Chapter 5: Indeterminate Structures – Slope-Deflection Method

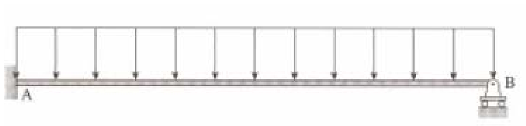
1. Introduction

- *Slope-deflection method* is the second of the two classical methods presented in this course. This method considers the deflection as the primary unknowns, while the redundant forces were used in the force method.

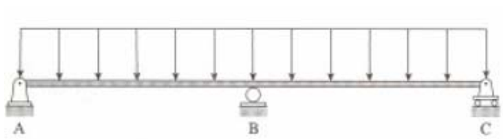
- In the slope-deflection method, the relationship is established between moments at the ends of the members and the corresponding rotations and displacements.

- The *basic assumption* used in the slope-deflection method is that a typical member can flex but the shear and axial deformation are negligible. It is no different from that used with the force method.

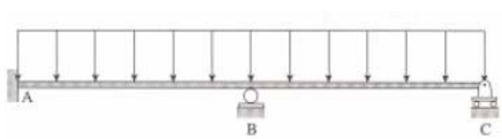
- *Kinematically indeterminate* structures versus statically indeterminate structures:



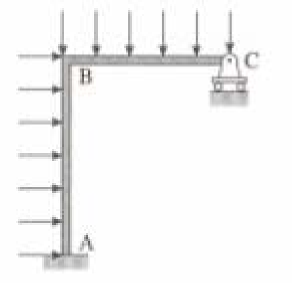
Redundants: 1 (B_y)
Kinematics: 1 (θ_B)



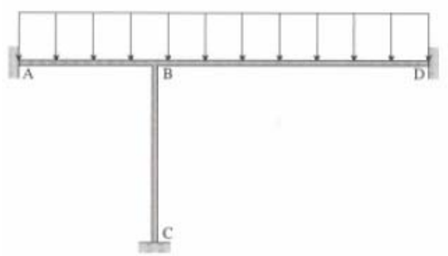
Redundants: 1 (B_y)
Kinematics: 3 ($\theta_A, \theta_B, \theta_C$)



Redundants: 2 (B_y, C_y)
Kinematics: 2 (θ_B, θ_C)



Redundants: 1 (C_y)
Kinematics: 3 ($\theta_B, \theta_C, \Delta_x$)



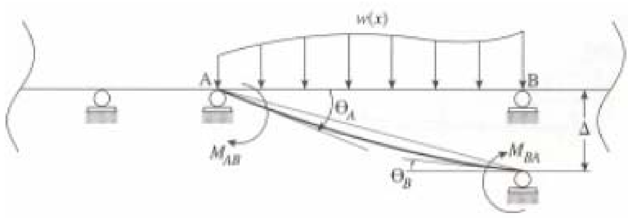
Redundants: 6
Kinematics: 1 (θ_B)

- <u>**Sign convention**</u>: **All clockwise internal moments and end rotation are positive**.

Basic Idea of Slope Deflection Method

The basic idea of the slope deflection method is to write the equilibrium equations for each node in terms of the deflections and rotations. Solve for the generalized displacements. Using moment-displacement relations, moments are then known. The structure is thus reduced to a determinate structure.

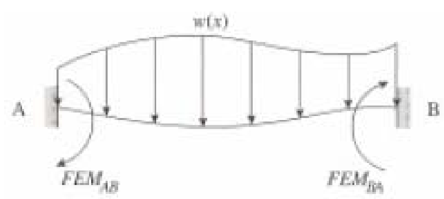
Fundamental Slope-Deflection Equations:



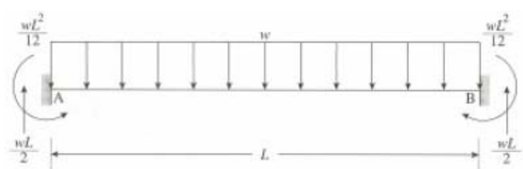
$$\begin{cases} M_{AB} = \dfrac{2EI}{L}\left[2\theta_A + \theta_B - 3\dfrac{\Delta}{L}\right] + FEM_{AB} \\ \\ M_{BA} = \dfrac{2EI}{L}\left[2\theta_B + \theta_A - 3\dfrac{\Delta}{L}\right] + FEM_{BA} \end{cases}$$

where

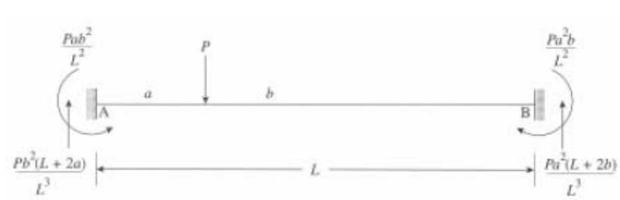
Case A: fixed-end moments



$$M_{AB} = FEM_{AB}; \quad M_{BA} = FEM_{BA}$$

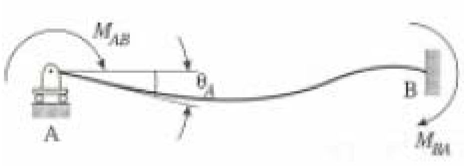


$$FEM_{AB} = -\frac{wL^2}{12}; \quad FEM_{BA} = +\frac{wL^2}{12}$$ **Example 5.1.11**



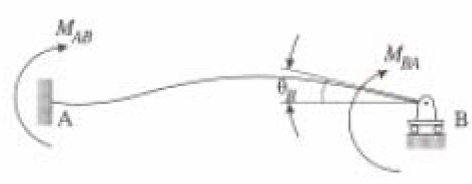
$$FEM_{AB} = -\frac{Pab^2}{L^2}; \quad FEM_{BA} = +\frac{Pa^2b}{L^2}$$ **Example 5.1.12**

Case B: rotation at A



$$M_{AB} = \frac{4EI}{L}\theta_A; \quad M_{BA} = \frac{2EI}{L}\theta_A$$ **Example 5.1.5**

Case C: rotation at B



$$M_{AB} = \frac{2EI}{L}\theta_B; \quad M_{BA} = \frac{4EI}{L}\theta_B$$ **Example 5.1.5**

Case D: displacement of end B related to end A

$$M_{AB} = -\frac{6EI}{L^2}\Delta; \quad M_{BA} = -\frac{6EI}{L^2}\Delta$$ **Example 5.1.13**

3. Analysis of Beams – Slope-Deflection Method

- General Procedure:

***Step 1*:** Scan the beam and identify the number of (a) segments and (b) kinematic unknowns. A segment is the portion of the beam between two nodes. Kinematic unknowns are those rotations and displacements that are not zero and must be computed. The support or end conditions of the beam will help answer the question.

***Step 2*:** For each segment, generate the two governing equations. Check the end conditions to see whether one of the end rotations is zero or not (it is not possible for both the end rotations and other deflection components to be zero). If there are no element loads, the FEM term is zero. If there are one or more element loads, use the appropriate formula to compute the FEM for each element load and then sum all the FEMs. If one end of the segment displace relative to the other, compute the chord rotation; otherwise it is zero.

***Step 3*:** **For each kinematic unknown, generate an equilibrium condition using the free-body diagram.**

Step 4: Solve for all unknowns by combining all the equations from steps 2 and 3. Now the equations are entirely in terms of the kinematic unknowns.

Step 5: Compute the support reactions with appropriate FBDs.

- **Example 1**: Compute the support reactions for a beam. (Rajan's book pages 301-302, Example 5.2.1)

- **Example 2**: Compute the support reactions for a continuous beam. (Rajan's book pages 302-305, Example 5.2.2)

- **Example 3**: Compute the support reactions for a continuous beam where the support B settles $0.5''$. (Rajan's book pages 305-307, Example 5.2.3)

4. Analysis of Frames without Sidesway – Slope-Deflection Method

- The analysis of frames via the slope-deflection method can also be carried out systematically by applying the two governing equations of beams. A sidesway will not occur if
  (a) the frame geometry and loading are symmetric, and
  (b) sidesway is prevented due to supports.

- **Example**: Compute the support reactions for a frame. (Rajan's book pages 308-310, Example 5.2.4)

5. Analysis of Frames with Sidesway – Slope-Deflection Method

- A sidesway will occur if
  (a) the frame geometry and loading are unsymmetrical, and
  (b) sidesway is not prevented due to supports.

- **Example 1**: Compute the support reactions for a frame. (Rajan's book pages 317-320, Example 5.2.7)

- **Example 2**: Compute the support reactions for a frame. (Rajan's book pages 320-324, Example 5.2.8)