Chapter No.3

NETWORK EQUATIONS:

Kirchhoff's Laws

Network equations are formulated from two simple laws that are; (1845)

- Kirchhoff's Voltage Law
- Kirchhoff's Current law.

These two laws are concerning the Algebraic sum of Voltages around a loop and currents entering and leaving a node.

Algebraic → is used to indicate that we take into account reference polarities and reference directions in the summation.

Kirchhoff's Voltage Law:

According to KVL the algebraic sum of all branch voltages around any closed loop of a network is zero at all instants of time.

Sources of Energy are given +ive (voltage rise) and Elements in which either energy is stored or exhaust are given -ive (voltage drop).

KVL can also be explained as "sum of the

Voltage rises are equal to sum of the Voltage drops in a closed Loop at all instants of time

Example:- .

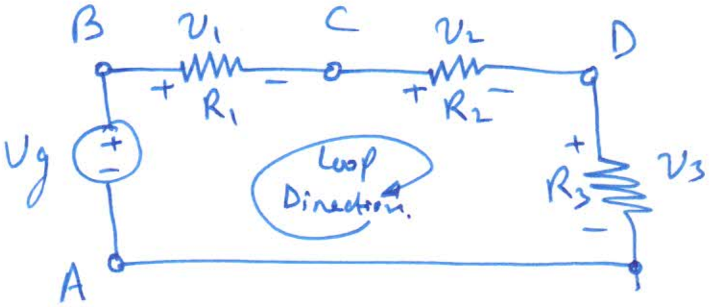


Fig: 3.1 One-loop resistive network to which K.V.L is applied.

→ Following the Loop current Direction,

(i) Put +ive sign if the Polarity marks occur in the order of + to − . (i.e arrow enters from +ive terminal)

(ii) Put −ive sign if the Polarity marks occur in the order of − to + . (i.e arrow enters from −ive terminal).

Hence Algebraic sum of all the voltages in a close Loop is equal to zero.

$$-V_g + V_1 + V_2 + V_3 = 0$$

OR

$$\boxed{V_1 + V_2 + V_3 = V_g}$$

The above equation can also be obtained if we state the second version of the law, which states that, sum of the voltage rises ~~are~~ are equal to sum of the voltage drops in a closed path

(i) Take sources of energy (i.e voltage sources in this case) as voltage rises

(ii) Take other passive elements as sources of voltage drops

Hence

$$\sum \text{voltage rises} = \sum \text{voltage drops}$$

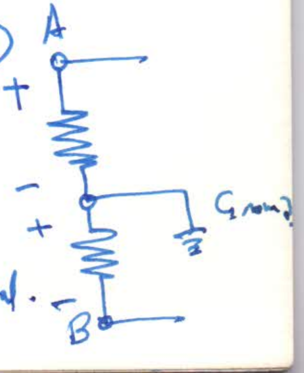
$$\boxed{V_g = V_1 + V_2 + V_3}$$

Reference Node / Datum node

In talking about rises and drops, it is convenient to have a reference node, also called a "datum node".

In the same way that we talk about sea level being the reference for elevations, so in talking about voltages we select a reference called "ground".

Fig: 3.2



In the figure, the voltage at node A is higher than ground potential, while that at node B is lower than (or below) ground potential.

Kirchhoff's Current Law:

KCL states that the algebraic sum of all branch currents leaving a node is zero at all instants of time.

The reference directions can be given +ive or -ive signs, such as currents leaving the node may be given -ive sign and currents entering the node may be given +ive sign.

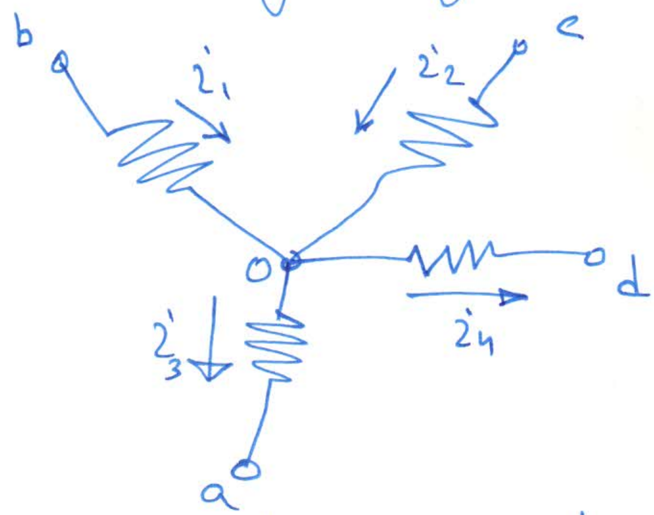


Fig: 3.3

Accordingly the equation can be written as

$$-i_3 - i_4 + i_1 + i_2 = 0$$

Manipulation of the equation

$$\boxed{i_1 + i_3 = i_3 + i_4}$$

The above equation may also result from another version of the KCL, which states that the sum of the currents entering a node are equal to sum of the currents leaving the node.

Application of Kirchhoff's Voltage Law:

→ We wish to determine the conditions under which the two networks of the figure are equivalent.

→ Two networks are said to be equivalent at a pair of terminals if the voltage-current relationships for the two networks are identical at these terminals.

→ For the networks of (a) and (b) of the figure to be equivalent, we must find the conditions under which $i_a = i_b$, when $v_a = v_b$.

From network (a)

$$v_a = v_1 + v_2 = R_1 i_a + R_2 i_a \longrightarrow \text{(i)}$$
$$= i_a (R_1 + R_2)$$

from network (b)

$$v_b = R_{eq} \cdot i_b \longrightarrow \text{(ii)}$$

equating above equations. (with condition $i_a = i_b$)

$$i_a (R_1 + R_2) = i_b (R_{eq})$$

$$\boxed{R_{eq} = R_1 + R_2}$$

Thus the Summation of resistors Connected in Series is equal to the equivalent resistance of the Combination.

Generalizing the result given for network (a) & (b), we obtained for Series Combination.

$$R_{eq} = R_1 + R_2 + R_3 + \cdots + R_n = \sum_{j=1}^{n} R_j.$$

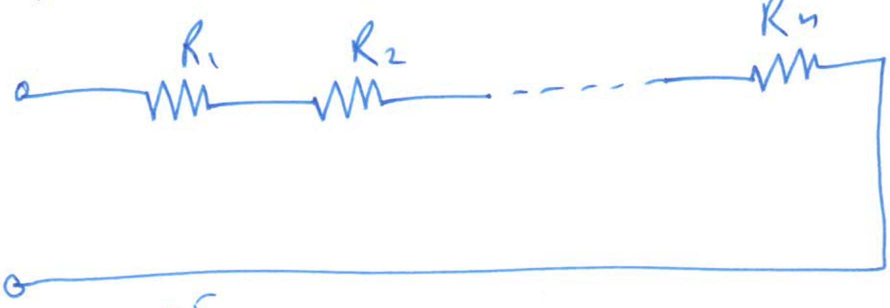
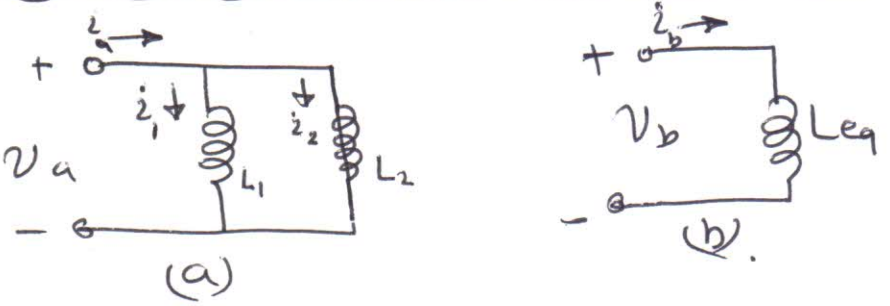


Fig. 3.5: Series Connected resistance/resistive network

Application of Kirchhoff's Current Law:



$$i_a = i_1 + i_2 = \frac{1}{L_1}\int v_a \cdot dt + \frac{1}{L_2}\int v_a \cdot dt \longrightarrow (a)$$

$$i_b = \frac{1}{L_{eq}}\int v_b \cdot dt. \longrightarrow (b)$$

$$v_a = v_b$$

$$i_a = i_b$$

equating (a) and (b)

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}. \quad \text{or} \quad \boxed{L_{eq} = \frac{L_1 L_2}{L_1 + L_2}}$$

The Number of Network Equations:

An important problem in analyzing a network concerns the number of equations that must be written in order to describe completely the voltages and currents in the network.

The Answer may appear to be obvious since we must always write the same number of equations as we have unknown quantities or variable.

$$\boxed{\text{No. of equations} = \text{No. of unknown quantities/variables}}$$

Note: smaller number may be chosen for simultaneous solution

① How may we properly choose our variables so that we have a minimum number of them?

② How can we be sure that the equations we write are independent?

We first make a number of restrictions for the discussion of this section.

→ We consider a branch to be the same as an element or one branch to represent a single element.

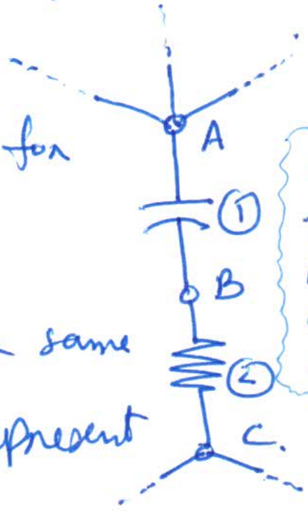
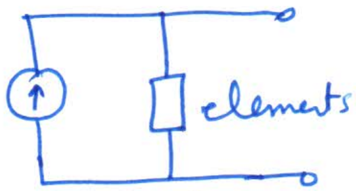


Fig: 3.7. Two Branches of a Network

Thus the part of the network shown in fig: 3.7 has 2-Branches, marked ① and ②. We will show later that under some conditions the two branches may be replaced by an equivalent branch for some calculations.

→ We will also assume that we have voltage sources in series with other elements, and current sources in parallel with other elements.

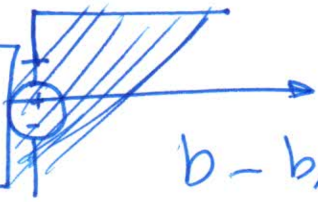
voltage source



Curnt source

→ We assume that there are no loops consisting only of voltage sources, nor can the network be separated into 2-parts joined only by curnt source.

Example



Consider a network composed of b-branches excited by Active elements for which we are to find responses in the Network. The unknown quantities of interest are the Branch-Voltages and Branch-Currents, making a total of 2b-unknown for the b branches.

Since the Voltage-Current Relationships are known for each of the elements by equations, like,

$$v = Ri, \quad v = L\frac{di}{dt}, \quad \text{and} \quad v = 1/C \int i \, dt,$$

we may reduce the number of unknown quantities from 2b to b. In other words, if we know the branch ~~elements~~ currents then we may routinely determine the Branch voltage and vice-versa.

we make use of Kirchhoff's Laws to write equations in b-unknowns. In preparation for writing these equations, we first select the datum or reference node. For the remaining nodes, we then write equations using Kirchhoff's current law.

This accomplished, it is necessary that we write

$$\boxed{\text{No. of Equations} = b - (n-1) = b - n + 1}$$ (b = no. of branches, n = no. of nodes

equations using K.V. Law in order that we have a total of b- equations.

we may then solve for the unknown voltages and currents, provided, of course, that the equations we have written are independent.

what do you mean by independent ?

A set of equations is said to be linearly dependent if at least one of the equations can be expressed as a linear combination of the others.

Thus, if we obtain an equation by adding or subtracting two other equations, one equation is dependent on the other two, and may not be used in finding a solution.

Thus, given the equations,

Linearly Dependent.

$$\begin{cases} 3i_1 + 2i_2 - i_3 = 4 & \longrightarrow (1) \\ -i_1 + 5i_2 + 3i_3 = -2 & \longrightarrow (2) \\ i_1 + 12i_2 + 5i_3 = 0. & \longrightarrow (3) \end{cases}$$

(1)
×2 (2)×2
+
(3)

we note that the 3rd equation may be obtained by multiplying the (2) by 2 and adding it to the first (1).

Then the equations are dependent and no Unique Solution may be found for i_1, i_2 and i_3.

66 The equations we write must be independent if we are to be able to solve the equation!!

In the interest of saving time and effort, we select the smallest number of Variables, consistent with the requirement that they be independent.

SOURCE TRANSFORMATION:

Voltage Sources may be transformed into equivalent Current sources and vice-versa.

And the Position of Sources in the network may be shifted.

Our objective in this network manipulation is to prepare the network for an analysis that is simple and Direct

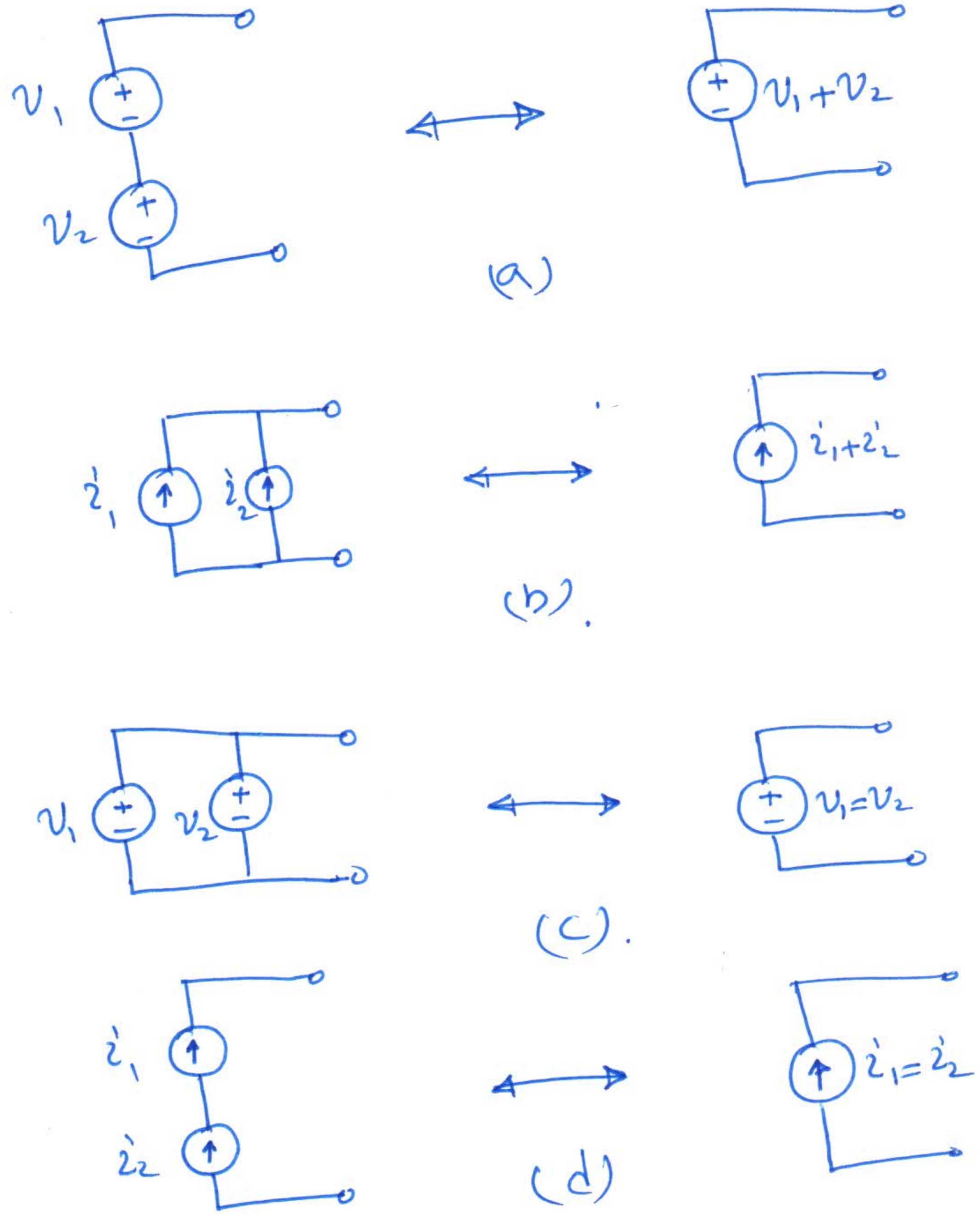


Fig: 3.14: Illustrating rules under which sources may be combined.

(a) Two voltage sources, v_1 and v_2, connected in series w.r.t polarities as indicated, are equivalent to a single voltage source, v_1+v_2

(b) Two current sources, i_1 and i_2, are equivalent to a single current source, i_1+i_2.

(c) Voltage sources cannot be connected in parallel unless the two sources have identical voltages.

(d) Current sources cannot be connected in series unless identical.

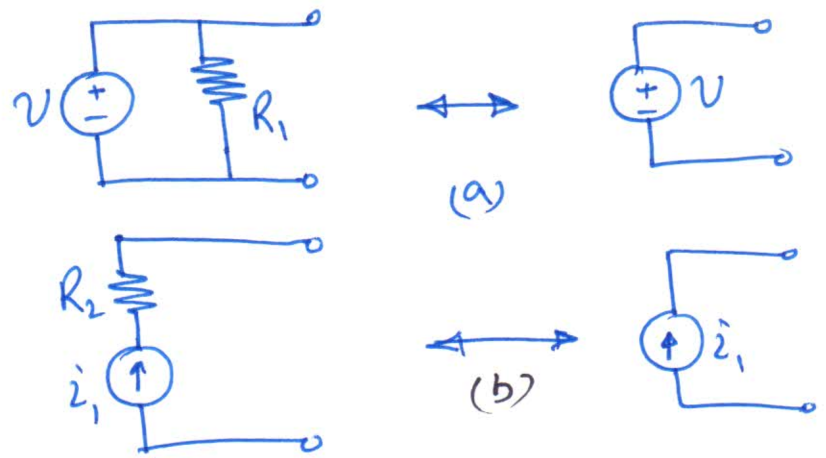


Fig: 3.15. Two examples of extraneous elements so far as terminal behavior is concerned.

Fig: (a) shows a resistor in parallel with a voltage source. The current through this resistor is determined only by the voltage source and not by the remainder of the network. As far as computations in the remainder of the network are concerned, a resistor in parallel with a voltage source may be ignored or omitted entirely from the network representation.

Fig: (b) The same situation applies to a resistor in series with a current source. This resistor in no way affects the current from the source. As far as computations in the remainder of the network are concerned, a resistor in series with a current source may be omitted from the network representation.

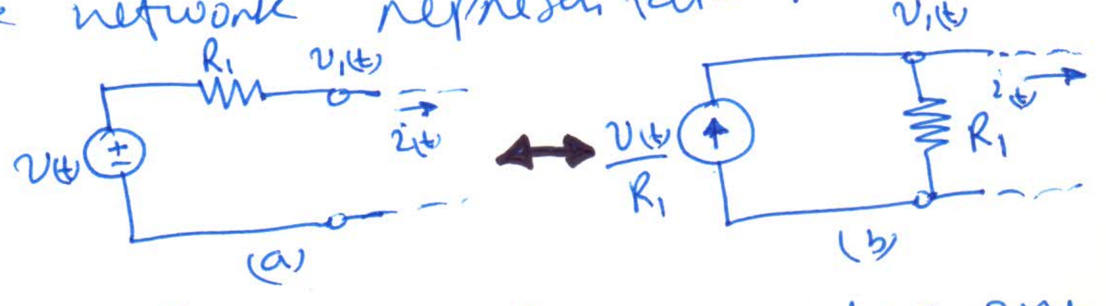


Fig: 3.16: Source Transformation involving one resistor

Ref. to fig. 3.16.

let $v(t)$ be the voltage of the source and $v_1(t)$ the voltage at the node located between resistor R_1 and the rest of the network.

K.V.L for the circuit (a)

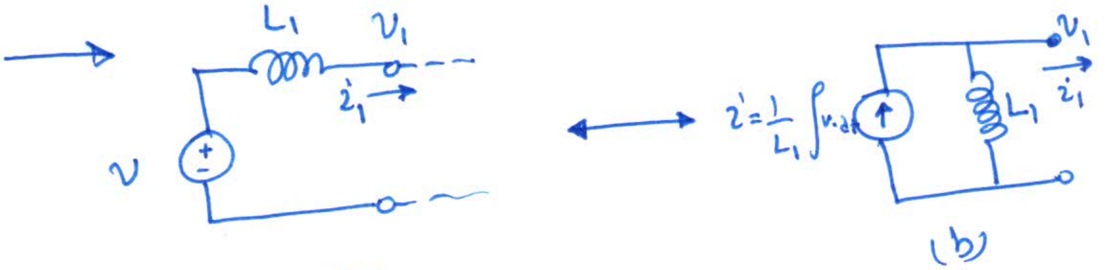
$$v(t) = R_1\, i(t) + v_1(t).$$

Solving this equation for $i(t)$ gives.

$$i(t) = \frac{v(t)}{R_1} - \frac{v_1(t)}{R_1}.$$

$v(t)/R_1$ = current from source.

$v_1(t)/R_1$ = current in the Resistor R_1 - connected in Parallel with the current source.



(a) (b)

Fig. 3.17. Source transformation for a network with a single inductor

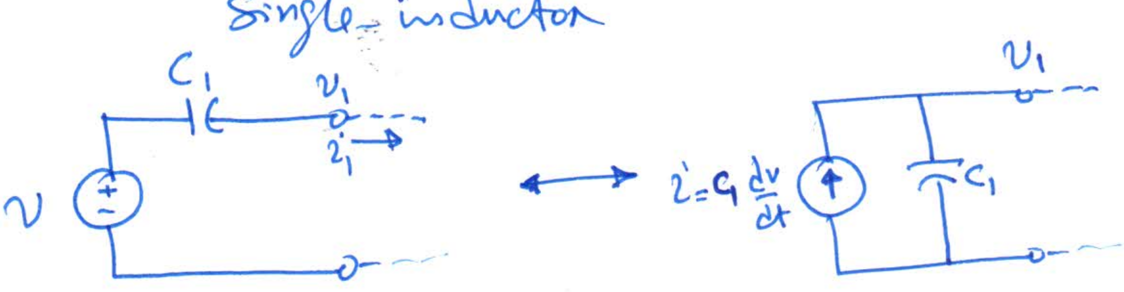


Fig. 3.18: Source transformation involving one Capacitor.

The reasoning we have just applied to the resistor in series with a voltage source may be extented to either an inductor or a Capacitor in the same position.

Note: This technique is not applied on networks involving more than one Passive elements in series.

In the analysis of networks, we often encounter Voltage sources without a series passive element, or current sources without a parallel passive element.
If it is desired to transform from one kind of source to the other, it's first necessary to "Shift" the source within the network.
The technique by which this is accomplished is explained in terms of the simple example of Fig. 3.19(a).

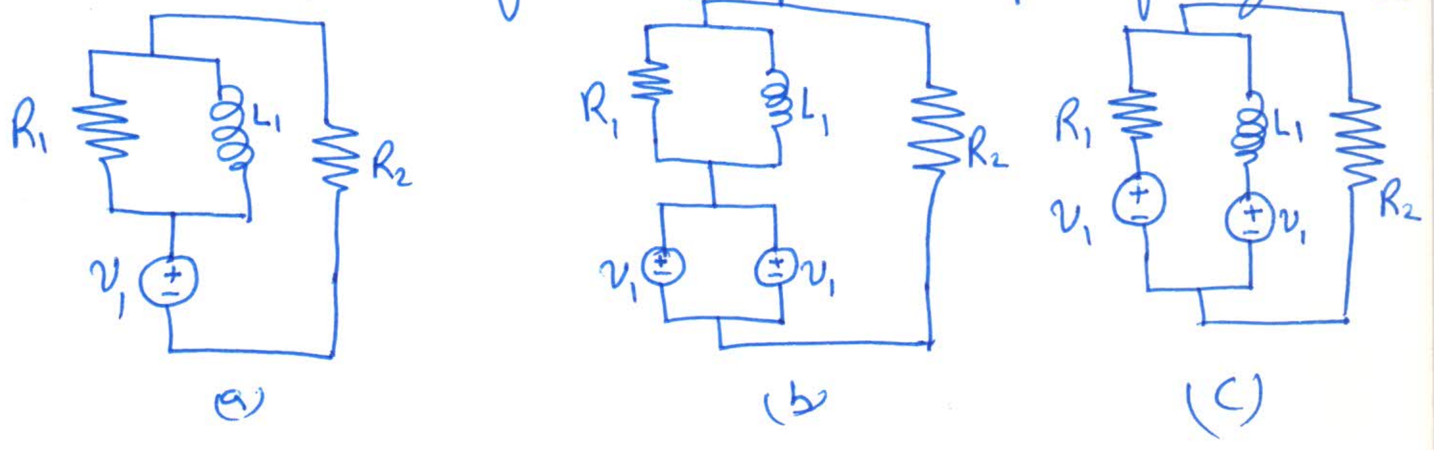


Fig: 3.19: Three equivalent networks illustrating a procedure for a source to be pushed through a node.

(a) The single Voltage source may be considered to be equivalent to two identical sources in parallel as in (b) of the figure. Now the network in (c) is identical to that of (b) since a connection from the +ive terminals of the two sources does not affect the network because there would be no current in such a connection.

Thus the network of (c) is seen to be equivalent to that of (b). If the equivalent current sources are requiring we can use fig: (c) for this purpose.

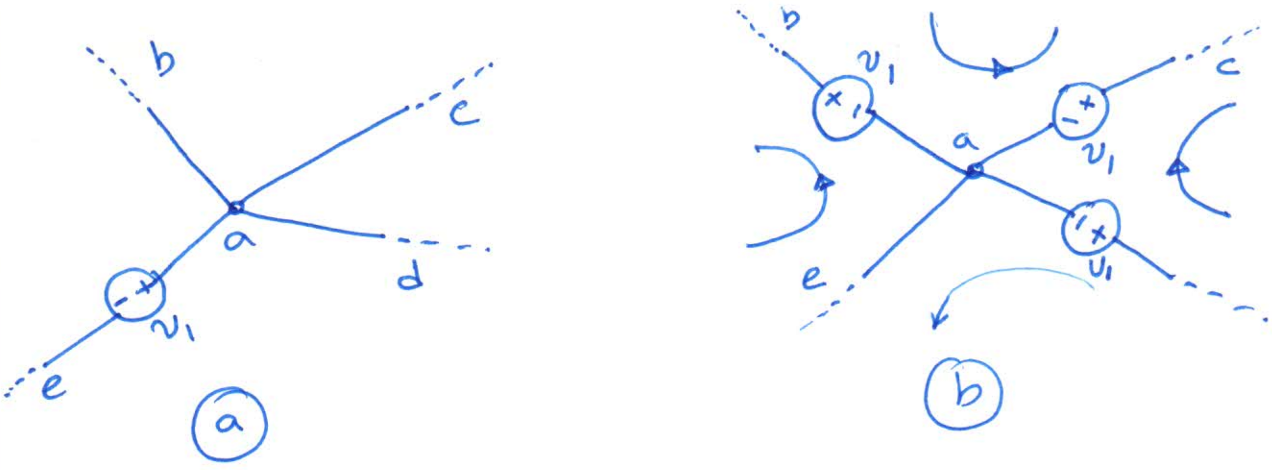


Fig: 3.20: Illustrating the procedure by which a voltage source is shifted in a network.

The example just given is a special case of a more general form of voltage source shifting illustrated in Fig 3.20

→ In traversing the four loops indicated by dashed lines of (b) of the figure, we observe that Kirchhoff Voltage Law gives the same equations for the two networks of (a) and (b).

Thus we may "push the voltage source through the node" with a new identical source appearing in every branch connected to the node, without affecting the current distribution in the network.

Note.
→ the voltage distribution in the network is changed since node a is now at the same voltage as node e, while before shifting the voltage difference of the two nodes was v_1.

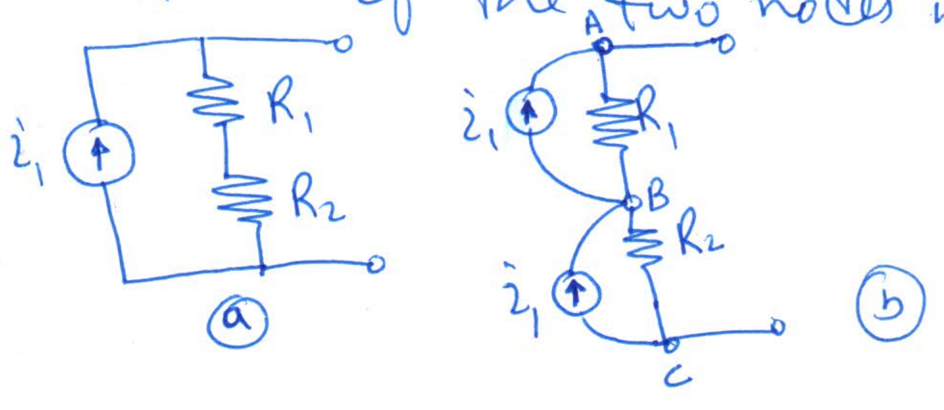
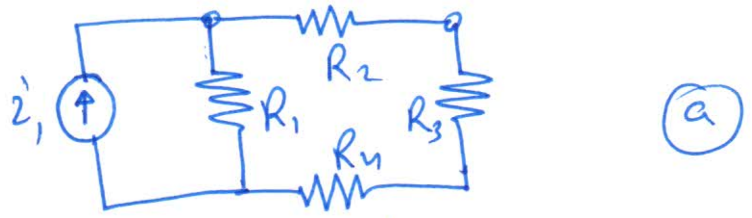


Fig: 3.21. Two equivalent networks illustrating the manner in which one source is replaced by two such that the KCL is still satisfied at each node.

Equivalent manipulations are also possible for the current source, following the pattern of duality. In the fig. 3.21, the network of (b) is equivalent to that of (a) in applying K.C.L at each of the nodes.



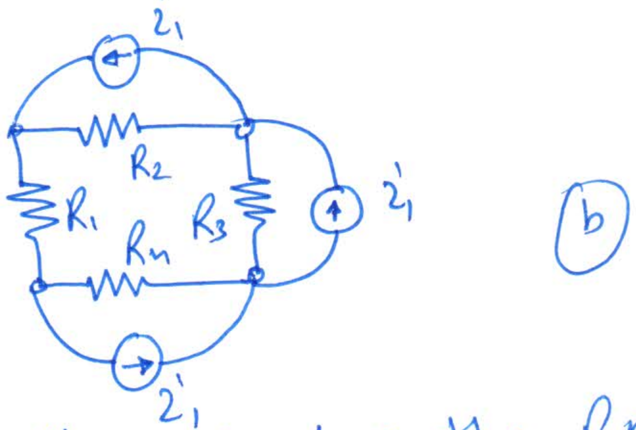


Fig. 3.22. An example illustrating the procedure by which a current source is shifted in a network.

Note: Same currents are maintained in each of the node of fig. (a) and (b).

The operations we have described may be employed successively in determining the simple equivalent of a complicated network. The resistive network of Fig: 3.23 has 3-voltage sources and one current source. By the step-by-step reduction shown in the figure the simplified equivalent network of (E) or (F) are found

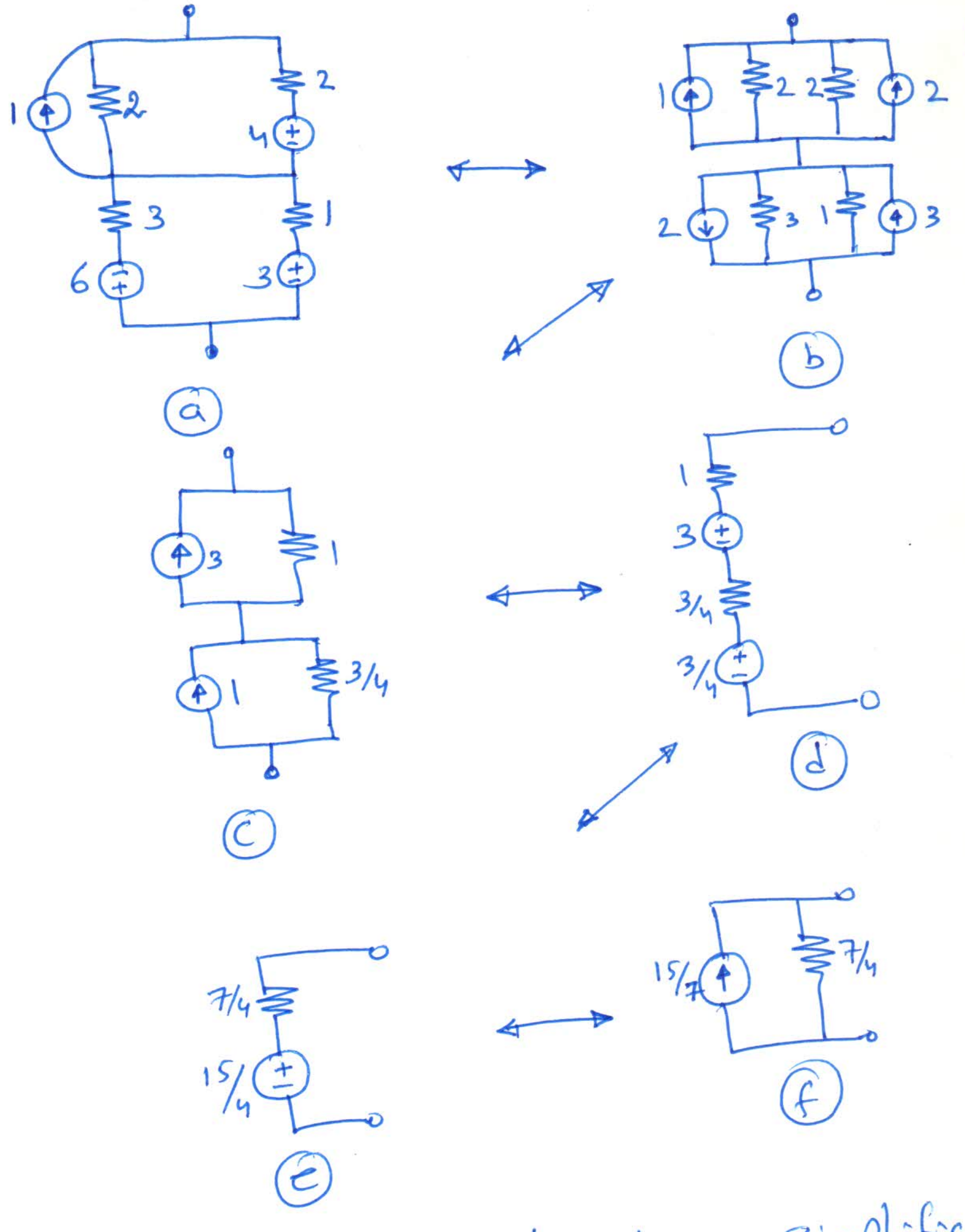


Fig: 3.23: An example of network simplification using successive source Transformations.

Source transformations have bearing on the representation of a network by a graph. Two observation may be made:

1. Elements in parallel with voltage source or in series with current sources can be eliminated from graph.
2. Since voltage sources may be shifted from one branch to others with the consequent elimination of

of that branch from the network, in the construction of a graph the Voltage source may be shorted out before analysis is started. Similarly, current sources may be open circuited and so eliminated from the graph which represents the network.

→ We will always follow the practice of preparing the network before writing equations from Kirchhoff's Law. This will involve the following practices. (Preparation)

1. If the network is to be analyzed on the node basis with Kirchhoff's current Law, we will manipulate the network sources so that only current sources are in the resulting network.
2. If the network is to be analyzed on the loop basis, using K.V. Law, then all sources will be manipulated until equivalent voltage sources are found.
3. If the network is to be analyzed using state variables, then we may prepare the network to have both voltage and current sources, depending upon the variable selected.

We are now prepared to write sets of equations describing networks. We understand that we may make use of node method, the loop method or

any other method in writing these equations. Which of the Choices shall we make?
Our Choice will be made in terms of such factors as these

1. Which method results in the smallest no. of Variables?
2. What is the objective of the analysis? One voltage? Several Currents?
3. Will solution be accomplished through hand Calculation or by digital computer?
   This question may, in turn, determine how Concerned we are about the independence of network equations.

If we will use pencil and pad of paper in obtaining the solution, we are very concerned about keeping the number of variables small because of no. of algebraic operations involved in the solution of simultaneous equations.
→ This number increases very rapidly with the number of variables, a point that will be dramatized by examples when we study the methods of determinant evaluation.