5

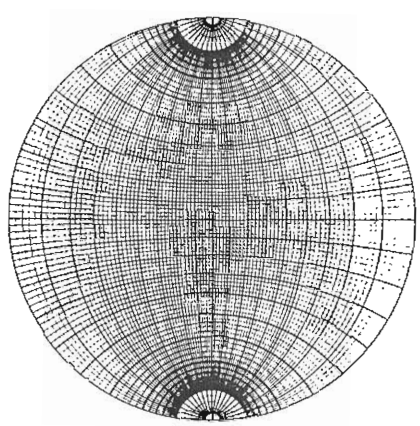
Stereographic Projection

OBJECTIVE

Use stereographic projection to determine the attitudes of lines and planes in various situations

An extremely fast and useful technique for solving many structural problems is stereographic projection. This involves the plotting of planes and lines on a circular grid or net. Two types of net are in common use. The net in Fig. 5.1a is called a *stereographic net*. It is also called a *Wulff net*, after G.V. Wulff, who adapted the net to crystallographic use. The net in Fig. 5.1b is called a *Lambert equal-area net*, or *Schmidt net*.

The two nets are constructed somewhat differently. On the equal-area net, equal areas on the reference sphere remain equal on the projection. This is not the case with the stereographic net. The situation is similar to map projections of the earth. Some projections sacrifice accuracy of area to preserve spatial relationships, while others do the opposite. In preserving area, the equal-area net does not preserve angular relationships. The construction of the equal-area net does allow the correct measurement of angles, however, and this net may reliably be used even when angular relationships are involved. Because structural geologists



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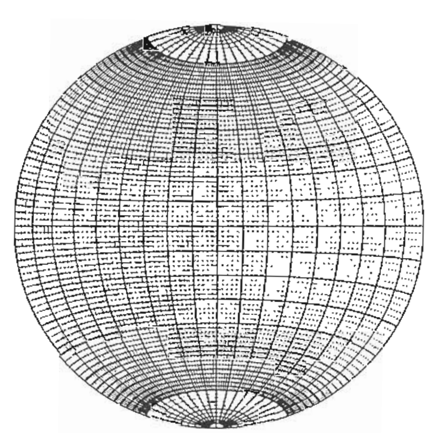


b

Fig. 5.1 Nets used for stereographic projection. (a) Stereographic net or Wulff net. (b) Lambert equal-area net or Schmidt net.

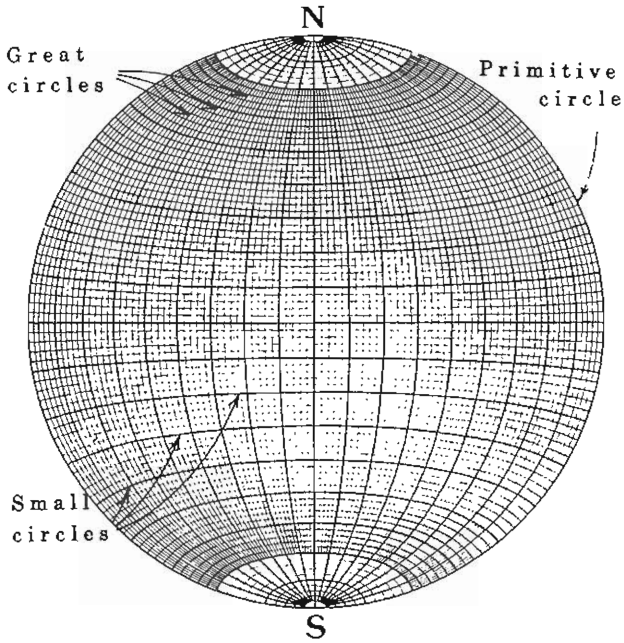


Fig. 5.2 Main elements of the equal-area projection.

are commonly concerned with the relative density of their data, a matter requiring accurate area projection, many use equal-area nets exclusively. In this book, only the equal-area net will be used.

The equal-area net is arranged rather like a globe of the earth, with north–south lines that are analogous to meridians of longitude and east–west lines that are analogous to parallels of latitude. The north–south lines are called *great circles* and the east–west lines are called *small circles*. The perimeter of the net is called the *primitive circle* (Fig. 5.2); here "primitive" has the mathematical sense of "fundamental."

Unlike crystallographers, who use the net as if it were an upper hemisphere, structural geologists use it as a lower hemisphere. To visualize how elements are projected onto the net, imagine looking down into a large bowl in which a cardboard half-circle has been snugly fitted at an angle. The exposed diameter of the half-circle is a straight line, and the curved part of the half-circle describes a curve on the bottom of the bowl. Figure 5.3a is an oblique view of a plane that strikes north–south and dips 50W. It is shown intersecting the lower hemisphere. Figure 5.3b is an equal-area projection of the same plane. Notice that the dip of the plane, 50° in this case, is measured from the perimeter of the net. The great circles on the net represent a set of planes having the same strike and all possible dips. The primitive circle represents a horizontal plane.

Stereographic projection is done by placing a piece of tracing paper over the net and rotating it about a thumbtack through the center of the net. Although north and south poles are labeled in Figs 5.2 and 5.3, geographical coordinates are actually attached to the tracing paper, not to the net. By means of rotating the tracing paper, a great circle corresponding to any plane can be drawn. Similarly, the straight line corresponding to the equator in Figs 5.2 and 5.3 will be referred to in the following examples as the "east–west line," even though it has no fixed geographic orientation.

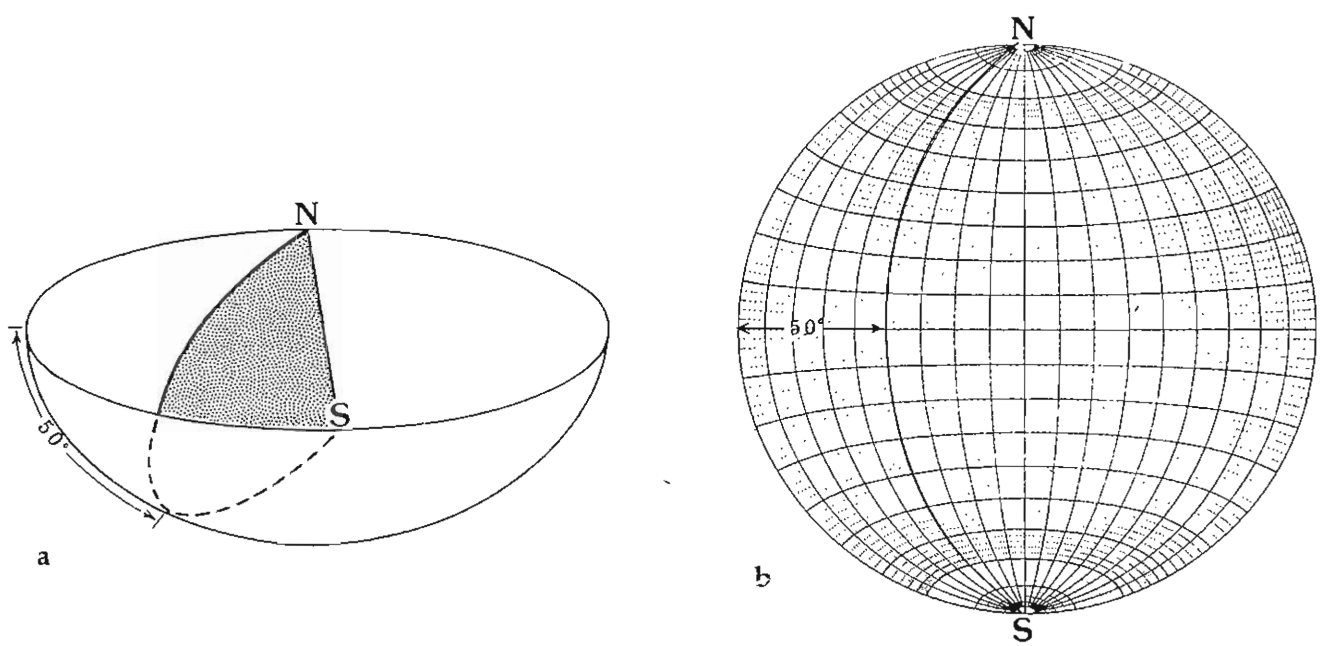
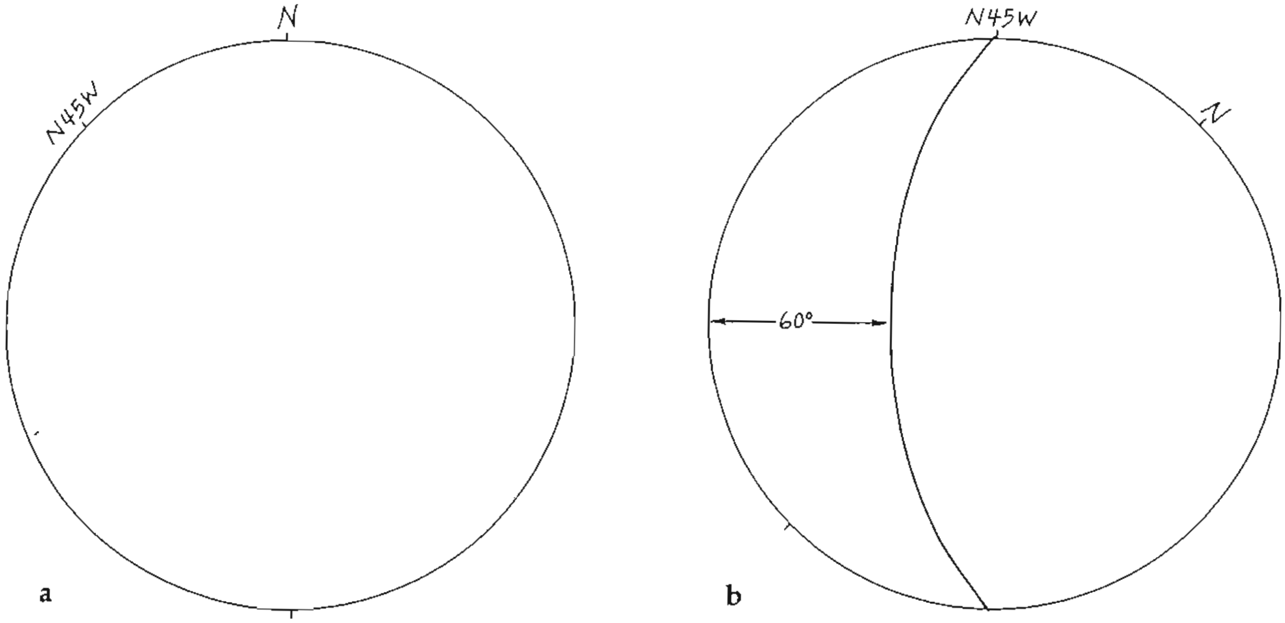


Fig. 5.3 Oblique lower-hemisphere view of the projection of a plane striking north–south and dipping 50° west. (a) Oblique view. (b) Equal-area projection.



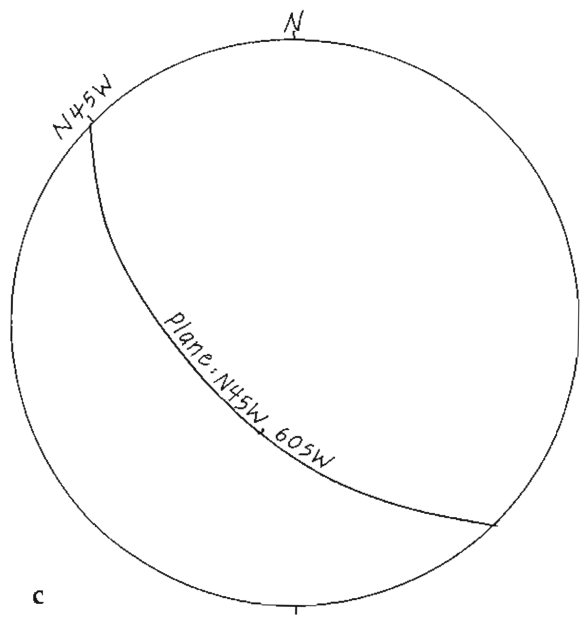


Fig. 5.4 Projection of a plane striking N45W (315) and dipping 60SW. (a) Plotting of strike. (b) Projection of plane with tracing paper rotated so that strike is at top of net (c) Tracing paper rotated back to original position.

At the back of this book is an equal-area net for use in this and succeeding chapters. Because your net will be heavily used, it is a good idea to tape it to a piece of thin cardboard to protect it and to ensure that the thumbtack hole does not get larger.

Following are several examples of stereographic projection. Work through each of them on your own net.

A plane

Suppose a plane has an attitude of N45W, 60SW. It is plotted on the equal-area net as follows.

1 Stick a thumbtack through the center of the net from the back, and place a piece of tracing paper over the net such that the tracing paper will rotate on the thumbtack. A small piece of clear tape in the center of the paper will prevent the hole from getting larger with use.

2 Trace the primitive circle on the tracing paper (this step may be eliminated later), and mark the north and south poles on the paper.

3 Find N45W on the primitive circle, mark it with a small tick mark, and label it on the tracing paper (Fig. 5.4a).

4 Rotate the tracing paper so that the N45W mark is at the north pole of the net (Fig. 5.4b).

5 Southwest is now on the left-hand side of the tracing

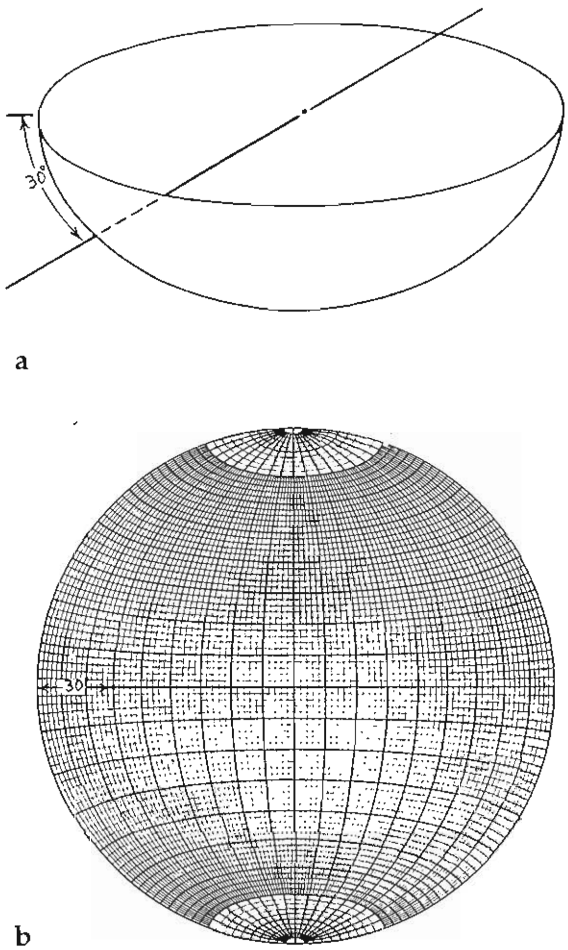


Fig. 5.5 Projection of a line that plunges 30° due west. (a) Oblique view. (b) Equal-area projection.

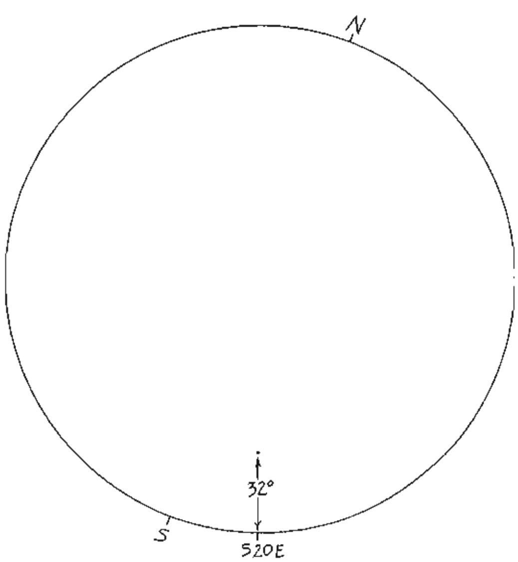


Fig. 5.6 Projection of a line that plunges 32, S20E (160).

paper, so count 60° down from the primitive circle along the east–west line of the net and put a mark on that point.

6 Without rotating the tracing paper, draw the great circle that passes through that point (Fig. 5.4b).

7 Finally, rotate the paper back to the original position (Fig. 5.4c).

A line

While a plane intersects the hemisphere as a line, a line intersects the hemisphere as a single point. Figure 5.5a is an oblique view of a line that trends due west and plunges 30°. Figure 5.5b is an equal-area net projection of the same line. Lines can be imagined to pass through the center of the sphere and then pierce through the lower hemisphere.

Consider a line of attitude 32, S20E.

1 Mark the north pole and S20E on the tracing paper.

2 Rotate the S20E point to the bottom ("south") point on the net. The top, left, or right point works just as well. It is only from one of these four points on the net that the plunge of a line may be measured.

3 Count 32° from the primitive circle inward, and mark that point (Fig. 5.6).

4 Rotate the paper to its original orientation.

Pole of a plane

It is possible to describe the orientation of a plane with a single point on the net. This is done by plotting the pole to the plane rather than the plane itself. The pole to a plane is the straight line perpendicular to the plane. As shown in Fig. 5.7, when a plane strikes north–south and dips 40° west, its pole plunges 50° due east.

Suppose a plane has an attitude of N74E, 80N. Its pole is plotted as follows.

1 Rotate N74E on the tracing paper to the north pole of the net, as if you were going to plot the plane itself. The great circle representing this plane is shown by the dashed lines in Fig. 5.8.

2 Find the point on the east–west line of the net where the great circle for this plane passes, and count 90° in a straight line across the net. This point is the pole to the plane. As shown in Fig. 5.8, the pole to this plane plunges 10, S16E.

In this way the orientation of numerous planes may be displayed on one diagram without cluttering up the tracing paper with a lot of lines.

Line of intersection of two planes

Many structural problems involve finding the orientation of a line common to two intersecting planes. Suppose we

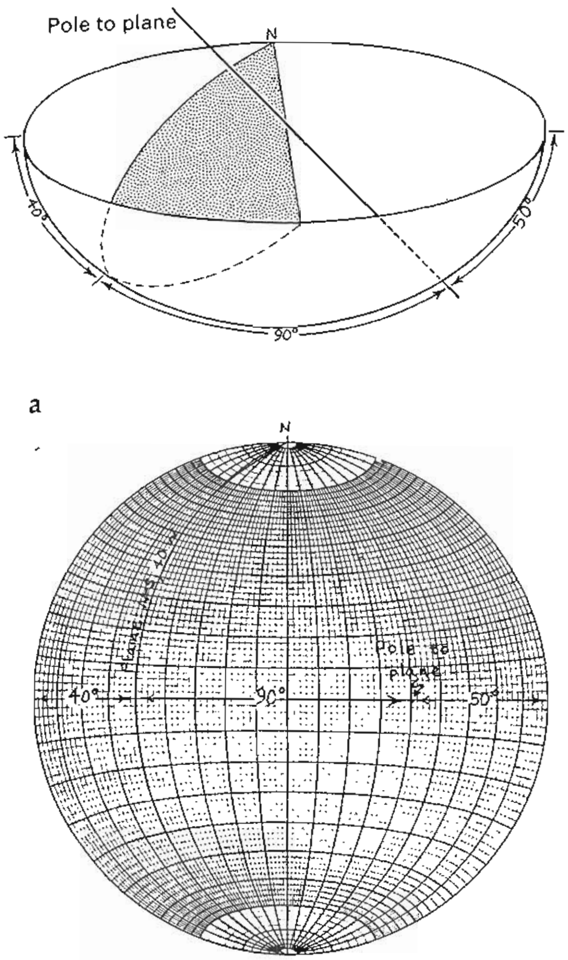


Fig. 5.7 Projection of a plane (N–S, 40W) and the pole to the plane. (a) Oblique view. (b) Equal-area projection.

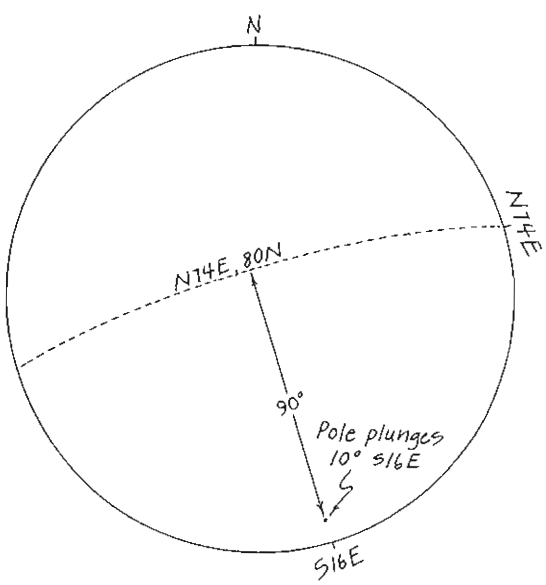


Fig. 5.8 Projection of pole to a plane.

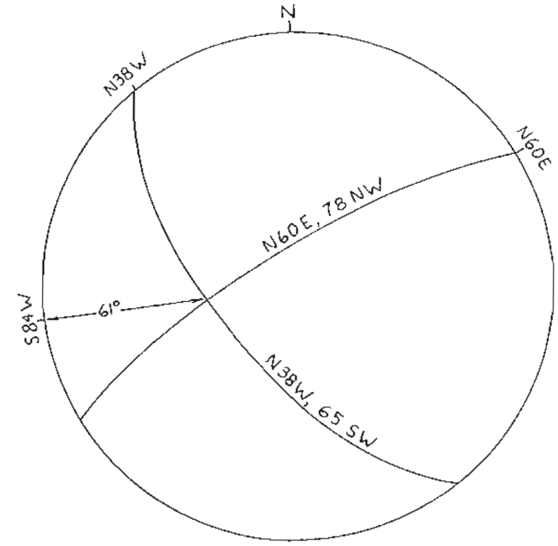


Fig. 5.9 Projection of the line of intersection of two planes. Attitude of the line is indicated.

wish to find the line of intersection of a plane N38W, 65SW with another plane N60E, 78NW.

1 Draw the great circle for each plane (Fig. 5.9).

2 Rotate the tracing paper so that the point of intersection lies on the east–west line of the net. Mark the primitive circle at the closest end of the east–west line.

3 Before rotating the tracing paper back, count the number of degrees on the east–west line from the primitive circle to the point of intersection. This is the plunge of the line of intersection.

4 Rotate the tracing paper back to its original orientation. Find the bearing of the mark made on the primitive circle in step 2. This is the trend of the line of

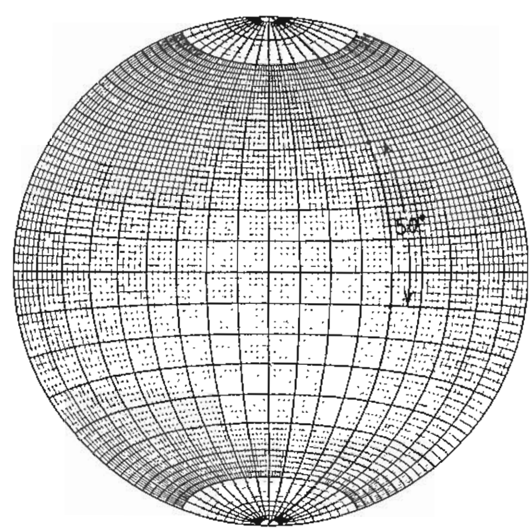


Fig. 5.10 Measuring the angle between two lines in a plane using its great circle.

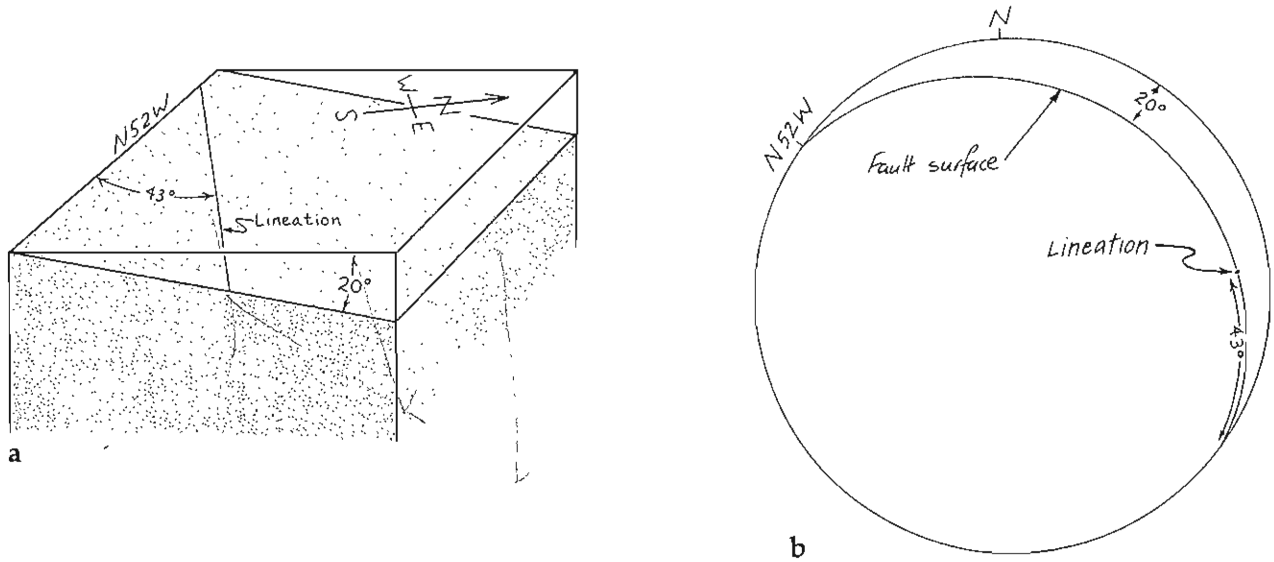


Fig. 5.11 Pitch of a line in a plane. (a) Block diagram. (b) Equal-area projection.

intersection. The line of intersection for this example plunges 61, S48W, as seen in Fig. 5.9.

Angles within planes

Angles within a plane are measured along the great circle of the plane. In Fig. 5.10, for example, each of the two points represents a line in a plane that strikes north–south and dips 50E. The angle between these two lines is 50°, measured directly along the plane's great circle.

The more common need is to plot the pitch or rake of a line within a plane. Plotting pitches may be useful when working with rocks containing lineations. The lineations must be measured in whatever outcrop plane (e.g. foliation or fault surface) they occur. Suppose, for example, that a fault surface of N52W, 20NE contains a slickenside lineation with a pitch of 43° to the east (Fig. 5.11a). Figure 5.11b shows the lineation plotted on the equal-area net.

True dip from strike and apparent dip

Two intersecting lines define a plane, so if the trend and plunge of an apparent dip are known, and if the strike of the plane is known, then these two lines can be used to determine the complete orientation of the plane.

Suppose a fault is known to strike N10E and an apparent dip is measured to have a trend of S26E and a plunge of 35. The true dip of the fault is determined as follows.

1 Draw a line representing the strike line of the plane. This will be a straight line across the center of the net intersecting the primitive circle at the strike bearing (Fig. 5.12a).

2 Put a mark on the primitive circle representing the trend of the apparent dip (Fig. 5.12a).

3 Rotate the tracing paper so that the point for apparent-dip trend lies on the east–west line of the net. Count the number of degrees of plunge toward the center of the net and mark that point. This point represents the apparent-dip line.

4 We now have two points on the primitive circle (the two ends of the strike line) and one point not on the primitive circle (the apparent-dip point), all three of which lie on the fault plane. Turn the strike line to lie on the north–south line of the net, and draw the great circle that passes through these three points.

5 Before rotating the tracing paper back, measure the true dip along the east–west line of the net. As shown in Fig. 5.12b, the true dip is 50°.

Strike and dip from two apparent dips

Even if the strike of a plane is not known, two apparent dips are sufficient to find the complete attitude. Suppose two apparent dips of a bed are 13, S18E and 19, S52W.

1 Plot points representing the two apparent-dip lines (Fig. 5.13a).

2 Rotate the tracing paper until both points lie on the same great circle. This great circle represents the plane of the bed, and the strike and dip are thus revealed. As shown in Fig. 5.13b, the attitude of the plane in this problem is N57W, 20SW.

Use stereographic projection to solve the following problems. Use a separate piece of tracing paper for each problem.

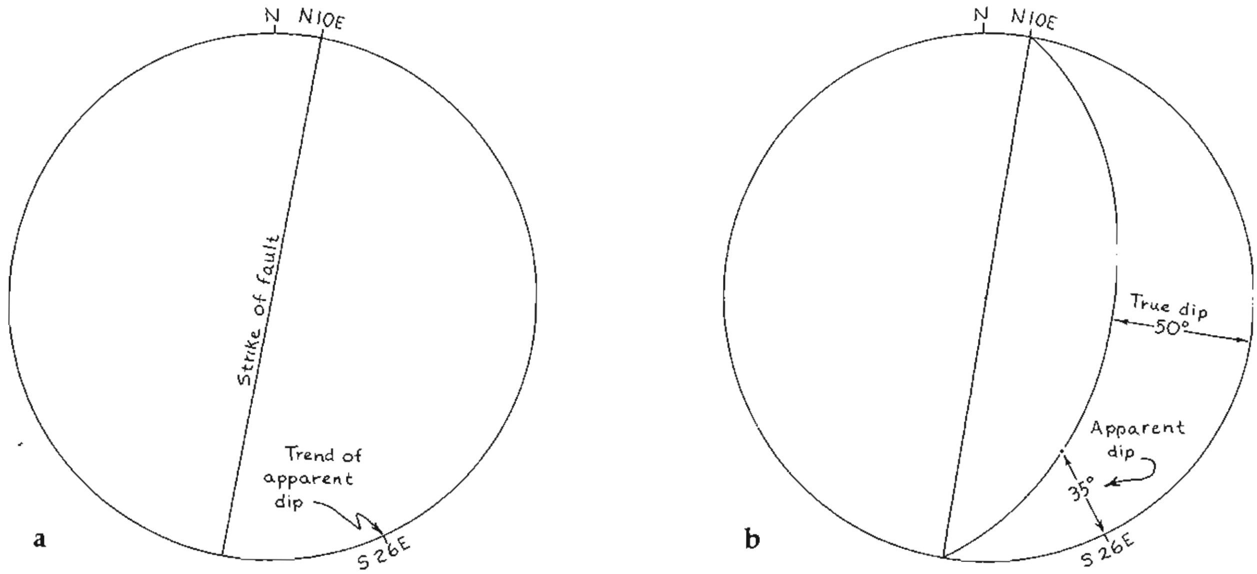


Fig. 5.12 Determining true dip from strike and apparent dip. (a) Draw strike line and trend of apparent dip. (b) Completed diagram showing direction and amount of true dip.

Problem 5.1
Along a vertical railroad cut a bed has an apparent dip of 20, N62W. The bed strikes N67E. What is the true dip?

Problem 5.2
In a mine a tabular dike has an apparent dip of 14, N90W in one tunnel and 25, S11E in another. What is the attitude of the dike?

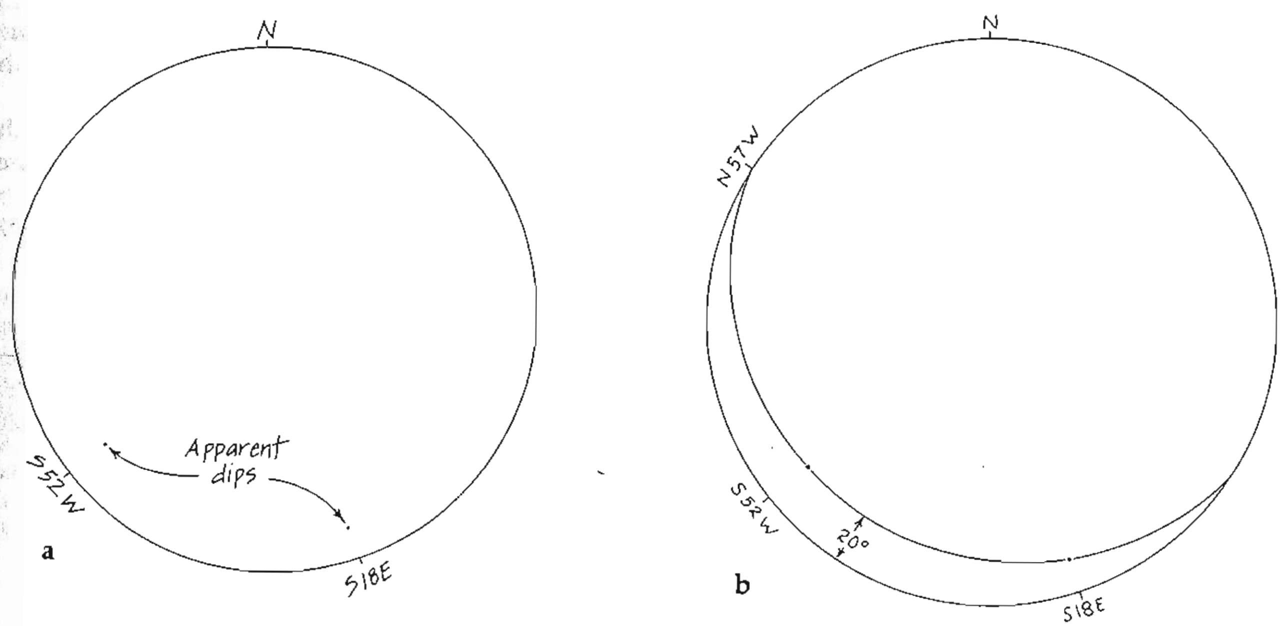


Fig. 5.13 Determining strike and dip from two apparent dips. (a) Apparent dips plotted as points on net. (b) Completed diagram showing strike and true dip.

Problem 5.3

A fault strikes due north and dips 70E. A limestone bed with an attitude of N35W, 25SW is cut by the fault. Hydrothermal alteration along the fault has resulted in an ore shoot at the intersection of the two planes.

1 What is the orientation of the ore shoot?

2 What is the pitch of the ore shoot in the plane of the fault?

3 What is the pitch of the ore shoot in the plane of the limestone bed?

Problem 5.4

A coal bed has an attitude of N68E, 40S. At what two bearings may mine adits be driven along the bottom of the bed such that the adits slope 10° and water drains out of the mine?

Problem 5.5

One limb of a fold has an attitude of N61E, 48SE and the other limb N28E, 55NW. What is the orientation of the fold axis?

Problem 5.6

The following are measurements of five lineations, taken at five different outcrops.

Locality	Attitude of outcrop surface	Pitch of lineation
1	N60W, 84NE	76°E
2	N10W, 30E	50°N
3	N40E, 70SE	63°SW
4	N23W, 30W	50°S
5	N88W, 45N	59°E

If these lineations are elements of a planar fabric within the rock, then they should all lie within the same plane. If you find this to be true, what is the attitude of this plane?

Problem 5.7

Two intersecting shear zones have the following attitudes: N80E, 75S and N60E, 52NW.

1 What is the orientation of the line of intersection?

2 What is the orientation of the plane perpendicular to the line of intersection?

continued

3 What is the obtuse angle between the shear zones within this plane?

4 What is the orientation of the plane that bisects the obtuse angle?

5 A mining adit is to be driven to the line of intersection of the two shear zones. For maximum stability the adit is to bisect the obtuse angle between the two shear zones and intersect the line of intersection perpendicularly. What should the trend and plunge of the adit be?

6 If the adit is to approach the line of intersection from the south, will the full ore carts be going uphill or downhill as they come out of the mine?

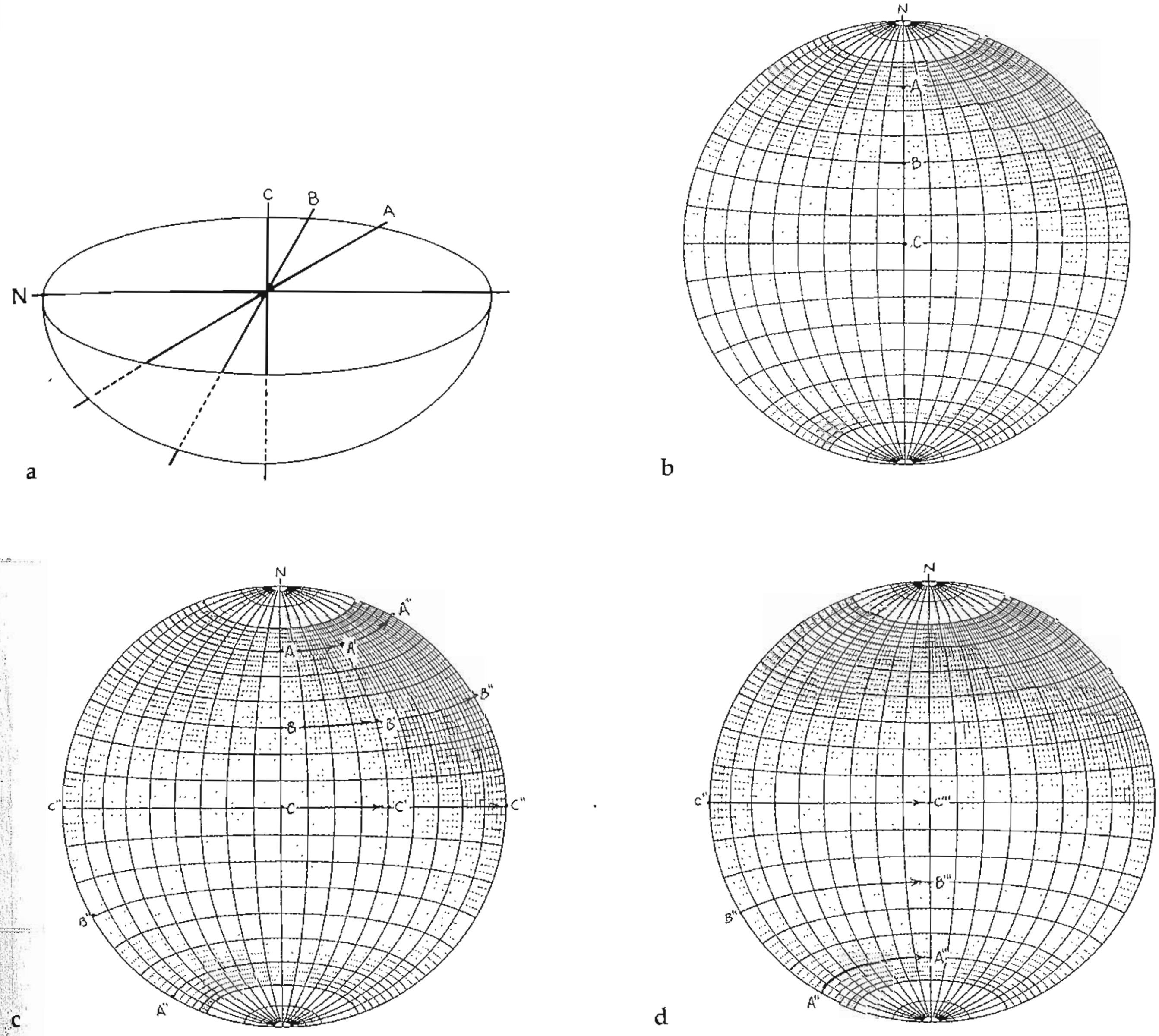
Rotation of lines

In subsequent examples it will be necessary to use stereographic projection to rotate lines and planes. Imagine three lines A, B, and C with plunges of 30°, 60°, and 90°, respectively, all lying within a vertical, north–south-striking plane. Figure 5.14a is an oblique view of these lines intersecting a lower hemisphere. Figure 5.14b shows the same three lines projected on the equal-area net.

As a plane rotates around the horizontal north–south axis of the net, the projection points of lines within the plane move along the small circles. Figure 5.14c shows points A, B, and C moving in unison 40° to points A′, B′, and C′ as the plane rotates 40° to the east. As the plane rotates 90° to a horizontal position, the projected points A″, B″, and C″ lie on the primitive circle. Notice from Fig. 5.14c that when the plane is horizontal each line is represented by two points 180° apart on the primitive circle, one in the northeast quadrant and one in the southwest quadrant.

As the plane continues to rotate, A and B leave the northern half of the hemisphere and appear in the lower half. Figure 5.14d shows the projection points of the three lines as the plane in which they lie rotates 180° from its original orientation in Figure 5.14a.

Although in this example the lines being rotated are coplanar, this need not be the case. Lines representing the poles of several variously oriented planes, for example, can be rotated together. The only requirement is that all points must move the same number of degrees along their respective small circles.



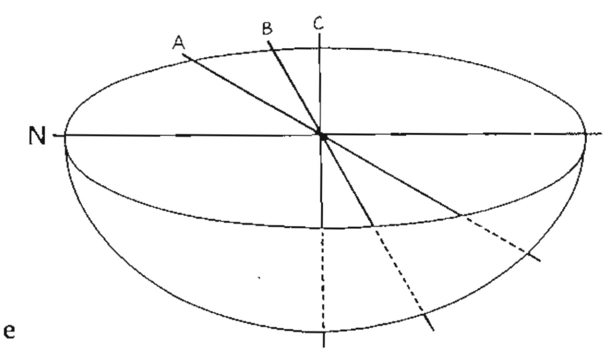


Fig. 5.14 Rotation of three coplanar lines. (a) Oblique view. (b) Projection of lines on net. (c) Projection of lines rotated 40° (A′, B′, C′) and 90° (A″, B″, C″). (d) Projection of lines rotated 180° from their original positions. (e) Oblique view of final position of lines rotated 180°. Lines A, B, C lie in a vertical north–south-striking plane.

The two-tilt problem

It is not uncommon to find rocks that have undergone more than one episode of deformation. In such situations it is sometimes useful to remove the effects of a later deformation in order to study an earlier one.

Consider the block diagram in Fig. 5.15a. An angular unconformity separates formation Y (N60W, 35NE) from formation O (N50E, 70SE). Formation O was evidently tilted and eroded prior to the deposition of formation Y, then tilted again. In order to unravel the structural history of this area we need to know the attitude of formation O at the time formation Y was being deposited. This problem is solved as follows.

1 Plot the poles of the two formations on the equal-area net (Fig. 5.15b).

2 We want to return formation Y to horizontal and measure the attitude of formation O. The pole of a horizontal bed is vertical, so if we move the Y pole point to the center of the net, formation Y will be horizontal. Rotate the tracing paper so that Y lies on the east–west line of the net.

3 Y can now be moved along the east–west line to the center of the net (Fig. 5.15c). This involves 35° of move-

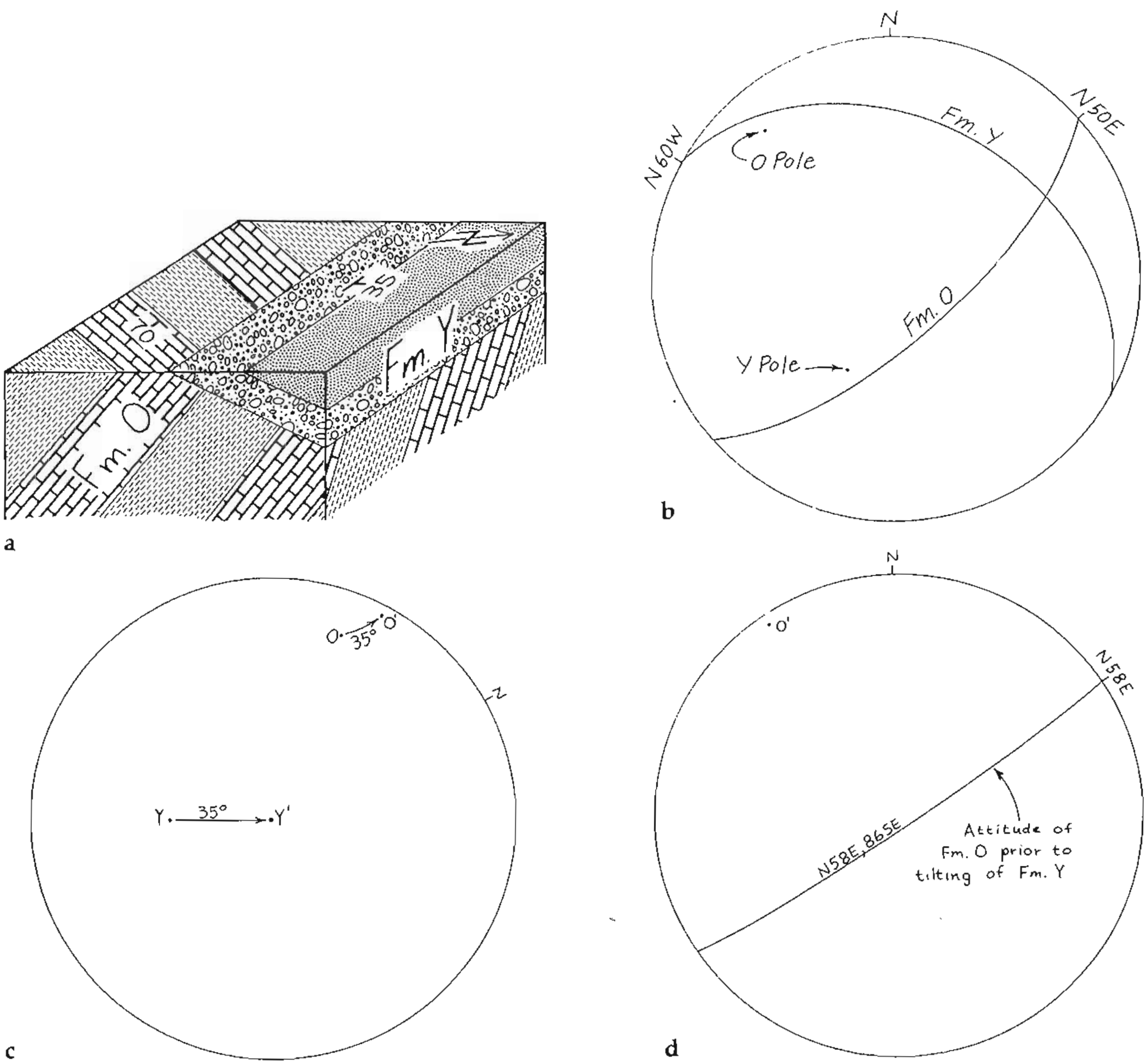


Fig. 5.15 Two-tilt problem. (a) Block diagram. (b) Plotting attitudes and poles of formations O and Y. (c) Removing dip of formation Y from that of formation O. (d) Plotting attitude of formation O before tilting of formation Y.

ment. O, therefore, must also be moved 35° along the small circle on which it lies, to O′ (Fig. 5.15c).

4 O′ is the pole of formation O prior to the last episode of tilting. As shown in Fig. 5.15d, the attitude of formation O at that time was N58E, 86SE.

Problem 5.8
The beds below an angular unconformity have an attitude of N26W, 74W, and those above N30E, 54NW. What was the attitude of the older beds while the younger were being deposited?

Problem 5.9
Iron-bearing minerals in volcanic rocks contain magnetic fields that were acquired when the magma flowed out onto the earth's surface and cooled. These magnetic fields can be measured to determine the orientation of the lines of force of the earth's magnetic field at the time of cooling. If the north-seeking paleomagnetic attitude in a basalt is 32, N67E, and the flow has been tilted to N12W, 40W, what was the attitude (trend and plunge) of the pretilt paleomagnetic orientation?

Problem 5.10
The orientation of cross-bedding in sandstones can be used to determine the direction that the current was flowing when the sand was deposited. If cross-beds indicating a current direction of S38E occur within an overturned sandstone bed whose attitude is N16W, 77W, what direction did the current flow?

Problem 5.11
In the northeastern fault block of the Bree Creek Quadrangle is a hill capped by Helm's Deep Sandstone overlying Rohan Tuff. In Problem 3.1 you determined the attitude of these two units at this locality. Using stereographic projection, determine the amount and direction of tilting that occurred in the northeastern fault block after deposition of the Rohan Tuff but before deposition of the Helm's Deep Sandstone.

Cones—the drill-hole problem

In certain problems it is necessary to project a cone onto the net. Suppose, for a simple example, that a hole has been drilled horizontally due north into a cliff. The drill core is shown in Fig. 5.16a. The rock is layered, and the angle between the core axis and the bedding plane is 30°. The angle between the pole to the bedding plane and the core axis is 60°, the complement of 30°.

Because the core rotated as it was extracted from the hole, the exact orientation of the bedding plane cannot be determined. A locus of possible orientations can be defined, however. Figure 5.16b is an oblique view of the situation, showing a cone with its axis horizontal and trending north–south. The cone represents all possible lines 30° from the axis. Lines perpendicular to the sides of the cone, representing poles to the bedding plane, pass through the center of the sphere and intersect the lower hemisphere as two half-circles. As shown in Fig. 5.16c, the equal-area plot of the possible poles to bedding is two small circles, each 60° from a pole. If a second hole is drilled, oriented differently from the first, two more small circles can be drawn. The second set of circles will have two, three, or four points in common with the first pair of small circles. A third hole results in a unique solution, establishing the pole to bedding.

Consider the following data from three drill holes.

Hole no.	Orientation of hole	Angle between axis of core and bedding	Angle between axis and pole to bedding
1	74, N80W	17°	73°
2	70, S30E	18°	72°
3	62, N67E	51°	39°

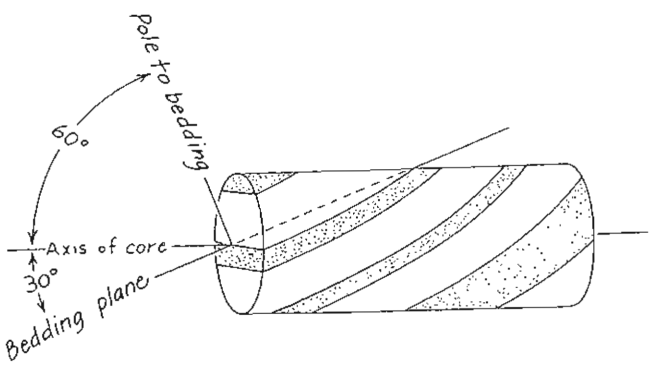
What is the attitude of the beds? The solution involves consideration of the holes in pairs. In part A we will consider holes 1 and 2 together, and in part B holes 1 and 3 together. The two parts should be done on separate pieces of tracing paper.

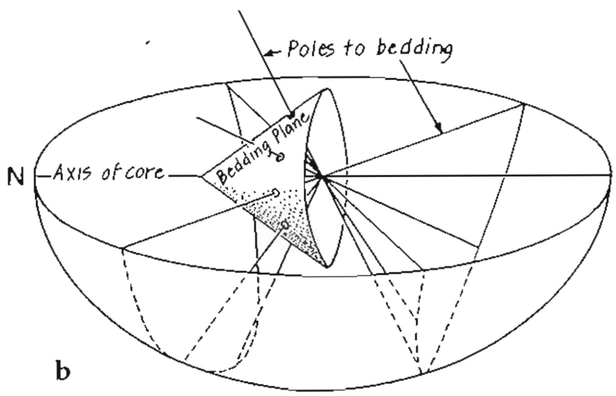
A.1 Plot each hole (Fig. 5.17a).

A.2 Rotate the tracing paper so that both points lie on the same great circle. In this position the two holes lie in a common plane that in this example dips 82SW.

A.3 Rotate the common plane to horizontal. This step moves points 1 and 2 along their respective small circles 82° to points 1′ and 2′ on the primitive circle (Fig. 5.17b).

A.4 Rotate the tracing paper so that point 1′ lies at a pole of the net. In this position hole 1 has effectively been oriented horizontal and north–south, similar to the core





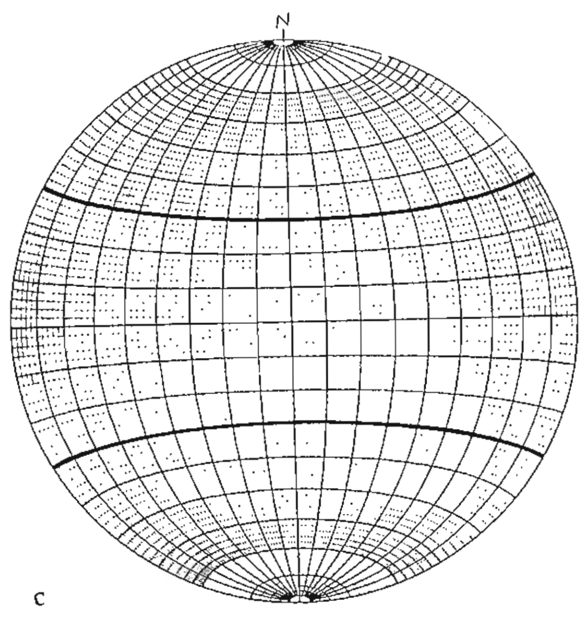


Fig. 5.16 Drill-hole problem. (a) Drill core. (b) Oblique view of projection of all possible poles to bedding of drill core. (c) Equal-area projection of all possible poles to bedding of drill core.

shown in Fig. 5.16a. The angle between the axis of hole 1 and the pole to bedding is 73°, so two small circles, each 73° from a pole, describe all of the possible pole orientations. Figure 5.17c shows these two small circles.

A.5 Now rotate the tracing paper so that point 2′ is at a pole of the net. The angle between the axis of hole 2 and the pole to bedding is 72°, so two small circles, each 72° from a pole, describe the possible orientations of the pole to bedding. As shown in Fig. 5.17d, the two sets of small circles cross at points A and B. Depending on the orientations of the two cores, there may be from one to four points of intersection.

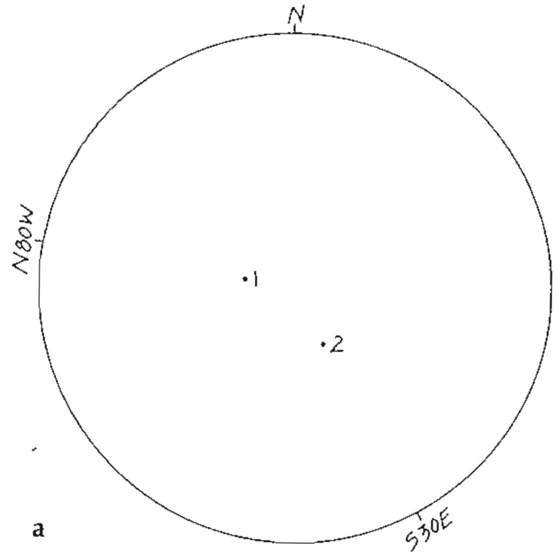
A.6 The points of intersection have been determined with the cores in a horizontal plane. The cores must be returned to their proper orientation for the orientations of the points of intersection to be determined. To do this, rotate the tracing paper so that points 1 and 2 again lie on a common great circle, as in step A.2. In this position points 1′ and 2′ are imagined to move 82° back to 1 and 2, and points A and B move 82° along small circles in the same direction to points A′ and B′, as shown in Fig. 5.17e. Notice that point A, after moving 72°, encounters the primitive circle. In order to complete its 82° excursion it reappears 180° around the primitive circle and travels 10° more. Points A′ and B′ thus located are both possible poles to bedding in this problem.

B.1 Holes 1 and 3 will now be considered together on a separate piece of tracing paper. Figure 5.18a shows cores 1 and 3 plotted, rotated to a common great circle, and moved to the primitive circle as 1′ and 3′. Notice that 1′ here is in a different location than 1′ in step A.3.

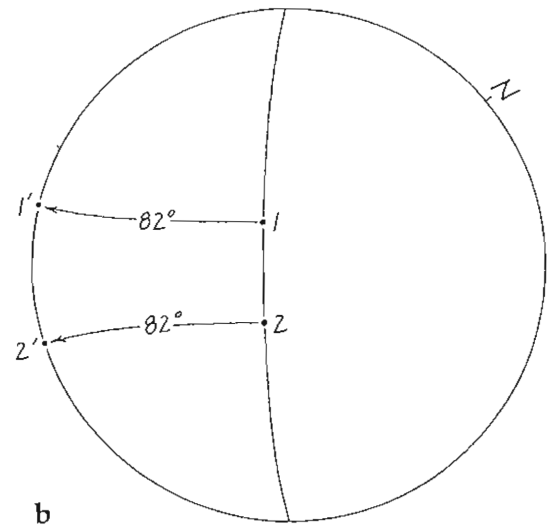
B.2 Small circles 73° and 39° from points 1′ and 3′, respectively, are drawn, as shown in Figure 5.18b. The intersection of the two pairs of small circles are points C and D.

B.3 As points 1′ and 3′ travel 83° back to their original positions as points 1 and 3, points C and D travel on small circles 83° also (Fig. 5.18c).

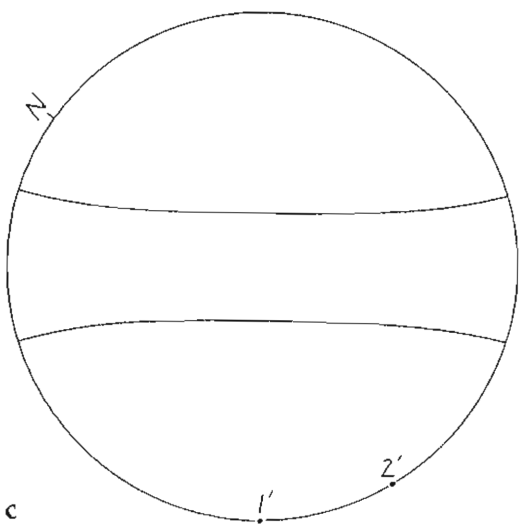
One of the points C′ or D′ should be in the same position on the net as A′ or B′. In this example points B′ and D′ are the same point with an orientation of 26, N41E. This is the pole to the bedding plane, thus establishing a bedding attitude of N50W, 64SW. A third solution could be done, with points 2 and 3 considered together, for further confirmation. If no two points are coincident, then either a mistake has been made or the attitude is not consistent from one hole to the next.



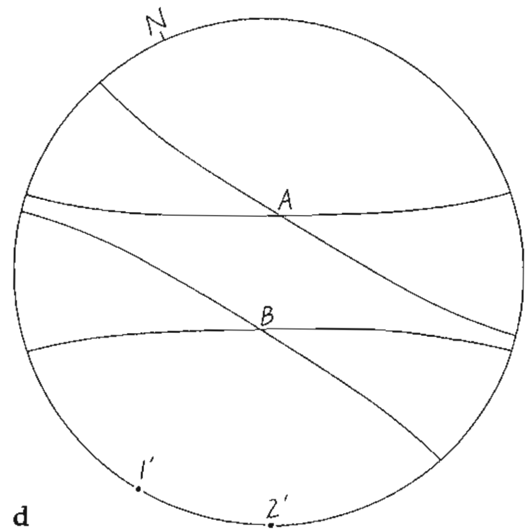
a



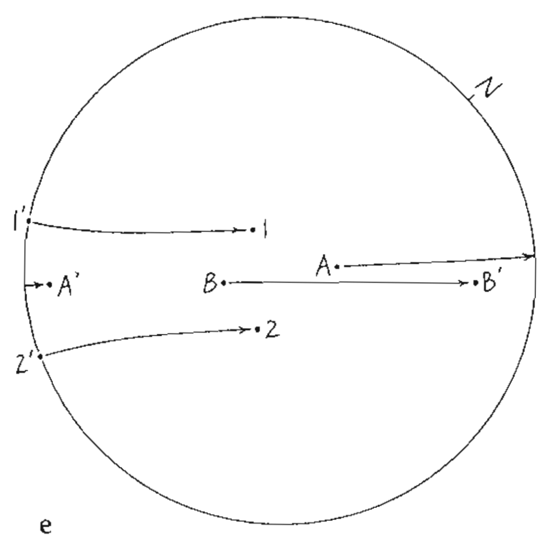
b



c



d



e

Fig. 5.17 Part A of drill-hole solution. (a) Plot orientation of holes 1 and 2. (b) Locate common plane and rotate to horizontal. (c) Plot cone of possible bedding-plane poles relative to hole 1. (d) Do the same for hole 2. (e) Determine orientation of common poles A and B. Another hole must be analyzed to choose the correct pole.

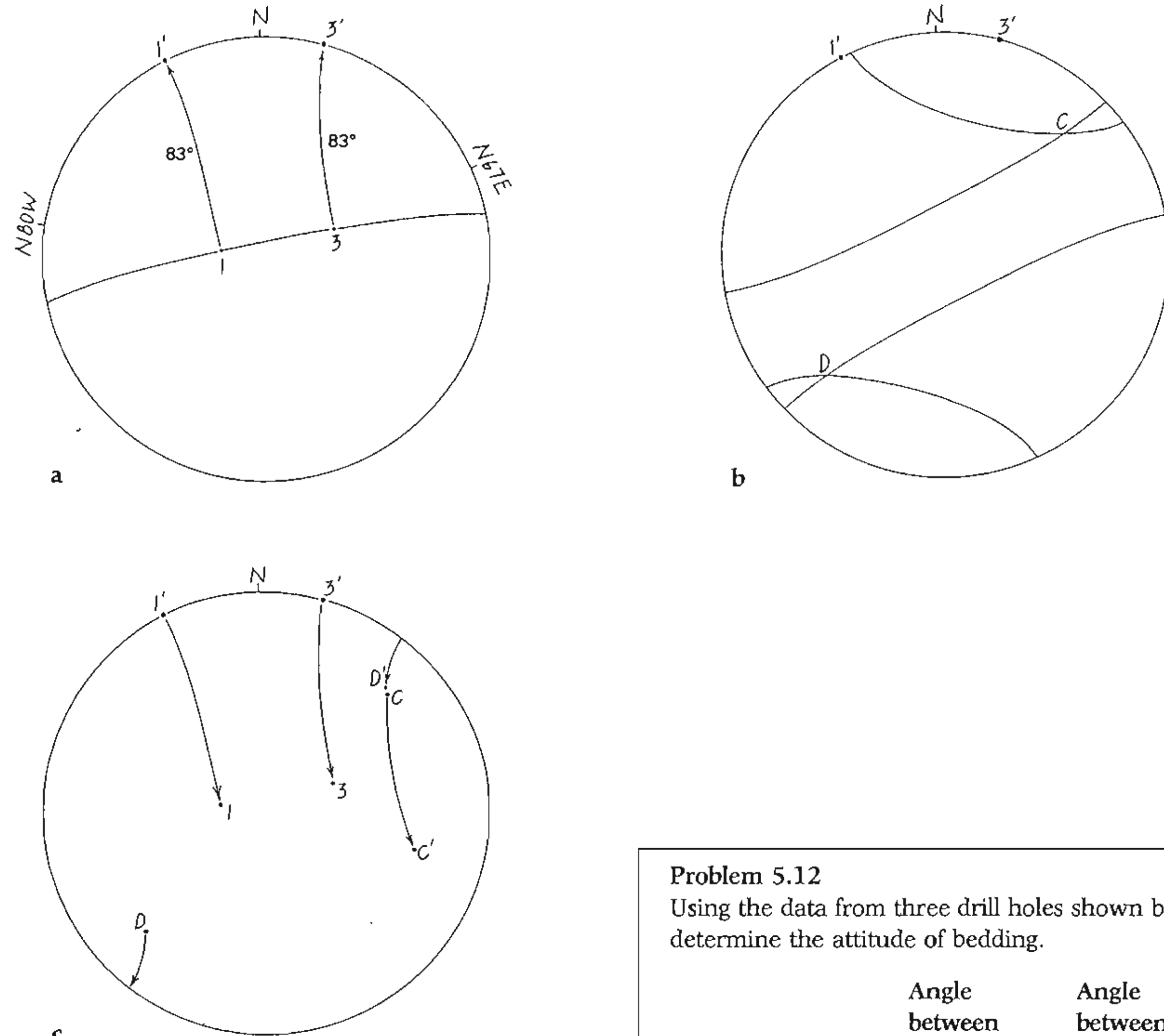


Fig. 5.18 Part B of drill-hole solution. (a) Plot orientation of holes 1 and 3 and rotate common plane to horizontal. (b) Plot cones of possible bedding-plane poles relative to holes 1 and 3. (c) Determine orientation of common poles C and D; pole D′ matches pole B′ and is the solution.

Problem 5.12
Using the data from three drill holes shown below, determine the attitude of bedding.

Hole no.	Orientation of hole	Angle between axis of core and bedding	Angle between axis and pole to bedding
1	70, N20W	40°	50°
2	76, N80E	65°	25°
3	68, S30W	54°	36°