

Chapter 8 Friction

8.1 Introduction

in engineering application problem, there are no perfectly frictionless surface exists

when two surface are in contact, tangential forces, called friction forces, will always develop if one attempt to move with respect to the other

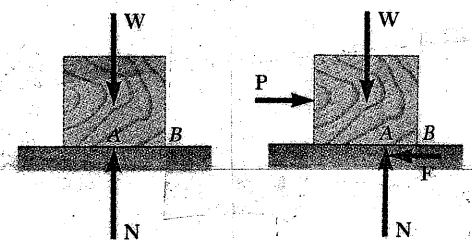
two types of friction are introduced : dry friction (Coulomb friction) and fluid friction

fluid friction develops between layers of fluid, they are moving at different velocities

we shall limit out present study to dry friction, i.e. problems involving rigid bodies which are in contact along nonlubricated surfaces, such as wedges, square-threaded screws, journal bearing, thrust bearing, rolling resistance, belt friction etc.

8.2 The Laws of Dry Friction, Coefficients of Friction

consider a block of weight W subjected to a horizontal force P , if P is small, the block will not move, the static friction force F will exist to balance P



if P increased, F also increased, until its magnitude reaches a certain maximum value F_m , if P is further increased, the friction force cannot balance it, the block starts sliding, and the magnitude of F drops from F_m to a lower value F_k , [where F_k is called kinetic friction force, F_k remains approximately constant, and the block sliding with increasing velocity]

experimental evidence shows F_m is proportional to N

i.e.
$$F_m = \mu_s N$$

where μ_s is a constant called coefficient of static friction, and N is the normal reaction force

also
$$F_k = \mu_k N$$

where μ_k is a constant called coefficient of kinetic friction

μ_s and μ_k do not depend upon the contact area, but depend strongly on the nature of the surface in contact

the values of μ_s are listed in table 8-1

μ_k would be about 25% smaller than μ_s in general

μ_s and μ_k are dimensionless

four different situations may occur when a rigid body is in contact with a horizontal surface

1. no friction force $P_x = 0$

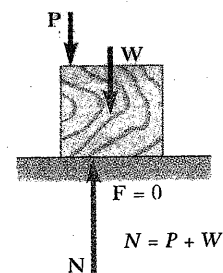
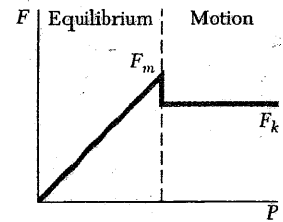
2. no motion $P_x < F_m$

there is not evidence that the maximum friction force has been reached

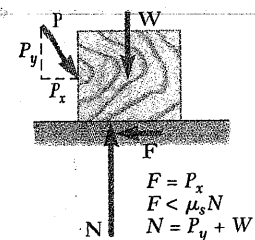
$$F \neq F_m = \mu_s N$$

3. motion impending

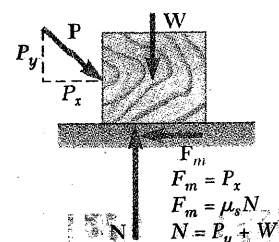
if the body just about to slide, $F = F_m = \mu_s N$ may be used



(a) No friction ($P_x = 0$)



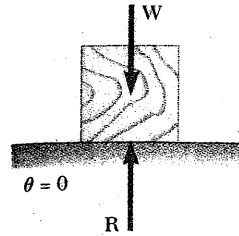
(b) No motion ($P_x < F_m$)



(c) Motion impending $\rightarrow (P_x = F_m)$

another example will show that the angle of friction can be used, consider a block resting on a board and subjected to no other force than W and R

1. $\theta = 0$ no friction



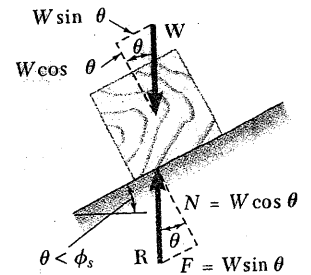
(a) No friction

2. $\theta < \phi_s$

$$N = W \cos \theta \quad F = W \sin \theta$$

$$\tan \phi = F / N = \cos \theta / \sin \theta = \tan \theta < \tan \phi_s$$

i.e. $\phi < \phi_s$ no motion



(b) No motion

3. $\theta = \phi_s$

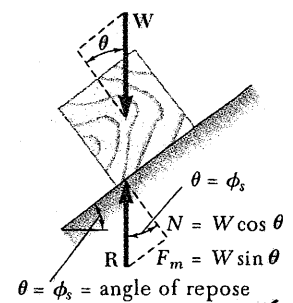
$$N = W \cos \theta \quad F = W \sin \theta$$

$$\tan \phi = F / N = \cos \theta / \sin \theta = \tan \theta$$

$$= \tan \phi_s = \mu_s$$

$$F = F_m = \mu_s N$$

i.e. $\phi = \phi_s$ motion impending



(c) Motion impending

4. $\theta > \phi_s$

$$N = W \cos \theta$$

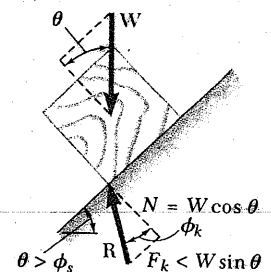
$$F_{\max} = F_m = \mu_s N = N \tan \phi_s < N \tan \theta$$

$$F_{\max} < W \sin \theta \quad \text{motion starts}$$

$$F = F_k = \mu_k N$$

$$\phi = \phi_k < \phi_s < \theta$$

R is not vertical, force acting on block are unbalanced



(d) Motion

8.4 Problems Involving Dry Friction

problems involving dry friction are found in many engineering applications rigid bodies in acceleration will be studied in dynamics

a number of common machines and mechanisms can be analyzed by applying the laws of dry friction, such as wedged, screws, journal bearing, thrust bearing and belt transmissions etc.

most problems involving friction fall into three groups

1. all applied forces are given, and μ_s is given, determine whether the body will remain at rest or slide

applied forces $\rightarrow N$ and F

- | | | |
|-------|---------------------|------------------|
| check | $F < F_m = \mu_s N$ | no motion |
| | $F = F_m = \mu_s N$ | motion impending |
| | $F > F_m = \mu_s N$ | motion occurs |

2. all applied forces are given, motion is impending, determine μ_s

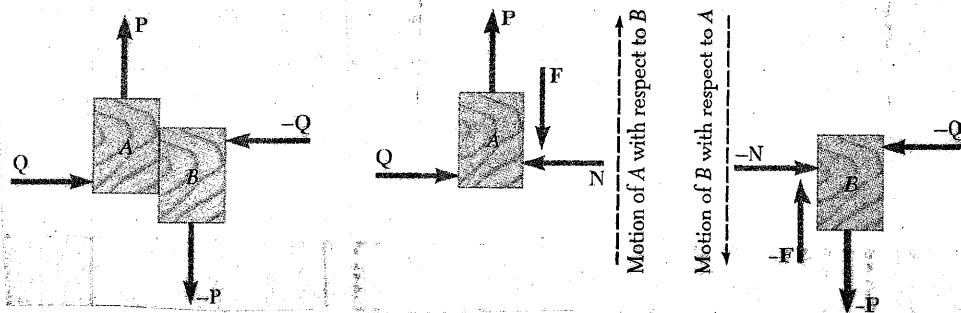
applied forces $\rightarrow N$ and F

$$\mu_s = F / N \quad [F = F_m \text{ at this time}]$$

3. μ_s is given, motion is impending in given direction, determine the magnitude and the direction of one of the applied forces

consider two bodies A and B are in contact

in drawing the free-body diagram of one of the bodies, the sense of friction force acting on A is opposite to that on the moving (or impending motion) of A as observed from B

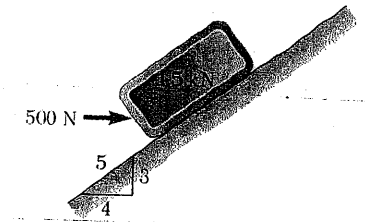


Sample Problem 8.1

$W = 1.5 \text{ kN}$

$\mu_s = 0.25 \quad \mu_k = 0.2$

determine whether the block is in equilibrium, and find the friction force



force required to equilibrium

$\Sigma F_x = 0 \quad 500 \times 4/5 - 1500 \times 3/5 - F = 0$

$F = -500 \text{ N} \nearrow$

$\Sigma F_y = 0 \quad N - 1500 \times 4/5 - 500 \times 3/5 = 0$

$N = 1500 \text{ N} \nwarrow$

$F_m = \mu_s N = 0.25 \times 1500 = 375 \text{ N}$

$F_m < 50 \text{ N}$ [force required to maintain equilibrium]

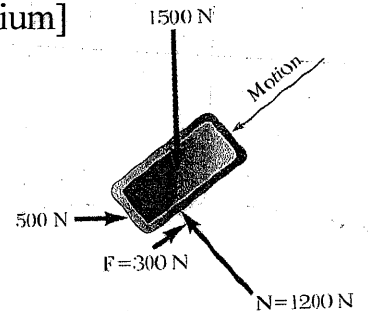
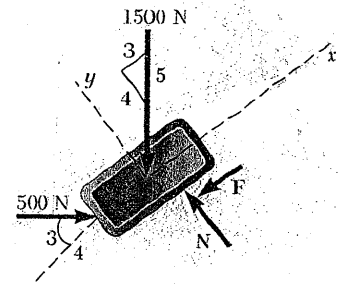
the block will slide down the plane

thus the actual friction force is

$F_k = \mu_k N = 0.2 \times 1500 = 300 \text{ N} \nearrow$

the unbalanced force is $500 - 300 = 200 \text{ N} \swarrow$

it will caused the body to an accelerated motion down the plane



Sample Problem 8.2

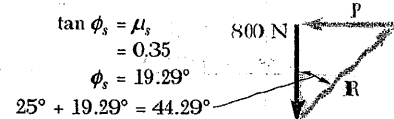
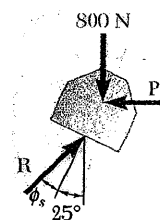
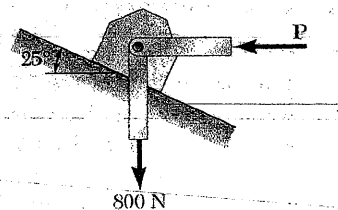
$\mu_s = 0.35 \quad \mu_k = 0.25$

determine P required

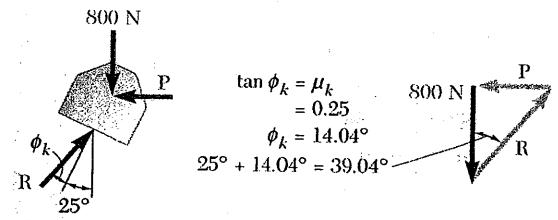
- (a) to start the block moving up
- (b) to keep it moving up
- (c) to prevent it from sliding down

(a) $\tan \phi_s = \mu_s = 0.35 \quad \phi_s = 19.29^\circ$

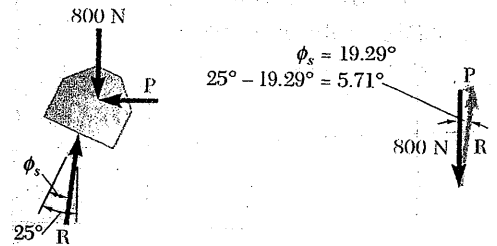
$P = 800 \tan 44.29^\circ = 780 \text{ N} \leftarrow$



(b) $\tan \phi_k = \mu_k = 0.25 \quad \phi_s = 14.04^\circ$
 $P = 800 \tan 39.04^\circ = 649 \text{ N} \leftarrow$



(c) $P = 800 \tan 5.71^\circ = 80 \text{ N} \leftarrow$
 if $\phi_s > 25^\circ$, P is not required



Sample Problem 8.3

$\mu_s = 0.25$, determine x at which the load W can be supported

$$F_A = \mu_s N_A = 0.25 N_A$$

$$F_B = \mu_s N_B = 0.25 N_B$$

$$\Sigma F_x = 0 \quad N_A = N_B$$

$$\Sigma F_y = 0 \quad 0.25 (N_A + N_B) = W$$

i.e. $0.5 N_A = W \quad N_A = N_B = 2W$

this is the condition for the bracket in equilibrium

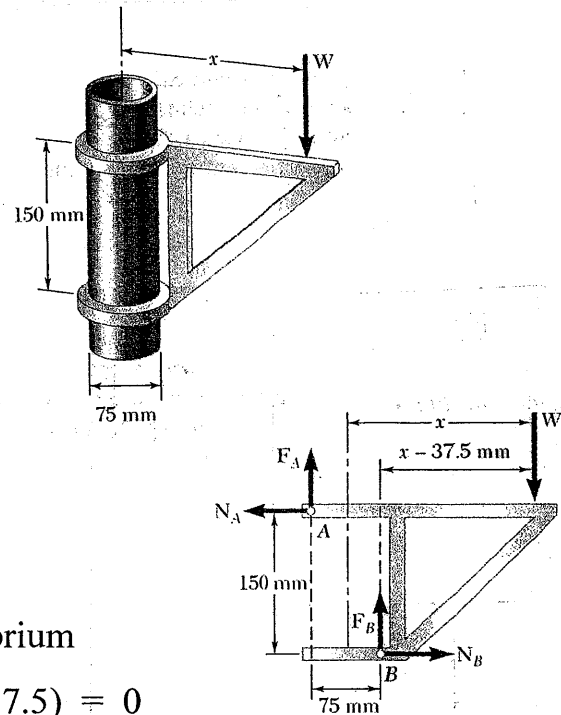
$$\Sigma M_B = 0 \quad N_A \times 150 - F_A \times 75 - W(x - 37.5) = 0$$

$$150 N_A - 75 (0.25 N_A) - Wx + 37.5 W = 0$$

$$150 (2W) - 18.75 (2W) - Wx + 37.5 W = 0$$

$$x = 300 \text{ mm}$$

that is the minimum value of x for the bracket in equilibrium



8.5 Wedges

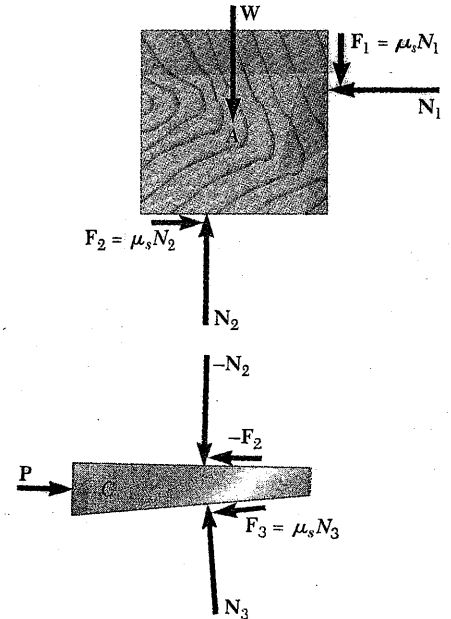
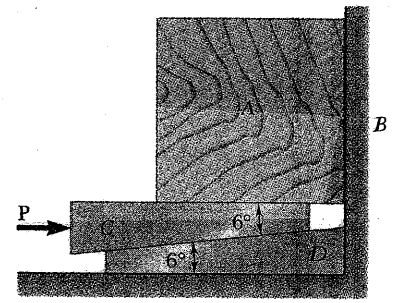
wedge are simple machines used to raise large stone block and other heavy loads

consider a wedges C and the block A , we want to find the minimum force of P which must be applied to the wedge C to move the block A up

the weight of the wedge is small compared with the other force involved and may be neglected

consider the free-body diagrams of the block and the wedge, there are 4 equations of equilibrium ($\Sigma F_x = 0$ and $\Sigma F_y = 0$ for each free-body) to solve 4 unknowns N_1 , N_2 , N_3 and P

if $\theta < \phi_s$, the block A will remains rest when P is removed



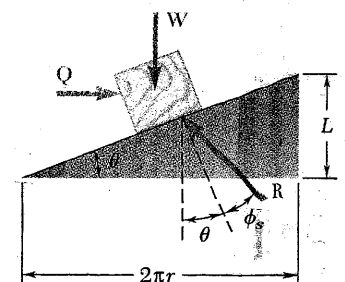
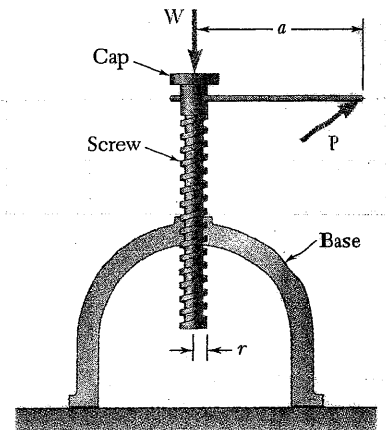
8.6 Square-Threaded Screws

square-threaded screws are frequently used in jacks, presses and other mechanism

consider the screw carries a load W and is supported by the base of the jack, the thread of the base has been unwrapped and shown as a straight line

the slope was obtained by plotting horizontally the product $2\pi r$ and the vertically the lead L of the screw, where θ is called the lead angle

since the friction force does not depend upon the area in contact, the two threads may be assumed to be in contact over a much smaller area may be represented by the block



the horizontal force Q having the same effect as the force P exerted on the handle, the moment of P and Q must have the same moment about the screw axis, i.e.

$$P a = Q r$$

$$Q = P a / r$$

the force Q (or P) required to raise the load W may be obtained when $P > P_{\min}$, $\phi = \phi_k$, then using ϕ_k to calculate Q and P

if the friction angle $\phi_s > \theta$, the screw is said to be self-locking, it will remain in place under the load, to lower the load, we must apply force

if the friction angle $\phi_s < \theta$, the screw will unwind under the load, it is then necessary to apply some force to maintain equilibrium

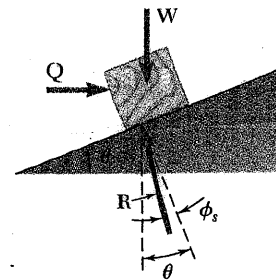
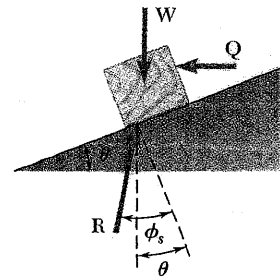
lead p : the distance through which the screw advance in one turn

pitch L : the distance measured between two consecutive threads

single-threaded screw : $L = p$

double-threaded screw : $L = 2p$

triple-threaded screw : $L = 3p$



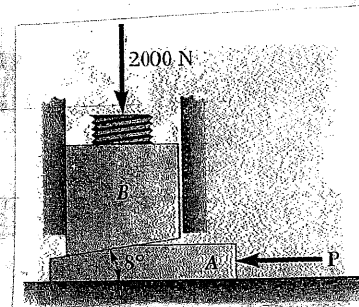
Sample Problem 8.4

$\mu_s = 0.35$, determine

(a) to rise the block

(b) to lower the block

$$\tan \phi_s = \mu_s = 0.35 \quad \phi_s = 19.3^\circ$$



(a) free-body for block B

$$R_1 / \sin 109.3^\circ = 2000 / \sin 43.4^\circ$$

$$R_1 = 2747 \text{ N}$$

free-body for block A

$$P / \sin 46.6^\circ = 2747 / \sin 70.7^\circ$$

$$P = 2120 \text{ N} \leftarrow$$

(b) free-body for block B

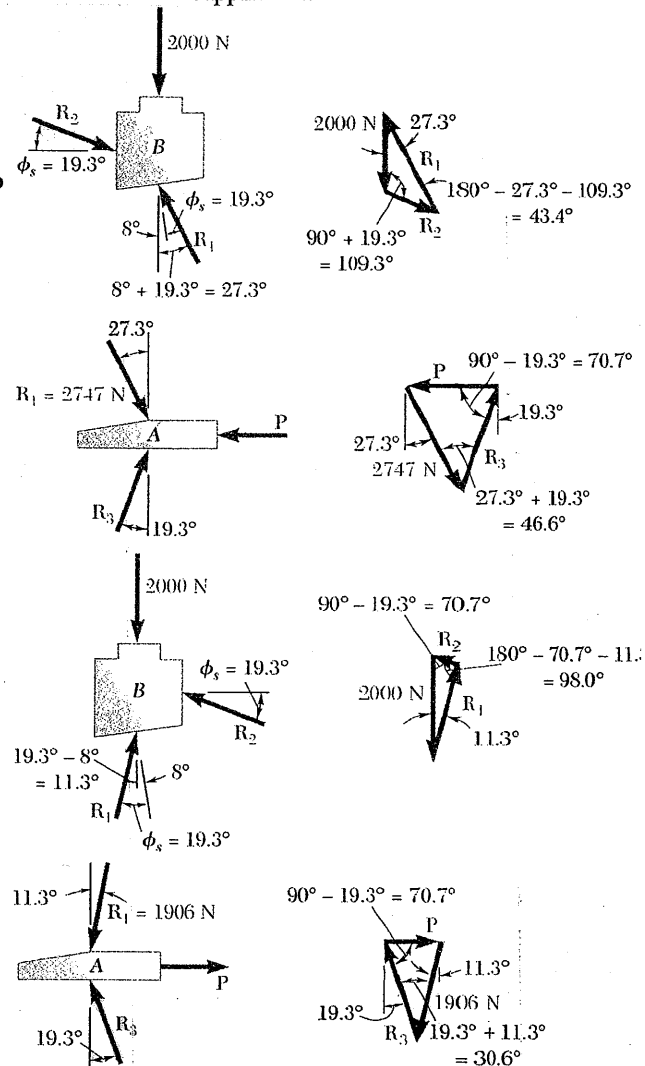
$$R_1 / \sin 70.7^\circ = 2000 / \sin 98.0^\circ$$

$$R_1 = 1906 \text{ N}$$

free-body for block A

$$P / \sin 30.6^\circ = 1906 / \sin 70.7^\circ$$

$$P = 1028 \text{ N} \rightarrow$$



Sample Problem 8.5

a clamp of double-threaded screw

$$d = 10 \text{ mm} \quad \mu_s = 0.3$$

if $T = 40 \text{ N-m}$ is applied in tightening the clamp

determine (a) the force exerted on the wood and

(b) the torque required to loosen the clamp

(a) $L = 2p = 4 \text{ mm}$

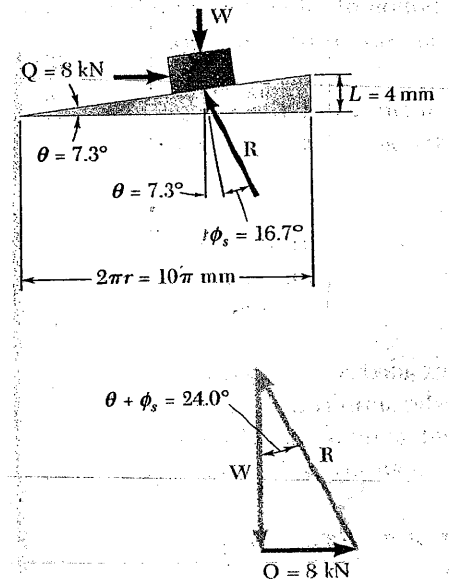
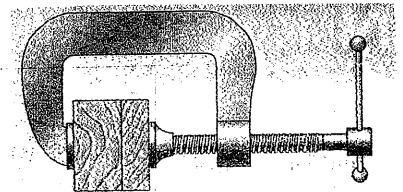
$$\tan \theta = 4 / 10\pi = 0.1273$$

$$\theta = 7.3^\circ$$

$$\mu_s = 0.3 \quad \phi_s = 16.7^\circ$$

$$Qr = 40 \quad Q = 8 \text{ kN}$$

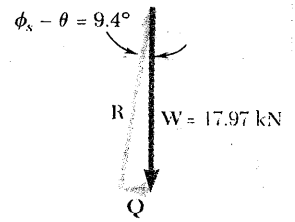
free-body on the block



free-body on the block

$$W = Q / \tan(\theta + \phi_s) = 8 / \tan 24^\circ = 17.97 \text{ kN}$$

W is the magnitude of the force exerted on the wood



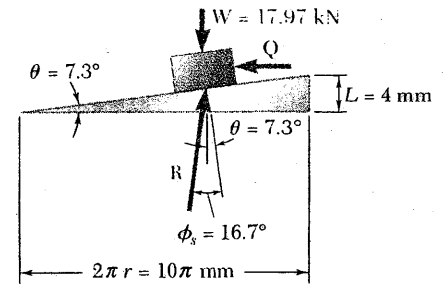
(b) $Q = W \tan(\phi_s - \theta)$

$$= 17.97 / \tan 9.4^\circ$$

$$= 2.975 \text{ kN}$$

$$T = Qr = 2.975 \times 5 = 14.87 \text{ N-m}$$

$\therefore \phi_s > \theta$, the screw is self-locking



8.7 Journal Bearings, Axle Friction

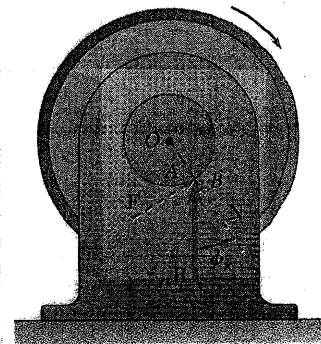
journal bearings : provide lateral support to rotate shaft (axle friction)

thrust bearings : provide axial support to shaft and axles (disk friction)

if the journal bearing is fully lubricated, friction depends upon ω , clearance, and viscosity of the lubricant

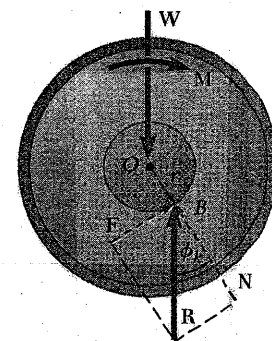
consider the axle friction when the bearing is not lubricated or only partially lubricated, to apply a couple M , to keep the while rotating at constant speed

free-body for the while



$$R = W$$

but the reaction R does not pass through the center, the contact between axle and bearing does not take place at the lowest point when the axle rotates



the angle between R and the normal to the surface of the bearing is equal to the angle of kinetic friction ϕ_k

$$\Sigma M_o = 0 \quad M = R r \sin \phi_k$$

for small value of ϕ_k

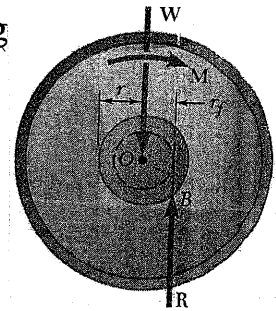
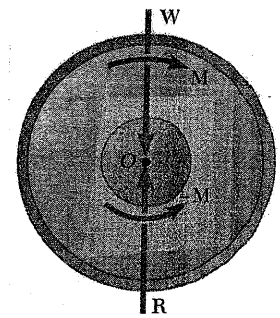
$$\sin \phi_k \doteq \tan \phi_k = \mu_k$$

and
$$M = R r \mu_k$$

this couple represent the friction resistance of the bearing
the line of action of R must be tangent to a
circle centered at O and of radius

$$r_f = r \sin \phi_k \doteq r \mu_k$$

this circle is called the circle of friction



8.8 Thrust Bearings, Disk Friction

end bearings : friction developed over full circular areas, or over ring-shaped areas when the end of the shaft is hollow

collar bearing : friction developed between the two ring-shaped areas which are in contact

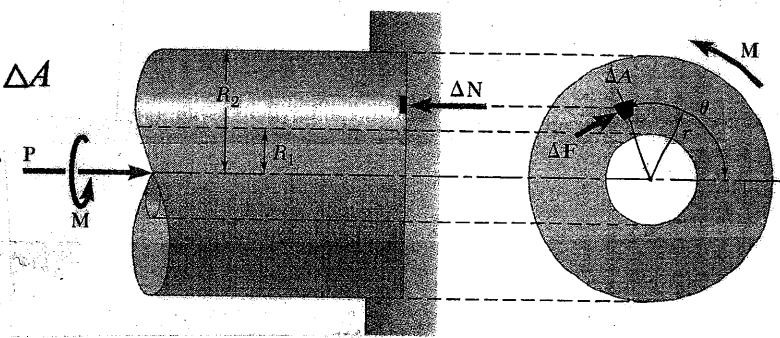
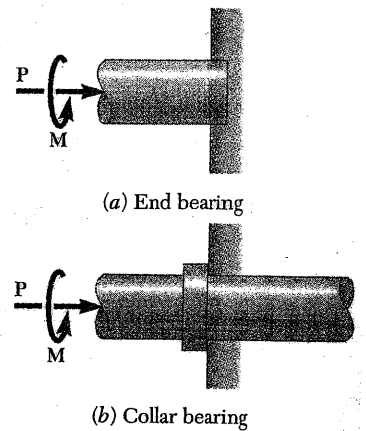
consider an end bearing of hollow shape
for a small area ΔA , the normal force is

$$\begin{aligned} \Delta N &= (P / A) \Delta A \\ &= [P / \pi (R_2^2 - R_1^2)] \Delta A \end{aligned}$$

R_1 and R_2 are inner and outer radii of the ring-shaped area

the friction force acting on the area is

$$\Delta F = \mu_k \Delta N$$



and the moment due to this friction force is

$$\Delta M = r \Delta F = r \mu_k P \Delta A / \pi (R_2^2 - R_1^2)$$

the moment required to overcome the friction resistance is

$$\begin{aligned} M &= \int dM = \frac{\mu_k P}{\pi (R_2^2 - R_1^2)} \int_A r dA && [dA = r dr d\theta] \\ &= \frac{\mu_k P}{\pi (R_2^2 - R_1^2)} \int_0^{2\pi} \int_{R_1}^{R_2} r^2 dr d\theta \\ &= \frac{\mu_k P}{\pi (R_2^2 - R_1^2)} \int_0^{2\pi} \frac{1}{3} (R_2^3 - R_1^3) d\theta \\ &= \frac{2}{3} \mu_k P \frac{(R_2^3 - R_1^3)}{(R_2^2 - R_1^2)} \end{aligned}$$

for $R_1 = 0$ and $R_2 = R$, i.e. contact in the whole circular area

$$M = \frac{2}{3} \mu_k P R$$

and $M_{\max} = \frac{2}{3} \mu_s P R$

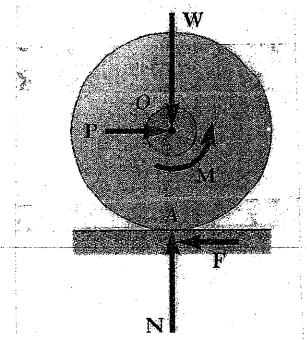
8.9 Wheel Friction, Rolling Resistance

because the point of the wheel in contact with the ground, the wheel eliminates the large friction forces which would arise if the load were in direct contact with ground

in practice, the wheel is not perfect, and some resistance to its motion exists, two types of resistance exists

1. it is due to combined effect of axle friction and friction of the rim
2. it is due to the fact that the wheel and the ground deform (contact not at a single point, but an area)

consider a wheel of the car is mounted on axle and bearing, the car is moving to the right with constant speed



- (a) if $P = W$, R applied at A
 when P is increased, R will applied at B

$$r_f = r \sin \phi \simeq r \mu_s = 25 \times 0.2 = 5 \text{ mm}$$

$$\Sigma M_B = 0 \quad 55 \times 2500 - 45 \times P = 0$$

$$P = 3060 \text{ N} \downarrow$$

- (b) as force P is decreased, R will applied at C

$$\Sigma M_B = 0 \quad 45 \times 2500 - 55 \times P = 0$$

$$P = 2050 \text{ N} \downarrow$$

- (c) three forces W , P , and R must be concurrent

$$OE = r_f = 5 \text{ mm}$$

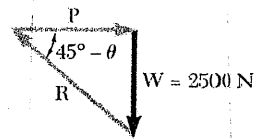
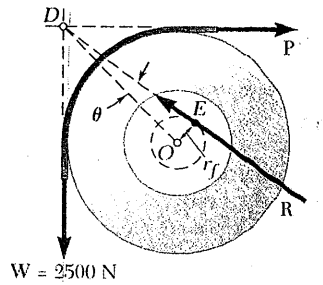
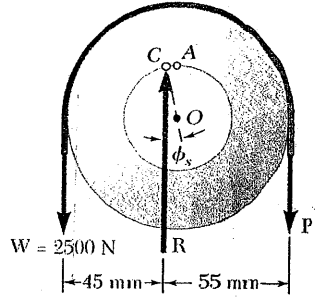
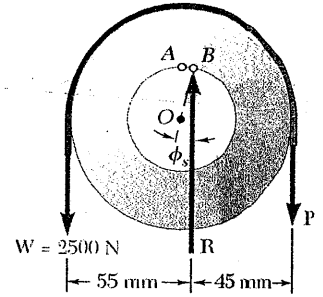
$$OD = 50 \sqrt{2} \text{ mm}$$

$$\sin \theta = OE / OD = 0.0707$$

$$\theta = 4.1^\circ$$

$$P = W \cot (45^\circ - \theta) = 2500 \cot 40.9^\circ$$

$$= 2890 \text{ N} \rightarrow$$



8.10 Belt Friction

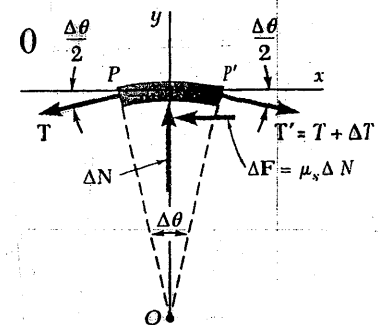
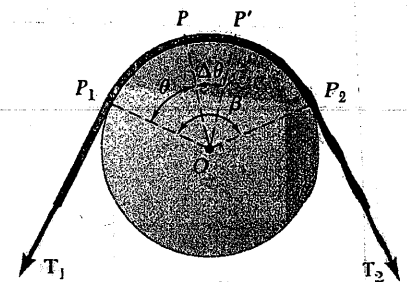
consider a flat belt passing over a fixed cylindrical drum, the motion between the belt and the drum is assumed to be impending

consider a small element PP' subtending an angle $\Delta\theta$, the equation of equilibrium are

$$\Sigma F_x = 0 \quad (T + \Delta T) \cos \Delta\theta/2 - T \cos \Delta\theta/2 - \mu_s \Delta N = 0$$

$$\Sigma F_y = 0 \quad \Delta N - (T + \Delta T) \sin \Delta\theta/2 - T \sin \Delta\theta/2 = 0$$

eliminate ΔN and obtained



$$\Delta T \cos \Delta\theta/2 - \mu_s (2T + \Delta T) \sin \Delta\theta/2 = 0$$

$$\text{or } \frac{\Delta T}{\Delta\theta} \cos \Delta\theta/2 - \mu_s (T + \Delta T/2) \frac{\sin \Delta\theta/2}{\Delta\theta/2} = 0$$

$$\text{for } \Delta\theta \rightarrow 0, \quad \Delta N, \Delta F, \Delta T \rightarrow 0$$

$$\text{and } \cos \Delta\theta/2 \rightarrow 1$$

$$\lim_{\Delta\theta \rightarrow 0} \frac{\sin \Delta\theta/2}{\Delta\theta/2} = \lim_{\Delta\theta \rightarrow 0} \frac{1/2 \cos \Delta\theta/2}{1/2} = 1$$

and the equation becomes

$$dT/d\theta - \mu_s T = 0$$

integrating from P_1 to P_2

$$\int_{T_1}^{T_2} \frac{dT}{T} = \mu_s \int_0^\beta d\theta$$

$$\text{at } P_1 : T = T_1, \theta = 0$$

$$\text{at } P_2 : T = T_2, \theta = \beta$$

it is obtained

$$\ln T_2 - \ln T_1 = \mu_s \beta$$

$$\ln T_2 / T_1 = \mu_s \beta$$

$$\text{or } T_2 / T_1 = e^{\mu_s \beta}$$

T_2 is always larger than T_1 , T_2 therefore represents the tension in that part of the belt which pulls, while T_1 is the tension in the part which resists

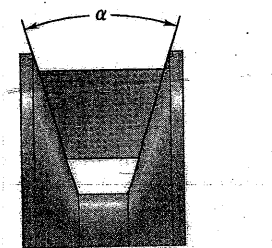
β may be expressed in radian, and may be larger than 2π

if the belt is actually slipping, μ_s should be changed to μ_k

consider a V-shaped belt, drawing the free body diagram of an element of the belt

the equations of equilibrium in x - and y -

directions are



$$\Sigma F_y = 0 \quad 2 \Delta N \sin \alpha/2 = (2T + \Delta T) \sin \Delta\theta/2$$

$$\Sigma F_x = 0 \quad 2 \mu_s \Delta N = \Delta T \cos \Delta\theta/2$$

eliminate ΔN and it is obtained

$$\Delta T \cos \Delta\theta/2 = \mu_s \frac{(2T + \Delta T) \sin \Delta\theta/2}{\sin \alpha/2}$$

$$\frac{\Delta T}{\Delta\theta} \cos \Delta\theta/2 = \frac{\mu_s (2T + \Delta T) \sin \Delta\theta/2}{\sin \alpha/2}$$

as $\Delta\theta \rightarrow 0$, the equation can be reduced

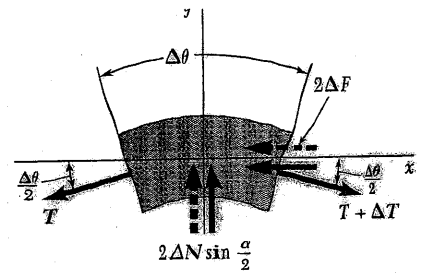
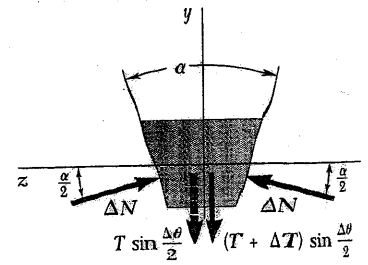
$$dT/d\theta = \mu_s T / \sin \alpha/2$$

or $dT/T = \mu_s d\theta / \sin \alpha/2$

integrating and obtained

$$T_2 / T_1 = e^{\mu_s \beta / \sin \alpha/2}$$

for $\alpha = \pi$, $\sin \alpha/2 = 1$, it will reduce to the flat belt equation



Sample 8.7

a hawser thrown two full turns around a bollard

(a) determine μ_s for the hawser just keep from slipping

(b) determine the number of turns if $T_1 = 400 \text{ N}$, $T_2 = 75 \text{ kN}$

(a) $\ln T_2 / T_1 = \mu_s \beta$

$$\beta = 2 \times 2\pi = 12.57 \text{ rad}$$

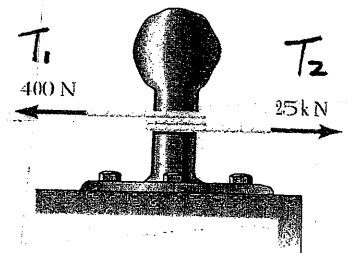
$$T_1 = 400 \quad T_2 = 25000$$

$$\ln (25000 / 400) = \mu_s \times 12.57$$

$$\mu_s = 0.329$$

(b) $\mu_s \beta = \ln (T_2 / T_1)$

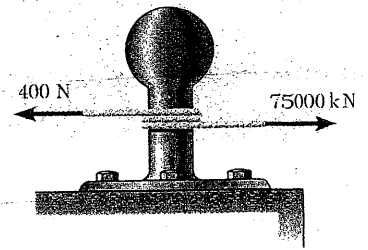
$$T_2 / T_1 = 75000 / 400 = 187.5$$



$$\mu_s \beta = \ln(187.5) = 5.234$$

$$\beta = 5.234 / 0.329 = 15.91 \text{ rad}$$

$$\text{number of turns} = \beta / 2\pi = 2.53 \text{ turns}$$



Sample Problem 8.8

a flat belt connect pulley *A* and motor *B*

$$\mu_s = 0.25 \quad \mu_k = 0.2 \quad T_{\max} = 300 \text{ N}$$

determine the largest torque

at motor *B*

$$T_2 / T_1 = e^{\mu_s \beta} = e^{0.25 (\frac{3}{2}\pi)} = 1.688$$

$$\text{for } T_2 = 3000 \text{ N then } T_1 = 1777.3 \text{ N}$$

at pulley *A*

$$\Sigma M_A = 0 \quad M_A - 3000 \times 200 + 1777.3 \times 200 = 0$$

$$M_A = 244,540 \text{ N-mm} = 244.54 \text{ N-m}$$

check μ_s required to prevent slipping at *A*

$$\mu_s \beta = \ln T_2 / T_1 = \ln 3000 / 1777.3 = 0.524$$

$$\beta = 240^\circ = 4\pi / 3 \text{ rad} = 4.189 \text{ rad}$$

$$\text{then } \mu_s = 0.125 < 0.25 \quad [\text{O.K.}]$$

