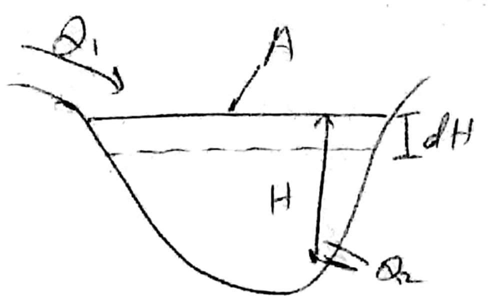
Unsteady flow

$V = A \times h$

$V = Qt$ $\quad Q_1 \neq Q_2$



$A\,dh$ = Incoming volume − Outgoing volume

$= Q_1\,dt - Q_2\,dt$

$A\,dh = (Q_1 - Q_2)\,dt$

$$dt = \frac{A\,dh}{Q_1 - Q_2} \quad \text{— (1)}$$

For steady
$Q_1 = Q_2$
i.e. h = constant

For orifice
$Q_{act} = C_d A \sqrt{2gh}$

$$\int_0^t dt = \int_{h_1}^{h_2} \frac{A\,dh}{Q_1 - Q_2}$$

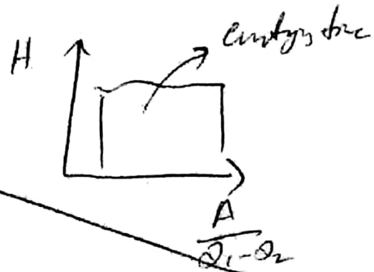
$$\boxed{t = \int_{h_1}^{h_2} \frac{A\,dh}{Q_1 - Q_2}} \quad \text{(2)}$$ Used to compute emptying time of reservoir

h_1 initial

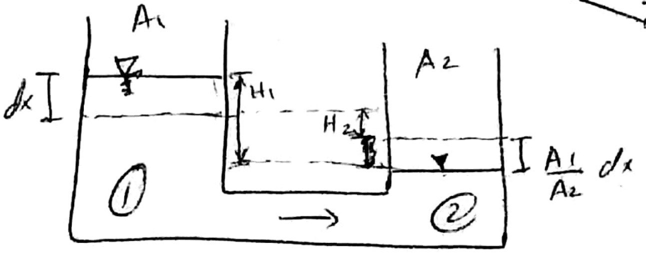
If $Q_1 = 0$

$$t = \int_{h_1}^{h_2} -\frac{A\,dh}{Q_2} \quad \text{— (3)}$$

A and Q are computed in terms of H



Discharge b/w 2 vessels:



$V_1 = V_2$

$A_1\,dx = A_2\left(\frac{A_1}{A_2}\right)dx$

Let h = diff of water level b/w 2 tanks
$= \Sigma$ losses
= entrance loss + friction loss + exit loss.

$$h = \frac{fLV^2}{2gd}$$

$$V = \sqrt{\frac{2gd}{fL}}\,\sqrt{h} \quad \text{— (1)}$$

$dh = h_1 - h_2$

$\boxed{h_1 - h_2 = dx + \frac{A_1}{A_2}dx}$

$$dh = dx + \frac{A_1}{A_2}dx \quad \text{— (2)}$$

$$dx = \left(\frac{A_2}{A_1 + A_2}\right)dh \quad \text{— (3)}$$ By taking L.C.M

$\boxed{dx\,A = V}$

Volume out of tank

$Q = Q$

$Q = AV$

$\frac{A_1\,dx}{dt} = \left(\frac{\pi d^2}{4}\right) V$ — (4)

$Q = \frac{V}{t}$

$dt = \frac{4A_1\,dx}{\pi d^2 V}$ — (5)

putting eq (1) and (3) in eq (5)

$$\int_0^t dt = \int_{h_1}^{h_2} \frac{4A_1 \left(\frac{A_2}{A_1+A_2}\right) dh}{\pi d^2 \sqrt{\frac{2gd}{fL}}\sqrt{h}}$$

$$t = \int_{h_1}^{h_2} \frac{4A_1A_2}{(A_1+A_2)\pi d^2}\sqrt{\frac{fL}{2gd}}\,(h)^{-\frac{1}{2}}\,dh$$

=

$$t = \frac{8A_1A_2}{(A_1+A_2)\pi d^2}\sqrt{\frac{fL}{2gd}}\left(\sqrt{h_1}-\sqrt{h_2}\right)$$

$\int (h)^{-\frac{1}{2}} dh$

$\left[\frac{h^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\right]_{h_1}^{h_2}$

$\frac{h^{\frac{1}{2}}}{\frac{1}{2}}$

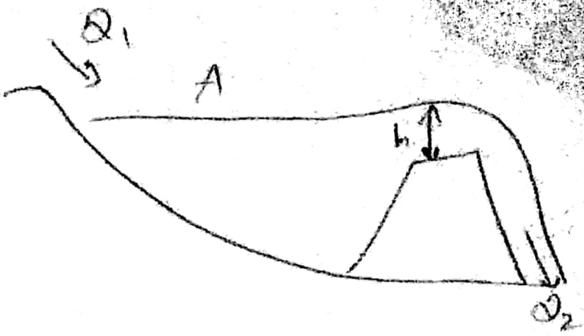
$2\left[h^{\frac{1}{2}}\right]_{h_1}^{h_2}$

$2\left(\sqrt{h_1}-\sqrt{h_2}\right)$

Scanned by CamScanner

Flow over weirs under varying head

As $Q = \frac{V}{t}$



$$Q_1 - Q_2 = \frac{dS}{dt} \longrightarrow \text{storage volume} \quad \text{--- (1)}$$

$$\frac{A\,dh}{dt} = \frac{dS}{dt} \quad \text{--- (2)}$$

Q for spillway

$$Q = \frac{2}{3} C_d \sqrt{2g}\, B \left(H + \frac{V^2}{2g}\right)^{3/2}$$

Velocity of approach is very small hence ignoring

$$Q = \frac{2}{3} C_d \sqrt{2g}\, B H^{3/2} \quad \text{--- (3)}$$

From (1) and (2)

$$Q_1 - Q_2 = \frac{A\,dh}{dt}$$

$$\int_0^t dt = \int_{h_1}^{h_2} \left(\frac{A}{Q_1 - Q_2}\right) dh$$

$$\boxed{t = \int_{h_1}^{h_2} \frac{A}{Q_1 - \frac{2}{3} C_d \sqrt{2g}\, B H^{3/2}}\, dh} \quad \text{--- (4)}$$

Water hammer

$$F = ma$$

$$PA = \rho V \cancel{A} \frac{V}{t}$$

$$P\cancel{A} = \rho \cancel{A} L \frac{V}{t}$$

Pressure $P = \dfrac{\gamma L V}{g\,t}$ — (1)

Pressure head $h = \dfrac{P}{\gamma} = \dfrac{\cancel{\gamma} L V}{g t \cancel{\gamma}} = \dfrac{LV}{gt}$ → Put in (1)

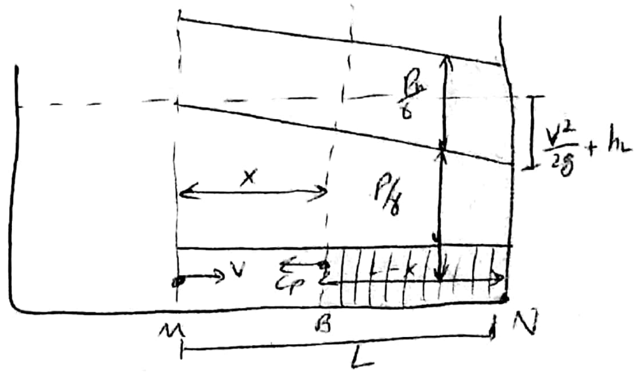
If $t = 0$ in closing the valve

$$P = \frac{\gamma L V}{gt} = \infty \text{ and } h = \infty$$

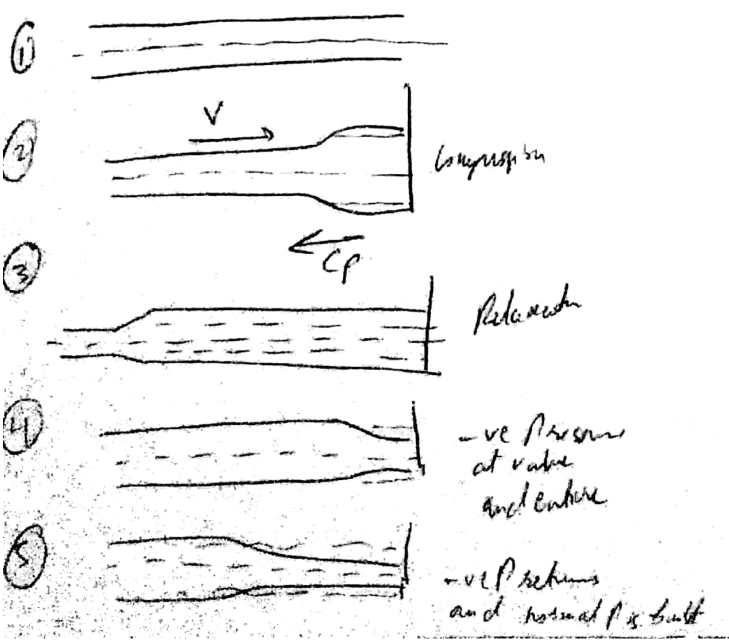
Assumptions
i) $t = 0$
ii) Water is incompressible
iii) Pipe is rigid.

Practically not approach infinity because
i) difficult to close valve in 0 time
ii) Water act as compressible fluid at high 'P'
iii) Material of pipe behave elastically

Water hammer pressure diagram



Process



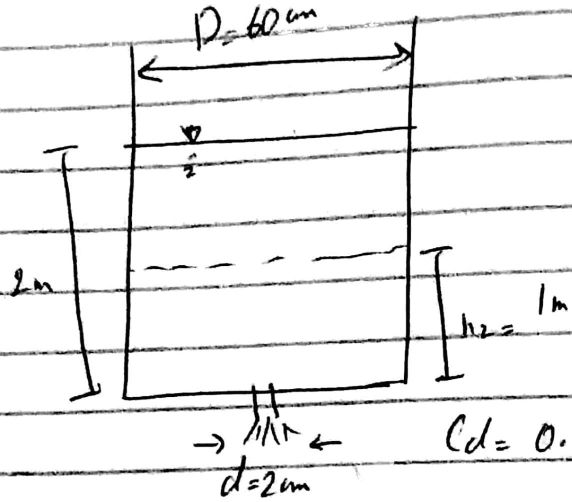
20/12/17

Ch# 12 Unsteady flow

Numerical (1)

A cylindrical vessel with its axis vertical is filled with water and discharges through a 2cm dia orifice at bottom with coefficient of discharge $C_D = 0.62$ for orifice. If dia of vessel is 60cm find the time required to change water level from 2m to 1m above the orifice. There is no inflow

Data:



$d = 2cm = 0.02m$ $\quad$ $Cd = 0.62$

after change ← h_2

$$t = \int_{h_1}^{h_2} \frac{A\,dh}{Q_1 - Q_2}$$

initial head ← h_1

$$A = \frac{\pi D^2}{4} = \frac{\pi (0.6)^2}{4} = 0.283\,m^2$$

$Q_1 = 0$

$Q_2 = Cd \cdot A \cdot V$

$$Q_2 = Cd\, \frac{\pi d^2}{4} \sqrt{2gh}$$

$$= 0.62\, \frac{\pi\, 0.02^2}{4} \sqrt{2 \times 9.81}\sqrt{h}$$

$$Q_2 = 8.63 \times 10^{-4} \sqrt{h}$$

$$t = \int_2^1 \frac{0.283}{0 - 8.63 \times 10^{-4}\sqrt{h}}\,dh$$

$t = 271?.7\,s$

②

Two water tanks A and B having (surface) cross sectional area of 8m² and 4m² respectively are connected to it by a 5cm diameter pipe of length 120m and friction factor f = 0.04. If the initial difference of water level of 2 tanks is 15m, find the time taken for 2m³ of water to pass from tank A to tank B.

<u>Data</u>

$h_1 = 15m$

$V_1 = A_1 \times dx$ $\qquad h_2 =$

$2 = 8 \times dx$

$dx = 0.25m$

$h_1 = dx + h_2 + \frac{A_1}{A_2} dx$

$15 = 0.25 + h_2 + \frac{8}{4}(0.25)$

$h_2 = 0.75m$

$\Rightarrow t = \frac{8A_1A_2}{\pi d^2 (A_1 + A_2)} \sqrt{\frac{fL}{2gd}} \left(\sqrt{h_1} - \sqrt{h_2}\right)$

$t = \frac{8 \times 8 \times 4}{\pi \times 0.05^2 (8+4)} \sqrt{\frac{0.04 \times 120}{2 \times 9.81 \times 0.05}} \left(\sqrt{15} - \sqrt{0.75}\right)$

$t = 2155.3s$

$t = 35.92min$

Flow over weirs under varying head

$$t = \int_{h_1}^{h_2} \frac{A}{Q_1 - \frac{2}{3} C_d \sqrt{2g}\, B H^{3/2}}\, dH$$

Problem Inslide 3

Data

Surface area = 900,000 m^2

B = Length of spillway = 30m

C_d = 0.72

t in hrs = ?

h_1 = 0.6m

h_2 = 0.15m

Q_1 = 0

$$t = \int_{0.6}^{0.15} \frac{900,000}{0 - \frac{2}{3} \times 0.72 \sqrt{2 \times 9.81} \times 30 \times H^{3/2}}\, dH$$

= 36432.126 s

= 10.12 hrs

Water hammer

c Celerity of wave