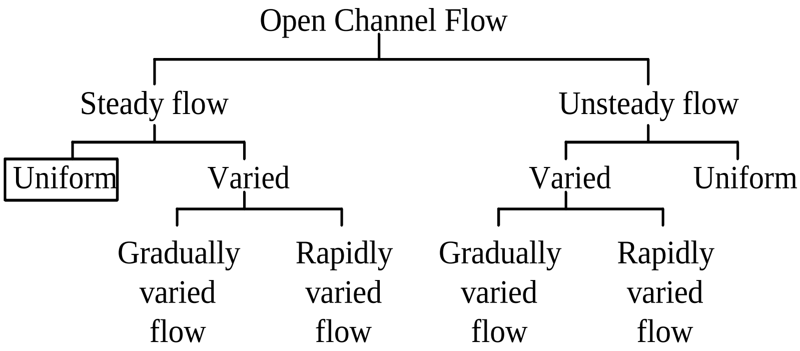
8 OPEN CHANNEL FLOW

8.1 Classification & Definition

- Open channel flows are flows in rivers, streams, artificial channels, irrigation ditches, partially filled pipe etc.

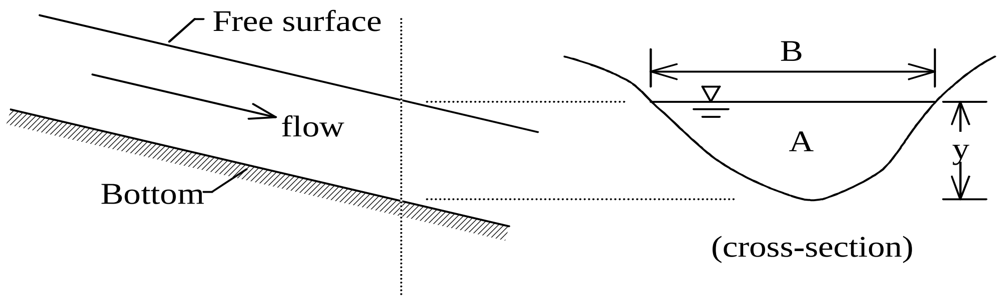
- Basically, it is a flow with **free surface**.
  (Free surface is a surface with atmospheric pressure)



Classifications of Open Channel Flow

8.1.1 Open Channel Geometry

1. Depth of flow, **y**: **vertical** distance from the bottom to surface.



2 Top width, **B**:
– the width of the channel at the free surface

3 Flow area, **A**:
– cross-sectional area of the flow

4 Wetted perimeter, **P**:
– the length of the channel cross-section in contact with the fluid

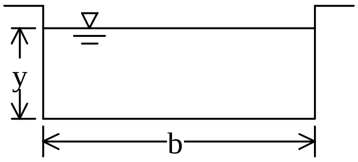
5 Hydraulic radius (hydraulic mean depth), **R**:

$$R = \frac{\text{Flow area}}{\text{Wetted perimeter}} = \frac{A}{P}$$

6 Average depth (hydraulic average depth), $\mathbf{y_{ave}}$:

$$y_{ave} = \frac{\text{Flow area}}{\text{Top width}} = \frac{A}{B}$$

8.1.2 Rectangular channel



– $B = b$

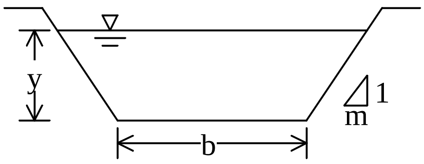
– $A = b*y$

– $P = b+2*y$

– $R = \dfrac{b*y}{b+2*y}$

– $y_{ave} = y$

8.1.3 Trapezoidal channel



– $B = b + 2*m*y$

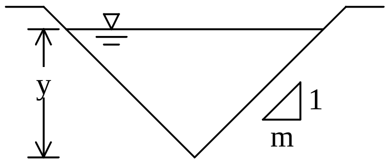
– $A = y*(b+m*y)$

– $P = b+2*y*\sqrt{1+m^2}$

– $R = \dfrac{y*(b+m*y)}{b+2*y*\sqrt{1+m^2}}$

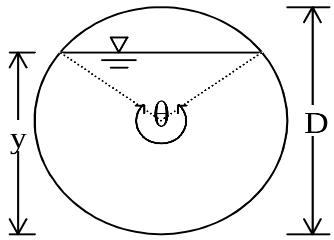
– $y_{ave} = \dfrac{y*(b+m*y)}{b+2*m*y}$

8.1.4 Triangular channel



- B $= 2*m*y$
- A $= m*y^2$
- P $= 2*y*\sqrt{1+m^2}$
- R $= \dfrac{m*y}{2*\sqrt{1+m^2}}$
- y_{ave} $= \dfrac{y}{2}$

8.1.5 Circular channel



(θ in radian)

- B $= 2*\sqrt{y*(D-y)}$
- A $= \dfrac{D^2*(\theta-\sin\theta)}{8}$
- P $= \dfrac{\theta*D}{2}$
- R $= \dfrac{D}{4}\left(1-\dfrac{\sin\theta}{\theta}\right)$
- y_{ave} $= \dfrac{D*(\theta-\sin\theta)}{8*\sin\theta/2}$

8.2 Steady Uniform Flow

- For a steady uniform flow
  - **depth** is **constant** along the flow
  - **velocity** is **constant** over the cross-section
  - **time independent**

8.2.1 Manning Equations

- In 1890, Manning, an Irish engineer derived a better and more accurate relationship, **Manning equation**, based on many field measurement.

$$V = \frac{1}{n} * R^{2/3} * S^{1/2} \tag{8.1}$$

  - **n - Manning's coefficient, s/m$^{1/3}$**
    (can be found in most of the hydraulic handbooks)

- To incorporate the continuity equation, Manning equation becomes

$$Q = \frac{A}{n} * R^{2/3} * S^{1/2} \tag{8.2}$$

- As the flow according to Manning equations is for normal steady uniform flow,
  - the flow is **Normal Flow**
  - the depth is **Normal Depth**

Worked examples:

1. Water flows in a rectangular, concrete, open channel that is 12 m wide at a depth of 2.5m. The channel slope is 0.0028. Find the water velocity and the flow rate. (n = 0.013)

Answer

By Manning equation,

$$V = \frac{1}{n} * R^{2/3} * S^{1/2}$$

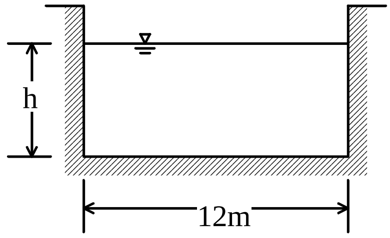
with n = 0.013
S = 0.0028

$A = 12 * 2.5 \text{ m}^2 = 30 \text{ m}^2$
$P = 12 + 2*2.5 \text{ m} = 17 \text{ m}$
$\therefore$ $R = A/P$
$= 30 / 17 \text{ m} = 1.765 \text{ m}$

hence $V = \frac{1}{0.013} * (1.765)^{2/3} * (0.0028)^{1/2}$
$= 5.945 \text{ m/s}$

Discharge, $Q = A*V$
$= 30*5.945 \text{ m}^3/\text{s}$
$= 178.3 \text{ m}^3/\text{s}$

2. Water flows in a rectangular, concrete, open channel that is 12 m wide. The channel slope is 0.0028. If the velocity of the flow is 6 m/s, find the depth of the flow. (n = 0.013)



Answer

By Manning equation,

$$V = \frac{1}{n} * R^{2/3} * S^{1/2}$$

with V = 6 m/s
n = 0.013
S = 0.0028

$$A = 12 * h \quad m^2$$

$$P = 12 + 2*h \ m$$

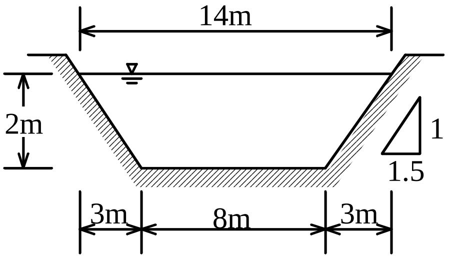
$$\therefore \quad R = \frac{A}{P} = \frac{12*h}{12+2*h} = \frac{6*h}{6+h}$$

$$\frac{6*h}{6+h} = 1.790$$

$$h = 2.551 \ m$$

Depth of the flow = 2.551m

3. A trapezoidal channel with side slopes of 2/3, a depth of 2 m, a bottom width of 8 m and a channel slope of 0.0009 has a discharge of 56 m^3/s. Find the Manning's n.



Answer

$A = (14+8)*2/2\ m^2$
$= 22\ m^2$

$P = 8 + 2* \sqrt{2^2+3^2}\ m$
$= 15.211\ m$

$A/P = 22 / 15.211\ m$
$= 1.446\ m$

By Manning equation,

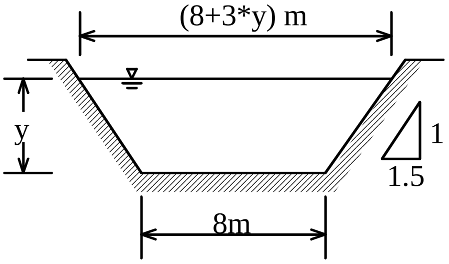
$$Q = \frac{A}{n} * R^{2/3} * S^{1/2}$$

$Q = 56\ m^3/s$, $S = 0.0009$

$$56 = \frac{22}{n} * (1.446)^{2/3} * (0.0009)^{1/2}$$

$n = 0.01507$

4. Determine the depth in a trapezoidal channel with side slopes of 1 to 1.5, a bottom width of 8 m and a channel slope of 0.0009. The discharge is 56 m^3/s and n = 0.017.



Answer

$A = (8+8+3*y)*y/2 \ m^2$

$= (8+1.5*y)*y \ m^2$

$P = 8 + 2*y*\sqrt{1^2 + 1.5^2} \quad m$

$= 8+3.6056*y \quad m$

$R = A/P$

$= (8+1.5*y)*y \,/\, 8+3.6056*y$

By Manning equation,

$$Q = \frac{A}{n} * R^{2/3} * S^{1/2}$$

$Q = 56 \ m^3/s, \quad S = 0.0009$

$$\therefore \quad 56 = \frac{(8+1.5*y)*y}{0.017} * \left[\frac{(8+1.5*y)*y}{8+3.6056*y}\right]^{2/3} * (0.0009)^{1/2}$$

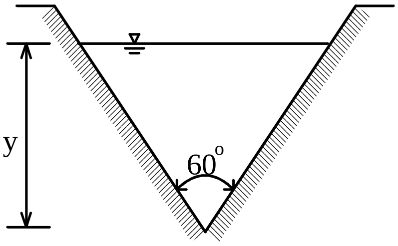
or

$$\frac{[(8+1.5*y)*y]^{5/3}}{[8+3.6056*y]^{2/3}} = 31.7333$$

$$\frac{[(1+0.1875*y)*y]^{5/3}}{[1+0.4507*y]^{2/3}} - 3.9667 = 0$$

By trial & error, y = 2.137 m.
The depth of the trapezoidal channel is 2.137m.

5. Water flows in the triangular steel channel shown in the figure below. Find the depth of flow if the channel slope is 0.0015 and the discharge is 0.22 m^3/s. (n=0.014)



Answer

$A = 2y\tan 30^\circ * y/2 \ m^2$

$\quad = y^2 * \tan 30^\circ \ m^2$

$P = 2y/\cos 30^\circ \ m$

$R = A/P = y^2 * \tan 30^\circ / 2y/\cos 30^\circ \ m$

$\quad = y\sin 30^\circ/2 \ m$

By Manning equation,

$$Q = \frac{A}{n} * R^{2/3} * S^{1/2}$$

$Q = 0.22 \ m^3/s, \quad S = 0.0015$

$$0.22 = \frac{y^2 \tan 30^\circ}{0.014} * \left(\frac{y\sin 30^\circ}{2}\right)^{2/3} * \sqrt{0.0015}$$

$\quad = y^{8/3} * 0.6338$

or $$y = \left(\frac{0.22}{0.6338}\right)^{3/8} \ m$$

$\quad = 0.672 \ m$

Depth of the channel is 0.672 m.

8.2.2 Optimum Hydraulic Cross-sections (REFERENCE ONLY)

- From Manning equation,

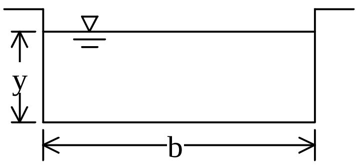
$$Q = \frac{1}{n} * \frac{A^{5/3} * \sqrt{S}}{P^{2/3}}$$

Hence, Q will be **maximum** when P is a minimum.

- For a given cross-sectional area, A of an open channel, the discharge, Q is maximum when the wetted perimeter, P is minimum. Hence if the wetted perimeter, P for a given flow area is minimised, the area, A will give the least expensive channel to be construct.

- This corresponding cross-section is the **optimum hydraulic section** or the **best hydraulic section**.

8.2.2.1 Rectangular section



width = b

depth = y

area, A = by

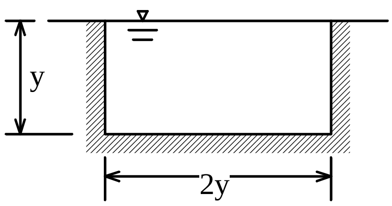
$$P = b + 2*y$$

$$= \frac{A}{y} + 2y$$

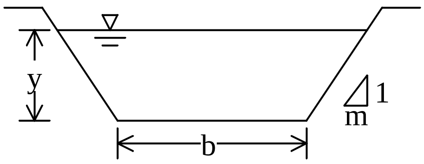
Hence $\frac{dP}{dy} = -\frac{A}{y^2} + 2 = 0$

i.e. $y = \sqrt{\frac{A}{2}}$ or $b = 2y$

Therefore, the optimum rectangular section is



8.2.2.2 Trapezoidal section



$$B = b+2*m*y$$

$$A = (b+m*y)*y$$

$$P = b+2*y*\sqrt{1+m^2}$$

By eliminating b from P,

$$P = \frac{A}{y} + (2*\sqrt{1+m^2} - m)*y$$

For a minimum value of P, $\delta P = 0$,

i.e. $\frac{dP}{dy} = 0$ and $\frac{dP}{dm} = 0$

From $\frac{dP}{dy} = 0$, $y^2 = \frac{A}{\sqrt{3}}$

From $\frac{dP}{dm} = 0$, $m = \frac{1}{\sqrt{3}}$

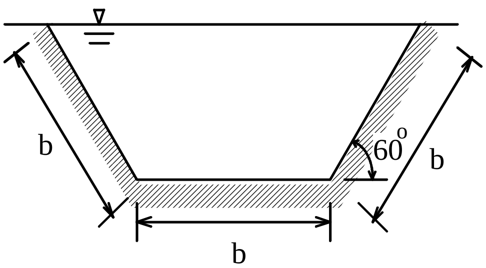
It implies the side slope of the channel is 60° to horizontal.

$$b = \frac{A}{y} - my = \sqrt{3}y - \frac{y}{\sqrt{3}} = \frac{2\sqrt{3}}{3}y$$

and $P = \frac{2\sqrt{3}}{3}y + \frac{4}{\sqrt{3}}y = 2\sqrt{3}y$

i.e. $P = 3*b$

The optimum section is given as follow:

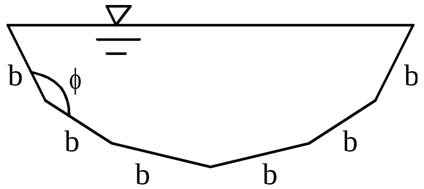


8.2.2.3 Other sections

N-side Channel

- from the conclusion of the previous two sections
  - reflection of the rectangular optimum section about the water surface will form a **square** of side b.
  - reflection of the trapezoidal optimum section about the water surface will form a regular **hexagon** of side b.

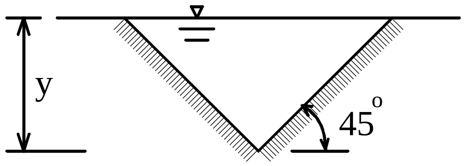
- For a N-side channel, the optimum hydraulic section should be in a form of half a **2N-side regular polygon**.



$$\phi = \left(\frac{N-1}{N}\right)*180°$$

Triangular Section

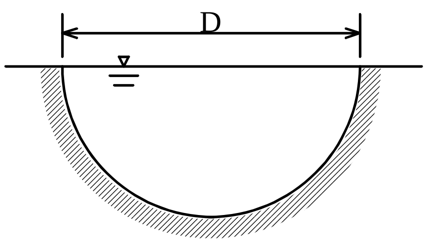
- N = 2, hence $\phi = 90°$



Circular Section

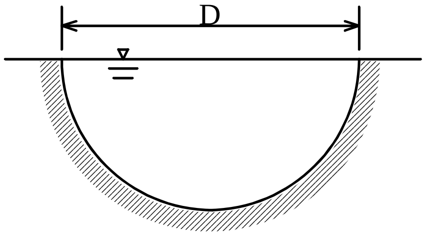
- From the result of N-side channel, it can be concluded that the optimum section of a circular channel is a **semi-circle**.

- It is the most optimum section for all the possible open-channel cross-section.



Worked examples

1. An open channel is to be designed to carry $1 m^3/s$ at a slope of 0.0065. The channel material has an n value of 0.011. Find the optimum hydraulic cross-section for a semi-circular section.



Answer

The optimum circular section is a semi-circular section with diameter D which can discharge $1\ m^3/s$.

For a semi-circular section,

$A = \pi * D^2/8$

$P = \pi * D/2$

$R = A/P$

$= D/4$

As n = 0.011, S = 0.0065 and Q = $1\ m^3/s$.

$$Q = \frac{A}{n} * R^{2/3} * S^{1/2}$$

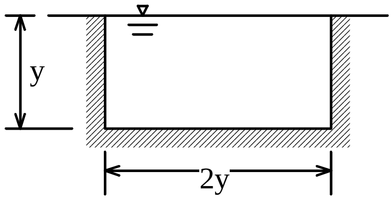
i.e. $$1 = \frac{\pi * D^2}{8 * 0.011} * \left(\frac{D}{4}\right)^{2/3} * \sqrt{0.0065}$$

$$D^{8/3} = \frac{8 * 0.011}{\pi} * 4^{\frac{2}{3}} * 0.0065^{-\frac{1}{2}}$$

$D = 0.951\ m$

The diameter of this optimum section is 951mm.

2. Find the optimum rectangular section from the last example.



Answer

A = $2*y^2$
P = $4*y$
R = $A/P = y/2$

By Manning equation,

$$Q = \frac{A}{n} * R^{2/3} * S^{1/2}$$

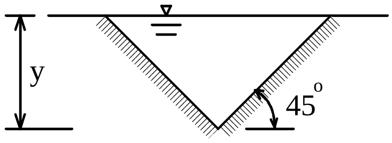
$$1 = \frac{2*y^2}{0.011} * \left(\frac{y}{2}\right)^{\frac{2}{3}} * \sqrt{0.0065}$$

$$y^{8/3} = \frac{1*0.011*2^{\frac{2}{3}}}{2*\sqrt{0.0065}}$$

y = 0.434 m

The optimum rectangular section has dimension of width 0.868m and depth 0.434m.

3. Find the optimum triangular section from the last example.



Answer

$$A = y^2$$
$$P = 2\sqrt{2} * y$$
$$R = A/P = y/2\sqrt{2}$$

By Manning equation,

$$Q = \frac{A}{n} * R^{2/3} * S^{1/2}$$

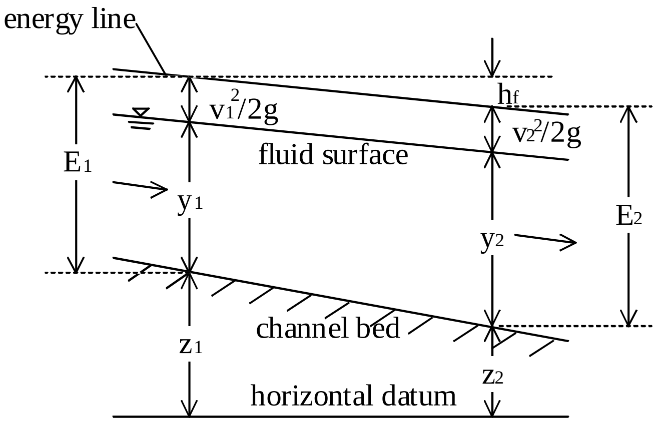
$$1 = \frac{y^2}{0.011} * \left(\frac{y}{2\sqrt{2}}\right)^{\frac{2}{3}} * \sqrt{0.0065}$$

$$y^{8/3} = \frac{0.011 * (2\sqrt{2})^{\frac{2}{3}}}{\sqrt{0.0065}}$$

$$y = 0.614 \text{ m}$$

The optimum triangular section is a right angle triangle with depth 0.614 m.

8.3 Non-Uniform flow - Specific Energy in Open Channel & Critical Flow



- In open channel, the solution of many problems are greatly assisted by the concept of **specific energy**, i.e.

$$E = \frac{v^2}{2g} + y \qquad (8.3)$$

In terms of flow rate, Q,

$$E = \frac{1}{2g}\left(\frac{Q}{A}\right)^2 + y \qquad (8.4)$$

- The minimum energy will be given as

$$\frac{dE}{dy} = 0 \qquad (8.5)$$

8.3.1 Rectangular Channel

- Let $$q = \frac{Q}{b} = v*y \qquad (8.6)$$

q - the discharge per unit width of a rectangular channel

$$\therefore \quad E = \frac{q^2}{2gy^2} + y \qquad (8.7)$$

- By assuming q is constant

$$\frac{dE}{dy} = 1 - \frac{q^2}{gy^3} = 0 \qquad (8.8)$$

or $$y = y_c$$

$$= \left(\frac{q^2}{g}\right)^{1/3} \qquad (8.9)$$

y_c - critical depth at which the energy is minimum.

- The corresponding energy, E is

$$E_{min} = \frac{3}{2} y_c \qquad (8.10)$$

- From (8.6), $v = \frac{q}{y}$.

Substitute into (8.8),

$$1 - \frac{v_c^2}{gy_c} = 0$$

$$\frac{v_c^2}{gy_c} = 1 \qquad (8.11)$$

or $$v_c = \sqrt{gy_c} \qquad (8.12)$$

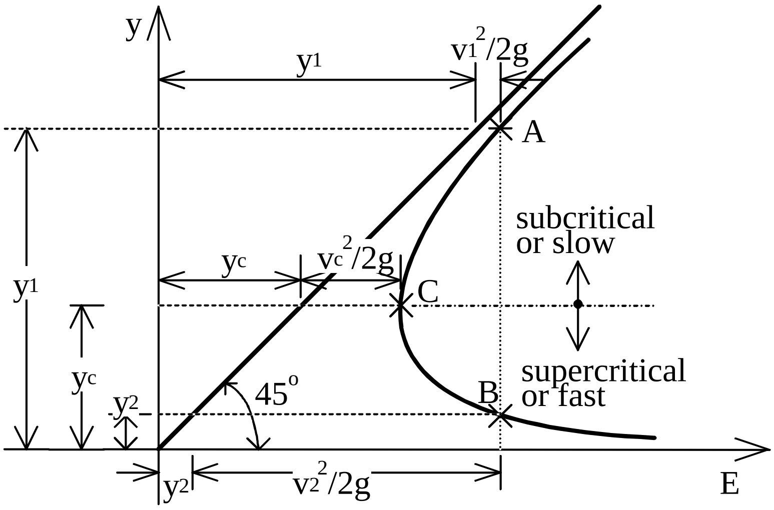
- Since Froude number, Fr is defined as

$$Fr = \frac{v}{\sqrt{gy_{ave}}} \qquad (8.13)$$

Hence, the minimum energy is occurred when

$$Fr^2 = 1 \qquad (8.14)$$

- For a given discharge, Q, if the flow is such that E is a min., the flow is **critical flow.**
  - critical flow - flow with E_{min}
  - critical depth, yc - the depth of the critical flow
  - critical velocity - $v_c = \sqrt{gy_c}$



- If the flow with $E > E_{min}$, there are two possible depths (y_1, y_2).
  - (y_1, y_2) are called **alternate depths.**

- C divides the curve AB into AC and CB regions.
  - AC - **subcritical** flow region
  - CB - **supercritical** flow region

	Subcritical	Critical	Supercritical
Depth of flow	$y > y_c$	$y = y_c$	$y < y_c$
Velocity of flow	$v < v_c$	$v = v_c$	$v > v_c$
Slope	Mild $S < S_c$	Critical $S = S_c$	Steep $S > S_c$
Froude number	$Fr < 1.0$	$Fr = 1.0$	$Fr > 1.0$
Other	$\frac{v^2}{2g} < \frac{y_c}{2}$	$\frac{v^2}{2g} = \frac{y_c}{2}$	$\frac{v^2}{2g} > \frac{y_c}{2}$

8.3.2 Non - Rectangular Channel

- If the channel width varies with y, the specific energy must be written in the form

$$E = \frac{Q^2}{2gA^2} + y \qquad (8.15)$$

- The minimum energy also occurs where

$$\frac{dE}{dy} = 0 \quad \text{at constant } Q$$

- Since A = A(y), therefore (8.15) becomes

$$1 - \frac{2Q^2A^{-3}}{2g}\frac{dA}{dy} = 0$$

or
$$\frac{dA}{dy} = \frac{gA^3}{Q^2} \qquad (8.16)$$

- Since $\frac{dA}{dy} = B$ - the channel width at the free surface,

$$\therefore \quad B = \frac{gA^3}{Q^2}$$

or
$$A = \left(\frac{BQ^2}{g}\right)^{1/3} \qquad (8.17)$$

$$v_c = \frac{Q}{A}$$

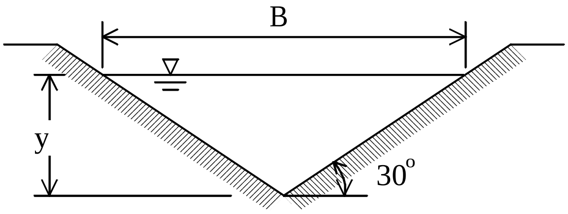
$$= \left(\frac{gA}{B}\right)^{1/2} \qquad (8.18)$$

- For a given channel shape, A(y) & B(y), and a given Q, (8.17) & (8.18) have to be solved by trial and error to find the A and then v_c.

- If a critical channel flow is also moving uniformly (at constant depth), it must correspond to a critical slope, S_c, with $y_n = y_c$. This condition can be analysed by Manning formula.

Worked examples:

1. A triangular channel with an angel of 120° made by 2 equal slopes. For a flow rate of 3 m^3/s, determine the critical depth and hence the maximum depth of the flow.



Answer

For critical flow,

$$v^2 = g*y_{ave}$$

$$Q^2 = g*y_{ave}*A^2$$

$$= \frac{gA^3}{B} \qquad (y_{ave} = \frac{A}{B})$$

For critical flow,

$$B = 2*y*\cot 30°$$

$$\& \quad A = y^2*\cot 30°$$

$$\therefore \quad Q^2 = \frac{3g}{2}y^5$$

Hence

$$y = \left(\frac{2Q^2}{3g}\right)^{1/5}$$

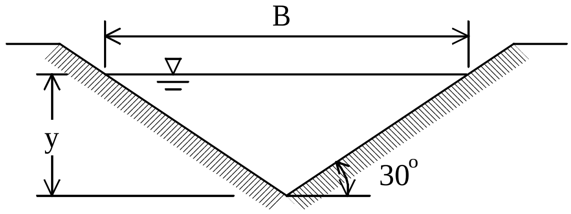
$$= \left(\frac{2*3^2}{3*9.81}\right)^{1/5} \text{ m}$$

$$= \underline{0.906 \text{ m}}$$

The maximum depth is 0.906 m.

The critical depth, $y_c = y_{ave} = \frac{A}{B} = \frac{(B*y)/2}{B} = \frac{y}{2} = \frac{0906}{2} = 0.453m$.

2. In the last example, the channel Manning roughness coefficient is 0.012 and the flow rate is 3 m^3/s. What is the value of the channel slope if the flow is critical, subcritical or supercritical?



Answer

$$B = 2\sqrt{3}*y$$

$$A = \frac{1}{2}*B*y$$

$$P = 4*y$$

Using Manning equation,

$$Q = \frac{A}{n}*R^{2/3}*S^{1/2}$$

$$= \frac{1}{n}*(\frac{1}{2}*B*y)*(\frac{B}{8})^{2/3}*S^{1/2}$$

$$S_c^{1/2} = \frac{2nQ}{By}(\frac{B}{8})^{-2/3}$$

$$= \frac{2nQ}{2\sqrt{3}*y^2}(\frac{\sqrt{3}*y}{4})^{-2/3}$$

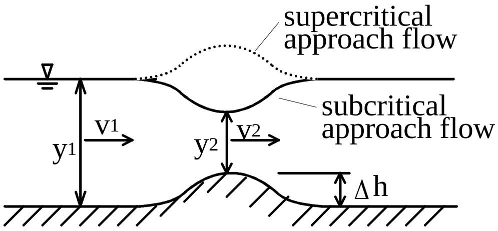
For critical flow, y = y

$$\therefore \quad S_c^{1/2} = \frac{2*0.012*3}{2\sqrt{3}*(0.906)^2}(\frac{\sqrt{3}*0.906}{4})^{-2/3}$$

$$S_c = 0.0472$$

For flow is	critical,	S = 0.0472	- critical slope
	subcritical,	S < 0.0472	
	supercritical,	S > 0.0472	

8.4 Frictionless Flow over a Bump



- When fluid is flowing over a bump, the behaviour of the free surface is sharply different according to whether the approach flow is subcritical or supercritical.

- The height of the bump can change the character of the results.

- Applying Continuity and Bernoulli's equations to sections 1 and 2,

$$v_1 * y_1 = v_2 * y_2$$

$$\& \qquad \frac{v_1^2}{2g} + y_1 = \frac{v_2^2}{2g} + y_2 + \Delta h$$

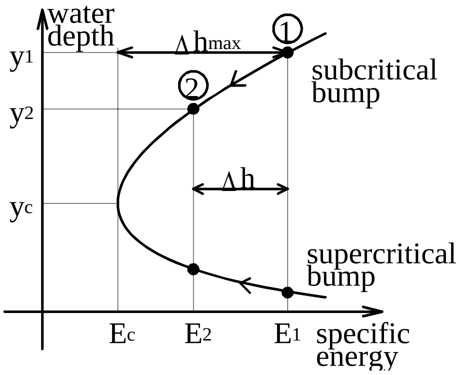
- Eliminating v_2 between these two gives a cubic polynomial equation for the water depth y_2 over the bump,

$$y_2^3 - E_2 * y_2^2 + \frac{v_1^2 * y_1^2}{2g} = 0 \qquad (8.19)$$

$$\text{where} \quad E_2 = \frac{v_1^2}{2g} + y_1 - \Delta h \qquad (8.20)$$

This equation has one negative and two positive solutions if Δh is not too large.

- The free surface's behaviour depends upon whether condition 1 is in subcritical or supercritical flow.



The specific energy E_2 is exactly Δh less than the approach energy, E_1, and point 2 will lie on the same leg of the curve as E_1.

- A subcritical approach, $Fr_1 < 1$, will cause the water level to decrease at the bump.

- Supercritical approach flow, $Fr_1 > 1$, causes a water level increase over the bump.

- If the bump height reaches $\Delta h_{max} = E_1 - E_c$, the flow at the crest will be exactly critical ($Fr = 1$).

- If the bump > Δh_{max}, there are no physical correct solution. That is, a bump to large will choke the channel and cause frictional effects, typically a hydraulic jump.

Worked example:

Water flow in a wide channel approaches a 10 cm high bump at 1.5 m/s and a depth of 1 m. Estimate

(a) the water depth y_2 over the bump, and
(b) the bump height which will cause the crest flow to be critical.

Answer

(a) For the approaching flow,

$$Fr = \frac{v_1}{\sqrt{gy_1}} = \frac{1.5}{\sqrt{9.81*1}}$$

$$= 0.479 \Rightarrow \text{subcritical}$$

For subcritical approach flow, if Δh is not too large, the water level over the bump will depress and a higher subcritical Fr at the crest.

$$E_1 = \frac{v_1^2}{2g} + y_1 = \frac{1.5^2}{2*9.81} + 1.0 \text{ m}$$

$$= 1.115 \text{ m}$$

Hence $E_2 = E_1 - \Delta h = 1.115 - 0.1$ m

$$= 1.015 \text{ m}$$

Substitute E_2 into (8.24),

$$y_2^3 - 1.015*y_2^2 + 0.115 = 0$$

By trial and error,

y_2 = 0.859 m, 0.451 m and –0.296 m (inadmissible)

The second (smaller) solution is the supercritical condition for E_2 and is not possible for this subcritical bump.

Hence y_2 = 0.859 m

Checking: v_2 = 1.745 m/s (By continuity)

Fr_2 = 0.601 ($> Fr_1$ and < 1) (OK)

(b) By considering per m width of the channel,

$$q = v*y = 1.5*1 \ m^2/s$$

For critical flow,

$$E_2 = E_{min} = \frac{3}{2} y_c$$

$$y_c = \left(\frac{q^2}{g}\right)^{1/3}$$

$$= \left(\frac{1.5^2}{9.81}\right)^{1/3}$$

$$= 0.612 \ m$$

$$E_2 = \frac{3}{2} * 0.612 \quad m$$

$$= 0.918 \ m$$

$$\Delta h_{max} = E_1 - E_{min}$$

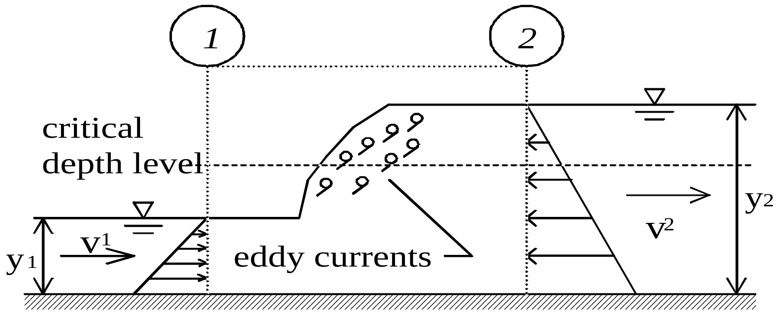
$$= 1.115 - 0.918 \ m$$

$$= \underline{0.197 \ m}$$

8.5 Hydraulic Jump in Rectangular Channel

- A hydraulic jump is a **sudden** change from a supercritical flow to subcritical flow.

- Assumptions:
  - the bed is horizontal.
  - the velocity over each cross-section is uniform.
  - the depth is uniform across the width.
  - frictionless boundaries.
  - surface tension effects are neglect.



- Considering the control volume between 1 and 2, the forces are

$$F_{31} = \rho g y_1 * \frac{b}{2} * y_1 \qquad = \rho g b * \frac{y_1^2}{2} \qquad (8.21a)$$

Similarly $$F_{32} = \rho g b * \frac{y_2^2}{2} \qquad (8.21b)$$

- By continuity equation,

$$Q = b*y_1*v_1 \qquad = b*y_2*v_2 \qquad (8.22)$$

- By the momentum equation,

$$F_1 = F_2 = 0$$

hence $$F_{31} - F_{32} = \rho*Q*(v_2 - v_1) \qquad (8.23)$$

- Sub. (8.21a, b) and (8.22) into (8.23), then

$$\frac{\rho g b}{2}(y_1^2 - y_2^2) = \rho Q\left(\frac{Q}{y_2 b} - \frac{Q}{y_1 b}\right)$$

$$= \frac{\rho Q^2}{b}\left(\frac{y_1 - y_2}{y_1 y_2}\right) \qquad (8.24)$$

- In a hydraulic jump, $y_1 \neq y_2$,

$$y_1{*}y_2{*}(y_1+y_2) = \frac{2Q^2}{gb^2}$$

$$\therefore \qquad y_1^2 y_2 + y_1 y_2^2 = \frac{2Q^2}{gb^2}$$

i.e.
$$\left(\frac{y_2}{y_1}\right)^2 + \left(\frac{y_2}{y_1}\right) - \frac{2Q^2}{gb^2 y_1^3} = 0 \qquad (8.25)$$

Solving (8.25),

$$\frac{y_2}{y_1} = \frac{1}{2}\left[-1 + \sqrt{1 + \frac{8Q^2}{gb^2 y_1^3}}\right] \qquad (8.26)$$

This is the hydraulic jump equation.

- Using Froude number,

$$Fr_1^2 = \frac{v_1^2}{gy_1} = \frac{Q^2}{gy_1^3 b^2} \qquad (8.27)$$

then,
$$\frac{y_2}{y_1} = \frac{1}{2}\left[-1 + \sqrt{1 + 8Fr_1^2}\right] \qquad (8.28a)$$

or
$$\frac{y_1}{y_2} = \frac{1}{2}\left[-1 + \sqrt{1 + 8Fr_2^2}\right] \qquad (8.28b)$$

- (y_1, y_2) are called **conjugate depths.**

- The energy loss in a jump is given by

$$\frac{v_1^2}{2g} + y_1 = \frac{v_2^2}{2g} + y_2 + h_f$$

i.e.
$$h_f = \left(\frac{v_1^2 - v_2^2}{2g}\right) + (y_1 - y_2) \qquad (8.29)$$

- Sub. (8.22) into above,

$$h_f = \left[-\frac{Q^2}{2gb^2}\left(\frac{y_1 + y_2}{y_1^2 y_2^2}\right) + 1\right](y_1 - y_2) \qquad (8.29)$$

◆ Using (8.25), (8.29) becomes

$$h_f = \frac{(y_2 - y_1)^3}{4y_1y_2} \qquad (8.30)$$

This is the energy loss equation for the hydraulic jump ($y_2>y_1$, $h_f>0$).

◆ The power loss in a jump is

$$P = \rho g h_f * Q$$

◆ This energy loss is useful for getting away with the unwanted energy of a flow. The energy loss is due to the frictional forces amount the eddy currents in the pump. It will increase the temperature of the fluid.

Worked example:

Water flows in a wide channel at $q = 10\ m^2/s$ and $y_1 = 1.25$ m. If the flow undergoes a hydraulic jump, calculate

(a) y_2,
(b) v_2,
(c) Fr_2,
(d) h_f, and
(e) the percentage dissipation of the energy.

Answer

(a)

$$v_1 = \frac{q}{y_1} = \frac{10}{1.25}\ \text{m/s} = 8\ \text{m/s}$$

$$Fr_1 = \frac{v_1}{\sqrt{gy_1}} = \frac{8}{\sqrt{9.81*1.25}} = 2.285$$

Since

$$\frac{y_2}{y_1} = \frac{1}{2}[-1+\sqrt{1+8Fr_1^2}] = \frac{1}{2}[-1+\sqrt{1+8*(2.285)^2}] = 2.77$$

or $y_2 = 2.77*1.25$ m $=$ <u>3.46 m</u>

(b) By Continuity equation,

$$v_2 = v_1*(\frac{y_1}{y_2}) = 8*\frac{1.25}{3.46}\ \text{m/s} = \underline{2.89\ \text{m/s}}$$

(c) $Fr_2 = \dfrac{v_2}{\sqrt{gy_2}}$

$= \dfrac{2.89}{\sqrt{9.81 * 3.46}}$

$= \underline{0.496}$

(d) $h_f = \dfrac{(y_2 - y_1)^3}{4y_1y_2}$

$= \dfrac{(3.46 - 1.25)^3}{4 * 3.46 * 1.25}$

$= \underline{0.625 \text{ m}}$

(e) $E_1 = \dfrac{{v_1}^2}{2g} + y_1$

$= \dfrac{8^2}{2 * 9.81} + 1.25 \quad \text{m}$

$= \underline{4.51 \text{ m}}$

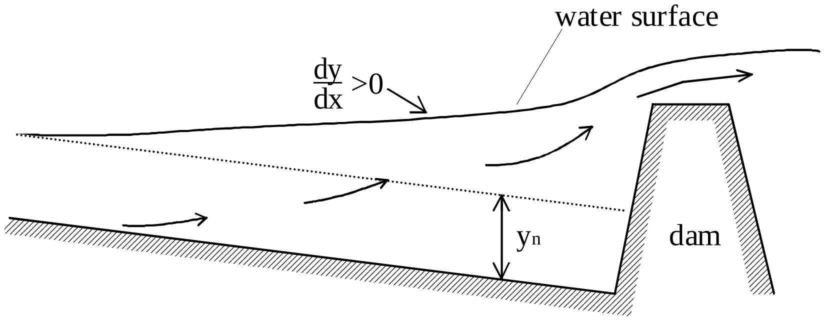
percentage loss $= \dfrac{h_f}{E_1} * 100\%$

$= \dfrac{0.625}{4.51} * 100\%$

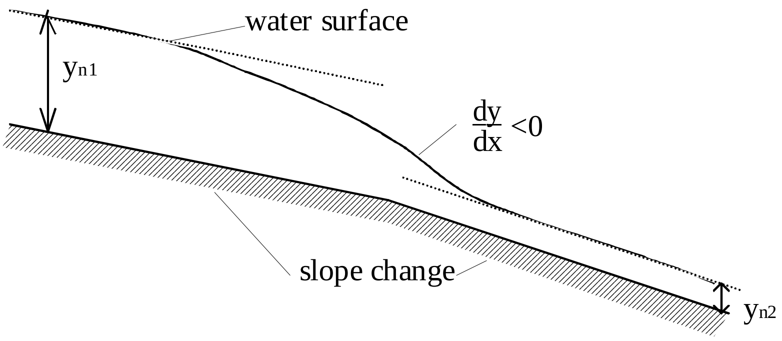
$= \underline{14 \ \%}$

8.6 Gradually Varied Flow

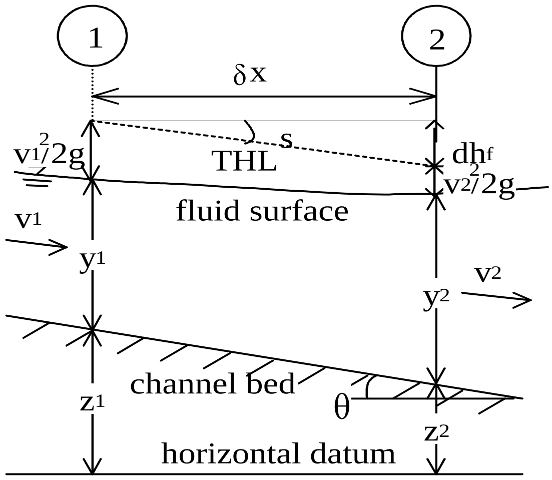
- It is not always possible to have uniform depth across the flow i.e. normal flow with normal depth.
- The depth of flow can be changed by the conditions along the channel.
- Examples of Gradually Varied Flow are:
  - backwater curve



  - Downdrop curve



- In a **uniform flow**, the body weight effect in balanced out by the wall friction.
- In **gradually varied flow**, the weight and the friction effects are unable to make the flow uniform.



- Basic assumptions are
  - slowly changing bottom slope
  - slowly changing water depth (no hydraulic jump)
  - slowly changing cross section
  - one dimensional velocity distribution
  - pressure distribution approximately hydrostatic

- Denoting

v_1	$= v;$	v_2	$= v + dv$
z_1	$= z,$	z_2	$= z + dz$
y_1	$= y,$	y_2	$= y + dy$
p_1	$= p,$	p_2	$= p$

Apply Bernoulli's equation between section 1 and 2,

$$\frac{p}{\gamma} + \frac{v^2}{2g} + y + z = \frac{p}{\gamma} + \frac{(v + dv)^2}{2g} + (y + dy) + (z + dz) + dh_f$$

Neglecting higher order terms,

$$dh_f + dy + dz + \frac{v}{g} du \quad = 0$$

When lim dx → 0,

$$\frac{dh_f}{dx} + \frac{dy}{dx} + \frac{dz}{dx} + \frac{v}{g}\frac{dv}{dx} = 0 \qquad (8.30)$$

- In (8.30), the four terms are

$\frac{dh_f}{dx}$ - rate of head loss along the channel

= S (head loss gradient in Manning equation)

$\frac{dy}{dx}$ - rate of change of water depth

- water surface profile's gradient

$\frac{dz}{dx}$ - rate of vertical change along channel

= -sinθ

$\frac{v}{g}\frac{dv}{dx}$ - rate of change of velocity head along the channel

- By Continuity equation

$$v*A = \text{constant}$$

i.e.

$$A\frac{dv}{dx} + v\frac{dA}{dx} = 0$$

$$A\frac{dv}{dx} + v\frac{dA}{dy}\frac{dy}{dx} = 0$$

$$A\frac{dv}{dx} + vB\frac{dy}{dx} = 0$$

or

$$\frac{dv}{dx} = -\frac{vB}{A}\frac{dy}{dx}$$

$$= -\frac{v}{y}\frac{dy}{dx}$$

Therefore

$$\frac{v}{g}\frac{dv}{dx} = -\frac{v^2}{gy}\frac{dy}{dx}$$

$$= -Fr^2\frac{dy}{dx} \qquad (8.31)$$

- Hence (8.30) becomes

$$s + \frac{dy}{dx} - \sin\theta - Fr^2\frac{dy}{dx} = 0$$

or

$$\frac{dy}{dx} = \left[\frac{\sin\theta - S}{1 - Fr^2}\right] \qquad (8.32)$$

- general equation of gradually varied flow.

- (8.32) is a 1st order non-linear differential equation. Numerical method is used to solve the equation.

  The equation is rewritten as

$$\frac{dx}{dy} = \left[\frac{1-Fr^2}{\sin\theta - S}\right]$$

$$\int_{x_1}^{x_2} dx = \int_{y_1}^{y_2}\left(\frac{1-Fr^2}{\sin\theta - S}\right)dy$$

$$x_2 = x_1 + \int_{y_1}^{y_2}\left(\frac{1-Fr^2}{\sin\theta - S}\right)dy \qquad (8.33)$$

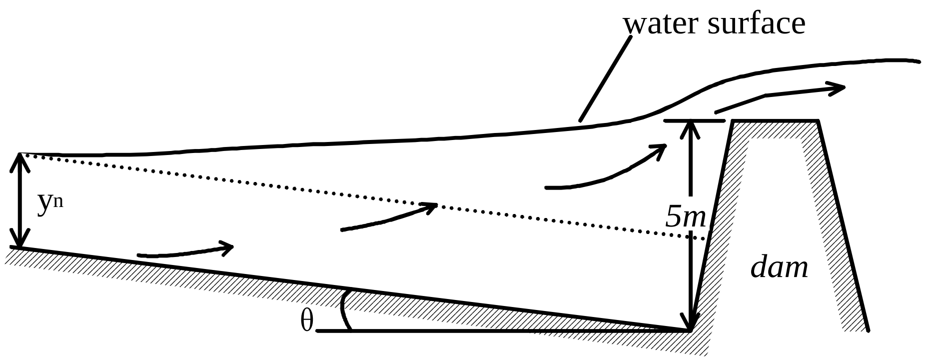
  The simplest solution is the direct mid-point solution of the integral.

  i.e. $$x_2 = x_1 + \left(\frac{1-Fr^2}{\sin\theta - S}\right)_{\left(\frac{y_1+y_2}{2}\right)} *(y_2-y_1) \qquad (8.34)$$

- (8.34) may be used to calculate the water profile in a step-by-step sequence from a known (x_1, y_1) value.

Worked example:

Determine the upstream profile of a backwater curve given:
$Q = 10\ m^3/s$, $b = 3m$, $\sin\theta = 0.001$, $n = 0.022$.



Answer

For normal flow, $(S \rightarrow \sin\theta)$

$$Q = \frac{A}{n} * R^{2/3} * S^{1/2}$$

i.e.
$$10 = \frac{(3y_n)}{0.022} * \left(\frac{3y_n}{3+2y_n}\right)^{2/3} * \sqrt{0.001}$$

$$y_n = 2.44\ m$$

The water profile is from 2.44 m to 5 m along the channel.

From Manning equation,

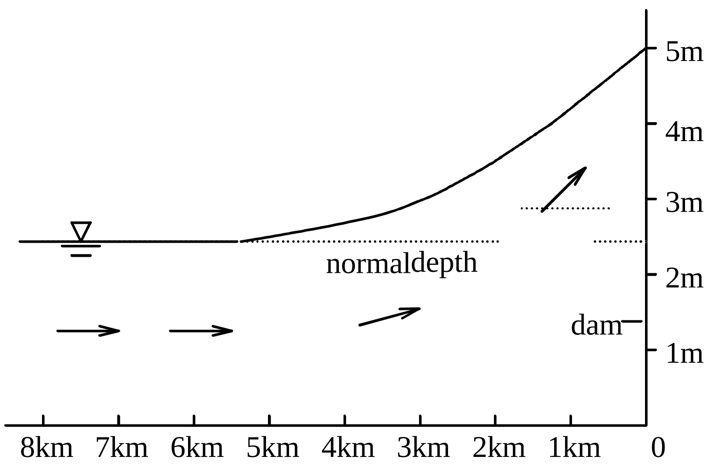
$$v = \frac{1}{n} * R^{2/3} * S^{1/2}$$

$$S = \frac{n^2 v^2}{R^{4/3}}$$

Hence
$$x_2 = x_1 + \left(\frac{1 - Fr^2}{0.001 - \frac{n^2 v^2}{R^{4/3}}}\right) * (y_2 - y_1)$$

section, I	yi (m)	dy (m)	yave (m)	v (m/s)	Fr	1-Fr*Fr	R (m)	So - Sf	dx (m)	x (m)
1	5									0
		0.25	4.875	0.684	0.099	0.990	1.147	0.000812	305	
2	4.75									305
		0.25	4.625	0.721	0.107	0.989	1.133	0.000787	314	
3	4.5									619
		0.25	4.375	0.762	0.116	0.986	1.117	0.000758	326	
4	4.25									945
		0.25	4.125	0.808	0.127	0.984	1.100	0.000722	341	
5	4									1285
		0.25	3.875	0.860	0.140	0.981	1.081	0.000677	362	
6	3.75									1647
		0.25	3.625	0.920	0.154	0.976	1.061	0.000622	392	
7	3.5									2040
		0.25	3.375	0.988	0.172	0.971	1.038	0.000551	440	
8	3.25									2480
		0.25	3.125	1.067	0.193	0.963	1.014	0.000459	524	
9	3									3004
		0.25	2.875	1.159	0.218	0.952	0.986	0.000337	707	
10	2.75									3711
		0.31	2.595	1.285	0.255	0.935	0.951	0.000146	1992	
11	2.44									5703

From the table, the water level is not affected by the dam at 5.7 km upstream.



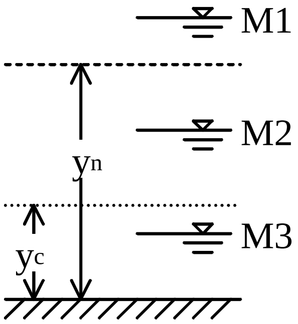
From the graph, the water depth at any location can be obtained.

8.6.1 Classifications of Surface Profile of Gradually Varied Flow

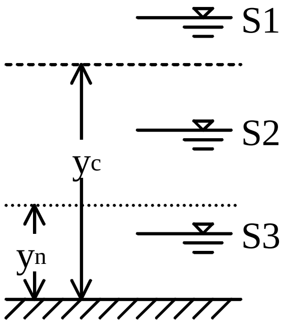
- It is customary to compare the actual channel slope, sinθ or S_o with the critical slope S_c for the same Q.

- There are **five** classes of channel slope giving rise to twelve distinct types of solution curves.
  - $S_o > S_c$ - Steep (S)
  - $S_o = S_c$ - Critical (C)
  - $S_o < S_c$ - Mild (M)
  - $S_o = 0$ - Horizontal (H)
  - $S_o < 0$ - Adverse (A)

- There are **three** number designators for the type of profile relates to the position of the actual water surface in relation to the position of the water for normal and critical flow in a channel.
  - 1 the surface of stream lies above both normal and critical depth
  - 2 the surface of stream lies between normal and critical depth
  - 3 the surface of stream lies below both normal and critical depth



Mild slope

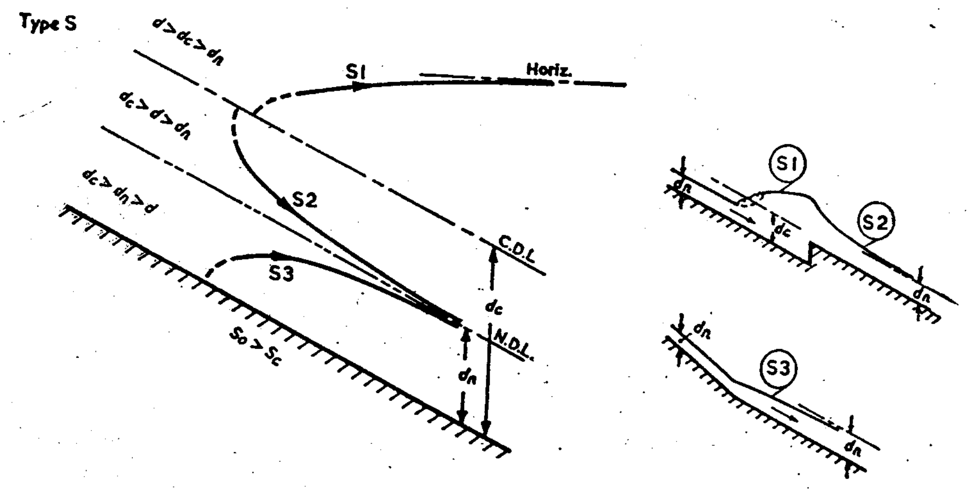


Steep slope

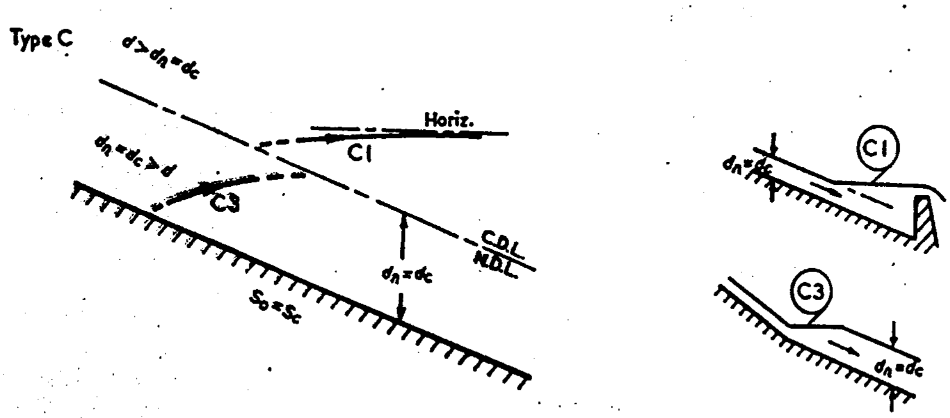
- Combining the two designators, we have

Slope class	Slope notation	Depth class	Froude number	Actual depth	Profile
$\sin\theta > S$	Steep (S)	$y_c > y_n$	Fr < 1	$y > y_n; y > y_c$	S1
			Fr > 1	$y_c > y > y_n$	S2
			Fr > 1	$y < y_n; y < y_c$	S3
$\sin\theta = S$	Critical (C)	$y_c = y_n$	Fr < 1	$y > y_n = y_c$	C1
			Fr > 1	$y < y_n = y_c$	C3
$\sin\theta < S$	Mild (M)	$y_c < y_n$	Fr < 1	$y > y_n; y > y_c$	M1
			Fr < 1	$y_n > y > y_c$	M2
			Fr > 1	$y < y_n; y < y_c$	M3
$\sin\theta = 0$	Horizontal (H)	$y_n = \infty$	Fr < 1	$y > y_c$	H2
			Fr > 1	$y < y_n; y < y_c$	H3
$\sin\theta < 0$	Adverse (A)	$y_n = \text{Im}$	Fr < 1	$y > y_c$	A2
			Fr > 1	$y < y_c$	A3

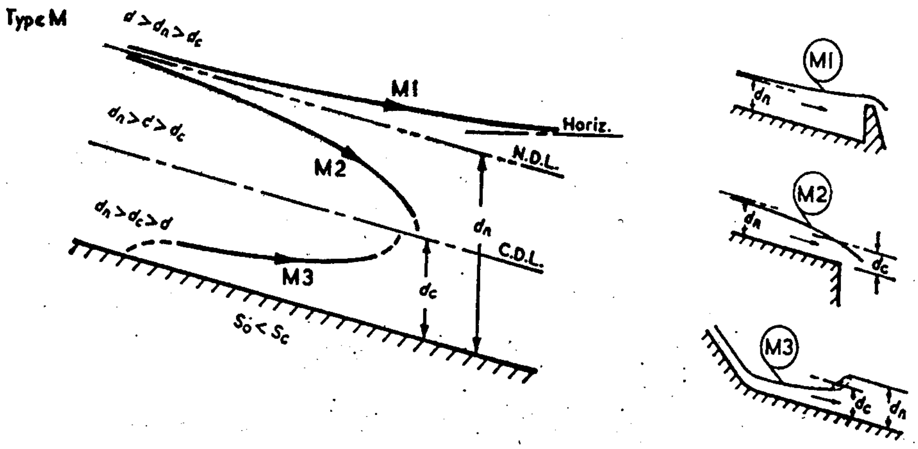
- For type S



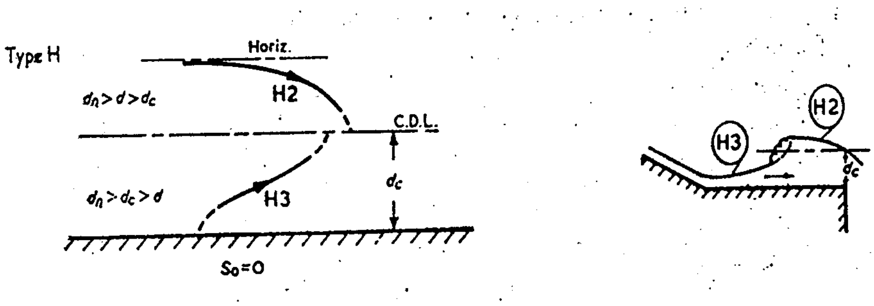
- For type C



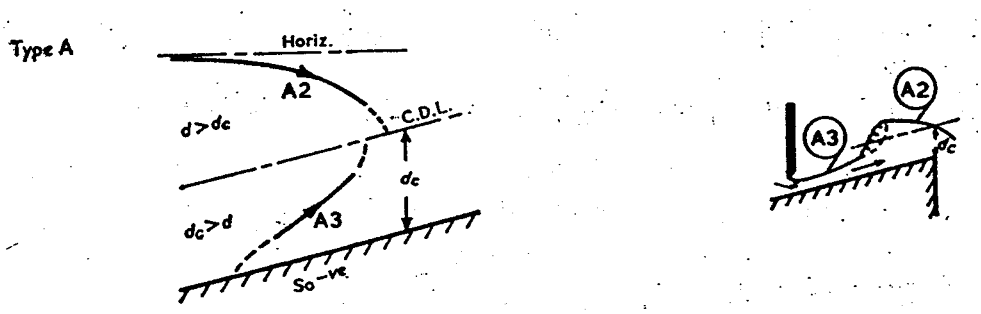
- For type M



- For type H

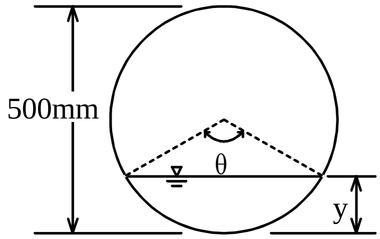


- For type A



Class Exercise 8.1:

A 500 mm-diameter concrete pipe on a 1:500 slope is to carry water at a velocity of 0.18 m/s. Find the depth of the flow. (n=0.013)



(y = 18 mm)

Class Exercise 8.2:

What are the dimensions for an optimum rectangular brick channel (n = 0.015) designed to carry 5 m^3/s of water in uniform flow with s = 0.001? What will be the percentage increase in flow rate if the channel is a semi-circle but retained the same sectional area? (increase = 8.4%)

Class Exercise 8.3:

A trapezoidal channel has a bottom width of 6.0 m and side slopes of 1:1. The depth of flow is 1.5 m at a discharge of 15 m^3/s. Determine the specific energy and alternate depth. (E = 1.59 m, y = 0.497 m)

Class Exercise 8.4:

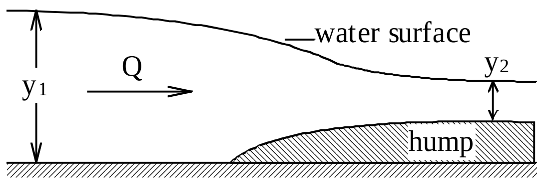
A triangular channel has an apex angle of 60° and carries a flow with a velocity of 2.0 m/s and depth of 1.25 m.

(a) Is the flow subcritical or supercritical?
(b) What is the critical depth?
(c) What is the specific energy?
(d) What is the alternate depth possible for this specific energy?

(y_c = 1.148 m, E = 1.454 m, y = 1.06 m)

Class Exercise 8.5:

A rectangular channel is 4.0 m wide and carries a discharge of 20 m^3/s at a depth of 2.0 m. At a certain section it is proposed to build a hump. Calculate the water surface elevations at upstream of the hump and over the hump if the hump height is 0.33 m. (Assume no loss of energy at the hump.)

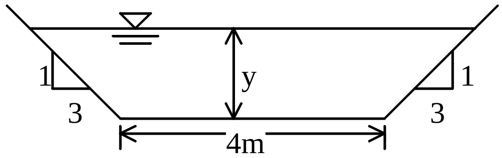


Class Exercise 8.6:

In a hydraulic jump occurring in a horizontal, rectangular channel it is desired to have an energy head loss equal to 6 times the supercritical flow depth. Calculate the Froude number of the flow necessary to have this jump. ($Fr_1 = 4.822$)

Tutorial – Open Channel Flow

1. Calculate the normal depth in a concrete trapezoidal channel with side slope of 1 to 3, a bed slope of 0.00033, a bottom width of 4.0 m and a water discharge of 39 m^3/s. Manning coefficient is 0.013.



2. Determine the critical depth of the trapezoidal channel for a discharge of 15 m^3/s. The width of the channel bottom, b = 6 m, and the side slope is 45°.

3. Consider a flow in a wide channel over a bump with an approaching velocity, v_1 at the upstream is 1 m/s and the depth, y_1 is 1 m. If the maximum bump height is 15 cm, determine

   (a) the Froude number over the top of the bump, and
   (b) the depression in the water surface. ($y_2 > 0.5$ m)

4. Water flows in a trapezoidal channel at a rate of 8.5 m^3/s. The channel has a bottom width of 3 m and side slope of 1:1. If a hydraulic jump is forced to occur where the upstream depth is 0.3 m, what will be the downstream depth and velocity? What are the values of Fr_1 and Fr_2?

5. A wide canal has a bed slope of 1 in 1000 and conveys water at a normal depth of 1.2 m. A weir is to be constructed at one point to increase the depth of flow to 2.4 m. How far upstream of the weir will the depth be 1.35 m? (Take n in the Manning equation as 0.013)