Hydraulic Jump

SEISMIC GROUP UET LHR

Prob 11.76

Data:

Rectangular channel

$b = 3m$

$Q = 3m^3/s$

$y_o = 0.5m$

Obstruction = 15cm from bottom

a) Will obstruction cause H.J? Why?

b) Water depth over obstruction = ?
Classify surface profile
Sketch resulting water surface profile and E.L showing y_c and y_o.

Sol:

a)

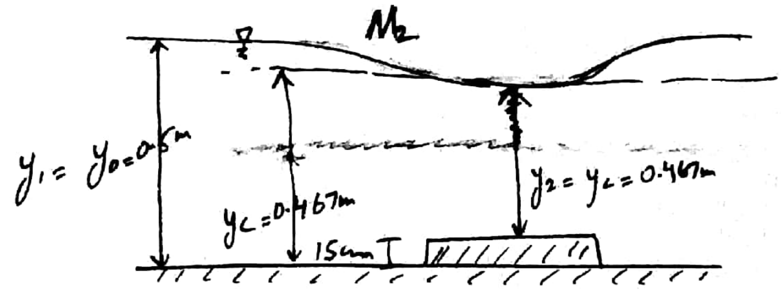
$$y_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}} \qquad q = \frac{Q}{b} = \frac{3}{3} = 1 m^2/s$$

$$= \left(\frac{1^2}{9.81}\right)^{\frac{1}{3}}$$

$$y_c = 0.467 \ m$$

As $y_o > y_c$, $\Rightarrow$ subcritical flow

As flow is not super critical, hence No H.J will be formed.



b) Water depth over obstruction:

$$y_o + \frac{V_o^2}{2g} = \Delta Z_c + y_c + \frac{V_c^2}{2g}$$

$$0.5 + \frac{2^2}{2 \times 9.81} = \Delta Z_c + 0.467 + \frac{2.14^2}{2 \times 9.81}$$

$$\because V_o = \frac{Q}{A} = \frac{3}{3 \times 0.5} = 2 m/s$$

$$\because V_c = \frac{Q}{A_c} = \frac{3}{3 \times 0.467} = 2.14 m/s$$

$$\Delta Z_c = 3.46 \times 10^{-3}$$
$$= 0.00346 \ m$$

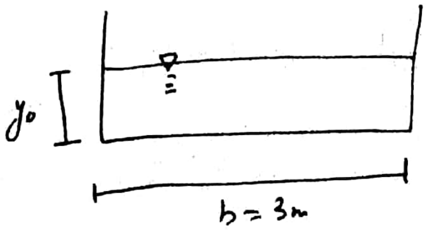
As $\Delta Z = 0.15m > \Delta Z_c = 0.00346m$

Damming action will be formed and $y_2 = y_c$ will occur on hump

$$\boxed{y_2 = 0.467m}$$

Example 11.5

Data: Analyze water surface profile
Rectangular channel
$n = 0.013$
$b = 3m$
$Q = 10 m^3/s$



Abrupt Change in slope from 0.015 → 0.0016 (S_1 → S_2)
Find power loss in jump.

Sol:

Before break

$$Q = \frac{A}{n} R^{2/3} S^{\frac{1}{2}}$$

$$Q = \frac{by_{o_1}}{n}\left(\frac{by_{o_1}}{b+2y_{o_1}}\right)^{\frac{2}{3}} S_1^{1/2}$$

$$10 = \frac{3y_{o_1}}{0.013}\left(\frac{3y_{o_1}}{3+2y_{o_1}}\right)^{\frac{2}{3}} (0.015)^{1/2}$$

$$\boxed{y_{o_1} = 0.62\ m}$$ (normal depth on upper slope)

After break

Using similar procedure, normal depth on lower slope can be found,

$$10 = \frac{3y_{o_2}}{0.013}\left(\frac{3y_{o_2}}{3+2y_{o_2}}\right)^{\frac{2}{3}} (1.36)^{\frac{1}{2}}$$

$$\boxed{y_{o_2} = 1.36m}$$

Finding critical depth:

$$y_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}} = \left(\frac{3.33^2}{9.81}\right)^{\frac{1}{3}}$$

$\because q = \frac{Q}{b} = \frac{10}{3} = 3.33 m^2/s$

$$y_c = 1.04m$$

$y_{o1} < y_c$ Flow is supercritical before break
$y_{o2} > y_c$ Flow is subcritical after break

H.J must occur.

Now we have to check whether H.J is forming before break or after:

Determine conjugate depth to the 0.62 m (Upper slope) normal depth

$$\frac{q^2}{g} = y_1 y_2 \left(\frac{y_1 + y_2}{2}\right)$$

$$\frac{3.33^2}{9.81} = 0.62\, y_2 \left(\frac{0.62 + y_2}{2}\right) = 1.625\text{ m} = 1.63\text{ m}$$

On upper slope $y_{01} = 0.62\text{m}$

$y_2' = 1.63\text{m}$

H.J cannot occur because y_{02} on lower slope is less than 1.63m
i.e 1.36m

· Determining conjugate depth to the 1.36m (lower slope) normal depth, we get.

$$y_{o2} = 1.36m$$
$$y_1' = ?$$

$$\frac{q^2}{g} = y_1' y_2 \left(\frac{y_1' + y_2}{2} \right)$$

$$\frac{3.33^2}{9.81} = y_1' (1.36) \left(\frac{y_1' + 1.36}{2} \right)$$

$$y_1' = 0.78m$$

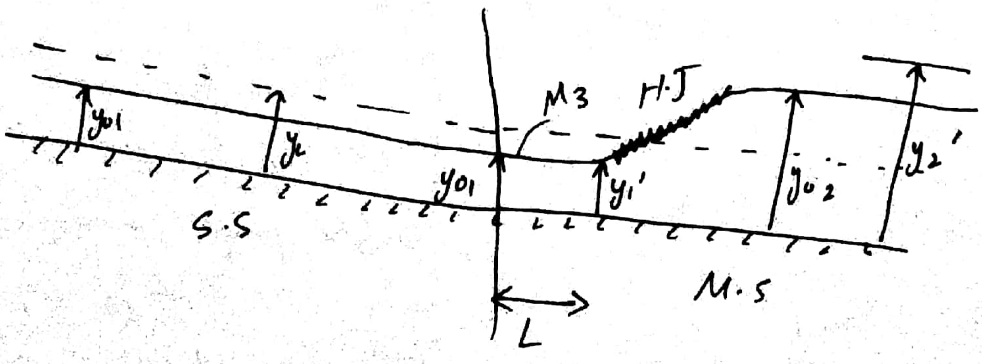
On lower slope

$$y_1' = 0.78m$$
$$y_2 = 1.36m$$

Since $y_1' > y_{o1}$

and $y_1' < y_c$

Hence H.J will form:



y_{o1} and y_1' will be used to find 'L'

The distance below the break in slope may be found by:

$$L = \frac{E_1 - E_2}{S - S_0}$$

$y_1 = y_{01} = 0.62$ m
$y_2 = y' = 0.78$ m
$S_0 =$ Steep slope $= 0.0016$

$$E_1 = y_1 + \frac{V_1^2}{2g}$$

$$= 0.62 + \frac{5.371^2}{2 \times 9.81}$$

$$= 2.09\text{ m}$$

$$V_1 = \frac{q}{y_1} = \frac{3.33}{0.62} = 5.371\text{ m/s}$$

$$E_2 = y_2 + \frac{V_2^2}{2g}$$

$$= 0.78 + \frac{2.449^2}{2 \times 9.81}$$

$$= 0.78 + \frac{4.27^2}{2 \times 9.81}$$

$$= 1.71\text{ m}$$

$$V_2 = \frac{q}{y_2} = \frac{3.33}{0.78}$$

$$= 4.27\text{ m/s}$$

$$S = \frac{V_m^2 n^2}{R_m^{4/3}}$$

$$S = \frac{4.82^2 \times 0.013^2}{(0.476)^{4/3}}$$

$$S = 0.0106$$

$$V_m = \frac{V_1 + V_2}{2} = \frac{5.371 + 4.27}{2} = 4.82\text{ m/s}$$

$$R_m = \frac{R_1 + R_2}{2} = \frac{1}{2}\left[\frac{A_1}{P_1} + \frac{A_2}{P_2}\right]$$

$$= \frac{1}{2}\left[\frac{b y_1}{b + 2y_1} + \frac{b y_2}{b + 2y_2}\right]$$

$$= \frac{1}{2}\left[\frac{3 \times 0.62}{3 + 2 \times 0.62} + \frac{3 \times 0.78}{3 + 2 \times 0.78}\right]$$

$$= 0.476\text{ m}$$

$$L = \frac{2.09 - 1.71}{0.0106 - 0.0016}$$

$$\boxed{L = 42.22\text{ m}}$$

Power loss

$\Delta P = \gamma Q \Delta E$

Before jump at $y_1 = 0.78$ m
$E = 1.71$ m

After jump at $y_2 = 1.36$ m
$E = 1.67$ m

$$\Delta P = \frac{9810 \times 10 \times (1.71 - 1.67)}{1000}$$

$\Delta P = 3.92$ kW

Prob. 11.78

Data: Analyze water surface profile

Make sketch showing
i) normal depths
ii) Critical depths
iii) Water surface profile types

Rectangular channel

$n = 0.013$

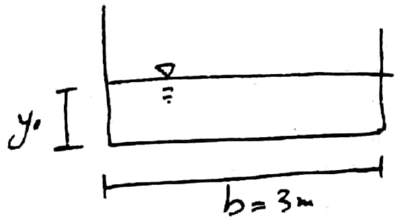
$b = 3m$

$Q = 12 m^3/s$

Abrupt Change in slope from $0.0016 \rightarrow 0.015$ ($S_1 \rightarrow S_2$)

Sol:

Before Break:



$$Q = \frac{A}{n} R^{\frac{2}{3}} S_1^{\frac{1}{2}}$$

$$Q = \frac{b y_{01}}{n} \left(\frac{b y_{01}}{b + 2y_{01}} \right)^{\frac{2}{3}} S_1^{\frac{1}{2}}$$

$$12 = \frac{3 y_{01}}{0.013} \left(\frac{3 y_{01}}{3 + 2y_{01}} \right)^{\frac{2}{3}} (0.0016)^{\frac{1}{2}}$$

$y_{01} = 1.56\ m$ (normal depth on Upper slope)

After Break:

$$12 = \frac{3 y_{02}}{0.013} \left(\frac{3 y_{02}}{3 + 2y_{02}} \right)^{\frac{2}{3}} (0.015)^{\frac{1}{2}}$$

$y_{02} = 0.7\ m$ (normal depth on Down slope)

Find critical depth:

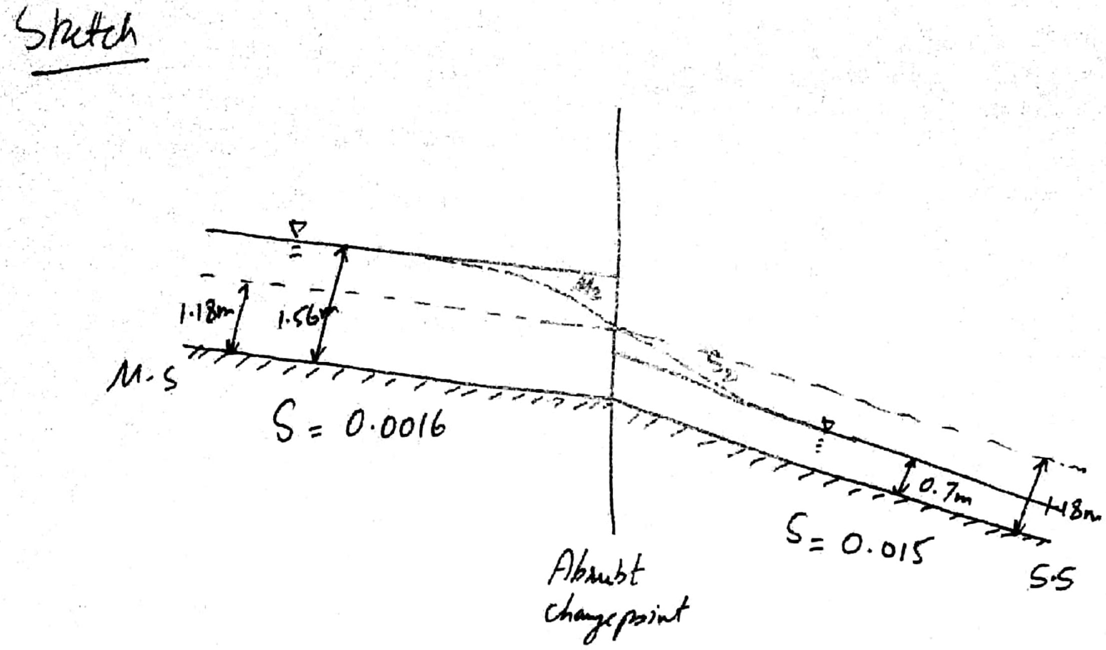
$$y_c = \left(\frac{q^2}{g} \right)^{\frac{1}{3}} = \left(\frac{4^2}{9.81} \right)^{\frac{1}{3}} \qquad \because q = \frac{Q}{b} = \frac{12}{3} = 4 m^2/s$$

$y_c = 1.18\ m$

As $y_{01} > y_c$ Flow is subcritical before break

and $y_{02} < y_c$ Flow is supercritical after break

Hence H.J will not occur.



Prob 11.79

Data:

$Q = 4\ m^3/s$

other data is same like 11.78

Sol:

$y_{01} = 0.71m$ $\qquad q = 1.33 m^2/s$

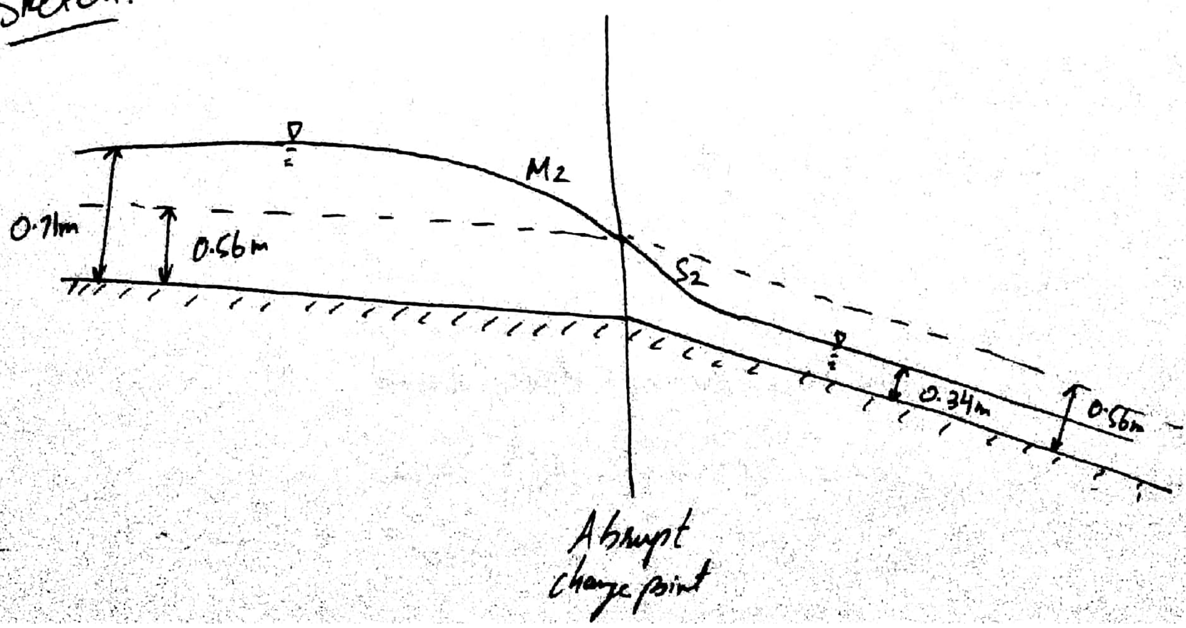
$y_{02} = 0.34m$ $\qquad y_c = 0.56\ m$

$y_{01} > y_c$ Flow is subcritical before break

$y_{02} < y_c$ Flow is supercritical after break.

No. H.J

Sketch:



Prob 11.80

Data.

$Q = 4\ m^3/s$

$S_1 = 0.0016$

$S_2 = 0.0006$

other data is same like 11-78 Problem

Find distance U/S from Break to point where y_0 occurs.

Sol:

$y_{01} = 0.71m$ $\quad$ $q = 1.33 m^2/s$

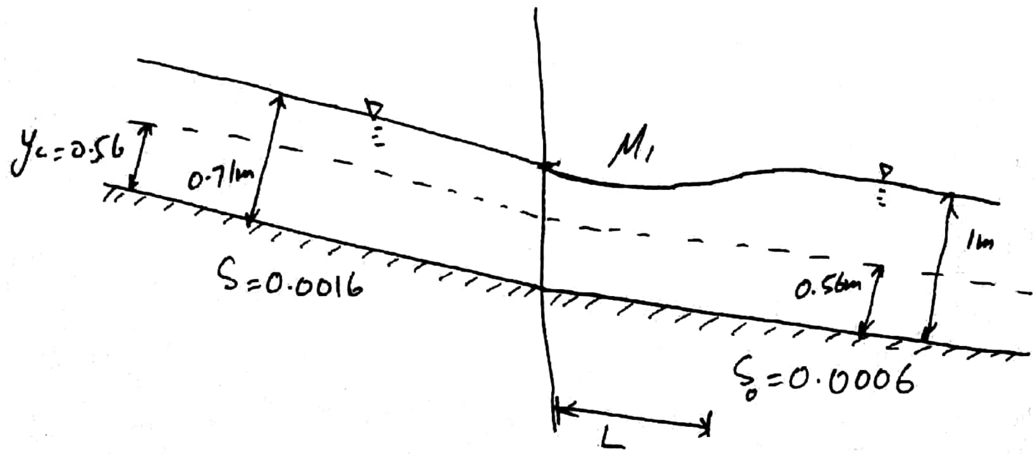
$y_{02} = 1m$ $\quad$ $y_c = 0.56m$

$y_{01} > y_c$ Flow is subcritical before break

$y_{02} > y_c$ Flow is " " after break

No H.J

Sketch



$y_1 = 0.71m$ $\quad$ $y_2 = 1m$

$$L = \frac{E_1 - E_2}{S - S_0}$$

$$R_m = \frac{R_1 + R_2}{2} = 0.541m$$

$$E_1 = y_1 + \frac{V^2}{2g} = y_1 + \frac{q^2}{y_1^2 2g}$$

$$E_2 = y_2 + \frac{q^2}{y_2^2 2g}$$

$$L = \frac{0.89 - 1.09}{0.00098 - 0.0006}$$

$E_2 = 1.09m$

$L = -526.3\ m$

$E_1 = 0.89m$

$$V_m = \frac{V_1 + V_2}{2} = \frac{1.87 + 1.33}{2}$$

$$S = \frac{V_m^2 n^2}{R_m^{4/3}} = \frac{1.6^2 \times 0.013^2}{(0.541)^{4/3}}$$

$V_m = 1.6 m/s$

$S = 0.00098$

Prob 11.83 Class prob # 1

Data: Rectangular channel:

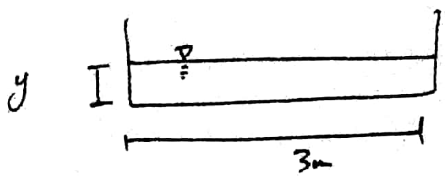
$b = 3m$

$Q = 5.65 m^3/s$

$y_1 = 0.3m$

a) What will be the depth after the H.J ?

b) Energy loss = ?



Sol:

a)

$$\frac{q^2}{g} = \left(\frac{y_1+y_2}{2}\right) y_1 y_2$$

$$\because q = \frac{Q}{b} = \frac{5.65}{3} = 1.88 m^2/s$$

$$\frac{1.88^2}{9.81} = \frac{(0.3 + y_2)}{2} 0.3 y_2$$

$$\boxed{y_2 = 1.41m}$$

b) $\Delta P = \gamma Q \Delta E$

$\Delta P = 9810 \times 5.65 \times 0.81$

$\Delta P = 44895 W$

$= 44.895 KW$

$$\boxed{\Delta P = 60.1 hp}$$

$$\Delta E = \left(y_1 + \frac{V_1^2}{2g}\right) - \left(y_2 + \frac{V_2^2}{2g}\right)$$

$$\Delta E = \left(0.3 + \frac{6.28^2}{2 \times 9.81}\right) - \left(1.41 + \frac{1.33^2}{2 \times 9.81}\right)$$

$\Delta E = 0.81m$

$$\because V_1 = \frac{q}{y_1} = \frac{1.88}{0.3} = 6.28 m/s$$

$$\because V_2 = \frac{q}{y_2} = \frac{1.88}{1.41} = 1.33 m/s$$

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Prob 11.84 Class prob #3

Data:

$b = 1.5m$

$y_1 = 0.2m$

$y_2 = 0.9m$

a) $Q = ?$

b) $\Delta P = ?$

Sol:

a) $\frac{q^2}{g} = y_1 y_2 \left(\frac{y_1 + y_2}{2}\right)$

$\frac{q^2}{9.81} = 0.2 \times 0.9 \left(\frac{0.2 + 0.9}{2}\right)$

$q = 0.985 \ m^2/s$

As $q = \frac{Q}{b}$

$Q = q b$

$= 0.985 \times 1.5$

$Q = 1.478 \ m^3/s$

b) $\Delta P = \gamma Q \Delta E$

$\Delta P = 9810 \times 1.478 \times 0.475$

$= 6887.11 \ W$

$\Delta P = 9.23 \ hp$

$\Delta E = \left(y_1 + \frac{V_1^2}{2g}\right) - \left(y_2 + \frac{V_2^2}{2g}\right)$

$= \left(0.2 + \frac{4.925^2}{2 \times 9.81}\right) - \left(0.9 + \frac{1.094^2}{2 \times 9.81}\right)$

$= 0.475m$

$\because V_1 = \frac{q}{y_1} = \frac{0.985}{0.2} = 4.925 m/s$

$\because V_2 = \frac{q}{y_2} = \frac{0.985}{0.9} = 1.094 m/s$

Prob 11.87   Class problem # 2

Data: Triangular flume

H : V

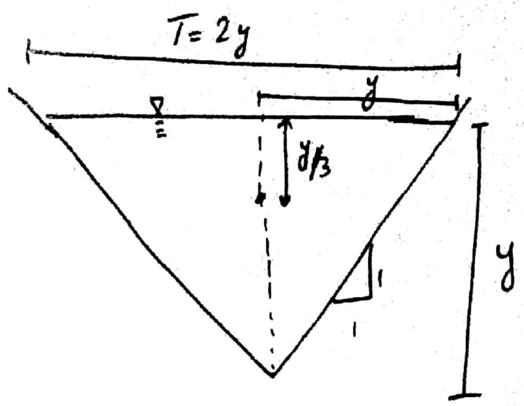
$S/s = 1:1$

$Q = 0.45\ m^3/s$

$y_1 = 0.3\ m$

a) $y_2 = ?$

b) $\Delta P\ (h.p) = ?$



Sol:

a) $T = 2y$

$$A = \frac{1}{2}(y)(y) + \frac{1}{2}(y)(y) = y^2$$

As

$$\frac{Q^2}{gA_1} + A_1 h_{c1} = \frac{Q^2}{gA_2} + A_2 h_{c2}$$

Now

$$\frac{Q^2}{g y_1^2} + y_1^2 \frac{y_1}{3} = \frac{Q^2}{g y_2^2} + y_2^2 \frac{y_2}{3}$$

$$\frac{0.45^2}{9.81 \times 0.3^2} + \frac{0.3^3}{3} = \frac{0.45^2}{9.81 \times y_2^2} + \frac{y_2^3}{3}$$

$$\boxed{y_2 = 0.86\ m}$$

b)

$\Delta P = \gamma Q \Delta E$

$\Delta P = 9810 \times 0.45 \times 0.7$

$= 3090.15\ W$

$\Delta P = 4.14\ hp$

$$\Delta E = \left(y_1 + \frac{V_1^2}{2g}\right) - \left(y_2 + \frac{V_2^2}{2g}\right)$$

$$= \left(0.3 + \frac{5^2}{2 \times 9.81}\right) - \left(0.86 + \frac{0.61^2}{2 \times 9.81}\right)$$

$= 0.70\ m$

$\because V_1 = \frac{Q}{A} = \frac{Q}{y_1^2} = \frac{0.45}{0.3^2}$

$\boxed{V_1 = 5\ m/s}$

$\because V_2 = \frac{Q}{y_2^2} = \frac{0.45}{0.86} = 0.61\ m/$

Prob 11.86

Data:

Rectangular Channel

$b = 5\,m$

$Q = 6\,m^3/s$

$S = 0.005$

$y_2 = 1.4\,m$

a) $y_1 = ?$

b) losses in energy and power in jump?

Sol:

$\because q = \frac{Q}{b} = \frac{6}{5} = 1.2\,m^2/s$

a)

$$\frac{q^2}{g} = \left(\frac{y_1 + y_2}{2}\right) y_1 y_2$$

$$\frac{1.2^2}{9.81} = \left(\frac{y_1 + 1.4}{2}\right) y_1 (1.4)$$

$$\boxed{y_1 = 0.14\,m}$$

b)

$$\Delta P = \gamma Q \Delta E$$

$$\Delta P = \frac{9810 \times 6 \times 2.45}{1000}$$

$$\Delta P = 144.21\,KW$$

$$\Delta P = \frac{144.21}{746}$$

$$\boxed{\Delta P = 193\,hp}$$

$$\Delta E = E_1 - E_2$$

$$\Delta E = \left(y_1 + \frac{V_1^2}{2g}\right) - \left(y_2 + \frac{V_2^2}{2g}\right)$$

$$\Delta E = \left(0.14 + \frac{8.57^2}{2 \times 9.81}\right) - \left(1.4 + \frac{0.86^2}{2 \times 9.81}\right)$$

$$\Delta E = 2.45\,m$$

$\because V_1 = \frac{q}{y_1} = \frac{1.2}{0.14} = 8.57\,m/s$

$\because V_2 = \frac{q}{y_2} = \frac{1.2}{1.4} = 0.86\,m/s$

Prob 16.89

Data:

Rectangular channel very wide

$S_0 = 0.0003$

$n = 0.02$

$Q = 5 m^3/s$ /m of width $\quad q = 5 m^2/s$

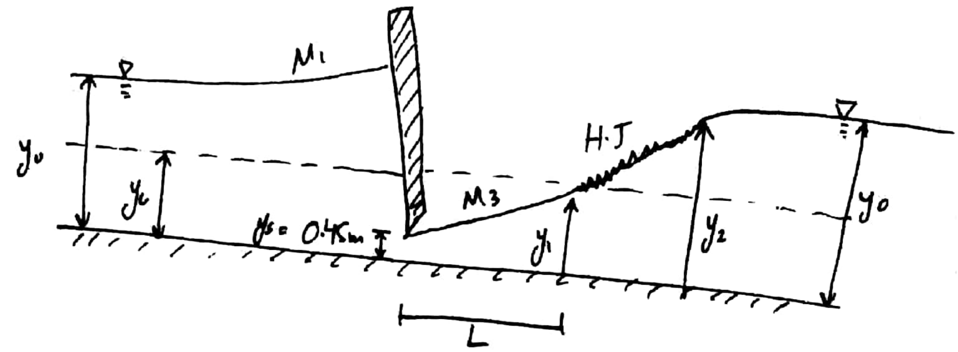
By sluice gate $y_s = 0.45$ m

a) Determine whether an H.J will form D/S = ?

Also Find distance from gate to the jump = ?

Sol:

Sketch



⇒ Finding normal depth

$$V = \frac{1}{n} R^{2/3} S^{\frac{1}{2}} \qquad \because V = \frac{q}{y}$$

$$\frac{q}{y_0} = \frac{1}{n} y_0^{\frac{2}{3}} S^{\frac{1}{2}} \qquad \because R \approx y_0 \text{ (In wide rectangular channels)}$$

$$\frac{5}{y_0} = \frac{1}{0.02} y_0^{\frac{2}{3}} 0.0003^{\frac{1}{2}}$$

$$\boxed{y_0 = 2.86 \text{ m}}$$

⇒ Finding critical depth

$$y_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}}$$

$$= \left(\frac{5^2}{9.81}\right)^{\frac{1}{3}}$$

$$\boxed{y_c = 1.37 m}$$

$\because y_s < y_c$

$\therefore$ Hence, H.J will be formed on D/S

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In this case $y_2 \simeq y_0$

Hence

$$\frac{q^2}{g} = y_1 y_2 \left(\frac{y_1 + y_2}{2}\right)$$

$$\frac{5^2}{9.81} = y_1 \times 2.86 \left(\frac{y_1 + 2.86}{2}\right)$$

$$\boxed{y_1 = 0.53 m}$$

Now

$$L = \frac{E_s - E_1}{S - S_0}$$

$$E_s = y_s + \frac{V_s^2}{2g}$$

$$\because V_s = \frac{q}{y_s} = \frac{5}{0.45}$$

$$= 0.45 + \frac{11.1^2}{2 \times 9.81}$$

$$V_s = 11.1 m/s$$

$$E_s = 6.73 m$$

$$E_1 = y_1 + \frac{V_1^2}{2g}$$

$$V_1 = \frac{q}{y_1} = \frac{5}{0.53}$$

$$= 0.53 + \frac{9.43^2}{2 \times 9.81}$$

$$V_1 = 9.43 m/s$$

$$E_1 = 5.06 m$$

$$S = \frac{V_m^2 n^2}{R_m^{4/3}}$$

$$V_m = \frac{V_s + V_1}{2} = \frac{11.1 + 9.43}{2}$$

$$= 10.265 m/s$$

$$S = \frac{10.265^2 \times 0.02^2}{0.49^{4/3}}$$

$$S = 0.1091$$

$$R_m = \frac{R_1 + R_2}{2}$$

$$\because R \simeq y$$

$$R_m = \frac{y_s + y_1}{2} = \frac{0.45 + 0.53}{2}$$

$$R_m = 0.49 m$$

$$L = \frac{6.73 - 5.06}{0.1091 - 0.0003}$$

$$\boxed{L = 15.35 m}$$

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