open channel flow, like Ranga Raju (2003) or Subramanya (2002).



## 2.8.7 Non-uniform gradually varied flow calculation

A representative non-uniform gradually varied flow is shown in Figure 18.

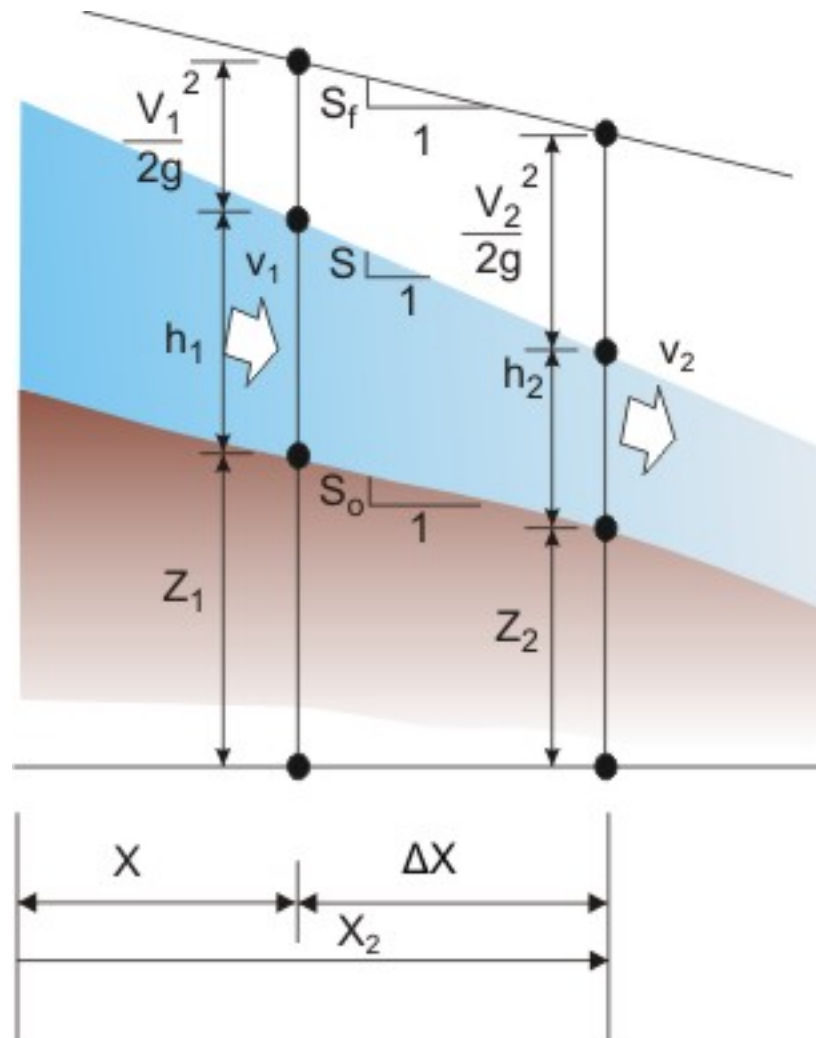


FIGURE 18. Non-uniform gradually varied flow

Over the incremental distance  $\Delta x$ , the depth and velocity are known to change slowly. The slope of the energy grade line is designated as  $\alpha$  in contrast to uniform flow, the slopes of the energy grade line, water surface, and channel bottom are no longer parallel. Since the changes in the water depth  $h$  and velocity  $V$  are gradual, the energy lost over the incremental  $\Delta x$  can be represented by Manning equation. This means that equation 11, which is valid for uniform flow can also be used to evaluate  $S$  for a gradual varied flow situation, and that the roughness coefficients discussed in Section 2.8.3 are applicable.

Additional assumption includes a regular cross section, small channel slope, hydrostatic pressure distribution and one-dimensional flow.

Applying the equivalence of energy between locations 1 and 2, and assuming the loss term as  $h_L$  given by  $S_f \cdot \Delta x$  one obtains

$$Z_1 + h_1 + \alpha \frac{V_1^2}{2g} = Z_2 + h_2 + \alpha \frac{V_2^2}{2g} + S_f \Delta x \quad (31)$$

In the above equation,  $\Delta x$  is the distance between two consecutive sections  $x_1$  and  $x_2$  such that  $\Delta x = x_2 - x_1$ .

The energy coefficient  $\alpha$  has been used along with the  $\frac{V^2}{2g}$  term, as it may be much different from 1.0 for natural sections. The term  $S_f$  in equation (31) may be evaluated by the expression for uniform flow, equation (11), where  $S_0$  may be replaced by  $S_f$ . Since equation (31) relates the energy between the sections,  $S_f$  may be taken either of the following:

$$\text{Arithmetic mean: } \overline{S_f} = \frac{1}{2}(S_{f1} + S_{f2}) \quad (32)$$

$$\text{Geometric mean: } \overline{S_f} = \sqrt{S_{f1} S_{f2}} \quad (33)$$

$$\text{Harmonic mean: } \overline{S_f} = \frac{2 S_{f1} S_{f2}}{S_{f1} + S_{f2}} \quad (34)$$

Where  $S_{f1}$  and  $S_{f2}$  are the friction slopes evaluated at section 1 and 2 by using the Mannings formula equation (12).

Equation (29) may be used by starting from one end of the channel where the flow depth and velocity are known and working backward or forward in steps. Here, two, methods are used of which we shall discuss one, called the standard step method. A very popular computer program called HEC-2 developed by hydrologic engineering center of the US Army Corps of Engineers is based on this method. It may be freely downloaded from the website: [www.hec.usace.army.mil/software/legacysoftware/hec2/hec2-download.htm](http://www.hec.usace.army.mil/software/legacysoftware/hec2/hec2-download.htm).

In the standard step method, for any given discharge the depth of flow would be known at the control section. It is then required to calculate the depth of flow at the section immediately next to the control section. Two examples are illustrated in Figure 19.

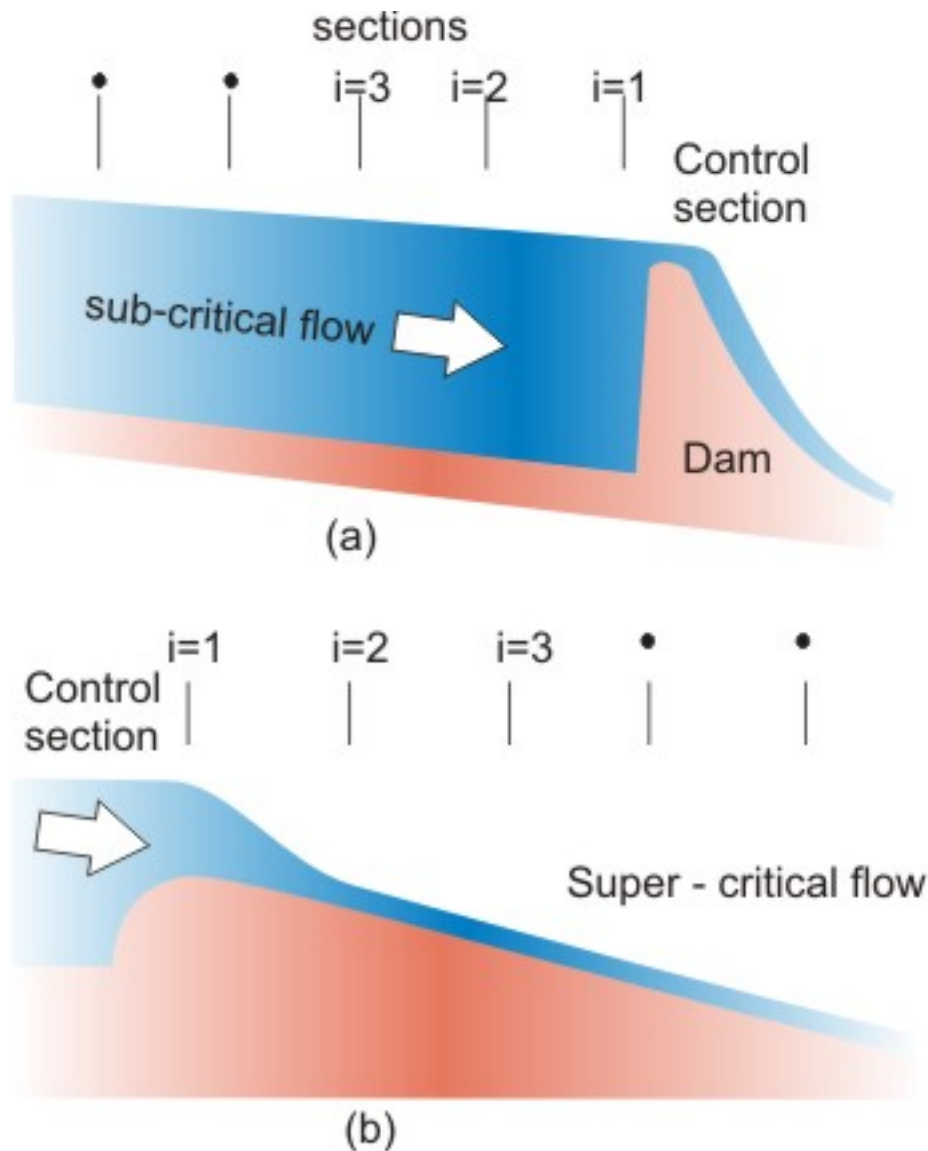


FIGURE 19. Computation grid by the standard step method.

(a) profile behind a dam: calculation proceeds upstream from control section

(b) profile in a steep channel : calculation proceeds downstream from control section.

The distance between the two successive sections ( $i$  and  $i+1$ ) is taken as constant, say  $\Delta x$ . It may be observed from the Figure 19a since the water is flowing above the dam the water depth above the dam crest can be found out for the given discharge. Hence the water level at the control section just upstream of

the dam is known. Similarly, in Figure 19b, since the water is flowing down from the reservoir into the steep channel critical depth corresponding to the given discharge would exist at the control section. Here too, the water level at the control section is then known.

Starting at the control section ( $i=1$ ), the total energy of water is found out to be

$$H_1 = Z_1 + h_1 + \alpha \frac{V_1^2}{2g} \quad (35)$$

Next, consider the first reach, that is, between sections  $i=1$  and  $i=2$ . A depth of flow is assumed at section 2 and the energy there, that is,

$$H_2 = Z_2 + h_2 + \alpha \frac{V_2^2}{2g} \quad (36)$$

is evaluated. Now, one of the equations for finding  $\overline{S_f}$  (the average friction slope) in the reach is found out by, say equation 31.

As may be observed from Figure 18 the numerical value of  $H_2$  found from equation (33) should be equal to that of  $H_1$  found from equation (33) +  $S_f$ . If the depth at the section 2 has been correctly assumed if the two don't match, a new depth  $h_2$  is assumed and the calculations are repeated till the two values match.

Once a correct depth is found at section 2, a similar procedure is used to find the depth at section 3, and so on.

These are the two other methods to find out water surface profiles of gradually varied flow situations, namely; method of direct integration and method of graphical integration. Interested reader may refer to standard textbooks on Hydraulics of open channel flow, like the following for details about these methods.

1. Ranga Raju (2003)
2. Subramanya (2002)

## 2.8.8 Gradually varied flow profiles

In many flow problems it is enough to make a qualitative sketch of water surface profile for a given flow that is taking place between two locations. It is not necessary therefore to find out the exact level of water at different points but the general shape of the free surface has to be drawn as accurately as possible. An

analysis of water surface profile may be done by studying the governing equation, which can be derived from the sketch in Figure 20.

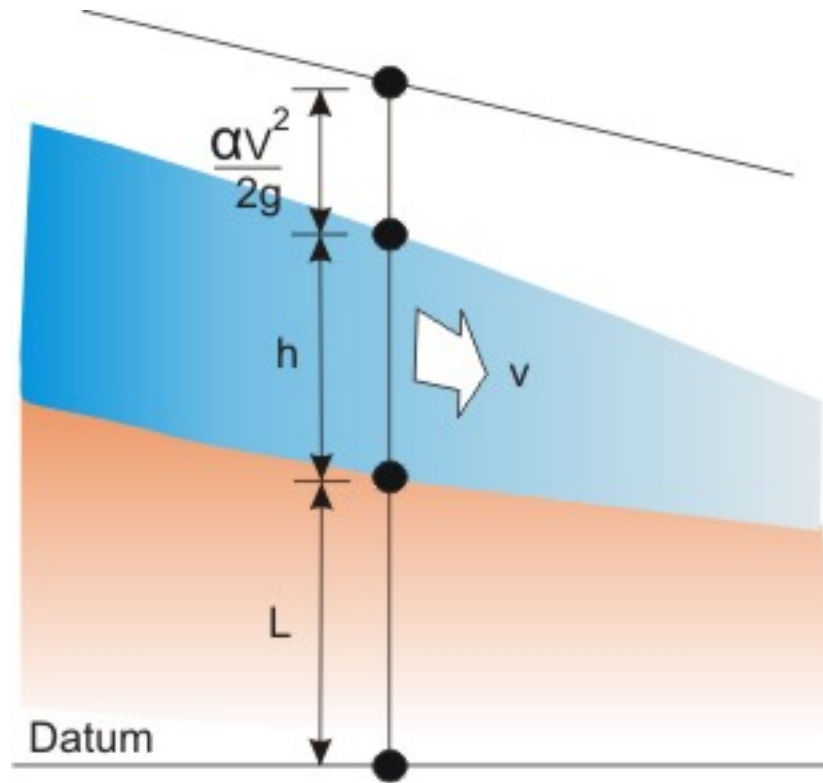


FIGURE 20. Definition sketch

The total energy  $H$  at a channel section is given as

$$H = Z + h + \alpha \frac{V^2}{2g} \quad (37)$$

Where

- $H$ : Elevation of energy line above the datum
- $Z$ : Elevation of channel bottom above datum
- $h$ : Flow depth
- $V$ : Mean flow velocity
- $\alpha$ : Velocity head coefficient

Considering  $x$  as the space coordinate, taken positive in the direction of flow one obtains by differentiating both sides of the equation (36) with respect to  $x$  and expressing  $V$  in terms of discharge  $Q$ .

$$\frac{dH}{dx} = \frac{dZ}{dx} + \frac{dh}{dx} + \alpha \frac{Q^2}{2g} \frac{d}{dx} \left( \frac{1}{A^2} \right) \quad (38)$$

Again, we know by definition:

$$\frac{dH}{dx} = -S_f \quad (39)$$

$$\text{And } \frac{dZ}{dx} = -S_o \quad (40)$$

In which

- $S_f$ : Slope of the energy grade line
- $S_o$ : Slope of the channel bottom.

The negative sign of  $S_f$  and  $S_o$  indicates that both  $H$  and  $Z$  decrease as  $x$  increases. In equation (37) an expression for the derivative of  $A^{-2}$  may be found out as follows:

$$\frac{d}{dx} \left( \frac{1}{A^2} \right) = \frac{d}{dA} \left( \frac{1}{A^2} \right) \frac{dA}{dx} \quad (41)$$

$$= \frac{d}{dA} \left( \frac{1}{A^2} \right) \frac{dA}{dh} \frac{dh}{dx} \quad (42)$$

$$= -\frac{2B}{A^3} \frac{dh}{dx} \quad (43)$$

Since  $\frac{dA}{dh} = B$

By substituting equations (39), (40) and (43) into equation (38), and rearranging the resulting equation one obtains

$$\frac{dh}{dx} = \frac{S_o - S_f}{1 - (\alpha B Q^2) / g A^3} \quad (44)$$

If the channel is not prismatic, then the cross sectional area  $A$  changes with distance, and may be expressed as:

$$\frac{dA}{dx} = \frac{\partial A}{\partial x} + \frac{\partial A}{\partial y} \frac{dh}{dx} \quad (45)$$

The above change would modify equations (40) and (43) accordingly.

We may express equation (43), which describes the variation of  $h$  with  $x$ , in terms of the Froude Number ( $Fr$ ) if we note the following:

$$\frac{\alpha B Q^2}{g A^3} = \frac{(Q/A)^2}{(g A)/(\alpha B)} = Fr^2 \quad (46)$$

Hence, equation (39) may be written as

$$\frac{dh}{dx} = \frac{S_0 - S_f}{1 - Fr^2} \quad (47)$$

Equation (47) can give a general idea about the nature of the curve if one knows the relative inclinations of the channel bed slope and friction slope ( $S_f$ ) and the Froude Number ( $Fr$ ). This may be done by observing the water flow depth ( $h$ ) with respect to **normal depth** ( $h_n$ ) and **critical depth** ( $h_c$ ) for a given discharge, the following figures show the relative changes of  $h_n$  and  $h_c$  as channel bed slope is increased gradually from horizontal. It may be observed that for a given discharge  $h_c$  does not change but  $h_n$  goes on decreasing starting from an infinite value for a flat slope.

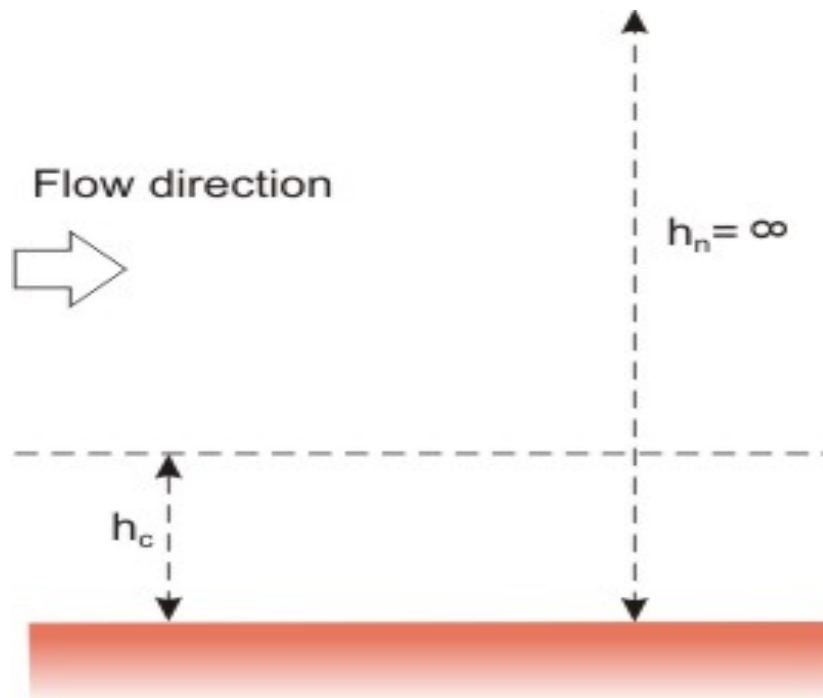


FIGURE 21. Normal depth ( $h_n$ ) and critical depth ( $h_c$ ) on horizontal sloped bed

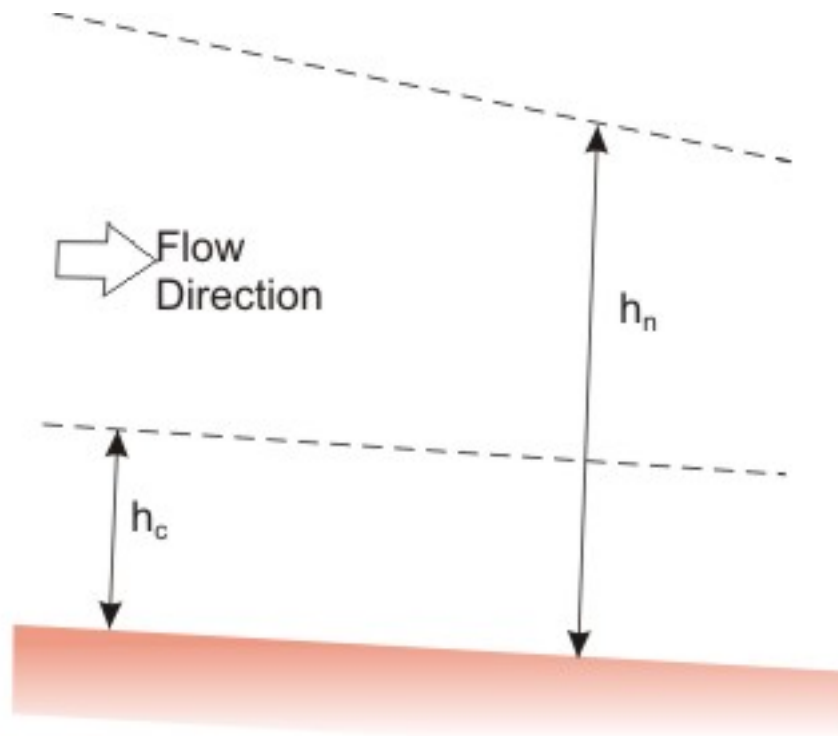


FIGURE 22. Slope increased slightly  
 $h_n > h_c$ ; mild slope

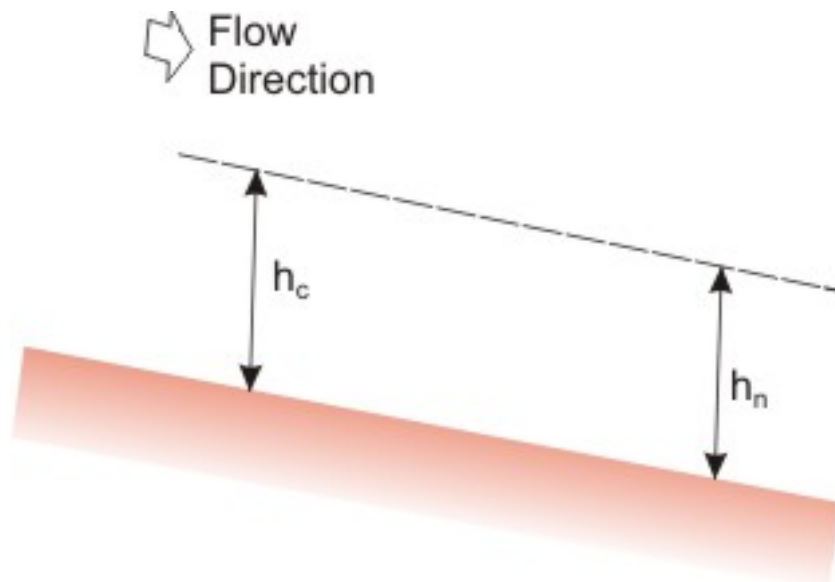


FIGURE 23. Slope increased further  
 $h_n = h_c$ ; critical slope



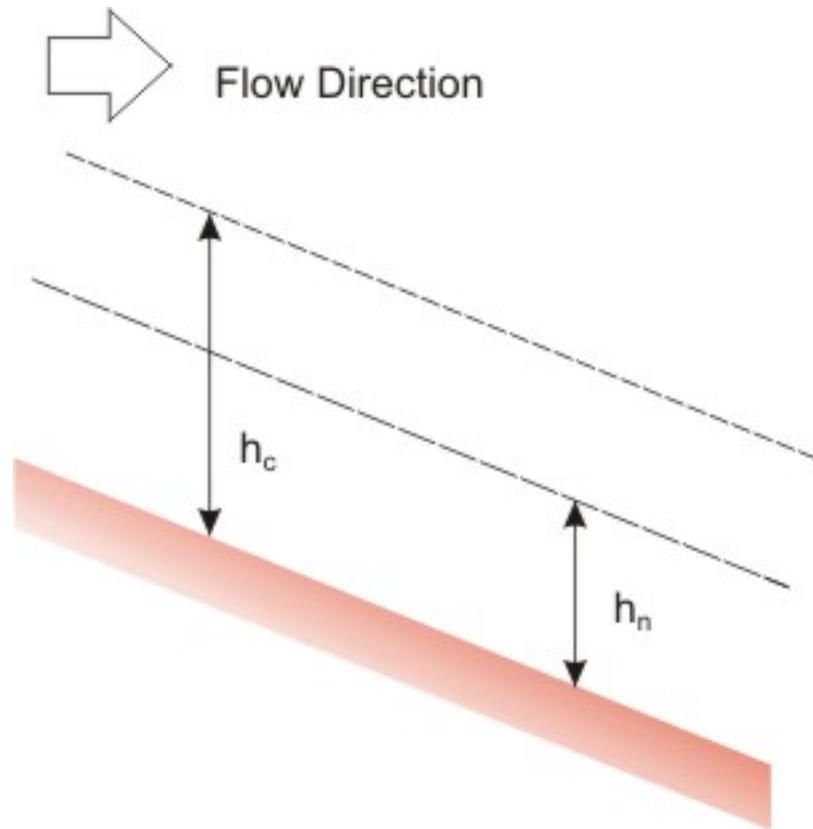


FIGURE 24. Slope increased substantially  $h_n < h_c$  ; Steep slope

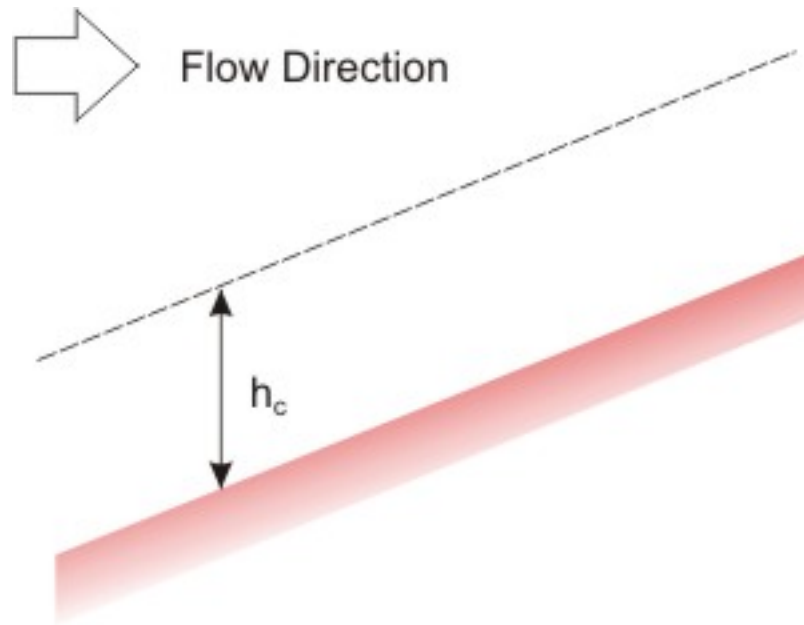


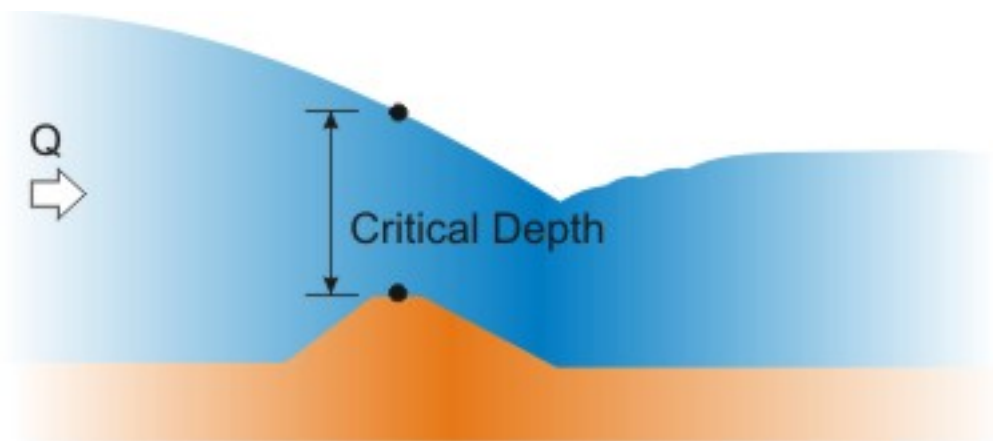
FIGURE 25 . Adverse slope .No defined  $h_n$

In water resources projects, one generally encounters slopes of channels that are either of the following:

- Mild, where  $h_n > h_c$  (Figure 22)
- Steep, where  $h_n < h_c$  (Figure 24)
- Critical, where  $h_n = h_c$  (Figure 23)
- Flat, where  $h_n = \infty$  (Figure 21)
- Adverse, where the slope is reversed (Figure 25)

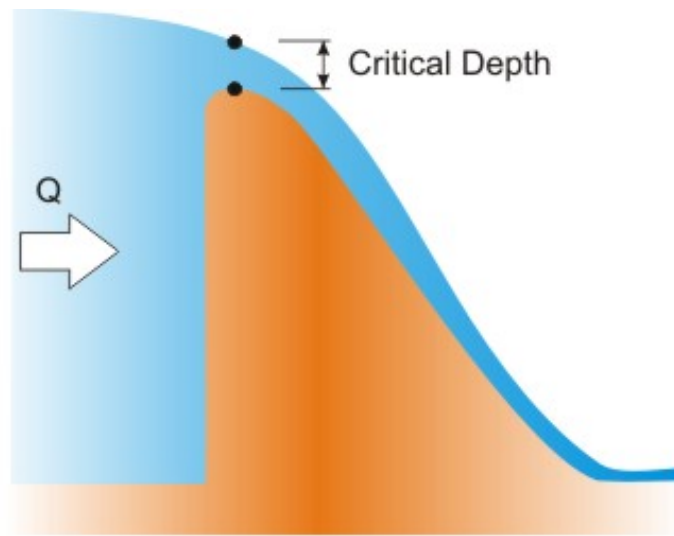
For each of these slopes, the actual water surface would vary depending upon a control that exist either at the upstream or downstream end of the channel. Some examples of controls are given below

- Weir or spillway (Figures 26 and 27)
- Gate (Figure 28)
- Free overfall (Figure 29)



Weir length :L  
Weir coefficient :  $C_d$   
 $V = Q/A$   
Q = Discharge  
A = Area

FIGURE 26. Flow over a weir



Spillway Length :  $L$   
 $V = Q/A$   
 $Q = \text{Discharge}$   
 $A = \text{Area}$

FIGURE 27. Flow over a dam spillway

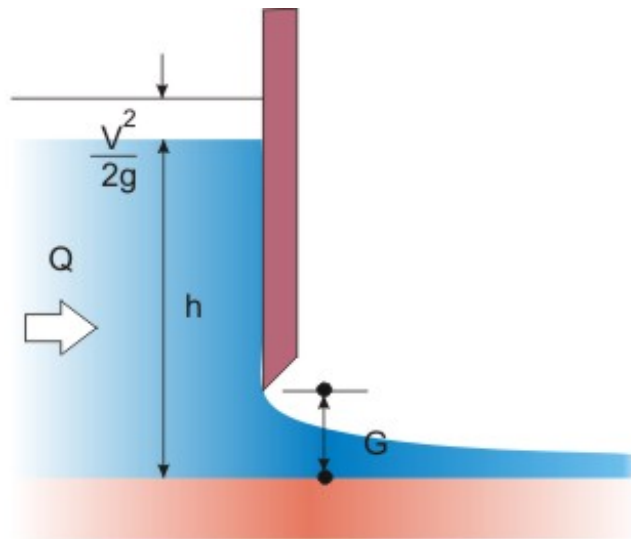
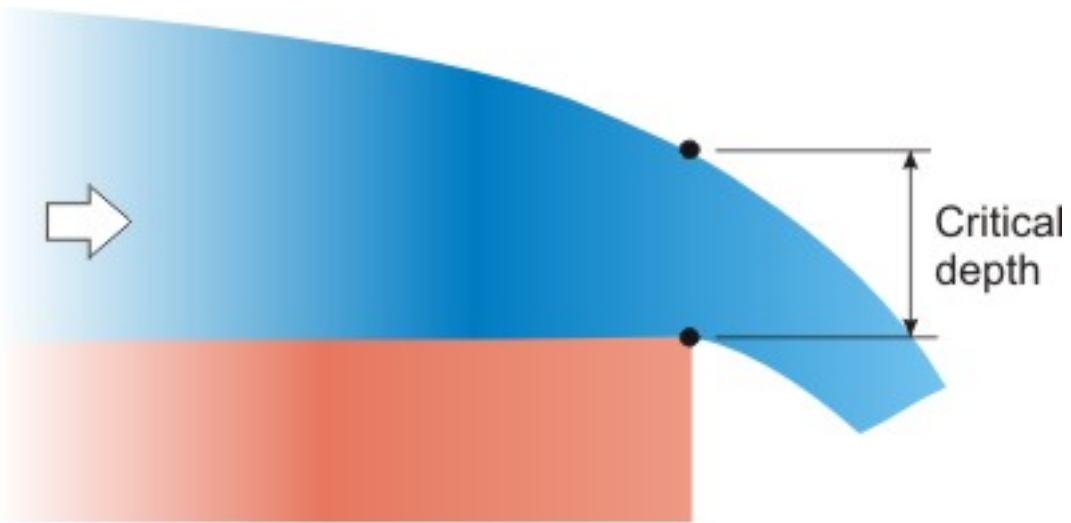
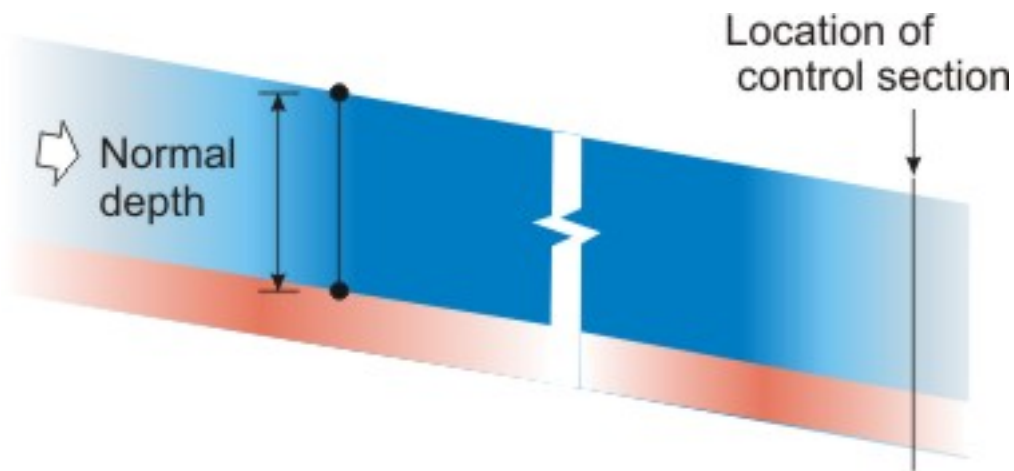


FIGURE 28. Flow below a gate .The depth of water just upstream of the gate ( $h$ ) may be determined from the formula  $Q=C.L.G.\sqrt{H}$  where  $H=h+\frac{v^2}{2g}$ . The formula is valid for small gate openings



**FIGURE 29. Free over fall**

Apart from the above a normal depth may be assumed to exist within a very long channel, for which the conditions at the far end may be neglected (Figure 30).



**FIGURE 30. Flow in a long channel**

The situation shown in Figure 30 is used often while analyzing flow in, say, at the tail end of long irrigation channels or in a long river. Examples illustrating the use

of equation (42) and a known control section in determining flow profiles where for a mildly slope channel. Similar profiles may be qualitatively sketched for other channels too.

### 2.8.9 Downstream control raising the water level above normal depth

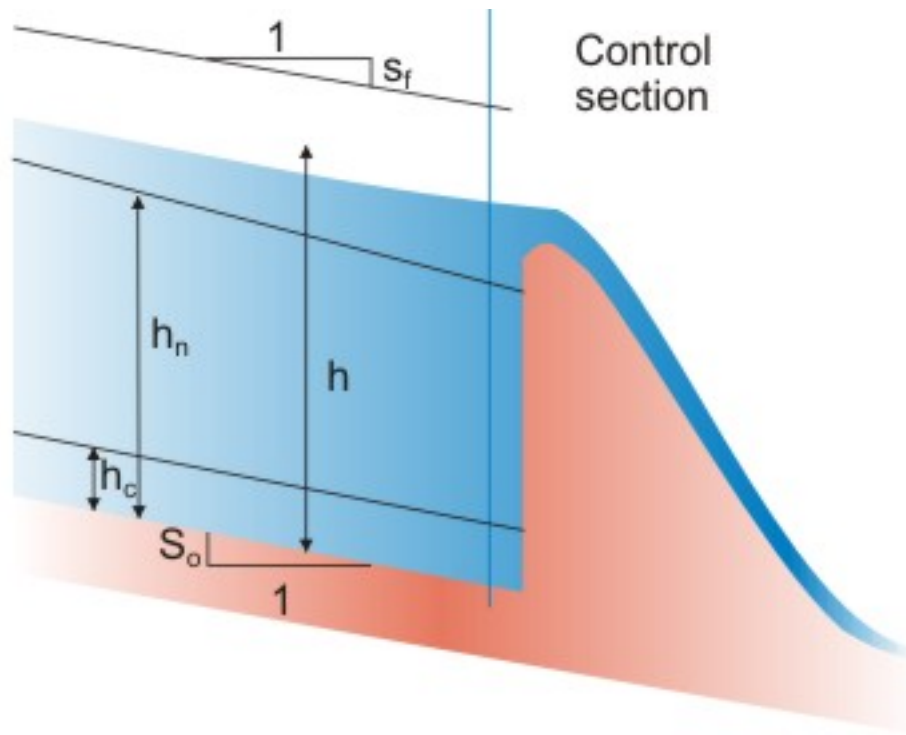


FIGURE 31. M1 type of water surface profile behind dam spillway

This situation is common for spillways of large dams. The flow profile in a mildly sloped channels where  $h > h_n > h_c$  as shown in Figure 31 is known as the  $M_1$  curve. Now, for uniform flow,  $S_f = S = S_o$  when  $h = h_n$ . Hence it is clear from Mannings formula (equation 11), that for a given discharge,  $Q$ ,

$$S_f < S_o \text{ if } h > h_n$$

Thus, in equation (47) i.e.,

$$\frac{dh}{dx} = \frac{S_o - S_f}{1 - Fr^2} \quad (47)$$

The numerator is positive  $Fr < 1$  since  $h > h_c$ . Therefore, the denominator of equation (47) is positive as well. Hence, it follows from this eqn that

$$\frac{dh}{dx} = \frac{S_0 - S_f}{1 - Fr^2} = \frac{+}{+} = +$$

This means that  $h$  increases with distance  $x$ .

Comparing with Figure 29 it may be inferred that quite some distance upstream of the spillway the flow depth nearly equals normal depth. And, since  $dh/dx$  for this profile is positive which means that the water depth goes on increasing towards the spillway, the flow depth becomes nearly horizontal. However very close to spillway the flow profile again changes which is due to the fact that the flow here is not really one-dimensional (Figure 32).

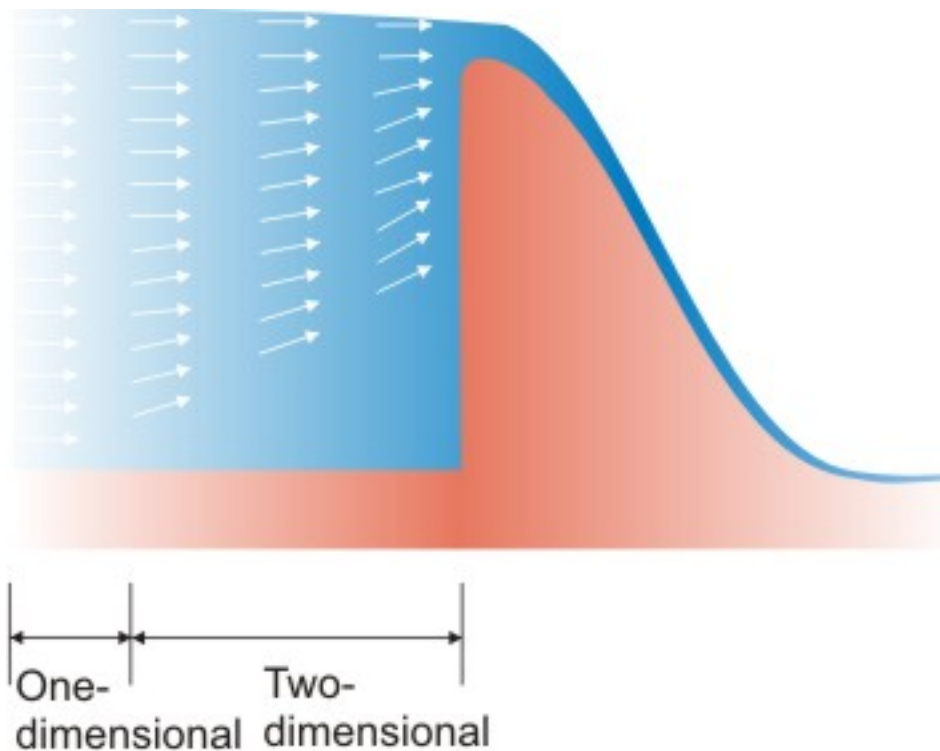


FIGURE 32. Flow situation very close to a spillway

## 2.8.10 Downstream controlled raising water level above critical depth but below normal depth

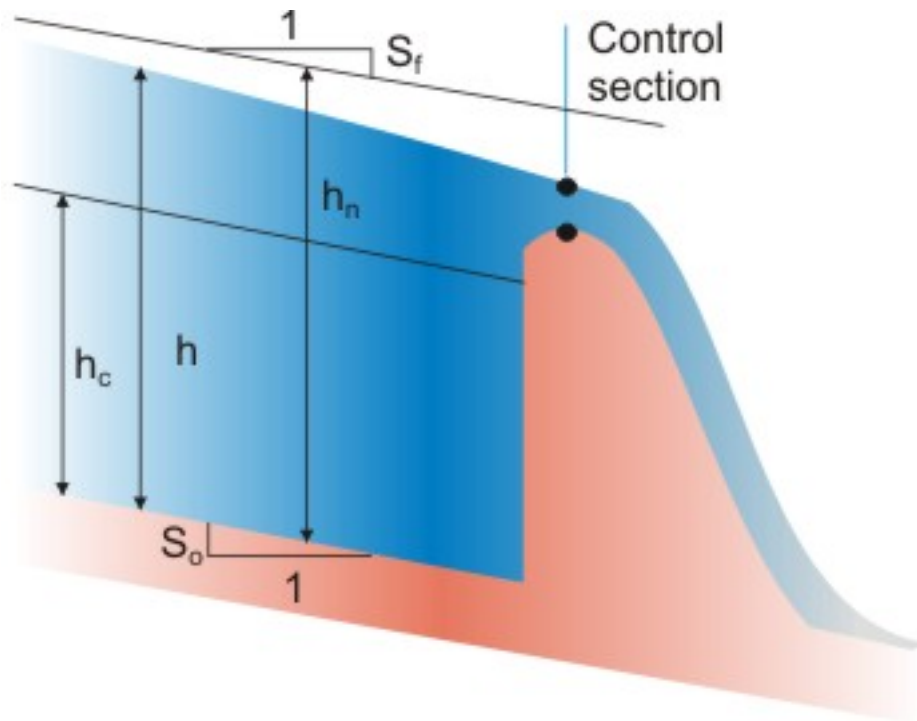


FIGURE 33. M<sub>2</sub> type of water surface profile behind a short-height spillway

The flow profile in a mildly sloping channel where  $h_n > h > h_c$ , has been shown in Figure 33 is known as the  $M_2$  curve. In this case  $S_f > S_0$  since  $h < h_n$  (from Mannings formula). Thus the numerator in equation (46) is negative. However, the denominator is positive, since  $Fr < 1$  because  $h > h_c$  hence it follows from equation 46 that

$$\frac{dh}{dx} = \frac{S_0 - S_f}{1 - Fr^2} = \frac{-}{-} = +$$

Thus  $h$  decreases as  $x$  increases for upstream of this spillway control section the flow depth would be asymptotic to normal depth  $h_n$ .

### 2.8.11 Upstream control causing water depth to be less than both normal and critical depths

This situation is shown in Figure 34 for flow taking place below a sluice gate. The reader is advised to check the trend of water surface profile using equation (47) in this case.

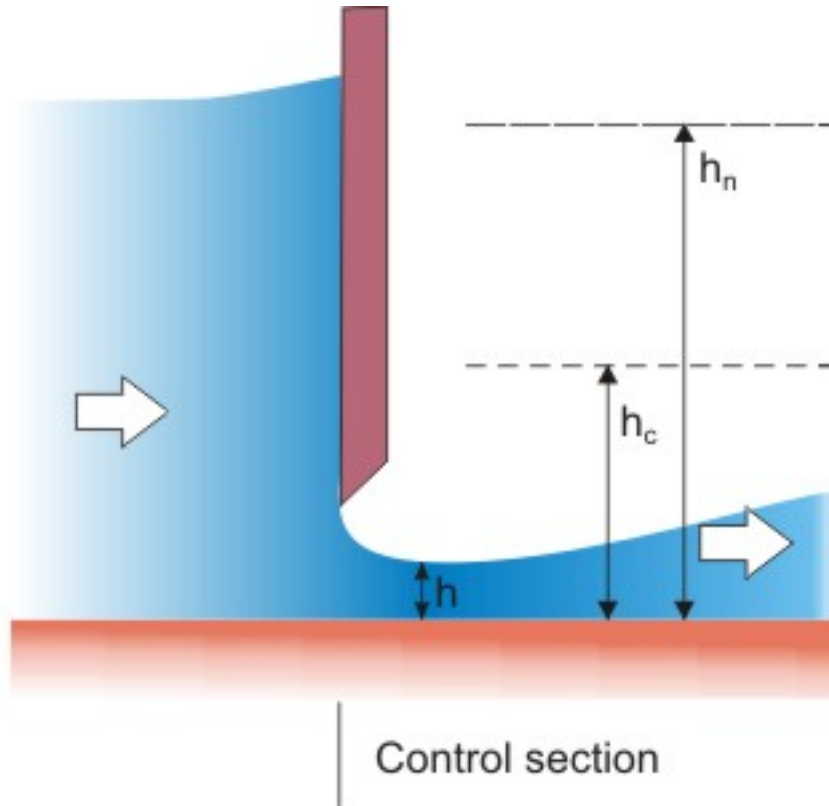


FIGURE 34. M3 type of water surface profile downstream of a gate

### 2.8.12 Important terms, definitions and procedures

This lesson has used certain terms, which are discussed to some detail here.

#### Newton's Method

This method is useful in finding a simple root of the function  $f(x) = 0$ , when the derivative of  $f(x)$  is easily obtainable. The iteration formula used in the method can be derived by the Taylor's series expansion of  $f(x)$  about  $x=x_0$ , the approximate value of the desired root. We have



$$f(x_0 + h) = f(x_0) + h f'(x_0) + \frac{h^2}{2!} f''(x_0) + \dots$$

Where h is the small correction to the root.

Now if h is relatively small, we may neglect terms containing  $h^2$  and higher powers of h. Then, we get

$$f(x_0) + h f'(x_0) = 0$$

This gives  $h = -\frac{f(x_0)}{f'(x_0)}$

Thus, we can take the improved value of the root as

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

The Newton-Raphson iteration can thus be written as

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n=0,1,2,\dots$$

The sequence  $\{x_n\}$ , if it converges, gives the root.

### Froude Number

This measures the ratio of inertia to gravity forces. In problems where there is an interface between two immiscible fluids the gravity forces are of importance. Froude number is defined by the relation

$$Fr = \frac{V}{\sqrt{gD}}$$

### Normal depth

For given values of channel roughness  $n$ , discharge  $Q$ , and the channel slope  $S$ , there is only one depth possible at which uniform flow occurs. It is known as normal depth.

### Critical depth

The depth of flow at which the specific energy  $\left(E = y + \frac{V^2}{2g}\right)$  attains a minimum value is called critical depth.

## 2.8.13 References

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