Chapter 5. Design of Beams – Flexure and Shear

5.1 Section force-deformation response & Plastic Moment (M_p)

- A beam is a structural member that is subjected primarily to transverse loads and negligible axial loads.
- The transverse loads cause internal shear forces and bending moments in the beams as shown in Figure 1 below.

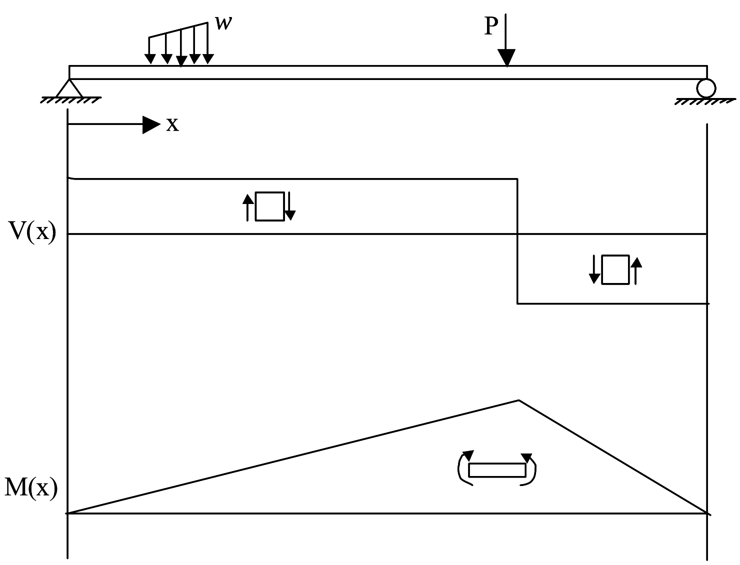
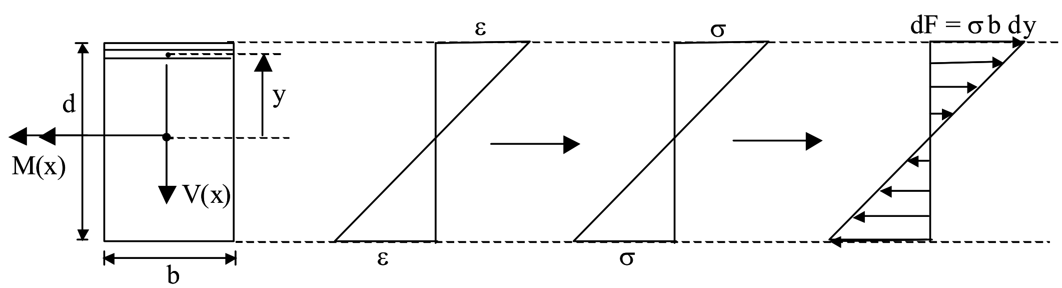


Figure 1. Internal shear force and bending moment diagrams for transversely loaded beams.

- These internal shear forces and bending moments cause longitudinal axial stresses and shear stresses in the cross-section as shown in the Figure 2 below.



Curvature = $\phi = 2\varepsilon/d$ $\quad$ $F = \int_{-d/2}^{+d/2} \sigma\, b\, dy$ $\quad$ $M = \int_{-d/2}^{+d/2} \sigma\, b\, dy\, y$ (Planes remain plane)

Figure 2. Longitudinal axial stresses caused by internal bending moment.

- Steel material follows a typical stress-strain behavior as shown in Figure 3 below.

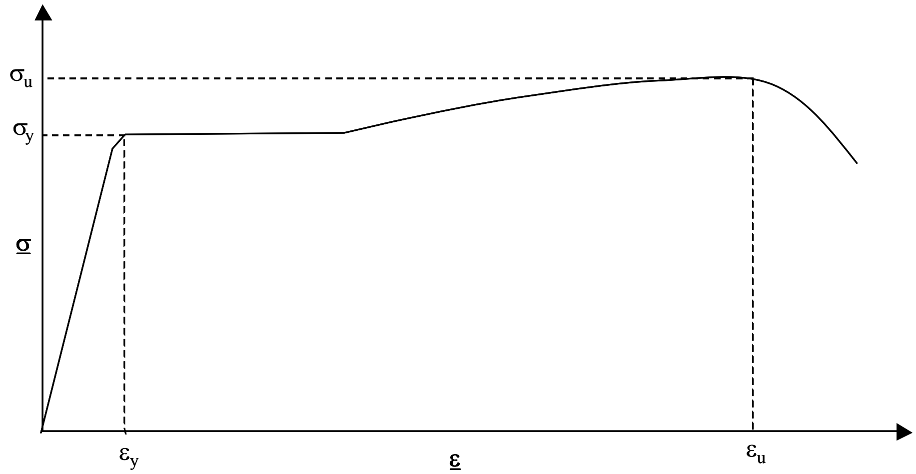
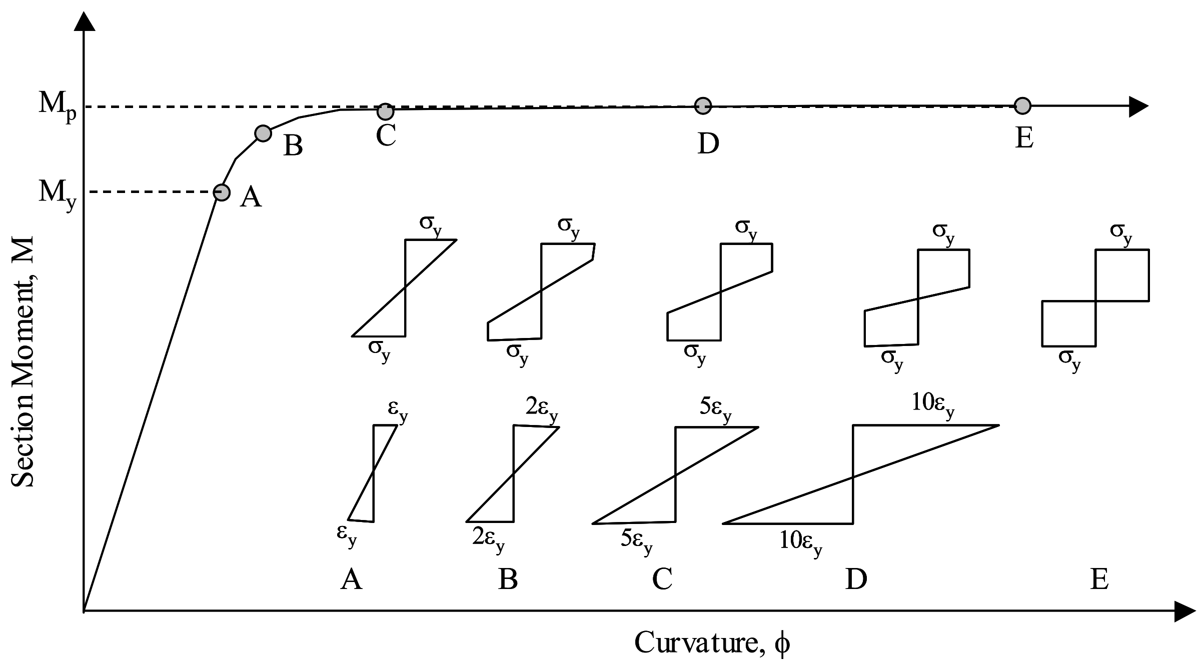


Figure 3. Typical steel stress-strain behavior.

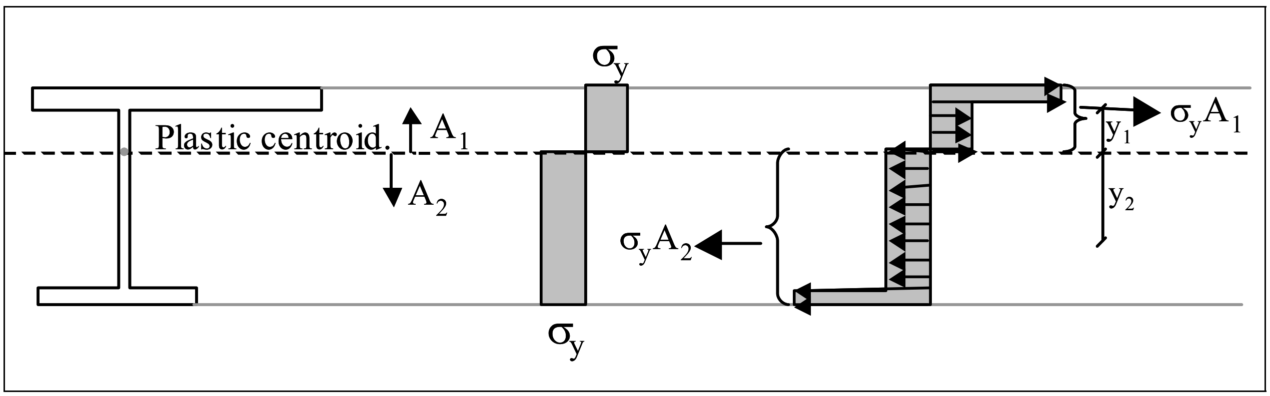
- If the steel stress-strain curve is approximated as a bilinear elasto-plastic curve with yield stress equal to σ_y, then the section Moment - Curvature (M-ϕ) response for monotonically increasing moment is given by Figure 4.



A: Extreme fiber reaches ε_y B: Extreme fiber reaches $2\varepsilon_y$ C: Extreme fiber reaches $5\varepsilon_y$
D: Extreme fiber reaches $10\varepsilon_y$ E: Extreme fiber reaches infinite strain

Figure 4. Section Moment - Curvature (M-ϕ) behavior.

- In Figure 4, M_y is the moment corresponding to first yield and M_p is the plastic moment capacity of the cross-section.
  - The ratio of M_p to M_y is called as the shape factor *f* for the section.
  - For a rectangular section, *f* is equal to 1.5. For a wide-flange section, *f* is equal to 1.1.
- Calculation of M_p: Cross-section subjected to either $+\sigma_y$ or $-\sigma_y$ at the plastic limit. See Figure 5 below.



(a) General cross-section (b) Stress distribution (c) Force distribution

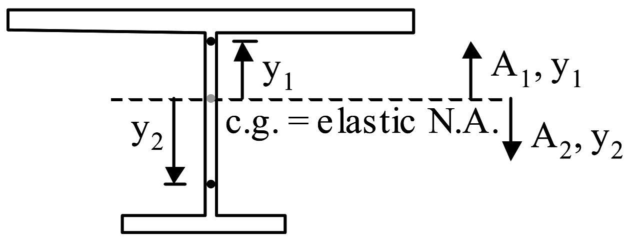
$$F = \sigma_y A_1 - \sigma_y A_2 = 0$$
$$\therefore A_1 = A_2 = A/2$$
$$\therefore M = \sigma_y \frac{A}{2} \times (y_1 + y_2)$$
$$\text{Where, } y_1 = \text{centroid of } A_1$$
$$y_2 = \text{centroid of } A_2$$

(d) Equations

Figure 5. Plastic centroid and M_p for general cross-section.

- The plastic centroid for a general cross-section corresponds to the axis about which the total area is equally divided, i.e., $A_1 = A_2 = A/2$
  - The plastic centroid is not the same as the elastic centroid or center of gravity (c.g.) of the cross-section.

- As shown below, the c.g. is defined as the axis about which $A_1y_1 = A_2y_2$.



About the c.g. $A_1y_1 = A_2y_2$

- For a cross-section with at-least one axis of symmetry, the neutral axis corresponds to the centroidal axis in the elastic range. However, at M_p, the neutral axis will correspond to the plastic centroidal axis.

- **For a doubly symmetric cross-section, the elastic and the plastic centroid lie at the same point.**
- $M_p = \sigma_y$ x A/2 x (y_1+y_2)
- As shown in Figure 5, y_1 and y_2 are the distance from the plastic centroid to the centroid of area A_1 and A_2, respectively.
- A/2 x (y_1+y_2) is called **Z,** the plastic section modulus of the cross-section. Values for Z are tabulated for various cross-sections in the properties section of the LRFD manual.
- $\phi M_p = 0.90\ Z\ F_y$ - See Spec. F1.1

  where,

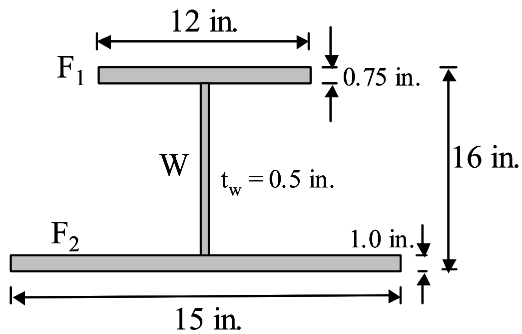
  M_p = plastic moment, which must be ≤ 1.5 M_y for homogenous cross-sections

  M_y = moment corresponding to onset of yielding at the extreme fiber from an elastic stress distribution = F_y S for homogenous cross-sections and = F_{yf} S for hybrid sections.

  Z = plastic section modulus from the Properties section of the AISC manual.

  S = elastic section modulus, also from the Properties section of the AISC manual.

Example 2.1 Determine the elastic section modulus, S, plastic section modulus, Z, yield moment, M_y, and the plastic moment M_p, of the cross-section shown below. What is the design moment for the beam cross-section. Assume 50 ksi steel.



- $A_g = 12 \text{ x } 0.75 + (16 - 0.75 - 1.0) \text{ x } 0.5 + 15 \text{ x } 1.0 = 31.125 \text{ in}^2$

  $A_{f1} = 12 \text{ x } 0.75 = 9 \text{ in}^2$

  $A_{f2} = 15 \text{ x } 1.0 = 15.0 \text{ in}^2$

  $A_w = 0.5 \text{ x } (16 - 0.75 - 1.0) = 7.125 \text{ in}^2$

- distance of elastic centroid from bottom = $\bar{y}$

  $$\bar{y} = \frac{9 \times (16 - 0.75/2) + 7.125 \times 8.125 + 15 \times 0.5}{31.125} = 6.619 \text{ in.}$$

  $I_x = 12 \times 0.75^3/12 + 9.0 \times 9.006^2 + 0.5 \times 14.25^3/12 + 7.125 \times 1.506^2 + 15.0 \times 1^3/12 +$

  $15 \times 6.119^2 = 1430 \text{ in}^4$

  $S_x = I_x / (16\text{-}6.619) = 152.43 \text{ in}^3$

  $M_{y\text{-}x} = F_y\, S_x = 7621.8$ kip-in. = 635.15 kip-ft.

- distance of plastic centroid from bottom = $\bar{y}_p$

  $$\therefore \quad 15.0 \times 1.0 + 0.5 \times (\bar{y}_p - 1.0) = \frac{31.125}{2} = 15.5625$$

  $$\therefore \quad \bar{y}_p = 2.125 \text{ in.}$$

y_1=centroid of top half-area about plastic centroid = $\dfrac{9\times 13.5 + 6.5625\times 6.5625}{15.5625} = 10.5746$ in.

y_2=centroid of bottom half-area about plas. cent. = $\dfrac{0.5625\times 0.5625 + 15.0\times 1.625}{15.5625} = 1.5866$ in.

$Z_x = A/2 \text{ x } (y_1 + y_2) = 15.5625 \text{ x } (10.5746 + 1.5866) = 189.26 \text{ in}^3$

$M_{p\text{-}x} = Z_x\, F_y = 189.26 \text{ x } 50 = 9462.93$ kip-in. = 788.58 kip-ft.

- Design strength according to AISC Spec. F1.1= $\phi_b M_p$= 0.9 x 788.58 = 709.72 kip-ft.
- Check = $M_p \leq 1.5\ M_y$

  Therefore, 788.58 kip-ft. < 1.5 x 635.15 = 949.725 kip-ft. - OK!

- Reading Assignment

5.2 Flexural Deflection of Beams – Serviceability

- Steel beams are designed for the factored design loads. The moment capacity, i.e., the factored moment strength ($\phi_b M_n$) should be greater than the moment (M_u) caused by the factored loads.
- A *serviceable* structure is one that performs satisfactorily, not causing discomfort or perceptions of unsafety for the occupants or users of the structure.
  - For a beam, being serviceable usually means that the deformations, primarily the vertical slag, or deflection, must be limited.
  - The maximum deflection of the designed beam is checked at the service-level loads. The deflection due to service-level loads must be less than the specified values.
- The AISC Specification gives little guidance other than a statement in Chapter L, "*Serviceability Design Considerations*," that deflections should be checked. Appropriate limits for deflection can be found from the governing building code for the region.
- The following values of deflection are typical maximum allowable total (service dead load plus service live load) deflections.
  - Plastered floor construction – L/360
  - Unplastered floor construction – L/240
  - Unplastered roof construction – L/180
- In the following examples, we will assume that local buckling and lateral-torsional buckling are not controlling limit states, i.e, the beam section is compact and laterally supported along the length.

Example 5.2 Design a simply supported beam subjected to uniformly distributed dead load of 450 lbs/ft. and a uniformly distributed live load of 550 lbs/ft. The dead load does not include the self-weight of the beam.

- **Step I.** Calculate the factored design loads (without self-weight).

  $w_U = 1.2\ w_D + 1.6\ w_L = 1.42$ kips / ft.

  $M_U = w_u\ L^2 / 8 = 1.42 \times 30^2 / 8 = 159.75$ kip-ft.

- **Step II.** Select the lightest section from the AISC Manual design tables.

  From page ____________ of the AISC manual, select **W16 x 26** made from 50 ksi steel with $\phi_b M_p = 166.0$ kip-ft.

- **Step III.** Add self-weight of designed section and check design

  $w_{sw} = 26$ lbs/ft

  Therefore, $w_D = 476$ lbs/ft = 0.476 lbs/ft.

  $w_u = 1.2 \times 0.476 + 1.6 \times 0.55 = 1.4512$ kips/ft.

  Therefore, $M_u = 1.4512 \times 30^2 / 8 = 163.26$ kip-ft. $< \phi_b M_p$ of W16 x 26.

  <u>**OK!**</u>

- **Step IV.** Check deflection at service loads.

  $w = 0.45 + 0.026 + 0.55$ kips/ft. = 1.026 kips/ft.

  $\Delta = 5\ w\ L^4 / (384\ E\ I_x) = 5 \times (1.026/12) \times (30 \times 12)^4 / (384 \times 29000 \times 301)$

  $\Delta = 2.142$ in. $> L/360$ - for plastered floor construction

- **Step V.** Redesign with service-load deflection as design criteria

  $L/360 = 1.0$ in. $> 5\ w\ L^4/(384\ E\ I_x)$

  Therefore, $I_x > 644.8$ in^4

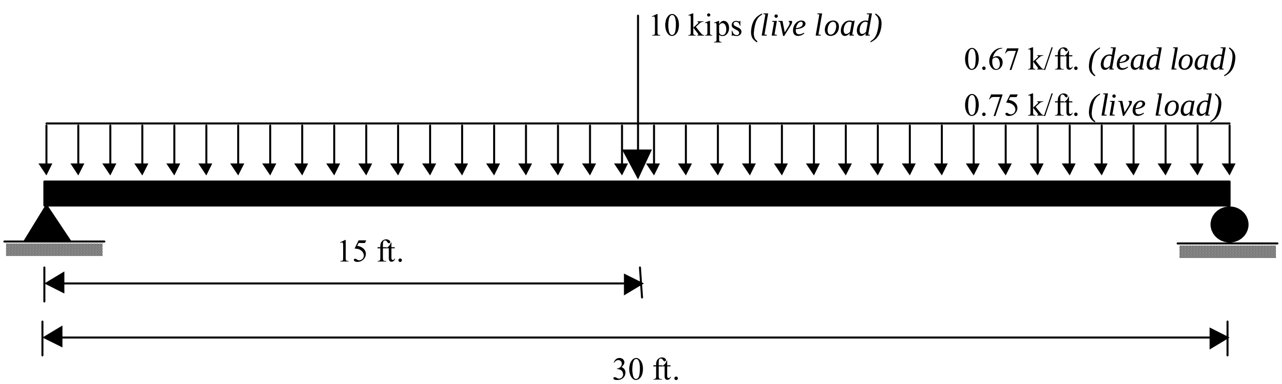
Select the section from the *moment of inertia* selection tables in the AISC manual. See page – select **W21 x 44.**

W21 x 44 with $\mathbf{I_x}$ = 843 in^4 and $\boldsymbol{\phi_b M_p}$ = 358 kip-ft. (50 ksi steel).

Deflection at service load = Δ = 0.765 in. < L/360 - **OK!**

Note that the serviceability design criteria controlled the design and the section

Example 5.3 Design the beam shown below. The unfactored dead and live loads are shown in the Figure.



- **Step I.** Calculate the factored design loads (without self-weight).

  $w_u = 1.2\ w_D + 1.6\ w_L = 1.2 \times 0.67 + 1.6 \times 0.75 = 2.004$ kips / ft.

  $P_u = 1.2\ P_D + 1.6\ P_L = 1.2 \times 0 + 1.6 \times 10 = 16.0$ kips

  $M_u = w_U\ L^2 / 8 + P_U\ L / 4 = 225.45 + 120 = 345.45$ kip-ft.

- **Step II.** Select the lightest section from the AISC Manual design tables.

  From page ____________ of the AISC manual, select **W21 x 44** made from 50 ksi steel with $\phi_b M_p$ = 358.0 kip-ft.

  Self-weight = w_{sw} = 44 lb/ft.

- **Step III.** Add self-weight of designed section and check design

  $w_D = 0.67 + 0.044 = 0.714$ kips/ft

  $w_u = 1.2 \times 0.714 + 1.6 \times 0.75 = 2.0568$ kips/ft.

  Therefore, $M_u = 2.0568 \times 30^2 / 8 + 120 = 351.39$ kip-ft. $< \phi_b M_p$ of W21 x 44.

  <u>OK!</u>

- **Step IV.** Check deflection at service loads.

  Service loads

  – Distributed load = w = 0.714 + 0.75 = 1.464 kips/ft.

  – Concentrated load = P = D + L = 0 + 10 kips = 10 kips

  Deflection due to uniform distributed load = $\Delta_d = 5\ w\ L^4 / (384\ EI)$

  Deflection due to concentrated load = $\Delta_c = P\ L^3 / (48\ EI)$

Therefore, service-load deflection = $\Delta = \Delta_d + \Delta_c$

$\Delta = 5 \times 1.464 \times 360^4 / (384 \times 29000 \times 12 \times 843) + 10 \times 360^3 / (48 \times 29000 \times 843)$

$\Delta = 1.0914 + 0.3976 = 1.49$ in.

Assuming unplastered floor construction, $\Delta_{max} = L/240 = 360/240 = 1.5$ in.

Therefore, $\Delta < \Delta_{max}$ **<u>- OK!</u>**

5.3 Local buckling of beam section – Compact and Non-compact

- M_p, the plastic moment capacity for the steel shape, is calculated by assuming a plastic stress distribution (+ or - σ_y) over the cross-section.
- The development of a plastic stress distribution over the cross-section can be hindered by two different length effects:

  (1) *Local buckling* of the individual plates (flanges and webs) of the cross-section before they develop the compressive yield stress σ_y.

  (2) *Lateral-torsional buckling* of the unsupported length of the beam / member before the cross-section develops the plastic moment M_p.

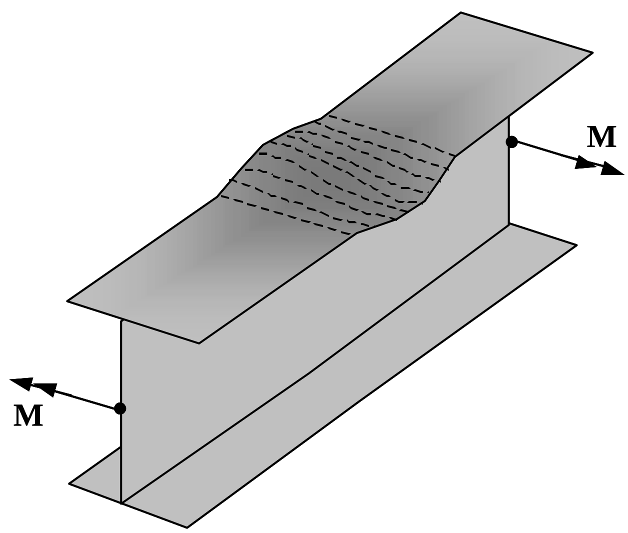


Figure 7. Local buckling of flange due to compressive stress (σ)

- The analytical equations for local buckling of steel plates with various edge conditions and the results from experimental investigations have been used to develop limiting slenderness ratios for the individual plate elements of the cross-sections.
- See Spec. B5 (page 16.1 – 12), Table B5.1 (16.1-13) and Page 16.1-183 of the AISC-manual
- Steel sections are classified as compact, non-compact, or slender depending upon the slenderness (λ) ratio of the individual plates of the cross-section.

- *Compact section* if all elements of cross-section have $\lambda \leq \lambda_p$
- *Non-compact sections* if any one element of the cross-section has $\lambda_p \leq \lambda \leq \lambda_r$
- *Slender section* if any element of the cross-section has $\lambda_r \leq \lambda$

- It is important to note that:
  - If $\lambda \leq \lambda_p$, then the individual plate element can develop and sustain σ_y for large values of ε before local buckling occurs.
  - If $\lambda_p \leq \lambda \leq \lambda_r$, then the individual plate element can develop σ_y but cannot sustain it before local buckling occurs.
  - If $\lambda_r \leq \lambda$, then elastic local buckling of the individual plate element occurs.

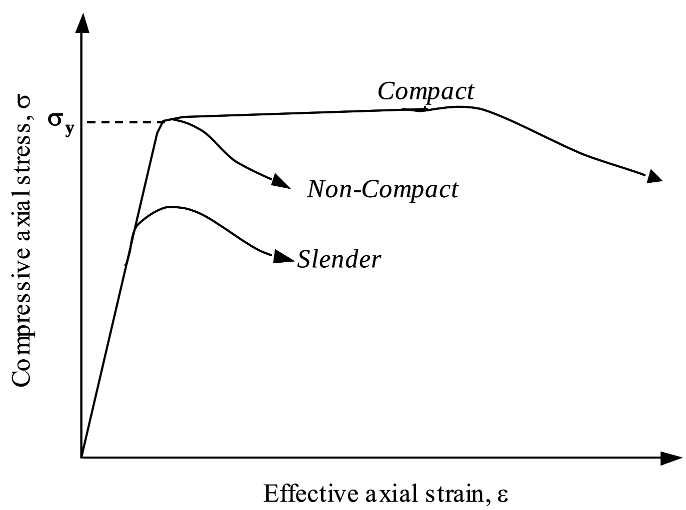


Figure 8. Stress-strain response of plates subjected to axial compression and local buckling.

- Thus, slender sections cannot develop M_p due to elastic local buckling. Non-compact sections can develop M_y but not M_p before local buckling occurs. Only compact sections can develop the plastic moment M_p.
- All rolled wide-flange shapes are <u>**compact**</u> with the following exceptions, which are non-compact.

- W40x174, W14x99, W14x90, W12x65, W10x12, W8x10, W6x15 (made from A992)

- The definition of λ and the values for λ_p and λ_r for the individual elements of various cross-sections are given in Table B5.1 and shown graphically on page 16.1-183. For example,

Section	**Plate element**	λ	λ_p	λ_r
Wide-flange	Flange	$b_f/2t_f$	$0.38\sqrt{E/F_y}$	$0.38\sqrt{E/F_L}$
	Web	h/t_w	$3.76\sqrt{E/F_y}$	$5.70\sqrt{E/F_y}$
Channel	Flange	b_f/t_f	$0.38\sqrt{E/F_y}$	$0.38\sqrt{E/F_L}$
	Web	h/t_w	$3.76\sqrt{E/F_y}$	$5.70\sqrt{E/F_y}$
Square or Rect. Box	Flange	(b-3t)/t	$1.12\sqrt{E/F_y}$	$1.40\sqrt{E/F_y}$
	Web	(b-3t)/t	$3.76\sqrt{E/F_y}$	$5.70\sqrt{E/F_y}$

In CE405 we will design all beam sections to be compact from a local buckling standpoint

5.4 Lateral-Torsional Buckling

- The laterally unsupported length of a beam-member can undergo lateral-torsional buckling due to the applied flexural loading (bending moment).

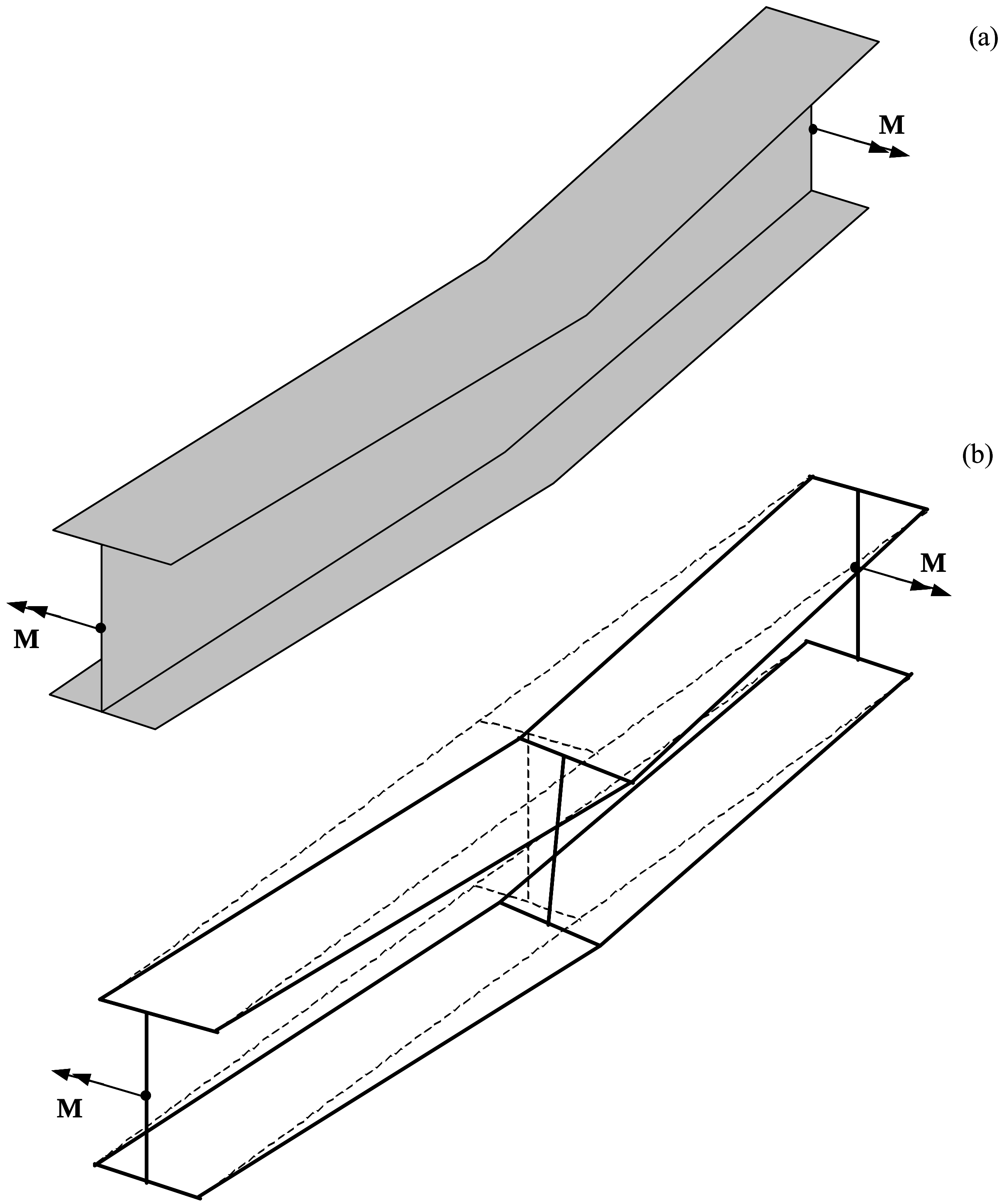


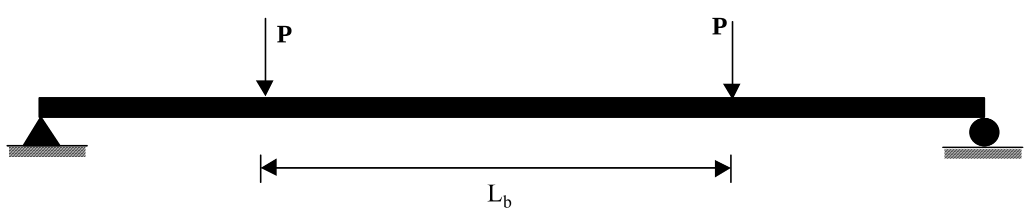
Figure 9. Lateral-torsional buckling of a wide-flange beam subjected to constant moment.

- Lateral-torsional buckling is fundamentally similar to the flexural buckling or flexural-torsional buckling of a column subjected to axial loading.
  - The similarity is that it is also a bifurcation-buckling type phenomenon.
  - The differences are that lateral-torsional buckling is caused by flexural loading (M), and the buckling deformations are coupled in the lateral and torsional directions.

  - There is one very important difference. For a column, the axial load causing buckling remains constant along the length. But, for a beam, usually the lateral-torsional buckling causing bending moment M(x) varies along the unbraced length.
  - The worst situation is for beams subjected to uniform bending moment along the unbraced length. Why?

5.4.1 Lateral-torsional buckling – Uniform bending moment

- Consider a beam that is simply-supported at the ends and subjected to four-point loading as shown below. The beam center-span is subjected to uniform bending moment M. Assume that lateral supports are provided at the load points.



- Laterally unsupported length = L_b.
- If the laterally unbraced length L_b is less than or equal to a plastic length L_p then lateral torsional buckling is not a problem and the beam will develop its plastic strength M_p.
- $L_p = 1.76\ r_y \times \sqrt{E/F_y}$ - for I members & channels (See Pg. **16.1**-33)

- If L_b is greater than L_p then lateral torsional buckling will occur and the moment capacity of the beam will be reduced below the plastic strength M_p as shown in Figure 10 below.

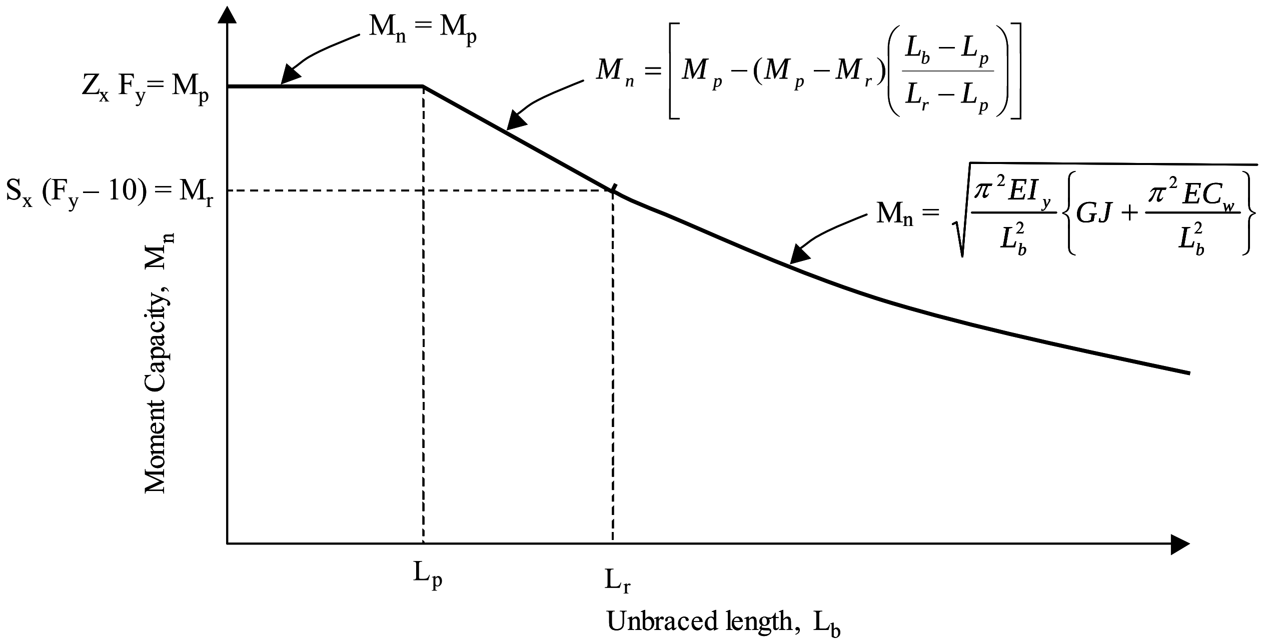


Figure 10. Moment capacity (M_n) versus unsupported length (L_b).

- As shown in Figure 10 above, the lateral-torsional buckling moment ($M_n = M_{cr}$) is a function of the laterally unbraced length L_b and can be calculated using the equation:

$$M_n = M_{cr} = \frac{\pi}{L_b}\sqrt{E \times I_y \times G \times J + \left(\frac{\pi \times E}{L_b}\right)^2 \times I_y \times C_w}$$

where, M_n = moment capacity

L_b = laterally unsupported length.

M_{cr} = critical lateral-torsional buckling moment.

E = 29000 ksi; G = 11,200 ksi

I_y = moment of inertia about minor or y-axis (in^4)

J = torsional constant (in^4) from the AISC manual pages _______________.

C_w = warping constant (in^6) from the AISC manual pages _______________.

- This equation is valid for **ELASTIC** lateral torsional buckling only (like the Euler equation). That is it will work only as long as the cross-section is elastic and no portion of the cross-section has yielded.
- As soon as any portion of the cross-section reaches the yield stress F_y, the elastic lateral torsional buckling equation cannot be used.
  - L_r is the unbraced length that corresponds to a lateral-torsional buckling moment $M_r = S_x\,(F_y - 10)$.
  - M_r will cause yielding of the cross-section due to residual stresses.
- When the unbraced length is less than L_r, then the elastic lateral torsional buckling equation cannot be used.
- When the unbraced length (L_b) is less than L_r but more than the plastic length L_p, then the lateral-torsional buckling M_n is given by the equation below:
  - If $L_p \le L_b \le L_r$, then $M_n = \left[M_p - (M_p - M_r)\left(\dfrac{L_b - L_p}{L_r - L_p}\right)\right]$
  - This is linear interpolation between (L_p, M_p) and (L_r, M_r)
  - See Figure 10 again.

5.4.2 Moment Capacity of beams subjected to non-uniform bending moments

- As mentioned previously, the case with uniform bending moment is worst for lateral torsional buckling.
- For cases with non-uniform bending moment, the lateral torsional buckling moment **is greater** than that for the case with uniform moment.
- The AISC specification says that:
  - The lateral torsional buckling moment for non-uniform bending moment case

    = $\mathbf{C_b}$ **x** lateral torsional buckling moment for uniform moment case.
- $\mathbf{C_b}$ is always greater than 1.0 for non-uniform bending moment.
  - $\mathbf{C_b}$ is equal to 1.0 for uniform bending moment.
  - Sometimes, if you cannot calculate or figure out $\mathbf{C_b}$, then it can be conservatively assumed as 1.0.
- $$C_b = \frac{12.5\ M_{max}}{2.5\ M_{max} + 3\ M_A + 4\ M_B + 3\ M_c}$$

  where, $\mathbf{M_{max}}$ = magnitude of maximum bending moment in $\mathbf{L_b}$

  $\mathbf{M_A}$ = magnitude of bending moment at quarter point of $\mathbf{L_b}$

  $\mathbf{M_B}$ = magnitude of bending moment at half point of $\mathbf{L_b}$

  $\mathbf{M_C}$ = magnitude of bending moment at three-quarter point of $\mathbf{L_b}$
- The moment capacity M_n for the case of non-uniform bending moment
  - $M_n = \mathbf{C_b}$ x {M_n for the case of uniform bending moment} $\leq \mathbf{M_p}$
  - Important to note that the increased moment capacity for the non-uniform moment case cannot possibly be more than $\mathbf{M_p}$.
  - Therefore, if the calculated values is greater than $\mathbf{M_p}$, then you have to reduce it to $\mathbf{M_p}$

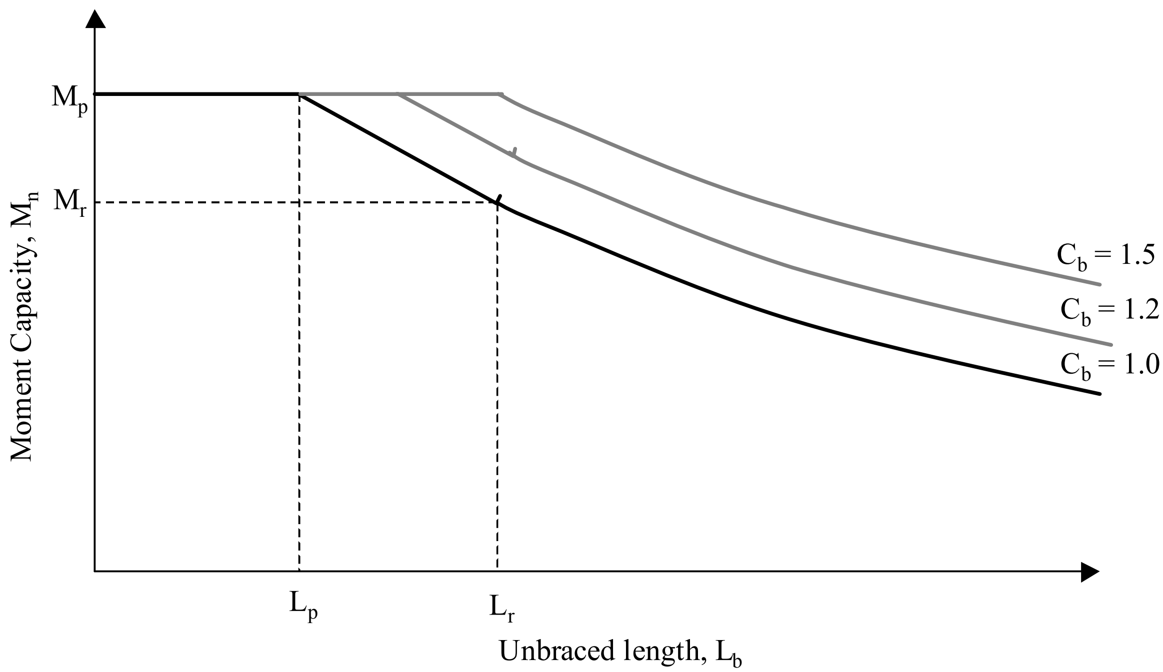
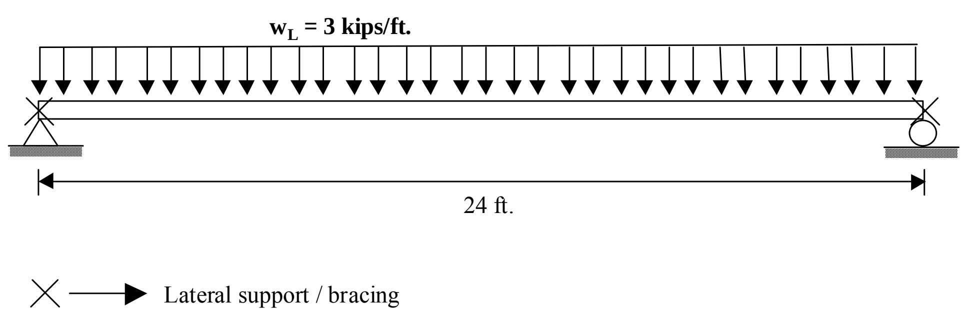


Figure 11. Moment capacity versus L_b for non-uniform moment case.

5.5 Beam Design

Example 2.4

Design the beam shown below. The unfactored uniformly distributed live load is equal to 3 kips/ft. There is no dead load. Lateral support is provided at the end reactions.



Step I. Calculate the factored loads assuming a reasonable self-weight.

Assume self-weight = w_{sw} = 100 lbs/ft.

Dead load = w_D = 0 + 0.1 = 0.1 kips/ft.

Live load = w_L = 3.0 kips/ft.

Ultimate load = $w_u = 1.2\ w_D + 1.6\ w_L$ = 4.92 kips/ft.

Factored ultimate moment = $M_u = w_u\ L^2/8$ = 354.24 kip-ft.

Step II. Determine unsupported length L_b and C_b

There is only one unsupported span with L_b = 24 ft.

C_b = 1.14 for the parabolic bending moment diagram, See values of C_b shown in Figure.

Step III. Select a wide-flange shape

The moment capacity of the selected section $\boldsymbol{\phi_b M_n > M_u}$ (Note $\boldsymbol{\phi_b = 0.9}$)

$\boldsymbol{\phi_b M_n}$ = moment capacity = $\boldsymbol{C_b}$ x ($\boldsymbol{\phi_b M_n}$ for the case with **uniform moment**) $\leq \boldsymbol{\phi_b M_p}$

- Pages __________________ in the AISC-LRFD manual, show the plots of $\boldsymbol{\phi_b M_n\text{-}L_b}$ for the case of uniform bending moment (C_b=1.0)
- Therefore, in order to select a section, calculate $\boldsymbol{M_u/C_b}$ and use it with L_b to find the first section with a <u>**solid line**</u> as shown in class.
- M_u/C_b = 354.24/1.14 = 310.74 kip-ft.
- Select W16 x 67 (50 ksi steel) with $\phi_b M_n$ =357 kip-ft. for L_b = 24 ft. and C_b =1.0
- For the case with C_b = 1.14,

  $\phi_b M_n$ = 1.14 x 357 = 406.7 kip-ft., **which <u>must</u> be $\leq \phi_b M_p$ = 491 kip-ft**.

  <u>**OK!**</u>

- *Thus, W16 x 67 made from 50 ksi steel with moment capacity equal to 406.7 kip-ft. for an unsupported length of 24 ft. is the designed section.*

Step IV. Check for local buckling.

$\lambda = b_f / 2t_f = 7.7$; Corresponding $\lambda_p = 0.38\,(E/Fy)^{0.5} = 9.192$

Therefore, $\lambda < \lambda_p$ - compact flange

$\lambda = h/t_w = 34.4$; Corresponding $\lambda_p = 3.76\,(E/F_y)^{0.5} = 90.5$

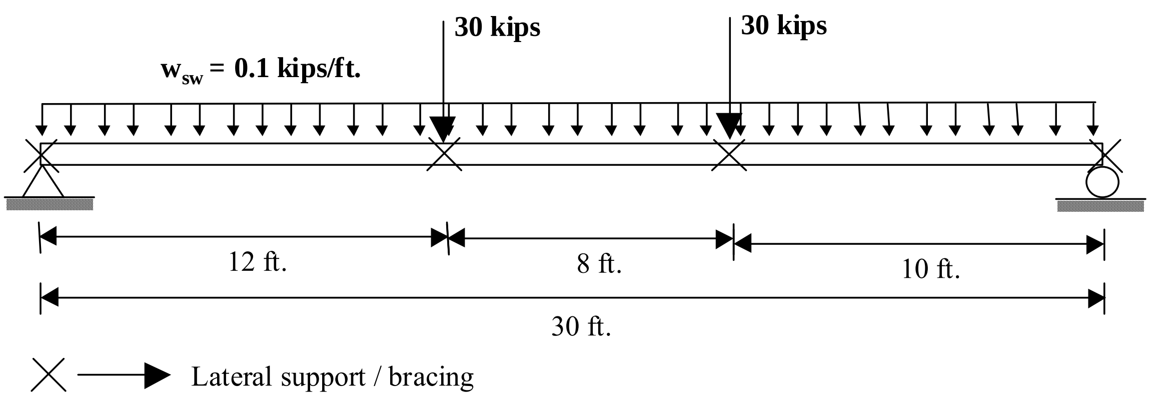
Therefore, $\lambda < \lambda_p$ - compact web

Compact section. - OK!

- *This example demonstrates the method for designing beams and accounting for $C_b > 1.0$*

Example 5.5

Design the beam shown below. The concentrated live loads acting on the beam are shown in the Figure. The beam is laterally supported at the load and reaction points.



Step I. Assume a self-weight and determine the factored design loads

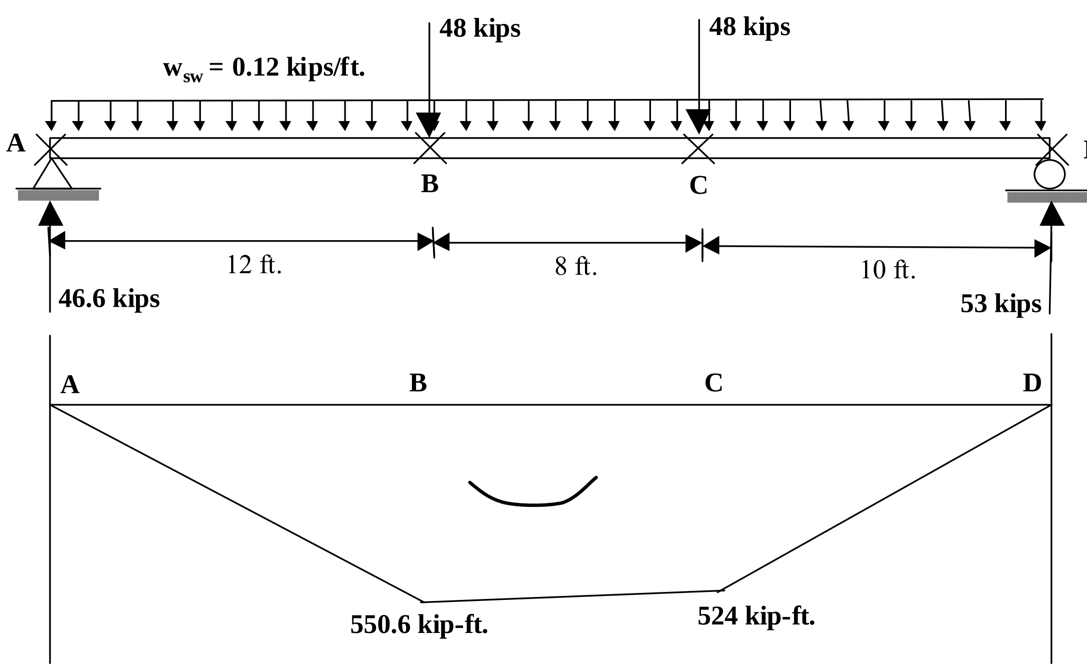
Let, w_{sw} = 100 lbs/ft. = 0.1 kips/ft.

P_L = 30 kips

$P_u = 1.6\ P_L$ = 48 kips

$w_u = 1.2 \times w_{sw}$ = 0.12 kips/ft.

The reactions and bending moment diagram for the beam are shown below.



Step II. Determine L_b, C_b, M_u, and M_u/C_b for all spans.

Span	L_b (ft.)	C_b	M_u (kip-ft.)	M_u/C_b (kip-ft.)
AB	12	1.67	550.6	329.7
BC	8	1.0 (assume)	550.6	550.6
CD	10	1.67	524.0	313.8

It is important to note that it is possible to have different L_b and C_b values for different laterally unsupported spans of the same beam.

Step III. Design the beam and check all laterally unsupported spans

Assume that **span BC** is the controlling span because it has the largest M_u/C_b although the corresponding L_b is the smallest.

From the AISC-LRFD manual select W21 x 68 made from 50 ksi steel (page _____)

Check the selected section for spans **AB, BC, and CD**

Span	L_b (ft.)	$\phi_b M_n$ for $C_b = 1.0$ *from _______*	C_b	$\phi_b M_n$ for C_b value *col. 3 x col. 4*	$\phi_b M_p$ *limit*
AB	12	507	1.67	846.7	600 kip-ft
BC	8	572	1.0	572.0	
CD	10	540	1.67	901.8	600 kip-ft.

Thus, for span AB, $\phi_b M_n = 600$ kip-ft. $> M_u$ - OK!

for span BC, $\phi_b M_n = 572.0$ kip-ft. $> M_u$ -OK!

For span CD, $\phi_b M_n = 600$ kip-ft. $> M_u$ -OK!

Step IV. Check for local buckling

$\lambda = b_f / 2t_f = 6.0$; Corresponding $\lambda_p = 0.38\ (E/Fy)^{0.5} = 9.192$

Therefore, $\lambda < \lambda_p$ - compact flange

$\lambda = h/t_w = 43.6$; Corresponding $\lambda_p = 3.76\ (E/F_y)^{0.5} = 90.55$

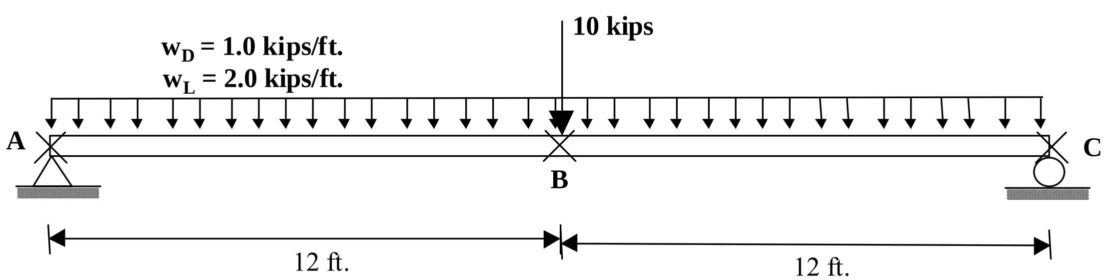
Therefore, $\lambda < \lambda_p$ - compact web

Compact section. - OK!

This example demonstrates the method for designing beams with several laterally unsupported spans with different L_b and C_b values.

Example 5.6

Design the simply-supported beam shown below. The uniformly distributed dead load is equal to 1 kips/ft. and the uniformly distributed live load is equal to 2 kips/ft. A concentrated live load equal to 10 kips acts at the mid-span. Lateral supports are provided at the end reactions and at the mid-span.



Step I. Assume the self-weight and calculate the factored design loads.

Let, w_{sw} = 100 lbs/ft. = 0.1 kips/ft.

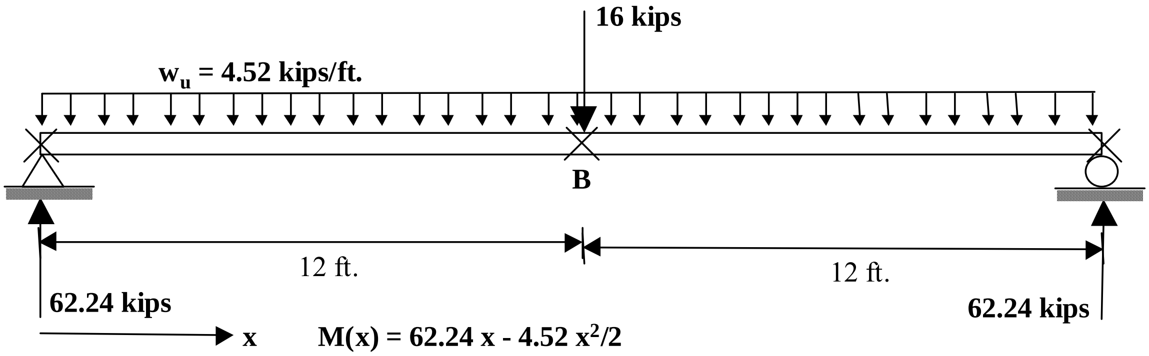
$w_D = 1 + 0.1 = 1.1$ kips/ft.

$w_L = 2.0$ kips/ft.

$w_u = 1.2\ w_D + 1.6\ w_L = 4.52$ kips/ft.

$P_u = 1.6 \times 10 = 16.0$ kips

The reactions and the bending moment diagram for the factored loads are shown below.



Step II. Calculate L_b and C_b for the laterally unsupported spans.

Since this is a symmetric problem, need to consider only span AB

$$L_b = 12 \text{ ft.};\ C_b = \frac{12.5\, M_{max}}{2.5\, M_{max} + 3\, M_A + 4\, M_B + 3\, M_c}$$

$M(x) = 62.24\ x - 4.52\ x^2/2$

Therefore,

$M_A = M(x = 3 \text{ ft.}) = 166.38$ kip-ft. - quarter-point along L_b = 12 ft.

$M_B = M(x = 6 \text{ ft.}) = 292.08$ kip-ft. - half-point along L_b = 12 ft.

$M_C = M(x = 9\text{ft.}) = 377.1$ kip-ft -three-quarter point along L_b= 12 ft.

$M_{max} = M(x = 12 \text{ ft.}) = 421.44$ kip-ft. - maximum moment along L_b =12ft.

Therefore, $C_b = 1.37$

Step III. Design the beam section

$M_u = M_{max} = 421.44$ kip-ft.

$L_b = 12.0$ ft.; $C_b = 1.37$

$M_u/C_b = 421.44/1.37 = 307.62$ kip-ft.

- Select W21 x 48 made from 50 ksi with $\phi_b M_n$ = 322 kip-ft. for L_b = 12.0 ft. and C_b =1.0
- For C_b = 1.37, $\phi_b M_n$ = 441.44 k-ft., but must be < or = $\phi_b M_p$ = 398 k-ft.

- Therefore, for C_b =1.37, $\phi_b M_n$ = <u>398 k-ft.</u> < M_u

Step IV. Redesign the section

- Select the next section with greater capacity than W21 x 48
- Select W18 x 55 with $\phi_b M_n$ = 345 k-ft. for L_b = 12 ft. and C_b = 1.0

  For C_b = 1.37, $\phi_b M_n$ = 345 x 1.37 = 472.65 k-ft. but must be $\leq \phi_b M_p$ = 420 k-ft.

  Therefore, for C_b = 1.37, $\phi_b M_n$ = 420 k-ft., which is < M_u (421.44 k-ft), (**NOT OK!**)

- Select W 21 x 55 with $\phi_b M_n$ = 388 k-ft. for L_b = 12 ft. and C_b = 1.0

  For C_b 1.37, $\phi_b M_n$ = 388 x 1.37 = 531.56 k-ft., but must be $\leq \phi_b M_p$ = 473 k-ft.

  Therefore, for C_b = 1.37, $\phi_b M_n$ = 473 k-ft, which is > M_u (421.44 k-ft). (<u>**OK!**</u>)

Step V. Check for local buckling.

$\lambda = b_f / 2t_f$ = <u>7.87</u>; Corresponding $\lambda_p = 0.38\ (E/F_y)^{0.5} = 9.192$

Therefore, $\lambda < \lambda_p$ - compact flange

$\lambda = h/t_w$ = <u>50.0</u>; Corresponding $\lambda_p = 3.76\ (E/F_y)^{0.5} = 90.55$

Therefore, $\lambda < \lambda_p$ - compact web

Compact section. **- OK!**

This example demonstrates the calculation of C_b and the iterative design method.