Problem 4.2

A concrete sewer pipe 4 ft in diameter is laid so it has a drop in elevation of 1.00ft per 1000 ft of length. If sewage (assume the properties are the same as those of water) flows at a depth of 2 ft in the pipe, what will be the discharge?

Solution:

$$R_h = \frac{A}{P} = \frac{\frac{1}{8}D^2\pi}{\frac{1}{2}D\pi} = \frac{D}{4} = 1.0 \text{ ft}$$

Assuming $n = 0.013$, we then have

$$Q = \frac{1.49}{n} A R_h^{\frac{2}{3}} S_0^{\frac{1}{2}} = \frac{1.49}{0.013}\frac{\pi}{4}(4 \text{ ft})^2(1.0 \text{ ft})^{\frac{2}{3}}\left(\frac{1.00 \text{ ft}}{1000 \text{ ft}}\right)^{\frac{1}{2}} = 22.8 \text{ cfs}$$

Problem 4.4

A rectangular troweled concrete channel 12 ft wide with a slope of 10 ft in 8000 ftis designed for a discharge of 600 cfs. For a water temperature of 40°F, estimate the depth of the flow.

Solution:

Assuming $n = 0.015$,

$$Q = \frac{1.49}{n} A R_h^{\frac{2}{3}} S_0^{\frac{1}{2}} = \frac{1.49}{n} by\left(\frac{by}{b+2y}\right)^{\frac{2}{3}} S_0^{\frac{1}{2}}$$

where, b is the channel width and y is the flow depth

$$600 \text{ cfs} = \frac{1.49}{0.015} \cdot (12 \text{ ft})^{\frac{5}{3}} \cdot \frac{y^{\frac{5}{3}}}{(12 \text{ ft} + y)^{\frac{2}{3}}} \cdot \left(\frac{10 \text{ ft}}{8000 \text{ ft}}\right)^{\frac{1}{2}}$$

Solving this for y we get the solutions of −4.11 ft and 5.60 ft. Obviously, only the latter one is possible, so $y = 5.60$ ft.

Problem 4.7

A concrete-lined trapezoidal channel with bottom width of 10 ft and side slopes of 1 vertical to 2 horizontal is designed to carry a flow of 3000 cfs. If the slope of the channel is 0.001, what would be the depth of flow in the channel?

Solution:

Assuming $n = 0.012$

$$\frac{AR_h^{\frac{2}{3}}}{b^{\frac{8}{3}}} = \frac{Qn}{1.49 S_0^{\frac{1}{2}} b^{\frac{8}{3}}} = \frac{3000\text{ cfs} \cdot 0.012}{1.49 \cdot 0.001^{\frac{1}{2}} \cdot (10\text{ ft})^{\frac{8}{3}}} = 1.65$$

Then from Fig. 4-7, $y/b = 0.90$ or $y = 9$ ft.

Problem 4.12

Estimate the discharge in the Moyie River near Eastport, Idaho, when the depth is 4 ft. Assume $S_0 = 0.0032$.

Solution:

$$Q = \frac{1.49}{n} AR_h^{\frac{2}{3}} S_0^{\frac{1}{2}}$$

First calculate A and R_h from the given figure. By approximating the area as several triangles and rectangles the area is found to be $A = 300\text{ ft}^2$. Likewise, by approximation it is found that $P = 125$ ft. Thus

$$R_h = \frac{A}{P} = \frac{300\text{ ft}^2}{125\text{ ft}} = 2.40\text{ ft}.$$

Assume $n = 0.038$ as given in Fig. 4-3. Then

$$Q = \frac{1.49}{n} AR_h^{\frac{2}{3}} S_0^{\frac{1}{2}} = \frac{1.49}{0.038} \cdot 300\text{ ft} \cdot (2.40\text{ ft})^{\frac{2}{3}} \cdot \left(\frac{5\text{ ft}}{5280\text{ ft}}\right)^{\frac{1}{2}} = 1200\text{ cfs}$$

Problem 4.16

A trapezoidal irrigation canal is to be excavated in soil and lined with coarse gravel. The canal is to be designed for a discharge of 200 cfs, and it will have slope of 0.0016. What should be the magnitude of the cross-sectional area and hydraulic radius for the canal if it is to be designed so that erosion of the canal will not occur? Choose a canal cross section that will satisfy the limitations.

Solution:

From Table 4-3 the maximum permissible velocity for coarse gravel is given as 4.00 ft/s and $n = 0.025$. Manning equation:

$$V = \frac{1.49}{n} R_h^{\frac{2}{3}} S_0^{\frac{1}{2}}$$

or

$$R_h = \left(\frac{Vn}{1.49 S_0^{\frac{1}{2}}} \right)^{\frac{3}{2}} = 2.17 \text{ ft.}$$

Also

$$A = \frac{Q}{V} = \frac{200 \text{ cfs}}{4.00 \text{ ft/s}} = 50 \text{ ft}^2$$

Assume side slopes will be 1 vertical to 2 horizontal

$$P = \frac{A}{R_h} = \frac{50 \text{ ft}^2}{2.17 \text{ ft}} = 23.0 \text{ ft}$$

Also

$$P = b + 2y\sqrt{1+4} = b + 2\sqrt{5}y = 23.0 \text{ ft}$$

$$A = by + 2y^2 = 50 \text{ ft}^2$$

Solving the above two equations for the bottom width b and depth y yields $b = 7.59$ ft and $y = 3.45$ ft.

Problem 4.22

Water flows at a depth of 10 cm with a velocity of 6 m/s in a rectangular channel. Is the flow subcritical or supercritical? What is the alternate depth?

Solution:

Check Froude number

$$\mathrm{Fr} = \frac{V}{\sqrt{gy}} = \frac{6\ \mathrm{m/s}}{\sqrt{9.81\ /\mathrm{s}^2 \cdot 0.1\ \mathrm{m}}} = 6.06 > 1$$

so the flow is supercritical.

$$E = y + \frac{V^2}{2g} = 0.1\ \mathrm{m} + \frac{(6\ \mathrm{m/s})^2}{2 \cdot 9.81\ \mathrm{m/s}^2} = 1.935\ \mathrm{m}$$

Solving for the alternate depth for an $E = 1.935$ m yields $y_{\mathrm{alt}} = 1.93$ m

Problem 4.31

Derive a formula for critical depth, d_c in the V-shaped channel shown below.

Solution:

$$\frac{A_c^3}{T_c} = \frac{Q^2}{g}$$

where for this channel $A_c = d_c^2$ and $T_c = 2d_c$ so

$$\frac{(d_c^2)^3}{2d_c} = \frac{1}{2}d_c^5 = \frac{Q^2}{g}$$

or

$$d_c = \left(\frac{2Q^{@}}{g}\right)^{\frac{1}{5}}.$$

Problem 4.34

A 10-ft wide rectangular channel is very smooth except for a small reach that is roughened with angle irons attached to the bottom of the channel. Water flows in the channel at a rate of 200 cfs and at a depth of 1.00 ft. Assume frictionless flow except over the roughened part where the total drag of all the roughness (all the angle irons) is assumed to be 2000 lb. Determine the depth at the end of the roughness elements for the assumed conditions.

Solution:

Use the momentum equation written from the section upstream of the angle irons (call it 1) to a section downstream of them (section 2). Write it per foot of width of channel.

$$\frac{\gamma y_1^2}{2} - \frac{\gamma y_2^2}{2} - \frac{F_a}{b} = -\rho V_1^2 y_1 + \rho V_2^2 y_2$$

or

$$y_1^2 - y_2^2 - \frac{2F_a}{b\gamma} = -\frac{2}{g} V_1^2 y_1 + \frac{2}{g}\left(\frac{Q}{by_2}\right)^2 y_2$$

where F_a is the force on the angle irons and b is the channel width. Substituting the known values we get

$$(1\text{ ft})^2 - y_2^2 - \frac{2\cdot 2000\text{ lb}}{10\text{ ft}\cdot 62.4\text{ lb/ft}^3} = -\frac{2\cdot(20\text{ ft/s})^2\cdot 1\text{ ft}}{32.2\text{ ft/s}^2} + \frac{2\cdot(200\text{ cfs})^2}{32.2\text{ ft/s}^2\cdot(10\text{ ft})^2}\frac{1}{y_2}$$

Solving this equation for y_2 we get three solutions, −4.95 ft, 1.43 ft and 3.52 ft. The first one is of course impossible, but the other two represent the supercritical and subcritical solutions. In this case $y_2 = 1.43$ ft.

Problem 4.36

Water flows with a velocity of 2 m/s and at a depth of 3 m in a rectangular channel. What is the change in depth and in water surface elevation produced by a gradual upward change in bottom elevation (upstep) of 60 cm? What would be the depth and elevation changes if there were a gradual downstep of 15 cm? What is the maximum size of upstep that could exist before upstream depth changes would result? Neglect head losses.

Solution:

$$E_1 = y_1 + \frac{V_1^2}{2g} = 3\text{ m} + \frac{(2\text{ m/s})^2}{2 \cdot 9.81\text{ m/s}^2} = 3.20\text{ m}.$$

$$E_2 = E_1 - \Delta z = 3.20\text{ m} - 0.60\text{ m} = 2.60\text{ m}.$$

Also

$$E_2 = y_2 + \frac{q^2}{2gy_2^2} = y_2 + \frac{(6\text{ m}^3/\text{s/m})^2}{2 \cdot 9.81\text{ m/s}^2 \cdot y_2^2} = 2.60\text{ m}$$

so $y_2 = 2.24$ m. $\Delta y = y_2 - y_1 = -0.76$ m so water surface drops 0.16 m.

For a downward step of 15 cm we have

$$E_2 = E_1 - \Delta z = 3.20\text{ m} - (-0.15\text{ m}) = 3.35\text{ m}.$$

giving $y_2 = 3.17$ m and $\Delta y = y_2 - y_1 = 0.17$ m so water surface rises 0.02 m.

The maximum upstep possible before affecting upstream water surface levels is for $y_2 = y_c$

$$y_c = \sqrt[3]{\frac{q^2}{g}} = \sqrt[3]{\frac{(6\text{ m}^3/\text{s/m})^2}{9.81\text{ m/s}^2}} = 1.54\text{ m}.$$

Problem 4.39

The spillway shown has a discharge of 1.2 m^3/s per meter of width occurring over it. What depth y_2 will exist downstream of the hydraulic jump? Assume negligible energy loss over the spillway.

Solution:

$$y_0 + \frac{q^2}{2gy_0^2} = y_1 + \frac{q^2}{2gy_1^2}$$

$$5\text{ m} + \frac{(1.2\text{ m}^3/\text{s/m})^2}{2 \cdot 9.81\text{ m/s}^2 \cdot (5\text{ m})^2} = y_1 + \frac{(1.2\text{ m}^3/\text{s/m})^2}{2 \cdot 9.81\text{ m/s}^2 \cdot y_1^2}$$

solving for y_1 we get $y_1 = 0.123$ m.

$$\mathrm{Fr}_1 = \frac{q}{\sqrt{g y_1^3}} = \frac{(1.2\ \mathrm{m^3/s/m})^2}{\sqrt{9.81\ \mathrm{m/s^2} \cdot (0.123\ \mathrm{m})^3}} = 8.88.$$

$$y_2 = \frac{y_1}{2}\left(\sqrt{1 + 8\mathrm{Fr}_1^2} - 1\right) = \frac{0.123\ \mathrm{m}}{2}\left(\sqrt{1 + 8 \cdot 8.88^2} - 1\right) = 1.48\ \mathrm{m}.$$

Problem 4.43

Water is flowing as shown under the sluice gate in a horizontal rectangular channel that is 6 ft wide. The depths y_0 and y_1 are 65 ft and 1 ft respectively. What will be the horsepower lost in the hydraulic jump?

Solution:

Assume negligible energy loss for flow under the sluice gate. Write the bernoulli equation from a section upstream of the sluice gate to a section immediately downstream of the sluice gate.

$$y_0 + \frac{V_0^2}{2g} = y_1 + \frac{V_1^2}{2g}$$

$$65\ \mathrm{ft} + 0 = 1\ \mathrm{ft} + \frac{V_1^2}{2 \cdot 32.2\ \mathrm{ft/s^2}}$$

$$V_1 = \sqrt{(65\ \mathrm{ft} - 1\ \mathrm{ft}) \cdot 2 \cdot 32.2\ \mathrm{ft/s^2}} = 64.2\ \mathrm{ft/s}$$

$$\mathrm{Fr}_1 = \frac{V_1}{\sqrt{g y_1}} = \frac{64.2\ \mathrm{ft/s}}{\sqrt{32.2\ \mathrm{ft/s^2} \cdot 1\ \mathrm{ft}}} = 11.3$$

Now solve for the depth after the jump

$$y_2 = \frac{y_1}{2}\left(\sqrt{1 + 8\mathrm{Fr}_1^2} - 1\right) = \frac{1\ \mathrm{ft}}{2}\left(\sqrt{1 + 8 \cdot 11.3^2} - 1\right) = 15.5\ \mathrm{ft}$$

$$h_L = \frac{(y_2 - y_1)^2}{4 y_1 y_2} = \frac{(15.5\ \mathrm{ft} - 1\ \mathrm{ft})^2}{4 \cdot 1\ \mathrm{ft} \cdot 15.5\ \mathrm{ft}} = 49.2\ \mathrm{ft}$$

$$P = Q\gamma h_L = Vby\gamma h_L = \frac{64.2\ \text{ft/s} \cdot 6\ \text{ft} \cdot 1\ \text{ft} \cdot 62.4\ \text{lb/ft}^3 \cdot 49.2\ \text{ft}}{550\ \text{ft} \cdot \text{lbs/s/HP}} = 2150\ \text{HP}.$$

Problem 4.48

The partial water surface profile shown is for a rectangular channel that is 3 m wide and has water flowing in at a rate of 5 m^3/s. Sketch in the missing part of water surface profile and identify the type(s).

Solution:

Flow over weir:

$$Q = (0.40 + 0.05\frac{H}{P})L\sqrt{2g}H^{\frac{3}{2}} = (0.40 + 0.05\frac{H}{1.6\ \text{m}}) \cdot 3\ \text{m}\sqrt{2 \cdot 9.81\ \text{m/s}^2}H^{\frac{3}{2}} = 5\ \text{m}^3/\text{s}$$

giving $H = 0.917$ m so the depth upstream of the weir is 0.917 m + 1.60 m = 2.52 m.

$$\text{Fr}_1 = \frac{q}{\sqrt{gy^3}} = \frac{\frac{5\ \text{m}^3/\text{s}}{3\ \text{m}}}{\sqrt{9.81\ \text{m/s}^2 \cdot (0.3\ \text{m})^3}} = 3.24 > 1 \quad \text{supercritical}$$

$$\text{Fr}_1 = \frac{q}{\sqrt{gy^3}} = \frac{1.67\ \text{m}^3/\text{s/m}}{\sqrt{9.81\ \text{m/s}^2 \cdot (0.52\ \text{m})^3}} = 0.133 < 1 \quad \text{subcritical}$$

Hydraulic jump forms. Depth downstream of jump

$$y = \frac{0.3\ \text{m}}{2}\left(\sqrt{1 + 8 \cdot 3.24^2} - 1\right) = 1.23\ \text{m}$$

The water surface profile downstream of the hydraulic jump and above the slope is S1 and above the horizontal bottom is H2.

Problem 4.50

A horizontal rectangular concrete channel terminates in a free outfall. The channel is 4 m wide and carries a discharge of water of 12 m^3/s. What is the water depth 300 m upstream from the outfall?

Solution:

$$q = \frac{Q}{b} = \frac{12 \text{ m}^3/\text{s}}{4 \text{ m}} = 3 \text{ m}^3/\text{s/m}$$

$$y_c = \sqrt[3]{\frac{q^2}{g}} = \sqrt[3]{\frac{(3 \text{ m}^3/\text{s/m})^2}{9.81 \text{ m/s}^2}} = 0.972 \text{ m}$$

so start at $x = 4y_c = 3.89$ m

$$\text{Re} = \frac{V \cdot 4R_h}{\nu} = \frac{3.09 \text{ m/s} \cdot 4 \cdot 0.654 \text{ m}}{10^{-6} \text{ m}^2/\text{s}} = 8 \cdot 10^6$$

using values from the first section, giving $f = 0.013$. Carrying out a step solution gives the below table. Depth at $x = 300$ m is then 1.57 m.

section	Depth m	V m/s	$\overline{V}$ m/s	V^2 m^2/s^2	E	R_h m	$\overline{R_h}$ m	S_f	Δx m	x m
1	0.972	3.09			1.458	0.654				3.89
			3.07	9.448			0.656	0.0024	0.04	3.93
2	0.980	3.06			1.458	0.658				
			3.05	9.277			0.660	0.0023	0.17	4.11
3	0.990	3.03			1.458	0.662				
			2.99	8.915			0.669	0.0022	1.30	5.41
4	1.020	2.94			1.461	0.675				
			2.89	8.327			0.684	0.0020	3.65	9.1
5	1.060	2.83			1.468	0.693				
			2.78	7.721			0.701	0.0018	5.95	15.0
6	1.100	2.73			1.479	0.710				
			2.61	6.831			0.730	0.0016	25.44	40.4
7	1.200	2.50			1.519	0.750				
			2.40	5.778			0.769	0.0012	42.48	83
8	1.300	2.31			1.571	0.788				
			2.23	4.952			0.806	0.0010	61.50	144
9	1.400	2.14			1.634	0.824				
			2.07	4.291			0.840	0.0008	82.57	227
10	1.500	2.00			1.704	0.857				
			1.95	3.821			0.869	0.0007	72.70	300
11	1.571	1.91			1.757	0.880				

Problem 4.51

Given the hydraulic jump for the long horizontal rectangular channel, what kind of water surface profile (classification) is upstream of the jump? What kind of water surface profile is downstream of the jump? If baffle blocks are put on the bottom of the channel in the vicinity of A to increase the bottom resistance. What changes are apt to occur given the same gate opening? Explain or sketch the changes.

Solution:

Upstream of jump the profile will be H3. Downstream of jump the profile will be H2. The baffle blocks will cause the depth upstream of A to increase, therefore the jump will move towards the sluice gate.

Problem 4.59

Theory and experimental verification indicate that the mean velocity along a vertical line in a wide stream is closely approximated by the velocity at 0.6 depth. If the indicated velocities at 0.6 depth in a river cross section are measured, what is the discharge in the river?

Solution:

$$Q = \sum V_i A_i$$

V	A	VA
m/s	m^2	m^3/s
1.32	7.6	10.0
1.54	21.7	33.4
1.68	18.0	30.2
1.69	33.0	55.8
1.71	24.0	41.0
1.75	39.0	68.2
1.80	42.0	75.6
1.91	39.0	74.5
1.87	37.2	69.6
1.75	30.8	53.9
1.56	18.4	28.7
1.02	8.0	8.2
	$\sum VA =$	549 m^3/s

Problem 4.63

A flood caused water to flow over a highway as shown below. The water surface elevation upstream of the highway (at A) was measured to be 101.00 ft. The elevation at the top of the crown of the pavement of the highway is 100.10 ft. Estimate the discharge over a stretch of highway with this elevation, which is 100 ft long. What was the depth of flow at the crown of the highway?

Solution:

The flow over the highway is as if flow were occurring over a broad crested weir

$$Q = 0.385CL\sqrt{2g}H^{\frac{3}{2}}$$

Assume $C = 1$

$$Q = 0.385 \cdot 1 \cdot 100 \text{ ft} \sqrt{2 \cdot 32.2 \text{ ft/s}^2}(101.0 \text{ ft} - 100.1 \text{ ft})^{\frac{3}{2}} = 264 \text{ cfs}$$

Critical depth will occur at pt. B.

$$y_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}} = \left(\frac{(2.64 \text{ ft}^2/\text{s})^2}{32.2 \text{ ft/s}^2}\right)^{\frac{1}{3}} = 0.60 \text{ ft.}$$