CHAPTER 8

SHEAR STRENGTH OF SOIL

8.1 INTRODUCTION

One of the most important and the most controversial engineering properties of soil is its shear strength or ability to resist sliding along internal surfaces within a mass. The stability of a cut, the slope of an earth dam, the foundations of structures, the natural slopes of hillsides and other structures built on soil depend upon the shearing resistance offered by the soil along the probable surfaces of slippage. There is hardly a problem in the field of engineering which does not involve the shear properties of the soil in some manner or the other.

8.2 BASIC CONCEPT OF SHEARING RESISTANCE AND SHEARING STRENGTH

The basic concept of shearing resistance and shearing strength can be made clear by studying first the basic principles of friction between solid bodies. Consider a prismatic block B resting on a plane surface MN as shown in Fig. 8.1. Block B is subjected to the force P_n which acts at right angles to the surface MN, and the force F_a that acts tangentially to the plane. The normal force P_n remains constant whereas F_a gradually increases from zero to a value which will produce sliding. If the tangential force F_a is relatively small, block B will remain at rest, and the applied horizontal force will be balanced by an equal and opposite force F_r on the plane of contact. This resisting force is developed as a result of roughness characteristics of the bottom of block B and plane surface MN. The angle δ formed by the resultant R of the two forces F_r and P_n with the normal to the plane MN is known as the *angle of obliquity*.

If the applied horizontal force F_a is gradually increased, the resisting force F_r will likewise increase, always being equal in magnitude and opposite in direction to the applied force. Block B will start sliding along the plane when the force F_a reaches a value which will increase the angle of obliquity to a certain maximum value δ_m. If block B and plane surface MN are made of the same

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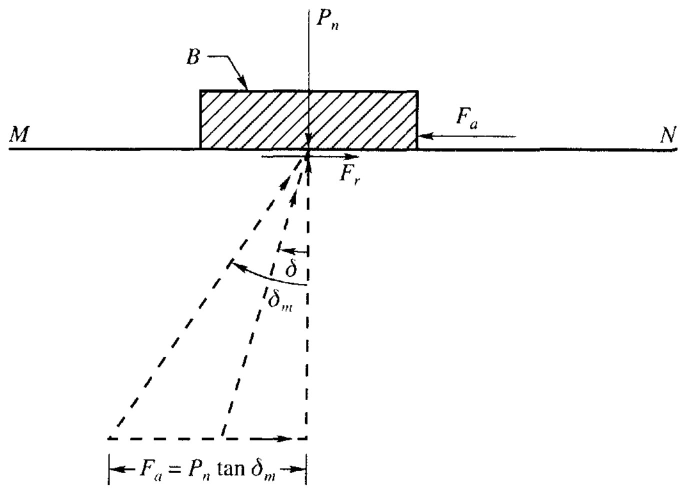

Figure 8.1 Basic concept of shearing resistance and strength.

material, the angle δ_m is equal to ϕ which is termed the *angle of friction*, and the value tan ϕ is termed the *coefficient of friction*. If block B and plane surface MN are made of dissimilar materials, the angle δ is termed the *angle of wall friction*. The applied horizontal force F_a on block B is a shearing force and the developed force is friction or *shearing resistance*. The maximum shearing resistance which the materials are capable of developing is called the *shearing strength*.

If another experiment is conducted on the same block with a higher normal load P_n the shearing force F_a will correspondingly be greater. A series of such experiments would show that the shearing force F_a is proportional to the normal load P_n, that is

$$F_a = P_n \tan \phi \tag{8.1}$$

If A is the overall contact area of block B on plane surface MN, the relationship may be written as

$$\text{shear strength, } s = \frac{F_a}{A} = \frac{P_n}{A} \tan \phi$$

or

$$s = \sigma \tan \phi \tag{8.2}$$

8.3 THE COULOMB EQUATION

The basic concept of friction as explained in Sect. 8.2 applies to soils which are purely granular in character. Soils which are not purely granular exhibit an additional strength which is due to the cohesion between the particles. It is, therefore, still customary to separate the shearing strength s of such soils into two components, one due to the cohesion between the soil particles and the other due to the friction between them. The fundamental shear strength equation proposed by the French engineer Coulomb (1776) is

$$s = c + \sigma \tan \phi \tag{8.3}$$

This equation expresses the assumption that the cohesion c is independent of the normal pressure σ acting on the plane of failure. At zero normal pressure, the shear strength of the soil is expressed as

$$s = c \tag{8.4}$$

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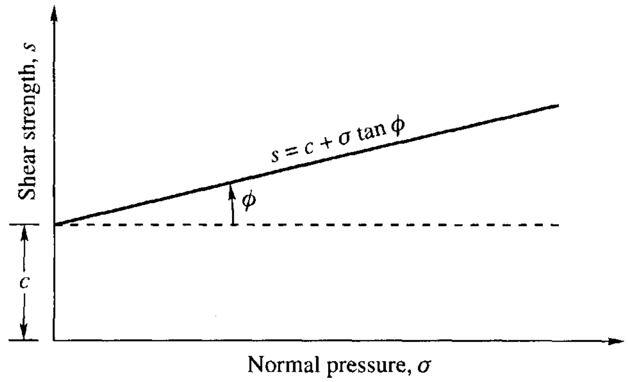

Figure 8.2 Coulomb's law

According to Eq. (8.4), the cohesion of a soil is defined as the shearing strength at zero normal pressure on the plane of rupture.

In Coulomb's equation c and ϕ are empirical parameters, the values of which for any soil depend upon several factors; the most important of these are :

1. The past history of the soil.
2. The initial state of the soil, i.e., whether it is saturated or unsaturated.
3. The permeability characteristics of the soil.
4. The conditions of drainage allowed to take place during the test.

Since c and ϕ in Coulomb's Eq. (8.3) depend upon many factors, c is termed as *apparent cohesion* and ϕ the angle of shearing resistance. For cohesionless soil $c = 0$, then Coulomb's equation becomes

$$s = \sigma \tan \phi \tag{8.5}$$

The relationship between the various parameters of Coulomb's equation is shown diagrammatically in Fig. 8.2.

8.4 METHODS OF DETERMINING SHEAR STRENGTH PARAMETERS

Methods

The shear strength parameters c and ϕ of soils either in the undisturbed or remolded states may be determined by any of the following methods:

1. *Laboratory methods*
   (a) Direct or box shear test
   (b) Triaxial compression test
2. *Field method:* Vane shear test or by any other indirect methods

Shear Parameters of Soils in-situ

The laboratory or the field method that has to be chosen in a particular case depends upon the type of soil and the accuracy required. Wherever the strength characteristics of the soil in-situ are required, laboratory tests may be used provided undisturbed samples can be extracted from the

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stratum. However, soils are subject to disturbance either during sampling or extraction from the sampling tubes in the laboratory even though soil particles possess cohesion. It is practically impossible to obtain undisturbed samples of cohesionless soils and highly pre-consolidated clay soils. Soft sensitive clays are nearly always remolded during sampling. Laboratory methods may, therefore, be used only in such cases where fairly good undisturbed samples can be obtained. Where it is not possible to extract undisturbed samples from the natural soil stratum, any one of the following methods may have to be used according to convenience and judgment :

1. Laboratory tests on remolded samples which could at best simulate field conditions of the soil.
2. Any suitable field test.

The present trend is to rely more on field tests as these tests have been found to be more reliable than even the more sophisticated laboratory methods.

Shear Strength Parameters of Compacted Fills

The strength characteristics of fills which are to be constructed, such as earth embankments, are generally found in a laboratory. Remolded samples simulating the proposed density and water content of the fill materials are made in the laboratory and tested. However, the strength characteristics of existing fills may have to be determined either by laboratory or field methods keeping in view the limitations of each method.

8.5 SHEAR TEST APPARATUS

Direct Shear Test

The original form of apparatus for the direct application of shear force is the shear box. The box shear test, though simple in principle, has certain shortcomings which will be discussed later on. The apparatus consists of a square brass box split horizontally at the level of the center of the soil sample, which is held between metal grilles and porous stones as shown in Fig. 8.3(a). Vertical load is applied to the sample as shown in the figure and is held constant during a test. A gradually increasing horizontal load is applied to the lower part of the box until the sample fails in shear. The shear load at failure is divided by the cross-sectional area of the sample to give the ultimate shearing strength. The vertical load divided by the area of the sample gives the applied vertical stress σ. The test may be repeated with a few more samples having the same initial conditions as the first sample. Each sample is tested with a different vertical load.

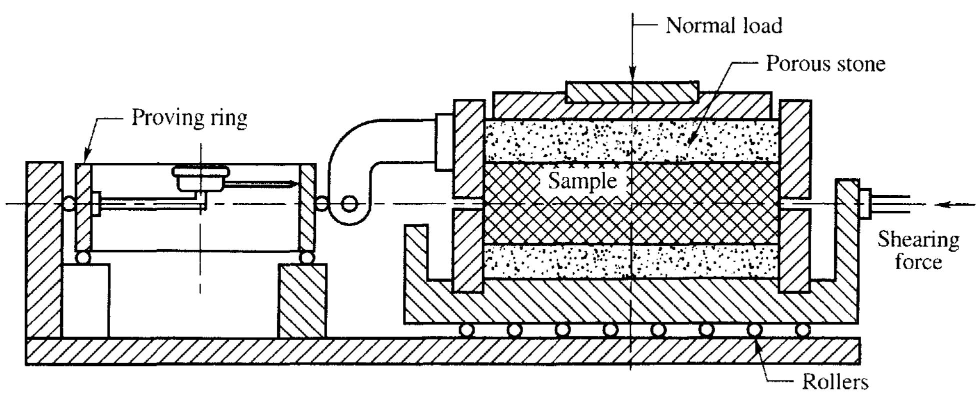

Figure 8.3(a) Constant rate of strain shear box

Figure 8.3(b) Strain controlled direct shear apparatus (Courtesy: Soiltest)

The horizontal load is applied at a constant rate of strain. The lower half of the box is mounted on rollers and is pushed forward at a uniform rate by a motorized gearing arrangement. The upper half of the box bears against a steel proving ring, the deformation of which is shown on the dial gauge indicating the shearing force. To measure the volume change during consolidation and during the shearing process another dial gauge is mounted to show the vertical movement of the top platen. The horizontal displacement of the bottom of the box may also be measured by another dial gauge which is not shown in the figure. Figure 8.3(b) shows a photograph of strain controlled direct shear test apparatus.

Procedure for Determining Shearing Strength of Soil

In the direct shear test, a sample of soil is placed into the shear box. The size of the box normally used for clays and sands is 6 × 6 cm and the sample is 2 cm thick. A large box of size 30 × 30 cm with sample thickness of 15 cm is sometimes used for gravelly soils.

The soils used for the test are either undisturbed samples or remolded. If undisturbed, the specimen has to be carefully trimmed and fitted into the box. If remolded samples are required, the soil is placed into the box in layers at the required initial water content and tamped to the required dry density.

After the specimen is placed in the box, and all the other necessary adjustments are made, a known normal load is applied. Then a shearing force is applied. The normal load is held constant throughout the test but the shearing force is applied at a constant rate of strain (which will be explained later on). The shearing displacement is recorded by a dial gauge.

Dividing the normal load and the maximum applied shearing force by the cross-sectional area of the specimen at the shear plane gives respectively the unit normal pressure σ and the shearing strength s at failure of the sample. These results may be plotted on a shearing diagram where σ is the abscissa and s the ordinate. The result of a single test establishes one point on the graph representing the Coulomb formula for shearing strength. In order to obtain sufficient points to draw the Coulomb graph, additional tests must be performed on other specimens which are exact duplicates of the first. The procedure in these additional tests is the same as in the first, except that a different normal stress is applied each time. Normally, the plotted points of normal and shearing stresses at failure of the various specimens will approximate a straight line. But in the case of saturated, highly cohesive clay soils in the undrained test, the graph of the relationship between the normal stress and shearing strength is usually a curved line, especially at low values of normal stress. However, it is the usual practice to draw the best straight line through the test points to establish the Coulomb Law. The slope of the line gives the angle of shearing resistance and the intercept on the ordinate gives the apparent cohesion (See. Fig. 8.2).

Triaxial Compression Test

A diagrammatic layout of a triaxial test apparatus is shown in Fig. 8.4(a). In the triaxial compression test, three or more identical samples of soil are subjected to uniformly distributed fluid pressure around the cylindrical surface. The sample is sealed in a watertight rubber membrane. Then axial load is applied to the soil sample until it fails. Although only compressive load is applied to the soil sample, it fails by shear on internal faces. It is possible to determine the shear strength of the soil from the applied loads at failure. Figure 8.4(b) gives a photograph of a triaxial test apparatus.

Advantages and Disadvantages of Direct and Triaxial Shear Tests

Direct shear tests are generally suitable for cohesionless soils except fine sand and silt whereas the triaxial test is suitable for all types of soils and tests. Undrained and consolidated undrained tests on clay samples can be made with the box-shear apparatus. The advantages of the triaxial over the direct shear test are:

1. The stress distribution across the soil sample is more uniform in a triaxial test as compared to a direct shear test.
2. The measurement of volume changes is more accurate in the triaxial test.
3. The complete state of stress is known at all stages during the triaxial test, whereas only the stresses at failure are known in the direct shear test.
4. In the case of triaxial shear, the sample fails along a plane on which the combination of normal stress and the shear stress gives the maximum angle of obliquity of the resultant with the normal, whereas in the case of direct shear, the sample is sheared only on one plane which is the horizontal plane which need not be the plane of actual failure.
5. Pore water pressures can be measured in the case of triaxial shear tests whereas it is not possible in direct shear tests.
6. The triaxial machine is more adaptable.

Advantages of Direct Shear Tests

1. The direct shear machine is simple and fast to operate.
2. A thinner soil sample is used in the direct shear test thus facilitating drainage of the pore water quickly from a saturated specimen.
3. Direct shear requirement is much less expensive as compared to triaxial equipment.

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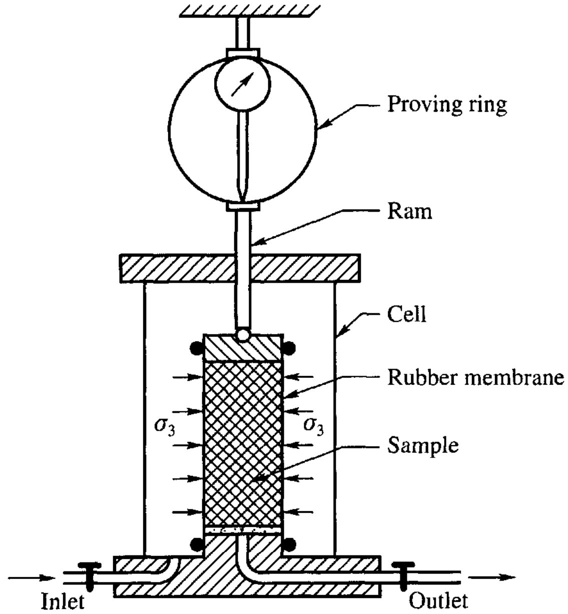


(a) Diagrammatic layout

(b) Multiplex 50-E load frame triaxial test apparatus (Courtesy: Soiltest USA)

Figure 8.4 Triaxial test apparatus

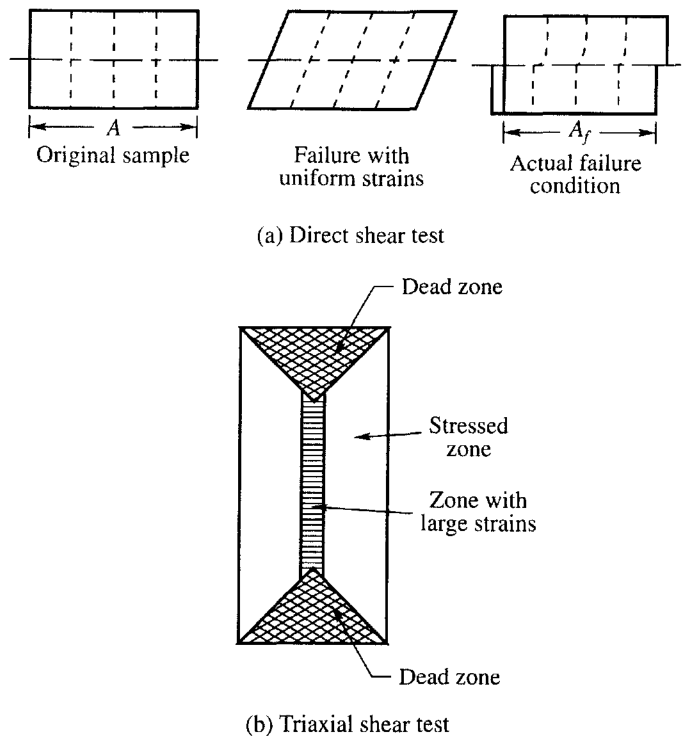


Figure 8.5 Condition of sample during shearing in direct and triaxial shear tests

The stress conditions across the soil sample in the direct shear test are very complex because of the change in the shear area with the increase in shear displacement as the test progresses, causing unequal distribution of shear stresses and normal stresses over the potential surface of sliding. Fig. 8.5(a) shows the sample condition before and after shearing in a direct shear box. The final sheared area A_f is less than the original area A.

Fig. 8.5(b) shows the stressed condition in a triaxial specimen. Because of the end restraints, dead zones (non-stressed zones) triangular in section are formed at the ends whereas the stress distribution across the sample midway between the dead zones may be taken as approximately uniform.

8.6 STRESS CONDITION AT A POINT IN A SOIL MASS

Through every point in a stressed body there are three planes at right angles to each other which are unique as compared to all the other planes passing through the point, because they are subjected only to normal stresses with no accompanying shearing stresses acting on the planes. These three planes are called *principal planes*, and the normal stresses acting on these planes are *principal stresses*. Ordinarily the three principal stresses at a point differ in magnitude. They may be designated as the major principal stress σ_1, the intermediate principal stress σ_2, and the minor principal stress σ_3. Principal stresses at a point in a stressed body are important because, once they are evaluated, the stresses on any other plane through the point can be determined. Many problems in foundation engineering can be approximated by considering only two-dimensional stress conditions. The influence of the intermediate principal stress σ_2 on failure may be considered as not very significant.

A Two-Dimensional Demonstration of the Existence of Principal Planes

Consider the body (Fig. 8.6(a)) is subjected to a system of forces such as F_1, F_2, F_3 and F_4 whose magnitudes and lines of action are known.

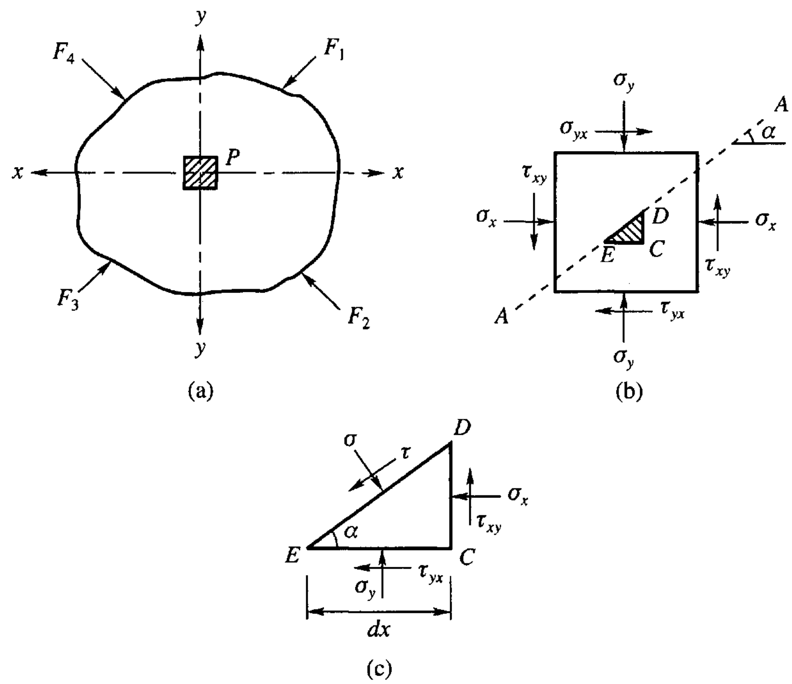


Figure 8.6 Stress at a point in a body in two dimensional space

Consider a small prismatic element P. The stresses acting on this element in the directions parallel to the arbitrarily chosen axes x and y are shown in Fig. 8.6(b).

Consider a plane AA through the element, making an angle α with the x-axis. The equilibrium condition of the element may be analyzed by considering the stresses acting on the faces of the triangle ECD (shaded) which is shown to an enlarged scale in Fig. 8.6(c). The normal and shearing stresses on the faces of the triangle are also shown.

The unit stress in compression and in shear on the face ED are designated as σ and τ respectively.

Expressions for σ and τ may be obtained by applying the principles of statics for the equilibrium condition of the body. The sum of all the forces in the x-direction is

$$\sigma_x dx \tan \alpha + \tau_{xy} dx + \tau\, dx \sec \alpha \cos \alpha - \sigma dx \sec \alpha \sin \alpha = 0 \tag{8.6}$$

The sum of all the forces in the y-direction is

$$\sigma_y dx + \tau_{xy} dx \tan \alpha - \tau\, dx \sec \alpha \sin \alpha - \sigma dx \sec \alpha \cos \alpha = 0 \tag{8.7}$$

Solving Eqs. (8.6) and (8.7) for σ and τ, we have

$$\sigma = \frac{\sigma_y + \sigma_x}{2} + \frac{\sigma_y - \sigma_x}{2} \cos 2\alpha + \tau_{xy} \sin 2\alpha \tag{8.8}$$

$$\tau = \frac{1}{2}\left(\sigma_y - \sigma_x\right) \sin 2\alpha - \tau_{xy} \cos 2\alpha \tag{8.9}$$

By definition, a principal plane is one on which the shearing stress is equal to zero. Therefore, when τ is made equal to zero in Eq. (8.9), the orientation of the principal planes is defined by the relationship

$$\tan 2\alpha = \frac{2\tau_{xy}}{\sigma_y - \sigma_x} \tag{8.10}$$

Equation (8.10) indicates that there are two principal planes through the point P in Fig. 8.6(a) and that they are at right angles to each other. By differentiating Eq. (8.8) with respect to α, and equating to zero, we have

$$\frac{d\sigma}{d\alpha} = -\sigma_y \sin 2\alpha + \sigma_x \sin 2\alpha + 2\tau_{xy} \cos 2\alpha = 0$$

or $$\tan 2\alpha = \frac{2\tau_{xy}}{\sigma_y - \sigma_x} \tag{8.11}$$

Equation (8.11) indicates the orientation of the planes on which the normal stresses σ are maximum and minimum. This orientation coincides with Eq. (8.10). Therefore, it follows that the principal planes are also planes on which the normal stresses are maximum and minimum.

8.7 STRESS CONDITIONS IN SOIL DURING TRIAXIAL COMPRESSION TEST

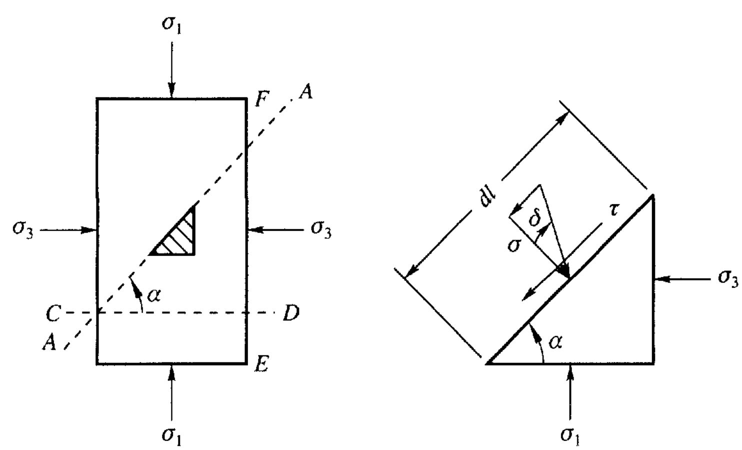
In triaxial compression test a cylindrical specimen is subjected to a constant all-round fluid pressure which is the minor principal stress σ_3 since the shear stress on the surface is zero. The two ends are subjected to axial stress which is the major principal stress σ_1. The stress condition in the specimen goes on changing with the increase of the major principal stress σ_1. It is of interest to analyze the state of stress along inclined sections passing through the sample at any stress level σ_1 since failure occurs along inclined surfaces.

Consider the cylindrical specimen of soil in Fig. 8.7(a) which is subjected to principal stresses σ_1 and σ_3 ($\sigma_2 = \sigma_3$).

Now *CD*, a horizontal plane, is called a principal plane since it is normal to the principal stress σ_1 and the shear stress is zero on this plane. *EF* is the other principal plane on which the principal stress σ_3 acts. *AA* is the inclined section on which the state of stress is required to be analyzed.

Consider as before a small prism of soil shown shaded in Fig. 8.7(a) and the same to an enlarged scale in Fig. 8.7(b). All the stresses acting on the prism are shown. The equilibrium of the prism requires

$$\Sigma \text{ Horizontal forces} = \sigma_3 \sin \alpha\, dl - \sigma \sin \alpha\, dl + \tau \cos \alpha\, dl = 0 \tag{8.12}$$



(a) (b)

Figure 8.7 Stress condition in a triaxial compression test specimen

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$$\Sigma \text{ Vertical forces} = \sigma_1 \cos\alpha\, dl - \sigma\cos\alpha\, dl - \tau\sin\alpha\, dl = 0 \tag{8.13}$$

Solving Eqs. (8.12) and (8.13) we have

$$\sigma = \frac{\sigma_1+\sigma_3}{2} + \frac{\sigma_1-\sigma_3}{2}\cos 2\alpha \tag{8.14}$$

$$\tau = \frac{1}{2}(\sigma_1-\sigma_3)\sin 2\alpha \tag{8.15}$$

Let the resultant of σ and τ make an angle δ with the normal to the inclined plane. One should remember that when α is less than 90°, the shear stress τ is positive, and the angle δ is also positive.

Eqs. (8.14) and (8.15) may be obtained directly from the general Eqs. (8.8) and (8.9) respectively by substituting the following:

$\sigma_y = \sigma_1$, $\sigma_x = \sigma_3$ and $\tau_{xy} = 0$

8.8 RELATIONSHIP BETWEEN THE PRINCIPAL STRESSES AND COHESION *c*

If the shearing resistance s of a soil depends on both friction and cohesion, sliding failure occurs in accordance with the Coulomb Eq. (8.3), that is, when

$$\tau = s = c + \sigma\tan\phi \tag{8.16}$$

Substituting for the values of σ and τ from Eqs. (8.14) and (8.15) into Eqs. (8.16) and solving for σ_1 we obtain

$$\sigma_1 = \sigma_3 + \frac{c + \sigma_3\tan\phi}{\sin\alpha\cos\alpha - \cos^2\alpha\tan\phi} \tag{8.17}$$

The plane with the least resistance to shearing along it will correspond to the minimum value of σ_1 which can produce failure in accordance with Eq. (8.17). σ_1 will be at a minimum when the denominator in the second member of the equation is at a maximum, that is, when

$$\frac{d}{d\alpha}(\sin\alpha\cos\alpha - \cos^2\alpha\tan\phi) = 0$$

Differentiating, and simplifying, we obtain (writing $\alpha = \alpha_c$)

$$\alpha_c = 45° + \phi/2 \tag{8.18}$$

Substituting for α in Eq. (8.17) and simplifying, we have

$$\sigma_1 = \sigma_3\tan^2(45° + \phi/2) + 2c\tan(45° + \phi/2) \tag{8.19}$$

or

$$\sigma_1 = \sigma_3 N_\phi + 2c\sqrt{N_\phi} \tag{8.20}$$

where $N_\phi = \tan^2(45° + \phi/2)$ is called the *flow value.*

If the cohesion $c = 0$, we have

$$\sigma_1 = \sigma_3 N_\phi \tag{8.21}$$

If $\phi = 0$, we have

$$\sigma_1 = \sigma_3 + 2c \tag{8.22}$$

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If the sides of the cylindrical specimen are not acted on by the horizontal pressure σ_3, the load required to cause failure is called the unconfined compressive strength q_u. It is obvious that an unconfined compression test can be performed only on a cohesive soil. According to Eq. (8.20), the unconfined compressive strength q_u is equal to

$$\sigma_1 = q_u = 2c\sqrt{N_\phi} \tag{8.23}$$

$$\text{If } \phi = 0, \text{ then } q_u = 2c \tag{8.24a}$$

or the shear strength

$$s = c = \frac{q_u}{2} \tag{8.24b}$$

Eq. (8.24b) shows one of the simplest ways of determining the shear strength of cohesive soils.

8.9 MOHR CIRCLE OF STRESS

Squaring Eqs. (8.8) and (8.9) and adding, we have

$$\left[\sigma - \frac{\sigma_y + \sigma_x}{2}\right]^2 + \tau^2 = \left(\frac{\sigma_y - \sigma_x}{2}\right)^2 + \tau_{xy}^2 \tag{8.25}$$

Now, Eq. (8.25) is the equation of a circle whose center has coordinates

$\left[\frac{1}{2}\left(\sigma_y + \sigma_x\right), 0\right]$ and whose radius is $\frac{1}{2}\sqrt{\left(\sigma_y - \sigma_x\right)^2 - 4\tau_{xy}^2}$

The coordinates of points on the circle represent the normal and shearing stresses on inclined planes at a given point. The circle is called the *Mohr circle of stress*, after Mohr (1900), who first recognized this useful relationship. Mohr's method provides a convenient graphical method for determining

1. The normal and shearing stress on any plane through a point in a stressed body.
2. The orientation of the principal planes if the normal and shear stresses on the surface of the prismatic element (Fig. 8.6) are known. The relationships are valid regardless of the mechanical properties of the materials since only the considerations of equilibrium are involved.

If the surfaces of the element are themselves principal planes, the equation for the Mohr circle of stress may be written as

$$\tau^2 + \sigma_y - \frac{\sigma_1 + \sigma_3}{2}^2 = \frac{\sigma_1 - \sigma_3}{2}^2 \tag{8.26}$$

The center of the circle has coordinates $\tau = 0$, and $\sigma = (\sigma_1 + \sigma_3)/2$, and its radius is $(\sigma_1 - \sigma_3)/2$. Again from Mohr's diagram, the normal and shearing stresses on any plane passing through a point in a stressed body (Fig. 8.7) may be determined if the principal stresses σ_1 and σ_3 are known. Since σ_1 and σ_3 are always known in a cylindrical compression test, Mohr's diagram is a very useful tool to analyze stresses on failure planes.

8.10 MOHR CIRCLE OF STRESS WHEN A PRISMATIC ELEMENT IS SUBJECTED TO NORMAL AND SHEAR STRESSES

Consider first the case of a prismatic element subjected to normal and shear stresses as in Fig. 8.8(a).

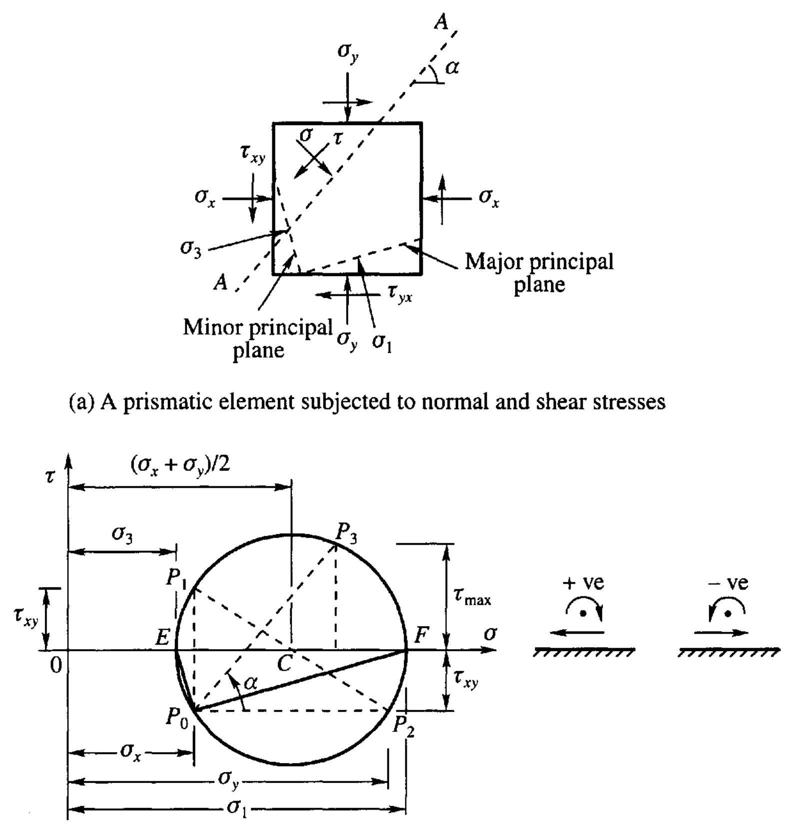
Sign Convention

1. Compressive stresses are positive and tensile stresses are negative.
2. Shear stresses are considered as positive if they give a clockwise moment about a point above the stressed plane as shown in Fig. 8.8(b), otherwise negative.

The normal stresses are taken as abscissa and the shear stresses as ordinates. It is assumed the normal stresses σ_x, σ_y and the shear stress τ_{xy} ($\tau_{xy} = \tau_{yx}$) acting on the surface of the element are known. Two points P_1 and P_2 may now be plotted in Fig. 8.8(b), whose coordinates are

$$P_1 = (\sigma_x, \tau_{xy})$$
$$P_2 = (\sigma_y, -\tau_{xy})$$

If the points P_1 and P_2 are joined, the line intersects the abscissa at point C whose coordinates are $[(\sigma_x + \sigma_y)/2, 0]$.



(a) A prismatic element subjected to normal and shear stresses

(b) Mohr circle of stress

Figure 8.8 Mohr stress circle for a general case

Point O is the origin of coordinates for the center of the Mohr circle of stress. With center C a circle may now be constructed with radius

$$\frac{1}{2}\sqrt{\left(\sigma_y - \sigma_x\right)^2 + 4\tau_{xy}^2}$$

This circle which passes through points P_1 and P_2 is called the *Mohr circle of stress*. The Mohr circle intersects the abscissa at two points E *and* F. The major and minor principal stresses are σ_1 (= OF) and σ_3 (= OE) respectively.

Determination of Normal and Shear Stresses on Plane AA [Fig. 8.8(a)]

Point P_1 on the circle of stress in Fig. 8. 8(b) represents the state of stress on the vertical plane of the prismatic element; similarly point P_2 represents the state of stress on the horizontal plane of the element. If from point P_1 a line is drawn parallel to the vertical plane, it intersects the circle at point P_0 and if from the point P_2 on the circle, a line is drawn parallel to the horizontal plane, this line also intersects the circle at point P_0. The point P_0 so obtained is called the *origin of planes* or the *pole*. If from the pole P_0 a line is drawn parallel to the plane AA in Fig. 8.8(a) to intersect the circle at point P_3 (Fig. 8.8(b)) then the coordinates of the point give the normal stress σ and the shear stress τ on plane AA as expressed by equations 8.8 and 8.9 respectively. This indicates that a line drawn from the pole P_0 at any angle α to the σ-axis intersects the circle at coordinates that represent the normal and shear stresses on the plane inclined at the same angle to the abscissa.

Major and Minor Principal Planes

The orientations of the principal planes may be obtained by joining point P_0 to the points E and F in Fig 8.8(b). P_0F is the direction of the major principal plane on which the major principal stress σ_1 acts; similarly P_0E is the direction of the minor principal plane on which the minor principal stress σ_3 acts. It is clear from the Mohr diagram that the two planes P_0E and P_0F intersect at a right angle, i.e., angle $EP_0F = 90°$.

8.11 MOHR CIRCLE OF STRESS FOR A CYLINDRICAL SPECIMEN COMPRESSION TEST

Consider the case of a cylindrical specimen of soil subjected to normal stresses σ_1 and σ_3 which are the major and minor principal stresses respectively (Fig. 8.9)

From Eqs. (8.14) and (8.15), we may write

$$\tau^2 + \left(\sigma - \frac{\sigma_1 + \sigma_3}{2}\right)^2 = \left(\frac{\sigma_1 - \sigma_3}{2}\right)^2 \qquad (8.27)$$

Again Eq. (8.27) is the equation of a circle whose center has coordinates

$$\sigma = \frac{\sigma_1 + \sigma_3}{2} \text{ and } \tau = 0 \text{ and whose radius is } \frac{\sigma_1 - \sigma_3}{2}$$

A circle with radius $(\sigma_1 - \sigma_3)/2$ with its center C on the abscissa at a distance of $(\sigma_1 + \sigma_3)/2$ may be constructed as shown in Fig. 8.9. This is the Mohr circle of stress. The major and minor principal stresses are shown in the figure wherein $\sigma_1 = OF$ and $\sigma_3 = OE$.

From Fig. 8.8, we can write equations for σ_1 and σ_3 and τ_{max} as follows

$$\sigma_1, \sigma_3 = \frac{\sigma_x + \sigma_y}{2} \pm \frac{1}{2}\sqrt{(\sigma_y - \sigma_x)^2 + 4\tau_{xy}^2} \qquad (8.28)$$

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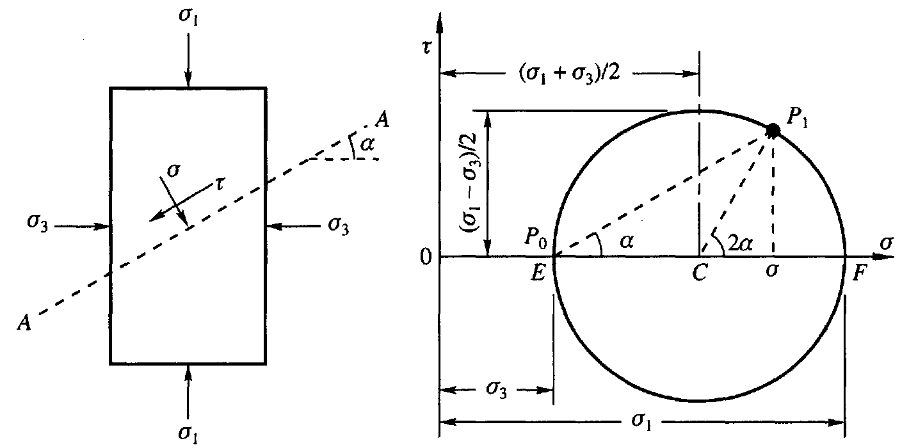

Figure 8.9 Mohr stress circle for a cylindrical specimen

$$\tau_{max} = \frac{1}{2}\sqrt{\left(\sigma_y - \sigma_x\right)^2 + 4\tau_{xy}^2} \tag{8.29}$$

where τ_{max} is the maximum shear stress equal to the radius of the Mohr circle.

The origin of planes or the pole P_0 (Fig. 8.9) may be obtained as before by drawing lines from points E and F parallel to planes on which the minor and major principal stresses act. In this case, the pole P_0 lies on the abscissa and coincides with the point E.

The normal stress σ and shear stress τ on any arbitrary plane AA making an angle α with the major principal plane may be determined as follows.

From the pole P_0 draw a line $P_0 P_1$ parallel to the plane AA (Fig. 8.9). The coordinates of the point P_1 give the stresses σ and τ. From the stress circle we may write

$$\angle P_1CF = 2\alpha$$

$$\sigma = \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2}\cos 2\alpha \tag{8.30}$$

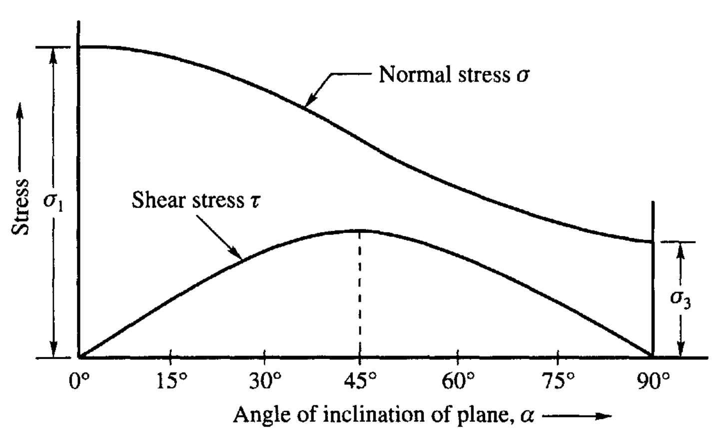

Figure 8.10 Variation of σ and τ with α

$$\tau = \frac{\sigma_1 - \sigma_3}{2} \sin 2\alpha \tag{8.31}$$

Equations (8.30) and (8.31) are the same as Eqs. (8.14) and (8.15) respectively.

It is of interest to study the variation of the magnitudes of normal and shear stresses with the inclination of the plane.

Eqs. (8.30) and (8.31) are plotted with α as the abscissa shown in Fig. 8.10. The following facts are clear from these curves:

1. The greatest and least principal stresses are respectively the maximum and minimum normal stresses on any plane through the point in question.
2. The maximum shear stress occurs on planes at 45° to the principal planes.

8.12 MOHR-COULOMB FAILURE THEORY

Various theories relating to the stress condition in engineering materials at the time of failure are available in the engineering literature. Each of these theories may explain satisfactorily the actions of certain kinds of materials at the time they fail, but no one of them is applicable to all materials. The failure of a soil mass is more nearly in accordance with the tenets of the Mohr theory of failure than with those of any other theory and the interpretation of the triaxial compression test depends to a large extent on this fact. The Mohr theory is based on the postulate that a material will fail when the shearing stress on the plane along which the failure is presumed to occur is a unique function of the normal stress acting on that plane. The material fails along the plane only when the angle between the resultant of the shearing stress and the normal stress is a maximum, that is, where the combination of normal and shearing stresses produces the maximum obliquity angle δ.

According to Coulomb's Law, the condition of failure is that the shear stress

$$\tau \leqq c + \sigma \tan \phi \tag{8.32}$$

In Fig 8.11(b) M_0N and M_0N_1 are the lines that satisfy Coulomb's condition of failure. If the stress at a given point within a cylindrical specimen under triaxial compression is represented by Mohr circle 1, it may be noted that every plane through this point has a shearing stress which is smaller than the shearing strength.

For example, if the plane AA in Fig. 8.11(a) is the assumed failure plane, the normal and shear stresses on this plane at any intermediate stage of loading are represented by point b on Mohr circle 1 where the line P_0b is parallel to the plane AA. The shearing stress on this plane is ab which is less than the shearing strength ac at the same normal stress Oa. Under this stress condition there is no possibility of failure. On the other hand it would not be possible to apply the stress condition represented by Mohr stress circle 2 to this sample because it is not possible for shearing stresses to be greater than the shearing strength. At the normal stress Of, the shearing stress on plane AA is shown to be fh which is greater than the shear strength of the materials fg which is not possible. Mohr circle 3 in the figure is tangent to the shear strength line M_0N and M_0N_1 at points e and e_1 respectively. On the same plane AA at normal stress Od, the shearing stress de is the same as the shearing strength de. Failure is therefore imminent on plane AA at the normal stress Od and shearing stress de. The equation for the shearing stress de is

$$s = de = de' + e'e = c + \sigma \tan \phi \tag{8.33}$$

where ϕ is the slope of the line M_0N which is the maximum angle of obliquity on the failure plane. The value of the obliquity angle can never exceed $\delta_m = \phi$, the angle of shearing resistance, without the occurrence of failure. The shear strength line M_0N which is tangent to Mohr circle 3 is called the

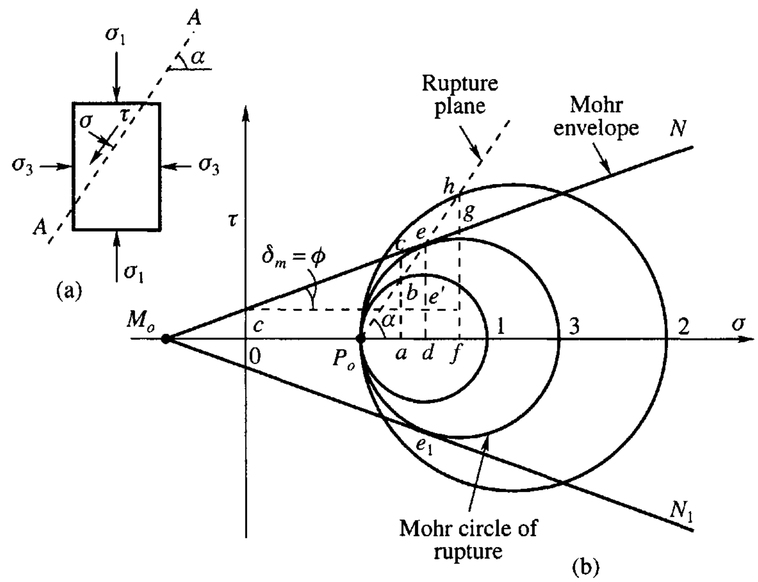

Figure 8.11 Diagram presenting Mohr's theory of rupture

Mohr envelope or *line of rupture*. The Mohr envelope may be assumed as a straight line although it is curved under certain conditions. The Mohr circle which is tangential to the shear strength line is called the *Mohr circle of rupture*. Thus the Mohr envelope constitutes a shear diagram and is a graph of the Coulomb equation for shearing stress. This is called the *Mohr-Coulomb Failure Theory*. The principal objective of a triaxial compression test is to establish the Mohr envelope for the soil being tested. The cohesion and the angle of shearing resistance can be determined from this envelope. When the cohesion of the soil is zero, that is, when the soil is cohesionless, the Mohr envelope passes through the origin.

8.13 MOHR DIAGRAM FOR TRIAXIAL COMPRESSION TEST AT FAILURE

Consider a cylindrical specimen of soil possessing both cohesion and friction is subjected to a conventional triaxial compression test. In the conventional test the lateral pressure σ_3 is held constant and the vertical pressure σ_1 is increased at a constant rate of stress or strain until the sample fails. If σ_1 is the peak value of the vertical pressure at which the sample fails, the two principal stresses that are to be used for plotting the Mohr circle of rupture are σ_3 and σ_1. In Fig. 8.12 the values of σ_1 and σ_3 are plotted on the σ-axis and a circle is drawn with $(\sigma_1 - \sigma_3)$ as diameter. The center of the circle lies at a distance of $(\sigma_1 + \sigma_3)/2$ from the origin. As per Eq. (8.18), the soil fails along a plane which makes an angle $\alpha = 45° + \phi/2$ with the major principal plane. In Fig. 8.12 the two lines P_0P_1 and P_0P_2 (where P_0 is the origin of planes) are the conjugate rupture planes. The two lines M_0N and M_0N_1 drawn tangential to the rupture circle at points P_1 and P_2 are called Mohr envelopes. If the Mohr envelope can be drawn by some other means, the orientation of the failure planes may be determined.

The results of analysis of triaxial compression tests as explained in Sect. 8.8 are now presented in a graphical form in Fig. 8.12. The various information that can be obtained from the figure includes

1. The angle of shearing resistance ϕ = the slope of the Mohr envelope.

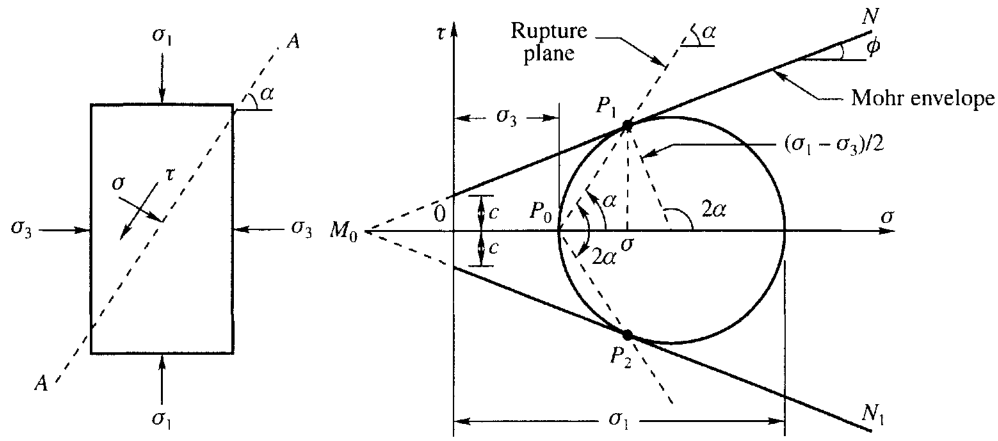

Figure 8.12 Mohr diagram for triaxial test at failure for c-ϕ soil

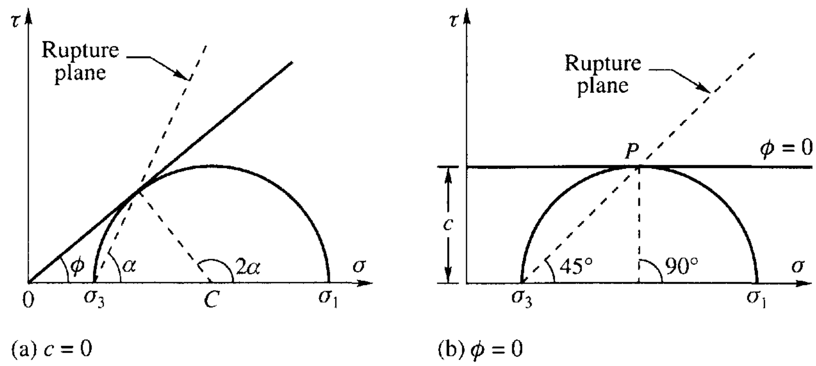

Figure 8.13 Mohr diagram for soils with $c = 0$ and $\phi = 0$

2. The apparent cohesion c = the intercept of the Mohr envelope on the τ-axis.
3. The inclination of the rupture plane = α.
4. The angle between the conjugate planes = 2α.

If the soil is cohesionless with $c = 0$ the Mohr envelopes pass through the origin, and if the soil is purely cohesive with $\phi = 0$ the Mohr envelope is parallel to the abscissa. The Mohr envelopes for these two types of soils are shown in Fig. 8.13.

8.14 MOHR DIAGRAM FOR A DIRECT SHEAR TEST AT FAILURE

In a direct shear test the sample is sheared along a horizontal plane. This indicates that the failure plane is horizontal. The normal stress σ on this plane is the external vertical load divided by the area of the sample. The shear stress at failure is the external lateral load divided by the area of the sample.

Point P_1 on the stress diagram in Fig. 8.14 represents the stress condition on the failure plane. The coordinates of the point are

normal stress = σ, shear stress $\tau = s$.

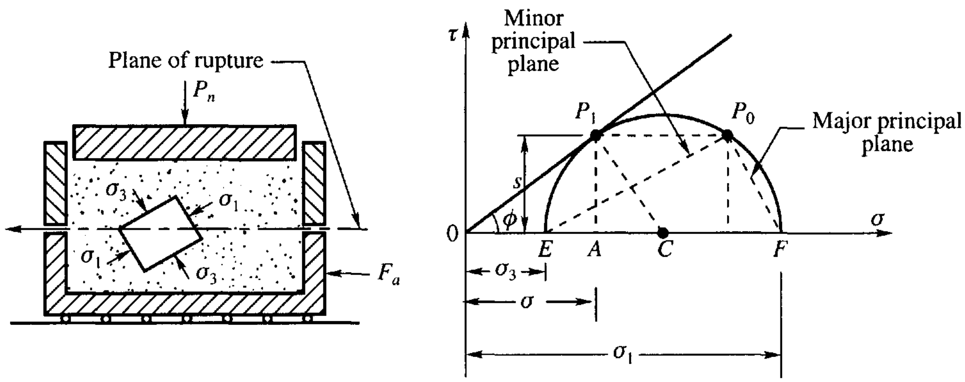

Figure 8.14 Mohr diagram for a direct shear test at failure

If it is assumed that the Mohr envelope is a straight line passing through the origin (for cohesionless soil or normally consolidated clays), it follows that the maximum obliquity δ_m occurs on the failure plane and $\delta_m = \phi$. Therefore the line OP_1 must be tangent to the Mohr circle, and the circle may be constructed as follows:

Draw P_1C normal to OP_1. Point C which is the intersection point of the normal with the abscissa is the center of the circle. CP_1 is the radius of the circle. The Mohr circle may now be constructed which gives the major and minor principal stresses σ_1 and σ_3 respectively.

Since the failure is on the horizontal plane, the origin of planes P_0 may be obtained by drawing a horizontal line through P_1 giving P_0. P_0F and P_0E give the directions of the major and minor principal planes respectively.

Example 8.1

What is the shearing strength of soil along a horizontal plane at a depth of 4 m in a deposit of sand having the following properties:

Angle of internal friction, $\phi = 35°$

Dry unit weight, $\gamma_d = 17$ kN/m^3

Specific gravity, $G_s = 2.7$.

Assume the ground water table is at a depth of 2.5 m from the ground surface. Also find the change in shear strength when the water table rises to the ground surface.

Solution

The effective vertical stress at the plane of interest is

$$\sigma' = 2.50 \times \gamma_d + 1.50 \times \gamma_b$$

Given $\gamma_d = 17$ kN/m^3 and $G_s = 2.7$

$$\text{We have } \gamma_d = 17 = \frac{G_s}{1+e}\gamma_w = \frac{2.7}{1+e} \times 9.81$$

$$\text{or} \quad 17e = 26.5 - 17 = 9.49 \quad \text{or} \quad e = \frac{9.49}{17} = 0.56$$

$$\text{Therefore, } \gamma_b = \frac{G_s - 1}{1+e}\gamma_w = \frac{2.7 - 1.0}{1 + 0.56} \times 9.81 = 10.7 \text{ kN/m}^3$$

Hence $\sigma' = 2.5 \times 17 + 1.5 \times 10.7 = 58.55$ kN/m^2

Hence, the shearing strength of the sand is

$s = \sigma' \tan \phi = 58.55 \times \tan 35° = 41$ kN/m^2

If the water table rises to the ground surface i.e., by a height of 2.5 m, the change in the effective stress will be,

$\Delta\sigma' = \gamma_d \times 2.5 - \gamma_b \times 2.5 = 17 \times 2.5 - 10.7 \times 2.5 = 15.75$ kN/m^2 (negative)

Hence the decrease in shear strength will be,

$= \Delta\sigma' \tan 35° = 15.75 \times 0.70 = 11$ kN/m^2

Example 8.2

Direct shear tests were conducted on a dry sand. The size of the samples used for the tests was 2 in. × 2 in. × 0.75 in. The test results obtained are given below:

Test No.	Normal load (lb)	Normal stress σ (lb/ft^2)	Shear force at failure (lb)	Shear stress (lb/ft^2)
1	15	540	12	432
2	20	720	18	648
3	30	1080	23	828
4	60	2160	47	1692
5	120	4320	93	3348

Determine the shear strength parameters c and ϕ.

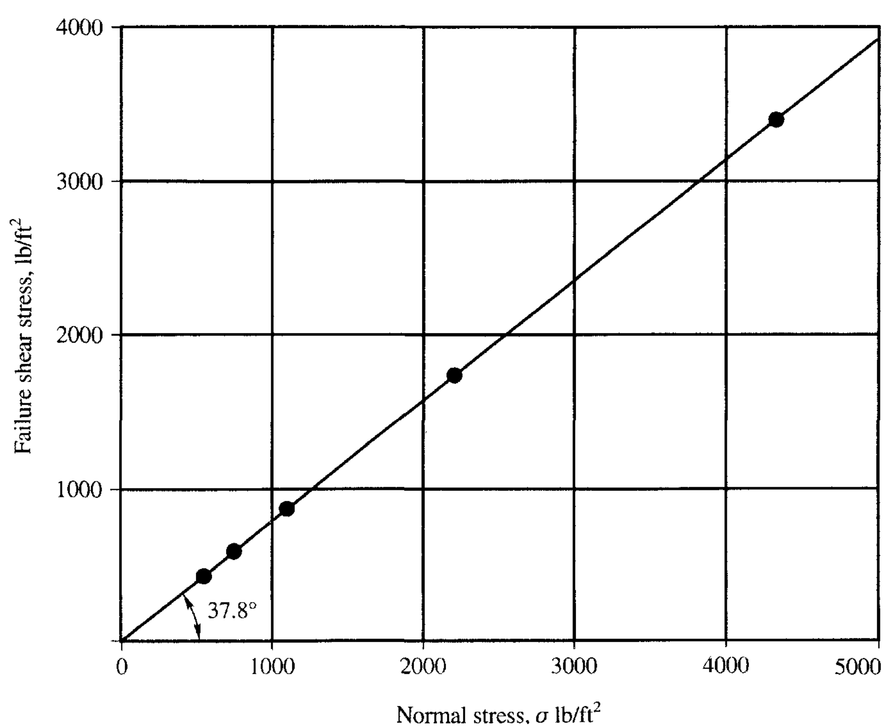

Figure Ex. 8.2

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Solution

The failure shear stresses τ_f, as obtained from the tests are plotted against the normal stresses σ, in Figure Ex 8.2. The shear parameters from the graph are: $c = 0$, $\phi = 37.8°$.

Example 8.3

A direct shear test, when conducted on a remolded sample of sand, gave the following observations at the time of failure: Normal load = 288 N; shear load = 173 *N*. The cross sectional area of the sample = 36 cm^2.

Determine: (i) the angle of internal friction, (ii) the magnitude and direction of the principal stresses in the zone of failure.

Solution

Such problems can be solved in two ways, namely graphically and analytically. The analytical solution has been left as an exercise for the students.

Graphical Solution

(i) Shear stress $\tau = \dfrac{173}{36} = 4.8 \text{ N/cm}^2 = 48 \text{ kN/m}^2$

Normal stress $\sigma = \dfrac{288}{36} = 8.0 \text{ N/cm}^2 = 80 \text{ kN/m}^2$

We know one point on the Mohr envelope. Plot point *A* (Fig. Ex. 8.3) with coordinates τ = 48 kN/m^2, and σ = 80 kN/m^2. Since cohesion $c = 0$ for sand, the Mohr envelope *OM* passes through the origin. The slope of *OM* gives the angle of internal friction $\phi = 31°$.

(ii) In Fig. Ex. 8.3, draw line *AC* normal to the envelope *OM* cutting the abscissa at point *C*. With *C* as center, and *AC* as radius, draw Mohr circle C_1 which cuts the abscissa at points *B* and *D*, which gives

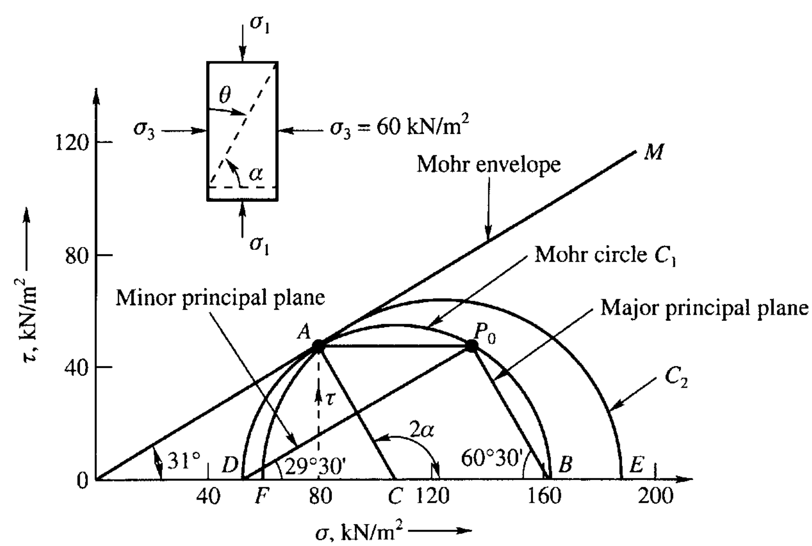

Figure Ex. 8.3

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major principal stress = $OB = \sigma_1 = 163.5$ kN/m^2

minor principal stress = $OD = \sigma_3 = 53.5$ kN/m^2

Now, $\angle ACB = 2\alpha$ = twice the angle between the failure plane and the major principal plane. Measurement gives

$2\alpha = 121°$ or $\alpha = 60.5°$

Since in a direct shear test the failure plane is horizontal, the angle made by the major principal plane with the horizontal will be 60.5°. The minor principal plane should be drawn at a right angle to the major principal plane.

The directions of the principal planes may also be found by locating the pole P_o. P_o is obtained by drawing a horizontal line from point A which is parallel to the failure plane in the direct shear test. Now P_oB and P_oD give the directions of the major and minor principal planes respectively.

8.15 EFFECTIVE STRESSES

So far, the discussion has been based on consideration of total stresses. It is to be noted that the strength and deformation characteristics of a soil can be understood better by visualizing it as a compressible skeleton of solid particles enclosing voids. The voids may completely be filled with water or partly with water and air. Shear stresses are to be carried only by the skeleton of solid particles. However, the total normal stresses on any plane are, in general, the sum of two components.

Total normal stress = component of stress carried by solid particles
+ pressure in the fluid in the void space.

This visualization of the distribution of stresses between solid and fluid has two important consequences:

1. When a specimen of soil is subjected to external pressure, the volume change of the specimen is not due to the total normal stress but due to the difference between the total normal stress and the pressure of the fluid in the void space. The pressure in the fluid is the pore pressure u. The difference which is called the effective stress σ' may now be expressed as

$$\sigma' = \sigma - u \tag{8.34}$$

2. The shear strength of soils, as of all granular materials, is largely determined by the frictional forces arising during slip at the contacts between the soil particles. These are clearly a function of the component of normal stress carried by the solid skeleton rather than of the total normal stress. For practical purposes the shear strength equation of Coulomb is given by the expression

$$s = c' + (\sigma - u)\tan\phi' = c' + \sigma'\tan\phi' \tag{8.35}$$

where c' = apparent cohesion in terms of effective stresses
ϕ' = angle of shearing resistance in terms of effective stresses
σ = total normal pressure to the plane considered
u = pore pressure.

The effective stress parameters c' and ϕ' of a given sample of soil may be determined provided the pore pressure u developed during the shear test is measured. The pore pressure u is developed when the testing of the soil is done under undrained conditions. However, if free

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drainage takes place during testing, there will not be any development of pore pressure. In such cases, the total stresses themselves are effective stresses.

8.16 SHEAR STRENGTH EQUATION IN TERMS OF EFFECTIVE PRINCIPAL STRESSES

The principal stresses may be expressed either as total stresses or as effective stresses if the values of pore pressure are known.

If u is the pore pressure developed during a triaxial test, we may write as before

$$\begin{aligned} \sigma_1' &= \sigma_1 - u \\ \sigma_3' &= \sigma_3 - u \end{aligned} \tag{8.36}$$

where σ_1' and σ_3' are the effective principal stresses. The equation for shear strength in terms of effective stresses is

$$s = \frac{\sigma_1' - \sigma_3'}{2} \sin 2\alpha = \frac{\sigma_1 - \sigma_3}{2} \sin 2\alpha = \frac{\sigma_1 - \sigma_3}{2} \cos \phi' \tag{8.37}$$

where $2\alpha = 90° + \phi'$

Coulomb's equation in terms of effective stresses is

$s = c' + (\sigma - u) \tan \phi'$

Therefore, $\dfrac{\sigma_1 - \sigma_3}{2} \cos \phi' = c' + (\sigma - u) \tan \phi'$

Since, $\sigma = \dfrac{\sigma_1 + \sigma_3}{2} + \dfrac{\sigma_1 - \sigma_3}{2} \cos(90 + \phi')$

we have $\dfrac{\sigma_1 - \sigma_3}{2} \cos \phi' = c' + \dfrac{\sigma_1 + \sigma_3}{2} \tan \phi'$

$$+ \frac{\sigma_1 - \sigma_3}{2} \cos(90 + \phi') \tan \phi' - u \tan \phi'$$

Simplifying

$$\frac{\sigma_1 - \sigma_3}{2} \cos^2 \phi' = c' \cos \phi' + \frac{\sigma_1 + \sigma_3}{2} \sin \phi' - \frac{\sigma_1 - \sigma_3}{2} \sin^2 \phi' - u \sin \phi'$$

or $$\frac{1}{2}(\sigma_1 - \sigma_3)_f = \frac{c' \cos \phi' + (\sigma_3 - u) \sin \phi'}{1 - \sin \phi'} \tag{8.38}$$

where $(\sigma_1 - \sigma_3)_f$ indicates the maximum deviator stress at failure. Eq (8.38) may also be expressed in a different form as follows by considering effective principal stresses

$$\frac{1}{2}(\sigma_1' - \sigma_3')_f = \frac{c' \cos \phi' + \sigma_3' \sin \phi'}{1 - \sin \phi'}$$

or $$\frac{1}{2}(\sigma_1' - \sigma_3')_f (1 - \sin \phi') = c' \cos \phi' + \sigma_3' \sin \phi'$$

Simplifying, we have

$$(\sigma_1' - \sigma_3')_f = (\sigma_1' + \sigma_3')\sin\phi' + 2c'\cos\phi' \tag{8.39}$$

8.17 STRESS-CONTROLLED AND STRAIN-CONTROLLED TESTS

Direct shear tests or triaxial compression tests may be carried out by applying stresses or strains at a particularly known rate. When the stress is applied at a constant rate it is called a *stress-controlled test* and when the strain is applied at a constant rate it is called a *strain-controlled test.* The difference between the two types of tests may be explained with respect to box shear tests.

In the stress-controlled test [Fig. 8.15(a)] the lateral load F_a which induces shear is gradually increased until complete failure occurs. This can be done by placing weights on a hanger or by filling a counterweighted bucket of original weight W at a constant rate. The shearing displacements are measured by means of a dial gauge G as a function of the increasing load F_a. The shearing stress at any shearing displacement, is

$$\tau = \frac{F_a}{A}$$

where A is the cross sectional area of the sample. A typical shape of a stress-strain curve of the stress-controlled test is shown in Fig. 8.15(a).

A typical arrangement of a box-shear test apparatus for the strain-controlled test is shown in Fig. 8.15(b). The shearing displacements are induced and controlled in such a manner that they occur at a constant fixed rate. This can be achieved by turning the wheel either by hand or by means of any electrically operated motor so that horizontal motion is induced through the worm gear B. The dial gauge G gives the desired constant rate of displacement. The bottom of box C is mounted on frictionless rollers D. The shearing resistance offered to this displacement by the soil sample is measured by the proving ring E. The stress-strain curves for this type of test have the shape shown in Fig. 8.15(b).

Both stress-controlled and strain-controlled types of test are used in connection with all the direct triaxial and unconfined soil shear tests. Strain-controlled tests are easier to perform and have the advantage of readily giving not only the peak resistance as in Fig. 8.15 (b) but also the ultimate resistance which is lower than the peak such as point b in the same figure, whereas the stress controlled gives only the peak values but not the smaller values after the peak is achieved. The stress-controlled test is preferred only in some special problems connected with research.

8.18 TYPES OF LABORATORY TESTS

The laboratory tests on soils may be on

1. Undisturbed samples, or
2. Remolded samples.

Further, the tests may be conducted on soils that are :

1. Fully saturated, or
2. Partially saturated.

The type of test to be adopted depends upon how best we can simulate the field conditions. Generally speaking, the various shear tests for soils may be classified as follows:

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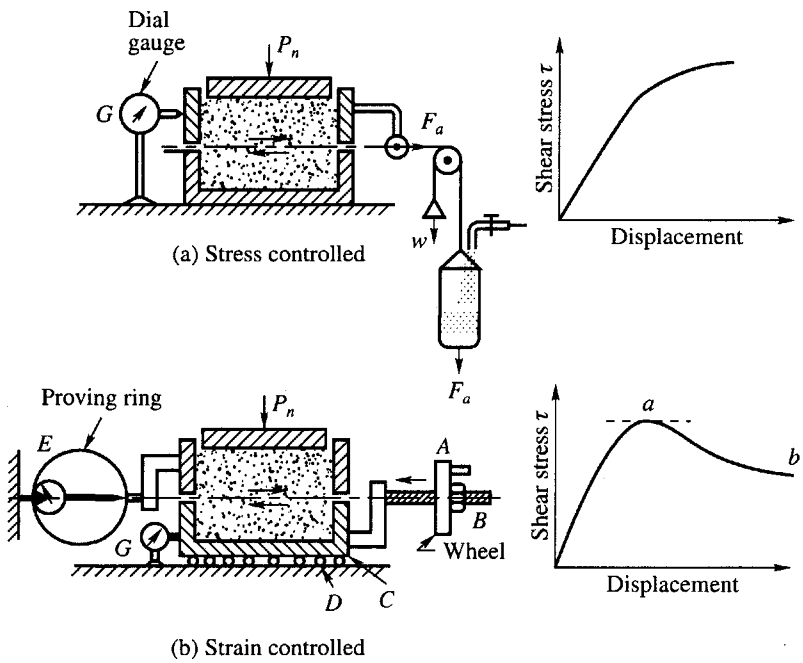

Figure 8.15 Stress and strain controlled box shear tests

1. Unconsolidated-Undrained Tests (UU)

The samples are subjected to an applied pressure under conditions in which drainage is prevented, and then sheared under conditions of no drainage.

2. Consolidated-Undrained or Quick Tests (CU)

The samples are allowed to consolidate under an applied pressure and then sheared under conditions of no drainage.

3. Consolidated-Drained or Slow Tests (CD)

The samples are consolidated as in the previous test, but the shearing is carried out slowly under conditions of no excess pressure in the pore space.

The drainage condition of a sample is generally the deciding factor in choosing a particular type of test in the laboratory. The purpose of carrying out a particular test is to simulate field conditions as far as possible. Because of the high permeability of sand, consolidation occurs relatively rapidly and is usually completed during the application of the load. Tests on sand are therefore generally carried out under drained conditions (drained or slow test).

For soils other than sands the choice of test conditions depends upon the purpose for which the shear strength is required. The guiding principle is that drainage conditions of the test should conform as closely as possible to the conditions under which the soils will be stressed in the field.

Undrained or quick tests are generally used for foundations on clay soils, since during the period of construction only a small amount of consolidation will have taken place and consequently the moisture content will have undergone little change. For clay slopes or cuts undrained tests are used both for design and for the investigation of failures.

Consolidated-undrained tests are used where changes in moisture content are expected to take place due to consolidation before the soil is fully loaded. An important example is the condition known as "sudden drawdown" such as that occurs in an earth dam behind which the water level is lowered at a faster rate than at which the material of the dam can consolidate. In the consolidated-undrained tests used in this type of problem, the consolidation pressures are chosen to represent the initial conditions of the soil, and the shearing loads correspond to the stresses called into play by the action of sudden drawdown.

As already stated, drained tests are always used in problems relating to sandy soils. In clay soils drained tests are sometimes used in investigating the stability of an earth dam, an embankment or a retaining wall after a considerable interval of time has passed.

Very fine sand, silts and silty sands also have poor drainage qualities. Saturated soils of these categories are likely to fail in the field under conditions similar to those under which consolidated quick tests are made.

Shearing Test Apparatus for the Various Types of Tests

The various types of shear tests mentioned earlier may be carried out either by the box shear test or the triaxial compression test apparatus. Tests that may be made by the two types of apparatus are:

Box Shear Test Apparatus

1. Undrained and consolidated- undrained tests on clay samples only.
2. Drained or Slow tests on any soil.

The box shear test apparatus is not suited for undrained or consolidated-undrained tests on samples other than clay samples, because the other soils are so permeable that even a rapid increase of the stresses in the sample may cause at least a noticeable change of the water content.

Triaxial Compression Test Apparatus

All types of tests can conveniently be carried out in this apparatus.

8.19 SHEARING STRENGTH TESTS ON SAND

Shear tests on sand may be made when the sand is either in a dry state or in a saturated state. No test shall be made when the soil is in a moist state as this state exists only due to apparent cohesion between particles which would be destroyed when it is saturated. The results of shear tests on saturated samples are almost identical with those on the same sand at equal relative density in a dry state except that the angle ϕ is likely to be 1 or 2 degrees smaller for the saturated sand.

The usual type of test used for coarse to medium sand is the slow shear test. However, consolidated undrained tests may be conducted on fine sands, sandy silts etc. which do not allow free drainage under changed stress conditions. If the equilibrium of a large body of saturated fine sand in an embankment is disturbed by rapid drawdown of the surface of an adjoining body of water, the change in water content of the fill lags behind the change in stress.

In all the shearing tests on sand, only the remolded samples are used as it is not practicable to obtain undisturbed samples. The soil samples are to be made approximately to the same dry density as it exists in-situ and tested either by direct shear or triaxial compression tests.

Tests on soils are generally carried out by the strain-controlled type apparatus. The principal advantage of this type of test on dense sand is that its peak-shear resistance, as well as the shear resistances smaller than the peak, can be observed and plotted.

Direct Shear Test

Only the drained or the slow shear tests on sand may be carried out by using the box shear test apparatus. The box is filled with sand to the required density. The sample is sheared at a constant

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vertical pressure σ. The shear stresses are calculated at various displacements of the shear box. The test is repeated with different pressures σ.

If the sample consists of loose sand, the shearing stress increases with increasing displacement until failure occurs. If the sand is dense, the shear failure of the sample is preceded by a decrease of the shearing stress from a peak value to an ultimate value (also known as residual value) lower than the peak value.

Typical stress-strain curves for loose and dense sands are shown in Fig. 8.16(a).

The shear stress of a dense sand increases from 0 to a peak value represented by point a, and then gradually decreases and reaches an ultimate value represented by point b. The sample of sand in a dense state is closely packed and the number of contact points between the particles are more than in the loose state. The soil grains are in an interlocked state. As the sample is subjected to shear stress, the stress has to overcome the resistance offered by the interlocked arrangement of the particles. Experimental evidence indicates that a significant percent of the peak strength is due to the interlocking of the grains. In the process of shearing one grain tries to slide over the other and the void ratio of the sample which is the lowest at the commencement of the test reaches the maximum value at point a, in the Fig 8.16(a). The shear stress also reaches the maximum value at this level. Any further increase of strain beyond this point is associated with a progressive disintegration of the structure of the sand resulting in a decrease in the shear stress. Experience shows that the change in void ratio due to shear depends on both the vertical load and the relative density of the sand. At very low vertical pressure, the void ratio at failure is larger and at very high pressure it is smaller than the initial void ratio, whatever the relative density of the sand may be. At

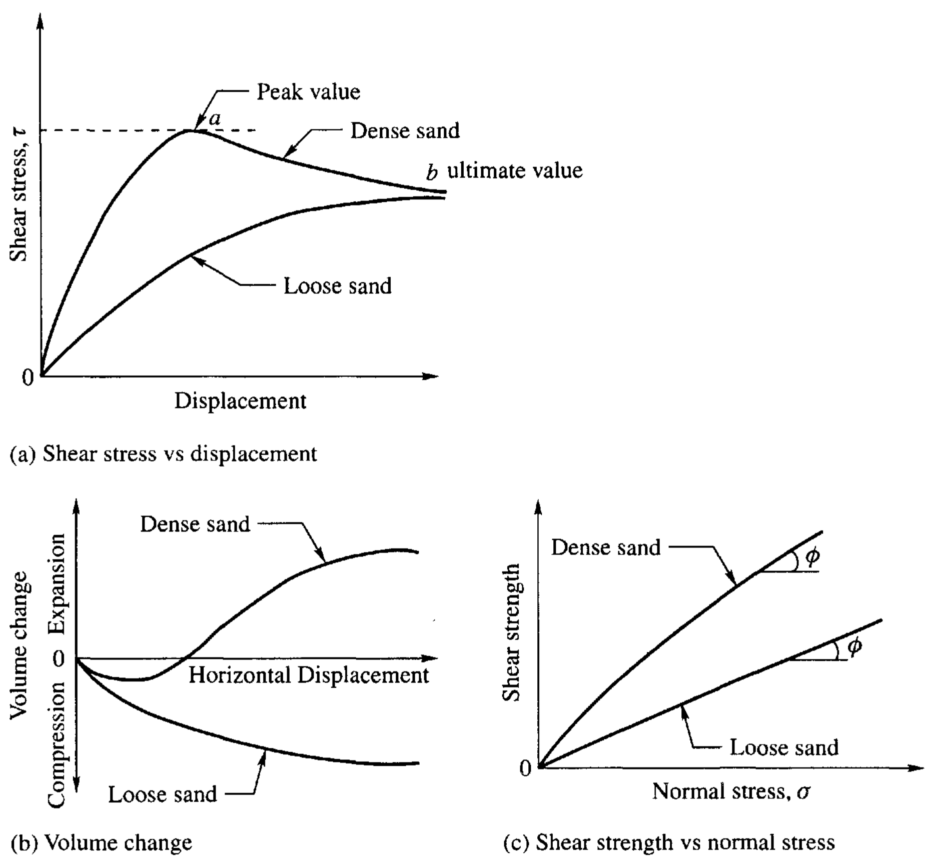

Figure 8.16 Direct shear test on sand

Table 8.1 Typical values of ϕ and $\bar{\phi}_u$ for granular soils

Types of soil	ϕ deg	$\bar{\phi}_u$ deg
Sand: rounded grains		
Loose	28 to 30	
Medium	30 to 35	26 to 30
Dense	35 to 38	
Sand: angular grains		
Loose	30 to 35	
Medium	35 to 40	30 to 35
Dense	40 to 45	
Sandy gravel	34 to 48	33 to 36

intermediate values of pressure, the shearing force causes a decrease in the void ratio of loose sand and an increase in the void ratio of dense sand. Fig 8.16(b) shows how the volume of dense sand decreases up to a certain value of horizontal displacement and with further displacement the volume increases, whereas in the case of loose sand the volume continues to decrease with an increase in the displacement. In saturated sand a decrease of the void ratio is associated with an expulsion of pore water, and an increase with an absorption of water. The expansion of a soil due to shear at a constant value of vertical pressure is called *dilatancy*. At some intermediate state or degree of density in the process of shear, the shear displacement does not bring about any change in volume, that is, density. The density of sand at which no change in volume is brought about upon the application of shear strains is called the *critical density*. The porosity and void ratio corresponding to the critical density are called the *critical porosity* and the *critical void ratio* respectively.

By plotting the shear strengths corresponding to the state of failure in the different shear tests against the normal pressure a straight line is obtained for loose sand and a slightly curved line for dense sand [Fig. 8.16(c)]. However, for all practical purposes, the curvature for the dense sand can be disregarded and an average line may be drawn. The slopes of the lines give the corresponding angles of friction ϕ of the sand. The general equation for the lines may be written as

$$s = \sigma \tan \phi$$

For a given sand, the angle ϕ increases with increasing relative density. For loose sand it is roughly equal to the *angle of repose*, defined as the angle between the horizontal and the slope of a heap produced by pouring clean dry sand from a small height. The angle of friction varies with the shape of the grains. Sand samples containing well graded angular grains give higher values of ϕ as compared to uniformly graded sand with rounded grains. The angle of friction ϕ for dense sand at peak shear stress is higher than that at ultimate shear stress. Table 8.1 gives some typical values of ϕ (at peak) and $\bar{\phi}_u$ (at ultimate).

Triaxial Compression Test

Reconstructed sand samples at the required density are used for the tests. The procedure of making samples should be studied separately (refer to any book on Soil Testing). Tests on sand may be conducted either in a saturated state or in a dry state. Slow or consolidated undrained tests may be carried out as required.

Drained or Slow Tests

At least three identical samples having the same initial conditions are to be used. For slow tests under saturated conditions the drainage valve should always be kept open. Each sample should be

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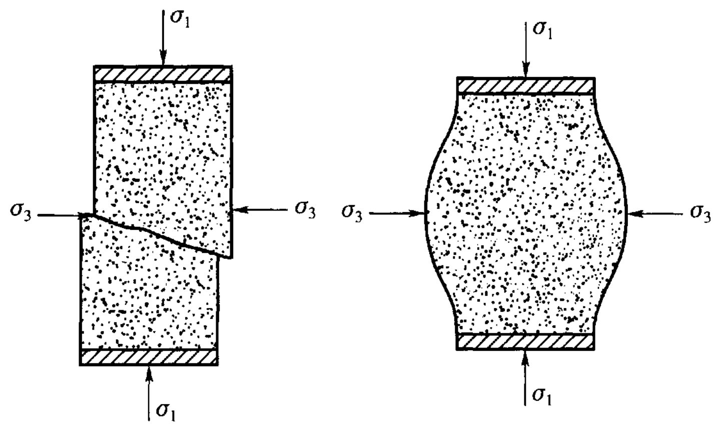
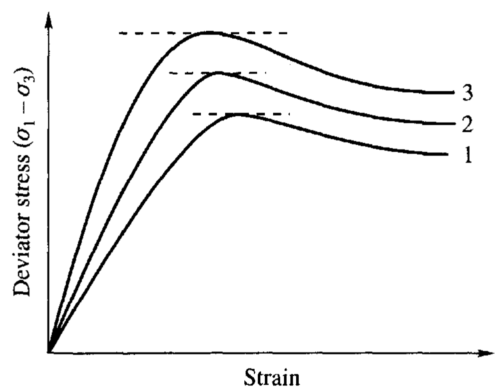
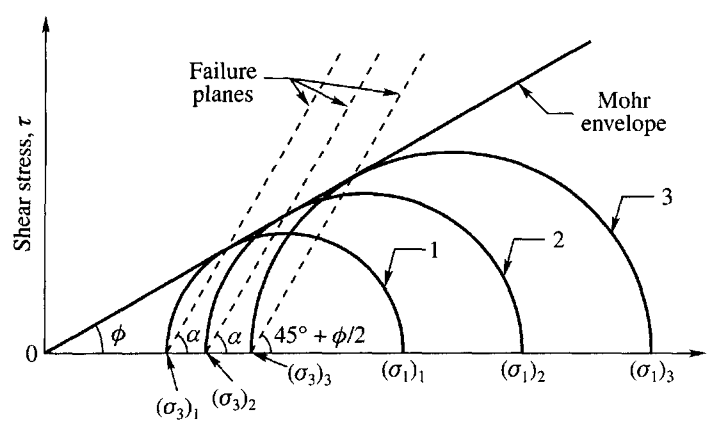


(a) Dense sand (b) Loose sand

Figure 8.17 Typical shapes of dense and loose sands at failure



(a) Stress-strain curves for three samples at dense state



(b) Mohr envelope

Figure 8.18 Mohr envelope for dense sand tested under different constant all-round pressures for example, 1, 2 and 3 kg/cm^2. Each sample is sheared to failure by increasing the vertical load at a sufficiently slow rate to prevent any build up of excess pore pressures.

At any stage of loading the major principal stress is the all-round pressure σ_3 plus the intensity of deviator stress $(\sigma_1 - \sigma_3)$. The actually applied stresses are the effective stresses in a slow test, that is $\sigma_1 = \sigma'_1$ and $\sigma_3 = \sigma'_3$, Dense samples fail along a clearly defined rupture plane whereas loose sand samples fail along many planes which result in a symmetrical bulging of the sample. The compressive strength of a sample is defined as the difference between the major and minor principal stresses at failure $(\sigma_1 - \sigma_3)_f$. Typical shapes of dense and loose sand samples at failure are shown in Fig. 8.17.

Typical stress-strain curves for three samples in a dense state and the Mohr circles for these samples at peak strength are shown in Fig. 8.18.

If the experiment is properly carried out there will be one common tangent to all these three circles and this will pass through the origin. This indicates that the Mohr envelope is a straight line for sand and the sand has no cohesion. The angle made by the envelope with the σ-axis is called the angle of internal friction. The failure planes for each of these samples are shown in Fig. 8.18(b). Each of them make an angle α with the horizontal which is approximately equal to

$$\alpha = 45° + \phi/2$$

From Fig. 8.18(b) an expression for the angle of internal friction may be written as

$$\sin\phi = \frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3} = \frac{\sigma_1 / \sigma_3 - 1}{\sigma_1 / \sigma_3 + 1} \tag{8.40}$$

Example 8.4

Determine the magnitude of the deviator stress if a sample of the same sand with the same void ratio as given in Ex. 8.3 was tested in a triaxial apparatus with a confining pressure of 60 kN/m^2.

Solution

In the case of a triaxial test on an identical sample of sand as given in Ex. 8.3, use the same Mohr envelope *OM* (Fig. Ex. 8.3). Now the point *F* on the abscissa gives the confining pressure $\sigma_3 = 60$ kN/m^2. A Mohr circle C_2 may now be drawn passing through point *F* and tangential to the Mohr envelope *OM*. The point *E* gives the major principal stress σ_1 for the triaxial test.

Now $\sigma_1 = OE = 188$ kN/m^2, $\sigma_3 = 60$ kN/m^2

Therefore $\sigma_1 - \sigma_3 = 188 - 60 = 128$ kN/m^2 = deviator stress

Example 8.5

A consolidated drained triaxial test was conducted on a granular soil. At failure $\sigma'_1/\sigma'_3 = 4.0$. The effective minor principal stress at failure was 100 kN/m^2. Compute ϕ' and the principal stress difference at failure.

Solution

$$\sin\phi' = \frac{\sigma'_1/\sigma'_3 - 1}{\sigma'_1/\sigma'_3 + 1} = \frac{4-1}{4+1} = 0.6 \quad \text{or} \quad \phi' = 37°$$

The principal stress difference at failure is

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$$(\sigma_1' - \sigma_3') = \sigma_3' \left(\frac{\sigma_{1f}'}{\sigma_{3f}'} - 1\right) = 100(4-1) = 300\ \text{kN/m}^2$$

Example 8.6

A drained triaxial test on sand with $\sigma_3' = 3150\ \text{lb/ft}^2$ gave $(\sigma_1'/\sigma_3')_f = 3.7$. Compute (a) σ_{1f}' (b) $(\sigma_1 - \sigma_3)_f$, and (c) ϕ'.

Solution

(a) $$\frac{\sigma_1'}{\sigma_3'} = 3.7$$

Therefore, $\sigma_1' = 3.7\sigma_3' = 3.7 \times 3150 = 11{,}655\ \text{lb/ft}^2$

(b) $(\sigma_1 - \sigma_3)_f = (\sigma_1' - \sigma_3')_f = 11{,}655 - 3150 = 8505\ \text{lb/ft}^2$

(c) $$\sin\phi' = \frac{\sigma_1'/\sigma_3' - 1}{\sigma_1'/\sigma_3' + 1} = \frac{3.7-1}{3.7+1} = 0.574 \quad \text{or} \quad \phi' = 35^\circ$$

Example 8.7

Assume the test specimen in Ex. 8.6 was sheared undrained at the same total cell pressure of $3150\ \text{lb/ft}^2$. The induced excess pore water pressure at failure u_f was equal to $1470\ \text{lb/ft}^2$. Compute:

(a) σ_{1f}'
(b) $(\sigma_1 - \sigma_3)_f$
(c) ϕ in terms of total stress,
(d) the angle of the failure plane α_f

Solution

(a) and (b): Since the void ratio after consolidation would be the same for this test as for Ex. 8.6, assume ϕ' is the same.

As before $$(\sigma_1 - \sigma_3)_f = \sigma_{3f}'\left(\frac{\sigma_1'}{\sigma_3'} - 1\right)$$

$$\sigma_{3f}' = \sigma_{3f} - u_f = 3150 - 1470 = 1680\ \text{lb/ft}^2$$

So $(\sigma_1 - \sigma_3)_f = 1680(3.7-1) = 4536\ \text{lb/ft}^2$

$$\sigma_{1f}' = (\sigma_1 - \sigma_3)_f + \sigma_{3f}' = 4536 + 1680 = 6216\ \text{lb/ft}^2$$

(c) $$\sin\phi_{\text{total}} = \frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3} = \frac{4536}{6216+1470} = 0.59 \quad \text{or} \quad \phi_{\text{total}} = 36.17^\circ$$

(d) From Eq. (8.18)

$$\alpha_f = 45° + \frac{\phi'}{2} = 45° + \frac{35}{2} = 62.5°$$

where ϕ' is taken from Ex. 8.6.

Example 8.8

A saturated specimen of cohesionless sand was tested under drained conditions in a triaxial compression test apparatus and the sample failed at a deviator stress of 482 kN/m^2 and the plane of failure made an angle of 60° with the horizontal. Find the magnitudes of the principal stresses. What would be the magnitudes of the deviator stress and the major principal stress at failure for another identical specimen of sand if it is tested under a cell pressure of 200 kN/m^2?

Solution

Per Eq. (8.18), the angle of the failure plane α is expressed as equal to

$$\alpha = 45° + \frac{\phi}{2}$$

Since $\alpha = 60°$, we have $\phi = 30°$.

From Eq. (8.40), $\sin\phi = \dfrac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3}$

with $\phi = 30°$, and $\sigma_1 - \sigma_3 = 482$ kN/m^2. Substituting we have

$$\sigma_1 + \sigma_3 = \frac{\sigma_1 - \sigma_3}{\sin\phi} = \frac{482}{\sin 30°} = 964 \text{ kN/m}^2 \tag{a}$$

$$\sigma_1 - \sigma_3 = 482 \text{ kN/m}^2 \tag{b}$$

solving (a) and (b) we have

$\sigma_1 = 723$ kN/m^2, and $\sigma_3 = 241$ kN/m^2

For the identical sample

$\phi = 30°$, $\sigma_3 = 200$ kN/m^2

From Eq. (8.40), we have

$$\sin 30° = \frac{\sigma_1 - 200}{\sigma_1 + 200}$$

Solving for σ_1 we have $\sigma_1 = 600$ kN/m^2 and $(\sigma_1 - \sigma_3) = 400$ kN/m^2

8.20 UNCONSOLIDATED-UNDRAINED TEST

Saturated Clay

Tests on saturated clay may be carried out either on undisturbed or on remolded soil samples. The procedure of the test is the same in both cases. A series of samples (at least a minimum of three) having the same initial conditions are tested under undrained conditions. With σ_3, the all-round pressure, acting on a sample under conditions of no drainage, the axial pressure is increased until failure occurs at a deviator stress $(\sigma_1 - \sigma_3)$. From the deviator stress, the major principal stress σ_1 is determined. If the other samples are tested in the same way but with different values of σ_3, it is found that for all types of saturated clay, the deviator stress at failure (compressive strength) is entirely independent of the magnitude of σ_3 as shown in Fig. 8.19. The diameters of all the Mohr circles are equal and the Mohr envelope is parallel to the σ-axis indicating that the angle of shearing resistance $\phi_u = 0$. The symbol ϕ_u represents the angle of shearing resistance under undrained conditions. Thus saturated clays behave as purely cohesive materials with the following properties:

$$\phi_u = 0, \text{ and } c_u = \frac{1}{2}(\sigma_1 - \sigma_3) \tag{8.41}$$

where c_u is the symbol used for cohesion under undrained conditions. Eq. (8.41) holds true for the particular case of an unconfined compression test in which $\sigma_3 = 0$. Since this test requires a very simple apparatus, it is often used, especially for field work, as a ready means of measuring the shearing strength of saturated clay, in this case

$$c_u = \frac{q_u}{2}, \text{ where } q_u = (\sigma_1 - \sigma_3)_f = (\sigma_1)_f \tag{8.42}$$

Effective Stresses

If during the test, pore-pressures are measured, the effective principal stresses may be written as

$$\sigma_1' = \sigma_1 - u$$
$$\sigma_3' = \sigma_3 - u \tag{8.43}$$

where u is the pore water pressure measured during the test. The effective deviator stress at failure may be written as

$$(\sigma_1' - \sigma_3')_f = (\sigma_1 - u)_f - (\sigma_3 - u)_f = (\sigma_1 - \sigma_3)_f \tag{8.44}$$

Eq. (8.44) shows that the deviator stress is not affected by the pore water pressure. As such the effective stress circle is only shifted from the position of the total stress circle as shown in Fig. 8.19.

Partially Saturated Clay

Tests on partially saturated clay may be carried out either on undisturbed or on remolded soil samples. All the samples shall have the same initial conditions before the test, i.e., they should possess the same water content and dry density. The tests are conducted in the same way as for saturated samples. Each sample is tested under undrained conditions with different all-round pressures σ_3.

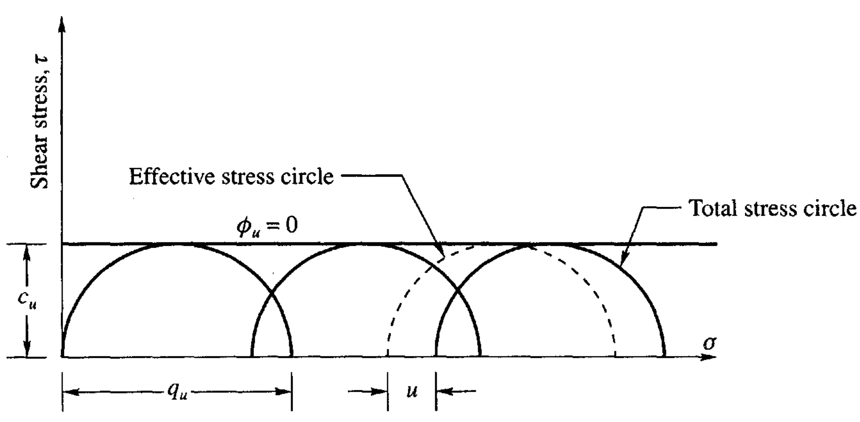

Figure 8.19 Mohr circle for undrained shear test on saturated clay

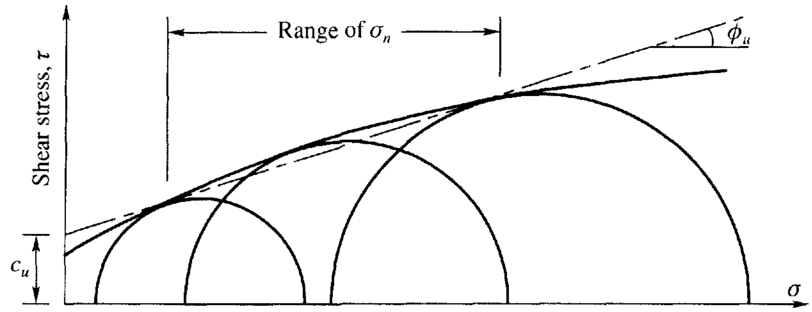

Figure 8.20 Mohr circle for undrained shear tests on partially saturated clay soils

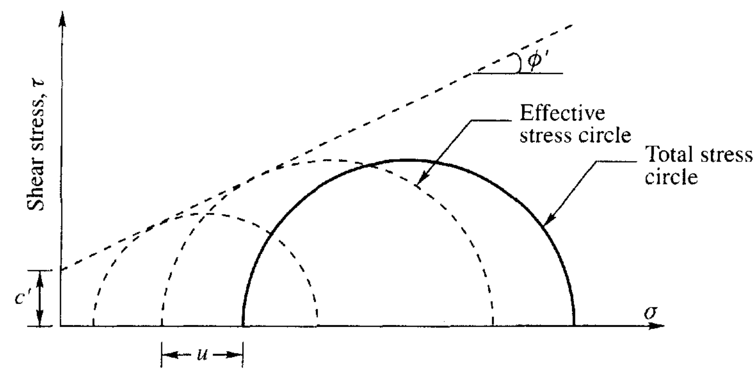

Figure 8.21 Effective stress circles for undrained shear tests on partially saturated clay soils

Mohr circles for three soil samples and the Mohr envelope are shown in Fig. 8.20. Though all the samples had the same initial conditions, the deviator stress increases with the increase in the all-round pressure σ_3 as shown in the figure. This indicates that the strength of the soil increases with increasing values of σ_3. The degree of saturation also increases with the increase in σ_3. The Mohr envelope which is curved at lower values of σ_3 becomes almost parallel to the σ-axis as full saturation is reached. Thus it is not strictly possible to quote single values for the parameters c_u and ϕ_u for partially saturated clays, but over any range of normal pressure σ_n encountered in a practical example, the envelope can be approximated by a straight line and the approximate values of c_u and ϕ_u can be used in the analysis.

Effective Stresses

If the pore pressures are measured during the test, the effective circles can be plotted as shown in Fig. 8.21 and the parameters c' and ϕ' obtained. The envelope to the Mohr circles, when plotted in terms of effective stresses, is linear.

Typical undrained shear strength parameters for partially saturated compacted samples are shown in Table 8.2.

8.21 UNCONFINED COMPRESSION TESTS

The unconfined compression test is a special case of a triaxial compression test in which the all-round pressure $\sigma_3 = 0$ (Fig. 8.22). The tests are carried out only on saturated samples which can stand without any lateral support. The test, is, therefore, applicable to cohesive soils only. The test

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Table 8.2 Probable undrained shear strength parameters for partially saturated soils

Types of soil	c_u (tsf)	ϕ_u	c' (tsf)	ϕ'
Sand with clay binder	0.80	23°	0.70	40°
Lean silty clay	0.87	13°	0.45	31°
Clay, moderate plasticity	0.93	9°	0.60	28°
Clay, very plastic	0.87	8°	0.67	22°

is an undrained test and is based on the assumption that there is no moisture loss during the test. The unconfined compression test is one of the simplest and quickest tests used for the determination of the shear strength of cohesive soils. These tests can also be performed in the field by making use of simple loading equipment.

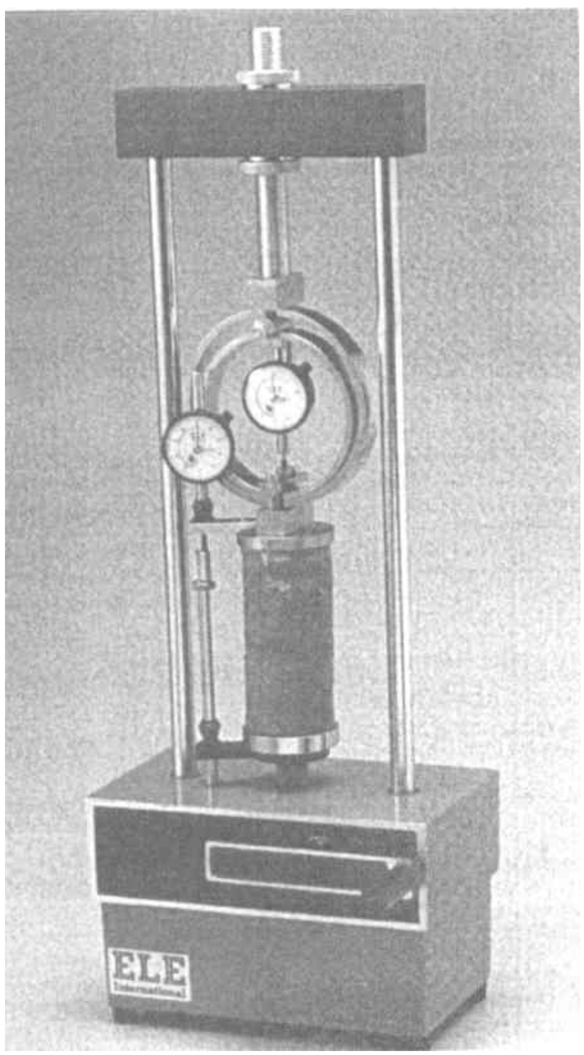

Figure 8.22 Unconfined compression test equipment (Courtesy: Soiltest)

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Any compression testing apparatus with arrangement for strain control may be used for testing the samples . The axial load σ_1 may be applied mechanically or pneumatically.

Specimens of height to diameter ratio of 2 are normally used for the tests. The sample fails either by shearing on an inclined plane (if the soil is of brittle type) or by bulging. The vertical stress at any stage of loading is obtained by dividing the total vertical load by the cross-sectional area. The cross-sectional area of the sample increases with the increase in compression. The cross-sectional area A at any stage of loading of the sample may be computed on the basic assumption that the total volume of the sample remains the same. That is

$$A_0 h_0 = Ah$$

where A_0, h_0 = initial cross-sectional area and height of sample respectively.

A, h = cross-sectional area and height respectively at any stage of loading

If Δh is the compression of the sample, the strain is

$$\varepsilon = \frac{\Delta h}{h_0} \quad \text{since} \quad \Delta h = h_0 - h, \text{ we may write}$$

$$A_0 h_0 = A(h_0 - \Delta h)$$

Therefore, $$A = \frac{A_0 h_0}{h_0 - \Delta h} = \frac{A_0}{1 - \Delta h / h_0} = \frac{A_0}{1 - \varepsilon} \tag{8.45}$$

The average vertical stress at any stage of loading may be written as

$$\sigma_1 = \frac{P}{A} = \frac{P(1-\varepsilon)}{A_0} \tag{8.46}$$

where P is the vertical load at the strain ε.

Using the relationship given by Eq. (8.46) stress-strain curves may be plotted. The peak value is taken as the unconfined compressive strength q_u, that is

$$(\sigma_1)_f = q_u \tag{8.47}$$

The unconfined compression test (UC) is a special case of the unconsolidated-undrained (UU) triaxial compression test (TX-AC). The only difference between the UC test and UU test is that a total confining pressure under which no drainage was permitted was applied in the latter test. Because of the absence of any confining pressure in the UC test, a premature failure through a weak zone may terminate an unconfined compression test. For typical soft clays, premature failure is not likely to decrease the undrained shear strength by more than 5%. Fig 8.23 shows a comparison of undrained shear strength values from unconfined compression tests and from triaxial compression tests on soft-Natsushima clay from Tokyo Bay. The properties of the soil are:

Natural moisture content w_n = 80 to 90%

Liquid limit w_n= 100 to 110%

Plasticity index I_p = 60%

There is a unique relationship between remolded undrained shear strength and the liquidity index, I_p, as shown in Fig. 8.24 (after Terzaghi et al., 1996). This plot includes soft clay soil and silt deposits obtained from different parts of the world.

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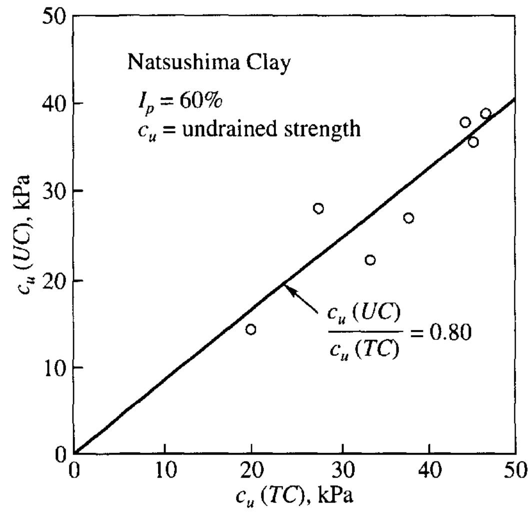

Figure 8.23 Relation between undrained shear strengths from unconfined compression and triaxial compression tests on Natsushima clay (data from Hanzawa and Kishida, 1982)

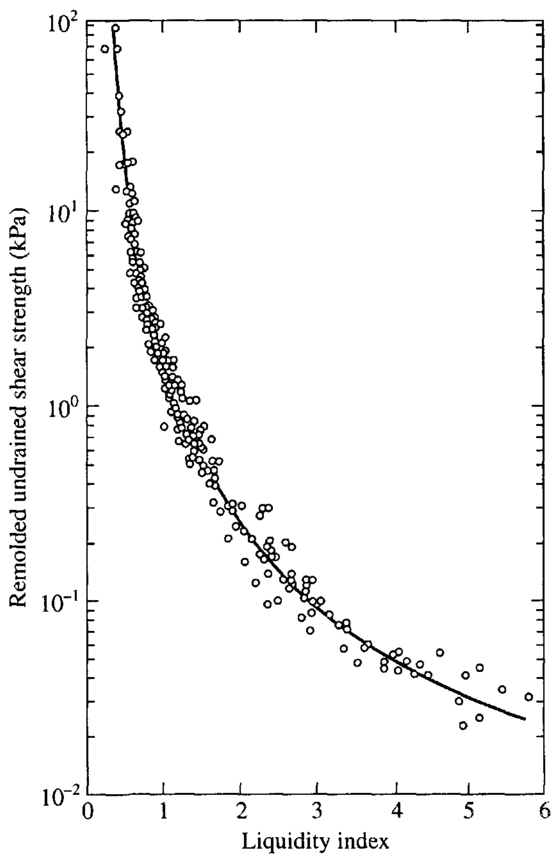

Figure 8.24 Relation between undrained shear strength and liquidity index of clays from around the world (after Terzaghi et al., 1996)

Example 8.9

Boreholes reveal that a thin layer of alluvial silt exists at a depth of 50 ft below the surface of the ground. The soil above this level has an average dry unit weight of 96 lb/ft^3 and an average water content of 30%. The water table is approximately at the surface. Tests on undisturbed samples give the following data: c_u = 1008 lb/ft^2, ϕ_u = 13°, c_d = 861 lb/ft^2, ϕ_d = 23°. Estimate the shearing resistance of the silt on a horizontal plane (a) when the shear stress builds up rapidly, and (b) when the shear stress builds up very slowly.

Solution

Bulk unit weight $\gamma_t = \gamma_d(1+w) = 96 \times 1.3 = 124.8 \text{ lb/ft}^3$

Submerged uint weight $\gamma_b = 124.8 - 62.4 = 62.4 \text{ lb/ft}^3$

Total normal pressure at 50 ft depth = 50 × 124.8 = 6240 lb/ft^2

Effective pressure at 50 ft depth = 50 × 62.4 = 3120 lb/ft^2

(a) For rapid build-up, use the properties of the undrained state and total pressure.

At a total pressure of 6240 lb/ft^2

shear strength, $s = c + \sigma \tan \phi$ = 1008 + 6240 tan 13° = 2449 lb/ft^2

(b) For slow build-up, use effective stress properties

At an effective stress of 3120 lb/ft^2,

shear strength = 861 + 3120 tan 23° = 2185 lb/ft^2

Example 8.10

When an undrained triaxial compression test was conducted on specimens of clayey silt, the following results were obtained:

Specimen No.	1	2	3
σ_3 (kN/m^2)	17	44	56
σ_1 (kN/m^2)	157	204	225
u (kN/m^2)	12	20	22

Determine the values of shear parameters considering (a) total stresses and (b) effective stresses.

Solution

(a) Total stresses

For a solution with total stresses, draw Mohr circles C_1, C_2 and C_3 for each of the specimens using the corresponding principal stresses σ_1 and σ_3.

Draw a Mohr envelope tangent to these circles as shown in Fig. Ex. 8.10. Now from the figure

$c = 48 \text{ kN/m}^2$, $\phi = 15°$

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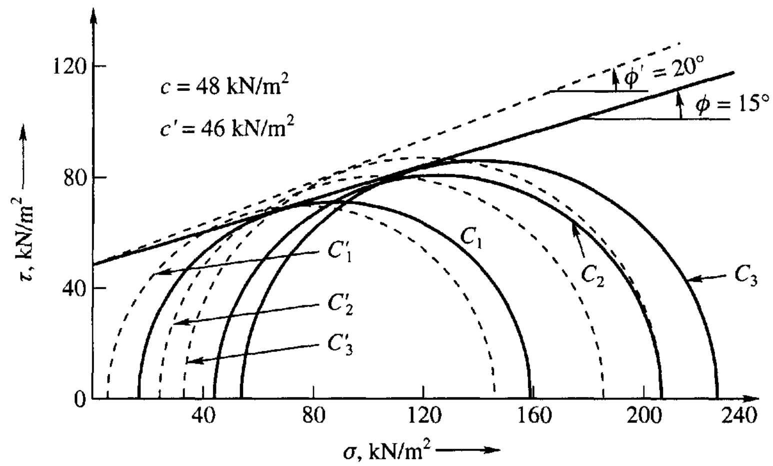


Figure Ex. 8.10

(b) With effective stresses

The effective principal stresses may be found by subtracting the pore pressures u from the total principal stresses as given below.

Specimen No.	1	2	3
$\sigma'_3 = (\sigma_3 - u)$ kN/m^2	5	24	34
$\sigma'_1 = (\sigma_1 - u)$ kN/m^2	145	184	203

As before draw Mohr circles C'_1, C'_2 and C'_3 for each of the specimens as shown in Fig. Ex. 8.10. Now from the figure

$c' = 46$ kN/m^2, $\phi' = 20°$

Example 8.11

A soil has an unconfined compressive strength of 120 kN/m^2. In a triaxial compression test a specimen of the same soil when subjected to a chamber pressure of 40 kN/m^2 failed at an additional stress of 160 kN/m^2. Determine:

(i) The shear strength parameters of the soil, (ii) the angle made by the failure plane with the axial stress in the triaxial test.

Solution

There is one unconfined compression test result and one triaxial compression test result. Hence two Mohr circles, C_1, and C_2 may be drawn as shown in Fig. Ex. 8.11. For Mohr circle C_1, $\sigma_3 = 0$ and $\sigma_1 = 120$ kN/m^2, and for Mohr circle C_2, $\sigma_3 = 40$ kN/m^2 and $\sigma_1 = (40 + 160) = 200$ kN/m^2. A common tangent to these two circles is the Mohr envelope which gives

(i) $c = 43$ kN/m^2 and $\phi = 19°$

(ii) For the triaxial test specimen, A is the point of tangency for Mohr circle C_2 and C is the center of circle C_2. The angle made by AC with the abscissa is equal to twice the angle between the failure plane and the axis of the sample = 2θ. From Fig. Ex. 8.11, $2\theta = 71°$ and $\theta = 35.5°$. The angle made by the failure plane with the σ-axis is $\alpha = 90° - 35.5° = 54.5°$.

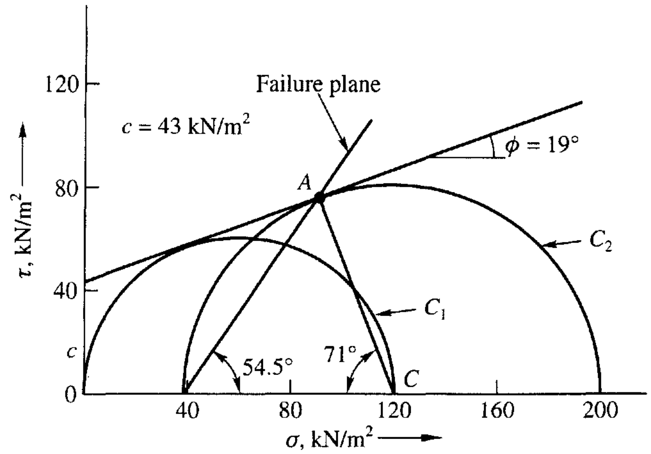


Figure Ex. 8.11

Example 8.12

A cylindrical sample of saturated clay 4 cm in diameter and 8 cm high was tested in an unconfined compression apparatus. Find the unconfined compression strength, if the specimen failed at an axial load of 360 N, when the axial deformation was 8 mm. Find the shear strength parameters if the angle made by the failure plane with the horizontal plane was recorded as 50°.

Solution

Per Eq. (8.46), the unconfined compression strength of the soil is given by

$$\sigma_1 = \frac{P(1-\varepsilon)}{A_o}, \quad \text{where } P = 360\ N$$

$$A_o = \frac{3.14}{4} \times (4)^2 = 12.56\ \text{cm}^2, \quad \varepsilon = \frac{0.8}{8} = 0.1$$

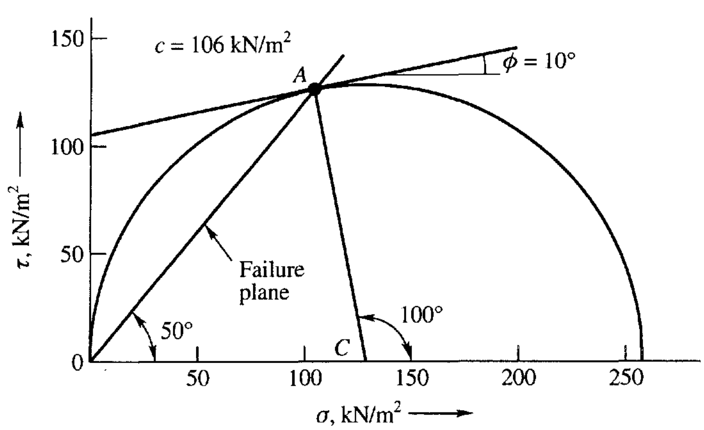


Figure Ex. 8.12

Therefore $\sigma_1 = \dfrac{360(1-0.1)}{12.56} = 25.8\ \mathrm{N/cm^2} = 258\ \mathrm{kN/m^2}$

Now $\phi = 2\alpha - 90°$ (Refer to Fig. 8.12) where $\alpha = 50°$. Therefore $\phi = 2 \times 50 - 90° = 10°$.

Draw the Mohr circle as shown in Fig. Ex. 8.12 ($\sigma_3 = 0$ and $\sigma_1 = 258$ kN/m^2) and from the center C of the circle, draw CA at $2\alpha = 100°$. At point A, draw a tangent to the circle. The tangent is the Mohr envelope which gives

$c = 106$ kN/m^2, and $\phi = 10°$

Example 8.13

An unconfined cylindrical specimen of clay fails under an axial stress of 5040 lb/ft^2. The failure plane was inclined at an angle of 55° to the horizontal. Determine the shear strength parameters of the soil.

Solution

From Eq. (8.20),

$$\sigma_1 = \sigma_3 N_\phi + 2c\sqrt{N_\phi}, \quad \text{where} \quad N_\phi = \tan^2 45° + \frac{\phi}{2}$$

since $\sigma_3 = 0$, we have

$$\sigma_1 = 2c\sqrt{N_\phi} = 2c\tan\left(45° + \frac{\phi}{2}\right), \quad \text{where } \sigma_1 = 5040\ \mathrm{lb/ft^2} \tag{a}$$

From Eq. (8.18), the failure angle α is

$$\alpha = 45° + \frac{\phi}{2}, \quad \text{since } \alpha = 55°, \quad \text{we have}$$

$$\phi = (55 - 45) \times 2 = 20°$$

From Eq. (a),

$$c = \frac{\sigma_1}{2\tan\ 45° + \dfrac{\phi}{2}} = \frac{5040}{2\tan 55°} = 1765\ \mathrm{lb/ft^2}$$

Example 8.14

A cylindrical sample of soil having a cohesion of 80 kN/m^2 and an angle of internal friction of 20° is subjected to a cell pressure of 100 kN/m^2.

Determine: (i) the maximum deviator stress ($\sigma_1 - \sigma_3$) at which the sample will fail, and (ii) the angle made by the failure plane with the axis of the sample.

Graphical solution

$\sigma_3 = 100$ kN/m^2, $\phi = 20°$, and $c = 80$ kN/m^2.

A Mohr circle and the Mohr envelope can be drawn as shown in Fig. Ex. 8.14(a). The circle cuts the σ-axis at B (= σ_3), and at E (= σ_1). Now $\sigma_1 = 433$ kN/m^2, and $\sigma_3 = 100$ kN/m^2.

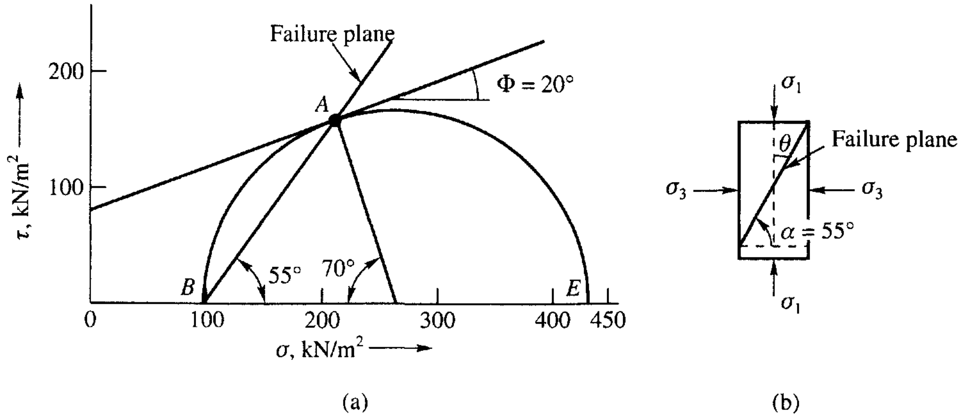


Figure Ex. 8.14

$(\sigma_1 - \sigma_3) = 433 - 100 = 333$ kN/m^2.

Analytical solution

Per Eq. (8.20)

$$\sigma_1 = \sigma_3 \tan^2\left(45° + \frac{\phi}{2}\right) + 2c\tan\left(45° + \frac{\phi}{2}\right)$$

Substituting the known values, we have

$\tan(45° + \phi/2) = \tan(45° + 10) = \tan 55° = 1.428$

$\tan^2(45° + \phi/2) = 2.04$.

Therefore,

$\sigma_1 = 100 \times 2.04 + 2 \times 80 \times 1.428 \approx 433$ kN/m^2

$(\sigma_1 - \sigma_3) = (433 - 100) = 333$ kN/m^2

If θ = angle made by failure planes with the axis of the sample, (Fig. Ex. 8.14(b))

$2\theta = 90 - \phi = 90 - 20 = 70°$ or $\theta = 35°$.

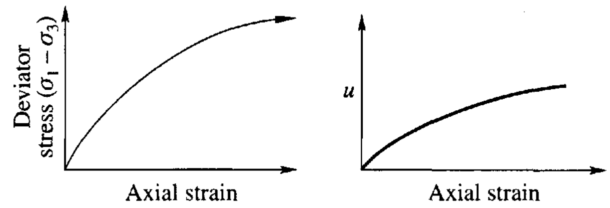
Therefore, the angle made by the failure plane with the σ-axis is

$\alpha = 90 - 35 = 55°$

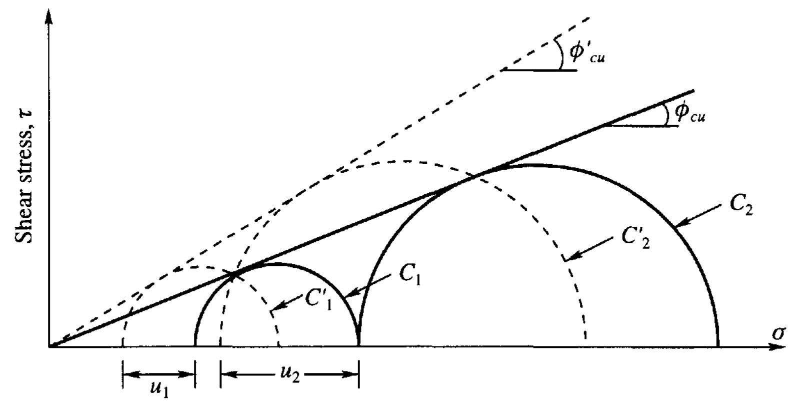
8.22 CONSOLIDATED-UNDRAINED TEST ON SATURATED CLAY

Normally Consolidated Saturated Clay

If two clay samples 1 and 2 are consolidated under ambient pressures of p_1 and p_2 and are then subjected to undrained triaxial tests without further change in cell pressure, the results may be expressed by the two Mohr circles C_1 and C_2 respectively as shown in Fig. 8.25(b). The failure envelope tangential to these circles passes through the origin and its slope is defined by ϕ_{cu}, the angle of shearing resistance in consolidated undrained tests. If the pore pressures are measured the effective stress Mohr circles C'_1 and C'_2 can also be plotted and the slope of this envelope is ϕ'_{cu}. The effective principal stresses are:



(a) Variation of $(\sigma_1 - \sigma_3)$ and u with axial strain



(b) Mohr envelope

Figure 8.25 Normally consolidated clay under undrained triaxial test

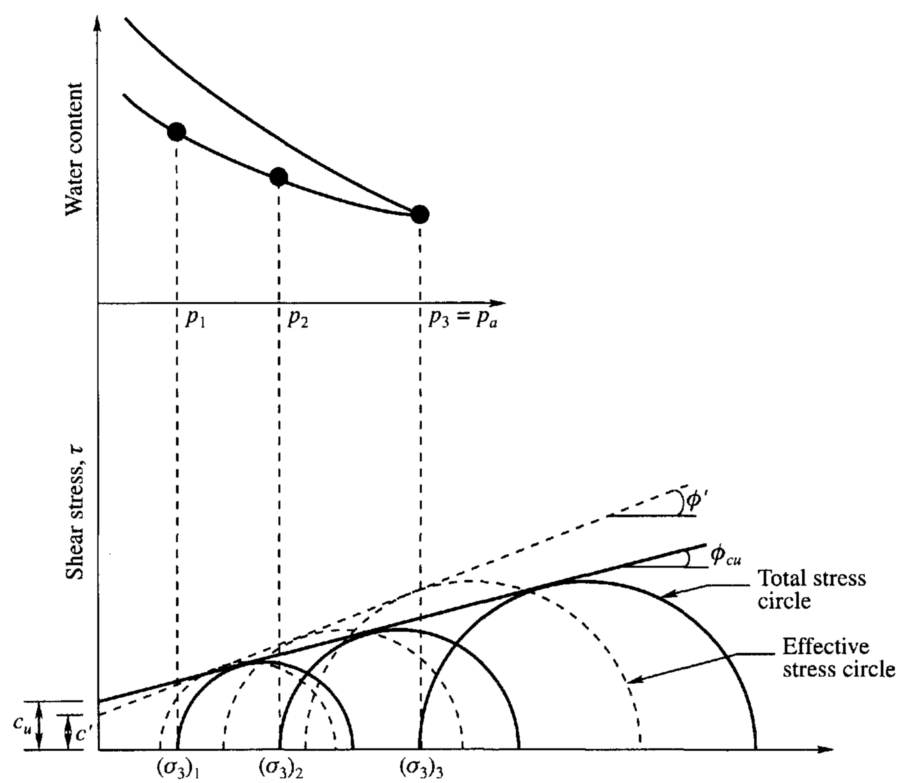


Figure 8.26 Consolidated-undrained tests on saturated overconsolidated clay

$$\sigma'_{11} = \sigma_{11} - u_1; \; \sigma'_{12} = \sigma_{12} - u_2$$

$$\sigma'_{31} = \sigma_{31} - u_1; \; \sigma'_{32} = \sigma_{32} - u_2$$

where u_1 and u_2 are the pore water pressures for the samples 1 and 2 respectively.

It is an experimental fact that the envelopes to the total and effective stress circles are linear. Fig. 8.25(a) shows the nature of the variation on the deviator stress ($\sigma_1 - \sigma_3$) and the pore water pressure u in the specimen during the test with the axial strain. The pore water pressure builds up during shearing with a corresponding decrease in the volume of the sample.

Overconsolidated Clay

Let a saturated sample 1 be consolidated under an ambient pressure p_a and then allowed to swell under the pressure p_1. An undrained triaxial test is carried out on this sample under the all-round pressure $p_1(= \sigma_{31})$. Another sample 2 is also consolidated under the same ambient pressure p_a and allowed to swell under the pressure $p_2(= \sigma_{32})$. An undrained triaxial test is carried out on this sample under the same all-round pressure p_2. The two Mohr circles are plotted and the Mohr envelope tangential to the circles is drawn as shown in Fig. 8.26. The shear strength parameters are c_u and ϕ_{cu}. If pore water pressure is measured, effective stress Mohr circles may be plotted as shown in the figure. The strength parameters for effective stresses are represented by c' and ϕ'.

8.23 CONSOLIDATED-DRAINED SHEAR STRENGTH TEST

In drained triaxial tests the soil is first consolidated under an ambient pressure p_a and then subjected to an increasing deviator stress until failure occurs, the rate of strain being controlled in such a way that at no time is there any appreciable pore-pressure in the soil. Thus at all times the applied stresses are effective, and when the stresses at failure are plotted in the usual manner, the failure envelope is directly expressed in terms of effective stresses. For normally consolidated clays and for sands the envelope is linear for normal working stresses and passes through the origin as shown in Fig. 8.27. The failure criterion for such soils is therefore the angle of shearing resistance in the drained condition ϕ_d.

The drained strength is

$$\frac{1}{2}(\sigma_1 - \sigma_3)_f = \frac{p \sin \phi_d}{1 - \sin \phi_d} \tag{8.48}$$

Eq. (8.48) is obtained from Eq. (8.38)

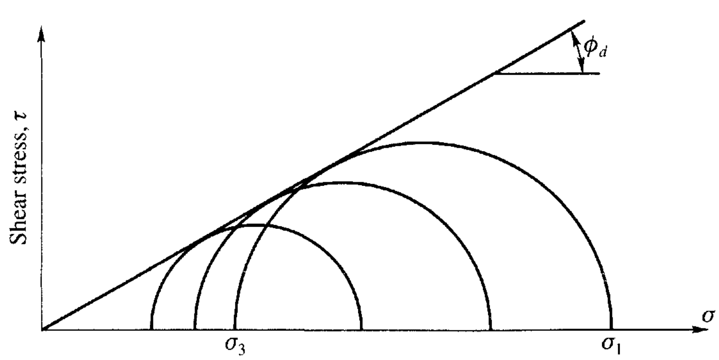


Figure 8.27 Drained tests on normally consolidated clay samples

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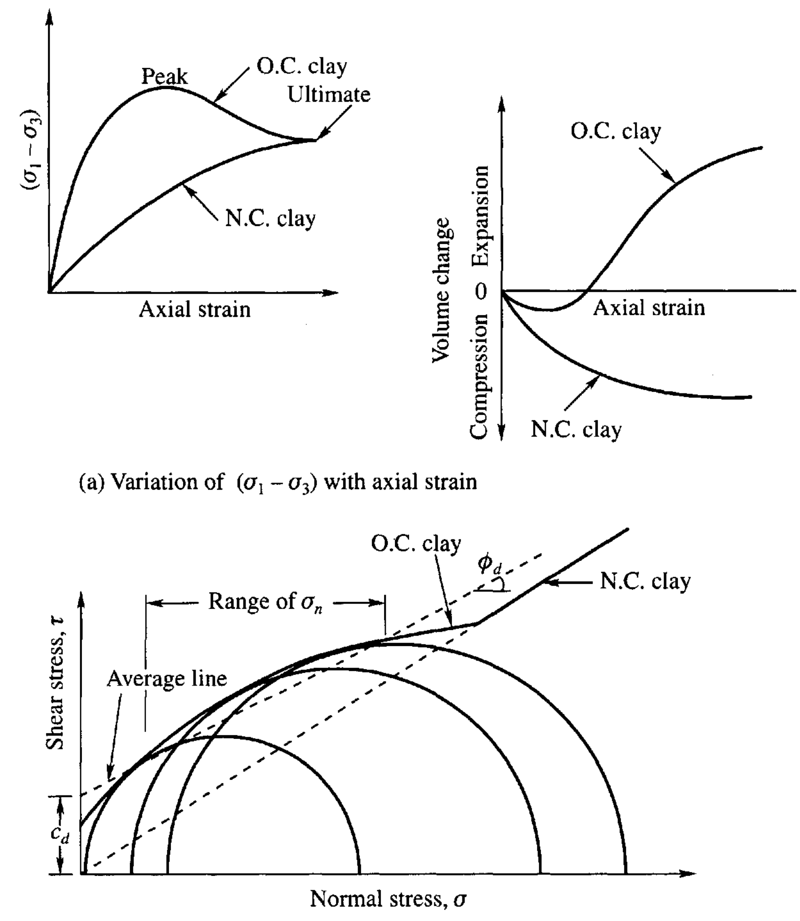

(a) Variation of $(\sigma_1 - \sigma_3)$ with axial strain

(b) Mohr envelope

Figure 8.28 Drained tests on overconsolidated clays

For overconsolidated clays, the envelope intersects the axis of zero pressure at a value c_d. The apparent cohesion in the drained test and the strength are given by the expression.

$$\frac{1}{2}(\sigma_1 - \sigma_3)_f = \frac{c_d \cos\phi_d + p\sin\phi_d}{1 - \sin\phi_d} \tag{8.49}$$

The Mohr envelope for overconsolidated clays is not linear as may be seen in Fig. 8.28(b). An average line is to be drawn within the range of normal pressure σ_n. The shear strength parameters c_d and ϕ_d are referred to this line.

Since the stresses in a drained test are effective, it might be expected that a given ϕ_d would be equal to ϕ' as obtained from undrained tests with pore-pressure measurement. In normally consolidated clays and in loose sands the two angles of shearing resistance are in fact closely equal since the rate of volume change in such materials at failure in the drained test is approximately zero and there is no volume change throughout an undrained test on saturated soils. But in dense sands and heavily overconsolidated clays there is typically a considerable rate of positive volume change at failure in drained tests, and work has to be done not only in overcoming the shearing resistance of the soils, but also in increasing the volume of the specimen against the ambient pressure. Yet in

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undrained tests on the same soils, the volume change is zero and consequently ϕ_d for dense sands and heavily overconsolidated clays is greater than ϕ'. Fig. 8.28(a) shows the nature of variation of the deviator stress with axial strain. During the application of the deviator stress, the volume of the specimen gradually reduces for normally consolidated clays. However, overconsolidated clays go through some reduction of volume initially but then expand.

8.24 PORE PRESSURE PARAMETERS UNDER UNDRAINED LOADING

Soils in nature are at equilibrium under their overburden pressure. If the same soil is subjected to an instantaneous additional loading, there will be development of pore pressure if drainage is delayed under the loading. The magnitude of the pore pressure depends upon the permeability of the soil, the manner of application of load, the stress history of the soil, and possibly many other factors. If a load is applied slowly and drainage takes place with the application of load, there will practically be no increase of pore pressure. However, if the hydraulic conductivity of the soil is quite low, and if the loading is relatively rapid, there will not be sufficient time for drainage to take place. In such cases, there will be an increase in the pore pressure in excess of the existing hydrostatic pressure. It is therefore necessary many times to determine or estimate the excess pore pressure for the various types of loading conditions. Pore pressure parameters are used to express the response of pore pressure to changes in total stress *under undrained conditions*. Values of the parameters may be determined in the laboratory and can be used to predict pore pressures in the field under similar stress conditions.

Pore Pressure Parameters Under Triaxial Test Conditions

A typical stress application on a cylindrical element of soil under triaxial test conditions is shown in Fig. 8.29 ($\Delta\sigma_1 > \Delta\sigma_3$). Δu is the increase in the pore pressure without drainage. From Fig. 8.29, we may write

$\Delta u_3 = B\Delta\sigma_3$, $\Delta u_1 = AB(\Delta\sigma_1 - \Delta\sigma_3)$, therefore,

$$\Delta u = \Delta u_1 + \Delta u_3 = B[\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)] \tag{8.50}$$

$$\text{or } \Delta u = B\Delta\sigma_3 + \bar{A}(\Delta\sigma_1 - \Delta\sigma_3) \tag{8.51}$$

where, $\bar{A} = AB$

for saturated soils $B = 1$, so

$$\Delta u = \Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3) \tag{8.52}$$

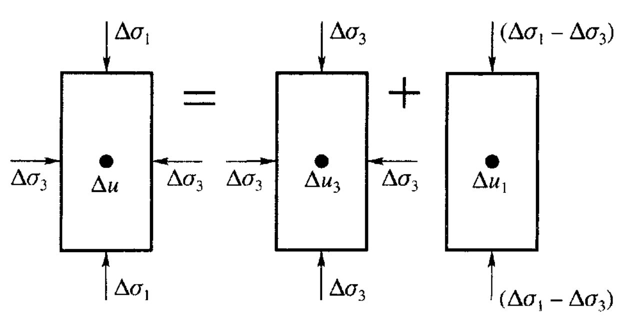

Figure 8.29 Excess water pressure under triaxial test conditions

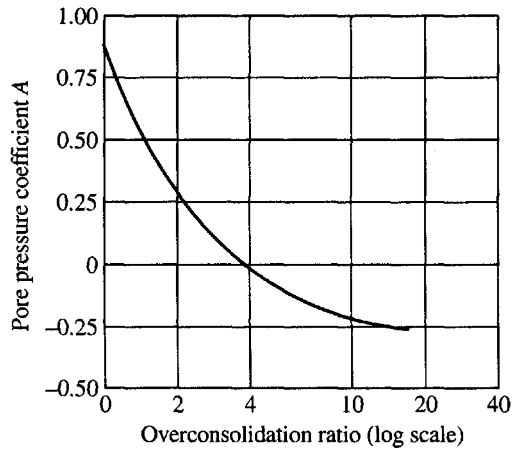

Figure 8.30 Relationship between overconsolidation ratio and pore pressure coefficient A

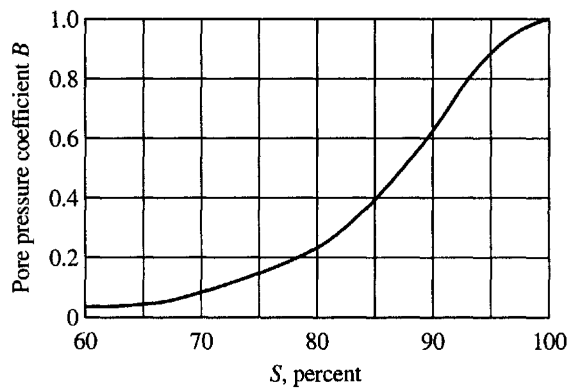

Figure 8.31 Typical relationship between B and degree of saturation S.

where A and B are called pore pressure parameters. The variation of A under a failure condition (A_f) with the overconsolidation ratio, O_{CR}, is given in Fig. 8.30. Some typical values of A_f are given in Table 8.3. The value of B varies with the degree of saturation as shown in Fig. 8.31.

Table 8.3 Typical values of A_f

Type of Soil	Volume change	A_f
Highly sensitive clay	large contraction	+ 0.75 to + 1.5
Normally consolidated clay	contraction	+ 0.5 to + 1.0
Compacted sandy clay	slight contraction	+ 0.25 to + 0.75
Lightly overconsolidated clay	none	+ 0.00 to + 0.5
Compacted clay gravel	expansion	− 0.25 to + 0.25
Heavily overconsolidated clay	expansion	− 0.5 to 0

8.25 VANE SHEAR TESTS

From experience it has been found that the vane test can be used as a reliable in-situ test for determining the shear strength of soft-sensitive clays. It is in deep beds of such material that the vane test is most valuable, for the simple reason that there is at present no other method known by which the shear strength of these clays can be measured. Repeated attempts, particularly in Sweden, have failed to obtain undisturbed samples from depths of more than about 10 meters in normally consolidated clays of high sensitivity even using the most modern form of thin-walled piston samplers. In these soils the vane is indispensable. The vane should be regarded as a method to be used under the following conditions:

1. The clay is normally consolidated and sensitive.
2. Only the undrained shear strength is required.

It has been determined that the vane gives results similar to those obtained from unconfined compression tests on undisturbed samples.

The soil mass should be in a saturated condition if the vane test is to be applied. The vane test cannot be applied to partially saturated soils to which the angle of shearing resistance is not zero.

Description of the Vane

The vane consists of a steel rod having at one end four small projecting blades or vanes parallel to its axis, and situated at 90° intervals around the rod. A post hole borer is first employed to bore a hole up to a point just above the required depth. The rod is pushed or driven carefully until the vanes are embedded at the required depth. At the other end of the rod above the surface of the ground a torsion head is used to apply a horizontal torque and this is applied at a uniform speed of about 0.1° per sec until the soil fails, thus generating a cylinder of soil. The area consists of the peripheral surface of the cylinder and the two round ends. The first moment of these areas divided by the applied moment gives the unit shear value of the soil. Fig. 8.32(*a*) gives a diagrammatic sketch of a field vane.

Determination of Cohesion or Shear Strength of Soil

Consider the cylinder of soil generated by the blades of the vane when they are inserted into the undisturbed soil in-situ and gradually turned or rotated about the axis of the shaft or vane axis. The turning moment applied at the torsion head above the ground is equal to the force multiplied by the eccentricity.

Let the force applied = P eccentricity (lever arm) = x units

Turning moment = Px

The surface resisting the turning is the cylindrical surface of the soil and the two end faces of the cylinder.

Therefore,

$$\text{resisting moment} = (2\pi r \times L \times c_u \times r + 2\pi r^2 \times c_u \times 0.67r) = 2\pi r^2 c_u(L + 0.67r)$$

where r = radius of the cylinder and c_u the undrained shear strength.

At failure the resisting moment of the cylinder of soil is equal to the turning moment applied at the torsion head.

Therefore, $Px = 2\pi r^2 c_u(L + 0.67r)$

$$c_u = \frac{Px}{2\pi r^2(L + 0.67r)} \tag{8.53}$$

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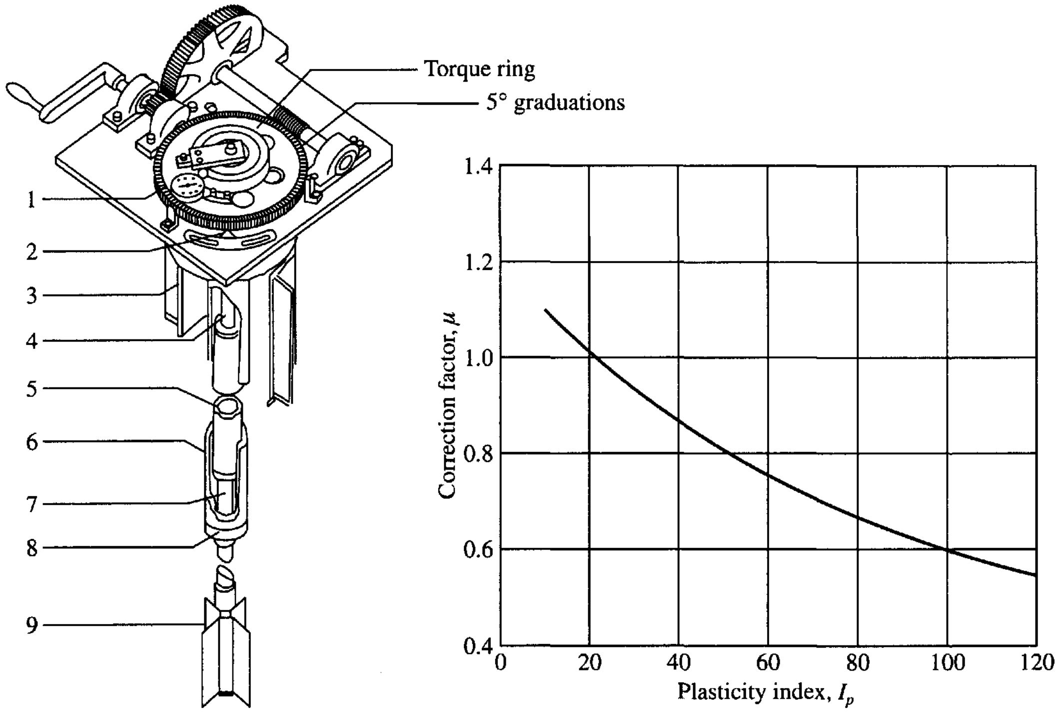

1. Straingauge for reading torque
2. Rotation indicator
3. 8-in casing with side fins for anchoring torque assembly
4. Torque rod
5. A-rod for applying torque to vane. Made up in 5-ft lengths
6. BX casing for housing torque rod and A rod
7. Vane rod
8. BX-casing-point containing bearing and water seals for vane rod
9. Vane varying sizes
   2 in dia by 4 in
   3 in dia by 6 in
   4 in dia by 8 in

(a) (b)

Figure 8.32 Vane shear test (a) diagrammatic sketch of a field vane, (b) correction factor μ (Bjerrum, 1973)

The standard dimensions of field vanes as recommended by ASTM (1994) are given in Table 8.4.

Some investigators believe that vane shear tests in cohesive soil gives a values of the shear strength about 15 per cent greater than in unconfined compression tests. There are others who believe that vane tests give lower values.

Table 8.4 Recommended dimensions of field vanes (ASTM, 1994)

Casing size	Height, mm (L)	Diameter, mm (d)	Blade thickness mm	Diameter of rod mm
AX	76.2	38.1	1.6	12.7
BX	101.6	50.8	1.6	12.7
NX	127.0	63.5	3.2	12.7

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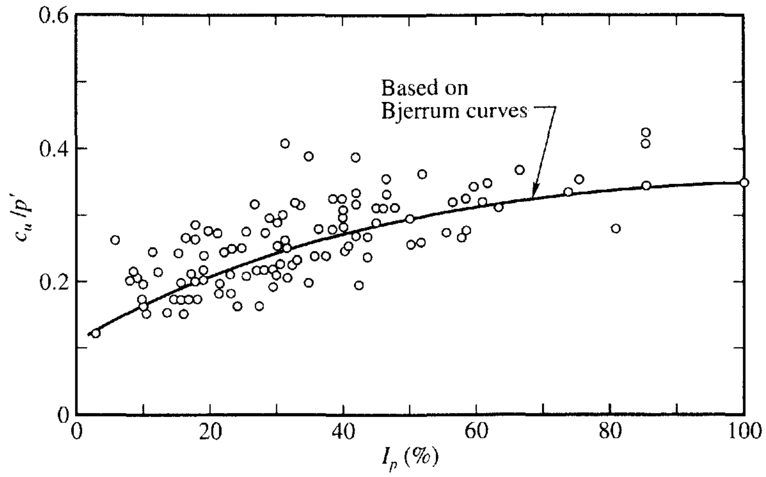


Figure 8.33 Undrained shear strengths from field vane tests on inorganic soft clays and silts (after Tavenas and Leroueil, 1987)

Bjerrum (1973) back computed a number of embankment failures on soft clay and concluded that the vane shear strength tended to be too high. Fig. 8.32(b) gives correction factors for the field vane test as a function of plasticity index, I_p (Ladd et al., 1977). We may write

$$c_u \text{ (field)} = \mu c_u \text{ (vane)} \tag{8.54}$$

where μ is the correction factor (Fig. 8.32b).

Fig. 8.33 give relationships between plasticity index I_p and c_u/p' where c_u is the undrained shear strength obtained by field vane and p' the effective overburden pressure. This plot is based on comprehensive test data compiled of Tavenas and Leroueil (1987). Necessary correction factors have been applied to the data as per Fig. 8.32 (b) before plotting.

8.26 OTHER METHODS FOR DETERMINING UNDRAINED SHEAR STRENGTH OF COHESIVE SOILS

We have discussed in earlier sections three methods for determining the undrained shear strength of cohesive soils. They are

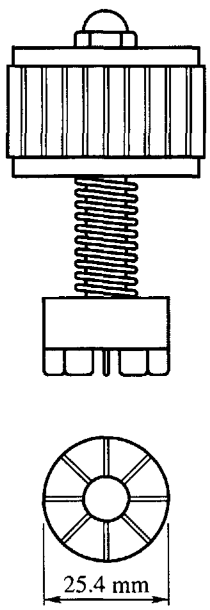
1. Unconfined compression test
2. UU triaxial test
3. Vane shear test

In this section two more methods are discussed. The instruments used for this purpose are

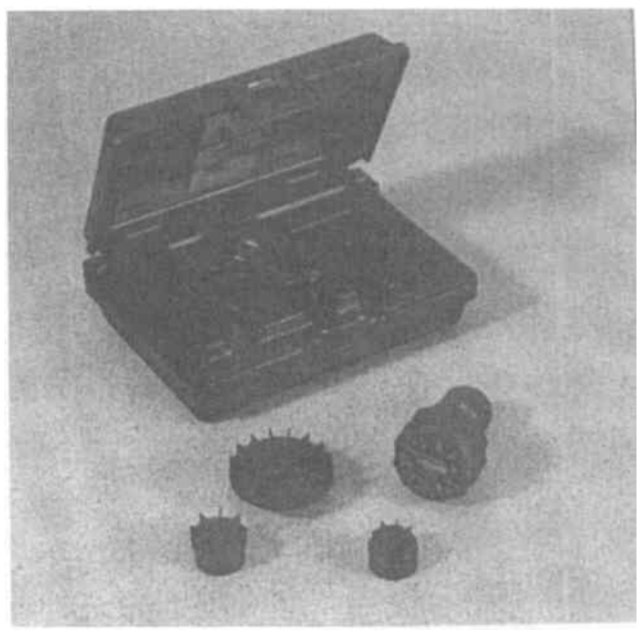
1. Torvane (TV)
2. Pocket penetrometer (PP)

Torvane

Torvane, a modification of the vane, is convenient for investigating the strength of clays in the walls of test pits in the field or for rapid scanning of the strength of tube or split spoon samples. Fig 8.34(a) gives a diagrammatic sketch of the instrument. Figure 8.34(b) gives a photograph of the same. The vanes are pressed to their full depth into the clay below a flat surface, whereupon a torque is applied through a calibrated spring until the clay fails along the cylindrical surface



(a)



(b)

Figure 8.34 Torvane shear device (a) a diagrammatic sketch, and (b) a photograph (Courtesy: Soiltest)

circumscribing the vanes and simultaneously along the circular surface constituting the base of the cylinder. The value of the shear strength is read directly from the indicator on the calibrated spring.

Specification for three sizes of vanes are given below (Holtz et al., 1981)

Diameter (mm)	Height of vane (mm)	Maximum shear strength (kPa)
19	3	250
25	5	100 (standard)
48	5	20

Pocket Penetrometer

Figure 8.35 shows a pocket penetrometer (Holtz et al., 1981) which can be used to determine undrained shear strength of clay soils both in the laboratory and in the field. The procedure consists in pushing the penetrometer directly into the soil and noting the strength marked on the calibrated spring.

Figure 8.35 Pocket penetrometer (PP), a hand-held device which indicates unconfined compressive strength (Courtesy: Soiltest, USA)

8.27 THE RELATIONSHIP BETWEEN UNDRAINED SHEAR STRENGTH AND EFFECTIVE OVERBURDEN PRESSURE

It has been discussed in previous sections that the shear strength is a function of effective consolidation pressure. If a relationship between undrained shear strength, c_u, and effective consolidation pressure p' can be established, we can determine c_u if p' is known and vice versa. If a soil stratum in nature is normally consolidated the existing effective overburden pressure p_0' can be determined from the known relationship. But in overconsolidated natural clay deposits, the preconsolidation pressure p_c' is unknown which has to be estimated by any one of the available methods. If there is a relationship between p_c' and c_u, c_u can be determined from the known value of p_c'. Alternatively, if c_u is known, p_c' can be determined. Some of the relationships between c_u and p' are presented below. A typical variation of undrained shear strength with depth is shown in Fig. 8.36 for both normally consolidated and heavily overconsolidated clays. The higher shear strength as shown in Fig. 8.36(a) for normally consolidated clays close to the ground surface is due to desiccation of the top layer of soil.

Skempton (1957) established a relationship which may be expressed as

$$\frac{c_u}{p'} = 0.10 + 0.004\, I_p \tag{8.55}$$

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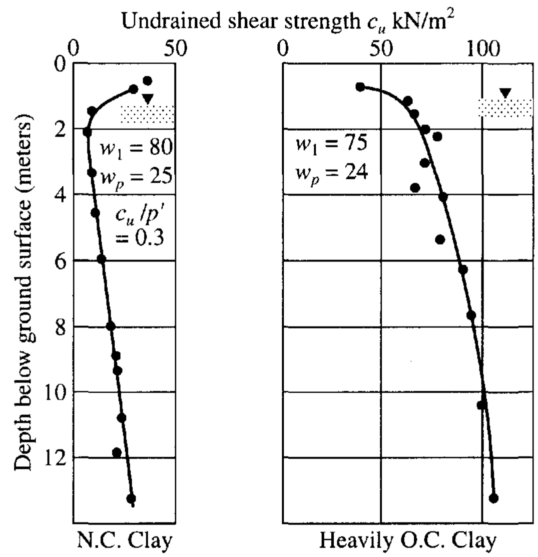

Figure 8.36 Typical variations of undrained shear strength with depth (After Bishop and Henkel, 1962)

He found a close correlation between c_u/p' and I_p as illustrated in Fig. 8.37. Though the Eq. (8.55) was originally meant for normally consolidated clays, it has been used for overconsolidated clays also. p' may be replaced by p_0' as the existing effective overburden pressure for normally consolidated clays, and by p_c' for overconsolidated clays. Peck et al., (1974) has extensively used this relationship for determining preconsolidation pressure p_c'. Eq. (8.55) may also be used for determining p_c' indirectly. If p_c' can be determined independently, the value of the undrained shear strength c_u for overconsolidated clays can be obtained from Eq. (8.55). The values of c_u so obtained may be checked with the values determined in the laboratory on undisturbed samples of clay.

Bjerrum and Simons (1960) proposed a relationship between c_u/p' and plasticity index I_p *as*

$$\frac{c_u}{p'} = 0.45\,(I_p)^{\frac{1}{2}} \text{ for } I_p > 5\% \tag{8.56}$$

The scatter is expected to be of the order of ± 25 percent of the computed value.

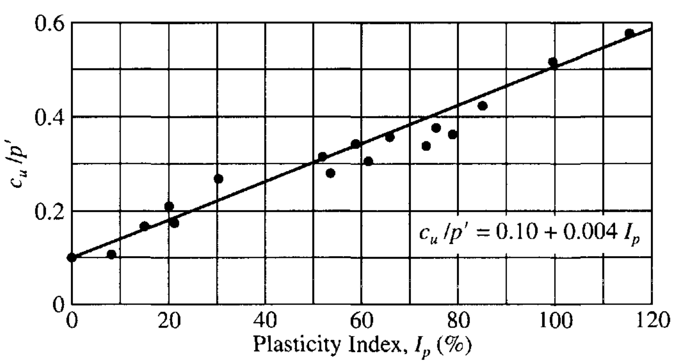

Figure 8.37 Relation between c_u/p' and plasticity index

Another relationship expressed by them is

$$\frac{c_u}{p'} = 0.18\,(I_l)^{-\frac{1}{2}} \text{ for } I_l > 0.5 \tag{8.57}$$

where I_l is the liquidity index. The scatter is found to be of the order of ± 30 percent.

Karlsson and Viberg (1967) proposed a relationship as

$$\frac{c_u}{p'} = 0.005w_l \text{ for } w_l > 20 \text{ percent} \tag{8.58}$$

where w_l is the liquid limit in percent. The scatter is of the order of ± 30 percent.

The engineer has to use judgment while selecting any one of the forms of relationships mentioned above.

c_u/p' Ratio Related to Overconsolidation Ratio p_c'/p_0'

Ladd and Foott (1974) presented a non-dimensional plot (Fig. 8.38) giving a relationship between a non-dimensional factor N_f and overconsolidation ratio OCR. Figure 8.38 is based on direct simple shear tests carried out on five clays from different origins. The plot gives out a trend but requires further investigation.

The non-dimensional factor N_f is defined as

$$N_f = \frac{(c_u/p_0')_{NC}}{(c_u/p_0')_{OC}} \tag{8.59}$$

where p_0' = existing overburden pressure

OC = overconsolidated

NC = normally consolidated

From the plot in Fig. 8.38 the shear strength c_u of overconsolidated clay can be determined if p_0' and $(c_u/p_0')_{NC}$ are known.

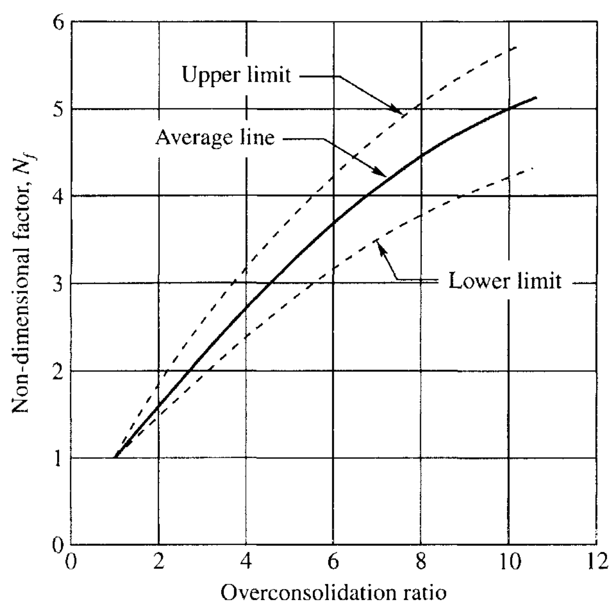

Figure 8.38 Relationship between N_f and overconsolidation ratio OCR (Ladd and Foott, 1974)

Example 8.15

A normally consolidated clay was consolidated under a stress of 3150 lb/ft^2, then sheared undrained in axial compression. The principal stress difference at failure was 2100 lb/ft^2, and the induced pore pressure at failure was 1848 lb/ft^2. Determine (a) the Mohr-Coulomb strength parameters, in terms of both total and effective stresses analytically. (b) compute $(\sigma_1/\sigma_3)_f$ and $(\sigma'_1/\sigma'_3)_f$, and (c) determine the theoretical angle of the failure plane in the specimen.

Solution

The parameters required are: effective parameters c' and ϕ', and total parameters c and ϕ.

(a) Given $\sigma_{3f} = 3150$ lb/ft^2, and $(\sigma_1 - \sigma_3)_f = 2100$ lb/ft^2. The total principal stress at failure σ_{1f} is obtained from

$\sigma_{1f} = (\sigma_1 - \sigma_3)_f + \sigma_{3f} = 2100 + 3150 = 5250$ lb/ft^2

Effective $\sigma'_{1f} = \sigma_{1f} - u_f = 5250 - 1848 = 3402$ lb/ft^2

$\sigma'_{3f} = \sigma_{3f} - u_f = 3150 - 1848 = 1302$ lb/ft^2

Now $\sigma_1 = \sigma_3 \tan^2(45° + \phi/2) + 2c \tan(45° + \phi/2)$

Since the soil is normally consolidated, $c = 0$. As such

$$\frac{\sigma_1}{\sigma_3} = \tan^2(45° + \phi/2) = \frac{1+\sin\phi}{1-\sin\phi}, \quad \text{or } \sin\phi = \frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3}$$

$$\text{Total } \phi = \sin^{-1}\frac{2100}{5250+3150} = \sin^{-1}\frac{2100}{8400} = 14.5°$$

$$\text{Effective } \phi' = \sin^{-1}\frac{2100}{3402+1302} = \sin^{-1}\frac{2100}{4704} = 26.5°$$

(b) The stress ratios at failure are

$$\frac{\sigma_1}{\sigma_3} = \frac{5250}{3150} = 1.67, \quad \frac{\sigma'_1}{\sigma'_3} = \frac{3402}{1302} = 2.61$$

(c) From Eq. (8.18)

$$\alpha_f = 45° + \frac{\phi'}{2} = 45° + \frac{26.5}{2} = 58.25°$$

The above problem can be solved graphically by constructing a Mohr-Coulomb envelope.

Example 8.16

The following results were obtained at failure in a series of consolidated-undrained tests, with pore pressure measurement, on specimens of saturated clay. Determine the values of the effective stress parameters c' and ϕ' by drawing Mohr circles.

σ_3 kN/m^2	$\sigma_1 - \sigma_3$ kN/m^2	u_w kN/m^2
150	192	80
300	341	154
450	504	222

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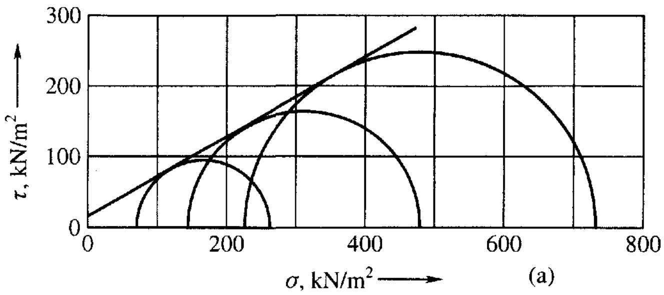

Figure Ex. 8.16

Solution

The values of the effective principal stresses σ'_1 and σ'_3 at failure are tabulated below

σ_3 kN/m²	σ_1 kN/m²	σ'_3 kN/m²	σ'_1 kN/m²
150	342	70	262
300	641	146	487
450	954	228	732

The Mohr circles in terms of effective stresses and the failure envelope are drawn in Fig. Ex. 8.16. The shear strength parameters as measured are :

$c' = 16$ kN/m²; $\phi' = 29°$

Example 8.17

The following results were obtained at failure in a series of triaxial tests on specimens of a saturated clay initially 38 mm in diameter and 76 mm long. Determine the values of the shear strength parameters with respect to (a) total stress, and (b) effective stress.

Type of test	σ_3 kN/m²	Axial load (N)	Axial compression (mm)	Volume change (cm³)
(a) Undrained	200	222	9.83	–
	400	215	10.06	–
	600	226	10.28	–
(b) Drained	200	467	10.81	6.6
	400	848	12.26	8.2
	600	1265	14.17	9.5

Solution

The principal stress difference at failure in each test is obtained by dividing the axial load by the cross-sectional area of the specimen at failure. The corrected cross-sectional area is calculated from Eq. (8.45). There is, of course, no volume change during an undrained test on a saturated clay. The initial values of length, area and volume for each specimen are $h_0 = 76$ mm, $A_0 = 11.35$ cm²; $V_0 = 86.0$ cm³ respectively.

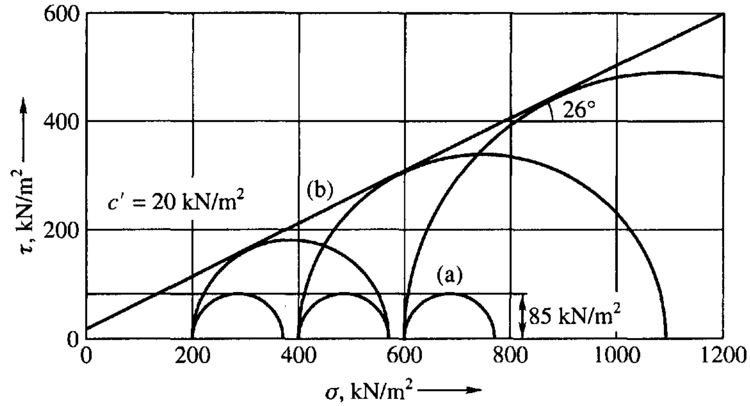

Figure Ex. 8.17

The Mohr circles at failure and the corresponding failure envelopes for both series of tests are shown in Fig. Ex. 8.17. In both cases the failure envelope is the line nearest to the common tangent to the Mohr circles. The total stress parameters representing the undrained strength of the clay are:

c_u = 85 kN/m^2; $\phi_u = 0$

The effective stress parameters, representing the drained strength of the clay, are:

c' = 20 kN/m^2; $\phi = 26°$

	σ_3 kN/m^2	$\Delta h/h_0$	$\Delta V/V_0$	Area (corrected) cm^2	$\sigma_1 - \sigma_3$ kN/m^2	σ_1 kN/m^2
a	200	0.129		13.04	170	370
	400	0.132		13.09	160	564
	600	0.135	-	13.12	172	772
b	200	0.142	0.077	12.22	382	582
	400	0.161	0.095	12.25	691	1091
	600	0.186	0.110	12.40	1020	1620

Example 8.18

An embankment is being constructed of soil whose properties are c' = 1071 lb/ft^2, ϕ' = 21° (all effective stresses), and γ = 99.85 lb/ft^3. The pore pressure parameters as determined from triaxial tests are A = 0.5, and B = 0.9. Find the shear strength of the soil at the base of the embankment just after the height of fill has been raised from 10 ft to 20 ft. Assume that the dissipation of pore pressure during this stage of construction is negligible, and that the lateral pressure at any point is one-half of the vertical pressure.

Solution

The equation for pore pressure is [Eq. (8.51)]

$$\Delta u = B\left[\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)\right]$$

$\Delta\sigma_1$ = Vertical pressure due to 10 ft of fill = 10 × 99.85 = 998.5 lb/ft^2

$$\Delta\sigma_3 = \frac{998.5}{2} = 499.25 \text{ lb/ft}^2$$

Therefore, $\Delta u = 0.9[499.25 + 0.5 \times 499.25] = 674 \text{ lb/ft}^2$

Original pressure, $\sigma_1 = 10 \times 99.85 = 998.5 \text{ lb/ft}^2$

Therefore, $\sigma' = \sigma_1 + \Delta\sigma_1 - \Delta u$

$$= 998.5 + 998.5 - 674 = 1323 \text{ lb/ft}^2$$

Shear strength, $s = c' + \sigma' \tan\phi' = 1071 + 1323 \tan 21^\circ = 1579 \text{ lb/ft}^2$

Example 8.19

At a depth of 6 m below the ground surface at a site, a vane shear test gave a torque value of 6040 N-cm. The vane was 10 cm high and 7 cm across the blades. Estimate the shear strength of the soil.

Solution

Per Eq. (8.53)

$$c_u = \frac{\text{Torque } (T)}{2\pi r^2 [L + (2/3) r]}$$

where T = 6040 N-cm, L = 10 cm, r = 3.5 cm.
substituting,

$$c_u = \frac{6040}{2 \times 3.14 \times 3.5^2 (10 + 0.67 \times 3.5)} = 6.4 \text{ N/cm}^2 = 64 \text{ kN/m}^2$$

Example 8.20

A vane 11.25 cm long, and 7.5 cm in diameter was pressed into soft clay at the bottom of a borehole. Torque was applied to cause failure of soil. The shear strength of clay was found to be 37 kN/m^2. Determine the torque that was applied.

Solution

From Eq. (8.53),

Torque, $T = c_u [2\pi r^2 (L + 0.67 r)]$ where $c_u = 37 \text{ kN/m}^2 = 3.7 \text{ N/cm}^2$

$$= 3.7[2 \times 3.14 \times (3.75)^2 (11.25 + 0.67 \times 3.75)] = 4500 \text{ N-cm}$$

8.28 GENERAL COMMENTS

One of the most important and the most controversial engineering properties of soil is its shear strength. The behavior of soil under external load depends on many factors such as arrangement of particles in the soil mass, its mineralogical composition, water content, stress history and many others. The types of laboratory tests to be performed on a soil sample depends upon the type of soil and its drainage condition during the application of external loads in the field. It is practically very difficult (if not impossible) to obtain undisturbed samples of granular soils from the field for laboratory tests. In such cases laboratory tests on remolded samples are mostly of academic interest. The angle of shearing resistance of granular soils is normally determined by the relationships established between ϕ and penetration resistance in the field. The accuracy of this method is sufficient for all practical purposes. The penetrometer used may be standard penetration equipment or static cone penetrometer. Shear strength tests on cohesive soils depend mostly on the accuracy with which undisturbed samples can be obtained from the field.

Undisturbed samples are extracted in sampling tubes of diameter 75 or 100 mm. Samples for triaxial tests are extracted in the laboratory from the samples in the sampling tubes by using sample extractors. Samples may be disturbed at both the stages of extraction. If we are dealing with a highly overconsolidated clay the disturbance is greater at both the stages. Besides there is another major disturbance which affects the test results very significantly. A highly overconsolidated clay is at equilibrium in its *in-situ* conditions. The overconsolidation pressures of such soils could be of the order 1000 kPa (10 tsf) or more. The standard penetration value N in such deposits could be 100 or more. The shear strength of such a soil under the *in-situ* condition could be in the order of 600 kPa or more. But if an undisturbed sample of such a soil is tested in standard triaxial equipment, the shear strength under undrained conditions could be very low. This is mostly due to the cracks that develop on the surface of the samples due to the relief of the *in-situ* overburden pressure on the samples. Possibly the only way of obtaining the *in-situ* strength in the laboratory is to bring back the state of the sample to its original field condition by applying all-around pressures on the samples equal to the estimated overconsolidation pressures. This may not be possible in standard triaxial equipment due to its limitations. The present practice is therefore to relate the *in-situ* shear strength to some of the field tests such as standard penetration tests, static cone penetration tests or pressuremeter tests.

8.29 QUESTIONS AND PROBLEMS

8.1 Explain Coulomb's equation for shear strength of a soil. Discuss the factors that affect the shear strength parameters of soil.

8.2 Explain the method of drawing a Mohr circle for a cylindrical sample in a triaxial test. Establish the geometrical relationships between the stresses on the failure plane and externally applied principal stresses.

8.3 Classify the shear tests based on drainage conditions. Explain how the pore pressure variation and volume change take place during these tests. Enumerate the field conditions which necessitate each of these tests.

8.4 What are the advantages and disadvantages of a triaxial compression test in comparison with a direct shear test?

8.5 For what types of soils, will the unconfined compression test give reliable results? Draw a Mohr circle for this test. How do you consider the change in the area of the specimen which takes place during the test in the final results?

8.6 What types of field tests are necessary for determining the shear strength parameters of sensitive clays? Derive the relationships that are useful for analyzing the observations of this test.

8.7 For loose and dense sands, draw the following typical diagrams:

(i) deviator stress vs. linear strain, and

(ii) volumetric strain vs. linear strain. Discuss them.

8.8 Discuss the effects of drainage conditions on the shear strength parameters of clay soil.

8.9 A direct shear test on specimens of fine sand gave the following results:

Normal stress (lb/ft^2)	2100	3700	4500
Shearing stress (lb/ft^2)	970	1700	2080

Determine :

(i) the angle of internal friction of the soil, and

(ii) shear strength of the soil at a depth of 15 ft from the ground surface.

The specific gravity of solids is 2.65, void ratio 0.7 and the ground water table is at a depth of 5 ft from the ground surface. Assume the soil above ground watar table is saturated.

8.10 A specimen of clean sand when subjected to a direct shear test failed at a stress of 2520 lb/ft^2 when the normal stress was 3360 lb/ft^2.

Determine:

(i) the angle of internal friction, and

(ii) the deviator stress at which the failure will take place, if a cylindrical sample of the same sand is subjected to a triaxial test with a cell pressure of 2000 lb/ft^2. Find the angle made by the failure plane with the horizontal.

8.11 A specimen of fine sand, when subjected to a drained triaxial compression test, failed at a deviator stress of 8400 lb/ft^2. It was observed that the failure plane made an angle of 30° with the axis of the sample. Estimate the value of the cell pressure to which this specimen would have been subjected.

8.12 A specimen of sandy silt, when subjected to a drained triaxial test failed at major and minor principal stresses of 120 kN/m^2 and 50 kN/m^2 respectively. At what value of deviator stress would another sample of the same soil fail,if it were subjected to a confining pressure of 75 kN/m^2?

8.13 A sand is hydrostatically consolidated in a triaxial test apparatus to 8820 lb/ft^2 and then sheared with the drainage valves open. At failure, $(\sigma_1 - \sigma_3)$ is 22 kips/ft^2. Determine the major and minor principal stresses at failure and the angle of shearing resistance.

8.14 The same sand as in Prob. 8.13 is tested in a direct shear apparatus under a normal pressure of 8820 lb/ft^2. The sample fails when a shear stress of 5880 lb/ft^2 is reached. Determine the major and minor principal stresses at failure and the angle of shearing resistance. Plot the Mohr diagram.

8.15 A sample of dense sand tested in a triaxial *CD* test failed along a well defined failure plane at an angle of 66° with the horizontal. Find the effective confining pressure of the test if the principal stress difference at failure was 100 kPa.

8.16 A drained triaxial test is performed on a sand with σ'_{3f} = 10.5 kips/ft^2. At failure $\sigma'_1/\sigma'_3 = 4$. Find σ'_{1f}, $(\sigma_1 - \sigma_3)_f$ and ϕ'.

8.17 If the test of Prob. 8.16 had been conducted undrained, determine $(\sigma_1 - \sigma_3)_f$, ϕ', ϕ_{total} and the angle of the failure plane in the specimen. The pore water pressure u = 2100 lb/ft^2.

8.18 If the test of Prob. 8.16 is conducted at an initial confining pressure of 21 kips/ft^2, estimate the principal stress difference and the induced pore pressure at failure.

8.19 A sample of silty sand was tested consolidated drained in a triaxial cell where σ_3 = 475 kPa. If the total axial stress at failure was 1600 kPa while σ_3 = 475 kPa, compute the angle of shearing resistance and the theoretical orientation of the failure plane with respect to the horizontal.

8.20 A drained triaxial test is to be performed on a uniform dense sand with rounded grains. The confining pressure is 4200 lb/ft^2. At what vertical pressure will the sample fail?

8.21 Compute the shearing resistance along a horizontal plane at a depth of 6.1 m in a deposit of sand. The water table is at a depth of 2.13 m. The unit weight of moist sand above the water

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table is 18.54 kN/m^3 and the saturated weight of submerged sand is 20.11 kN/m^3. Assume that the sand is drained freely and ϕ_d for the wet sand is 32°.

8.22 A sample of dry sand was tested in a direct shear device under a vertical pressure of 137.9 kN/m^2. Compute the angle of internal friction of the sand. Assume shearing resistance = 96.56 kN/m^2.

8.23 The sand in a deep natural deposit has an angle of internal friction of 40° in the dry state and dry unit weight of 17.28 kN/m^3. If the water table is at a depth of 6.1 m, what is the shearing resistance of the material along a horizontal plane at a depth of 3.05 m? Assume: $G_s = 2.68$.

8.24 Compute the shearing resistance under the conditions specified in Prob. 8.23, if the water table is at the ground surface.

8.25 A drained triaxial test was conducted on dense sand with a confining pressure of 3000 lb/ft^2. The sample failed at an added vertical pressure of 11,000 lb/ft^2. Compute the angle of internal friction ϕ and the angle of inclination α of the failure planes on the assumption that Coulomb's law is valid.

8.26 A saturated sample of dense sand was consolidated in a triaxial test apparatus at a confining pressure of 143.6 kN/m^2. Further drainage was prevented. During the addition of vertical load, the pore pressure in the sample was measured. At the instant of failure, it amounted to 115 kN/m^2. The added vertical pressure at this time was 138.85 kN/m^2. What was the value of ϕ for the sand?

8.27 An undrained triaxial test was carried out on a sample of saturated clay with a confining pressure of 2000 lb/ft^2. The unconfined compressive strength obtained was 7300 lb/ft^2. Determine the excess vertical pressure in addition to the all-round pressure required to make the sample fail.

8.28 A series of undrained triaxial tests on samples of saturated soil gave the following results

σ_3, kN/m^2	100	200	300
u, kN/m^2	20	70	136
$(\sigma_1 - \sigma_3)$, kN/m^2	290	400	534

Find the values of the parameters c and ϕ

(a) with respect to total stress, and (b) with respect to effective stress.

8.29 When an unconfined compression test was conducted on a specimen of silty clay, it showed a strength of 3150 lb/ft^2 . Determine the shear strength parameters of the soil if the angle made by the failure plane with the axis of the specimen was 35°.

8.30 A normally consolidated clay was consolidated under a stress of 150 kPa, then sheared undrained in axial compression. The principal stress difference at failure was 100 kPa and the induced pore pressure at failure was 88 kPa. Determine analytically (a) the slopes of the total and effective Mohr stress envelopes, and (b) the theoretical angle of the failure plane.

8.31 A normally consolidated clay sample was consolidated in a triaxial shear apparatus at a confining pressure of 21 kips/ft^2 and then sheared under undrained condition. The $(\sigma_1 - \sigma_3)$ at failure was 21 kips/ft^2 . Determine ϕ_{cu} and α.

8.32 A *CD* axial compression triaxial test on a normally consolidated clay failed along a clearly defined failure plane of 57°. The cell pressure during the test was 4200 lb/ft^2. Estimate ϕ', the maximum σ'/σ'_3, and the principal stress difference at failure.

8.33 Two identical samples of soft saturated normally consolidated clay were consolidated to 150 kPa in a triaxial apparatus. One specimen was sheared under drained conditions, and the principal stress difference at failure was 300 kPa. The other specimen was sheared undrained, and the principal stress difference at failure was 200 kPa. Determine ϕ_d and ϕ_{cu}.

8.34 When a triaxial compression test was conducted on a soil specimen, it failed at an axial pressure of 7350 lb/ft^2. If the soil has a cohesion of 1050 lb/ft^2 and an angle of internal friction of 24°, what was the cell pressure of the test? Also find the angle made by the failure plane with the direction of σ_3.

8.35 Given the following triaxial test data, plot the results in a Mohr diagram and determine ϕ.

σ_3 kN/m^2	Peak σ_1 kN/m^2	σ_3 kN/m^2	Peak σ_1 kN/m^2
69	190	276	759
138	376	345	959
207	580	414	1143

8.36 Two sets of triaxial tests were carried out on two samples of glacial silt. The results are (a) σ_{11} = 400 kN/m^2, σ_{31} = 100 kN/m^2 (b) σ_{12} = 680 kN/m^2, σ_{32} = 200 kN/m^2.
The angle of the failure plane in both tests was measured to be 59°. Determine the magnitudes of ϕ and c.

8.37 A triaxial compression test on a cylindrical cohesive sample gave the following effective stresses:
(a) Major principal stress, σ'_1 = 46,000 lb/ft^2
(b) Minor principal stress, σ'_3 = 14,500 lb/ft^2
(c) The angle of inclination of the rupture plane = 60° with the horizontal.
Determine analytically the (i) normal stress, (ii) the shear stress, (iii) the resultant stress on the rupture plane through a point, and (iv) the angle of obliquity of the resultant stress with the shear plane.

8.38 Given the results of two sets of triaxial shear tests:
σ_{11} = 1800 kN/m^2; σ_{31} = 1000 kN/m^2
σ_{12} = 2800 kN/m^2; σ_{32} = 2000 kN/m^2
Compute ϕ and c.

8.39 What is the shear strength in terms of effective stress on a plane within a saturated soil mass at a point where the normal stress is 295 kN/m^2 and the pore water pressure 120 kN/m^2 ? The effective stress parameters for the soil are c' = 12 kN/m^2, and ϕ' = 30°.

8.40 The effective stress parameters for a fully saturated clay are known to be c' = 315 lb/ft^2 and ϕ' = 29°. In an unconsolidated-undrained triaxial test on a sample of the same clay the confining pressure was 5250 lb/ft^2 and the principal stress difference at failure was 2841 lb/ft^2. What was the value of the pore water pressure in the sample at failure?

8.41 It is believed that the shear strength of a soil under certain conditions in the field will be governed by Coulomb's law, wherein c = 402 lb/ft^2, and ϕ = 22°. What minimum lateral pressure would be required to prevent failure of the soil at a given point if the vertical pressure were 9000 lb/ft^2?

8.42 The following data refer to three triaxial tests performed on representative undisturbed samples:

Test	Cell pressure kN/m^2	Axial dial reading (division) at failure
1	50	66
2	150	106
3	250	147

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The load dial calibration factor is 1.4 N per division. Each sample is 75 mm long and 37.5 mm diameter. Find by graphical means, the value of the apparent cohesion and the angle of internal friction for this soil.

8.43 In a triaxial test a soil specimen was consolidated under an allround pressure of 16 kips/ft^2 and a back pressure of 8 kips/ft^2 . Thereafter, under undrained conditions, the allround pressure was raised to 19 kips/ft^2, resulting in a pore water pressure of 10.4 kips/ft^2, then (with the confining pressure remaining at 19 kips/ft^2) axial load was applied to give a principal stress difference of 12.3 kips/ft^2 and a pore water pressure of 13.8 kips/ft^2. Calculate the values of the pore pressure coefficients A and B.

8.44 In an *in-situ* vane test on a saturated clay a torque of 35 N-m is required to shear the soil. The vane is 50 mm in diameter and 100 mm long. What is the undrained strength of the clay?

8.45 In a vane test a torque of 46 N-m is required to cause failure of the vane in a clay soil. The vane is 150 mm long and has a diameter of 60 mm. Calculate the apparent shear strength of the soil from this test when a vane of 200 mm long and 90 mm in diameter is used in the same soil and the torque at failure was 138 N-m. Calculate the ratio of the shear strength of the clay in a vertical direction to that in the horizontal direction.

8.46 A vane of 80 mm diameter and 160 mm height has been pushed into a soft clay stratum at the bottom of a bone hole. The torque required to rotate the vane was 76 N-m. Determine the undrained shear strength of the clay. After the test the vane was rotated several times and the ultimate torque was found to be 50 N-m. Estimate the sensitivity of the clay.

8.47 A normally consolidated deposit of undisturbed clay extends to a depth of 15 m from the ground surface with the ground water level at 5 m depth from ground surface. Laboratory test on the clay gives a plasticity index of 68%, saturated and dry unit weights of 19.2 kN/m^3 and 14.5 kN/m^3 respectively. An undisturbed specimen for unconfined compressive strength is taken at 10 m depth. Determine the unconfined compressive strength of the clay.

8.48 A triaxial sample was subjected to an ambient pressure of 200 kN/m^2, and the pore pressure recorded was 50 kN/m^2 at a fully saturated state. Then the cell pressure was raised to 300 kN/m^2. What would be the value of pore pressure? At this stage a deviator stress of 150 kN/m^2 was applied to the sample. Determine the pore pressure assuming pore pressure parameter $A = 0.50$.

8.49 In a triaxial test on a saturated clay, the sample was consolidated under a cell pressure of 160 kN/m^2. After consolidation, the cell pressure was increased to 350 kN/m^2, and the sample was then failed under undrained condition. If the shear strength parameters of the soil are $c' = 15.2$ kN/m^2, $\phi' = 26°$, $B = 1$, and $A_f = 0.27$, determine the effective major and minor principal stresses at the time of failure of the sample.

8.50 A thin layer of silt exists at a depth of 18 m below the surface of the ground. The soli above this level has an average dry unit weight of 15.1 kN/m^3 and an average water content of 36%. The water table is almost at the ground surface level. Tests on undisturbed samples of the silt indicate the following values:

$c_u = 45$ kN/m^2, $\phi_u = 18°$, $c' = 36$ kN/m^2 and $\phi' = 27°$.

Estimate the shearing resistance of the silt on a horizontal plane when (a) the shear stress builds up rapidly, and (b) the shear stress builds up slowly.

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