

Columns in Frames

The subject of *effective length* was introduced in the previous sections and some suggested K -values are listed in AISC/LRFD Specification Table C-C2.1. These factors were developed for columns with certain idealized conditions of end restraint which may be very different from practical design conditions. As discussed before, the connections between the beams and columns change the values of K . The deflection shape of the entire structure has to be evaluated using a statically indeterminate analysis.

Perhaps a few explanatory remarks should be made at this point, defining sidesway as it pertains to effective lengths. For this discussion sidesway refers to a type of buckling. In statically indeterminate structures sidesway occurs where the frames deflect laterally due to the presence of lateral loads or unsymmetrical vertical loads or where the frames themselves are unsymmetrical. Sidesway also occurs in columns whose ends can move transversely when they are loaded until buckling occurs.

Should frames with diagonal bracing or rigid shear walls be used, the columns will be prevented from sidesway and provided with some rotational restraint at their ends. For these situations, as illustrated below, the K -factors will fall somewhere between cases (a) and (d) of Table C-C2.1. The LRFD Specification C2 states that $K = 1.0$ should be used for columns in frames with sidesway inhibited unless an analysis shows that a smaller value can be used. $K = 1.0$ is often quite conservative, and an analysis made as described herein may result in some savings.

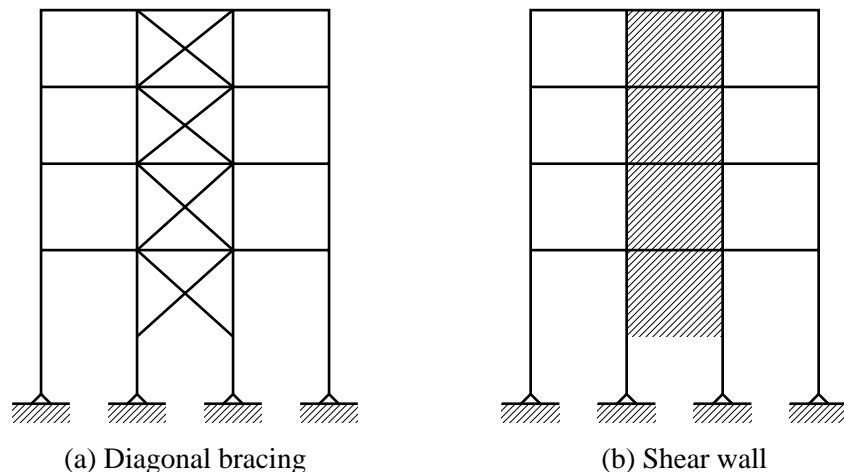
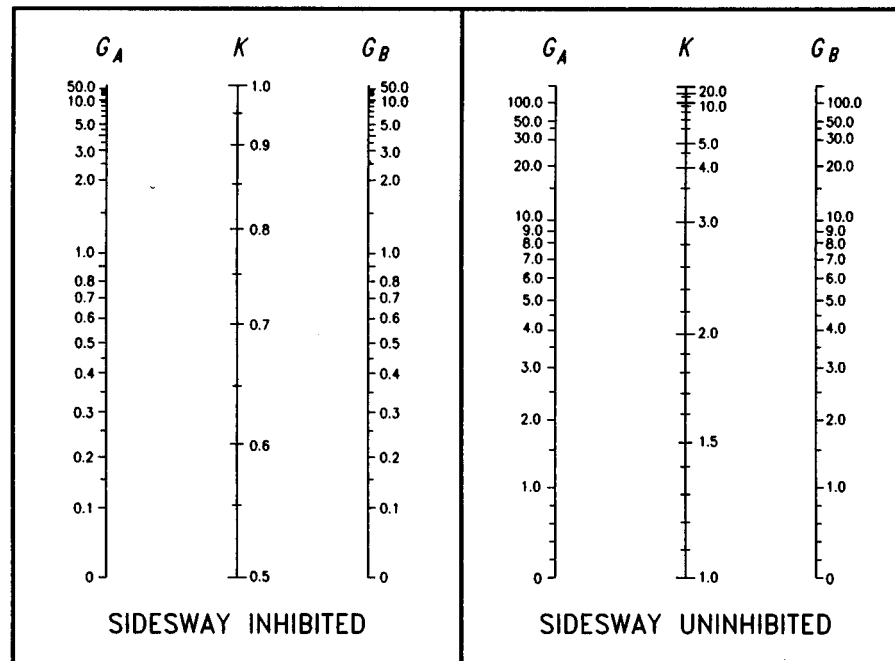


Figure 1: Sidesway inhibited

Theoretical mathematical analyses may be used to determine effective lengths, but such procedures are usually too lengthy and perhaps too difficult for the average designer. The usual procedure is to use Table C-C2.1, interpolating between the idealized values as the designer feels appropriate. To make analysis and design easier, AISC provides a chart to

calculate K . These charts, originally developed by Julian and Lawrence, present a practical method for estimating K -values. They were developed from a slope-deflection analysis of the frames that includes the effects of column loads. One chart was developed for columns braced against sidesway and one for columns subject to sidesway.



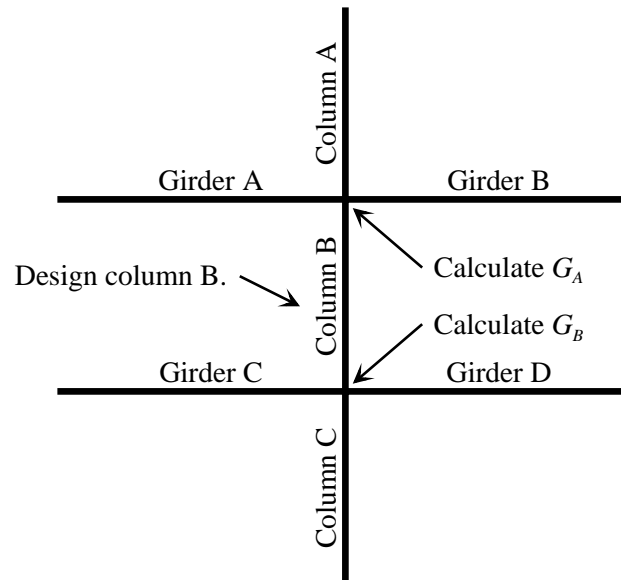
The subscripts A and B refer to the joints at the two ends of the column section being considered. G is defined as

$$G = \frac{\sum(I_c/L_c)}{\sum(I_g/L_g)}$$

in which \sum indicates a summation of all members rigidly connected to that joint and lying on the plane in which buckling of the column is being considered. I_c is the moment of inertia and L_c the unsupported length of a column section, and I_g is the moment of inertia and L_g the unsupported length of a girder or other restraining member. I_c and I_g are taken about axes perpendicular to the plane of buckling being considered.

For column ends supported by but not rigidly connected to a footing or foundation, G is theoretically infinity, but, unless actually designed as a true friction-free pin, may be taken as "10" for practical designs. If the column end is rigidly attached to a properly designed footing, G may be taken as 1.0. Smaller values may be used if justified by analysis.

Fig. C-C2.2 Alignment chart for effective length of columns in continuous frames.



The figure shown above illustrate columns (A, B, C) connected with beams (girders A, B, C, and D). First of all, it is necessary to find the length L and moment of inertia I of the columns and girders (beams) at each connection point.

Then the factor G (of the chart) should be calculated at the top and bottom of the column (local boundary conditions) using the formula:

$$G = \frac{\sum(I_c/L_c)}{\sum(I_g/L_g)}$$

where the summation is carried for all members connected to the point of interest. In this case,

$$G_A = \frac{I_{cA}/L_{cA} + I_{cB}/L_{cB}}{I_{gA}/L_{gA} + I_{gB}/L_{gB}}$$

$$G_B = \frac{I_{cB}/L_{cB} + I_{cC}/L_{cC}}{I_{gC}/L_{gC} + I_{gD}/L_{gD}}$$

The Structural Stability Research Council (SSRC) makes several recommendations concerning the use of the alignment charts.

1. For pinned columns, G is theoretically infinite, as where a column is connected to a footing with a frictionless hinge. Recognizing that such a connection is not frictionless, it is recommended that G be made equal to 10 where such nonrigid supports are used.
2. For rigid connections of columns to footing G theoretically approaches zero, but from a practical standpoint a value of 1.0 is recommended as no connections are perfectly rigid.

The K -factor is then obtained from the charts. There are two cases: (1) braced frames (sidesway inhibited) and (2) unbraced frames (sidesway uninhibited). After the K -value is

determined, the each column can be redesigned. Should the sizes change appreciably, new effective lengths can be determined, the column design repeated, and so on. Following shows a recommended design procedure for columns in frames.

1. Select the appropriate chart (sidesway inhibited or sidesway uninhibited).
2. Compute G at each end of the column and label the values G_A and G_B as desired.
3. Draw a straight line on the chart between the G_A and G_B values and read K where the line hits the center K scale.

The alignment chart C-C2.2 was made for stresses in the elastic region. If the elastic buckling controls the failure, the yielded region reduces the stiffness so that the K -factor should be corrected. Note that the AISC equations (E2-2) and (E2-3)

$$F_{cr} = \begin{cases} 0.658\lambda_c^2 F_y & \text{for } \lambda_c < 1.5 \\ 0.877F_E = \frac{0.877}{\lambda_c^2} F_y & \text{for } \lambda_c \geq 1.5 \end{cases} \quad (\text{E2-2, E2-3})$$

become equal ($F_{cf} = 0.39F_y$) when the slenderness parameter $\lambda_c = 1.5$. In other words, if the value P_u/A_g exceeds $0.39F_y$, the elastic buckling controls; otherwise, inelastic buckling is the probable mode of failure. In the case when the column behaves elastically, we can use the alignment chart directly to find G -value. However, for the case of inelastic buckling, the G -value should be corrected by multiplying the elastic G -value by the *stiffness reduction factor* (SRF; β_s) that can be found from the following table (suggested by Yura; Burns p. 114).

P_u/A_g ksi	$F_y = 36$ ksi	$F_y = 50$ ksi	P_u/A_g ksi	$F_y = 36$ ksi	$F_y = 50$ ksi
42	—	0.03	26	0.38	0.82
41	—	0.09	25	0.45	0.85
40	—	0.16	24	0.52	0.88
39	—	0.21	23	0.58	0.90
38	—	0.27	22	0.65	0.93
37	—	0.33	21	0.70	0.95
36	—	0.38	20	0.76	0.97
35	—	0.44	19	0.91	0.98
34	—	0.49	18	0.85	0.99
33	—	0.53	17	0.89	1.00
32	—	0.58	16	0.92	
31	—	0.63	15	0.95	
30	0.05	0.67	14	0.97	
29	0.14	0.71	13	0.99	
28	0.22	0.75	12	1.00	
27	0.30	0.79			

Design Procedure for Columns in Frames

Step 1: Determine the factored design loads.

Step 2: Guess initial column size: Since K_x is unknown, use $KL = K_y L_y$.

Step 3: Calculate design strength.

1. Find the properties of all girders and columns.
2. Calculate G_A and G_B using the equation

$$G = \frac{\sum(I_c/L_c)}{\sum(I_g/L_g)}$$

3. If inelastic buckling, i.e.,

$$\frac{P_u}{A_g} > 0.39F_y$$

then multiply G by the stiffness reduction factor found from the table; otherwise use G -values without modification.

4. Determine K_x from the chart.
5. Determine the effective length KL :

$$KL = \max \left\{ K_y L_y (\text{weak-axis}), \frac{K_x L_x}{r_x/r_y} (\text{strong-axis}) \right\}.$$

6. Enter into the column table to get the approximate design strength.

Step 4: Redesign. If the capacity is significantly different from the design load, it is necessary to pick a new column. Use the following approximate formula:

$$\text{Weight (new column)} = \frac{\text{Weight (old column)} \times \text{Load}}{\text{Capacity (old column)}}$$

Repeat Steps 3 and 4 until satisfactory conditions are met.

Step 5: Check the result using Table 3-36, 3-50, or the formula.