STEREOGRAPHIC PROJECTION

Introduction

Stereographic projection is a powerful method for solving geometric problems in structural geology. Unlike structure contouring and other map-based techniques, it preserves only the orientation of lines and planes with no ability to preserve position relationships. However, it is extremely useful, as orientation problems are very common in structural geology. Stereographic projection has been in use since the second century B.C. and is a popular method used by crystallographers as a tool for representing variations in crystal form. However, there is one important difference. Whereas crystallographers use an upper hemisphere projection, structural geologists always use the lower hemisphere. There is a good reason for this: the lower hemisphere represents the region beneath the Earth's surface where the rocks have not yet been eroded away. However, if you have already met the stereographic projection in mineralogy, the lower hemisphere may take a little getting used to.

Stereogram basics

There are two parts to any stereographic projection. The projection itself, or **sterogram,** is usually drawn on tracing paper, and represents a bowl-shaped surface embedded in the Earth. The **stereographic net** or **stereonet** is the 3-D equivalent of a protractor. It is used to measure angles on the projection. To measure angles, we need to rotate the net relative to the tracing paper. For practical reasons we usually turn the tracing paper and keep the net fixed, but it is important to remember that in reality, the projection has a fixed orientation and the net should be rotated to make measurements.

To construct a stereogram, take a sheet of tracing paper and draw a circle, with the same radius as an available stereonet. This circle is known as the **primitive**. Mark the centre with a cross, and mark a north arrow on the primitive at the top of the page. Mark E, S and W (or 090, 180 and 270) points at 90° intervals around the primitive. It is sometimes helpful to reinforce the centre with adhesive tape on the back of the tracing paper.
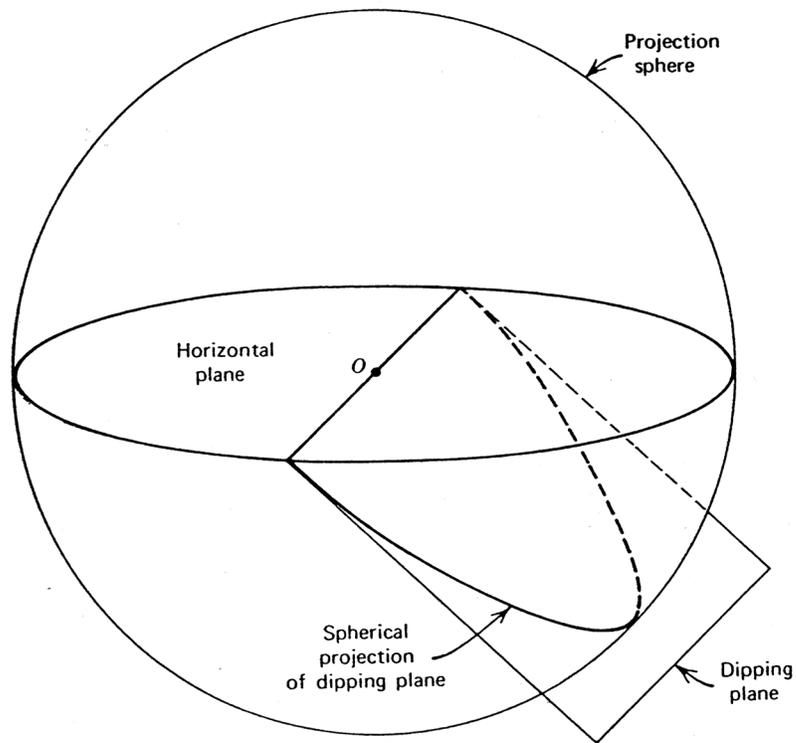
The stereonet may be reinforced with card to extend its life. It is convenient to place an old fashioned thumb tack through the centre of the net. Protect yourself from the thumb tack by keeping it embedded in an eraser while not in use. There are several varieties of stereonet available. We will start with a Wulff net, which is used for the construction of the true, or **equal-angle** stereographic projection. In later labs we will use a Schmidt net, which constructs an **equal area** projection.
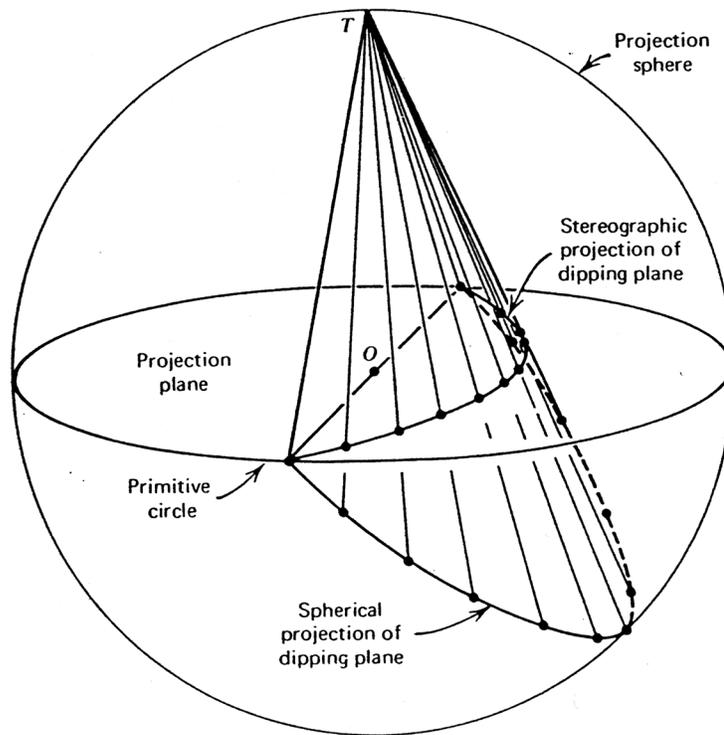
Principle of stereographic projection

For stereographic projection, a line or a plane is imagined to be surrounded by a **projection sphere** (Fig. 1a). A plane intersects the sphere in trace that is a **great circle** that bisects the sphere precisely. A line intersects the sphere in a point. To image features on a sheet of paper, these traces and points are projected from a point at the summit or **zenith** of the sphere onto the equatorial plane. This is clearer in a diagram (Fig. 1b), which shows the method for stereographic projection of a dipping plane. A family of planes dipping at various increments is shown in Fig. 2a. Planes project as curves that are actually perfectly circular arcs called **cyclographic traces** or just **great circles.** Lines project as points or **poles.**

As a general principle, planes that dip at low angles are represented by great circles having significant curvature and lying closer to the primitive, whereas steeply dipping planes are characterized by great circles having minimal curvature and passing close to the centre of the plot. All vertical planes will project as straight lines passing through the centre of the stereogram.

Sometimes we represent a plane by its **pole**. The pole to a plane is the plot of a line perpendicular to the plane.

(a)

(b)

Fig. 1. Stereographic projection of a dipping plane.

North

West 0° 10°   30°   50°   70° 90° 70°   50°   30°   10°   0° East

South

(a)

Small
circles of
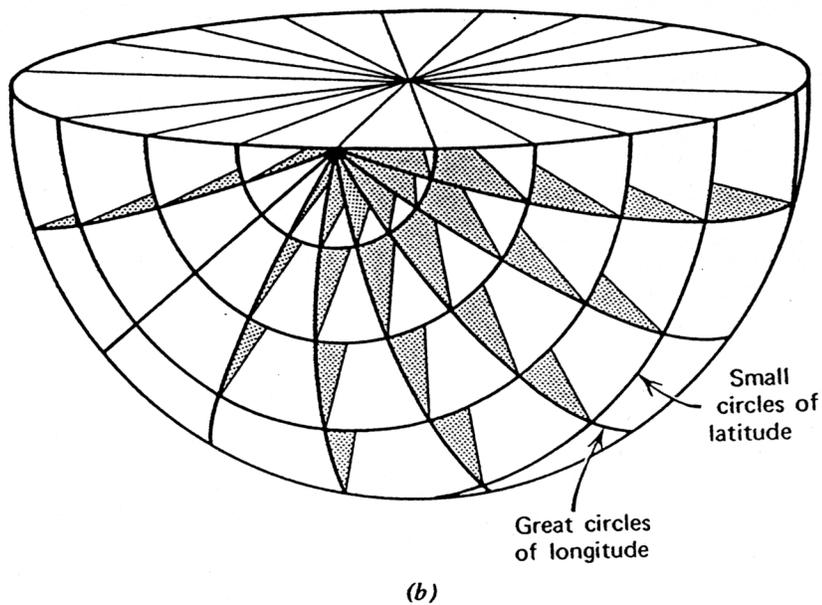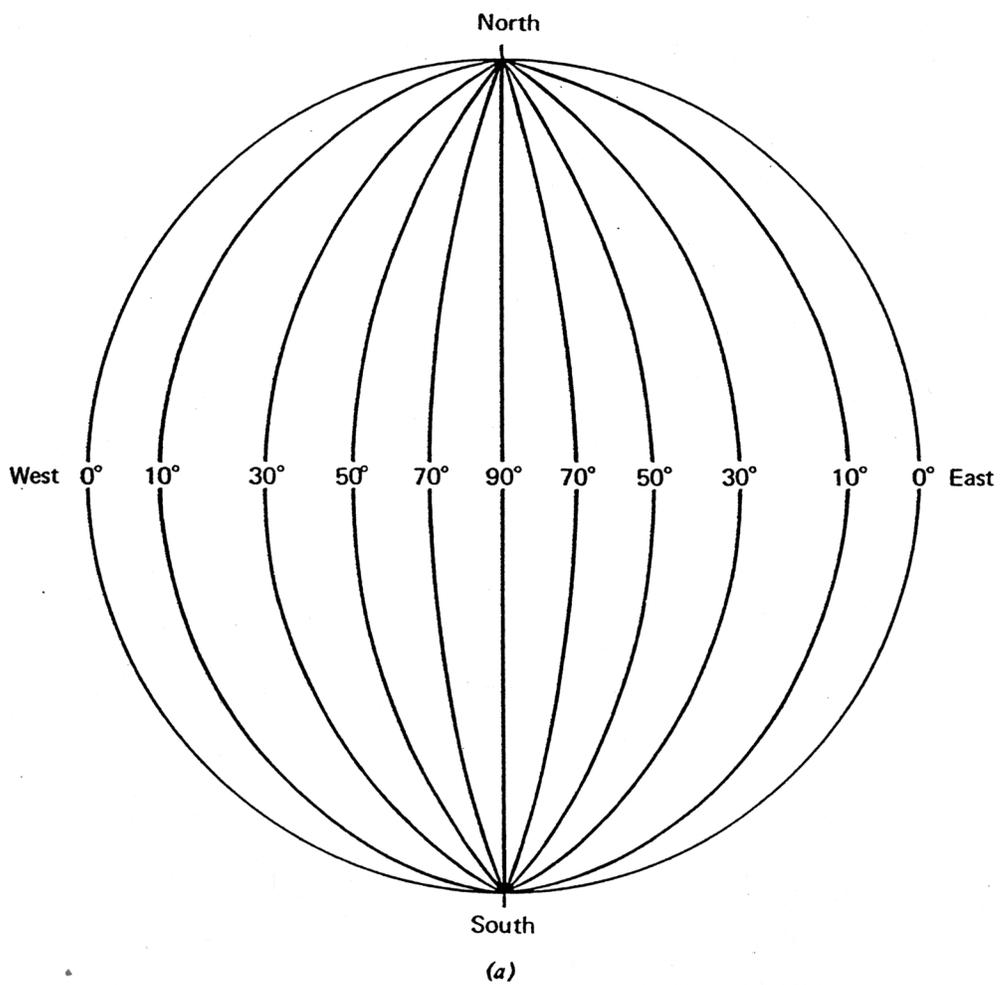latitude

Great circles
of longitude

(b)

Fig. 2. a) Stereographic projections of variously oriented planes, and b) construction of small circles.

For a horizontal plane the pole is in the centre of the net. Gently dipping planes have poles near the centre; steeply dipping planes have poles near the edge. The pole is always in the opposite quadrant to the great circle. Poles are used when plotting numerous great circles would make the plot cluttered and confusing.

Features of the net

The stereographic net assists in the construction of great circles and points. It contains a family of great circles intersecting at the top and bottom points of the net. Every fifth cyclographic trace is bolder so that ten degree increments can be easily counted. By rotating the net, a great circle can be manoeuvred into any desired strike and dip orientation. The net also contains **small circles** (Fig. 2b) that are helpful in solving rotation problems, and act as a scale of pitch angles along each great circle.

One great circle on the net corresponds to a vertical plane and is straight. It runs from top to bottom of the net. One trace in the family of small circles is also straight, from left to right on the net; it is actually a great circle too. These two intersecting lines form four **straight radii** which are crucial for counting angles of dip and plunge.

When the plot is located so that its north arrow coincides with the top of the net, it is said to be in **reference position.**

Ploting operations

The following plotting operations are described in detail in Chapter 5, pages 38-45, in the text by Rowland et al. (2007). The procedures here are slightly simplified because we assume that planes are specified by right hand rule, our preferred convention.

Plotting the Orientation of a Line

The simplest geometric information one can display on a stereonet is the orientation of a vertical line. It projects as a point in the middle of the stereonet. The next easiest information to portray is the orientation of a line that has a trend due north or due south. With the overlay in the reference position, count the amount of plunge from either the north or south index mark (as appropriate) on the primitive towards the center of the net along the N-S line, and place a dot on the tracing paper at this position. Remember, a line is represented by a point on the stereogram.

The procedure for a line of general orientation is as follows

      a) Visualize the problem first, using a pencil. Imagine that the pencil originates at the centre of the net, and extends downward to intersect a hemisphere below the net. In which quadrant will it intersect the hemisphere, and how far from the primitive will it be?

      b) With the overlay in the reference position, make a mark on the primitive that corresponds to the trend of the line.

      c) Rotate the overlay until the mark is aligned with a straight radius of the net; count the angle of plunge inwards from the primitive along the straight radius, and make a small cross.

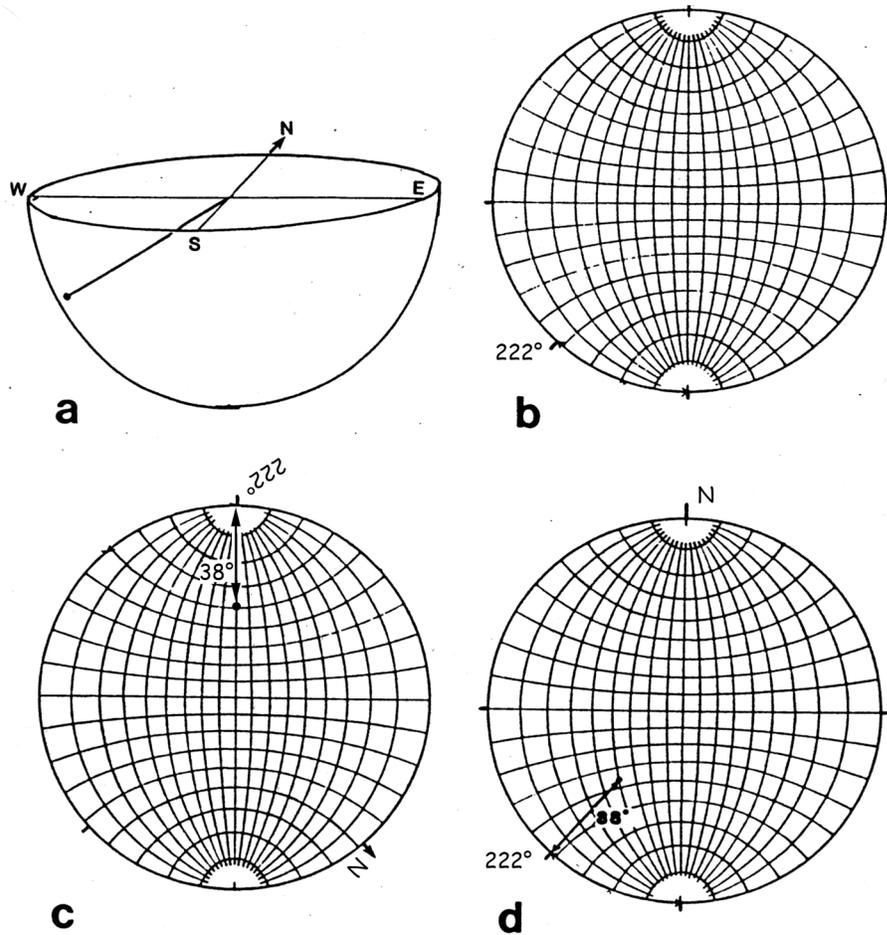   d) Return the overlay to the reference position and check that the cross is in the expected general position.

Fig. 3. Procedure for plotting the stereographic projection of a plunging line with plunge 38, trend 222.

Plotting a plane and its pole

Horizontal and vertical planes are straightforward to plot. A horizontal plane is represented by the primitive. N-S and E-W striking vertical planes are represented by the straight lines on the stereonet connecting the N-S poles and E-W positions, respectively. Other vertical planes are straight diameters oriented parallel to the strike.

To plot an inclined plane follow these steps:

a) Visualize the problem using your hand or a piece of paper. Imagine that this plane object passes through the centre of the plot and intersects a hemisphere below the net. In which quadrants will the curved line of intersection lie, and how close to the primitive will that curve be? Where will its pole be? The pole should be on the opposite side of the net. If the great circle is near the primitive then the pole will be near the centre, and vice versa.

b) With the overlay in the reference position make a mark on the primitive that corresponds to the strike direction of the plane. As a matter of sound practice, you should always make this mark on the strike direction according to the right hand rule.

c) Rotate the stereonet until the mark is aligned with the top point on the net. **From the right hand side of the net**, count degrees inward along the straight radius, until you reach the amount of dip. Trace the great circle which passes through this point; optionally, to plot the pole to the plane, count the same number of degrees **outward from the centre along the straight radius towards the left side of the net** and mark a small 'x'.

5

e) Return the overlay to the reference position and check that the great circle corresponds with the anticipated orientation.
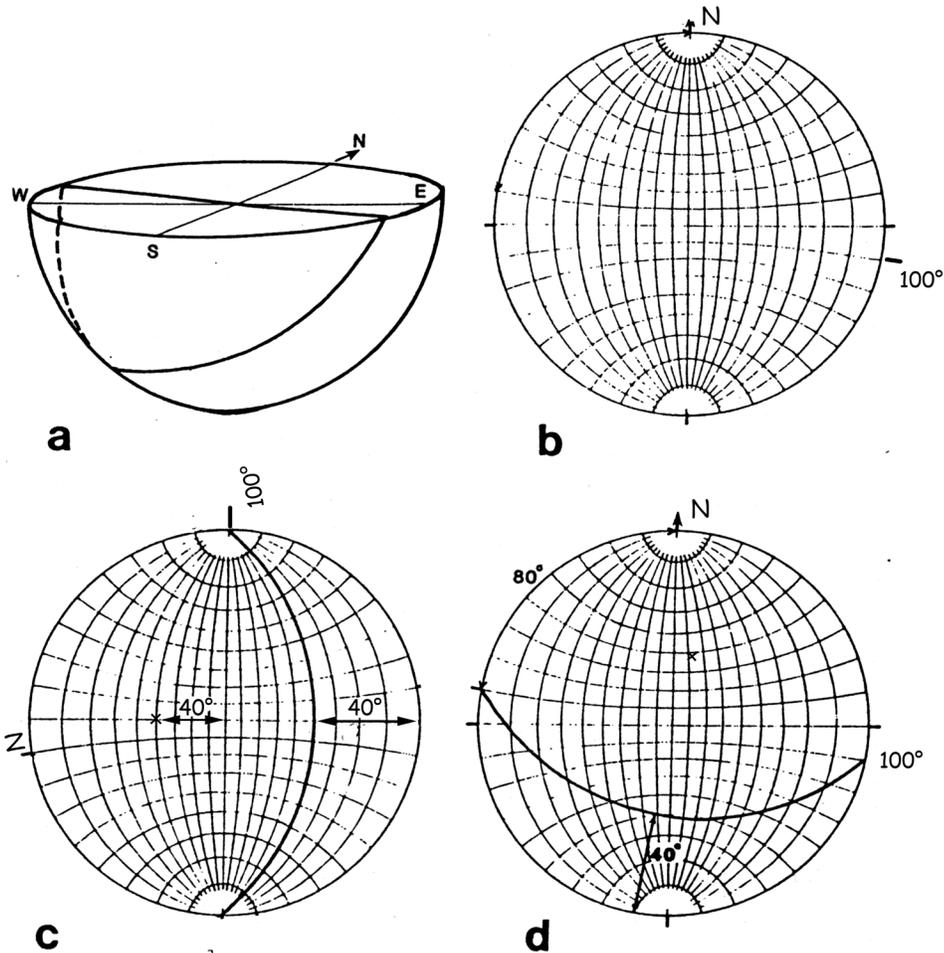

a

b

c

d

Fig. 4. Procedure for plotting the stereographic projection of an inclined plane with strike 100° dip 40°, and its pole

Plotting a line in a plane by pitch or rake

a) Plot the plane following steps a-c of the previous procedure.

b) From the top point of the net, count degrees *along the great circle you have just traced* until you reach the desired angle of pitch.

Measuring the orientation of a line from its pole

This is just the reverse of the plotting procedure

a) Rotate the net to place the pole on any of the four straight radii. Mark a point on the primitive at the end of this straight radius.

b) Count degrees of plunge inward from the primitive until you come to the pole. Record the plunge.

c) Return the net to the reference position. Count degrees clockwise round the primitive until you come to the mark made in step a. This is the trend.

Measuring the strike and dip of a plane from its trace

This is just the reverse of the plotting procedure for a plane

a) Rotate the net until the plane is on a great circle on the right hand half of the net. (This ensures an answer consistent with right-hand rule.)

b) Mark the end of the great circle that is at the top of the net

c) Count degrees of dip inward from the right hand side along the straight radius until you come to the great circle.

d) Return to reference position and note the azimuth of the mark made in b. This is the strike.

Measuring the strike and dip of a plane from its pole

A pole is perpendicular to the plane it represents, so steps a and c are opposite!

a) Rotate the net until the pole is on a the straight radius on the **left side** of the net.

b) Mark the end of the great circle that is at the top of the net

c) Count degrees of dip outward from the centre along the straight radius toward the left hand side of the net until you come to the pole.

d) Return to reference position and note the azimuth of the mark made in b. This is the strike.

Finding the plane common to two lines

a) Plot both planes as poles.

b) Rotate the net so that both poles lie on a single great circle.

c) Trace the great circle.

Angle between two lines

Repeat steps a-b above

c) Count degrees along the great circle between the two lines

Note that, unless the lines are at 90°, there will always be two answers: one greater and one less than 90°.