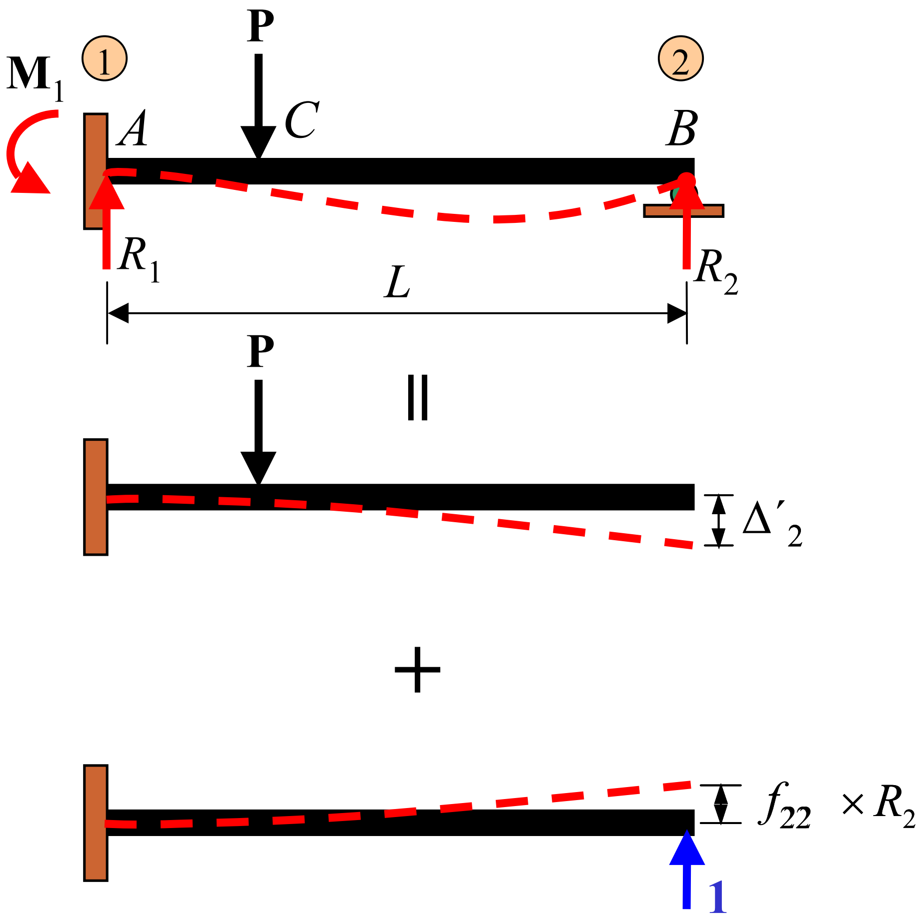
ANALYSIS OF STATICALLY INDETERMINATE STRUCTURES BY THE FORCE METHOD

- Force Method of Analysis: Beams
- Maxwell's Theorem of Reciprocal Displacements; Betti's Law
- Force Method of Analysis: Frames
- Force Method of Analysis: Trusses
- Force Method of Analysis: General
- Composite Structures

Force Method of Analysis : Beams

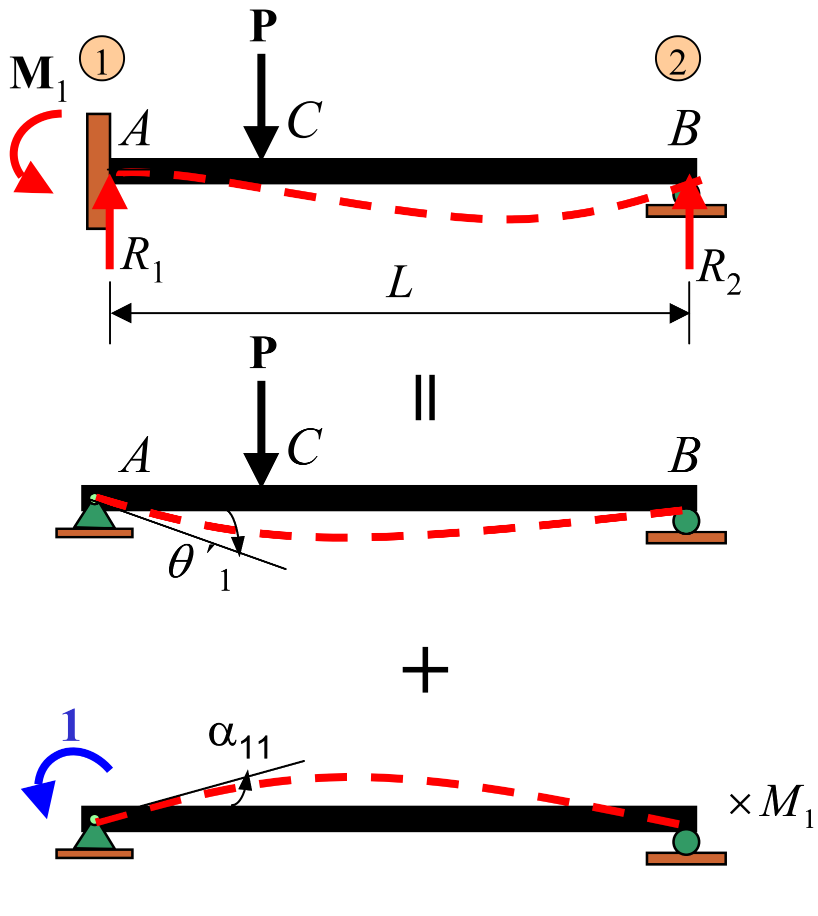
1 Degree of freedom

- **Compatibility of displacement**



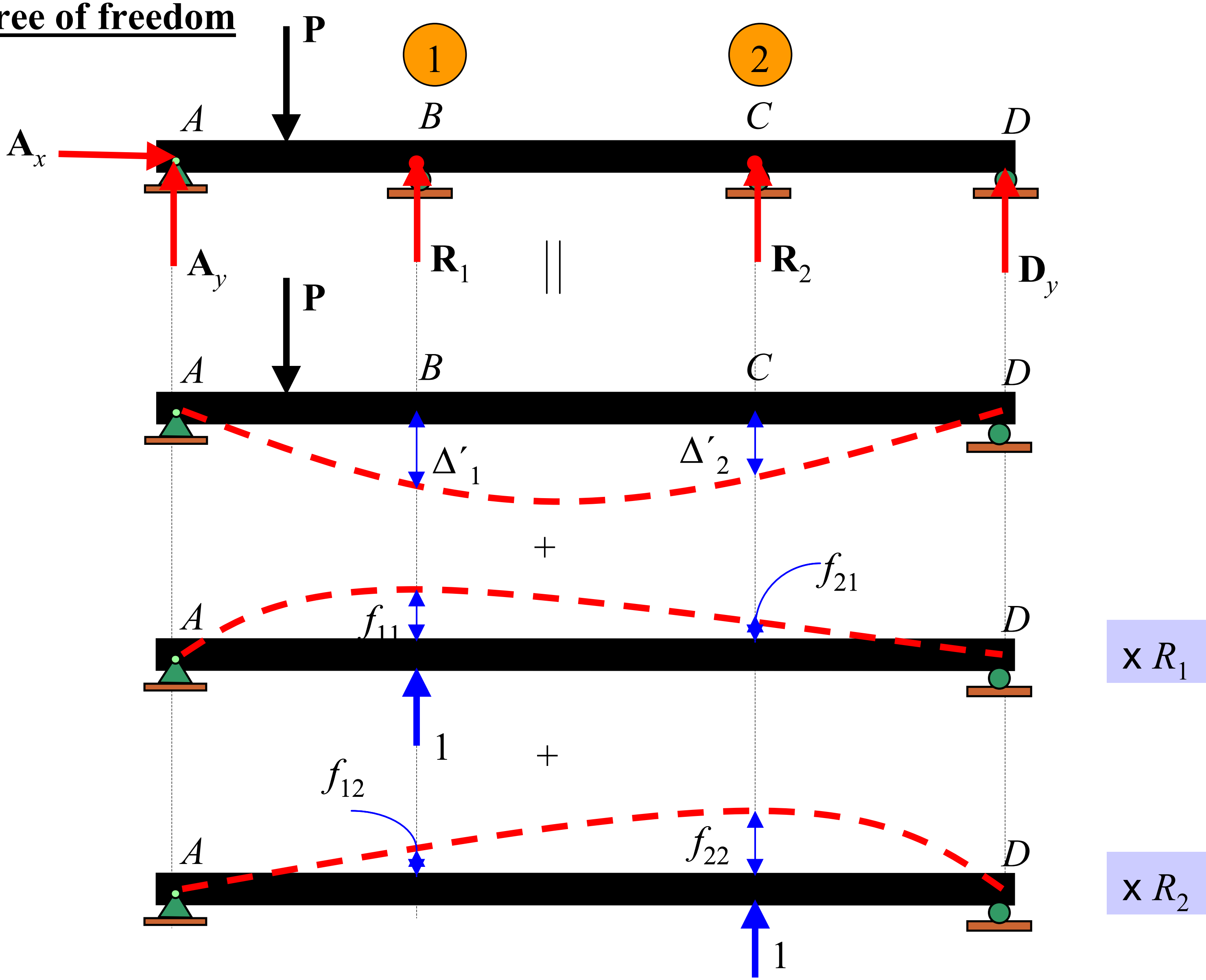
$$\Delta'_2 + f_{22}\mathbf{R}_2 = \Delta_2 = 0$$

- **Compatibility of slope**



$$\theta'_1 + \alpha_{11}\mathbf{M}_1 = \theta_1 = 0$$

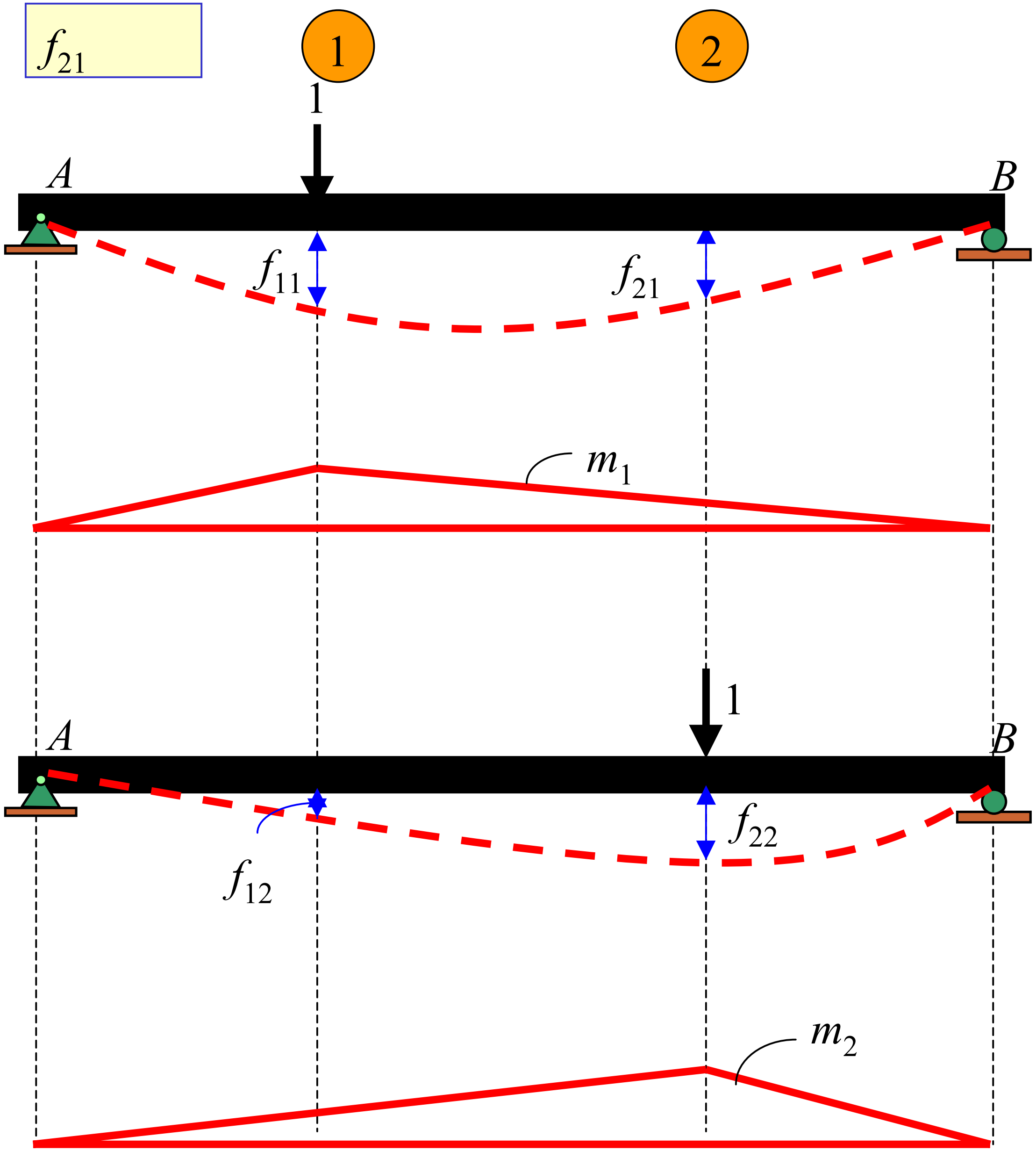
<u>**2 Degree of freedom**</u>



$$\Delta'_1 + f_{11}R_1 + f_{12}R_2 = \Delta_1 = 0$$

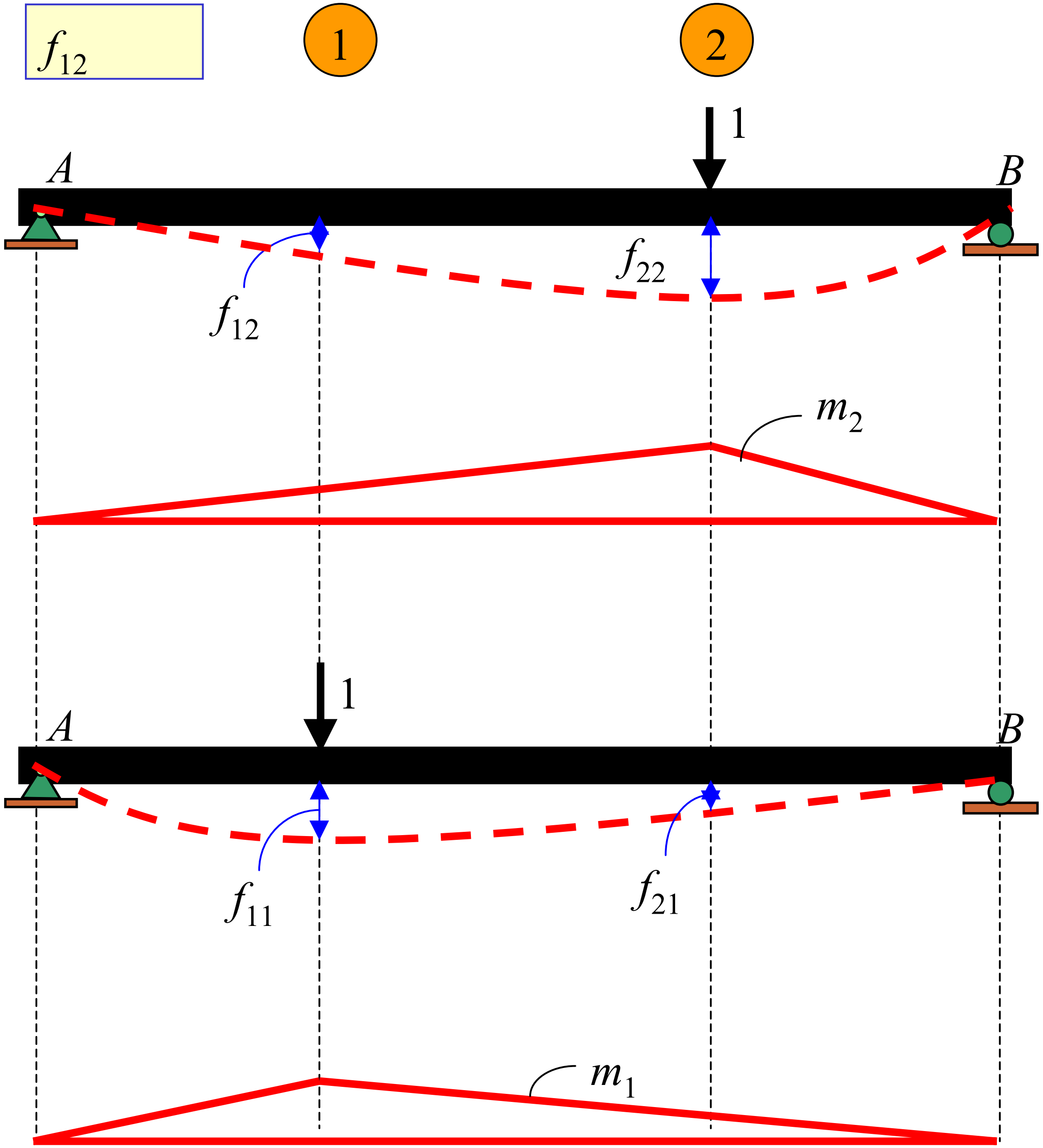
$$\Delta'_2 + f_{21}R_1 + f_{22}R_2 = \Delta_2 = 0$$

Maxwell's Theorem of Reciprocal Displacements; Betti's Law



$$1 \bullet f_{21} = \int_L \frac{m_2 M_1}{EI} dx = \int_L \frac{m_2 m_1}{EI} dx$$

$$f_{21} = \int_L \frac{m_2 m_1}{EI} dx$$



$$f_{21} = \int_L \frac{m_2 m_1}{EI} dx$$

$$1 \bullet f_{12} = \int_L \frac{m_1 M_2}{EI} dx = \int_L \frac{m_1 m_2}{EI} dx$$

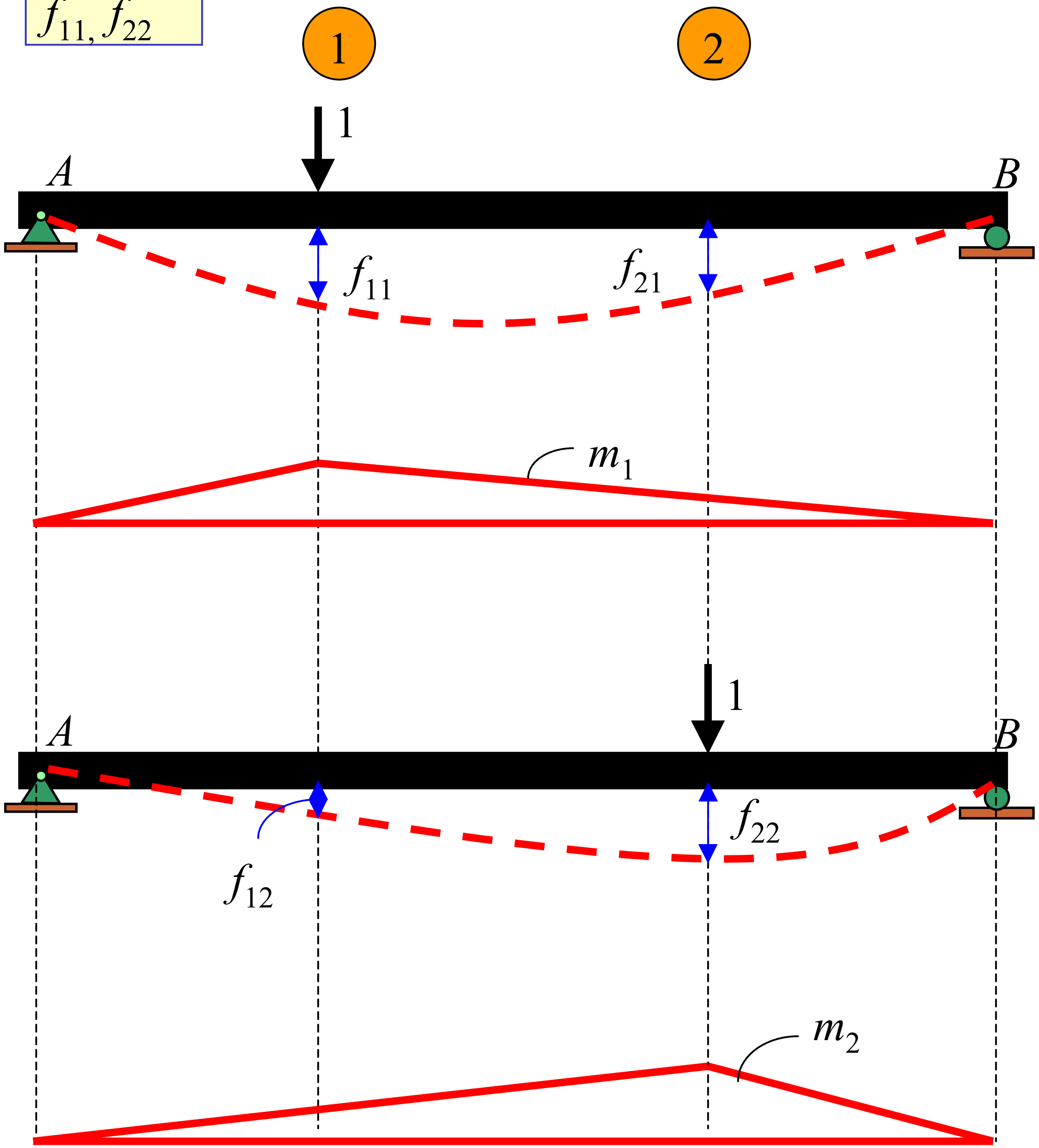
$$f_{12} = \int_L \frac{m_1 m_2}{EI} dx$$

$$f_{21} = f_{12}$$

Maxwell's Theorem:

$$f_{ij} = f_{ji}$$

f_{11}, f_{22}



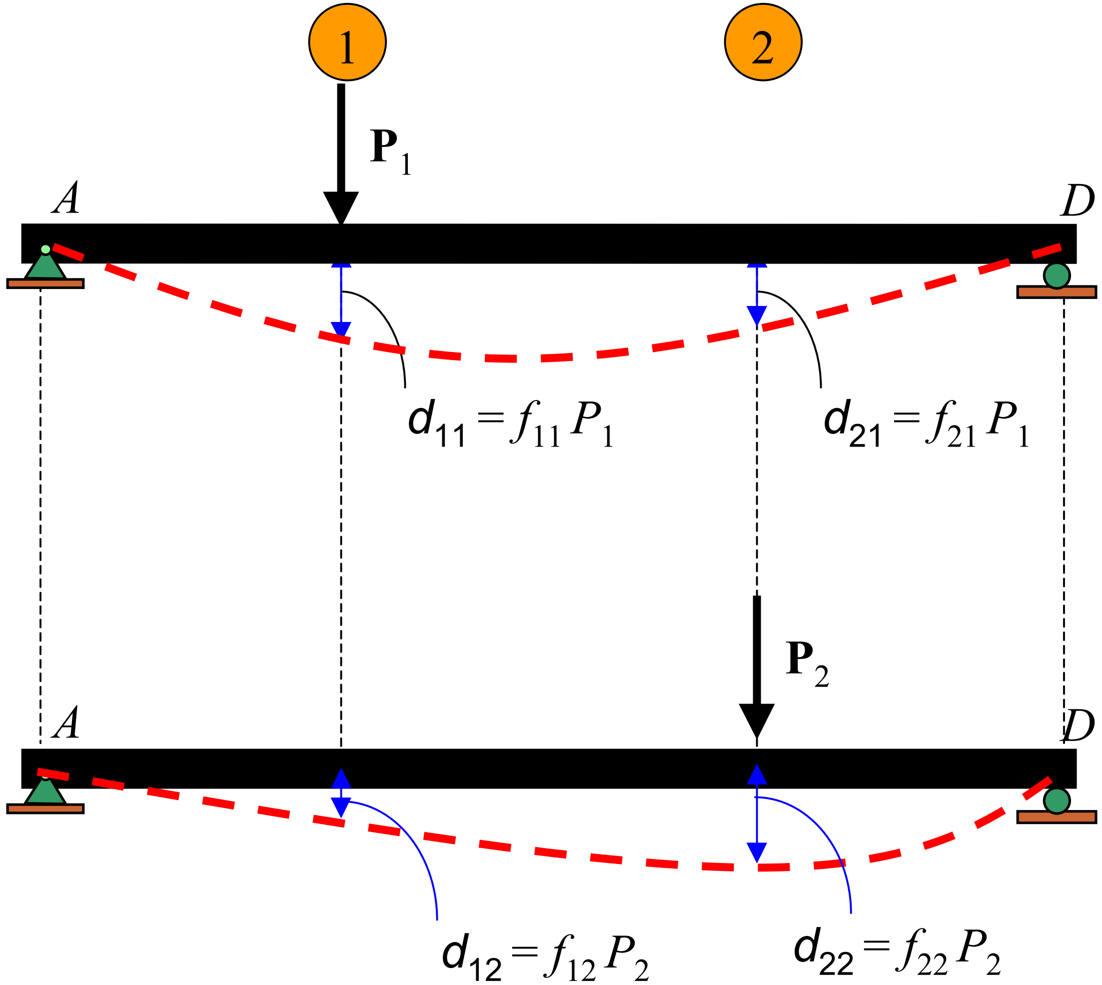
$$1 \bullet f_{11} = \int_L \frac{m_1 M_1}{EI} dx = \int_L \frac{m_1 m_1}{EI} dx$$

$$1 \bullet f_{22} = \int_L \frac{m_2 M_2}{EI} dx = \int_L \frac{m_2 m_2}{EI} dx$$

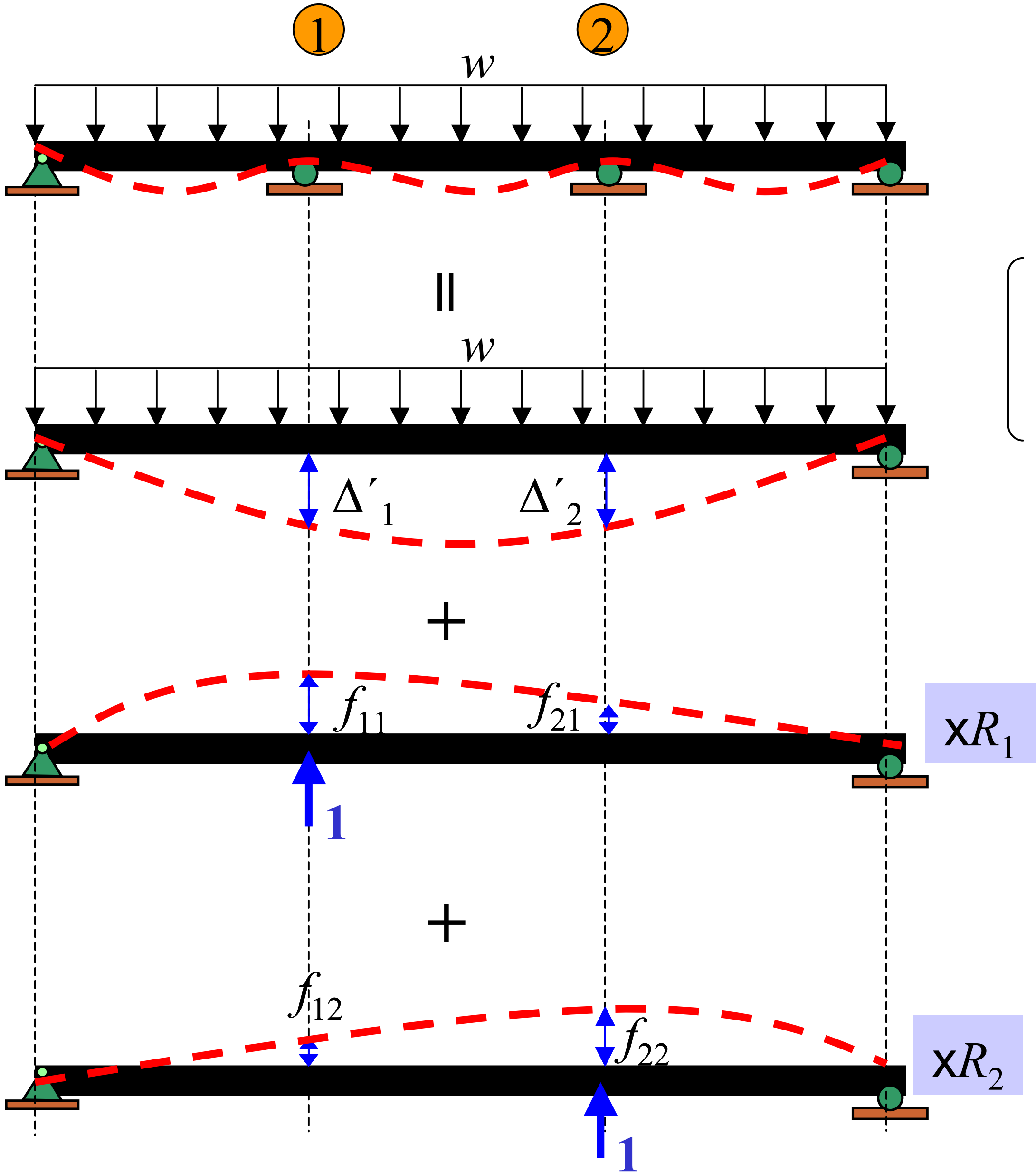
In general,

$$1 \bullet f_{ij} = f = \int_L \frac{m_i m_j}{EI} dx$$

$$1 \bullet f_{ji} = f_{ji} = \int_L \frac{m_j m_i}{EI} dx$$


1
2
$\mathbf{P}_1$
A
D
$d_{11} = f_{11} P_1$
$d_{21} = f_{21} P_1$
$\mathbf{P}_2$
A
D
$d_{12} = f_{12} P_2$
$d_{22} = f_{22} P_2$

Force Method of Analysis: General



Compatibility Eq.

$$\Delta'_1 + f_{11}R_1 + f_{12}R_2 = \Delta_1 = 0$$

$$\Delta'_2 + f_{21}R_1 + f_{22}R_2 = \Delta_2 = 0$$

$$\begin{pmatrix} \Delta'_1 \\ \Delta'_2 \end{pmatrix} + \begin{pmatrix} f_{11} & f_{12} \\ f_{12} & f_{22} \end{pmatrix} \begin{pmatrix} R_1 \\ R_2 \end{pmatrix} = \begin{pmatrix} \cancelto{0}{\Delta_1} \\ \cancelto{0}{\Delta_2} \end{pmatrix}$$

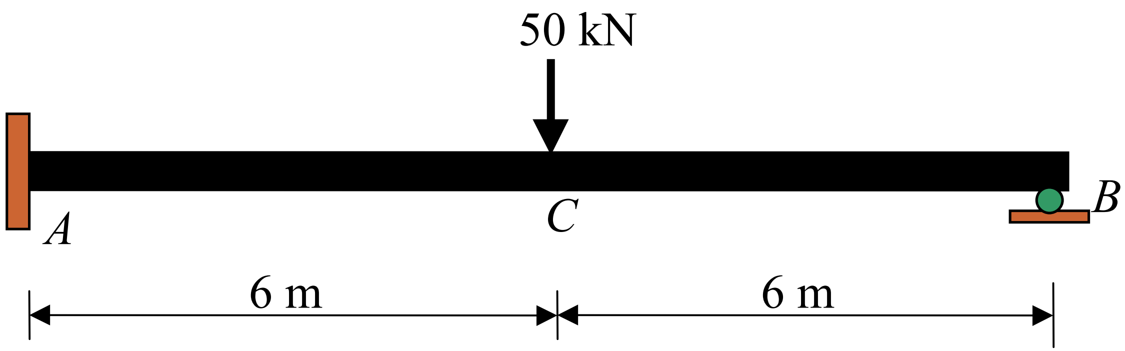
$$\begin{pmatrix} f_{11} & f_{12} \\ f_{12} & f_{22} \end{pmatrix} \begin{pmatrix} R_1 \\ R_2 \end{pmatrix} = - \begin{pmatrix} \Delta'_1 \\ \Delta'_2 \end{pmatrix}$$

General form:

$$\begin{pmatrix} f_{11} & f_{12} & f_{1n} \\ f_{21} & f_{22} & f_{2n} \\ & \vdots & \\ f_{n1} & f_{n2} & f_{nn} \end{pmatrix} \begin{pmatrix} R_1 \\ R_2 \\ \vdots \\ R_n \end{pmatrix} = - \begin{pmatrix} \Delta'_1 \\ \Delta'_2 \\ \vdots \\ \Delta'_n \end{pmatrix}$$

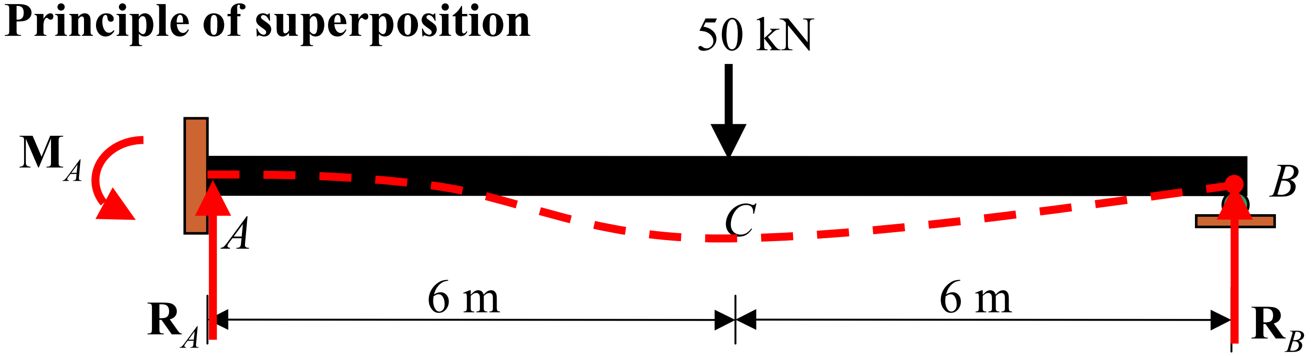
Example 9-1

Determine the **reaction at all supports** and **the displacement at *C***.



<u>SOLUTION</u> **Use compatibility of *displacement* for find reaction**

- **Principle of superposition**



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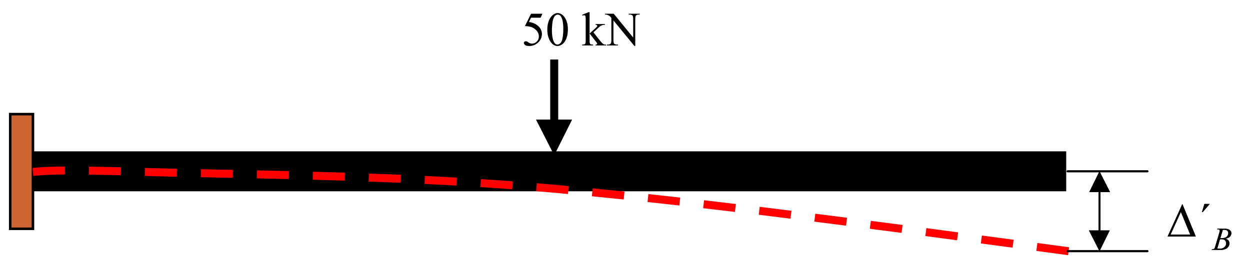

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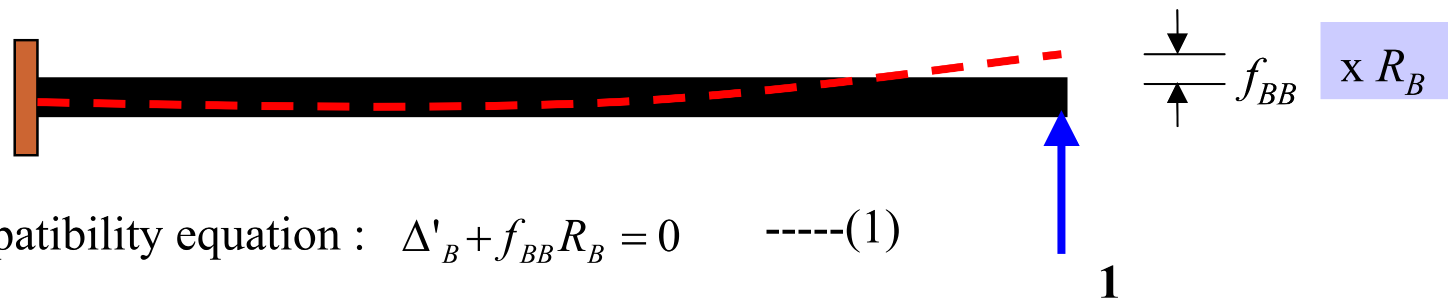
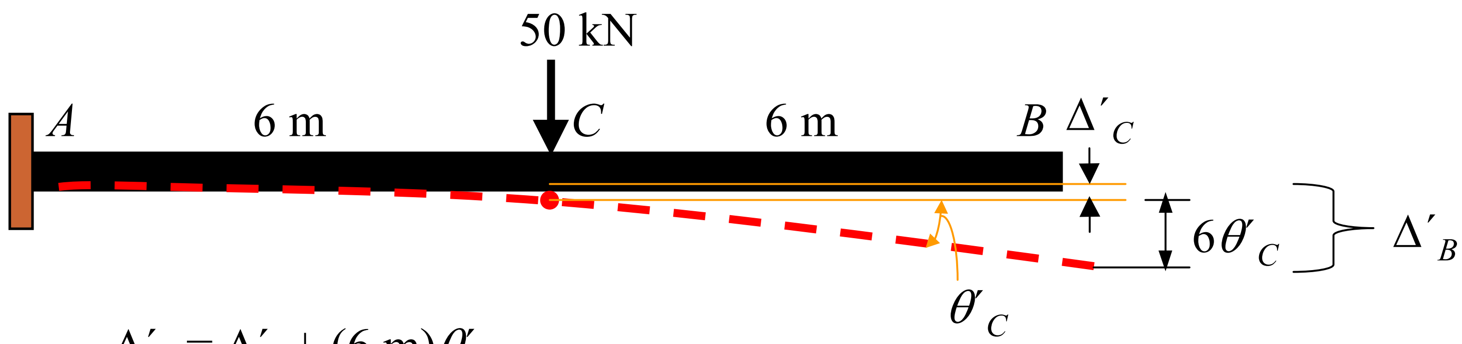


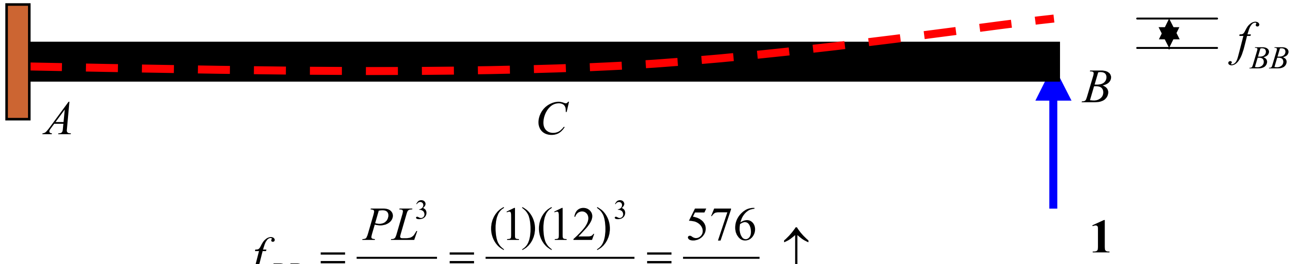
Compatibility equation : $\Delta'_B + f_{BB} R_B = 0$ -----(1)

• **Use formulation for** Δ'_B and f_{BB}



$$\Delta'_B = \Delta'_C + (6\text{ m})\theta'_C$$

$$\Delta'_B = \frac{P(6)^3}{3EI} + (6)\frac{P(6)^2}{2EI} \qquad = \frac{50(6)^3}{3EI} + (6)\frac{(50)(6)^2}{2EI} = \frac{9000}{EI}, \downarrow$$



$$f_{BB} = \frac{PL^3}{3EI} = \frac{(1)(12)^3}{3EI} = \frac{576}{EI}, \uparrow$$

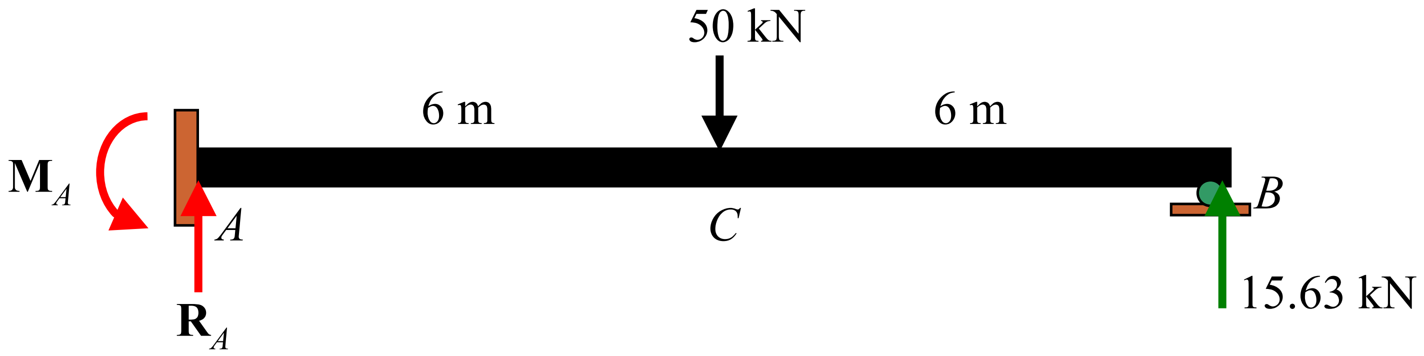
• **Substitute** Δ'_B and f_{BB} in Eq. (1): $\Delta'_B + f_{BB}R_B = 0$

$$\Delta'_B = \frac{9000}{EI}, \downarrow$$

$$f_{BB} = \frac{576}{EI}, \uparrow$$

$$+\uparrow: -\frac{9000}{\cancel{EI}} + (\frac{576}{\cancel{EI}})R_B = 0$$

$$R_B = 15.63 \text{ kN}, \uparrow$$

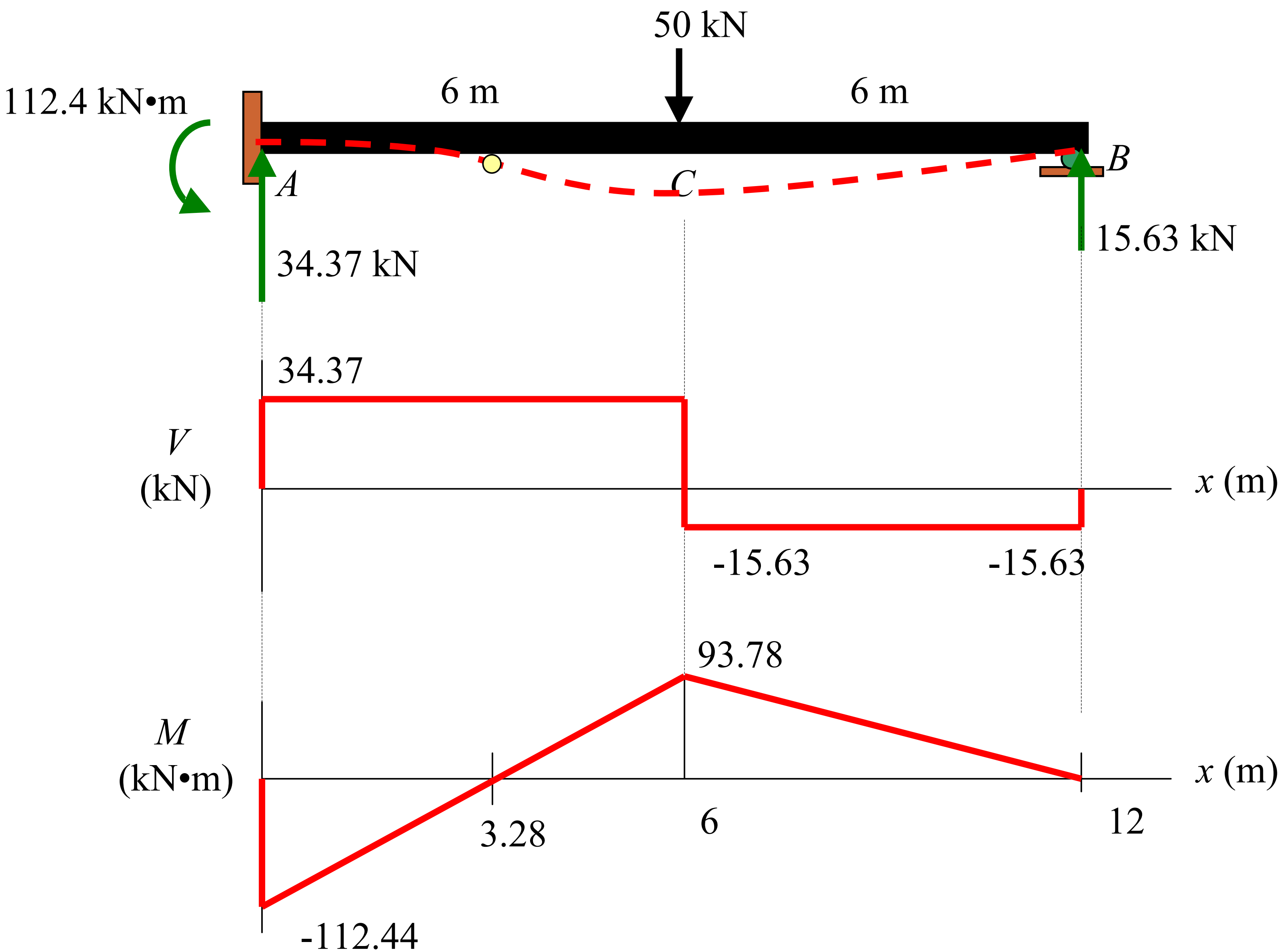


Equilibrium equation :

$$+\circlearrowright \Sigma M_A = 0: \quad M_A - 50(6) + 15.63(12) = 0, \qquad M_A = 112.4 \text{ kN}, \quad +\circlearrowright$$

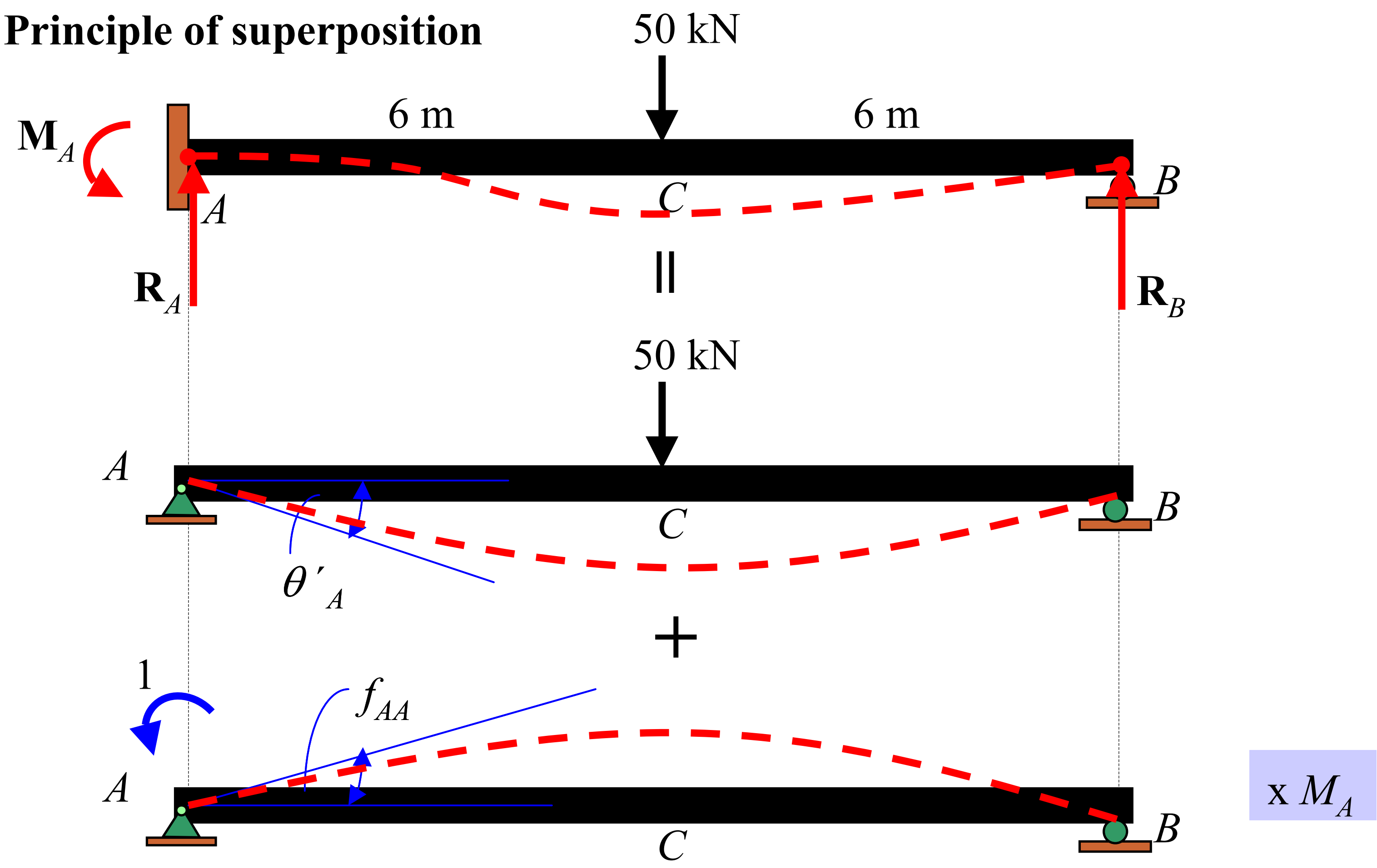
$$+\uparrow \Sigma F_y = 0: \quad +R_A - 50 + 15.63 = 0, \qquad R_a = 34.37 \text{ kN}, \uparrow$$

• **Quantitative shear and bending diagram and qualitative deflected curve**



Or use compatibility of *slope* to obtain reaction

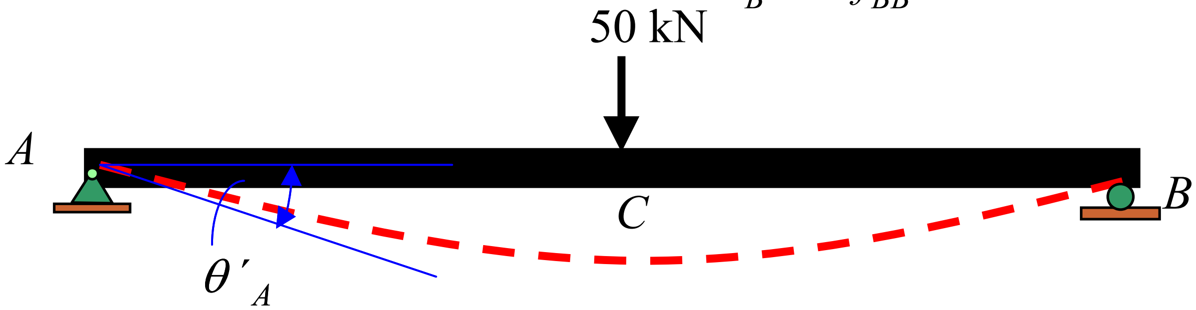
- **Principle of superposition**



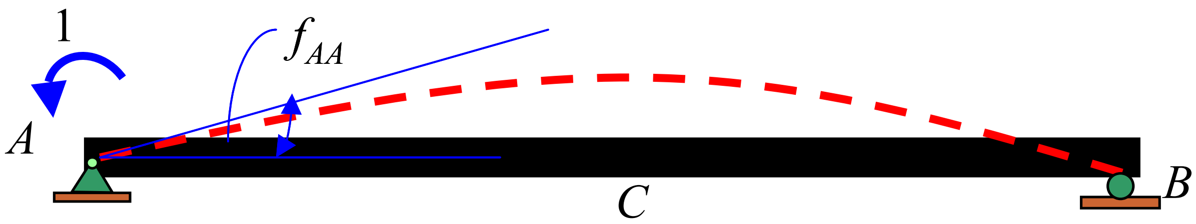
Compatibility equation :

$$\theta'_A + f_{AA} M_A = \theta_A = 0 \quad \text{-----(2)}$$

• **Use the table on the inside front cover for** θ'_B and f_{BB}



$$\theta_A' = \frac{PL^2}{16EI}$$



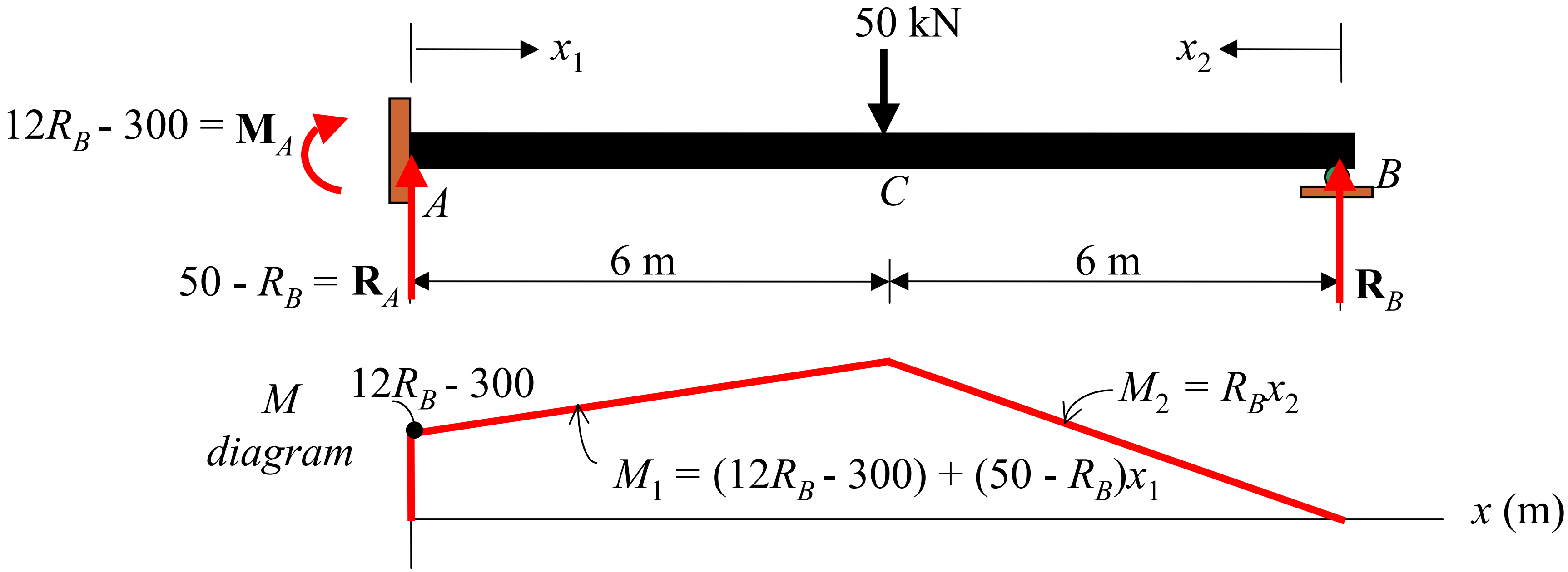
$$f_{CC} = \frac{L}{3EI}$$

Substitute the values in equation: $\theta'_A + f_{AA} M_A = \theta_A = 0$

$$+ : \qquad -\frac{PL^2}{16EI} + \frac{L}{3EI} M_A = 0$$

$$M_A = \frac{3PL}{16} = \frac{3(50)(12)}{16} = 112.5 \text{ kN•m}, \; +$$

Or use Castigliano least work method



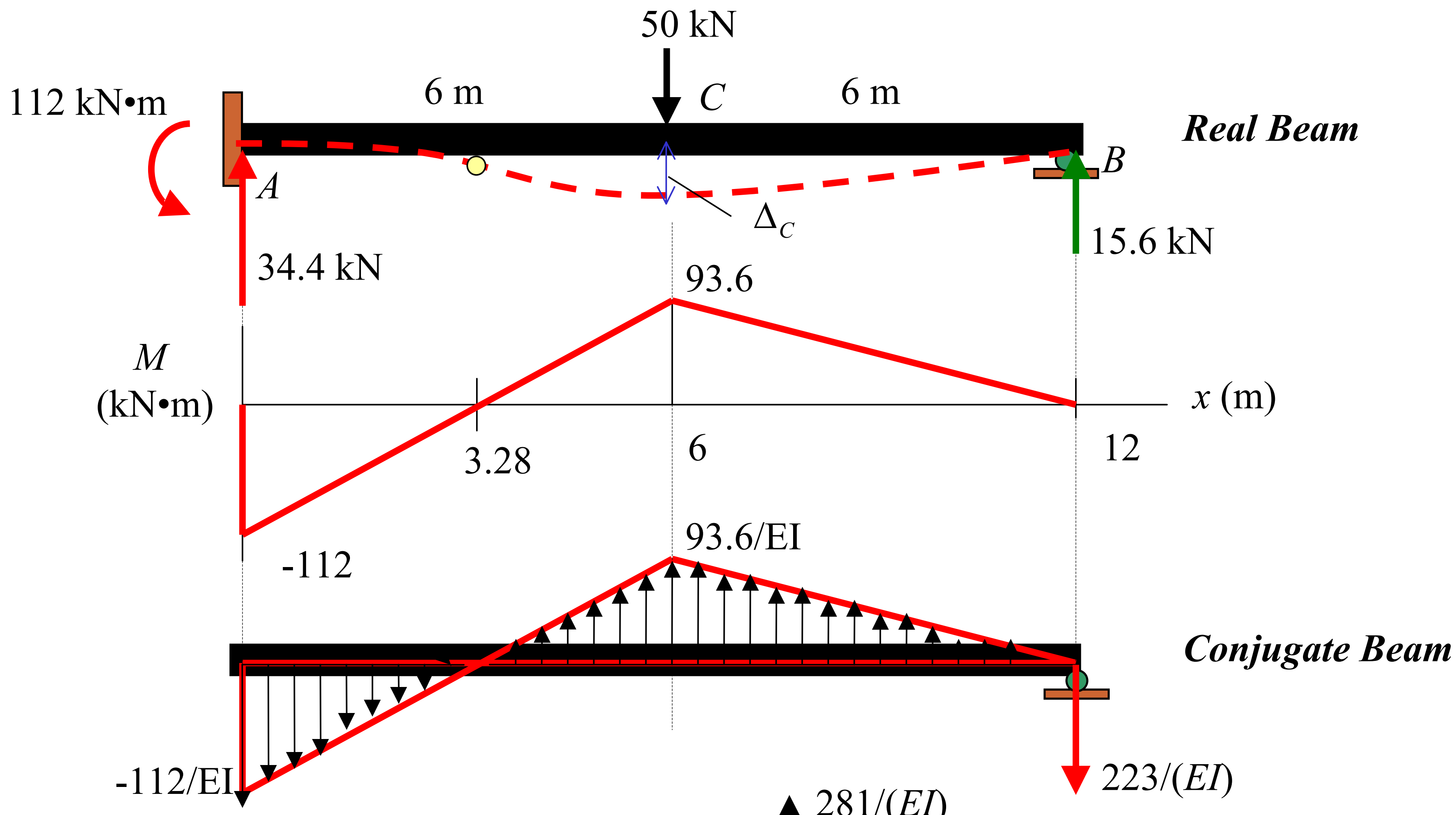
$$\Delta_B = 0 = \int_0^L \left(\frac{\partial M}{\partial R_B}\right)\frac{M}{EI}\,dx$$

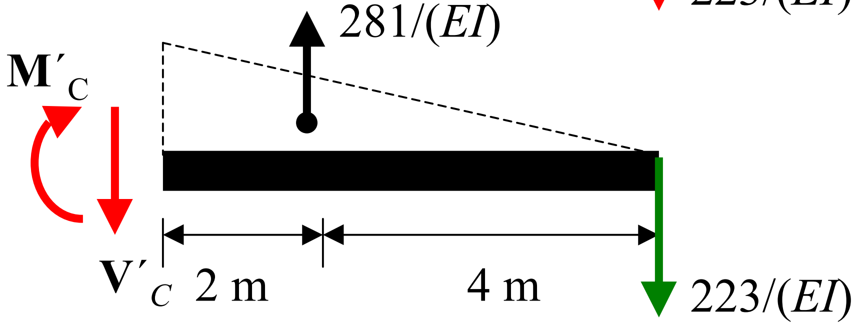
$$0 = \frac{1}{\cancel{EI}}\int_0^6 (12 - x_1)(12R_B - 300 + 50x_1 - R_B x_1)\,dx_1 + \frac{1}{\cancel{EI}}\int_0^6 x_2 (R_B x_2)\,dx_2$$

$$0 = \left(144R_B x_1 - 3600x_1 + \frac{900x_1^2}{2} - \frac{24x_1^2}{2}R_B - \frac{50x_1^3}{3} + \frac{x_1^3}{3}R_B\right)\Big|_0^6 + \frac{x_2^3}{3}R_B\Big|_0^6$$

R_B = 15.63 kN, ↑

Use conjugate beam for find the displacement

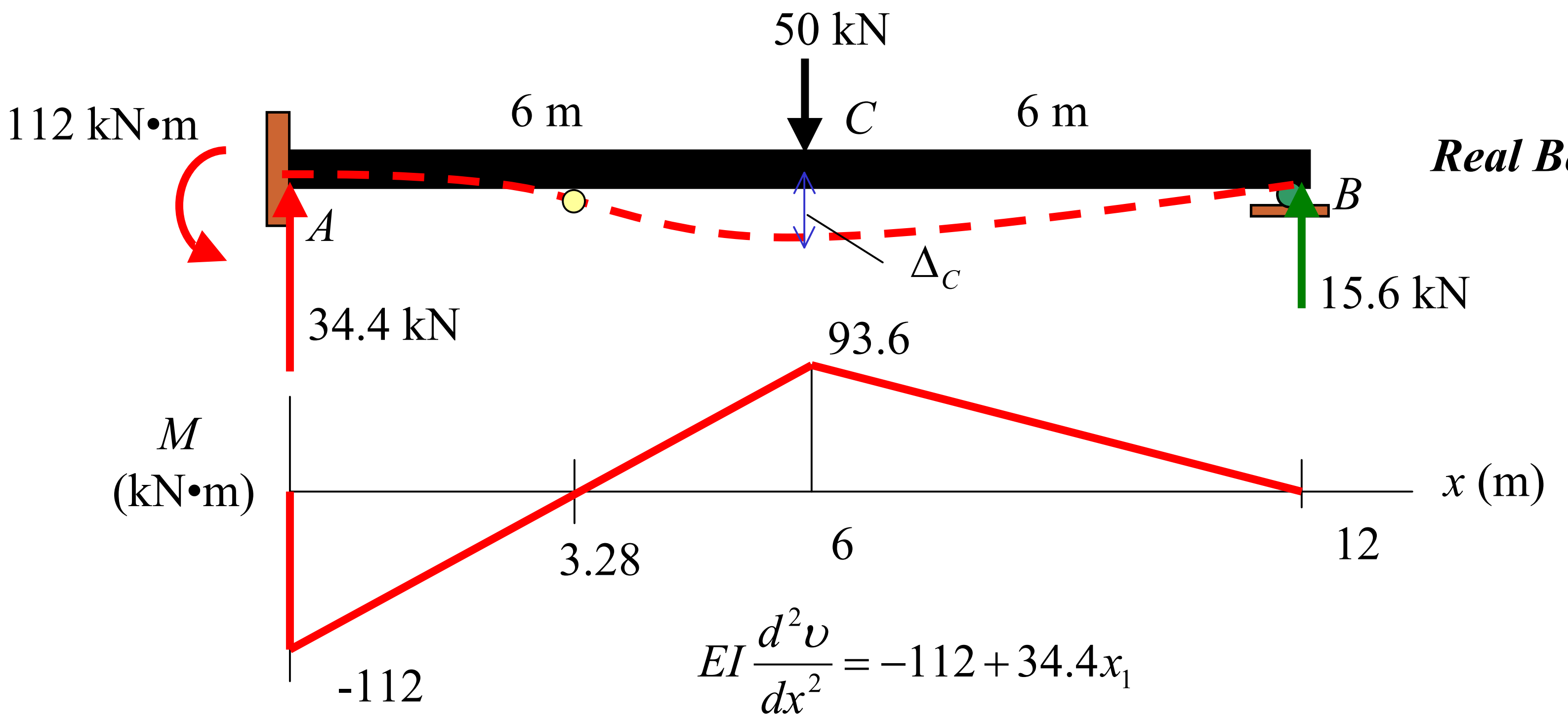




$$M'_C = \frac{281}{EI}(2) - \frac{223}{EI}(6) = -\frac{776}{EI}$$

$$\Delta_C = M'_C = -\frac{776}{EI}, \downarrow$$

Use double integration to obtain the displacement



$$EI\frac{d^2\upsilon}{dx^2} = -112 + 34.4x_1$$

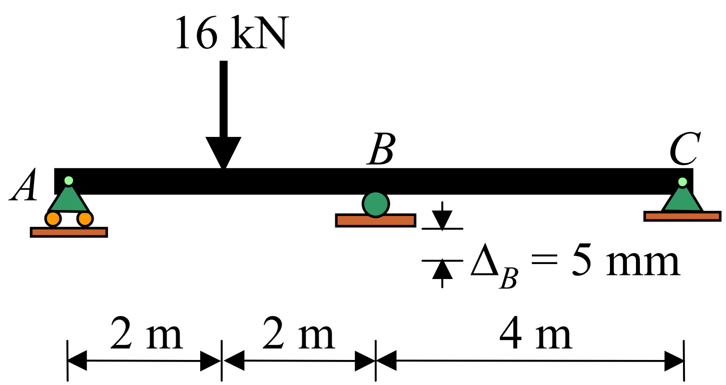
$$EI\frac{d\upsilon}{dx} = -112x_1 + 34.4\frac{x_1^2}{2} + C_1$$

$$EI\upsilon = -112.4\frac{x_1^2}{2} + 34.4\frac{x_1^3}{6} + C_1x_1 + C_2$$

$$\boxed{\Delta_C = \frac{1}{EI}(-112(\frac{6^2}{2}) + 34.4(\frac{6^3}{6}) + 0 + 0) = -\frac{778}{EI}, \downarrow}$$

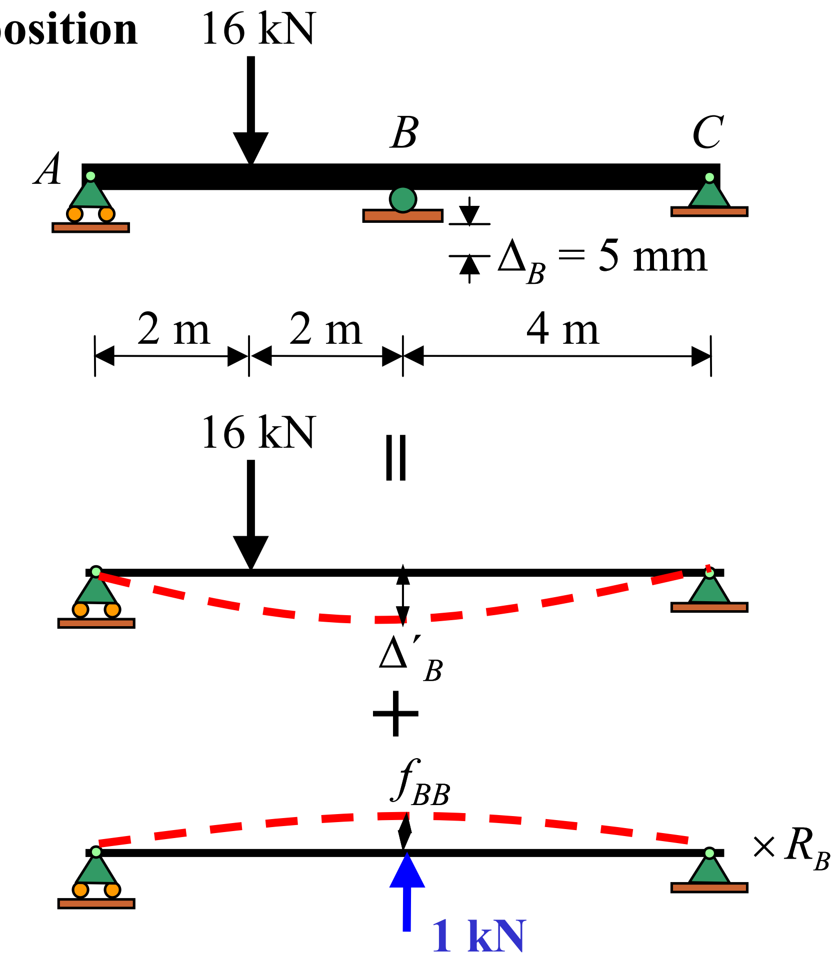
Example 9-2

Draw the quantitative Shear and moment diagram and the qualitative deflected curve for the beam shown below.The support at *B* settles 5 mm.
Take E = 200 GPa, I = 60(10^6) mm^4.



SOLUTION **Use compatibility of *displacement* to obain reaction**

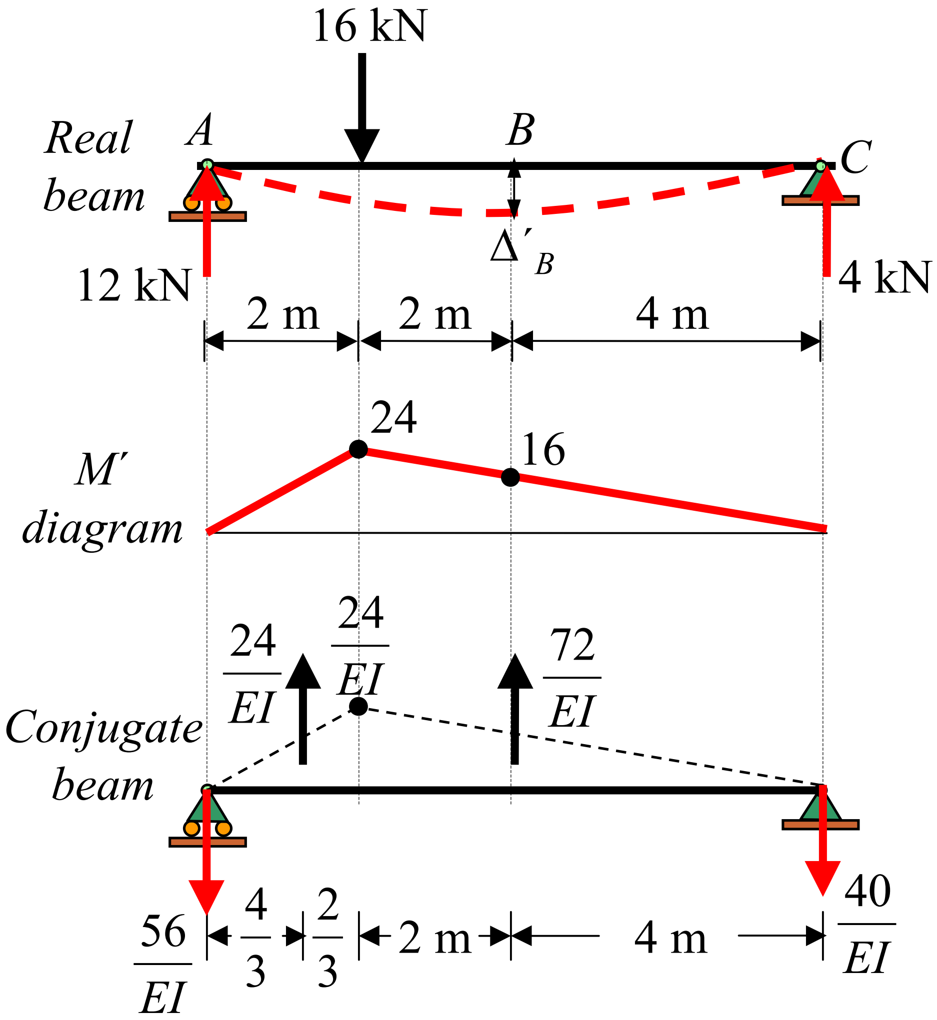
- **Principle of superposition**

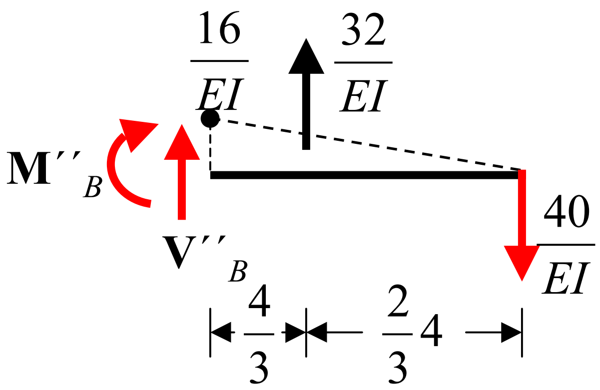


Compatibility equation :

$$\Delta_B = -0.005m = \Delta'_B + f_{BB}R_B \qquad \text{-----(1)}$$

• **Use conjugate beam method for Δ'_B**

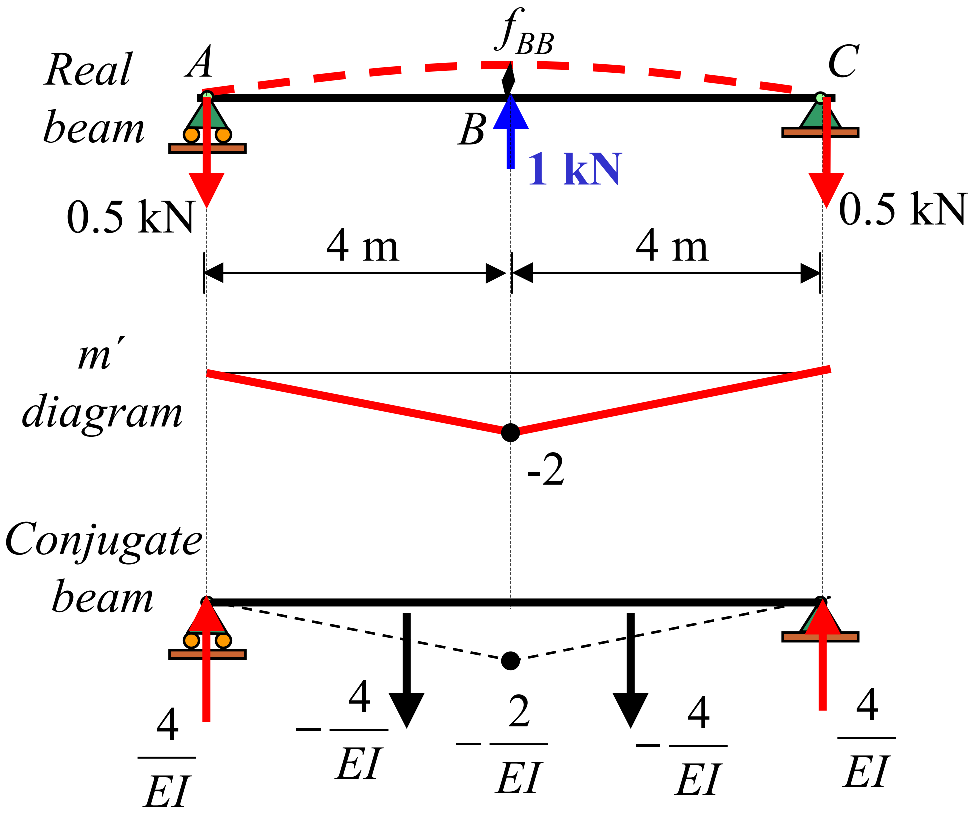


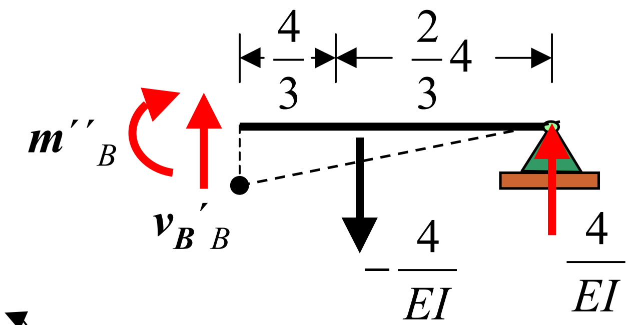


$$\curvearrowright + \ \Sigma M_B = 0: \ -M''_B + \frac{32}{EI}\left(\frac{4}{3}\right) - \frac{40}{EI}(4) = 0$$

$$\Delta'_B = M''_B = -\frac{117.33}{EI}, \downarrow$$

• **Use conjugate beam method for** f_{BB}





$$+\circlearrowright\ \Sigma M_B = 0:$$

$$-m_B'' - \frac{4}{EI}\left(\frac{4}{3}\right) + \frac{4}{EI}(4) = 0$$

$$f_{BB} = m_B'' = \frac{10.67}{EI}, \uparrow$$

- **Substitute** Δ'_B and f_{BB} in Eq. (1): $\Delta_B = -0.005\,m = \Delta'_B + f_{BB}R_B$

$$+\uparrow: \quad -0.005 = -\frac{117.33}{EI} + \frac{10.67}{EI}R_B$$

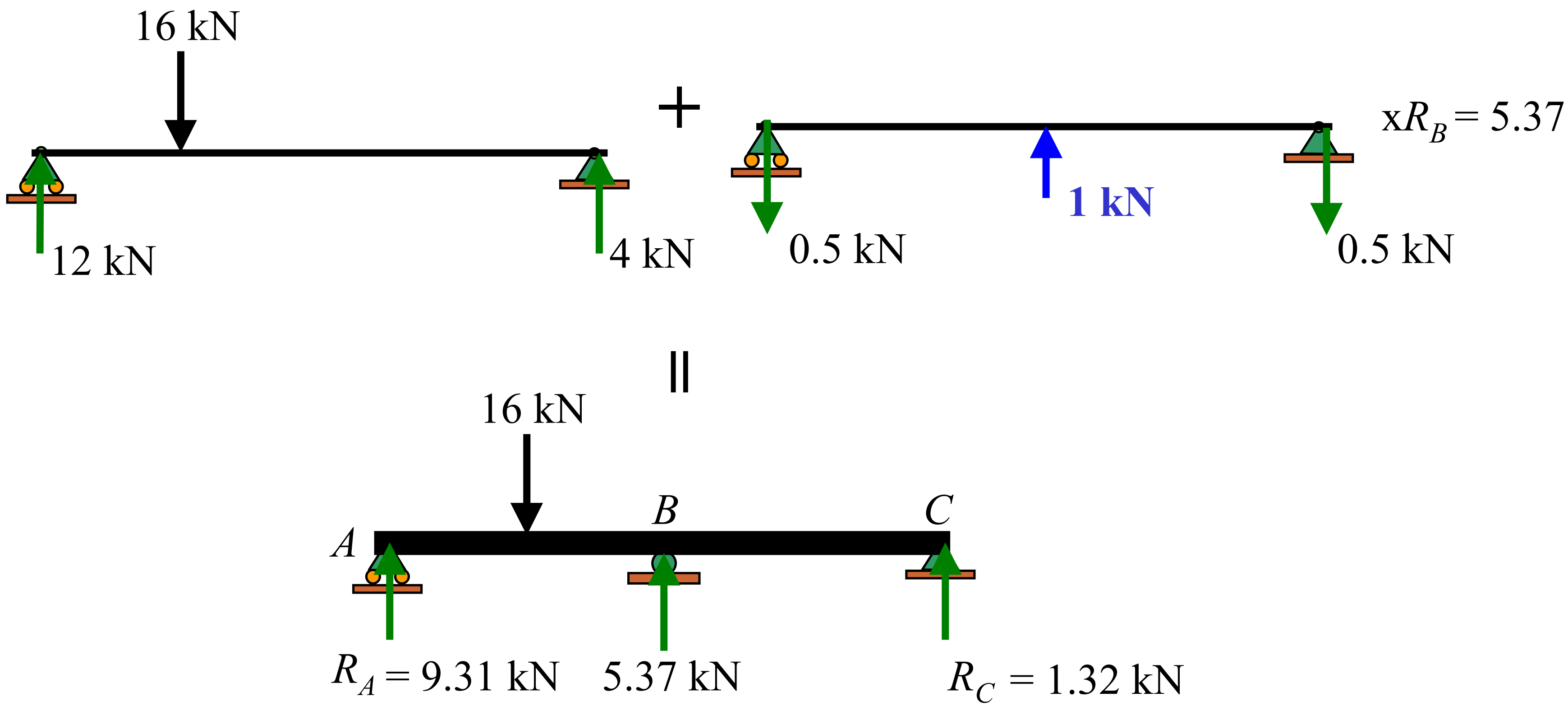
$$(-0.005)EI = -117.33 + 10.67R_B$$

$$(-0.005)(200\times 60) = -117.33 + 10.67R_B$$

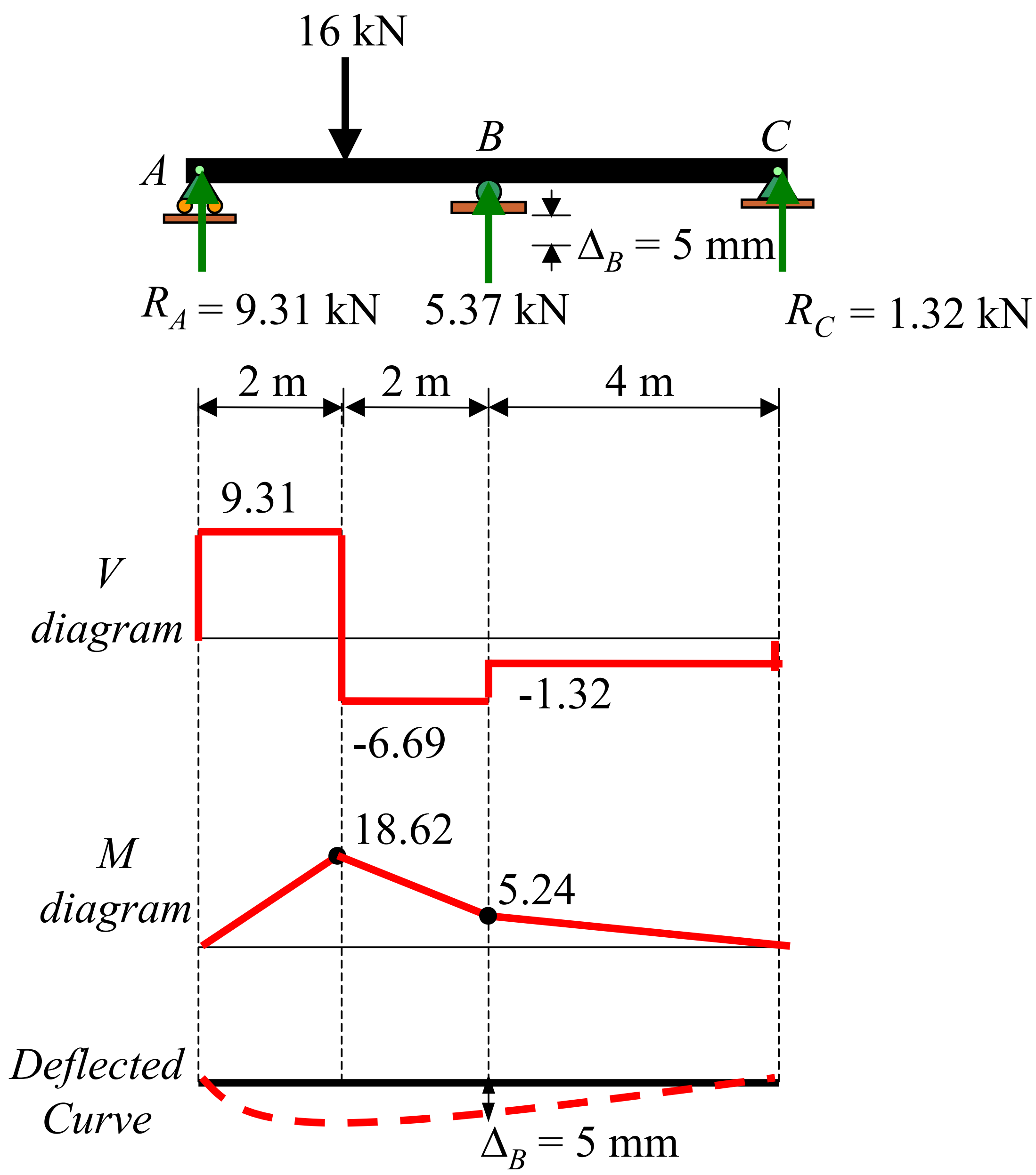
$$R_B = 5.37 \text{ kN}, \uparrow$$

$$\Delta'_B = M''_B = -\frac{117.33}{EI}, \downarrow$$

$$f_{BB} = m_B'' = \frac{10.67}{EI}, \uparrow$$

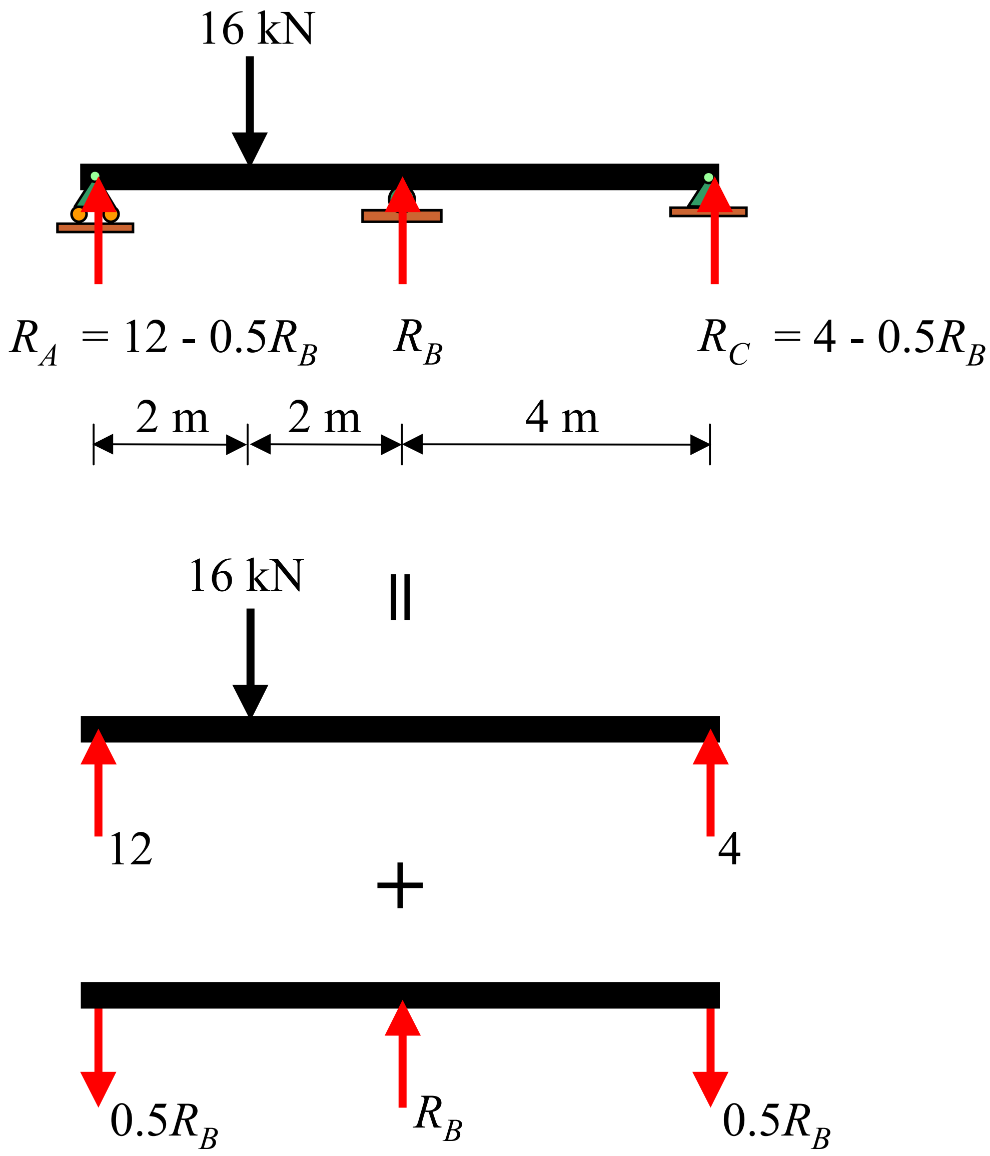


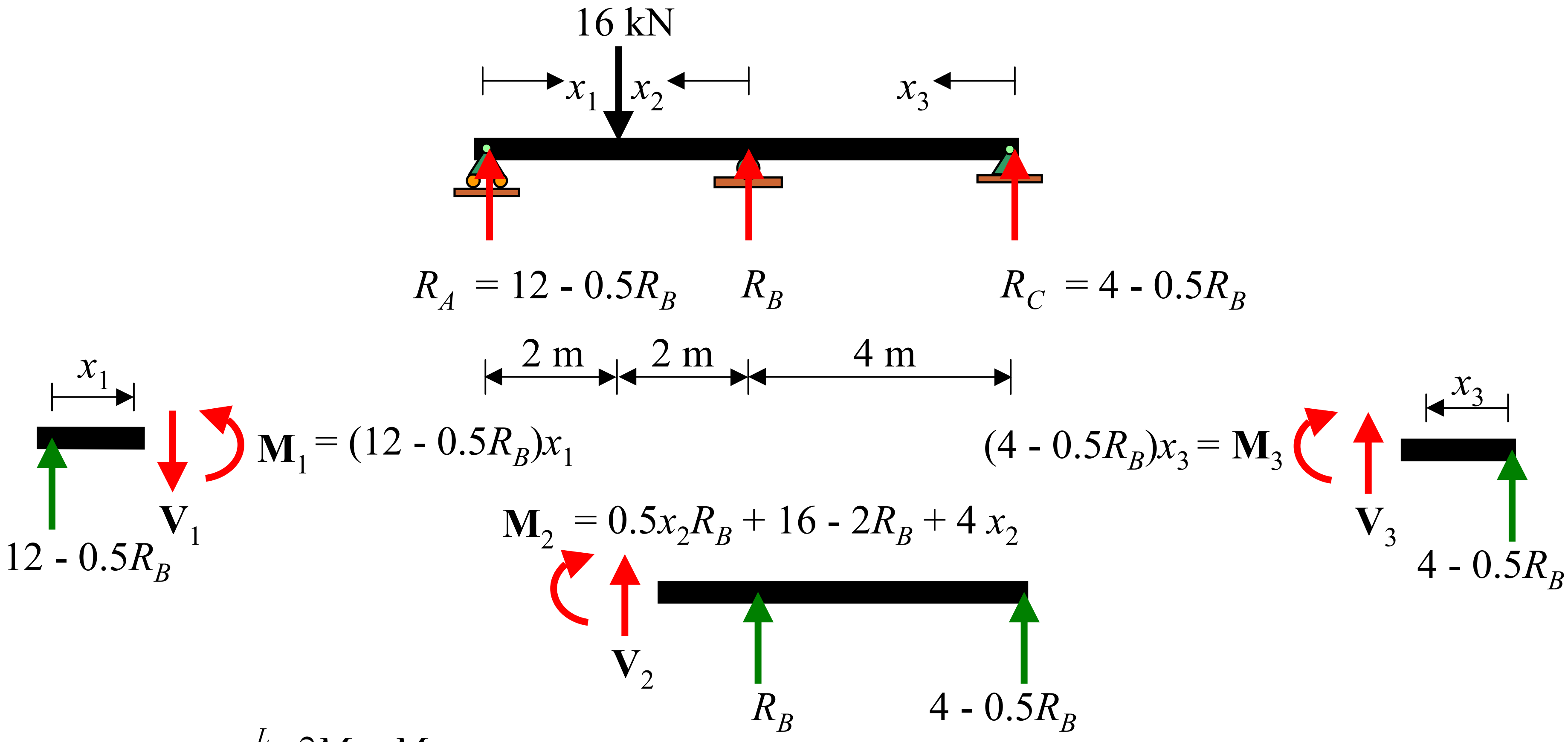
• **Quantitative shear and bending diagram and qualitative deflected curve**



Or use Castigliano least work method

- **Principle of superposition**





$$\Delta_B = -0.005 = \int_0^L \left(\frac{\partial M_i}{\partial R_B}\right)\frac{M_i}{EI}\,dx$$

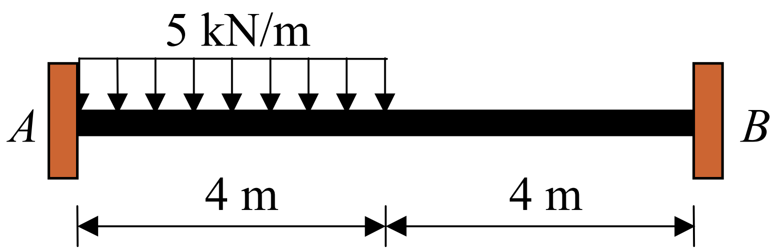
$$-0.005 = \frac{1}{EI}\int_0^2 (-0.5x_1)(12x_1 - 0.5x_1R_B)\,dx_1 + \frac{1}{EI}\int_0^2 (0.5x_2 - 2)(0.5x_2R_B + 16 - 2R_B + 4x_2)\,dx_2$$

$$+\frac{1}{EI}\int_0^4 (-0.5x_3)(4x_3 - 0.5x_3R_B)\,dx_3$$

$$-0.005EI = -117.34 + 10.66R_B, \quad R_B = 5.38\ \text{kN}, \uparrow$$

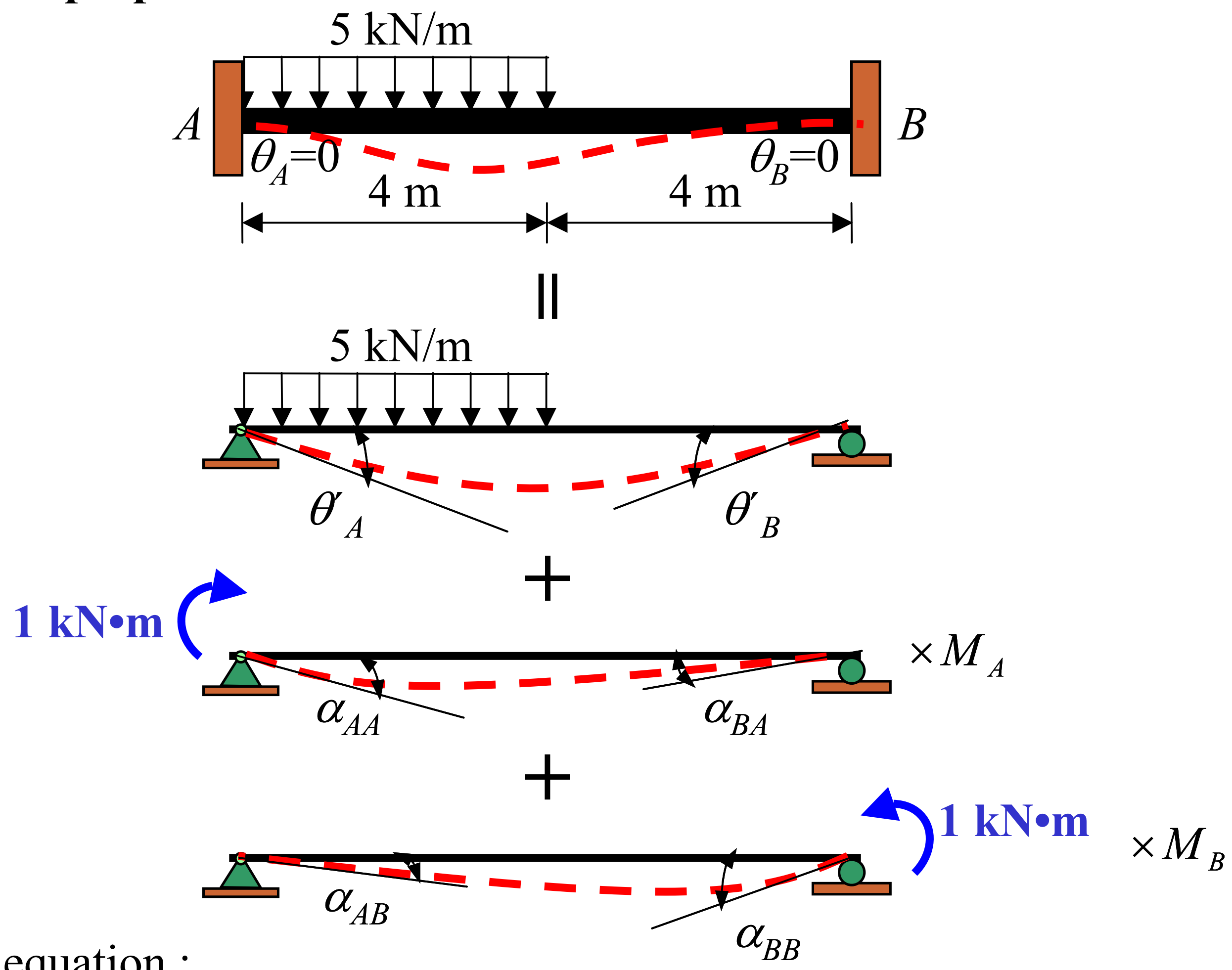
Example 9-3

Draw the quantitative Shear and moment diagram and the qualitative deflected curve for the beam shown below.*EI* is constant. Neglect the effects of axial load.



SOLUTION **Use compatibility of *displacement* to obtain reaction**

- **Principle of superposition**

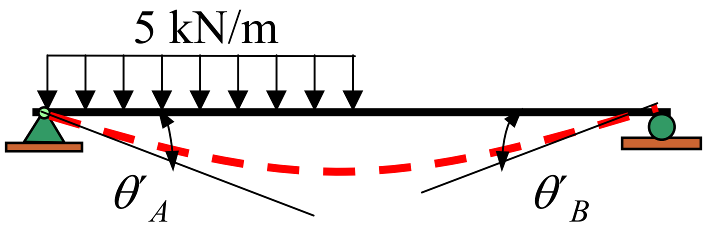


Compatibility equation :

$$\theta_A = 0 = \theta'_A + \alpha_{AA} M_A + \alpha_{AB} M_B \qquad \text{-----(1)}$$

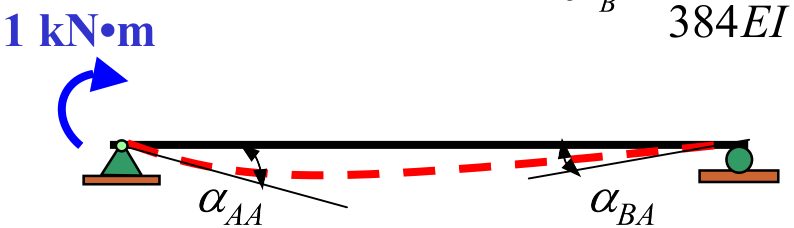
$$\theta_B = 0 = \theta'_B + \alpha_{BA} M_A + \alpha_{BB} M_B \qquad \text{-----(2)}$$

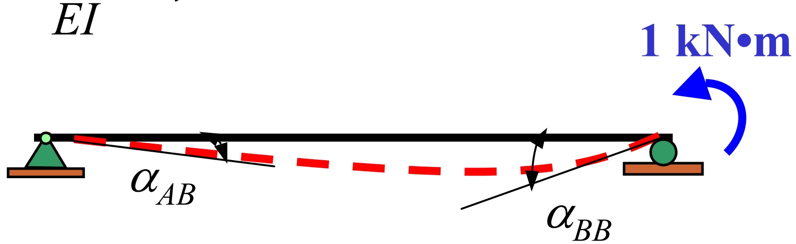
- **Use formulation:** θ'_A, θ'_B, α_{AA}, α_{BA}, α_{BB}, α_{AB},



$$\theta'_A = \frac{3wL^3}{128EI} = \frac{3(5)(8)^3}{128EI} = \frac{60}{EI}$$

$$\theta'_B = \frac{7wL^3}{384EI} = \frac{7(5)(8)^3}{384EI} = \frac{46.67}{EI}$$





$$\alpha_{AA} = \frac{M_oL}{3EI} = \frac{1(8)}{3EI} = \frac{2.67}{EI}$$

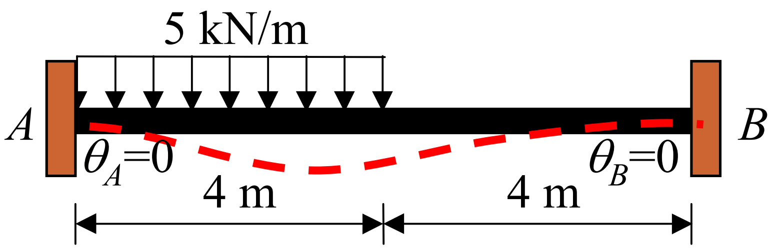
$$\alpha_{BA} = \frac{M_oL}{6EI} = \frac{1(8)}{6EI} = \frac{1.33}{EI}$$

$$\alpha_{BB} = \frac{M_oL}{3EI} = \frac{1(8)}{3EI} = \frac{2.67}{EI}$$

$$\alpha_{AB} = \frac{M_oL}{6EI} = \frac{1(8)}{6EI} = \frac{1.33}{EI}$$

Note : *Maxwell's theorem of reciprocal displacement is* $\alpha_{AB} = \alpha_{BA}$

- **Substitute** θ'_A, θ'_B, α_{AA}, α_{BA}, α_{BB}, α_{AB}, in Eq. (1) and (2)



$$\parallel$$

$$\theta_A = 0 = \theta'_A + \alpha_{AA} M_A + \alpha_{AB} M_B \qquad \text{-----(1)}$$

$$\theta_B = 0 = \theta'_B + \alpha_{BA} M_A + \alpha_{BB} M_B \qquad \text{-----(2)}$$

$$+ \qquad 0 = \frac{60}{\cancel{EI}} + \left(\frac{2.67}{\cancel{EI}}\right) M_A + \left(\frac{1.33}{\cancel{EI}}\right) M_B$$

$$+ \qquad 0 = \frac{46.67}{\cancel{EI}} + \left(\frac{1.33}{\cancel{EI}}\right) M_A + \left(\frac{2.67}{\cancel{EI}}\right) M_B$$

Solving these equations simultaneously, we haave

$$M_A = \text{-}18.31 \text{ kN•m}, \qquad +$$

$$M_B = \text{-}8.36 \text{ kN•m}, \qquad +$$

$$\theta'_A = \frac{60}{EI}$$

$$\alpha_{AA} = \frac{2.67}{EI}$$

$$\alpha_{AB} = \frac{1.33}{EI}$$

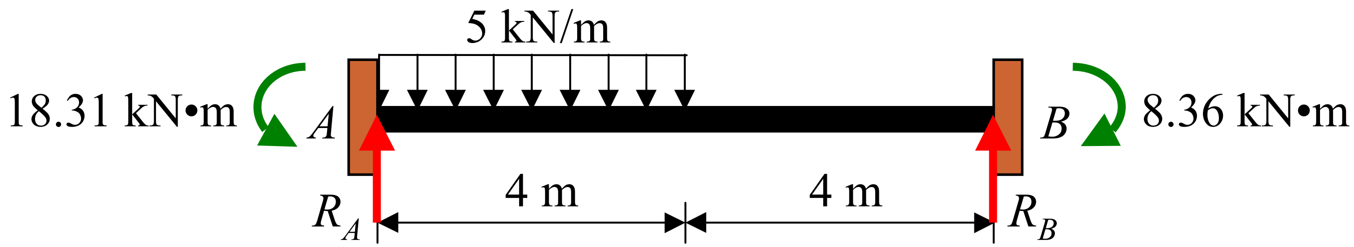
$$\theta'_B = \frac{46.67}{EI}$$

$$\alpha_{BA} = \frac{1.33}{EI}$$

$$\alpha_{BB} = \frac{2.67}{EI}$$

M_A = -18.31 kN•m,

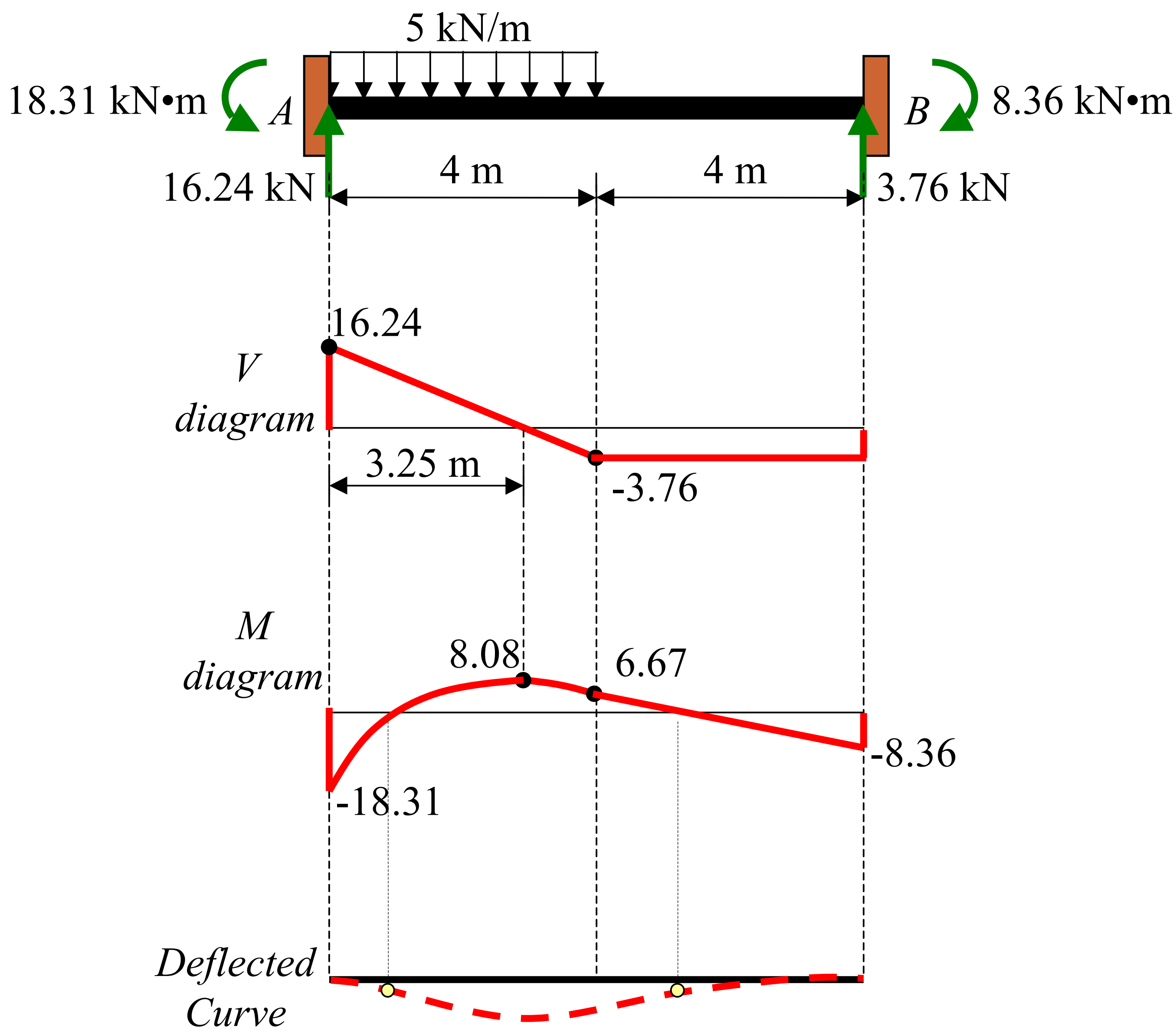
M_B = -8.36 kN•m,



$+\circlearrowright \; \Sigma M_A = 0: \quad 18.31 - 20(2) + R_B(8) - 8.36 = 0, \qquad R_B = 3.76\text{ kN}, \uparrow$

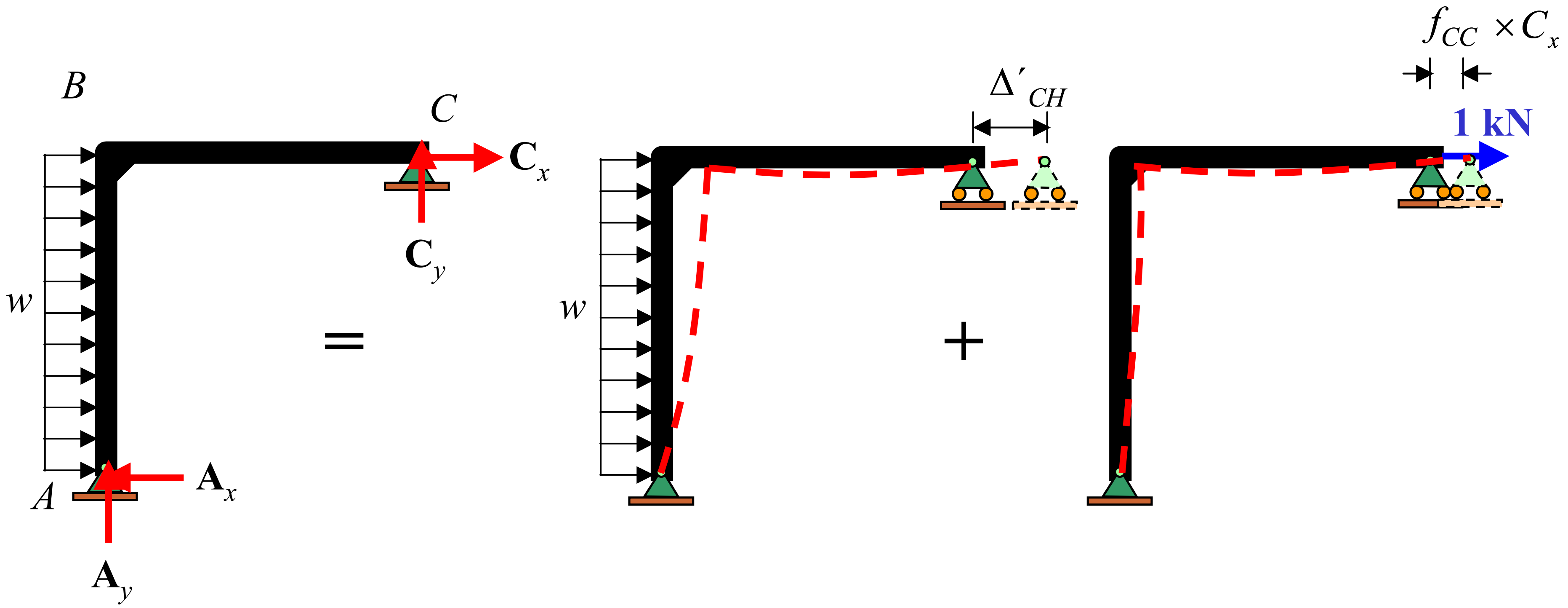
$+\uparrow \; \Sigma F_y = 0: \quad +R_A - 20 + \cancel{R_B}^{\,3.76} = 0, \qquad R_a = 16.24\text{ kN}, \uparrow$

• **Quantitative shear and bending diagram and qualitative deflected curve**



Force Method of Analysis : Frames

- **Principle of superposition**

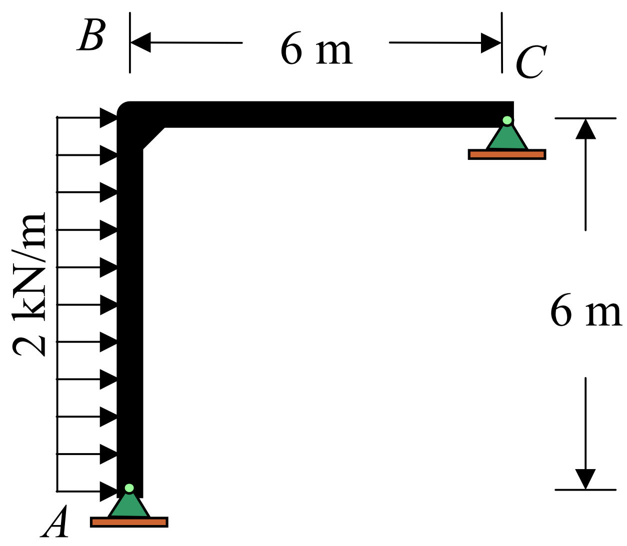


Compatibility equation :

$$\Delta_{CH} = 0 = \Delta'_{CH} + f_{CC}C_x$$

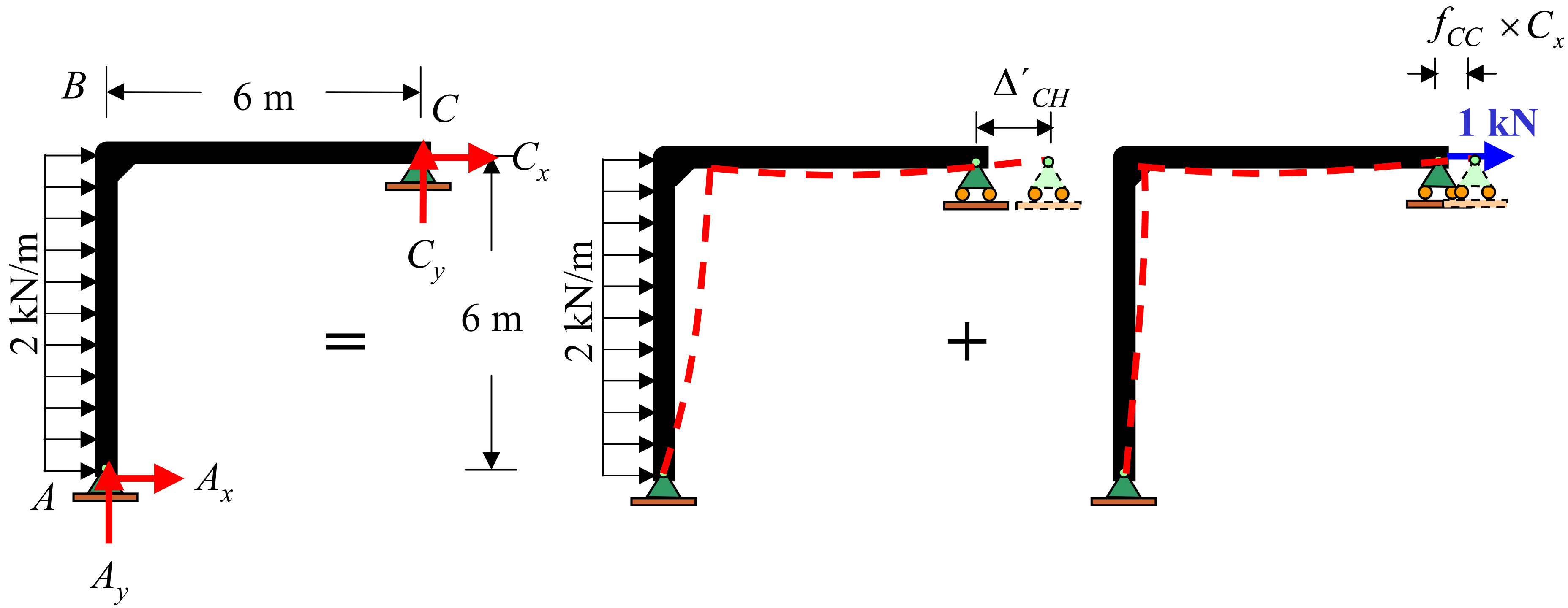
Example 9-4

Draw the quantitative Shear and moment diagram and the qualitative deflected curve for the Frame shown below.*EI* is constant.



<u>SOLUTION</u> **Use compatibility of *displacement* to obtain reaction**

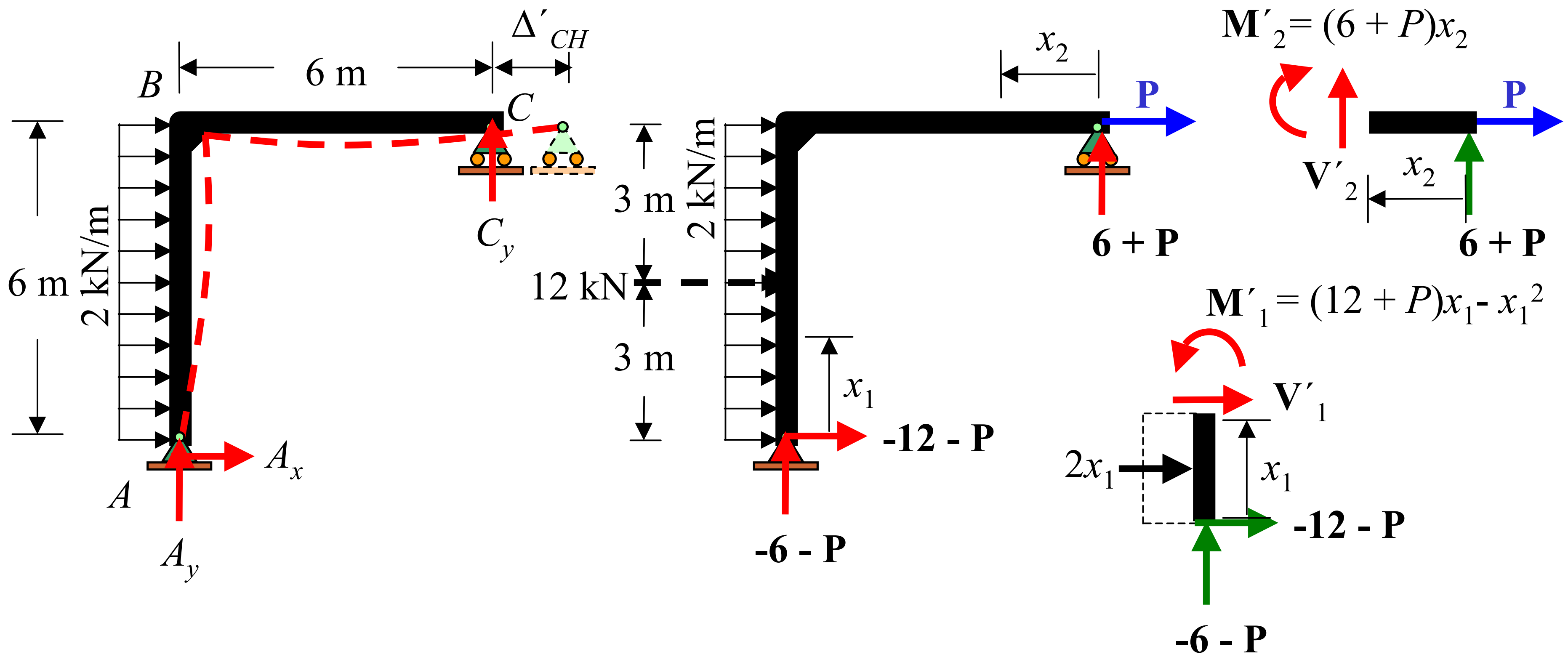
- **Principle of superposition**



Compatibility equation :

$$\Delta_{CH} = 0 = \Delta'_{CH} + f_{CC} C_x \qquad \text{-----(1)}$$

• **Use Castigliano's method for Δ'_{CH}**

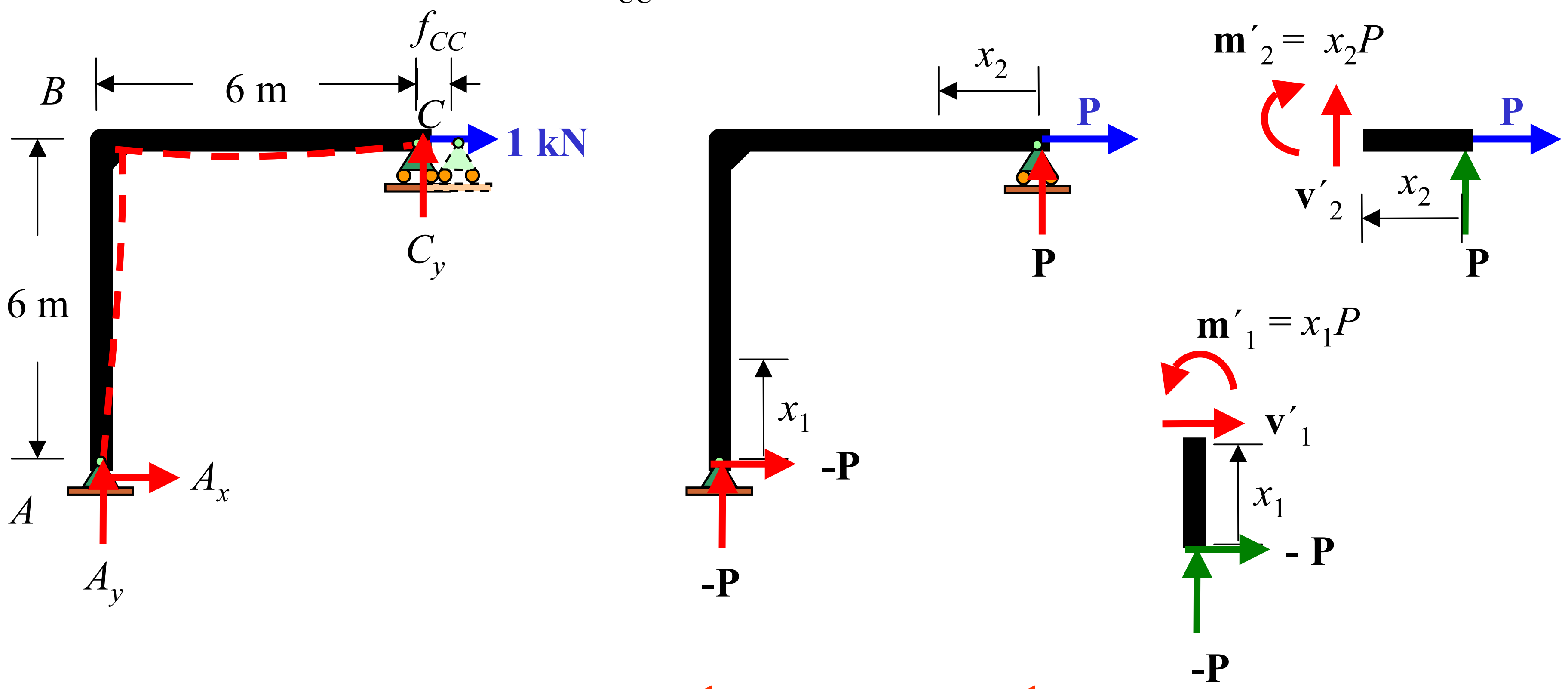


$$\Delta'_{CH} = \int_0^L \left(\frac{\partial M'_i}{\partial P}\right)\frac{M'_i}{EI}dx = \frac{1}{EI}\int_0^6 (x_1)(12x_1 + x_1\cancelto{0}{P} - x_1^{2})dx_1 + \frac{1}{EI}\int_0^6 (x_2)(6x_2 + x_2\cancelto{0}{P})dx_2$$

$$= \frac{1}{EI}\int_0^6 (12x_1^{2} - x_1^{3})dx_1 + \frac{1}{EI}\int_0^6 (6x_2^{2})dx_2$$

$$= \frac{1}{EI}\left(\frac{12x_1^{3}}{3} - \frac{x_1^{4}}{4}\right)\Bigg|_0^6 + \frac{1}{EI}\left(\frac{6x_2^{3}}{3}\right)\Bigg|_0^6 = \frac{972}{EI}, \rightarrow$$

• **Use Castigliano's method for f_{CC}**



$$f_{CC} = \int_0^L \left(\frac{\partial m'_i}{\partial P}\right)\frac{m'_i}{EI}\,dx \quad = \frac{1}{EI}\int_0^6 (x_1)(x_1 \overset{1}{P})dx_1 + \frac{1}{EI}\int_0^6 (x_2)(x_2 \overset{1}{P})dx_2$$

$$= \frac{1}{EI}\left(\frac{x_1^{\,3}}{3}\right)\Big|_0^6 + \frac{1}{EI}\left(\frac{x_2^{\,3}}{3}\right)\Big|_0^6 = \frac{144}{EI}, \rightarrow$$

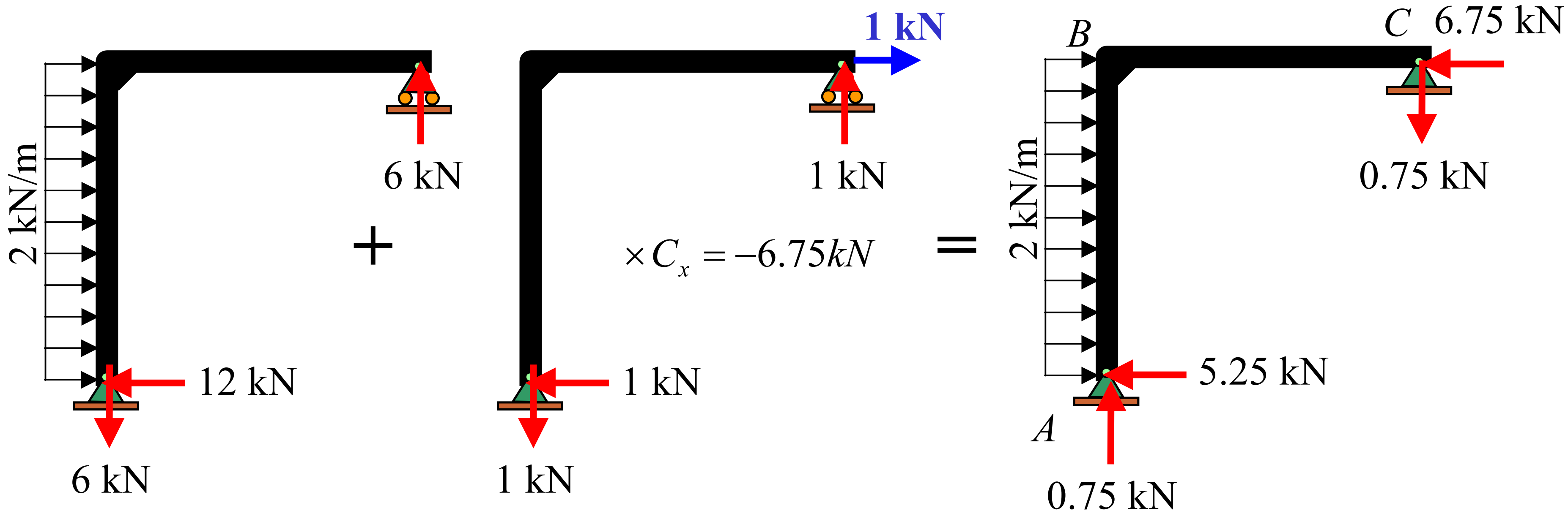
• **Substitute** Δ'_{CH} and f_{CC} in Eq. (1) $\Delta_{CH} = 0 = \Delta'_{CH} + f_{CC} C_x$ -----(1)

$$\Delta_{CH} = \frac{972}{EI}, \rightarrow$$

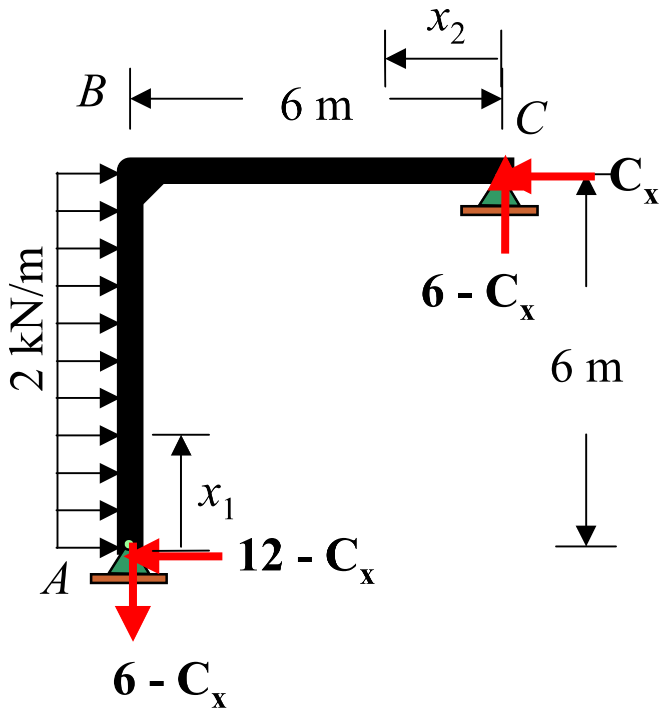
$$f_{cc} = \frac{144}{EI}, \rightarrow$$

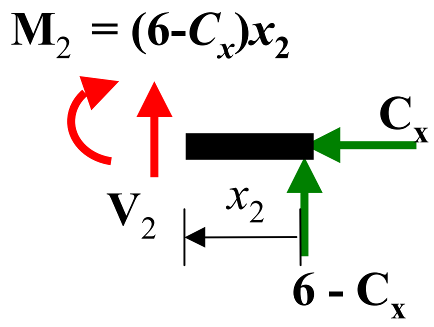
$$\xrightarrow{+}: \qquad 0 = \frac{972}{\cancel{EI}} + \frac{144}{\cancel{EI}} C_x$$

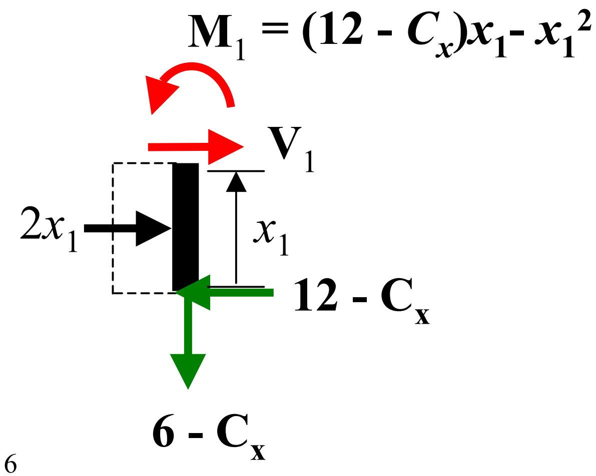
$$C_x = -6.75 \text{ kN}, \quad \leftarrow$$



Or use Castigliano least work method:







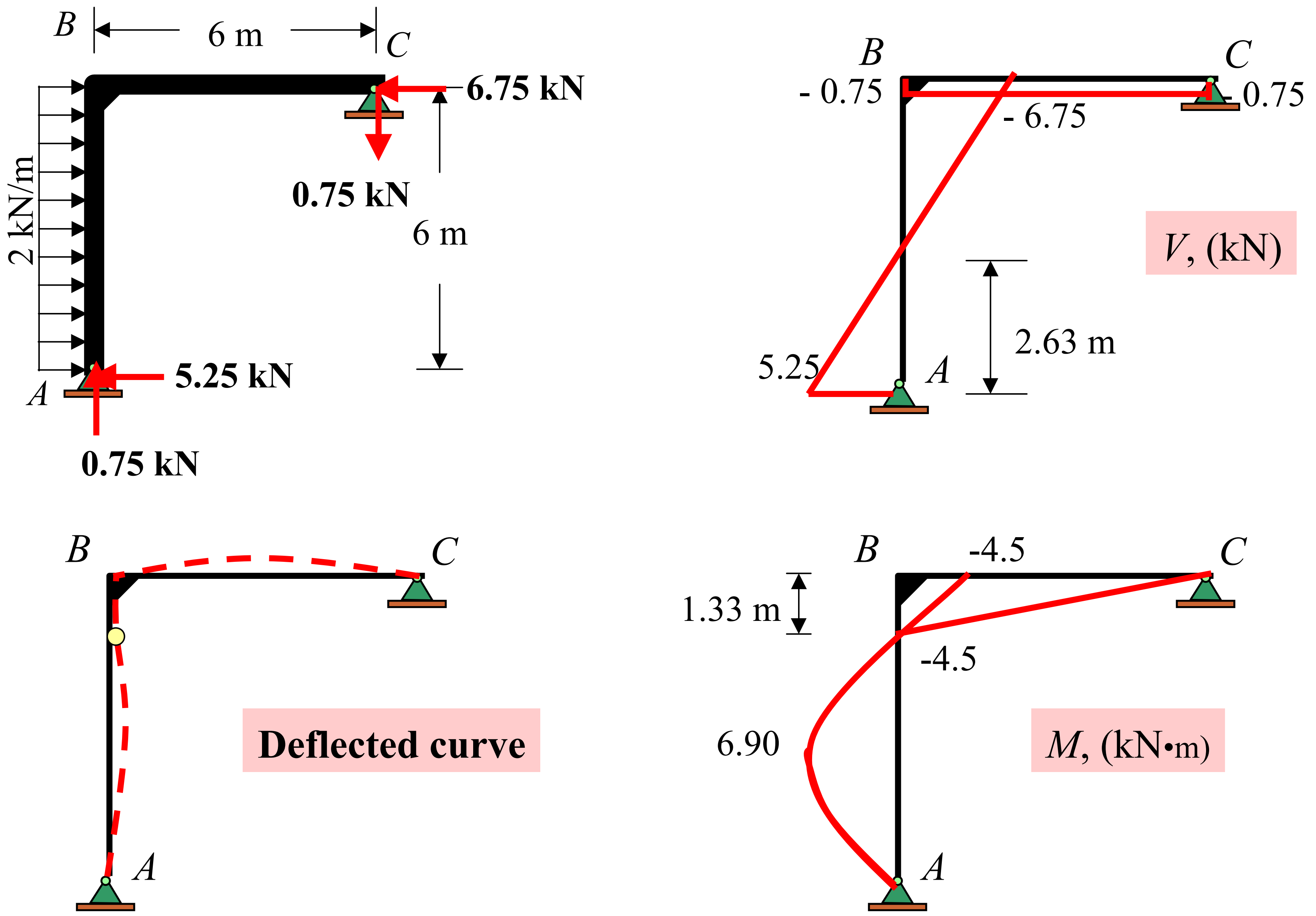
$$\frac{\partial U_i}{\partial C_x} = \int_0^L \left(\frac{\partial M_i}{\partial C_x}\right)\frac{M_i}{EI}\,dx = \Delta_{CH} = 0$$

$$0 = \frac{1}{\cancel{EI}}\int_0^6 (-x_1)(12x_1 - C_x x_1 - x_1^2)\,dx_1 + \frac{1}{\cancel{EI}}\int_0^6 (-x_2)(6x_2 - C_x x_2)\,dx_2$$

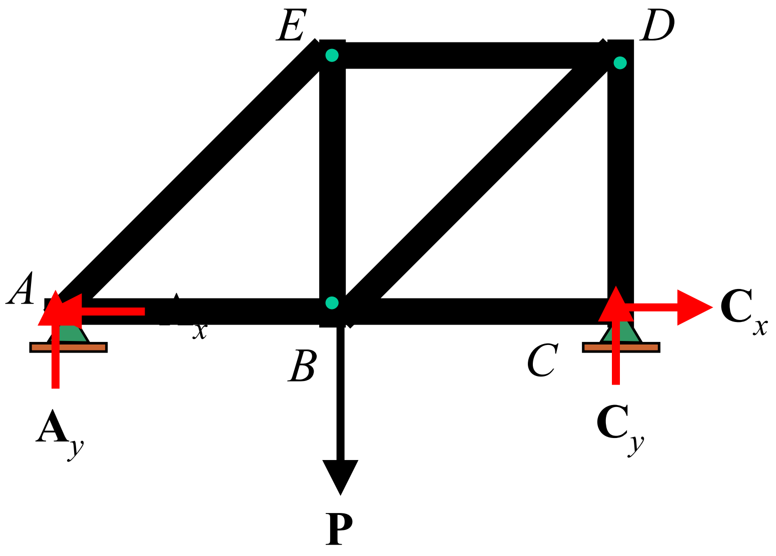
$$0 = \left(-\frac{12x_1^3}{3} + \frac{C_x x_1^3}{3} + \frac{x_1^4}{4}\right)\Bigg|_0^6 + \left(-\frac{6x_2^3}{3} + \frac{C_x x_2^3}{3}\right)\Bigg|_0^6$$

$$0 = -972 + 144C_x,\ \ C_x = 6.75\ \text{kN},\ \leftarrow$$

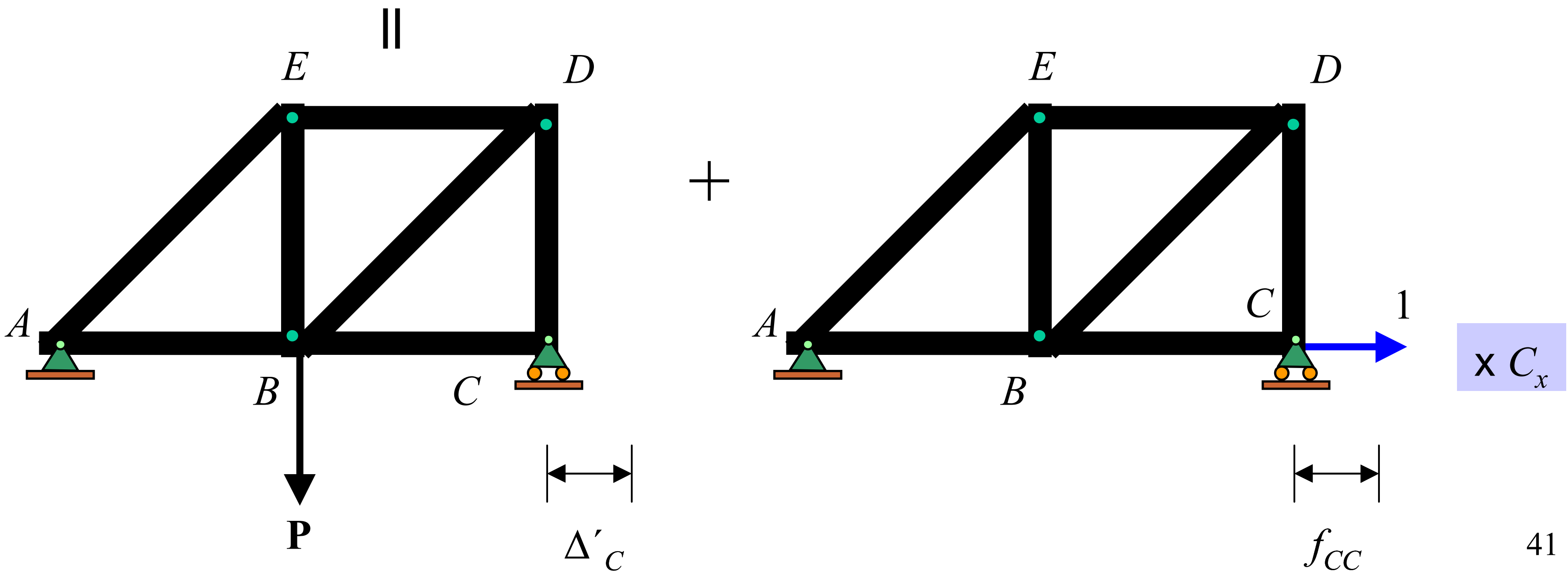
• **Quantitative shear and bending diagram and qualitative deflected curve**



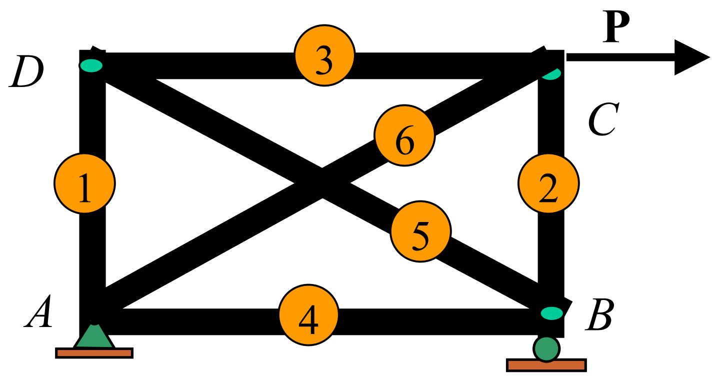
Force Method of Analysis : Truss (Externally indeterminate)



$$\Delta'_{CH} + f_{CC} C_x = \Delta_{CH} = 0$$

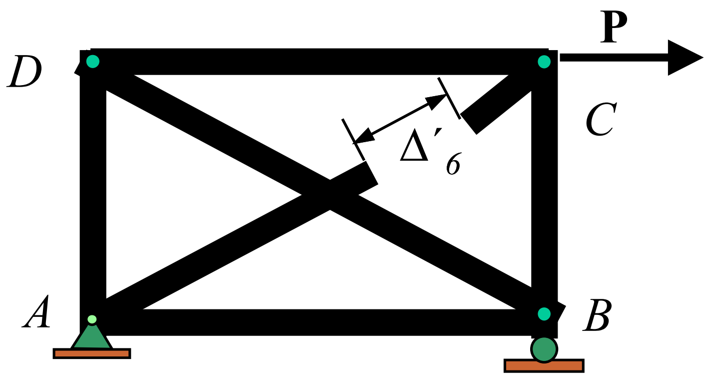


Truss (Internally indeterminate)



$$\Delta'_6 + f_{66} F_6 = \Delta_6 = 0$$

=



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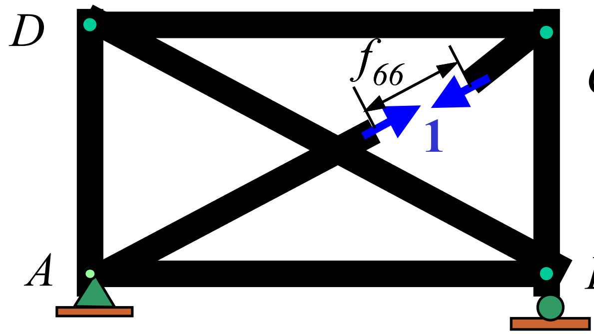
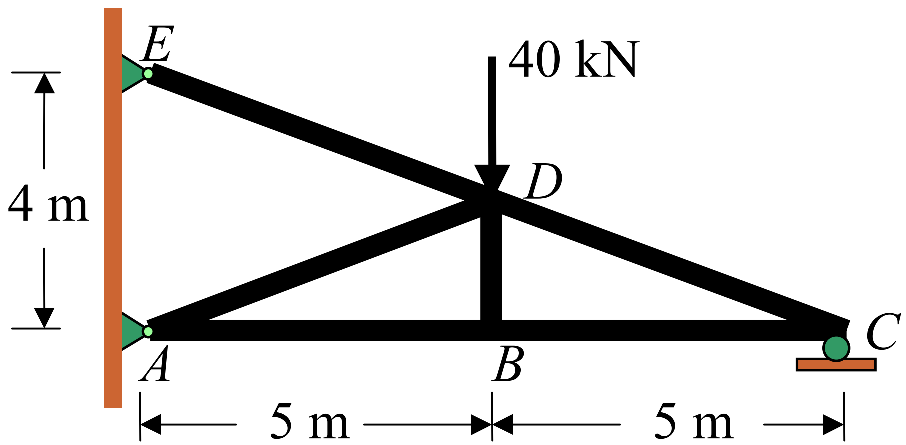


xF_6

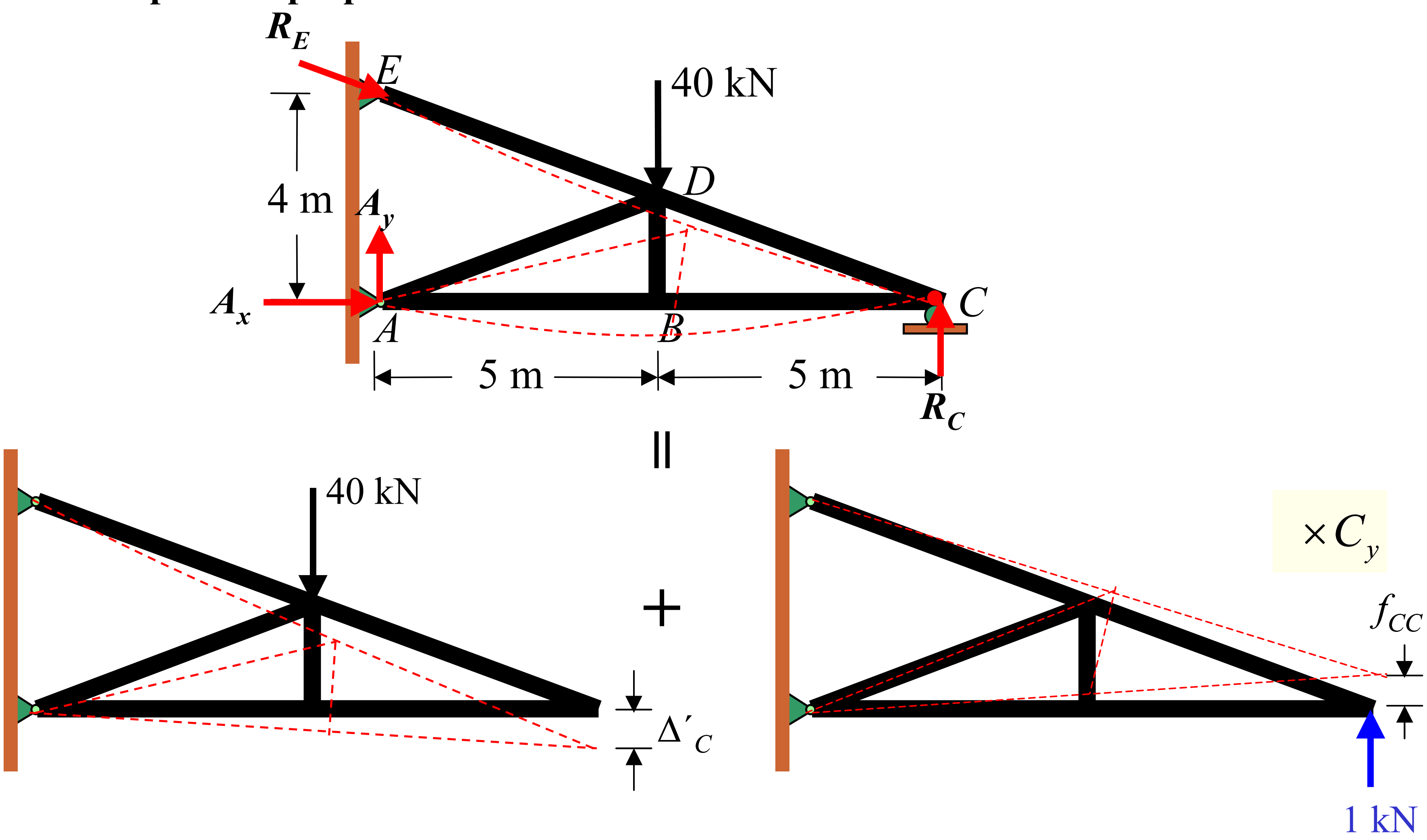
Example 9-5

Determine the reaction at support A, C, E and all the member forces. Take E = 200 GPa and A = 500. mm^2.



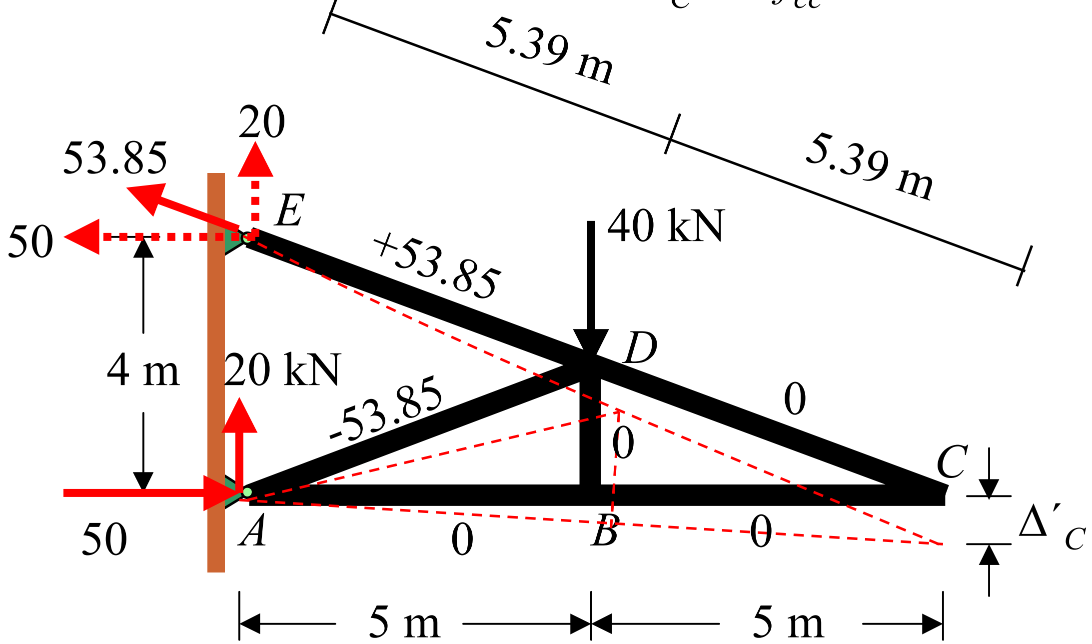
SOLUTION **Use compatibility of *displacement* to obtain reaction**

- **Principle of superposition**

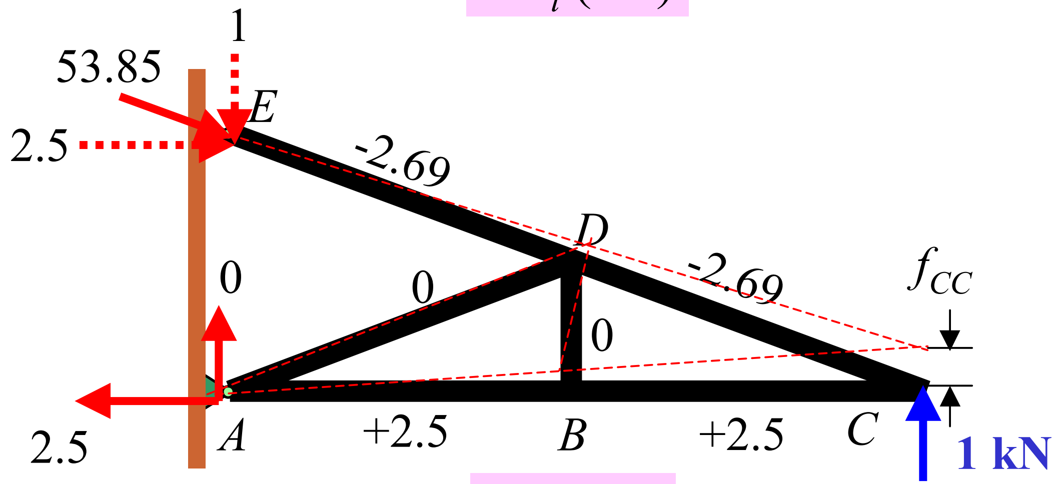


Compatibility equation : $\Delta_C = 0 = \Delta'_C + f_{CC} R_C$ -----(1)

- **Use unit load method for Δ'_C and f_{cc}**



N'_i (kN)



n'_i (kN)

$$\Delta'_C = \frac{\Sigma\, n'_i N'_i L_i}{A_i E_i}$$

$$= \frac{(53.85)(-2.69)(5.38)}{(200\text{x}10^6)(500\text{x}10^{-6})}$$

$$= -\ 7.81 \text{ mm}, \downarrow$$

$$f_{CC} = \frac{\Sigma\, n_i' n'_i L_i}{A_i E_i}$$

$$= \frac{2(-2.69)^2(5.385)}{(200\text{x}10^6)(500\text{x}10^{-6})}$$

$$+ \frac{2(2.5)^2(5)}{(200\text{x}10^6)(500\text{x}10^{-6})}$$

$$= 1.41 \text{ mm}, \uparrow$$

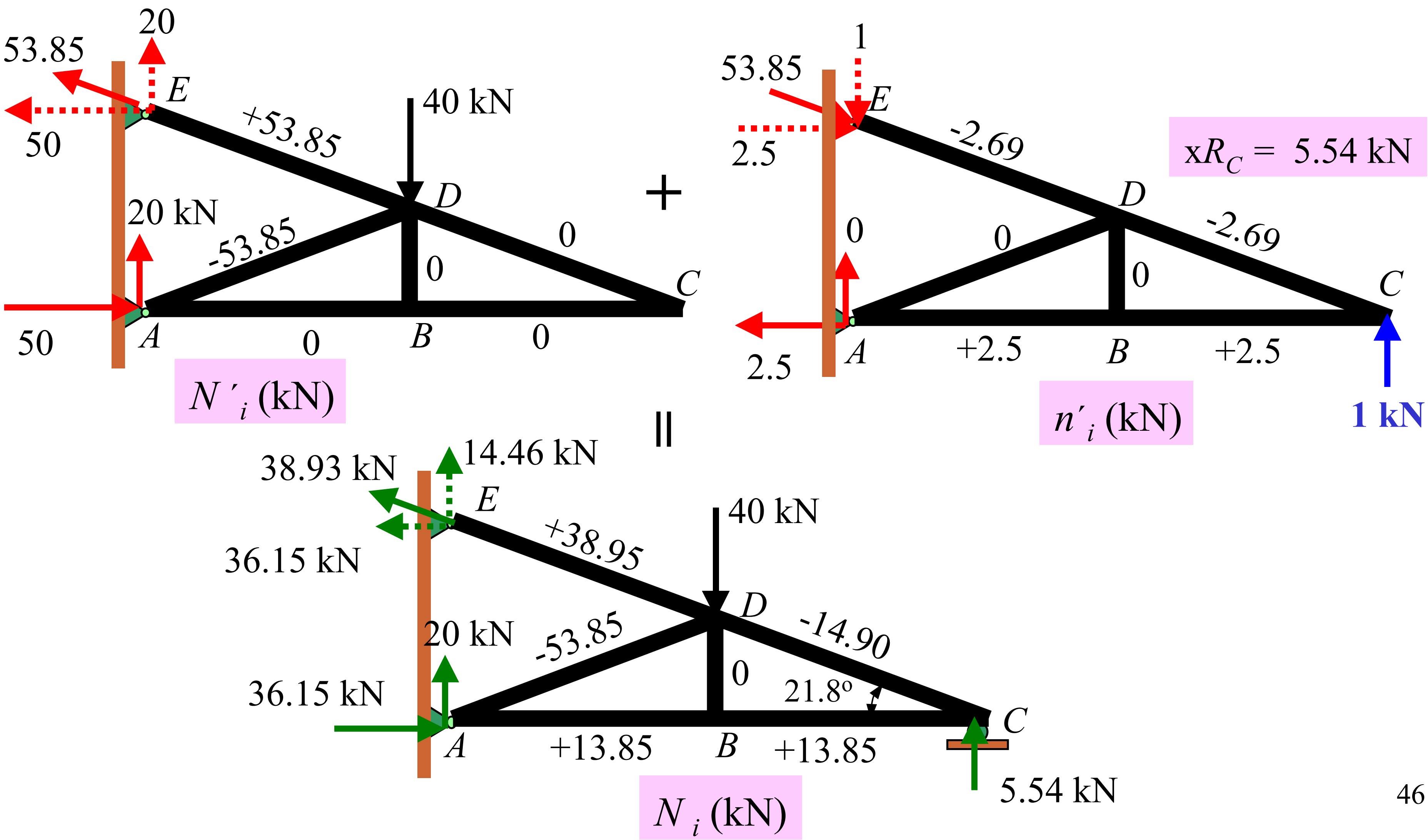
- **Substitute** Δ'_{Cv} and f_{CC} in Eq. (1): $\Delta_C = 0 = \Delta'_C + f_{CC} R_C$

$$+\uparrow: -7.81 + 1.41 R_C = 0$$

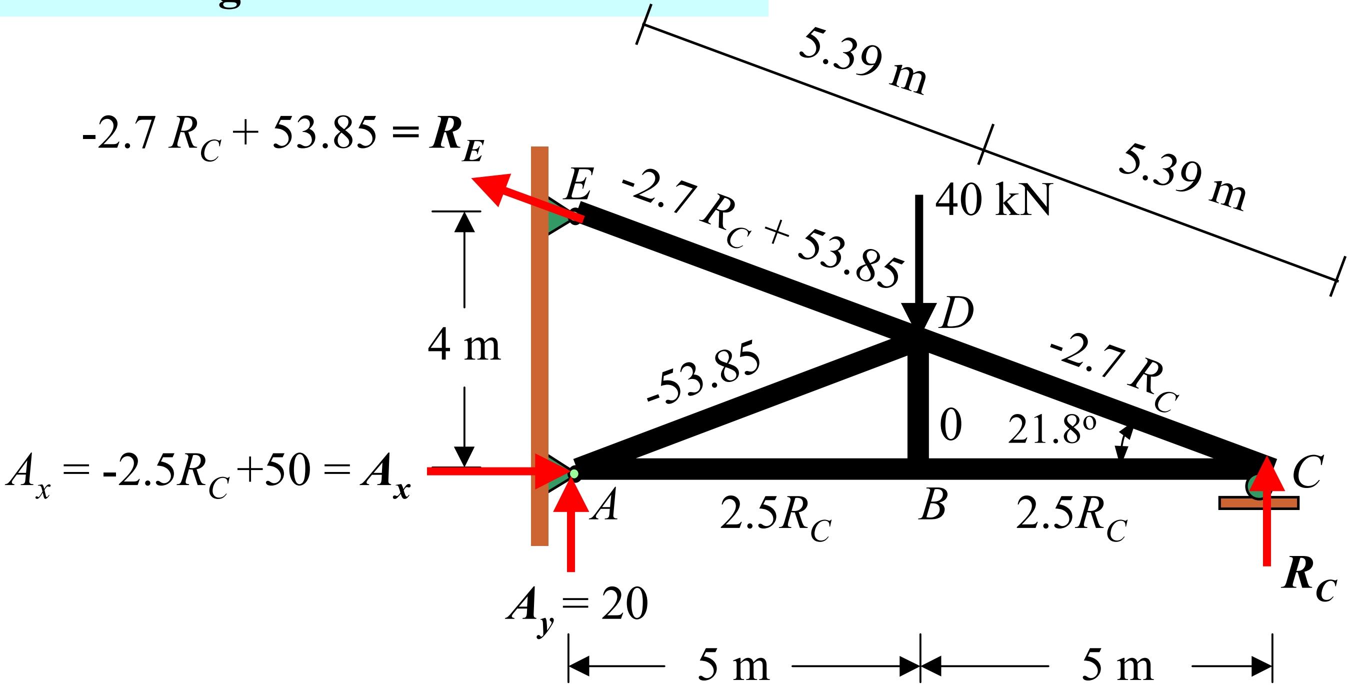
$$R_C = 5.54\ kN, \uparrow$$

$\Delta'_C = 7.81\ mm, \downarrow$

$f_{CC} = 1.41\ mm, \uparrow$



Or use Castigliano least work method:



Castigliano's Theorem of Least Work :

$$\Delta_{CV} = 0 = \sum \left(\frac{\partial N_i}{\partial R_C}\right)\frac{N_i L_i}{AE}$$

$$0 = \frac{1}{AE}[(-2.7)(-2.7R_C + 53.85)(5.39) + (-2.7)(-2.7R_C)(5.39) + 0 + 0 + 2[(2.5)(2.5R_C)(5)]]$$

$$0 = 39.3R_C - 783.68 + 39.3R_C + 62.5R_C$$

$R_C =$ 5.55 kN, $\uparrow$

Example 9-6

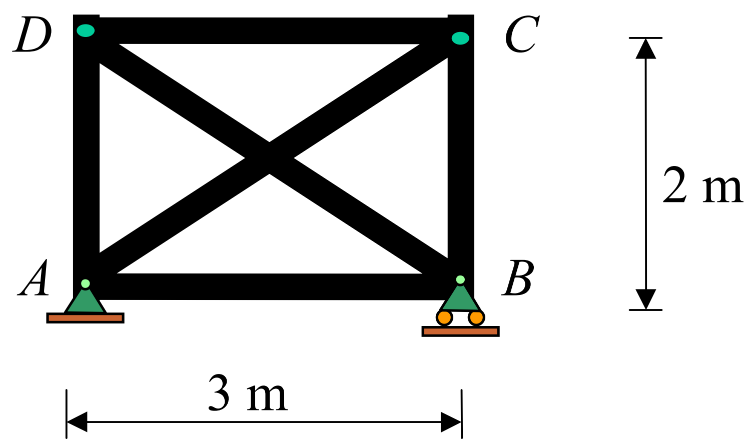
Determine the force in all member of the truss shown :

(a) If the horizontal force $P = 6$ kN is applied at joint C.

(b) If the turnbuckle on member AC is used to shorten the member by 1 mm.

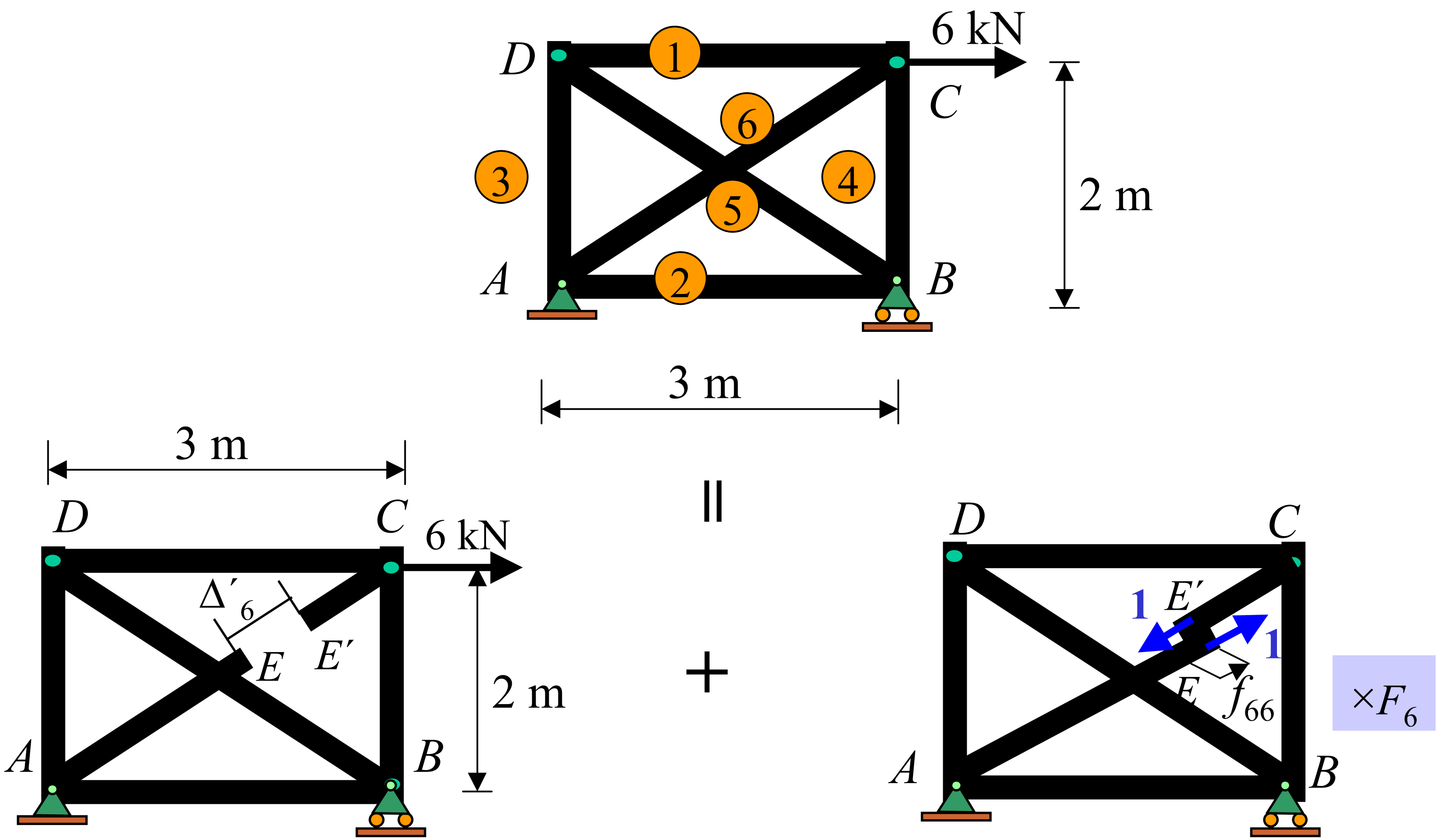
(c) If (a) and (b) are both accounted.

Each bar has a cross-sectional area of 500 mm^2 and $E = 200$ GPa.



<u>SOLUTION</u> ***Part (a) :*** *If the horizontal force P = 6 kN is applied at joint C.*

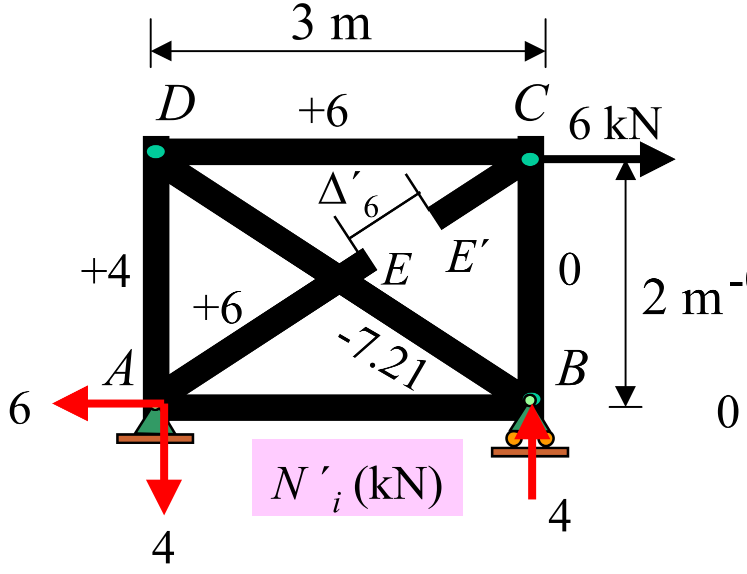
- **Principle of superposition**



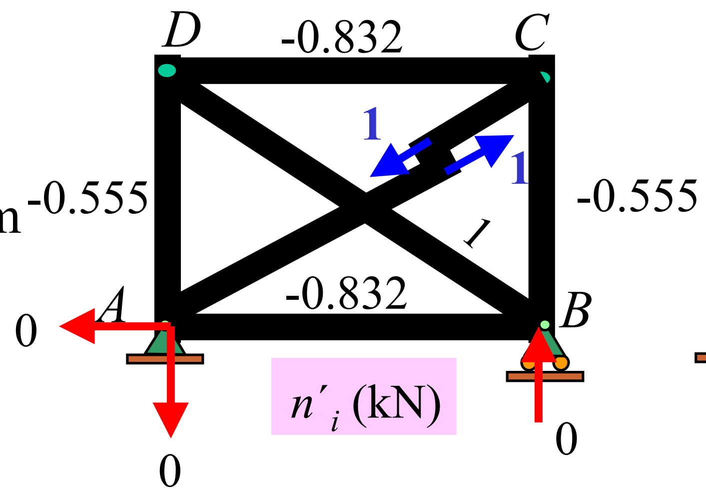
Compatibility equation : $\Delta'_6 + f_{66} F_6 = 0$ ----------(1)

Note : $AE + E'C = L$

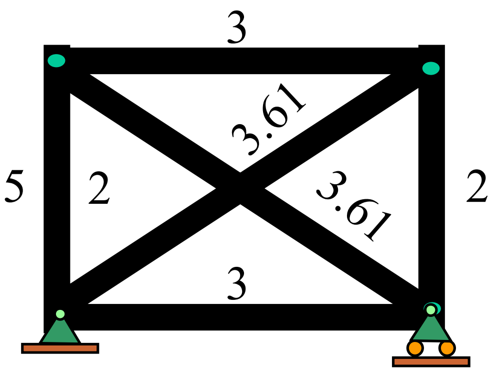
- **Use unit load method for** Δ'_6 and f_{66}



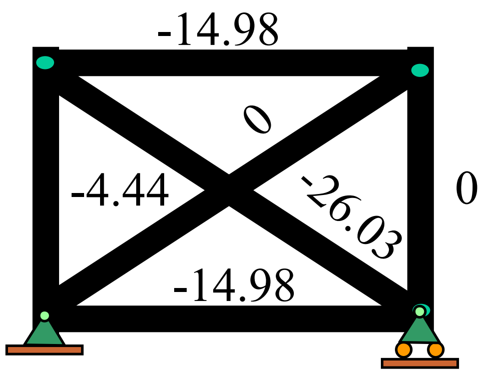
N'_i (kN)



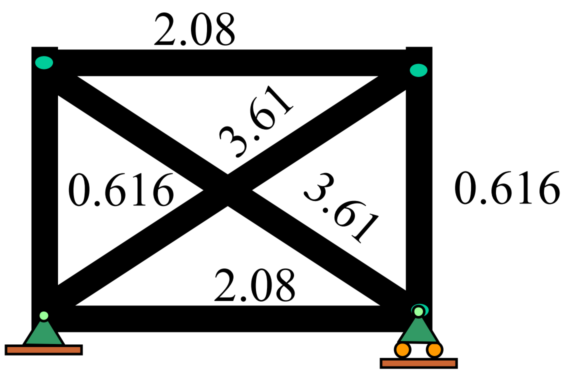
n'_i (kN)



L_i (m)



$n'_i N'_i L_i$ (kN²•m)



$n'^2_i L_i$ (kN²•m)

$$\Delta'_6 = \sum \frac{n'_i \, N'_i \, L_i}{A_i E_i}$$

$$= \frac{1}{AE}[-4.44 - 26.03 - 2(14.98)]$$

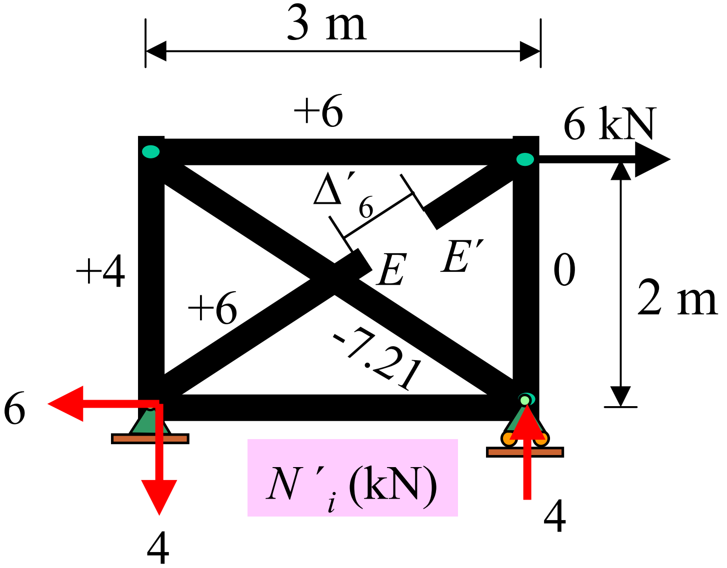
$$= \frac{-60.43}{AE}$$

$$f_{66} = \sum \frac{n'^{\,2}_i \, L_i}{A_i E_i} = \frac{1}{AE}[2(0.616) + 2(2.08) + 2(3.61)] = \frac{12.61}{AE}$$

• **Substitute** Δ'_6 and f_{66} in Eq. (1)

$$-\frac{60.43}{\cancel{AE}}+\frac{12.61}{\cancel{AE}}(F_6)=0$$

$$F_6 = 4.80 \text{ kN, (T)}$$



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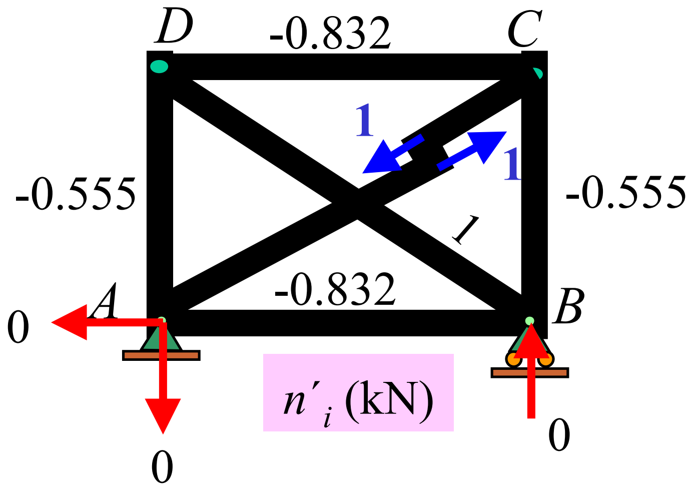
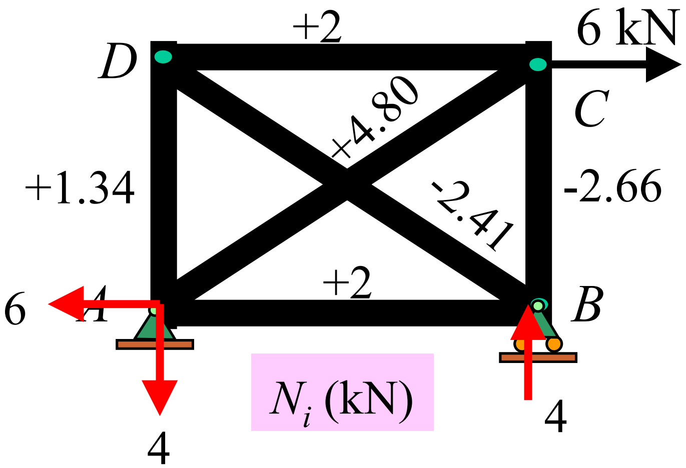


x F_6 = 4.80 kN

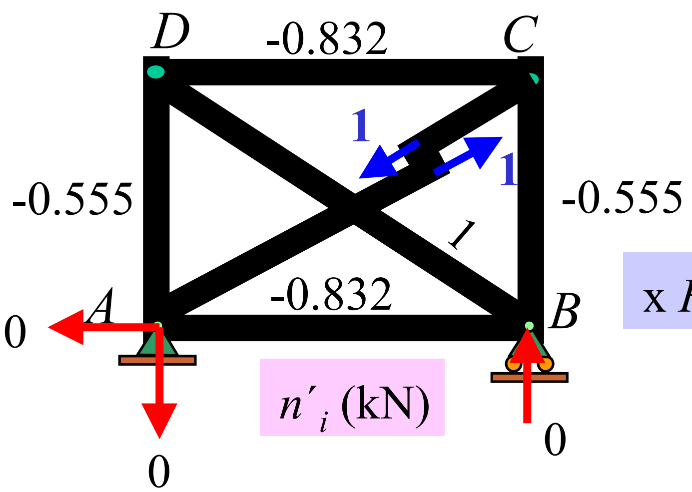
=



Part (b) : *If the turnbuckle on member AC is used to shorten the member by 1 mm.*

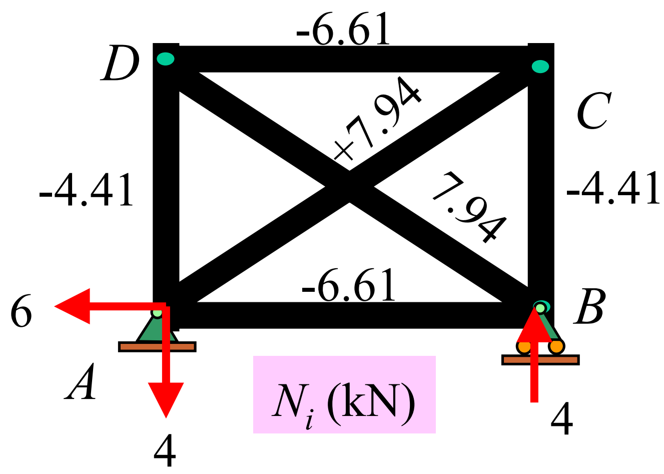
$$f_{66} = \frac{12.61}{AE} = \frac{12.61}{(500)(200)} = 1.26(10^{-4}) \text{ m} = 0.126 \text{ mm}$$

$$F_6 = \frac{1\ mm}{0.126\ mm}(1\ kN) = 7.94\ kN$$

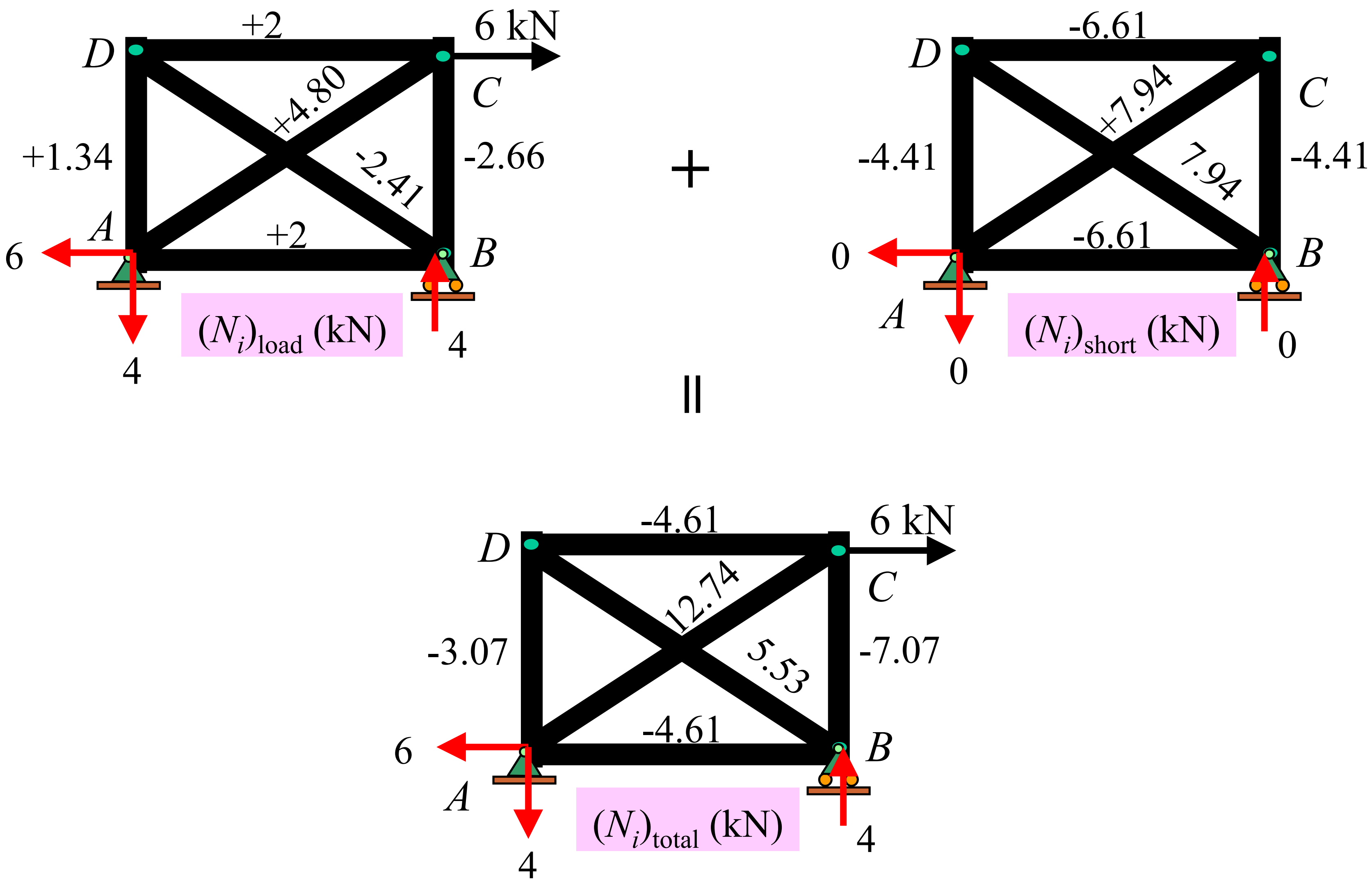


x F_6 = 7.94 kN

=



Part (c) : *If the horizontal force P = 6 kN is applied at joint C and the turnbuckle on member AC is used to shorten the member by 1 mm are both accounted.*

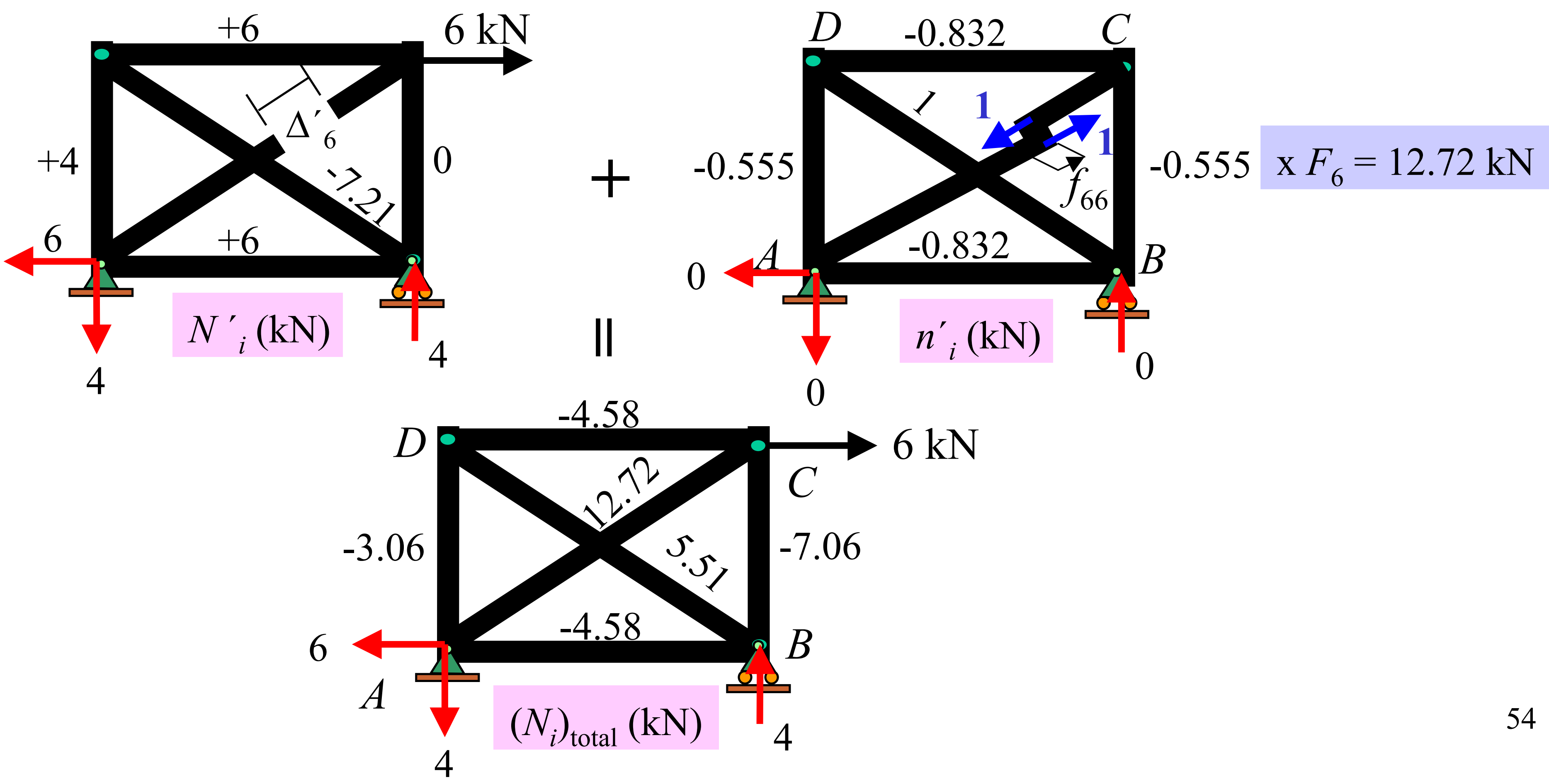


Or use compatibility equation :

$$\Delta'_6 + f_{66} F_6 = \Delta'_6 = 0.001$$

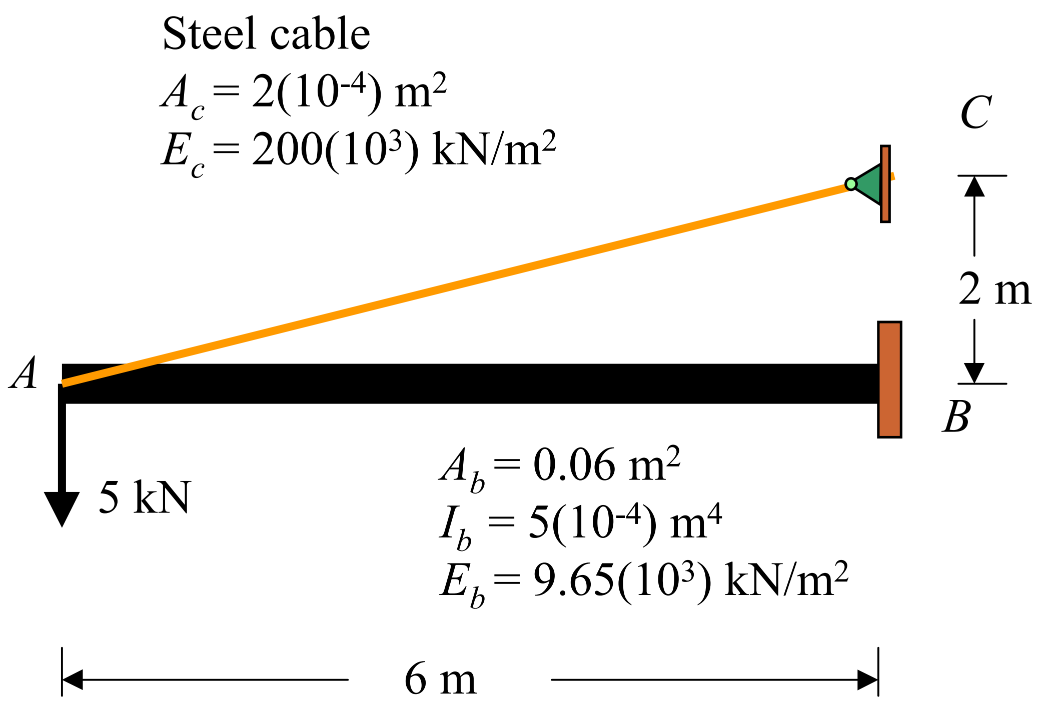
$$-\frac{60.43}{AE} + \frac{12.61}{AE}(F_6) = 0.001$$

$$F_6 = \frac{0.001AE + 60.43}{12.61} = \frac{0.001(500)(200) + 60.43}{12.61} = 12.72 \text{ kN, (T)}$$

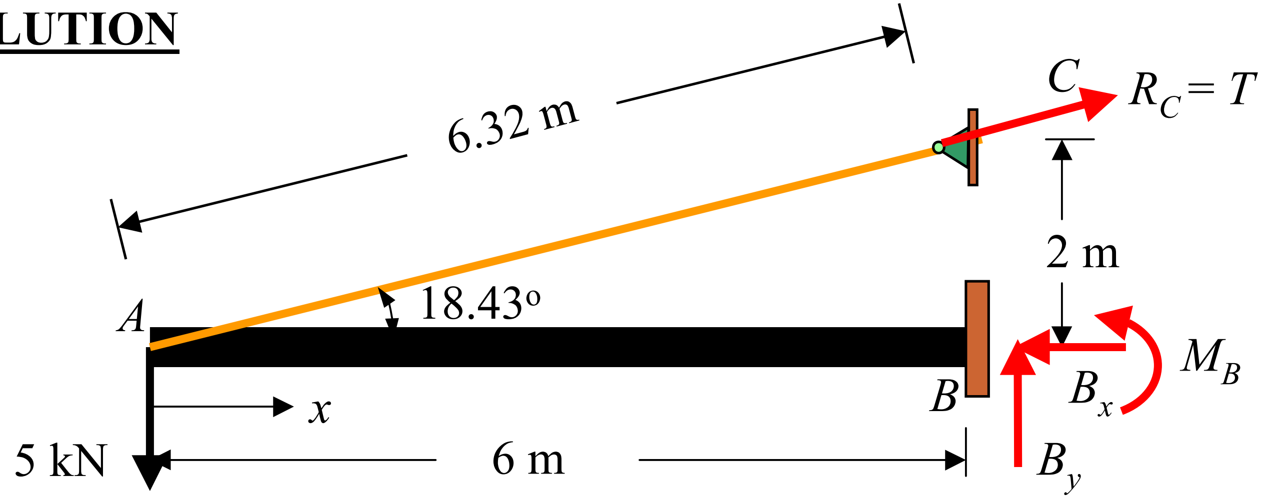


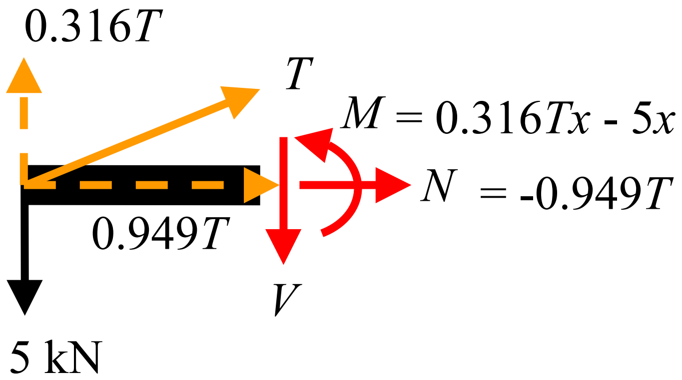
Example 9-7

Find all reaction and the tensile force in the steel support cable. Consider both bending and axial deformation.



SOLUTION





By Castigliano's Theorem of Least Work ;

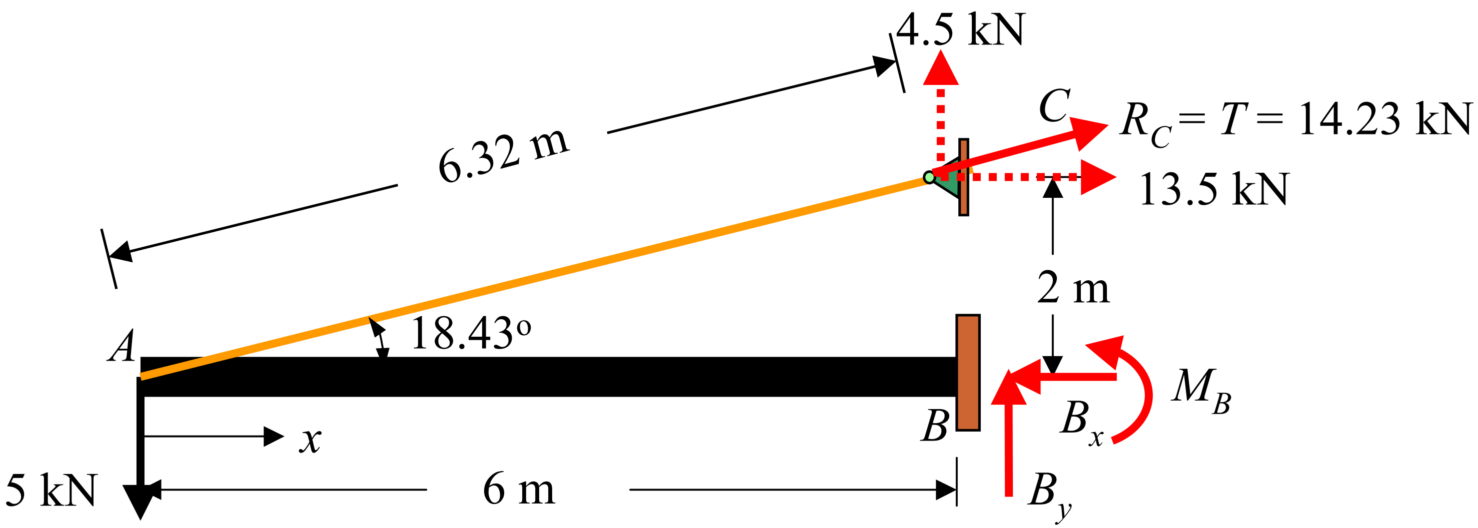
$$\Delta_C = 0 = \frac{\partial}{\partial T}(U_{ib} + U_{in})$$

$$\Delta_C = 0 = \int_0^L \left(\frac{\partial M}{\partial T}\right)\frac{M}{EI}dx + \int_0^L \left(\frac{\partial N}{\partial T}\right)\frac{N}{AE}dx$$

$$0 = \frac{1}{E_b I_b}\int_0^6 (0.316x)(0.316xT - 5x)dx + \frac{1}{A_b E_b}\int_0^6 (-0.949)(-0.949T)dx + \frac{1}{A_c E_c}\int_0^{6.32}(1)(T)dx$$

$$0 = \frac{1}{E_b I_b}\left[\left(\frac{0.316^2 x^3}{3}T\right) - \frac{(0.316\times 5)x^3}{3}\right]\Bigg|_0^6 + \frac{1}{A_b E_b}(0.949^2 xT)\Bigg|_0^6 + \frac{1}{A_c E_c}(xT)\Bigg|_0^{6.32}$$

$$0 = (1.49T - 23.58) + 9.33(10^{-3})T + 0.158T$$; $T = 14.23$ kN, (tension) #



$\xrightarrow{+}$ $\Sigma F_x = 0$: $B_x = R_c \cos\theta = 13.5$ kN, $\leftarrow$

$+\uparrow$ $\Sigma F_y = 0$: $B_y = 5 - R_c \sin\theta = 0.5$ kN, $\uparrow$

$+\circlearrowright$ $\Sigma M_B = 0$: $M_B = 13.5(2) - 5(6) = -3$ kN•m, $+\circlearrowright$