Chapter 4

Design of Slender Columns

By Murat Saatcioglu[1]

4.1 Introduction

The majority of reinforced concrete columns in practice are subjected to very little secondary stresses associated with column deformations. These columns are designed as short columns using the column interaction diagrams presented in Chapter 3. Rarely, when the column height is longer than typical story height and/or the column section is small relative to column height, secondary stresses become significant, especially if end restraints are small and/or the columns are not braced against side sway. These columns are designed as "slender columns." Fig. 3.1 eloquently illustrates the secondary moments generated in a slender column by P-Δ effects. Slender columns resist lower axial loads than short columns having the same cross-section. Therefore, the slenderness effect must be considered in design, over and above the sectional capacity considerations incorporated in the interaction diagrams. The significance of slenderness effect is expressed through *slenderness ratio*.

4.2 Slenderness Ratio

The degree of slenderness in a column is expressed in terms of "slenderness ratio," defined below:

Slenderness Ratio: $k\ell_u / r$

where, ℓ_u is unsupported column length; k is effective length factor reflecting the end restraint and lateral bracing conditions of a column; and *r* is the radius of gyration, reflecting the size and shape of a column cross-section.

4.2.1 Unsupported Length, ℓ_u

The unsupported length ℓ_u of a column is measured as the clear distance between the underside of the beam, slab, or column capital above, and the top of the beam or slab below. The unsupported length of a column may be different in two orthogonal directions depending on the supporting elements in

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respective directions. Figure 4.1 provides examples of different support conditions and corresponding unsupported lengths (ℓ_u). Each coordinate and subscript “x” and “y” in the figure indicates the plane of the frame in which the stability of column is investigated.

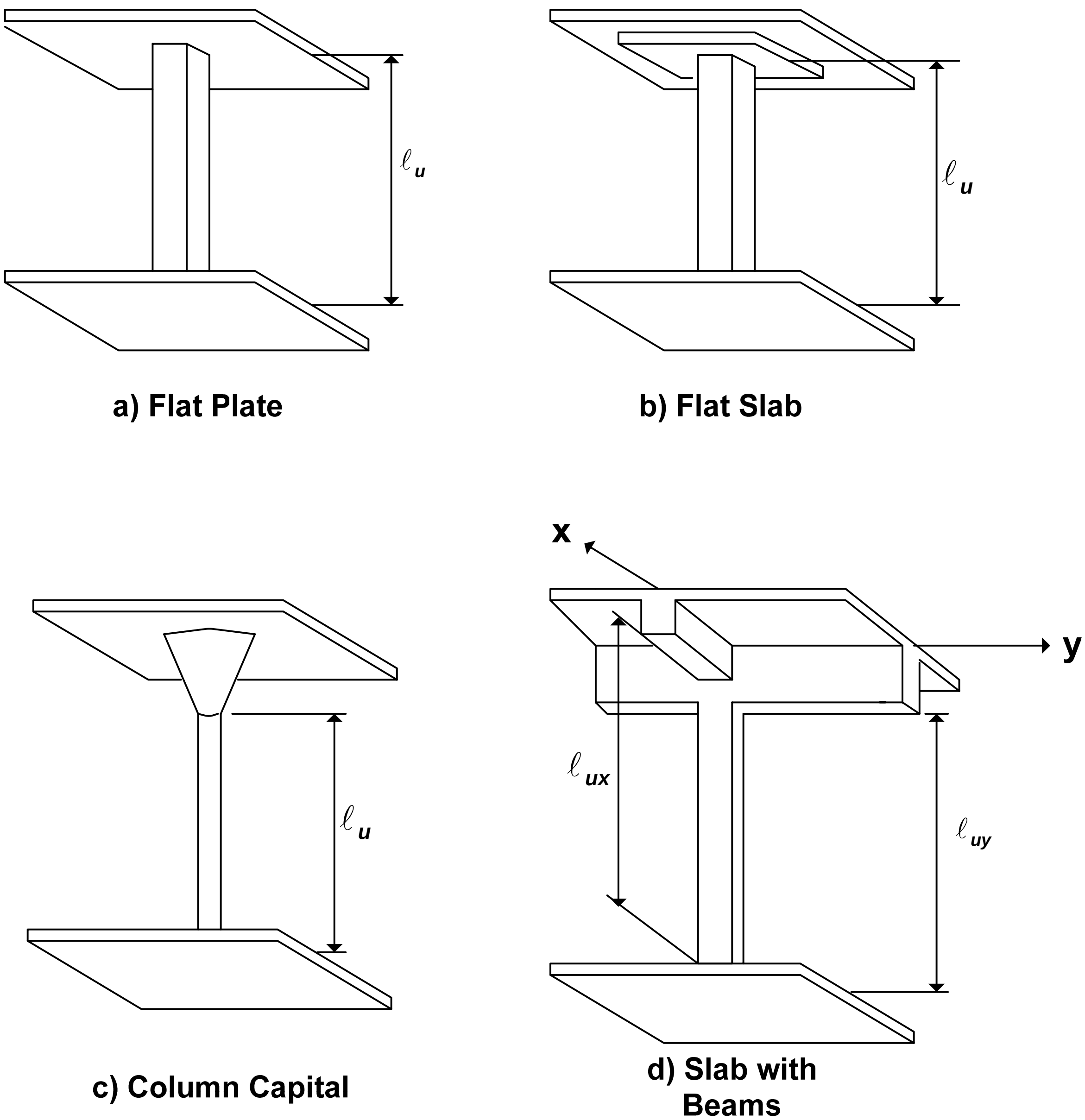


Fig. 4.1 Unsupported column length, ℓ_u

4.2.2 Effective Length Factor, k

The effective length factor k reflects the end restraint (support) and lateral bracing conditions of a column relative to a pin-ended and laterally braced "reference column." The reference column, shown in Fig. 4.2(a), follows a half sine wave when it buckles, and is assigned a *k* factor of 1.0. Therefore, the effective length $k\ell_u$ for this column is equal to the unsupported column length ℓ_u. A column with fully restrained end conditions develops the deflected shape illustrated in Fig. 4.2(b). The portion of the column between the points of contraflexure follows a half sine wave, the same deflected shape as that of the reference column. This segment is equal to 50% of the unsupported column length ℓ_u. Therefore, the effective length factor k for this case is equal to 0.5. Effective length factors for columns with idealized supports can be determined from Fig. 4.2. It may be of interest to note that *k* varies between 0.5 and 1.0 for laterally braced columns, and 1.0 and ∞ for unbraced columns. A discussion of lateral bracing is provided in Sec. 4.3 to establish whether a given column can be considered to be as part of a sway or a non-sway frame.

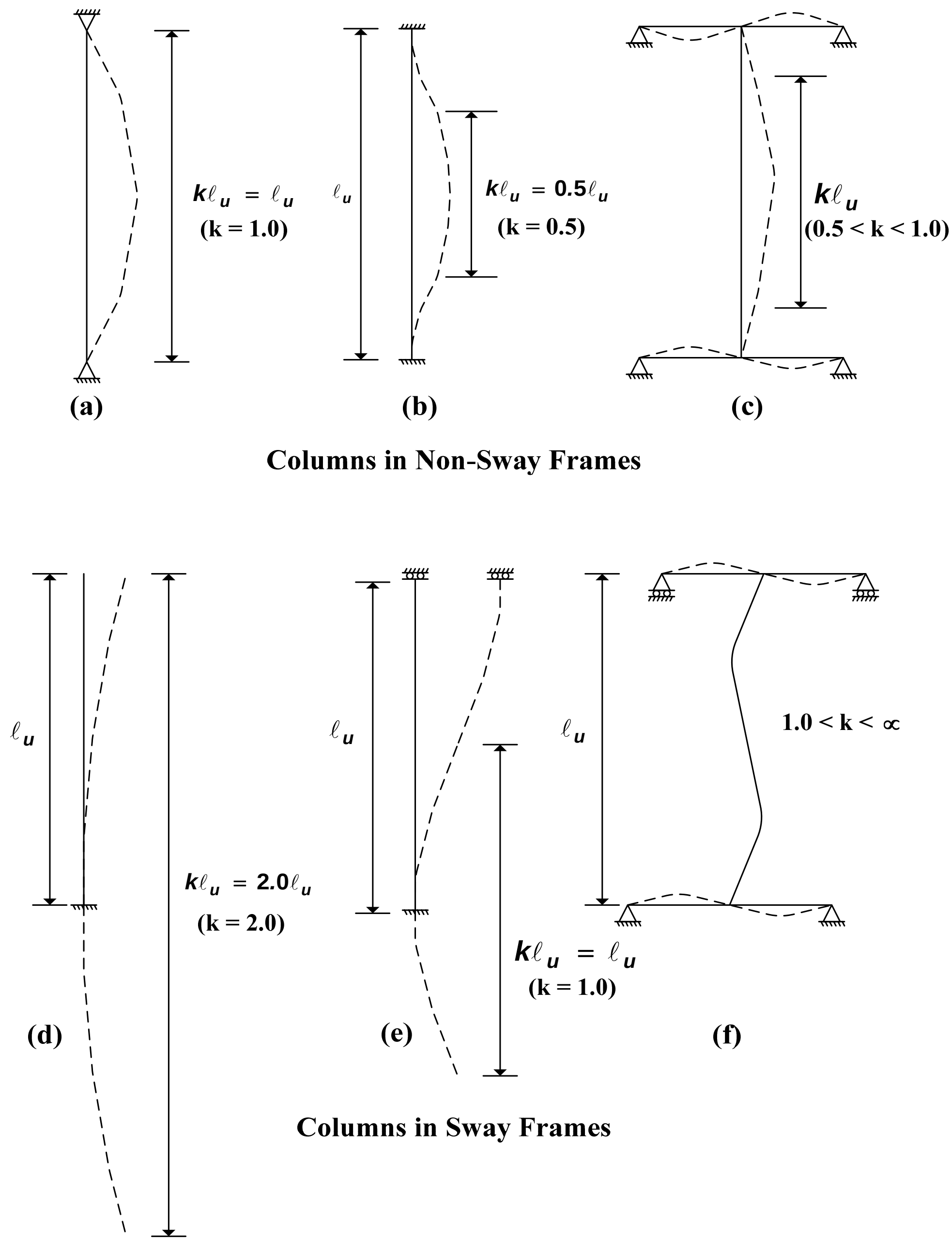


Fig. 4.2 Effective Length Factor *k* for Columns

Most columns have end restraints that are neither perfectly hinged nor fully fixed. The degree of end restraint depends on the stiffness of adjoining beams relative to that of the columns. Jackson and Moreland alignment charts, given in **Slender Columns 4.1** and **4.2** can be used to determine the effective length factor *k* for different values of relative stiffnesses at column ends. The stiffness ratios ψ_A and ψ_B used in **Slender Columns 4.1** and **4.2** should reflect concrete cracking, and the effects of sustained loading. Beams and slabs are flexure dominant members and may crack significantly more than columns which are compression members. The reduced stiffness values recommended by ACI 318-05 are given in **Slender Columns 4.3**, and should be used in determining *k*. Alternatively, **Slender Columns 4.4** may be used to establish conservative values of *k* for braced columns[2].

4.2.3 Radius of Gyration, r

The radius of gyration introduces the effects of cross-sectional size and shape to slenderness. For the same cross-sectional area, a section with higher moment of inertia produces a more stable column with a lower slenderness ratio. The radius of gyration r is defined below.

$$r = \sqrt{\frac{I}{A}} \tag{4-1}$$

It is permissible to use the approximations of $r = 0.3h$ for square and rectangular sections, and $r = 0.25h$ for circular sections, where "h" is the overall sectional dimension in the direction stability is being considered. This is shown in Fig. 4.3.

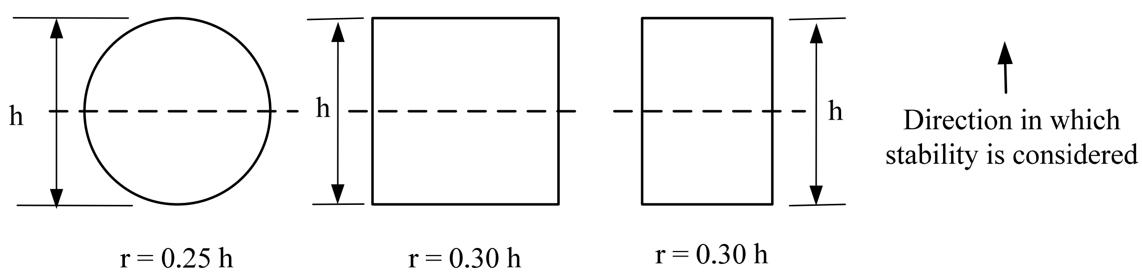


Fig. 4.3 Radius of gyration for circular, square and rectangular sections

4.3 Lateral Bracing and Designation of Frames as Non-Sway

A frame is considered to be "non-sway" if it is sufficiently braced by lateral bracing elements like structural walls. Otherwise, it may be designated as a "sway" frame. Frames that provide lateral resistance only by columns are considered to be sway frames. Structural walls that appear in the form of elevator shafts, stairwells, partial building enclosures or simply used as interior stiffening elements provide substantial drift control and lateral bracing. In most cases, even a few structural walls may be sufficient to brace a multi-storey multi-bay building. The designer can usually determine whether the frame is non-sway or sway by inspecting the floor plan. Frames with lateral bracing elements, where the total lateral stiffness of the bracing elements provides at least six times the summation of the stiffnesses of all the columns, may be classified as non-sway. ACI 318-05 permits columns to be designed as part of a non-sway frame if the increase in column end moments due to second-order

[2] "Concrete Design Handbook," Cement Association of Canada, third edition, 60 Queen Street, Ottawa, ON., Canada, K1P 5Y7, 2005.

effects does not exceed 5% of the first-order end moments (Sec. 10.11.4.1). Alternatively, Section 10.11.4.2 of ACI 318-05 defines a stability index "Q" (given in Eq. 4.2), where, $Q \leq 0.05$ indicates a non-sway column.

$$Q = \frac{\sum P_u \Delta_o}{V_{us} \ell_c} \tag{4.2}$$

Where, $\sum P_u$ is total factored axial load acting on all the columns in a story, V_{us} is total factored story shear, Δ_o is lateral story drift (deflection of the top of the story relative to the bottom of that story) due to V_{us}. The story drift Δ_o should be computed using the modified EI values given in **Slender Columns 4.3** with β_d defined as the ratio of the maximum factored sustained shear within a story to the maximum factored shear in that story. If Q exceeds approximately 0.2, the structure may have to be stiffened laterally to provide overall structural stability.

4.4 Design of Slender Columns

Design of a slender column should be based on a second-order analysis which incorporates member curvature and lateral drift effects, as well as material non-linearity and sustained load effects. An alternative approach is specified in ACI 318-05 for columns with slenderness ratios not exceeding 100. This approach is commonly referred to as the "Moment Magnification Method," and is based on magnifying the end moments to account for secondary stresses. The application of this procedure is outlined in the following sections.

4.4.1 Slender Columns in Non-Sway Frames

Slenderness effects may be neglected for columns in *non-sway frames* if the following inequality is satisfied:

$$\frac{k\ell_u}{r} \leq 34 - 12(M_1 / M_2) \tag{4-3}$$

Where $$(34 - 12M_1 / M_2) \leq 40 \tag{4-4}$$

M_1/M_2 is the ratio of smaller to larger end moments. This ratio is negative value when the column is bent in double curvature and positive when it is bent in single curvature. Fig. 4.4 illustrates columns in double and single curvatures. Columns in non-sway frames are more stable when they bend in double curvature, with smaller secondary effects, as compared to bending in single curvature. This is reflected in Eq. (4-3) through the sign of M_1/M_2 ratio. For negative values of this ratio the limit of slenderness in Eq. (4-3) increases, allowing a wider range of columns to be treated as short columns.

Slender columns in non-sway frames are designed for factored axial force P_u and amplified moment M_c. The amplified moment is obtained by magnifying the larger of the two end moments M_2 to account for member curvature and resulting secondary moments between the supports, while the supports are braced against sidesway. If M_c computed for the curvature effect between the ends is smaller than the larger end moment M_2, the design is carried out for M_2.

$$M_c = \delta_{ns} M_2 \tag{4-5}$$

$$\delta_{ns} = \frac{C_m}{1 - \frac{P_u}{0.75P_c}} \geq 1.0 \tag{4-6}$$

The critical column load, P_c (Euler buckling load) is;

$$P_c = \frac{\pi^2 EI}{(k\ell_u)^2} \tag{4-7}$$

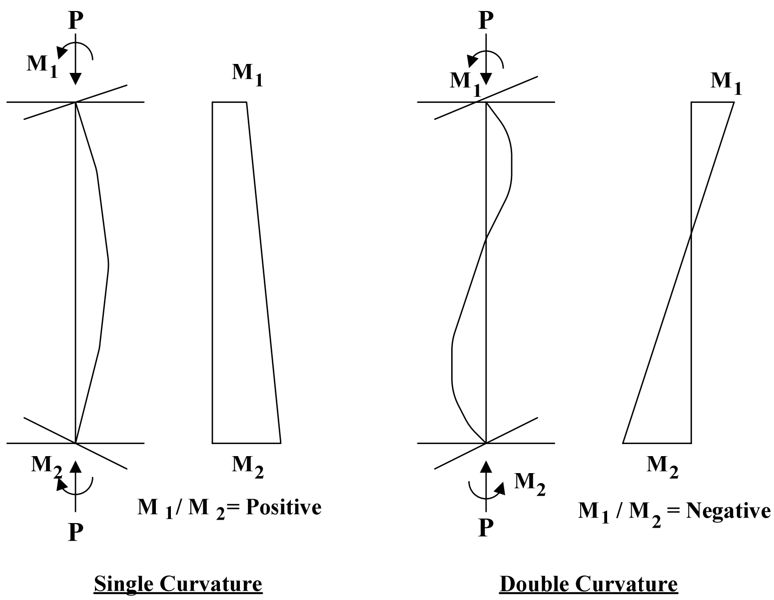


Fig. 4.4 Columns in Single and Double Curvature

EI in Eq. (4-7) is computed either with due considerations given to the presence of reinforcement in the section, as specified in Eq. (4-8), or approximately using Eq. (4-9).

$$EI = \frac{0.2E_cI_g + E_sI_{se}}{1 + \beta_d} \tag{4-8}$$

where β_d is the ratio of the maximum factored axial dead load to the total factored axial load. The moment of inertia of reinforcement about the cross-sectional centroid (I_{se}) can be computed using **Slender Columns 4.5**.

$$EI = \frac{0.4E_cI_g}{1 + \beta_d} \tag{4-9}$$

Note that Eq. (4-9) can be simplified further by assuming β_d = 0.6, in which case the equation becomes; $EI = 0.25E_cI_g$.

Coefficient C_m is equal to 1.0 for members with transverse loads between the supports. For the more common case of columns without transverse loads between the supports;

$$C_m = 0.6 + 0.4\frac{M_1}{M_2} \geq 0.4 \tag{4-10}$$

Where, M_1/M_2 is positive if the column is bent in single curvature.

When the maximum factored end moment M_2 is smaller than the minimum permissible design moment $M_{2,min}$, specified in Eq. (4-11), the magnification applies to $M_{2,min}$.

$$M_{2,min} \geq P_u(0.6 + 0.03h) \tag{4-11}$$

where h is the cross-sectional dimension in inches in the direction of the eccentricity of load. For columns for which $M_{2,min}$ is higher than M_2, the values of C_m, in Eq. (4-10) should either be taken 1.0 or determined based on the computed ratio of end moments (M_1/M_2). Once the amplified moment M_c is obtained, the designer can use the appropriate interaction diagrams given in Chapter 3 to determine the required percentage of longitudinal reinforcement.

4.4.2 Slender Columns in Sway Frames

Columns in sway frames are designed for the factored axial load P_u and the combination of factored gravity load moments and magnified sway moments. This is specified below, and illustrated in Fig. 4.5.

$$M_1 = M_{1ns} + \delta_s M_{1s} \tag{4-12}$$

$$M_2 = M_{2ns} + \delta_s M_{2s} \tag{4-13}$$

where, M_{1ns} and M_{2ns} are end moments due to factored gravity loads; and M_{1s} and M_{2s} are sway moments normally caused by factored lateral loads. All of these moments can be obtained from a first-order elastic frame analysis. Magnified sway moments $\delta_s M_{1s}$ and $\delta_s M_{2s}$ are obtained either from a second order frame analysis, with member flexural rigidity as specified in **Slender Columnss 4.3**, or by magnifying the end moments by sway magnification factor δ_s. The sway magnification factor is calculated either as given in Eq. (4-14) or Eq. (4-15).

$$\delta_s M_s = \frac{M_s}{1 - \dfrac{\sum P_u}{0.75\sum P_c}} \geq M_s \tag{4-14}$$

$$\delta_s M_s = \frac{M_s}{1 - Q} \geq M_s \tag{4-15}$$

However, if δ_s computed by Eq. (4-15) exceeds 1.5, $\delta_s M_s$ shall be calculated either through second order analysis or using Eq. (4-14).

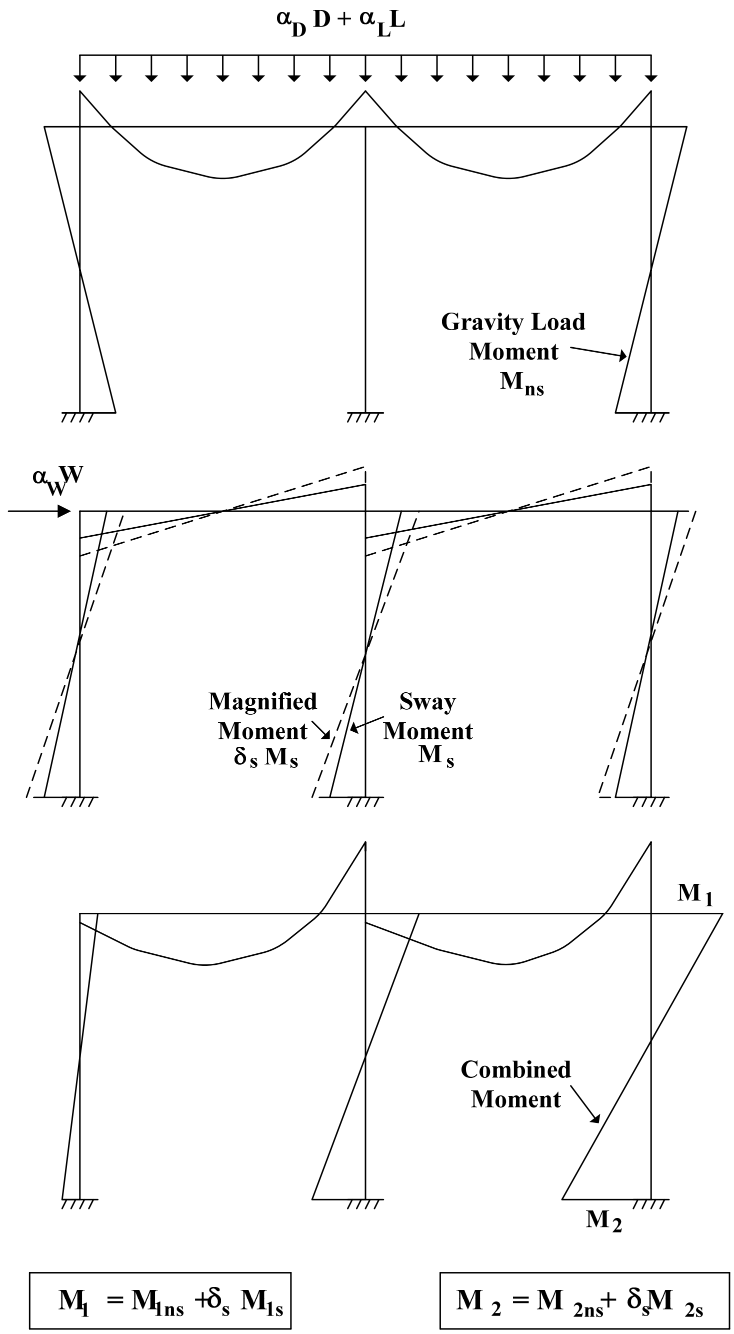


Fig. 4.5 Design moments in sway frames

In a sway frame, all the columns of a given story participate in the sway mechanism, and play roles in the stability of individual columns. Therefore, Eq. (4-14) includes $\sum P_u$ and $\sum P_c$ which give the summations of factored axial loads and critical loads for all the columns in the story, respectively. The critical column load P_c can be computed using Eqs. (4-7) through (4-9) with the effective length factor k computed for unbraced columns (for sway frames) and β_d as the ratio of the maximum factored sustained shear within the story to the maximum total factored shear in the story. Eq. (4-14) provides an average δ_s for all the columns in a story. Therefore, it yields acceptable results if all the columns in a story undergo the same story drift. When significant torsion is anticipated under lateral loading, a second order analysis is recommended for finding the amplified sway moment, $\delta_s M_s$.

The magnification of moments through Eq. (4-15) is applicable only if the sway magnification factor δ_s does not exceed 1.5. If it does, then either the second-order analysis or Eq. (4-14) should be employed (Sec. 10.13.4.2).

The sidesway magnification discussed above is intended to amplify the end moments associated with lateral drift. Although the amplified end moment is commonly the critical moment for most sway columns, columns with high slenderness ratios may experience higher amplification of moments between the ends (rather than at the ends) because of the curvature of the column along the column height. This is assumed to occur when the inequality given in Eq. (4-16) is satisfied.

$$\frac{\ell_u}{r} > \frac{35}{\sqrt{\dfrac{P_u}{f'_c\, A_g}}} \qquad (4\text{-}16)$$

The magnification of moment due to the curvature of column between the ends is similar to that for braced columns in non-sway frames. Therefore, if Eq. (4-16) is satisfied for a column, then the column should be designed for factored axial force P_u and magnified design moment (M_c) computed using Eqs. (4-5) and (4-6), with M_1 and M_2 computed from Eqs. (4-12) and (4-13).

Sometimes columns of a sway frame may buckle under gravity loads alone, without the effects of lateral loading. In this case one of the gravity load combinations may govern the stability of columns. The reduction of EI under sustained gravity loads may be another factor contributing to the stability of sway columns under gravity loads. Therefore, ACI 318-05 requires an additional check to safeguard against column buckling in sway frames under gravity loads alone (Sec. 10.13.6). Accordingly, the strength and stability of structure is reconsidered depending on the method of amplification used for sway moments. If a second order analysis was conducted to find $\delta_s M_{2s}$, two additional analyses are necessary using the reduced stiffness values given in **Slender Columns 4.3** with β_d taken as the ratio of the factored sustained axial dead load to total factored axial load. First, a second-order analysis is conducted under combined factored gravity loads and lateral loads equal to 0.5% of the gravity loads. Second, a first-order analysis is conducted under the same loading condition. The ratio of lateral drift obtained by the second-order analysis to that obtained by the first-order analysis is required to be limited to 2.5. If the sway moment was amplified by computing the sway magnification factor given in Eq. 4.14, as opposed to conducting second order analysis or using Eq. (4-15), then δ_s computed by using the gravity loads ($\sum P_u$ and $\sum P_c$ corresponding to the factored dead and live loads) is required to be positive and less than or equal to 2.5 to ensure the stability of the column. If the sway moment was amplified using Eq. (4-15), then the value of Q computed using $\sum P_u$ for factored dead and live loads should not exceed 0.60.

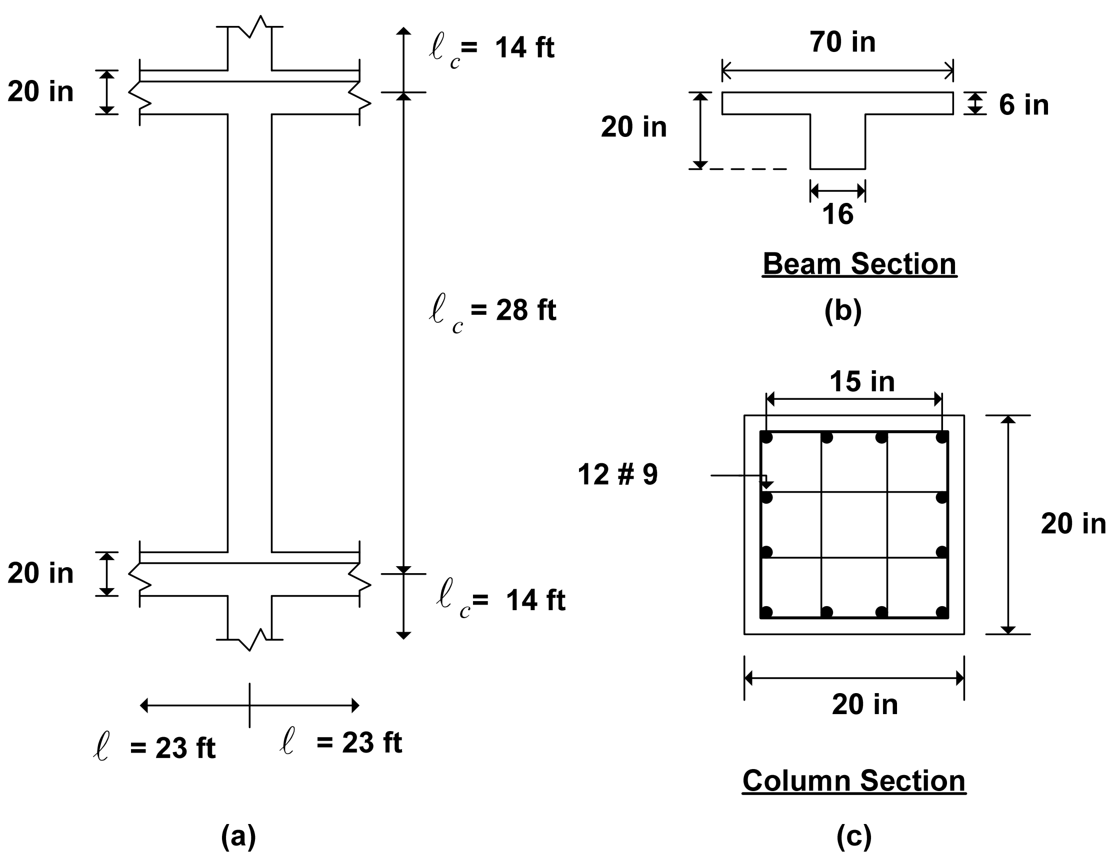
4.5 Slender Column Design Examples

SLENDER COLUMN EXAMPLE 1 - Design of an interior column braced against sidesway.

Consider a 10-story office building, laterally braced against sidesway by an elevator shaft (Q is computed to be much less than 0.05). The building has an atrium opening at the second floor level with a two-story high column in the opening to be designed. Design the column for the unfactored design forces given below, obtained from a first-order analysis. The framing beams are 16 in wide and 20 in deep with 23 ft (canter-to-centre) spans. The beam depth includes a slab thickness of 6 in. The story height is 14 ft (column height is 28 ft). It is assumed that the bracing elements provide full resistance to lateral forces and the columns only resist the gravity loads. Start the design with an initial column size of 20 in square. f'_c = 6,000 psi for all beams and columns; f_y = 60,000 psi.

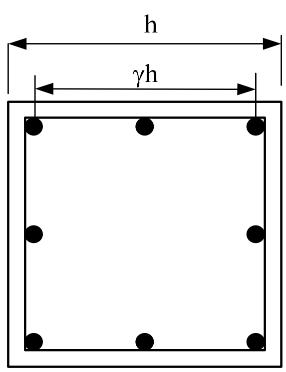
Unfactored Loads	Dead Load	Live Load
Axial load:	520 k	410 k
Top moment:	-1018 k-in	-620 k-in
Bottom moment:	-848 k-in	-540 k-in

Note: Moments are positive if counterclockwise at column ends. The column is bent in double curvature.



Slender Column Example 1

Procedure	Calculation	ACI 318-05 Section	Design Aid
Determine factored design forces: *Note:* M_1 is the lower and M_2 is the higher end moment.	i) U = 1.4D $P_u = 1.4\ P_D = 1.4\ (520) = 728$ k $M_2 = 1.4\ M_{D2} = 1.4\ (1018) = 1425$ k-in $M_1 = 1.4\ M_{D1} = 1.4\ (848) = 1187$ k-in ii) U = 1.2 D + 1.6 L $P_u = 1.2\ P_D + 1.6\ P_L = 1.2\ (520) + 1.6\ (410) = 1280$ k $M_2 = 1.2\ M_{D2} + 1.6\ M_{L2} = 1.2\ (1018) + 1.6\ (620) = 2214$ k-in $M_1 = 1.2\ M_{D1} + 1.6\ M_{L1} = 1.2\ (848) + 1.6\ (540) = 1882$ k-in *Note:* Load Combination (ii) governs the design.	9.2	
Calculate slenderness ratio $k\ell_u / r$ i) Find unsupported column length ii) Find the radius of gyration iii) Find effective length factor "k." This requires the calculation of stiffness ratios at the ends. First find beam and column stiffnesses. Read k from **Slender Columns 4.1**	$\ell_u = 28 - 20/12 = 26.3$ ft $r = 0.3\ h = 0.3\ (20) = 6$ in $(I_g)_{beam} = (I_g)_{T\text{-}beam} = 19{,}527\ in^4$ $(I_g)_{column} = bh^3/12 = (20)(20)^3/12 = 13{,}333\ in^4$ Cracked (reduced) EI values: $(EI)_{beam} = (1{,}545)(19{,}527) = 30x10^6$ k-in^2 $(EI)_{col} = (3{,}091)(13{,}333) = 41x10^6$ k.in^2 $(EI/\ell)_{beam} = (30x10^6) / (23x12) = 109x10^3$ k-in for both left and right beams $(EI/\ell_c)_{col} = (41x10^6 / (28x12) = 122x10^3$ k-in for the atrium column to be designed. $(EI/\ell_c)_{col} = (41x10^6) / (14x12) = 244x10^3$ k-in for columns above and below $\Psi = (\Sigma EI/\ell_c)_{col} / (\Sigma EI/\ell)_{beam}$ $\Psi = [(EI/\ell_c)_{col,\ above} + (EI/\ell_c)_{col,\ below}] / [(EI/\ell)_{beam,\ left} + (EI/\ell)_{beam,\ right}]$ $\Psi_A = (244 + 122)x10^3 / (109 + 109)x10^3$ $\Psi_A = 1.7 = \Psi_B$ for $\Psi_B = \Psi_A = 1.7$; select k = 0.83 from **Slender Columns 4.1** (Note that **Slender Columns 4.4** gives a	 10.11.1	Figure 4.1 Slender Cols. 4.3 Slender Cols. 4.1 Slender Col. 4.4

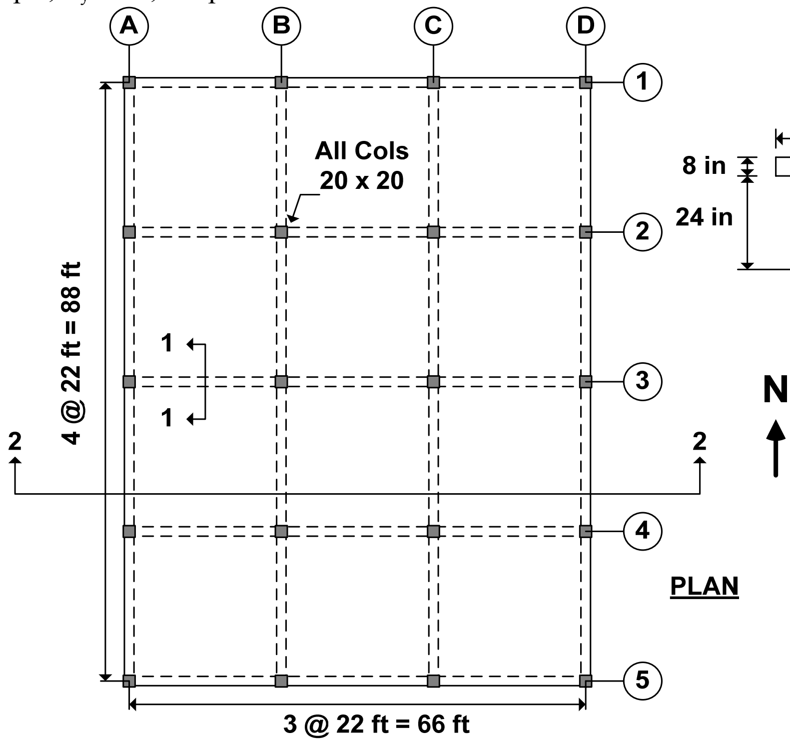
Compute the slenderness ratio	conservative value of k = 0.90) ℓ_u =28.9 – 1.67 = 26.3 ft $k\ell_u / r$ = 0.83 (26.3x12) / 6 = 45		
Check if slenderness can be neglected using Eq.(4-3): Apply the limit of Eq. (4-4)	$\frac{k\ell_u}{r} \leq 34 - 12(M_1 / M_2)$ $(34 - 12M_1 / M_2) \leq 40$ Note M_1/M_2 = - 1882/2214 = -0.85 (Bending in double curvature) or, for Load Combination I; M_1/M_2 = - 11871/1425 = -0.83 [34- 12 (-0.85)] = 44 > 40 use 40 $k\ell_u / r$ = 45 > 40 (limiting ratio for neglecting slenderness) Therefore, consider slenderness.	10.12.2 10.3.4 9.3.2	
Compute moment magnification factor (δ_{ns}) from Eq. (4-6): i) Compute critical load P_c from Eq (4-7) Use Eq. (4-8) to compute EI. Assume 2.5% column reinforcement, equally distributed along the perimeter of the square section with γ = 0.75 where γ is the ratio of the distance between the centres of the outermost bars to the column dimension perpendicular to the axis of bending.  Alternatively, compute EI from Eq,(4-9) Eq. (4-9) may further be simplified by assuming a value of β_d = 0.6. $EI = \frac{0.4E_cI_g}{1+\beta_d} = 0.25E_cI_g$	$\delta_{ns} = \frac{C_m}{1 - \frac{P_u}{0.75P_c}} \geq 1.0$ $P_c = \frac{\pi^2 EI}{(k\ell_u)^2}$ E_c = 4415 ksi E_s = 29,000 ksi $(I_g)_{column}$ = 13,333 in^4 $I_{se} = 0.18 \rho_t b h^3 \gamma^2$ (from **Slender Col. 4.5**) $I_{se} = 0.18(0.025)(20)(20)^3 (0.75)^2$ = 405 in^4 β_d = 1.2D / (1.2D + 1.6L) = 1.2 x 520 / (1.2 x 520 + 1.6 x 410) = 624 / 1280 = 0.49 EI = $(0.2E_cI_g + E_sI_{se}) / (1 + B_d)$ EI = [(0.2 x 4415 x 13,333) + (29,000 x 405)] / (1 + 0.49) = 16 x 10^6 k-in^2 EI = (0.4 x 4415 x 13,333) / (1 + 0.49) EI = 16 x 10^6 k-in^2 EI = 0.25 E_cI_g = 0.25 x 4415 x 13,333 EI = 15 x 10^6 k-in^2	10.12.3 10.12.3 8.5.2 10.12.3 10.12.3 R10.12.3	 Slender Cols. 4.3 Slender Cols. 4.5

ii) Compute C_m from Eq. (4-10):	$P_c = \pi^2 EI / (k \ell_u)^2$ $P_c = \pi^2 \times 16 \times 10^6 / (0.83 \times 26.3 \times 12)^2$ $P_c = 2301$ k $C_m = 0.6 + 0.4\ M_1/M_2 \geq 0.4$ $C_m = 0.6 + 0.4\ (-0.85) = 0.25 < 0.4$ use 0.4	10.12.3	
iii) Moment magnification factor	$\delta_{ns} = \dfrac{0.4}{1 - \dfrac{1280}{(0.75)(2301)}} = 1.55 \geq 1.0$	10.12.3	
Compute amplified moment M_c from Eq. (4-5)	$M_c = \delta_{ns} M_2$ = 1.55 (2214) = 3432 k-in	10.12.3	
Check against minimum design moment as per Eq. (4-11).	$M_{2,min} \geq P_u(0.6 + 0.03h)$ $M_{2,min}$ = 1280 (0.6 + 0.03 x 20) =1536 k-in M_c = 3432 k-in > $M_{2,min}$ = 1536 k-in Design for M_c = 3432 k-in	10.12.3.2	
Select reinforcement ratio and design the column section: Use Column Interaction Diagrams R6-60.7 and R6-60.9 for equal reinforcement on all sides and interpolate for $\gamma = 0.75$ (assumed above) A) Compute $K_n = \dfrac{P_n}{f'_c A_g}$ B) Compute $R_n = \dfrac{M_n}{f'_c A_g h}$ C) Read ρ_g for K_n and R_n values from the interaction diagrams D) Compute required A_{st} from $A_{st} = \rho_g A_g$ E) Find column reinforcement	Note: $\gamma = 0.75$ allows for more than 1.5 in clear cover required for interior columns, not exposed to weather. $K_n = \dfrac{P_u / \phi}{f'_c A_g} = \dfrac{1280 / 0.65}{(6)(20)^2} = 0.82$ $R_n = \dfrac{M_n / \phi}{f'_c A_g h} = \dfrac{3432 / 0.65}{(6)(20)^2(20)} = 0.11$ For $K_n = 0.82$ and $R_n = 0.11$ Read $\rho_g = 0.031$ for $\gamma = 0.7$ and $\rho_g = 0.029$ for $\gamma = 0.8$ Interpolating; $\rho_g = 0.030$ for $\gamma = 0.75$ (Note that the required steel ratio of 3% is slightly higher that the 2.5% assumed for computing EI. No revision is necessary). Required $A_{st} = 0.030 \times 400$ in.^{2} = 12.0 in.^{2} Try # 9 bars; 12.0 / 1.0 = 12.0 Use 12 # 9 Bars.	7.7.1	Flexure 9 Columns R6-60.7 and R6-60.8 Columns R6-60.7 and R6-60.8

SLENDER COLUMN EXAMPLE 2 - Design of an exterior column in a sway frame.

A typical floor plan and a section through a multi-story office building are shown below. Design column 3-A at the ground level for combined gravity and east-west wind loading. The results of first-order frame analysis under factored load combinations are given in the solution.

f'_c = 6,000 psi; f_y = 60,000 psi.



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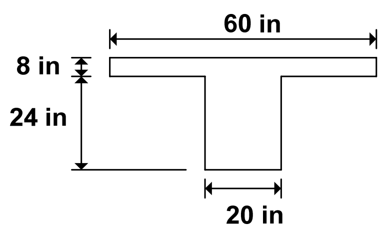
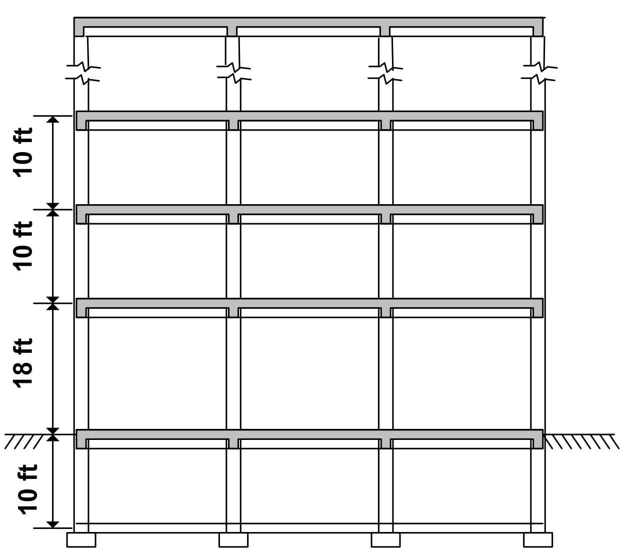
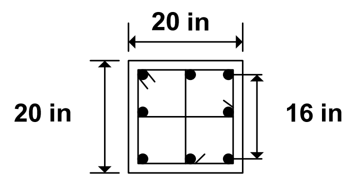


Section 1-1



SECTION 2-2



Column 3-A

Slender Column Example 2

Procedure	Calculation	ACI 318-05 Section	Design Aid
Consider the applicable load combinations: The structure is *not* braced against sidesway. Therefore, the column will be designed considering the loads that cause sidesway. Note that sidesway in this structure is caused by wind loading. No significant sidesway is anticipated due to gravity loads since the structure is symmetric. However, the possibility of sidesway instability under gravity loads alone shall be investigated as per Sec. 10.13.6.	a)Load combinations that include wind; Comb. I: $U = 1.2D + 1.6L_r + 0.8W$ Comb. II: $U = 1.2D + 1.6W + 1.0L + 0.5L_r$ Comb.III: $U = 0.9D + 1.6W$ b) Load combinations for gravity loads; Comb. IV: $U = 1.4D$ Comb. V: $U = 1.2D + 1.6L + 0.5L_r$ Comb. VI: $U = 1.2D + 1.6 L_r + 1.0L$	9.2.1 9.2.1	
Using the preliminary column section given in the figure, determine the effective length factor *k* for each column at the ground level. This requires the computation of beam and column stiffnesses. *Note:* All columns have the same section. Factor *k* reflects column end restraint conditions and depends on relative stiffnesses of columns to beams at top and bottom joints. Read *k* from **Slender Columns 4.2 and 4.1.**	$I_{beam} = 87{,}040 \text{ in}^4$ (for T-section) $I_{col} = (20)(20)^3/12 = 13{,}333 \text{ in}^4$ Find reduced EI values from **Slender Col. 4.3** for 6.0 ksi concrete; $(E_cI)_{beam} = 1545\ I_{beam} = (1545)(87{,}040) = 134 \times 10^6 \text{ k-in}^2$ $(E_cI)_{col} = 3091\ I_{col} = (3091)(13{,}333) = 41 \times 10^6 \text{ k-in}^2$ $(EI/\ell)_{beam} = 134 \times 10^6 / (22 \times 12) = 507{,}576 \text{ k-in}$ $(EI/\ell_c)_{col,\ typical} = 41 \times 10^6 / (10 \times 12) = 341{,}667 \text{ k-in}$ $(EI/\ell_c)_{col,\ atrium} = 41 \times 10^6 / (18 \times 12) = 189{,}815 \text{ k-in}$ $\Psi = (3EI/\ell_c)_{col} / (3EI/\ell)_{beam}$ $\Psi = [(EI/\ell_c)_{col,\ above} + (EI/\ell_c)_{col,\ below}] / [(EI/\ell)_{beam,\ left} + (EI/\ell)_{beam,\ right}]$ i) For exterior columns (columns on lines A and D): $\Psi_A = (341{,}667 + 189{,}815) / 507{,}576 = 1.05$ $\Psi_B = \Psi_A = 1.05$; from **Slender Cols. 4.2**: k = 1.35 (for unbraced frames) k = 0.78 for a braced column, from **Slender Cols. 4.1.** This value is computed for further magnification of moments, if necessary for column 3-A as per Sec. 10.13.6.	 10.11.1 10.13.6	 Slender Cols. 4.3 Slender Cols. 4.3 Slender Cols. 4.2 Slender Cols. 4.1

<table>
<tr>
<td>Compute the slenderness ratio</td>
<td>ii) For interior columns (columns on lines B and C):
Ψ_A = (341,667 + 189,815)/(507,576 + 507,576) = 0.52
$\Psi_B = \Psi_A$ = 0.52; from Slender Cols. 4.2;
k = 1.15 (for unbraced frames)</td>
<td></td>
<td>Slender Cols. 4.2</td>
</tr>
<tr>
<td>Compute critical load P_c from Eq. 4.7 and EI from either Eq. 4.8 or 4.9. Note, if Eq. 4.9 is used for simplicity with β_d = 0 (since wind loading is a short term load)
$$EI = \frac{0.4E_cI_g}{1+\beta_d} = 0.4E_cI_g$$
For braced columns, Eq. 4.9 can be simplified by substituting β_d = 0.6. Then;
$EI = 0.25\ E_c\ I_g$

P_c for braced columns may be needed if further magnification of moments is required as per Sec. 10.13.6..</td>
<td>i) For exterior columns (columns on lines A and D):
E_c = 4415 ksi for f'_c = 6 ksi

For sway columns;
EI = $0.4E_cI_g$ = 0.4(4415)(13,333)
= 23.5 x 10^6 k-in^2
ℓ_u = (18) (12) - 32 = 184 in
$P_c = \pi^2\ EI/(k\ell_u)^2$
= $\pi^2(23.5x10^6)/(1.35x184)^2$
= 3759 kips for a sway frame.

For braced columns;
EI = $0.25E_cI_g$ = 0.25(4415)(13,333)
= 14.7 x 10^6 k-in^2

$P_c = \pi^2EI/(k\ell_u)^2$
= $\pi^2(14.7x10^6)/(0.78x184)^2$
= 7044 kips for braced columns.

ii) For interior columns (columns on lines B and C):

$P_c = \pi^2\ EI\ /\ (k\ell_u)^2$
= $\pi^2\ (23.5\ x\ 10^6)\ /\ (1.15\ x\ 184)^2$
= 5180 kips for a sway frame.

ΣP_c = 10 (3759) + 10 (5180) = 89,390 kips</td>
<td>10.12.13</td>
<td>Slender Cols. 4.3</td>
</tr>
<tr>
<td>Compute magnified sway moment δ_sM_s Under Load Combination I.

Conduct first-order frame analysis using Load Combination I, and the stiffness values specified in Slender Cols. 4.3.

Note: Counterclockwise moment at column end is positive.</td>
<td>i) Load Comb. I: U = 1.2D + 1.6L_r + 0.8W
<table>
<tr><td>Load</td><td>1.2D +1.6L_r</td><td>0.8W</td></tr>
<tr><td>P_u (kips)
Corner Column</td><td>425</td><td>±12</td></tr>
<tr><td>P_u (kips)
Edge Column</td><td>682</td><td>±12</td></tr>
<tr><td>P_u (kips)
Interior Column</td><td>1134</td><td>±4</td></tr>
<tr><td>$(M_u)_{top}$ (k-in)
Column 3-A</td><td>-1296</td><td>±765</td></tr>
<tr><td>$(M_u)_{bot}$ (k-in)
Column 3-A</td><td>-1296</td><td>±1111</td></tr>
</table></td>
<td>9.2.1

10.11.1</td>
<td></td>
</tr>
</table>

<table>
<tr>
<td>Compute sway magnification factor δ_s from Eq. 8.12. This requires the computation of ΣP_f in addition to ΣP_c Obtained in the previous step.</td>
<td>Sway magnification factor δ_s:

ΣP_f = 4 (425 + 12) + 10 (682 + 12) + 6 (1134 + 4) = 15,516 kips

$\delta_s = 1 / [1 - \Sigma P_f / [(0.75)\Sigma P_c]$
= 1/[1– 15,516 /(0.75 x 89,390)] = 1.30

$\delta_s M_{1s}$ = 1.30 x 765 = 995 k-in
$\delta_s M_{2s}$ = 1.30 x 1111 = 1444 k-in</td>
<td>10.13.4.3</td>
<td></td>
</tr>
<tr>
<td>Compute design moments M_1 and M_2</td>
<td>$M_1 = M_{1ns}+\delta_s M_{1s}$ =1296 + 995 = 2291 k-in
$M_2 = M_{2ns}+\delta_s M_{2s}$ =1296 +1444= 2738 k-in</td>
<td>10.13.3</td>
<td>Fig. 4.5</td>
</tr>
<tr>
<td>Check if further magnification of moments is required for Column 3-A due to the curvature of columns between the ends as per Sec. 10.13.5</td>
<td>$\frac{\ell_u}{r} > \frac{35}{\sqrt{P_u/(f'_c A_g)}}$

P_u = 682 + 12 = 694 k
ℓ_u/r = 184 / (0.3 x 20) = 30.7
$35 / \sqrt{694/(6x(20)^2)}$ = 65.1 > 30.7
Therefore, no further magnification is required.</td>
<td>10.13.5</td>
<td></td>
</tr>
<tr>
<td>Compute magnified sway moment $\delta_s M_s$ Under Load Combination II.

Conduct first-order frame analysis using Load Combination I, and the stiffness values specified in Slender Cols. 4.4.

Note: Counterclockwise moment at column end is positive.

Compute sway magnification factor δ_s from Eq. 8.12. This requires the computation of ΣP_f in addition to ΣP_c Obtained in the previous step.</td>
<td>ii) Load Combination II:
$U=1.2D+1.6W+1.0L+0.5L_r$

<table>
<tr><td>Load</td><td>1.2D +1.0L + 0.5L_r</td><td>1.6W</td></tr>
<tr><td>P_u (kips)
Corner Column</td><td>493</td><td>±24</td></tr>
<tr><td>P_u (kips)
Edge Column</td><td>845</td><td>±24</td></tr>
<tr><td>P_u (kips)
Interior Column</td><td>1459</td><td>±8</td></tr>
<tr><td>$(M_u)_{top}$ (k-in)
Column 3-A</td><td>-1756</td><td>±1530</td></tr>
<tr><td>$(M_u)_{bot}$ (k-in)
Column 3-A</td><td>-1756</td><td>±2222</td></tr>
</table>

Sway magnification factor δ_s:

ΣP_f = 4 (493 + 24) + 10 (845 + 24) + 6 (1459 + 8) = 19,560 kips

$\delta_s = 1 / [1 - \Sigma P_f / (0.75\Sigma P_c)]$
= 1/[1– 19,560 /(0.75 x 89,390)] = 1.41

$\delta_s M_{1s}$ = 1.41 x 1530 = 2157 k-in
$\delta_s M_{2s}$ = 1.41 x 2222 = 3111 k-in</td>
<td>9.2.1

10.11.1

10.13.4.3</td>
<td></td>
</tr>
</table>

<table>
<tr><td></td><td></td><td></td><td></td></tr>
<tr><td>Compute design moments M_1 and M_2</td><td>$M_1 = M_{1ns}+\delta_s M_{1s}$ =1756 +2157 = 3913 k-in
$M_2 = M_{2ns}+\delta_s M_{2s}$ =1756 +3111= 4867 k-in</td><td>10.13.3</td><td>Fig. 4.5</td></tr>
<tr><td>Check if further magnification of moments is required for Column 3-A due to the curvature of columns between the ends as per Sec. 10.13.5</td><td>$\frac{\ell_u}{r} > \frac{35}{\sqrt{P_u/(f'_c A_g)}}$

P_u = 845 + 24 = 869 k
ℓ_u/r = 184 / (0.3 x 20) = 30.7
$35 / \sqrt{869/(6x(20)^2)}$ = 58.2 > 30.7
Therefore, no further magnification is required.</td><td>10.13.5</td><td></td></tr>
<tr><td>Compute magnified sway moment $\delta_s M_s$ Under Load Combination II.

Conduct first-order frame analysis using Load Combination I, and the stiffness values specified in Slender Cols. 4.4.

Note: Counterclockwise moment at column end is positive.

Compute sway magnification factor δ_s from Eq. 8.12. This requires the computation of ΣP_f in addition to ΣP_c Obtained in the previous step.</td><td>iii) Load Comb. III: U = 0.9D + 1.6W

<table>
<tr><td>Load</td><td>0.9D</td><td>1.6W</td></tr>
<tr><td>P_u (kips)
Corner Column</td><td>258</td><td>±24</td></tr>
<tr><td>P_u (kips)
Edge Column</td><td>452</td><td>±24</td></tr>
<tr><td>P_u (kips)
Interior Column</td><td>790</td><td>±8</td></tr>
<tr><td>$(M_u)_{top}$ (k-in)
Column 3-A</td><td>-972</td><td>±1530</td></tr>
<tr><td>$(M_u)_{bot}$ (k-in)
Column 3-A</td><td>-972</td><td>±2222</td></tr>
</table>
Sway magnification factor δ_s:
ΣP_f = 4 (258 + 24) + 10 (452 + 24) + 6 (790 + 8) = 10,676 kips

$\delta_s = 1 / [1 - \Sigma P_f / (0.75\Sigma P_c)]$
= 1/[1– 10,676 /(0.75 x 89,390)] = 1.19

$\delta_s M_{1s}$ = 1.19 x 1530 = 1821 k-in
$\delta_s M_{2s}$ = 1.19 x 2222 = 2644 k-in</td><td>9.2.1

10.11.1

10.13.4.3</td><td></td></tr>
<tr><td>Compute design moments M_1 and M_2</td><td>$M_1 = M_{1ns}+\delta_s M_{1s}$ = 972 +1821 = 2793 k-in
$M_2 = M_{2ns}+\delta_s M_{2s}$ = 972 +2644 = 3616 k-in</td><td>10.13.3</td><td>Fig. 4.5</td></tr>
<tr><td>Check if further magnification of moments is required for Column 3-A due to the curvature of columns between the ends as per Sec. 10.13.5</td><td>$\frac{\ell_u}{r} > \frac{35}{\sqrt{P_u/(f'_c A_g)}}$

P_u = 452 + 24 = 476 k
ℓ_u/r = 184 / (0.3 x 20) = 30.7
$35 / \sqrt{476/(6x(20)^2)}$ = 78.6 > 30.7
Therefore, no further magnification is required.</td><td>10.13.5</td><td></td></tr>
</table>

<table>
<tr>
<td>Check the stability of column under gravity loads only (Load combinations IV, V and VI) as per Sec. 10.13.6.

Consider factored axial loads and bending moments obtained from a first-order frame analysis, conducted using the flexural rigidities given in Slender Cols. 4.3

Note: Counterclockwise moment at column end is positive.

Compute the sway magnification factor δ_s

Critical load, from earlier calculation.</td>
<td>iv) Load Comb. IV: U = 1.4D

<table>
<tr><th>Load</th><th>1.4D</th></tr>
<tr><td>P_u (kips)
Corner Column</td><td>402</td></tr>
<tr><td>P_u (kips)
Edge Column</td><td>703</td></tr>
<tr><td>P_u (kips)
Interior Column</td><td>1229</td></tr>
<tr><td>$(M_u)_{top}$ kip-in
Column 3-A</td><td>-1512</td></tr>
<tr><td>$(M_u)_{bot}$ kip-in
Column 3-A</td><td>-1512</td></tr>
</table>

Sway magnification factor δ_s:

ΣP_f = 4 (402) + 10 (703) + 6 (1229) = 16,012 kips

ΣP_c = 10 (3759) + 10 (5180) = 89,390 kips

$\delta_s = 1 / [1 - \Sigma P_f / (0.75 \Sigma P_c)]$
$= 1 / [1 - 16{,}012 / (0.75 \times 89{,}390)]$
$= 1.31$

$\delta_s = 1.31 < 2.5$ O.K.</td>
<td>10.13.4.3</td>
<td>Slender Cols. 4.3</td>
</tr>
<tr>
<td>Check the stability of column under Load combination V as per Sec. 10.13.6.

Consider factored axial loads and bending moments obtained from a first-order frame analysis, conducted using the flexural rigidities given in Slender Cols. 4.3

Note: Counterclockwise moment at column end is positive.

Compute the sway magnification factor δ_s

Critical load, from earlier calculation.</td>
<td>v) Load Comb. V:U =1.2D + 1.6L + 0.5L_r

<table>
<tr><th>Load</th><th>1.2D + 1.6L + 0.5L_r</th></tr>
<tr><td>P_u (kips)
Corner Column</td><td>568</td></tr>
<tr><td>P_u (kips)
Edge Column</td><td>976</td></tr>
<tr><td>P_u (kips)
Interior Column</td><td>1687</td></tr>
<tr><td>$(M_u)_{top}$ kip-in
Column 3-A</td><td>-2032</td></tr>
<tr><td>$(M_u)_{bot}$ kip-in
Column 3-A</td><td>-2032</td></tr>
</table>

Sway magnification factor δ_s:
ΣP_f = 4 (568) + 10 (976) + 6 (1687) = 22,154 kips
ΣP_c = 10 (3759) + 10 (5180) = 89,390 kips
$\delta_s = 1 / [1 - \Sigma P_f / (0.75 \Sigma P_c)]$
$= 1 / [1 - 22{,}154 / (0.75 \times 89{,}390)]$
$= 1.49$
$\delta_s = 1.49 < 2.5$ O.K.</td>
<td>10.13.4.3</td>
<td>Slender Cols. 4.3</td>
</tr>
</table>

<table>
<tr>
<td>Check the stability of column under Load combination VI as per Sec. 10.13.6.

Consider factored axial loads and bending moments obtained from a first-order frame analysis, conducted using the flexural rigidities given in Slender Cols. 4.3

Note: Counterclockwise moment at column end is positive.

Compute the sway magnification factor δ_s

Critical load, from earlier calculation.</td>
<td>vi) Load Comb. VI: $U=1.2D+1.6\ L_r + 1.0L$
<table>
<tr><th>Load</th><th>$1.2D + 1.6\ L_r + 1.0L$</th></tr>
<tr><td>P_u (kips)
Corner Column</td><td>548</td></tr>
<tr><td>P_u (kips)
Edge Column</td><td>900</td></tr>
<tr><td>P_u (kips)
Interior Column</td><td>1514</td></tr>
<tr><td>$(M_u)_{top}$ kip-in
Column 3-A</td><td>-1756</td></tr>
<tr><td>$(M_u)_{bot}$ kip-in
Column 3-A</td><td>-1756</td></tr>
</table>
Sway magnification factor δ_s:

$\Sigma P_f = 4\ (548) + 10\ (900) + 6\ (1514)$
$= 20{,}276$ kips

$\Sigma P_c = 10\ (3759) + 10\ (5180) = 89{,}390$ kips

$\delta_s = 1 / [1 - \Sigma P_f / (0.75 \Sigma P_c)]$
$= 1 / [1 - 20{,}276 / (0.75 \times 89{,}390)]$
$= 1.43$

$\delta_s = 1.43 < 2.5$ O.K.</td>
<td>10.13.5</td>
<td>Slender Cols. 4.3</td>
</tr>
<tr>
<td>Design the Column for the governing load combination.

Note: Counterclockwise moment at column end is positive.

Select the interaction diagrams given in Columns 3.4.3 from Chapter 3 for equal reinforcement on all sides for $\gamma = 0.80$ (assumed)

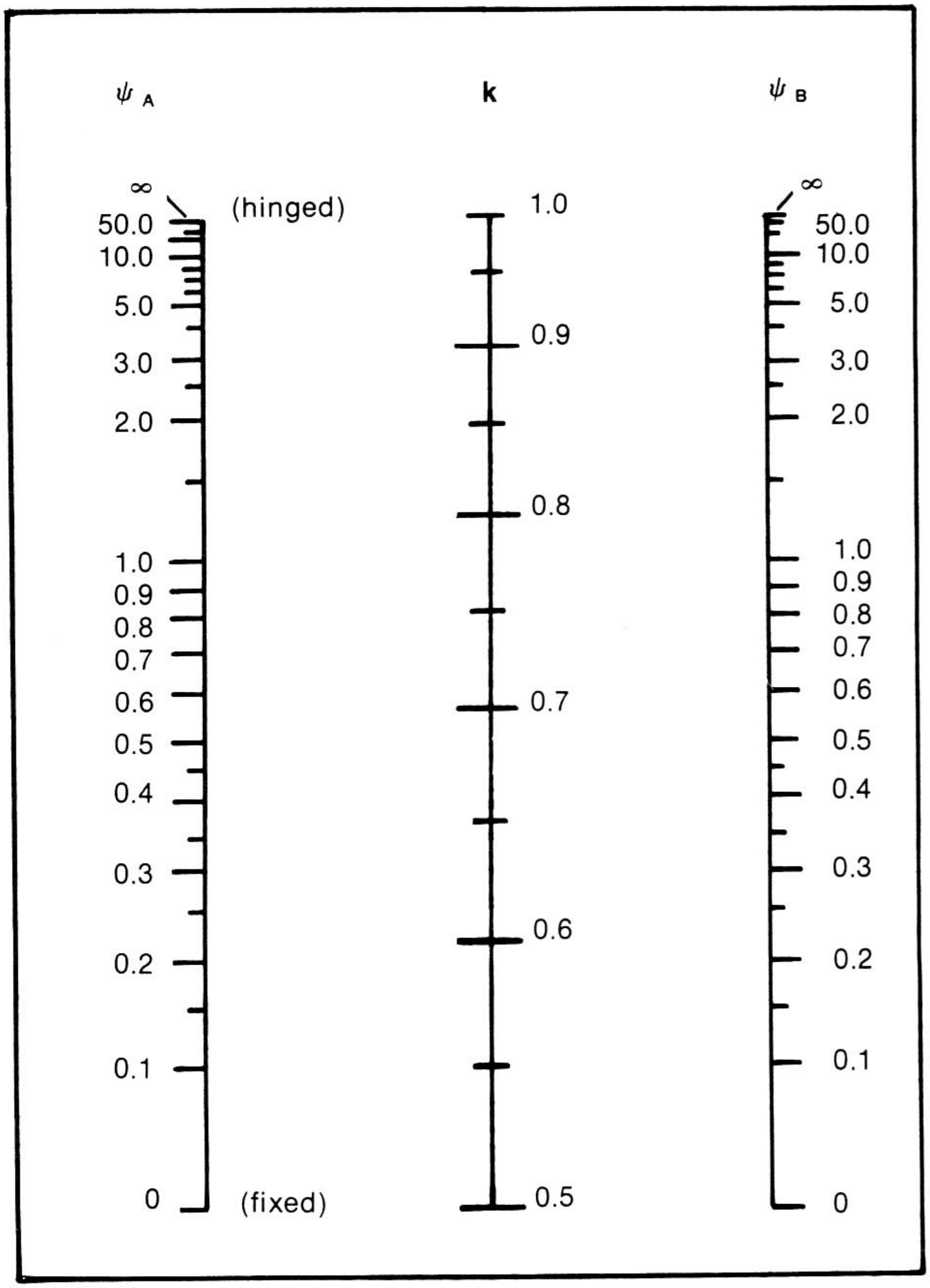
Compute; $K_n = \frac{P_n}{f'_c A_g}$

Compute; $R_n = \frac{M_n}{f'_c A_g h}$</td>
<td>Summary of Design Loads:
<table>
<tr><th>Load Combinations</th><th>P_u (kN)</th><th>(M_u) (kN.m)</th></tr>
<tr><td>I - U = 1.2D + 1.6L$_r$ + 0.8W</td><td>682</td><td>-2738</td></tr>
<tr><td>II - U =1.2D+1.6W +1.0L+0.5L_r</td><td>845</td><td>-4867</td></tr>
<tr><td>III - U = 0.9D + 1.6W</td><td>452</td><td>-3616</td></tr>
<tr><td>IV - U = 1.4D</td><td>703</td><td>-1512</td></tr>
<tr><td>V - U =1.2D + 1.6L + 0.5L_r</td><td>976</td><td>-2032</td></tr>
<tr><td>VI - U=1.2D+1.6 L_r + 1.0L</td><td>900</td><td>-1756</td></tr>
</table>
For Load Combination II;
$K_n = \frac{P_u/\varphi}{f'_c A_g} = \frac{845/0.65}{(6)(20)^2} = 0.54$
$R_n = \frac{M_n/\varphi}{f'_c A_g h} = \frac{4867/0.65}{(6)(20)^2(20)} = 0.16$</td>
<td></td>
<td>Columns 3.4.3</td>
</tr>
</table>

Read ρ_g for K_n and R_n values from the interaction diagrams	For K_n = 0.54 and R_n = 0.16 Read ρ_g = 0.030 Required A_{st} = 0.030 x 400 in.^{2} = 12.0 in.^{2} Try # 9 bars; 12.0 / 1.0 = 12.0 Try 12 # 9 Bars. Check for Load Combination V; $K_n = \frac{P_u/\varphi}{f_c^{'} A_g} = \frac{976/0.65}{(6)(20)^2} = 0.63$ $R_n = \frac{M_n/\varphi}{f_c^{'} A_g h} = \frac{2032/0.65}{(6)(20)^2(20)} = 0.065$ For K_n = 0.63 and R_n = 0.065 ρ_g = 0.030 is sufficient Therefore, use 12 # 9 Bars. *Note:* For further details of cross-sectional design refer to Chapter 3.		Columns 3.4.3 Columns 3.4.3

4.6 Slender Column Design Aids

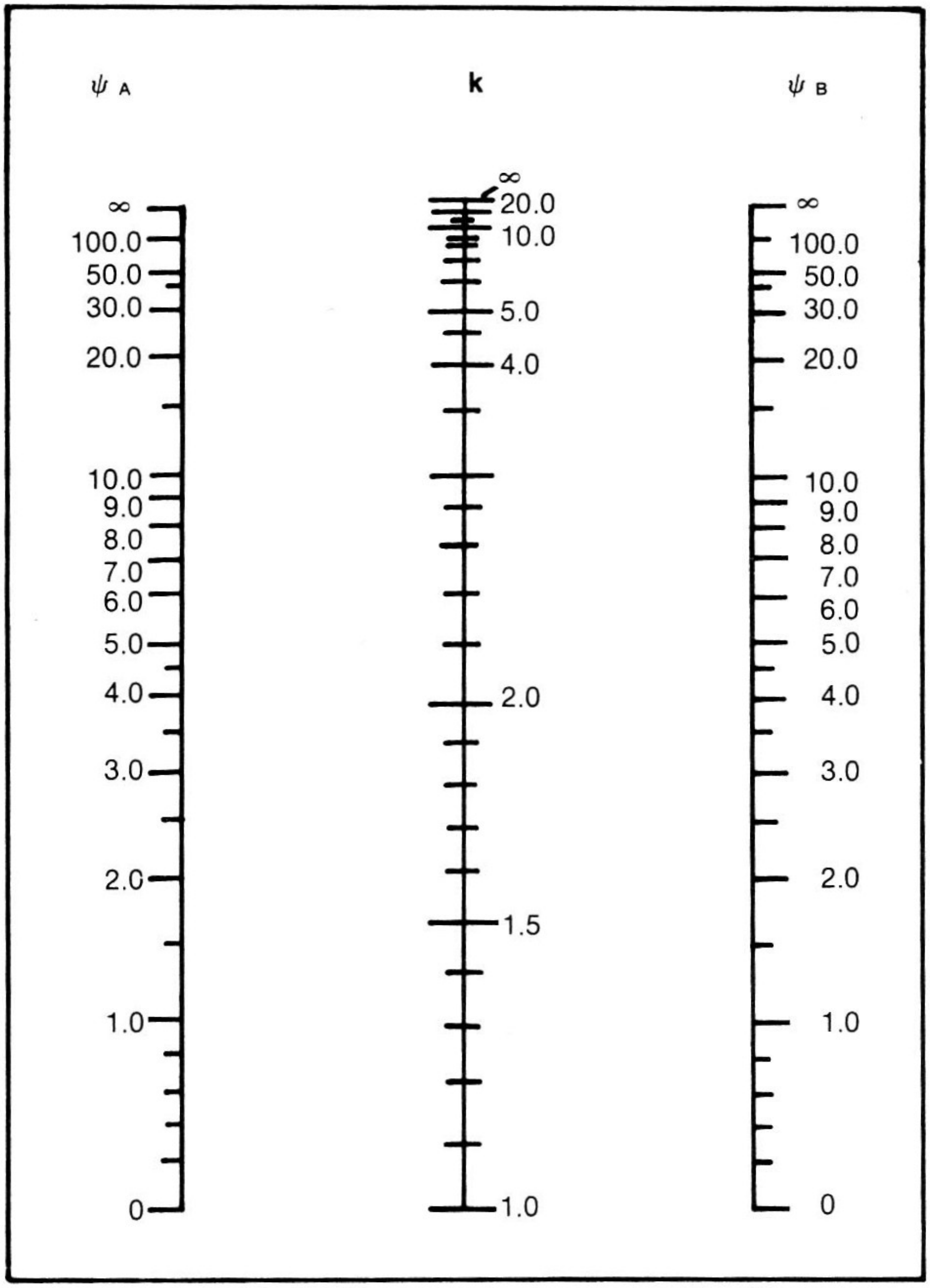
Slender Column - 4.1 Effective Length Factor – Jackson and Moreland Alignment Chart for Columns in Braced (Non-Sway) Frames[3]



$$\psi_i = \frac{(\Sigma EI/\ell_c)_{columns}}{(\Sigma EI/\ell)_{beams}}$$ at end i of column

[3] "Guide to Design Criteria for Metal Compression Members," 2nd Edition, Column Research Council, Fritz Engineering Laboratory, Lehigh University, Bethlehem, PA, 1966

Slender Columns - 4.2 Effective Length Factor – Jackson and Moreland Alignment Chart for Columns in Unbraced (Sway) Frames[4]



$$\psi_i = \frac{(\Sigma EI/\ell_c)_{columns}}{(\Sigma EI/\ell)_{beams}} \quad \text{at end i of column}$$

[4] "Guide to Design Criteria for Metal Compression Members," 2nd Edition, Column Research Council, Fritz Engineering Laboratory, Lehigh University, Bethlehem, PA, 1966

Slender Columns - 4.3 Recommended Flexural Rigidities (EI) for use in First-Order and Second Order Analyses of Frames for Design of Slender Columns

f'_c (ksi)	3	4	5	6	7	8	9	10	I/I_g
E_c (ksi)	3120	3605	4031	4415	4769	5098	5407	5700	
$E_c I / I_g$ (ksi)									
Beams	1092	1262	1411	1545	1669	1784	1892	1995	0.35
Columns	2184	2524	2822	3091	3338	3569	3785	3990	0.70
Walls (Uncracked)	2184	2524	2822	3091	3338	3569	3785	3990	0.70
Walls (Cracked)	1092	1262	1411	1545	1669	1784	1892	1995	0.35
Flat Plates Flat Slabs	780	901	1008	1104	1192	1275	1352	1425	0.25

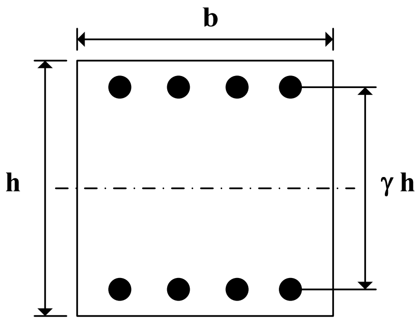
Notes:

1. The above values will be divided by $(1+\beta_d)$, when sustained lateral loads act or for stability checks made in accordance with Section 10.13.6 of ACI 318-05. For non-sway frames, β_d is ratio of maximum factored axial sustained load to maximum factored axial load associated with the same load combination, $\beta_d = 1.2D / (1.2D + 1.6L)$.
2. For sway frames, except as specified in Section 10.13.6 of ACI 318-05, β_d is ratio of maximum factored sustained shear within a story to the maximum factored shear in that story.
3. The above values are applicable to normal-density concrete with w_c between 90 and 155 lb/ft^3.
4. The moment of inertia of a T-beam should be based on the effective flange width, shown in **Flexure 6.** It is generally sufficiently accurate to take I_g of a T-beam as two times the I_g for the web.
5. Area of a member will *not* be reduced for analysis.

Slender Column - 4.4 Effective Length Factor “k” for Columns in Braced Frames

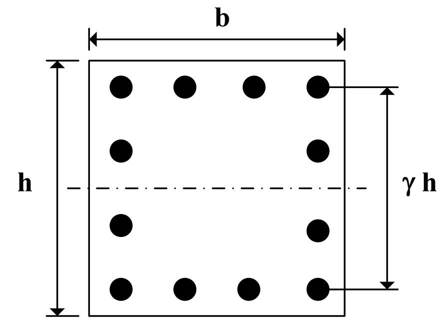
		k			
TOP	Hinged	0.81	0.91	0.95	1.00
	Elastic	0.77	0.86	0.90	0.95
	Elastic	0.74	0.83	0.86	0.91
	Stiff	0.67	0.74	0.77	0.81
		Stiff	Elastic	Elastic	Hinged
		BOTTOM			

Slender Columns - 4.5 Moment of Inertia of Reinforcement about Sectional Centroid[5]



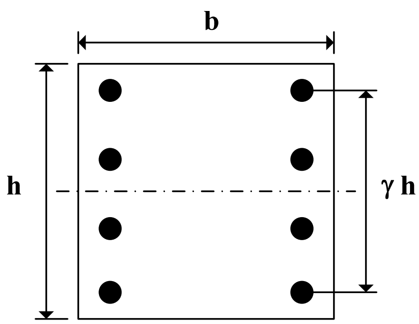
Bars in two end faces

$$I_{se} = 0.25\,\rho_t\, b\, h^3 \gamma^2$$



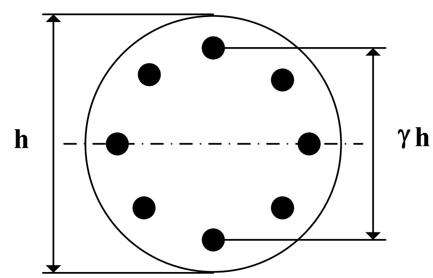
Equal reinforcement on four sides

$$I_{se} = 0.18\,\rho_t\, b\, h^3 \gamma^2$$



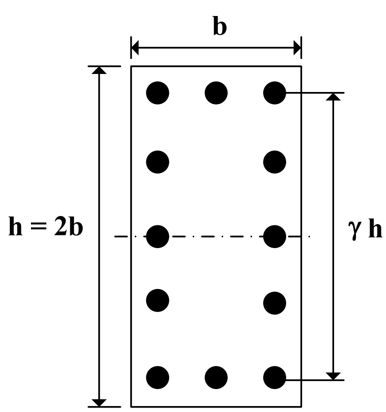
Bars in two side faces

$$I_{se} = 0.17\,\rho_t\, b\, h^3 \gamma^2 \quad \text{(3 bars per face)}$$
$$I_{se} = 0.12\,\rho_t\, b\, h^3 \gamma^2 \quad \text{(6 bars per face)}$$



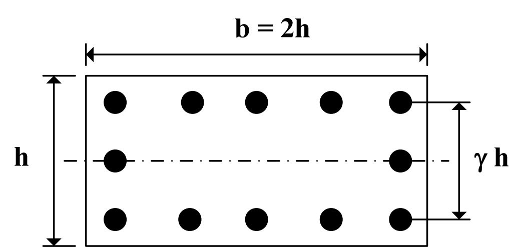
Uniformly distributed reinforcement

$$I_{se} = 0.10\,\rho_t\, h^4 \gamma^2$$



Bars uniformly spaced on all sides

$$I_{se} = 0.13\,\rho_t\, b\, h^3 \gamma^2$$



Bars uniformly spaced on all sides

$$I_{se} = 0.22\,\rho_t\, b\, h^3 \gamma^2$$

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[5] This table is based on Table 12-1 of MacGregor, J.G., Third Edition, Prentice Hall, Englewood Cliffs, New Jersey, 1997.