8.2 Problems Involving Dry Friction

Types of Friction Problems

- In all cases, geometry and dimensions are assumed to be known
- Three types of mechanics problem involving dry friction

  - Equilibrium

  - Impending motion at all points

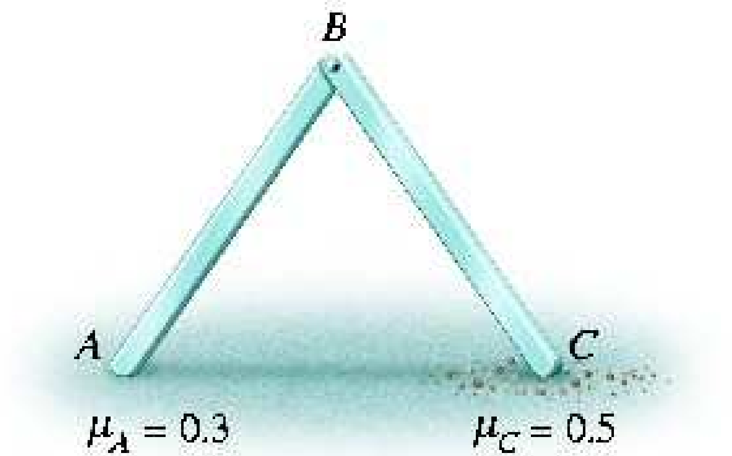
  - Impending motion at some points

8.2 Problems Involving Dry Friction

Types of Friction Problems

Equilibrium

- Total number of unknowns = Total number of available equilibrium equations
- Frictional forces must satisfy $F \leq \mu_s N$; otherwise, slipping will occur and the body will not remain in equilibrium
- We must determine the frictional forces at A and C to check for equilibrium

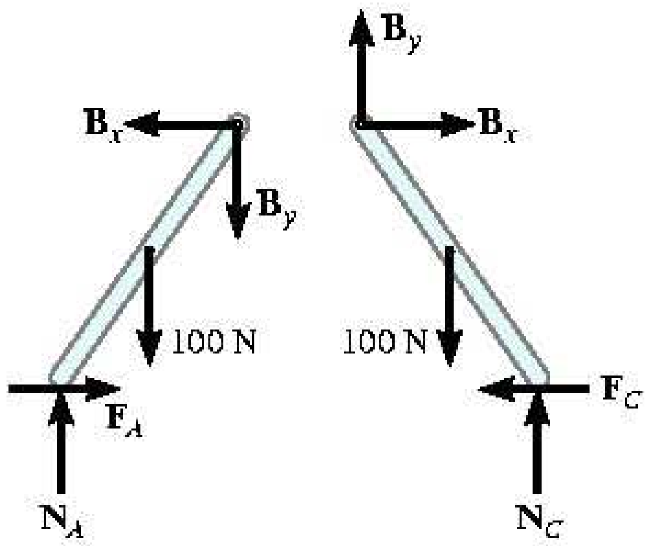


(a)

8.2 Problems Involving Dry Friction

Types of Friction Problems

- If the bars are uniform and have known weights of 100N each, FBD are shown below
- There are 6 unknown force components which can be determined strictly from the 6 equilibrium equations (three for each member)



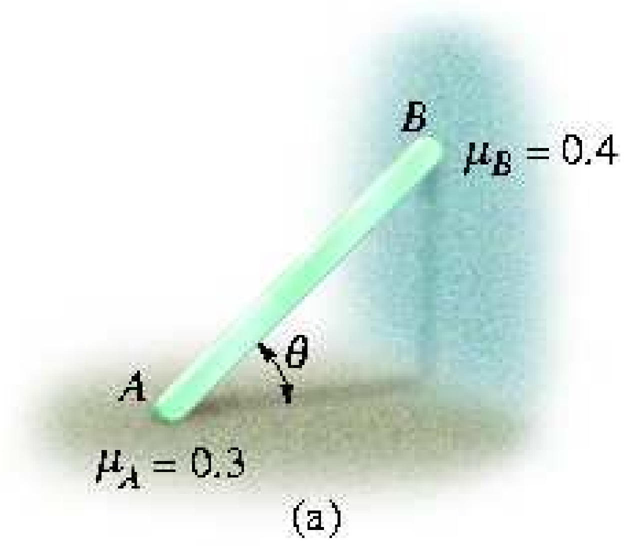
(b)

8.2 Problems Involving Dry Friction

Types of Friction Problems

Impending Motion at All Points

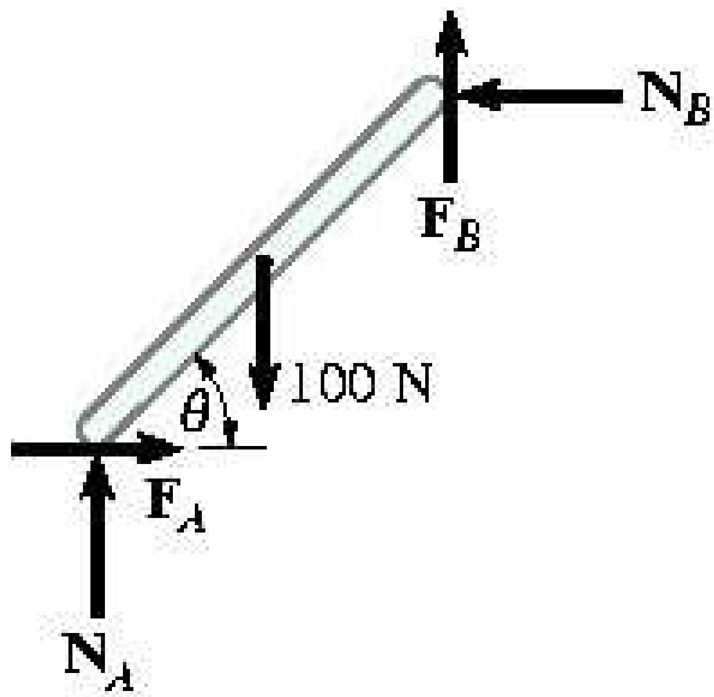
- Total number of unknowns = Total number of available equilibrium equations and available frictional equations
- If the motion is impending at the points of contact, $F_s = \mu_s N$
- If the body is slipping, $F_k = \mu_k N$
- Consider angle θ of the 100N bar for no slippage



8.2 Problems Involving Dry Friction

Types of Friction Problems

- FBD of the 100N bar
- 5 unknowns and 3 equilibrium equations and 2 static frictional equations which apply at both points of contact



(b)

8.2 Problems Involving Dry Friction

Types of Friction Problems

Impending Motion at Some Points

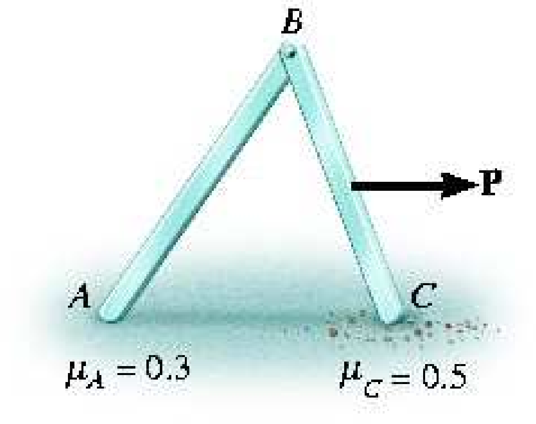
- Total number of unknowns < total number of available equilibrium equations and the frictional equations or conditional equations for tipping
- As a result, several possibilities for motion or impending motion will exist

8.2 Problems Involving Dry Friction

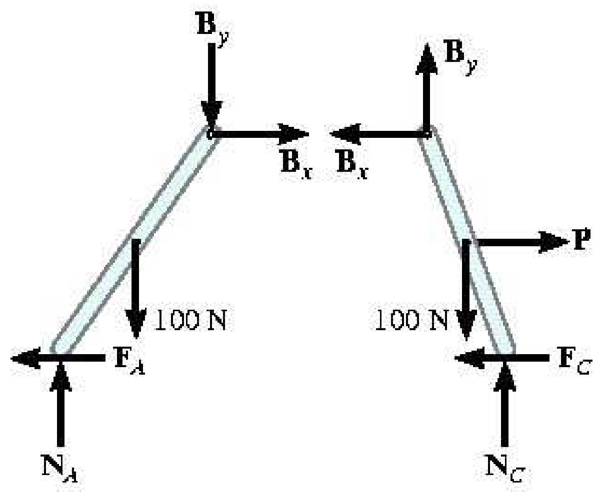
Types of Friction Problems

Example

- Consider 2-member frame to determine force **P** needed to cause movement
- Each member has a weight of 100N



(a)



(b)

8.2 Problems Involving Dry Friction

Types of Friction Problems

- 7 unknowns
- For unique solution, we must satisfy 6 equilibrium equations (three for each member) and only one of the two possible static frictional equations
- As P increases, it will either cause slipping at A and no slipping at C or slipping at C and no slipping at A

8.2 Problems Involving Dry Friction

Types of Friction Problems

- Actual situation can be determined by choosing the case for which P is smaller
- If in both cases, same value of P is obtained, slipping occur simultaneously at both points and the 7 unknowns will satisfy 8 equations

8.2 Problems Involving Dry Friction

Equilibrium Versus Frictional Equations

- Frictional force always acts so as to oppose the relative motion or impede the motion of the body over its contacting surface
- Assume the sense of the frictional force that require F to be an "equilibrium" force
- Correct sense is made after solving the equilibrium equations
- If F is a negative scalar, the sense of **F** is the reverse of that assumed

8.2 Problems Involving Dry Friction

Procedures for Analysis

FBD

- Draw the necessary FBD and unless it is stated that impeding motion or slipping occurs, always show the frictional forces as unknown
- Determine the number of unknowns and compare with the number of available equations
- If there are more unknowns that the equations of equilibrium, apply frictional equations at points of contact

8.2 Problems Involving Dry Friction

Procedures for Analysis

FBD

- If the equation $F = \mu N$ is used, show **F** acting in the proper direction on the FBD

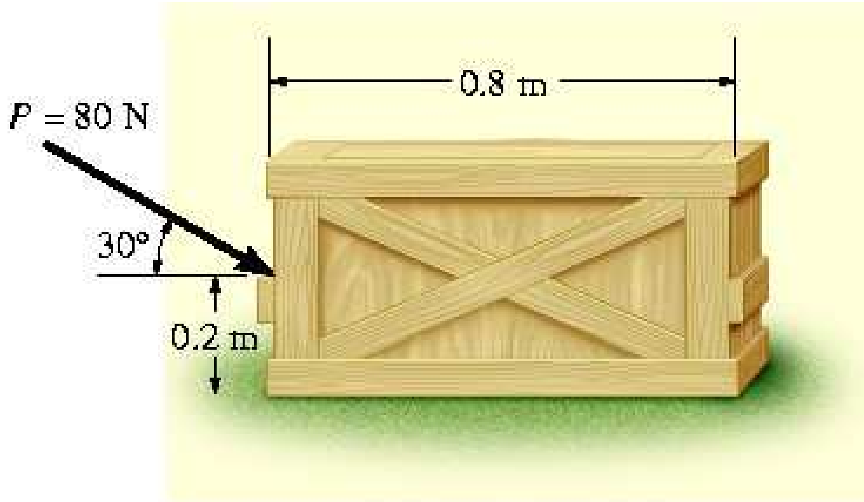
Equations of Equilibrium and Friction

- Apply the equilibrium equations and the necessary frictional equations and solve for unknowns
- If the problem is 3D, apply the equations using Cartesian coordinates

8.2 Problems Involving Dry Friction

Example 8.1

The uniform crate has a mass of 20kg. If a force P = 80N is applied on to the crate, determine if it remains in equilibrium. The coefficient of static friction is μ = 0.3.

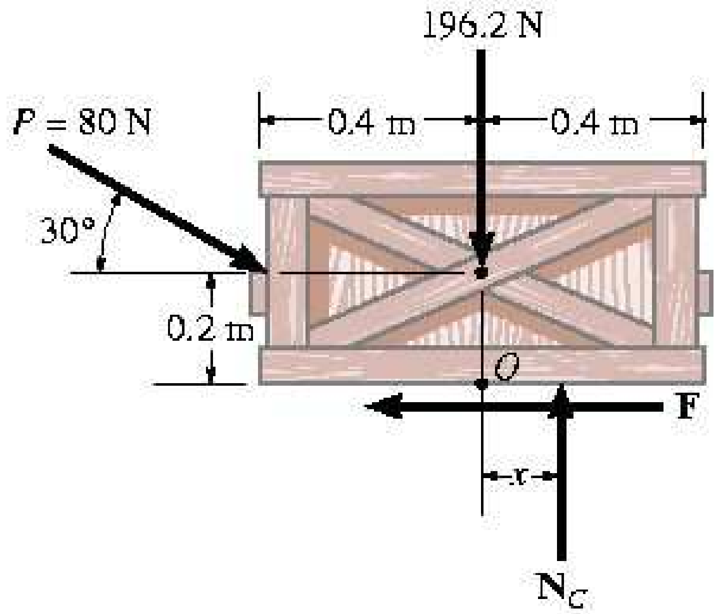


(a)

8.2 Problems Involving Dry Friction

Solution

- Resultant normal force **N**C act a distance x from the crate's center line in order to counteract the tipping effect caused by **P**
- 3 unknowns to be determined by 3 equations of equilibrium



(b)

8.2 Problems Involving Dry Friction

Solution

$$+\rightarrow \sum F_x = 0;$$

$$80\cos 30^\circ N - F = 0$$

$$+\uparrow \sum F_y = 0;$$

$$-80\sin 30^\circ N + N_C - 196.2N = 0$$

$$\sum M_O = 0;$$

$$80\sin 30^\circ N(0.4m) - 80\cos 30^\circ N(0.2m) + N_C(x) = 0$$

Solving

$$F = 69.3N, N_{C=236N, x=-0.00908m=-9.08mm}$$

8.2 Problems Involving Dry Friction

Solution

- Since x is negative, the resultant force acts (slightly) to the left of the crate's center line
- No tipping will occur since $x \le 0.4m$
- Maximum frictional force which can be developed at the surface of contact

$$F_{max} = \mu_s N_C = 0.3(236N) = 70.8N$$

- Since $F = 69.3N < 70.8N$, the crate will not slip thou it is close to doing so

4.2 Problems Involving Dry Friction

Example 8.2

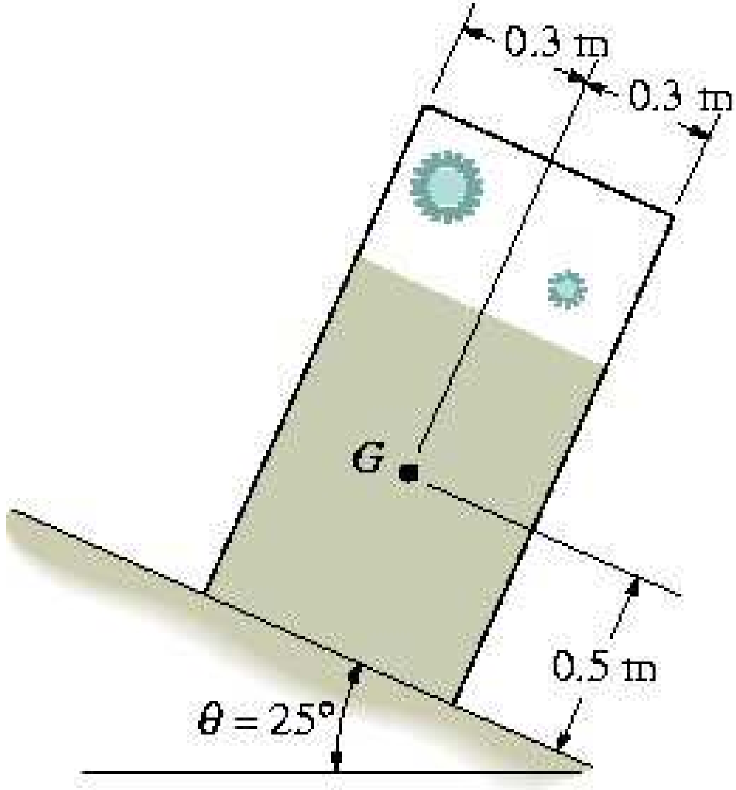
It is observed that when the bed of the dump truck is raised to an angle of $\theta = 25°$ the vending machines begin to slide off the bed. Determine the static of coefficient of friction between them and the surface of the truck

(a)

8.2 Problems Involving Dry Friction

Solution

- Idealized model of a vending machine lying on the bed of the truck
- Dimensions measured and center of gravity located
- Assume machine weighs W

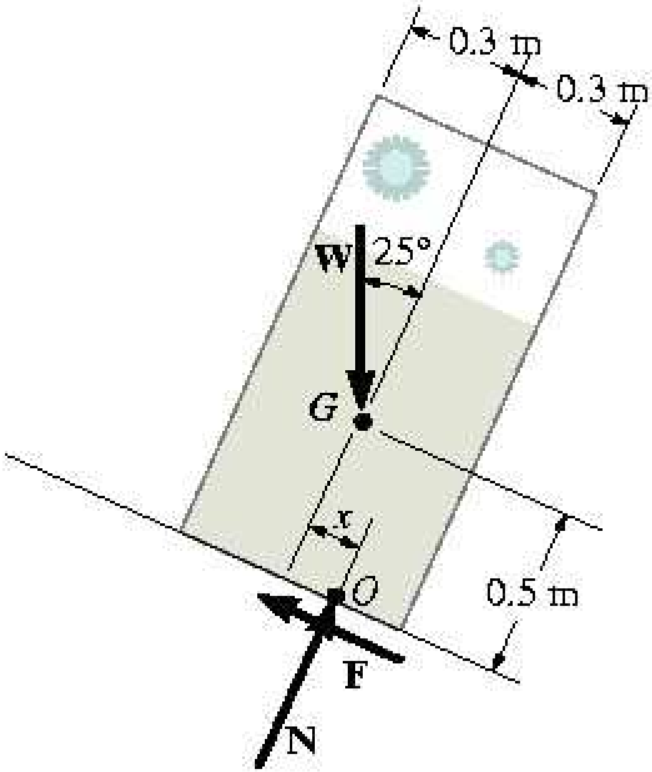


(b)

8.2 Problems Involving Dry Friction

Solution

- Dimension x used to locate position of the resultant normal force **N**
- 4 unknowns



(c)

8.2 Problems Involving Dry Friction

Solution

$$\sum F_x = 0;$$

$$W \sin 25^\circ N - F = 0$$

$$\sum F_y = 0;$$

$$N - W \cos 25^\circ N = 0$$

$$\sum M_O = 0;$$

$$-W \sin\theta(0.5m) + W \cos\theta(x) = 0$$

Slipping occurs at θ = 25°

$$F_s = \mu_s N; W \sin 25^\circ = \mu_s (W \cos 25^\circ N)$$

$$\mu_s = \tan 25^\circ = 0.466$$

8.2 Problems Involving Dry Friction

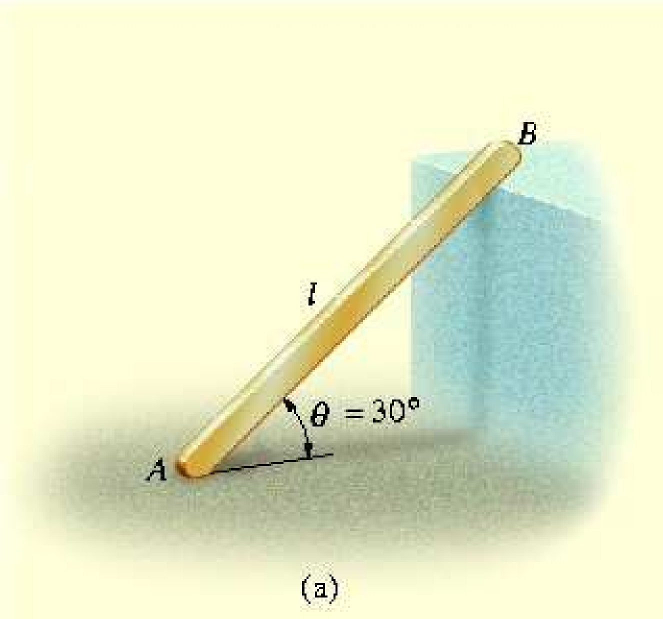
Solution

- Angle $\theta = 25°$is referred as the angle of repose
- By comparison, $\theta = \Phi_s$
- θ is independent of the weight of the vending machine so knowing θ provides a method for finding coefficient of static friction
- $\theta = 25°$, $x = 0.233m$
- Since $0.233m < 0.5m$the vending machine will slip before it can tip as observed

8.2 Problems Involving Dry Friction

Example 8.3

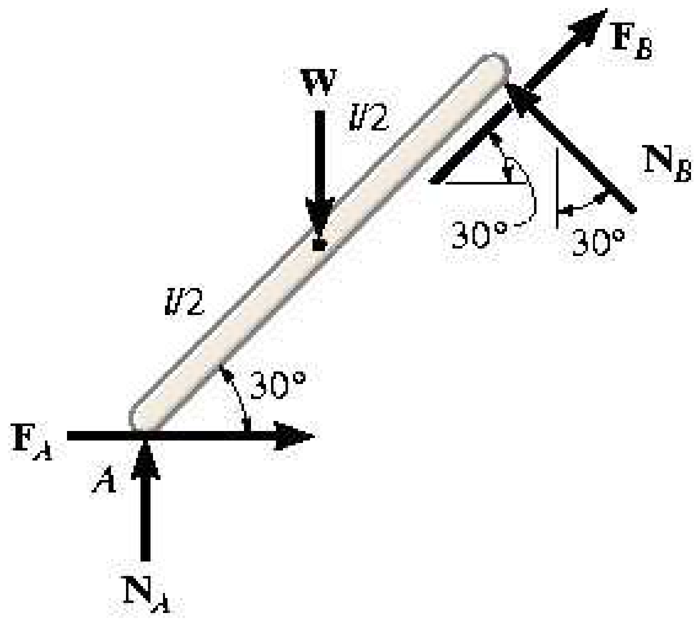
The uniform rob having a weight of W and length l is supported at its ends against the surfaces A and B. If the rob is on the verge of slipping when $\theta = 30°$, determine the coefficient of static friction μ_s at A and B. Neglect the thickness of the rob for calculation.



8.2 Problems Involving Dry Friction

Solution

- 5 unknowns
- 3 equilibrium equations and 2 frictional equations applied at A and B
- Frictional forces must be drawn with their correct sense so that they oppose the tendency for motion of the rod



(b)

8.2 Problems Involving Dry Friction

Solution

Frictional equations

$$F = \mu_s N;$$

$$F_A = \mu_s N_A, F_B = \mu_s N_B$$

Equilibrium equations

$$+ \rightarrow \sum F_x = 0;$$

$$\mu_s N_A + \mu_s N_B \cos 30^\circ - N_B \sin 30^\circ = 0$$

$$+ \uparrow \sum F_y = 0;$$

$$N_A - W + N_B \cos 30^\circ + \mu_s N_B \sin 30^\circ = 0$$

$$\sum M_A = 0;$$

$$N_B \ell - W\left(\frac{1}{2}\right)\cos 30^\circ = 0$$

8.2 Problems Involving Dry Friction

Solution

Solving

$$N_B = 0.4330W$$

$$\mu_s N_A = 0.2165W - (0.3750W)\mu_s$$

$$N_A = 0.6250W - (0.2165W)\mu_s$$

By division

$$0.6250\mu_s - 0.2165\mu_s^2 = 0.2165 - 0.375\mu_s$$

$$\mu_s^2 - 0.4619\mu_s + 1 = 0$$

Solving for the smallest root

$$\mu_s = 0.228$$

8.2 Problems Involving Dry Friction

Example 8.4

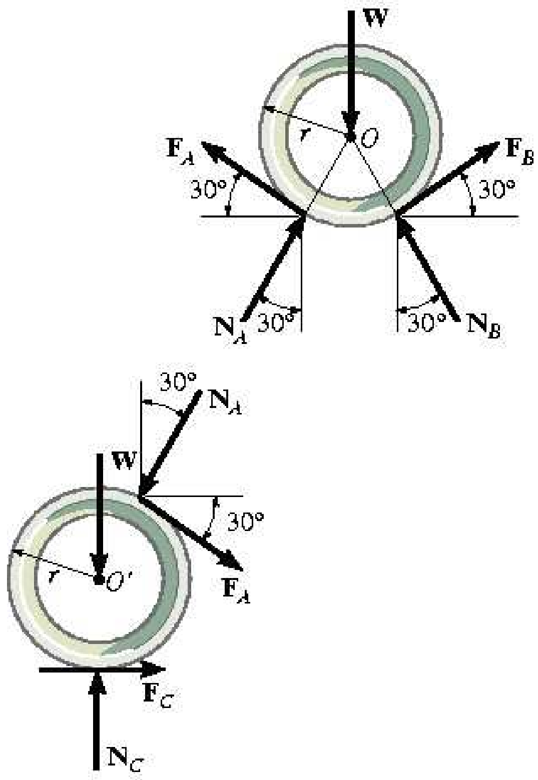
The concrete pipes are stacked in the yard. Determine the minimum coefficient of static friction at each point of contact so that the pile does not collapse.

(a)

8.2 Problems Involving Dry Friction

Solution

- Coefficient of static friction between the pipes A and B, and between the pipe and the ground, at C are different since the contacting surfaces are different
- Assume each pipe has an outer radius r and weight W
- 6 unknown, 6 equilibrium equations
- When collapse is about to occur, normal force at D = 0



(b)

8.2 Problems Involving Dry Friction

Solution

For the top pipe,

$$\sum M_O = 0;$$

$$-F_A(r) + F_B(r) = 0; F_A = F_B = F$$

$$+\rightarrow \sum F_x = 0;$$

$$N_A \sin 30^\circ - F \cos 30^\circ - N_B \sin 30^\circ + F \cos 30^\circ = 0$$

$$N_A = N_B = N$$

$$+\uparrow \sum F_y = 0;$$

$$2N \cos 30^\circ + 2F \sin 30^\circ - W = 0$$

8.2 Problems Involving Dry Friction

Solution

For the bottom pipe, using F_A = F and N_A = N,

$$\sum M_{O'} = 0;$$

$$F_C(r) - F(r) = 0; F_C = F$$

$$+ \rightarrow \sum F_x = 0;$$

$$-N\sin 30^\circ + F\cos 30^\circ + F = 0$$

$$+\uparrow \sum F_y = 0;$$

$$N_C - W - N\cos 30^\circ - F\sin 30^\circ = 0$$

Solving,

$$F = 0.268N$$

8.2 Problems Involving Dry Friction

Solution

Between the pipes,

$$(\mu_s)_{\min} = \frac{F}{N} = 0.268$$

$$N = 0.5W$$

$$N_C - W - (0.5W)\cos 30^\circ - 0.2679(0.5W)\sin 30^\circ = 0$$

$$N_C = 1.5W$$

For smallest required coefficient of static friction,

$$(\mu'_s)_{\min} = \frac{F}{N_C} = \frac{0.2679(0.5W)}{1.5W} = 0.0893$$

8.2 Problems Involving Dry Friction

Solution

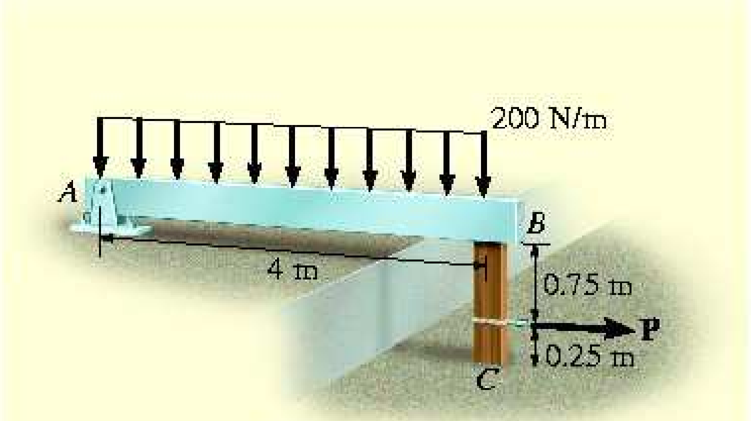
- A greater coefficient of static friction is required between the pipes than that required at the ground
- It is likely that the slipping would occur between the pipes at the bottom
- If the top pipes falls downwards, the bottom two pipes would roll away

8.2 Problems Involving Dry Friction

Example 8.5

Beam AB is subjected to a uniform load of 200N/m and is supported at B by post BC. If the coefficients of static friction at B and C are μ_B and $\mu_C = 0.5$, determine the force **P** needed to pull the post out from under the beam.

Neglect the weight of the members and the thickness of the post.

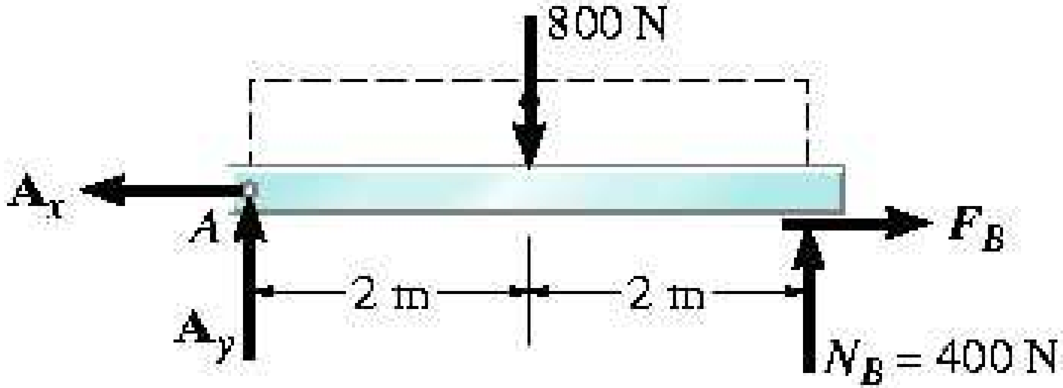


(a)

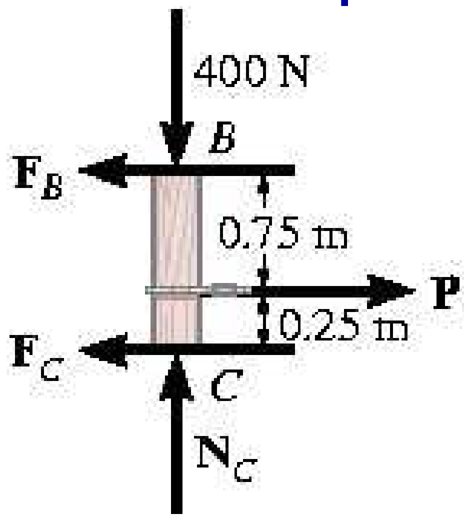
8.2 Problems Involving Dry Friction

Solution

- FBD of beam AB and the post
- Apply $\Sigma M_A = 0$, $N_B = 400N$
- 4 unknowns
- 3 equilibrium equations and 1 frictional equation applied at either B or C



(b)



(c)

8.2 Problems Involving Dry Friction

Solution

$$+\rightarrow \sum F_x = 0;$$

$$P - F_B - F_C = 0$$

$$+\uparrow \sum F_y = 0;$$

$$N_C - 400N = 0$$

$$\sum M_O = 0;$$

$$-P(0.25m) + F_B(1m) = 0$$

Post slips only at B

$$F_C \leq \mu_C N_C$$

$$F_B = \mu_B N_B; F_B = 0.2(400N) = 80N$$

8.2 Problems Involving Dry Friction

Solution

Solving

$$P = 320N, F_C = 240N, N_C = 400N$$

$$F_C = 240N > \mu_C N_C = 0.5(400N) = 200N$$

Post slips only at C

$$F_B \le \mu_B N_B$$

$$F_C = \mu_C N_C ; F_C = 0.5 N_C$$

Solving

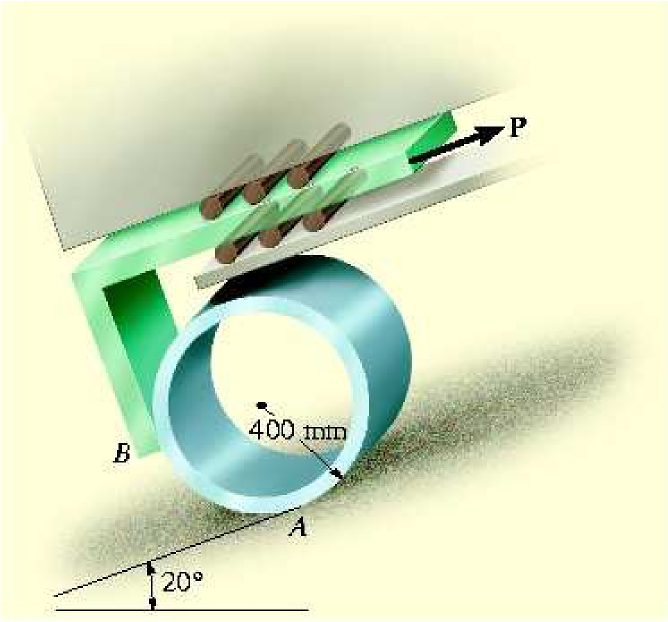
$$P = 267N, N_C = 400N, F_C = 200N, F_B = 66.7N$$

Choose second case as it requires a smaller value of P

8.2 Problems Involving Dry Friction

Example 8.6

Determine the normal force P that must be exerted on the rack to begin pushing the 100kg pipe up the 20° incline. The coefficients of static friction at points of contact are $(\mu_s)_A = 0.15$ and $(\mu_s)_B = 0.4$.



(a)

8.2 Problems Involving Dry Friction

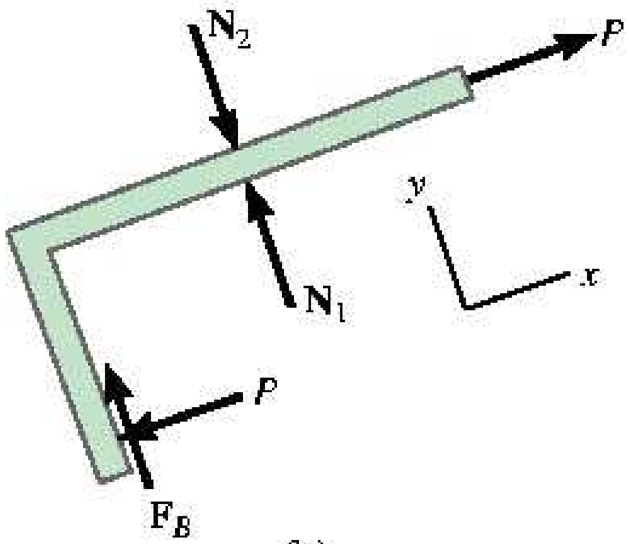
Solution

- The rack must exert a force P on the pipe due to force equilibrium in the x direction
- 4 unknowns
- 3 equilibrium equations and 1 frictional equation which apply at either A or B
- If slipping begins to occur at B, the pipe will roll up the incline
- If the slipping occurs at A, the pipe will begin to slide up the incline

8.2 Problems Involving Dry Friction

Solution

- FBD of the rack



(b)

8.2 Problems Involving Dry Friction

Solution

$$\sum F_x = 0;$$

$$-F_A + P - 981\sin 20^\circ = 0$$

$$\sum F_y = 0;$$

$$N_A - F_B - 981\cos 20^\circ = 0$$

$$\sum M_O = 0;$$

$$F_B(400mm) - F_A(400mm) = 0$$

Pipe rolls up incline

$$F_A \le 0.15 N_A$$

$$(F_S)_A = (\mu_S)_A; 224N \le 0.15(1146N) = 172N$$

8.2 Problems Involving Dry Friction

Solution

Inequality does not apply and slipping occurs at A

Pipe slides up incline

$$P \leq 0.4N_B$$

$$(F_s)_A = (\mu_s)_A N_A;$$

$$F_A = 0.15N_A$$

Solving

$$N_A = 1085N, F_A = 163N, F_B = 163N, P = 498N$$

Check: no slipping occur at B

$$F_B \leq (\mu_s)_B P;$$

$$163N \leq 0.4(498N) = 199N$$