Module 2

Analysis of Statically Indeterminate Structures by the Matrix Force Method

Lesson
10
The Force Method of Analysis: Trusses

Instructional Objectives

After reading this chapter the student will be able to

1. Calculate degree of statical indeterminacy of a planar truss
2. Analyse the indeterminate planar truss for external loads
3. Analyse the planar truss for temperature loads
4. Analyse the planar truss for camber and lack of fit of a member.

10.1 Introduction

The truss is said to be statically indeterminate when the total number of reactions and member axial forces exceed the total number of static equilibrium equations. In the simple planar truss structures, the degree of indeterminacy can be determined from inspection. Whenever, this becomes tedious, one could use the following formula to evaluate the static indeterminacy of static planar truss (see also section 1.3).

$$i = (m + r) - 2j \qquad (10.1)$$

where m, j and r are number of members, joints and unknown reaction components respectively. The indeterminacy in the truss may be external, internal or both. A planar truss is said to be externally indeterminate if the number of reactions exceeds the number of static equilibrium equations available (three in the present case) and has exactly $(2j-3)$ members. A truss is said to be internally indeterminate if it has exactly three reaction components and more than $(2j-3)$ members. Finally a truss is both internally and externally indeterminate if it has more than three reaction components and also has more than $(2j-3)$ members.

The basic method for the analysis of indeterminate truss by force method is similar to the indeterminate beam analysis discussed in the previous lessons. Determine the degree of static indeterminacy of the structure. Identify the number of redundant reactions equal to the degree of indeterminacy. The redundants must be so selected that when the restraint corresponding to the redundants are removed, the resulting truss is statically determinate and stable. Select redundant as the reaction component in excess of three and the rest from the member forces. However, one could choose redundant actions completely from member forces. Following examples illustrate the analysis procedure.

Example 10.1

Determine the forces in the truss shown in Fig.10.1a by force method. All the members have same axial rigidity.

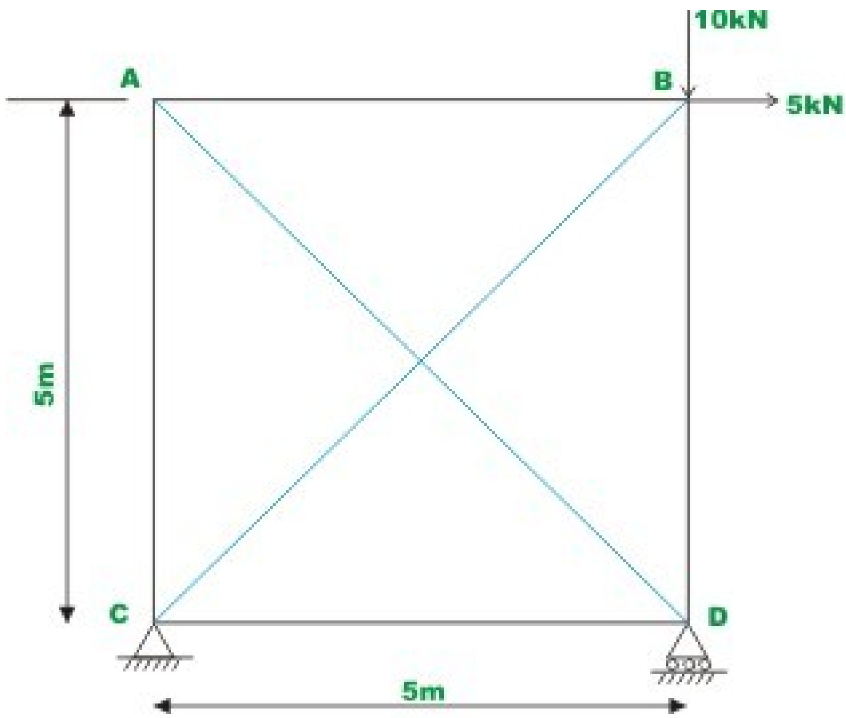

Fig.10.1 (a) Example 10.1

The plane truss shown in Fig.10.1a is statically indeterminate to first degree. The truss is externally determinate *i.e.* the reactions can be evaluated from the equations of statics alone. Select the bar force F_{AD} in member AD as the redundant. Now cut the member AD to obtain the released structure as shown in Fig. 10.1b. The cut redundant member AD remains in the truss as its deformations need to be included in the calculation of displacements in the released structure. The redundant (F_{AD}) consists of the pair of forces acting on the released structure.

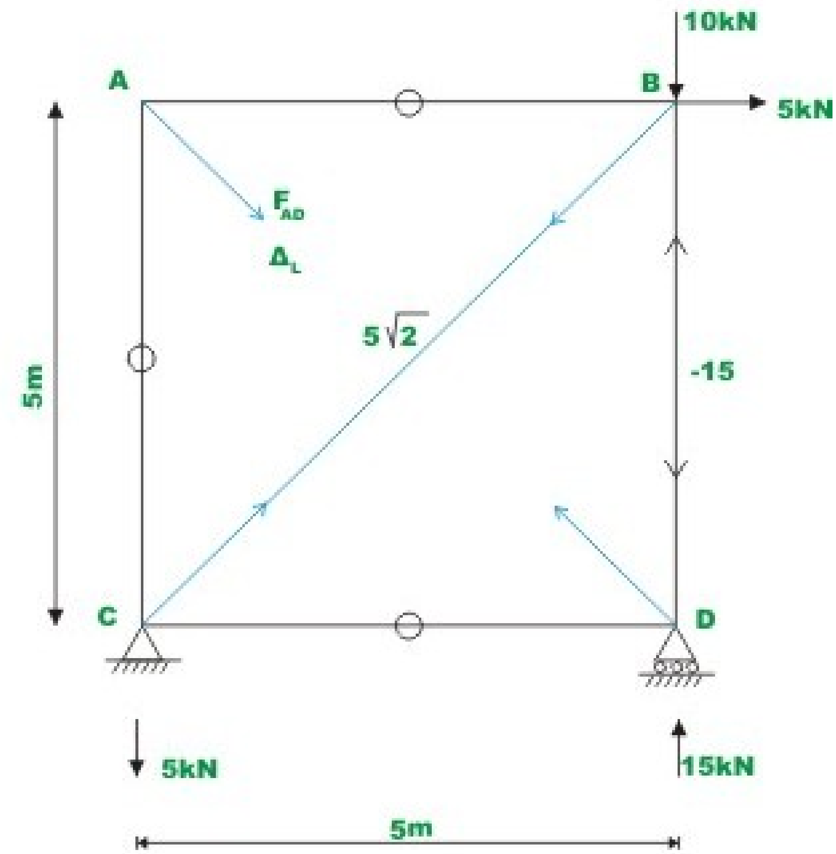

Fig.10.1 (b)

Evaluate reactions of the truss by static equations of equilibrium.

$$R_{Cy} = -5 \quad \text{kN} \quad \text{(downwards)}$$
$$R_{Cx} = -5 \quad \text{kN} \quad \text{(downwards)} \qquad (1)$$
$$R_{Dy} = 15 \quad \text{kN(upwards)}$$

Please note that the member tensile axial force is taken as positive and horizontal reaction is taken as positive to the right and vertical reaction is taken as positive when acting upwards. When the member cut ends are displaced towards one another then it is taken as positive.

The first step in the force method is to calculate displacement (Δ_L) corresponding to redundant bar force F_{AD} in the released structure due to applied external loading. This can be readily done by unit-load method.

To calculate displacement (Δ_L), apply external load and calculate member forces (P_i) as shown in Fig. 10.1b and apply unit virtual load along F_{AD} and calculate member forces $(P_v)_i$ (see Fig. 10.1c). Thus,

$$\Delta_L = \sum P_i (P_v)_i \frac{L_i}{AE} = \frac{103.03}{AE} \tag{2}$$

In the next step, apply a real unit load along the redundant F_{AD} and calculate displacement a_{11} by unit load method. Thus,

$$a_{11} = \sum (P_v)_i^2 \frac{L_i}{A_i E_i} = \frac{24.142}{AE} \tag{3}$$

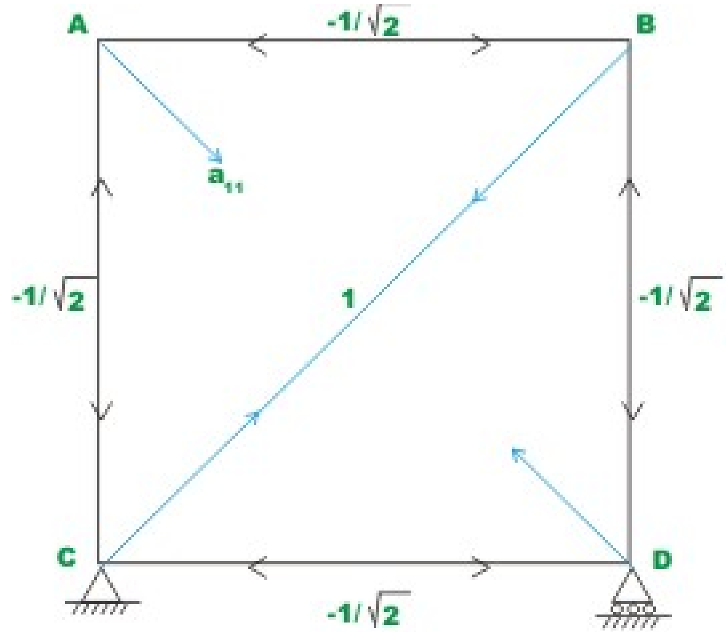

Fig. 10.1 (c) Plane truss of Example 10.1

The compatibility condition of the problem is that the relative displacement Δ_L of the cut member AD due to external loading plus the relative displacement of the member AD caused by the redundant axial forces must be equal to zero *i.e.*

$$\Delta_L + a_{11} F_{AD} = 0 \tag{4}$$

$$F_{AD} = \frac{-103.03}{24.142}$$
$$= -4.268 \quad \text{kN(compressive)}$$

Now the member forces in the members can be calculated by method of superposition. Thus,

$$F_i = P_i + F_{AD} (P_v)_i \tag{5}$$

The complete calculations can be done conveniently in a tabular form as shown in the following table.

Table 10.1 Computation for example 10.1

Member	Length L_i	Forces in the released truss due to applied loading P_i	Forces in the released truss due to unit load $(P_v)_i$	$P_i (P_v)_i \frac{L_i}{AE}$	$(P_v)_i^2 \frac{L_i}{A_i E_i}$	$F_i = P_i + F_{AD} (P_v)_i$
	m	kN	kN	m	m/kN	kN
AB	5	0	$-1/\sqrt{2}$	0	$5/2AE$	3.017
BD	5	-15	$-1/\sqrt{2}$	$75/\sqrt{2}AE$	$5/2AE$	-11.983
DC	5	0	$-1/\sqrt{2}$	0	$5/2AE$	3.017
CA	5	0	$-1/\sqrt{2}$	0	$5/2AE$	3.017
CB	$5\sqrt{2}$	$5\sqrt{2}$	1	$50/AE$	$5\sqrt{2}/AE$	2.803
AD	$5\sqrt{2}$	0	1	0	$5\sqrt{2}/AE$	-4.268
			Total	$\frac{103.03}{AE}$	$\frac{24.142}{AE}$	

Example 10.2

Calculate reactions and member forces of the truss shown in Fig. 10.2a by force method. The cross sectional areas of the members in square centimeters are shown in parenthesis. Assume $E = 2.0 \times 10^5 \text{ N/mm}^2$.

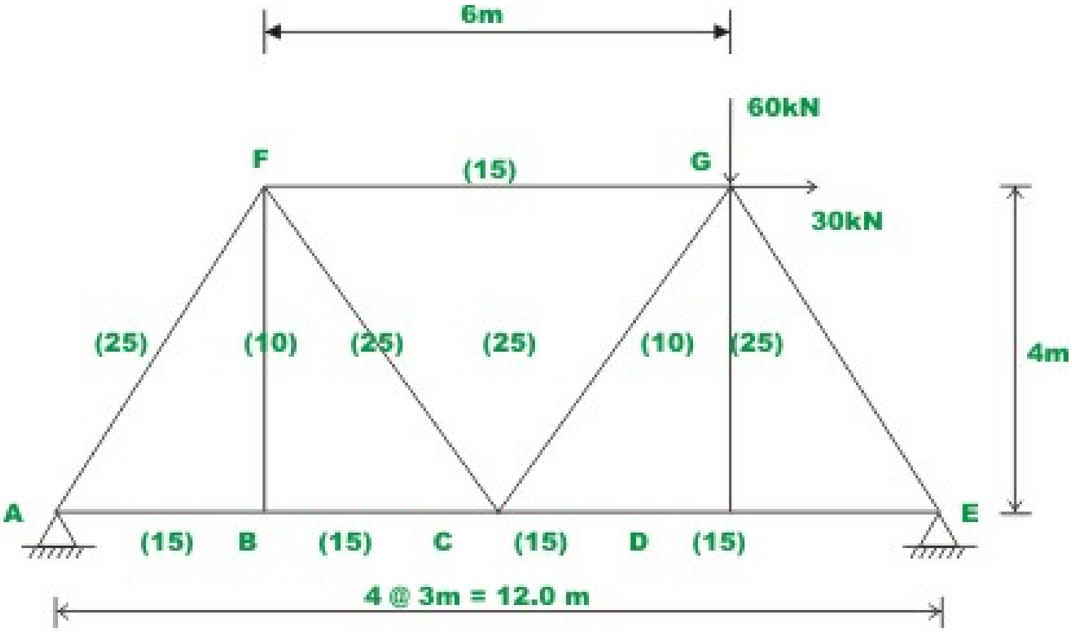

Fig.10.2 (a)

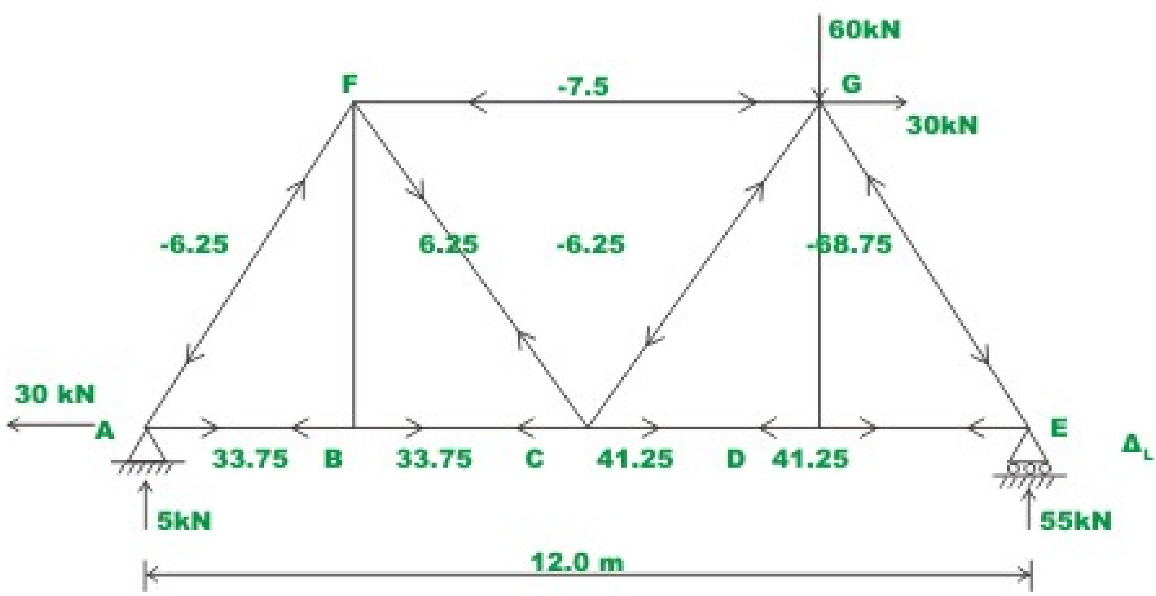

Fig.10.2 (b)

The plane truss shown in Fg.10.2a is externally indeterminate to degree one. Truss is internally determinate. Select the horizontal reaction at E, R_{Ex} as the redundant. Releasing the redundant (replacing the hinge at E by a roller support) a stable determinate truss is obtained as shown in Fig. 10.2b. The member axial forces and reactions of the released truss are shown in Fig. 10.2b.

Now calculate the displacement Δ_L corresponding to redundant reaction R_{Ex} in the released structure. This can be conveniently done in a table (see Figs. 10.2b, 10.2c and the table). Hence from the table,

$$\Delta_L = \sum P_i \left(P_v\right)_i \frac{L_i}{A_i E_i} \qquad (1)$$

$$= 15 \times 10^{-4} \quad \text{m}$$

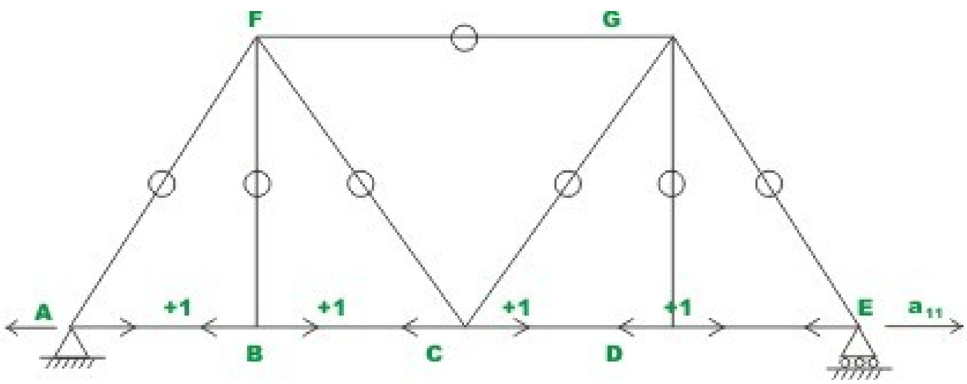

Fig. 10. 2 (c)

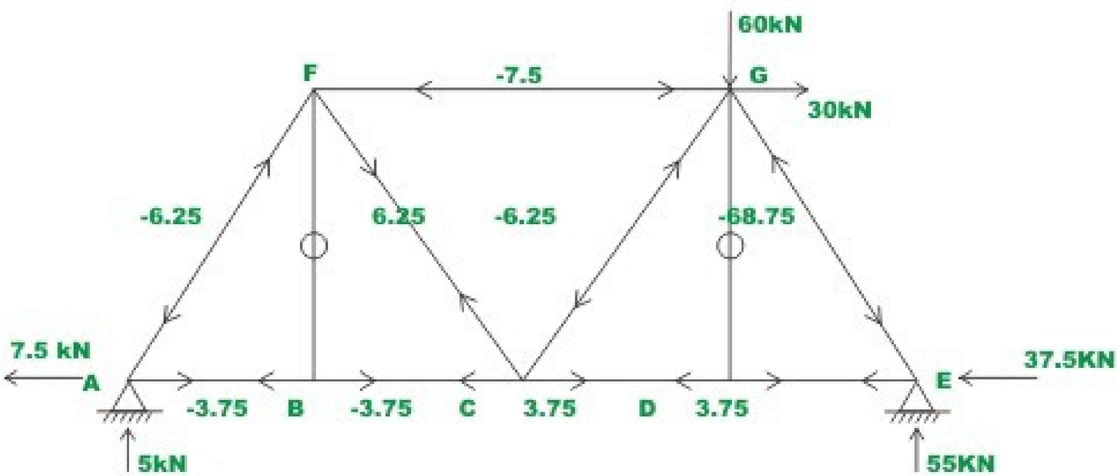

Fig. 10. 2 (d) Plane truss (Example 10.2)

In the next step apply a unit load, along the redundant reaction R_{Ex} and calculate the displacement a_{11} using unit load method.

$$a_{11} = \sum (P_v)_i^2 \frac{L_i}{A_i E_i} \qquad (2)$$
$$= 4 \times 10^{-5} \quad \text{m}$$

The support at E is hinged. Hence the total displacement at E must vanish. Thus,

$$\Delta_L + a_{11} F_{AD} = 0 \qquad (3)$$

$$15 \times 10^{-4} + 4 \times 10^{-5} R_{Ex} = 0$$

$$R_{Ex} = -\frac{15 \times 10^{-4}}{4 \times 10^{-5}}$$

$$= -37.5 \quad \text{kN(towards left)}$$

The actual member forces and reactions are shown in Fig. 10.2d.

Table 10.2 Numerical computation for example 10.2

Member	L_i	$A_i E_i$	Forces in the released truss due to applied loading P_i	Forces in the released truss due to unit load $(P_v)_i$	$P_i (P_v)_i \frac{L_i}{AE}$	$(P_v)_i^2 \frac{L_i}{A_i E_i}$	$F_i = P_i + F_{AD} (P_v)_i$
	m	(10^5)kN	kN	kN	(10^{-4})m	(10^{-5})m/Kn	kN
AB	3	3	33.75	+1	3.375	1	-3.75
BC	3	3	33.75	+1	3.375	1	-3.75
CD	3	3	41.25	+1	4.125	1	3.75
DE	3	3	41.25	+1	4.125	1	3.75
FG	6	3	-7.50	0	0	0	-7.5
FB	4	2	0.00	0	0	0	0
GD	4	2	0.00	0	0	0	0
AF	5	5	-6.25	0	0	0	-6.25
FC	5	5	6.25	0	0	0	6.25
CG	5	5	-6.25	0	0	0	-6.25
GE	5	5	-68.75	0	0	0	-68.75
				Total	15	4	

Example 10.3

Determine the reactions and the member axial forces of the truss shown in Fig.10.3a by force method due to external load and rise in temperature of member FB by $40°C$. The cross sectional areas of the members in square centimeters are shown in parenthesis. Assume $E = 2.0 \times 10^5 \text{ N/mm}^2$ and $\alpha = \frac{1}{75000}$ per °C.

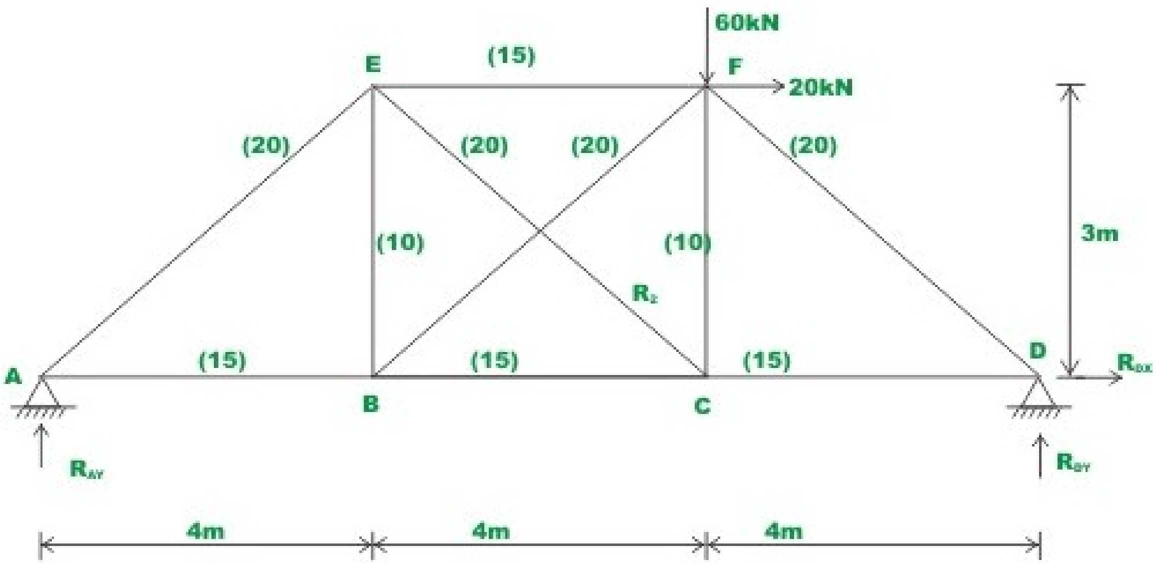

Fig. 10. 3 (a) Plane truss of Example 10.3

The given truss is indeterminate to second degree. The truss has both internal and external indeterminacy. Choose horizontal reaction at $D\ (R_1)$ and the axial force in member $EC\ (R_2)$ as redundant actions. Releasing the restraint against redundant actions, a stable determinate truss is obtained as shown in Fig. 10.3b.

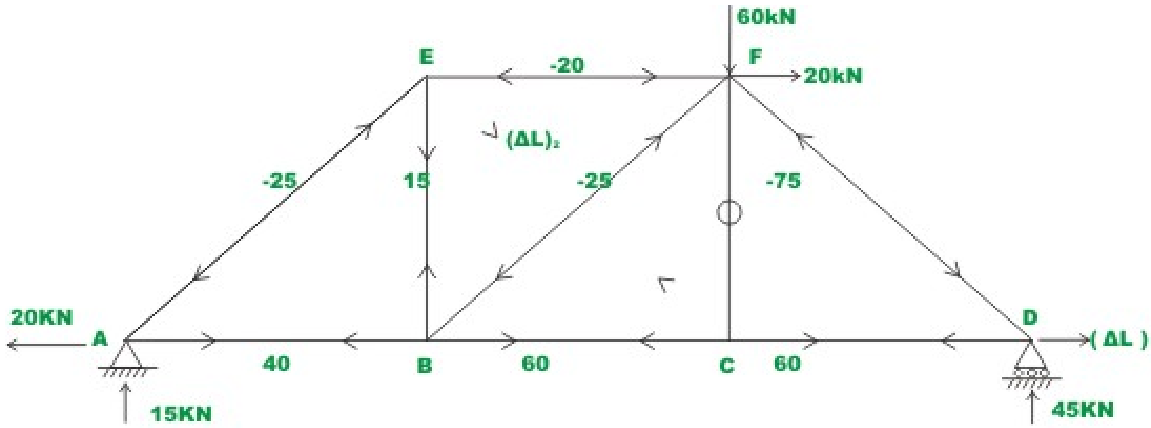

Fig. 10.3 (b)

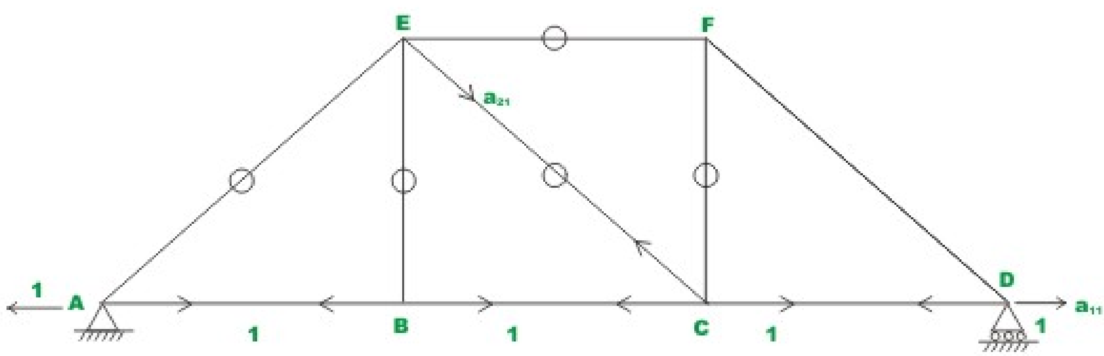

Fig. 10.3 (c)

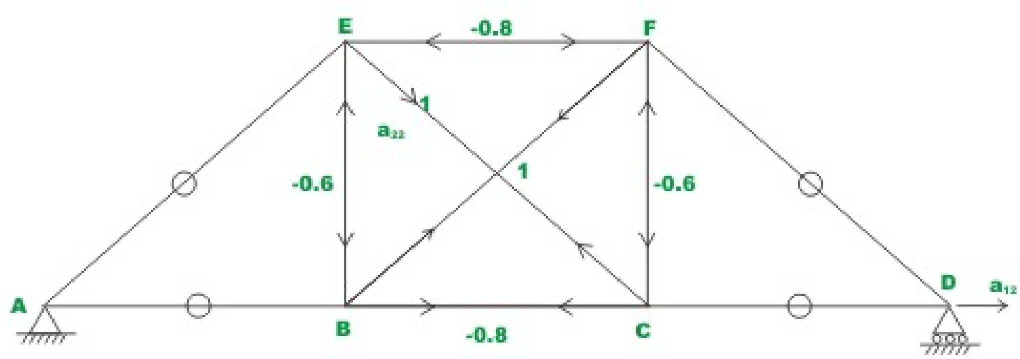

Fig. 10.3 (d)

Table 10.3a Deflection due to external loading

Member	L_i	$A_i E_i$	Forces in the released truss due to applied loading P_i	Forces in the released truss due to unit load $(P_v)_i$	Forces in the released truss due to unit load $(Q_v)_i$	$P_i (P_v)_i \frac{L_i}{AE}$	$P_i (Q_v)_i \frac{L_i}{AE}$
	m	(10^5)kN	kN	kN	kN	(10^{-4})m	(10^{-4})m
AB	4	3	40	+1	0	5.333	0.000
BC	4	3	60	+1	-0.8	8.000	-6.400
CD	4	3	60	+1	0	8.000	0.000
EF	4	3	-20	0	-0.8	0.000	2.133
EB	3	2	15	0	-0.6	0.000	-1.350
FC	3	2	0	0	-0.6	0.000	0.000
AE	5	4	-25	0	0	0.000	0.000
BF	5	4	-25	0	+1	0.000	-3.125
FD	5	4	-75	0	0	0.000	0.000
EC	5	4	0	0	+1	0.000	0.000
				Total		21.333	-8.742

Deflection of the released structure along redundant R_1 and R_2 respectively are,

$$(\Delta_L)_1 = 21.33 \times 10^{-4} \quad \text{m} \quad \text{(towards right)}$$

$$(\Delta_L)_2 = -8.742 \times 10^{-4} \quad \text{m} \quad \text{(shortening)} \qquad (1)$$

In the next step, compute the flexibility coefficients (ref. Fig. 10.3c and Fig. 10.3d and the accompanying table)

Table 10.3b Computation of flexibility coefficients

Member	L_i	A_iE_i	$(P_v)_i$	$(P_v)_i^2 \frac{L_i}{A_iE_i}$	$(Q_v)_i$	$(Q_v)_i^2 \frac{L_i}{A_iE_i}$	$(P_v)_i(Q_v)_i \frac{L_i}{AE}$
	m	(10^5)kN	kN	(10^{-5})m/kN	kN	(10^{-5})m/kN	(10^{-5})m/kN
AB	4	3	+1.00	1.333	0.000	0.000	0.000
BC	4	3	+1.00	1.333	-0.800	0.853	-1.067
CD	4	3	+1.00	1.333	0.000	0.000	0.000
EF	4	3	0	0.000	-0.800	0.853	0.000
EB	3	2	0	0.000	-0.600	0.540	0.000
FC	3	2	0	0.000	-0.600	0.540	0.000
AE	5	4	0	0.000	0.000	0.000	0.000
BF	5	4	0	0.000	1.000	1.250	0.000
FD	5	4	0	0.000	0.000	0.000	0.000
EC	5	4	0	0.000	1.000	1.250	0.000
			Total	4.000		5.286	-1.064

Thus,

$$a_{11} = 4 \times 10^{-5}$$

$$a_{12} = a_{21} = -1.064 \times 10^{-5} \qquad (2)$$

$$a_{22} = 5.286 \times 10^{-5}$$

Analysis of truss when only external loads are acting

The compatibility conditions of the problem may be written as,

$$(\Delta_L)_1 + a_{11}R_1 + a_{12}R_2 = 0$$

$$(\Delta_L)_2 + a_{21}R_1 + a_{22}R_2 = 0 \qquad (3)$$

Solving $R_1 = -51.73 \text{ kN (towards left)}$ and $R_2 = 6.136 \text{ kN (tensile)}$

The actual member forces and reactions in the truss are shown in Fig 10.3c. Now, compute deflections corresponding to redundants due to rise in temperature in the member FB. Due to rise in temperature, the change in length of member FB is,

$$\Delta_T = \alpha T L$$

$$= \frac{1}{75000} \times 40 \times 5 = 2.67 \times 10^{-3}\, m \qquad (4)$$

Due to change in temperature, the deflections corresponding to redundants R_1 and R_2 are

$$(\Delta_T)_1 = \sum (P_v)_i (\Delta_T)_i = 0$$

$$(\Delta_T)_2 = \sum (Q_v)_i (\Delta_T)_i = 2.67 \times 10^{-3}\, m \qquad (5)$$

When both external loading and temperature loading are acting

When both temperature loading and the external loading are considered, the compatibility equations can be written as,

$$(\Delta_L)_1 + (\Delta_T)_1 + a_{11} R_1 + a_{12} R_2 = 0$$

$$(\Delta_L)_2 + (\Delta_T)_2 + a_{21} R_1 + a_{22} R_2 = 0 \qquad (6)$$

Solving $R_1 = -65.92 \text{ kN(towards left)}$ and $R_2 = -47.26 \text{ kN (compressive)}$

The actual member forces and reactions are shown in Fig. 10.3f

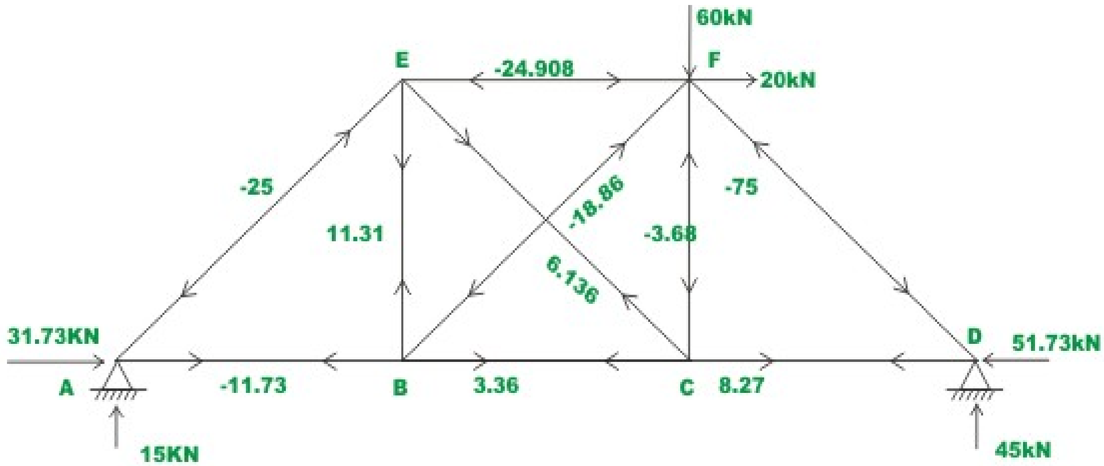

Fig. 10.3 (e)

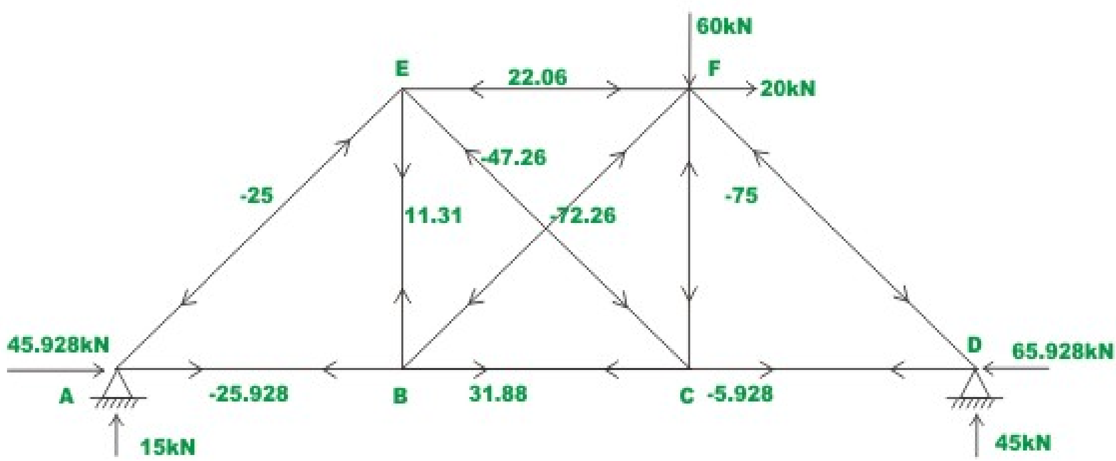

Fig. 10.3 (f)

Fig. 10.3 Plane truss of example 10.3

Summary

In this lesson, the flexibility matrix method is used to analyse statically indeterminate planar trusses. The equation to calculate the degree of statical indeterminacy of a planar truss is derived. The forces induced in the members due to temperature loading and member lack of fit is also discussed in this lesson. Few examples are solved to illustrate the force method of analysis as applied to statically indeterminate planar trusses.