Chapter (3)

SLOPE DEFLECTION METHOD

3.1 Introduction:-

The methods of three moment equation, and consistent deformation method are represent the FORCE METHOD of structural analysis, The slope deflection method use displacements as unknowns, hence this method is the displacement method.

In this method, if the slopes at the ends and the relative displacement of the ends are known, the end moment can be found in terms of slopes, deflection, stiffness and length of the members.

In- the slope-deflection method the rotations of the joints are treated as unknowns. For any one member bounded by two joints the end moments can be expressed in terms of rotations. In this method all joints are considered rigid; i.e the angle between members at the joints are considered not-to change in value as loads are applied, as shown in fig 1.

joint conditions:- to get θ_B & θ_C

$M_{BA}+M_{BC}+M_{BD} = 0$ ………… (1)

$M_{CB}+M_{CE} = 0$ ………… (2)

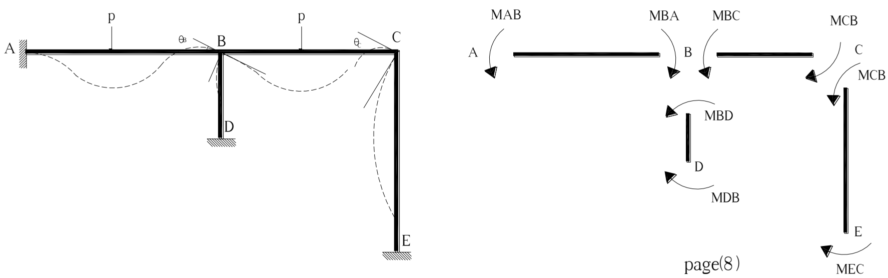


Figure (1)

3.2 ASSUMPTIONS IN THE SLOPE DEFLECTION METHOD

This method is based on the following simplified assumptions.

1- All the joints of the frame are rigid, i.e, the angle between the members at the joints do not change, when the members of frame are loaded.

2- Distortion, due to axial and shear stresses, being very small, are neglected.

3.2.1 Degree of freedom:-

The number of joints rotation and independent joint translation in a structure is called the degrees of freedom. Two types for degrees of freedom.

In rotation:-

For beam or frame is equal to D_r.

$$\boxed{D_r = j - f}$$

Where:

D_r = degree of freedom.

j = no. of joints including supports.

F = no. of fixed support.

<u>In translation:-</u>

For frame is equal to the number of independent joint translation which can be give in a frame. Each joint has two joint translation, the total number or possible joint translation = 2j. Since on other hand each fixed or hinged support prevents two of these translations, and each roller or connecting member prevent one these translations, the total number of the available translational restraints is;

2f + 2h + r + m where

f = no. of fixed supports.

h = no. of hinged supports.

r = no. of roller supports.

m = no. of supports.

The degree of freedom in translation, D_t, is given by:-

$$\mathbf{D_t = 2j-(2f+2h+r+m)}$$

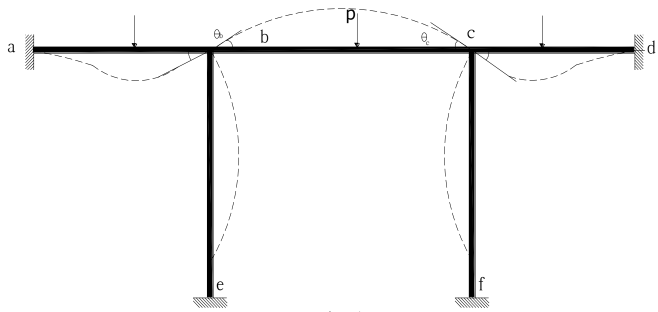
The combined degree of freedom for frame is:-

$$D = D_r + D_t$$

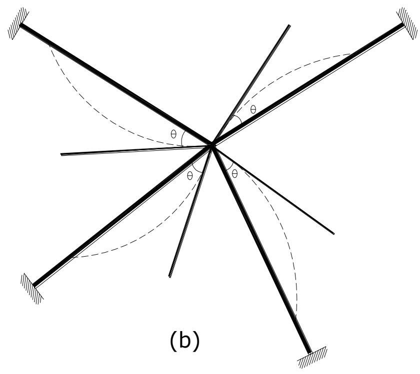
$$= j-f + 2j - (2f + 2h + r + m)$$

D = 3j – 3j – 2h – r - m

The slop defection method is applicable for beams and frames. It is useful for the analysis of highly statically indeterminate structures which have a low degree of kinematical indeterminacy. For example the frame shown in fig. 2.a



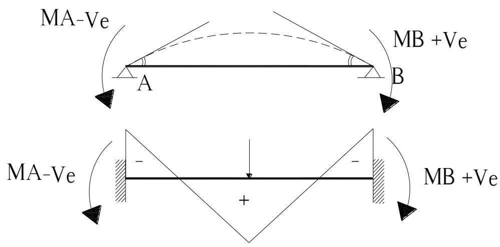
(a)

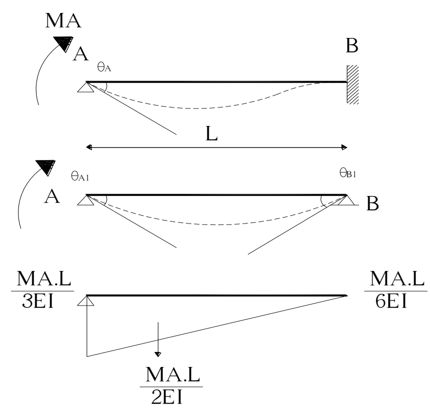

(b)

The frame (a) is nine times statically indeterminate. On other hand only tow unknown rotations, θ_b and θ_c i.e Kinematically indeterminate to second degree- if the slope deflection is used. The frame (b) is once indeterminate.

3.3 Sign Conventions:-

Joint rotation & Fixed and moments are considered positive when occurring in a clockwise direction.

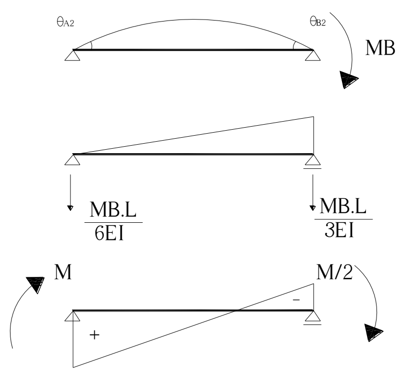




$$\theta_{A1} = \frac{2}{3}\frac{M_{A.L}}{2EI} = \frac{M_{A.L}}{3EI}$$

$$\theta_{BI} = \frac{1}{3} = \frac{M_{A.L}}{2EI} = \frac{-M_{A.L}}{6EI}$$

$$\text{hence} \quad \theta_{B1} = \frac{1}{2}\theta_{A1}$$



$$\theta_{A2} = \frac{1}{3}\frac{M_{B.L}}{2EI} = \frac{-M_{B.L}}{6EI}$$

$$\theta_{B2} = \frac{2}{3}\frac{MB.L}{2EI} = \frac{MB.L}{3EI}$$

$$\theta_{B1} + \theta_{B2} = 0$$

Hence: $M_A = 2M_B$

and $\theta_A = \theta_{A1} - \theta_{A2}$

$$= \frac{M_A.L}{3\,EL} - \frac{M_A.L}{12\,EL}$$

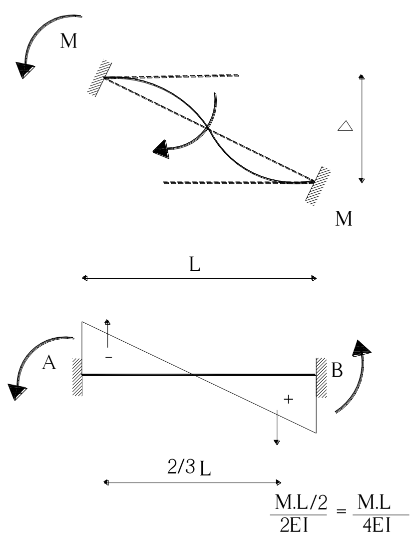
$$\theta_A = \frac{3MA.L}{12EI}$$

$MA = \frac{4EI}{L}.\theta A$
$MB = \frac{2EI}{L}.\theta A$

<u>Relation between Δ & M</u>

$$R = \frac{\Delta}{L}$$

by moment area method or by conjugate beammethod.



$$\Delta = \sum_{M\ at\ B}$$

$$= \frac{M.L}{4EI}\left(\frac{2L}{3}\right)$$

$$= \frac{M.L^2}{6EI}$$

$$M = \frac{6EI}{L^2}\Delta$$

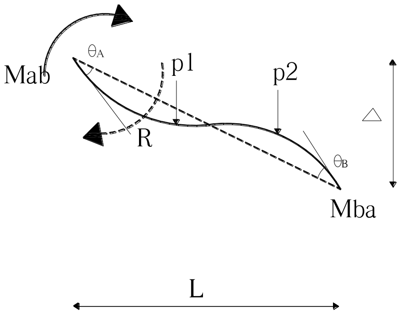
$$= \frac{6EI}{L}.R$$

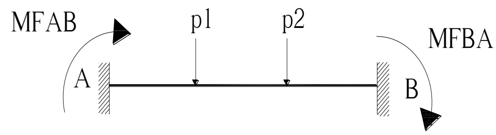
R (+ ve) when the rotation of member AB with clockwise.

3.4Fixed and moments:

As given in the chapter of Moment distribution method.

3.5 Derivation of slope deflection equation:-



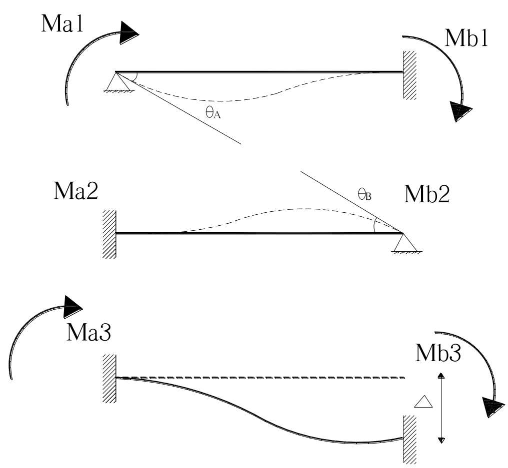


$$M_{a1} = \frac{4EI}{L}\theta_A$$

$$M_{b1} = \frac{2EI}{L}\theta_A$$

$$M_{a2} = \frac{2EI}{L}\theta_B$$

$$M_{b2} = \frac{4EI}{L}\theta_B$$



Required M_{ab} & M_{ba} in term of (1) θ_A, θ_B at joint

(2) rotation of member (R)

(3) loads acting on member

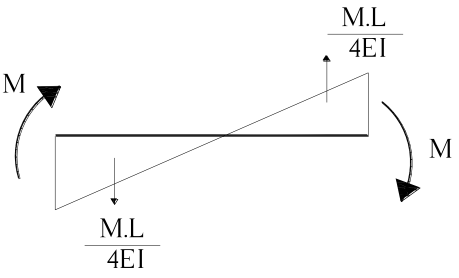
<u>First assume:-</u>

Get Mf_{ab} & Mf_{ba} due to acting loads. These fixed and moment must be corrected to allow for the end rotations θ_A,θ_B and the member rotation R.

The effect of these rotations will be found separately.

$$M_{a1} = \frac{4EI}{L} .\theta_A$$

$$M_{b1} = \frac{2EI}{L} .\theta_A$$



$$M_{b2} = \frac{4EI}{L} .\theta_B$$

$$M_{a2} = \frac{2EI}{L} .\theta_B$$

$$M_{b3} = M_{a3} = \frac{-6EI}{L^2} .\Delta$$

$$= \frac{-6EI}{L} .R$$

<u>by Superposition;</u>

$$M_{ab} = Mf_{ab} + M_{a1} + M_{a2} + M_{a3}$$

$$Mf_{ab} + \frac{4EI}{L} .\theta_A + \frac{2EI}{L} \theta_B + \frac{-6EI}{L} .R$$

$$\boxed{M_{ab} = Mf_{ab} + \frac{2EI}{L} (2\theta_A + \theta_B - 3R)}$$

In case of relative displacement between the ends of members, equal to zero (R = 0)

$$M_{ab} = Mf_{ab} + \frac{2EI}{L} (2\theta_a + \theta b)$$

$$M_{ba} = Mf_{ba} + \frac{2EI}{L} (2\theta_b + \theta_A)$$

The term ($\frac{2\ EI}{L}$) represents the relative stiffness of member say (K) <u>hence:</u>

$$M_{ab} = Mf_{ab} + K_{ab} (2\theta_A + \theta_b)$$

$$M_{ba} = Mf_{ba} + K_{ba}\ (2\theta_B + \theta_a)$$

<u>Note:</u>

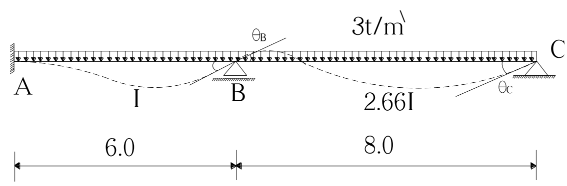
$\frac{\Delta}{L}$ = R is (+ ve) If the rotation of member with clockwise.

And (– ve) If anti clockwise.

$$M = \frac{-6\ EI}{L^2}.\Delta \quad (\text{with} + \text{ve}\ R)$$

$$M = \frac{-6\ EI}{L^2}.\Delta \quad (\text{with} - \text{ve}\ R)$$

3-5-1 <u>Example 1</u>



<u>Draw B.M.D. S.F.I</u>

<u>Solution:-</u>

1- Relative stiffness:- $K_{AB} : K_{BC} = \frac{1}{6} : \frac{2.66}{8}\ 1:2$

2- Fixed and Moment:-

$$MF_{BA} = \frac{3\times 6^2}{12} = -9\ t.m.$$

$$MF_{BA} = +\frac{3\times 6^2}{12} = +\ 9 \quad , \quad MF_{BC} = +\frac{3\times 8^2}{12} = -\ 18$$

$$MF_{CB} = +\frac{3\times 8^2}{12} = +18$$

3- Two unknown θ_B + θ_C then two static equations are required. 1) $\sum M_B = 0$

2) $M_C = 0$

<u>Hence:</u>

$M_{BA} + M_{BC} = 0$ ………………. (1)

$M_{BC} = 0$………………. (2)

<u>But:</u>

$M_{AB} = -9 + \theta_B$

$M_{BA} = 9 + 1\ (2\theta_{B)}$

$M_{BC} = -16 + 2\ (2\theta_B + \theta_C)$

$M_{CB} = +16 + 2\ (2\theta_C + \theta_B)$

From eqns. (1&2)

$9 + 2\theta_B + (-\ 16 + 2\ (2\theta_B + \theta_C) = 0$

$6\theta_B + 2\theta_C = 7$ …….(3)

and $4\theta_C + 2\theta_B = -16$

$2\theta_C + \theta_B = -8$ …….. (4)

from 3 & 4

$5\ \theta_B = 15$

$\theta_B = \frac{15}{5} = 3.0$

$\theta_C = -5.5$

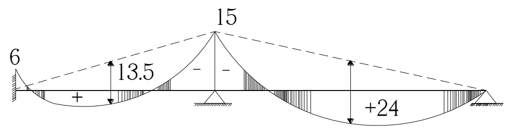
1.e. $M_{AB} = -9 + 3.4 = 5.6$ t.m

$M_{BA} = 9 + 2\times3.4 = 15.8$ t.m

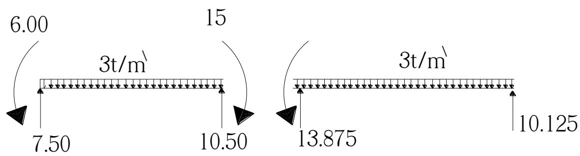
$M_{BC} = -18 + 2\ (2\times3.4) + (-\ 5.5) = -15.0$ t.m

$M_{CB} = 16 + 2\ (2.3 - 5.7 + 3.4) = 0.0$ (0.k)

B.M.D

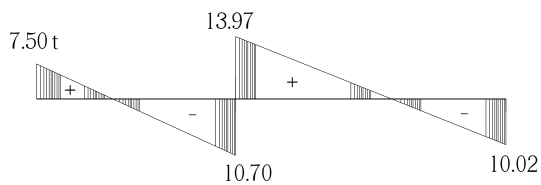


15
6
13.5
+
-
-
+24

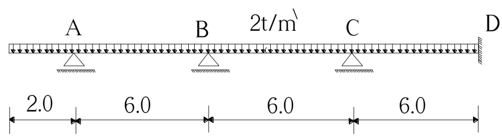


6.00
15
3t/m\
3t/m\
7.50
10.50
13.875
10.125

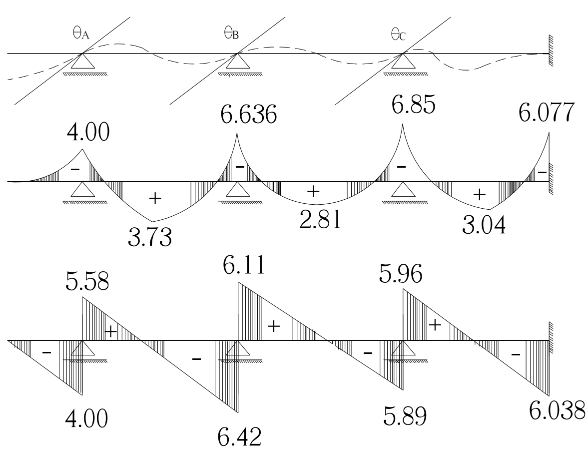
S.F.D



13.97
7.50 t
+
-
+
-
10.70
10.02 t



A
B
2t/m\
C
D
2.0
6.0
6.0
6.0



θA
θB
θC
6.636
6.85
6.077
4.00
3.73
2.81
3.04
5.58
6.11
5.96
4.00
6.42
5.89
6.038

1- **Unknowns** θ_A, θ_B, & θ_C

2- Fixed end Moment

$$MF_{AB} = MF_{BC} = MF_{CD} = \frac{2\times 6^2}{12} = -6 \text{ t.m} \ldots \text{etc}$$

3- Condition eqns.

M_{AB} = - 4 t.m, $M_{BA} + M_{BC} = 0$, & $M_{CB} + M_{CD} = 0$

4- Slope deflection equations

$M_{AB} = -6 + (2\theta_A + \theta_B) = -4$

$2\theta_A + \theta_B = 2$ ……….(1)

$M_{BA} + M_{BC} = 0$

$+6 + (2\theta_B + \theta_A) - 6 + (2\theta_B + \theta_C) = 0$

$4\theta_B + \theta_A + \theta_C = 0$ ……… (2)

$M_{CB} + M_{CD} = 0$

$= 6 + 2\theta_C + \theta_B - 6 + 2\theta_C = 0$

$4\theta_C + \theta_B = 0$ ……….(3)

From eqn.3 $\theta_C = -\frac{\theta B}{4}$

Substitute in eqn. (2)

Hence: $3.75\ \theta_B + \theta_A = 0$ ....(2)

$0.5\theta_B + \theta_A = 1$ ………… (2)

$3.25\theta_B = -1$

$\theta_B = -1$

$\theta_A = 1.15$

$\theta_C = 0.077$

Hence:

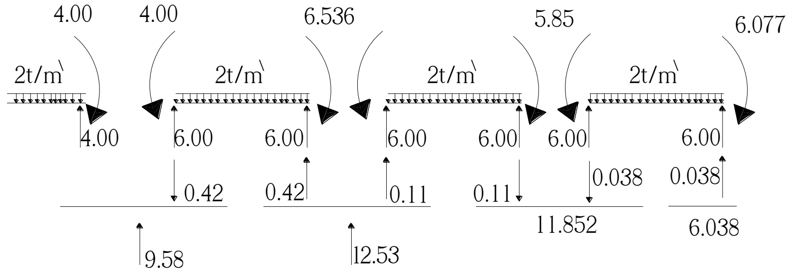
M_{AB} = - 6 + 2xl.15 + (- .307)

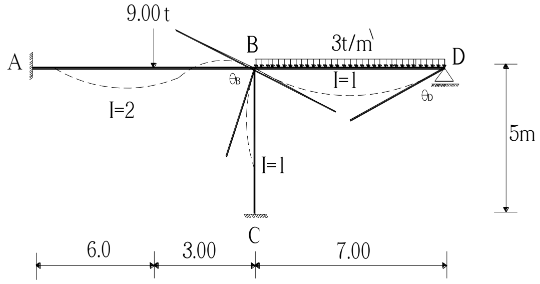
= - 4 t.m 0.K

M_{BA} = 6 + 2x (- .307) + 1.15 = 6.536 t.m

M_{CB} = 6 + 2x .77 + (- .307) = 5.85 t.m

M_{DC} = 6 + .077 = 6.077 t.m





Solution:-

1- Unknown displacements are θ_B & θ_D

2- Equations of equilibrium are:-

$M_{DB} = 0$ ……….(1)

$M_{BA} + M_{BD} + M_{BC} = 0$ ………(2)

3- **Relative Stiffness:-**

K_{AB}: K_{BC}: K_{BD} = 35:31. 5:22 ; 51. 56:1. 4:1.0.

4- Fixed and Moments:

$$MF_{AB} = \frac{-9\times6\times3\times3}{9\times9} = -6 \ \ t.m$$

$$MF_{BA} = \frac{9\times6\times3\times6}{9\times9} = 12 \ \ t.m$$

$$MF_{BD} = \frac{-3\times7^2}{12} = -12.25 \ \ t.m$$

$$MF_{DB} = \frac{-3\times7^2}{12} = -12.25 \ \ t.m$$

From the equations 1 & 2 hence;

$M_{DB} = MF_{DB} + (2\theta_D + \theta_B)$

$= 12.25 + 1\ (2\theta_D + \theta_B) \quad = 0$

$2\theta_D + \theta_B + 12.25 \quad = 0$ ------- (3)

and $M_{BA} \quad = 12 + 1.56\ (2\theta_B)$

$M_{BD} \quad = 12.25 + 1.0\ (2\theta_B + \theta_D)$

$M_{BC} \quad = 0 + 1.4 \quad (2\theta_B + 0)$

i.e.

$12+1.56\ (2\theta_B) - 12.25 + 2\theta_B + \theta_D + 1.4\ (2\theta_B) = o$

$7.92\theta_B + \theta_D - .25 \quad = 0$ ------------- (4)

$\underline{0.5\theta_B + \theta_D + 6.125} \quad = 0$ ------------- (3)

i.e $\quad 7.42\ \theta_B \quad - 6.375 \quad = o$

$\theta_B \quad = 0.86$

$\theta_D \quad = - 6.55$

<u>Hence:</u>

$M_{BA} = 12 + 1.56\ (2\times .86) = 14.68$ t.m

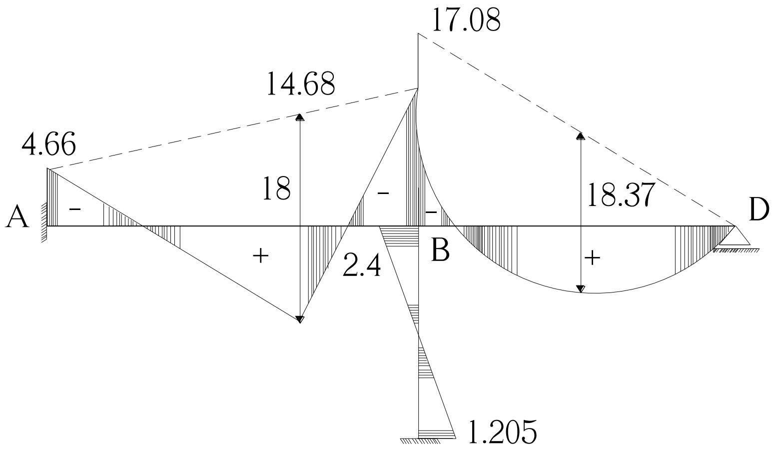
$$M_{BD} = -12.25 + (2 \times .86 - 6.55 \times 1) = -17.08$$

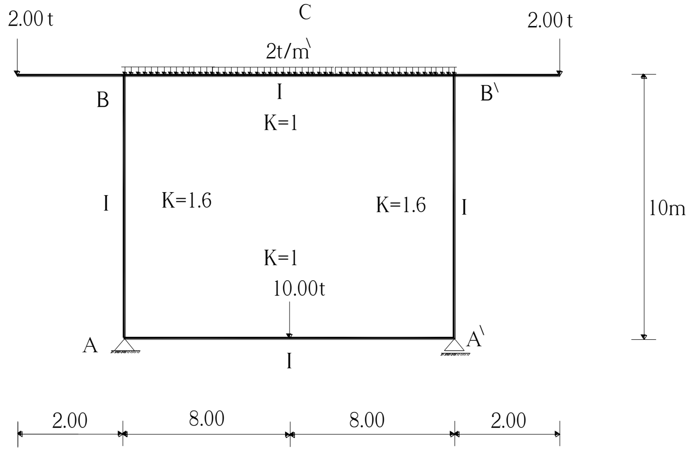
$$M_{BC} = 1.4\,(2 \times .86) = 2.41$$

$$M_{DB} = 12.25 + (2 \times -6.55) = \text{zero}$$

$$M_{CB} = \frac{1}{2} MBC = 1.205$$

$$M_{AB} = -6 + 1.56\,(.86) = -4.66$$





Two equilibrium eqns.

$M_{AB} + M_{AA} = 0$ ………………… (1)

$M_{BB} + M_{BA} + 4 = 0$ …………………(2)

Slope deflection eqns.

$M_{AB} = o + 1.6\ (2\theta_A + \theta_B)$

$M_{AA} = \frac{-10\times16}{8} + (2\theta_A + \theta_A)$

$M_{AA} = -20 + \theta_A$

$M_{BA} = o + 1.6\ (2\theta_B + \theta_A)$

$M_{BB} = -42.67 + (2\theta_B + \theta_B)$

$= -42.67 + \theta_B$

Hence:

$3.2\theta_A + 1.6\theta_B + \theta_A - 20 = o$

$4.2\theta_A + 1.6\theta_B = 20$ ……………… (1)

$-42.67 + 4.2\ \theta_B + 1.6\theta_A + 4 = 0$

$1.6\theta_A + 4.2\theta_B = 38.67$ ………………(2)

$1.6\theta_A + 0.61\theta_B = 7.62$ ………………(1)

$3.59\theta_B = 31.05$

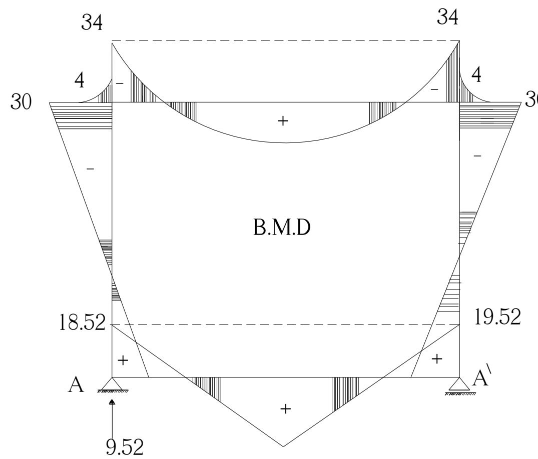
$\theta_B = 8.65$

$\theta_A = 1.46$

$M_{AB} = -18.52$

$M_{BA} = 30$

$M_{BB} = -34$



<u>Example 5</u>

Draw B.M.D for the shown frame

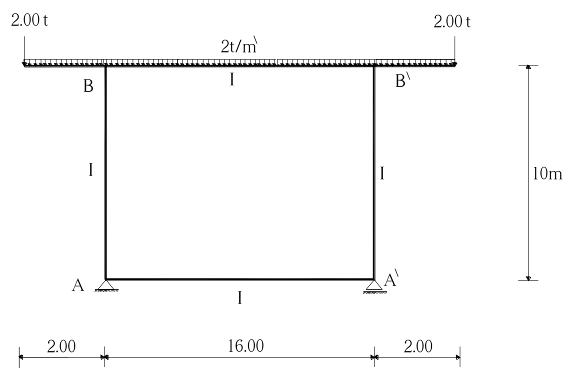
<u>Solution:-</u>

- **Two condition equations.**

$$M_{AA} + M_{AB} = 0 \quad \dots\dots\dots\dots\dots\dots\dots(1)$$

$$M_{BA} + M_{BB} + 8 = 0 \quad \dots\dots\dots\dots\dots\dots\dots(2)$$

- **Relative stiffness $\frac{1}{16} : \frac{1}{10}$ = 1:1.6**



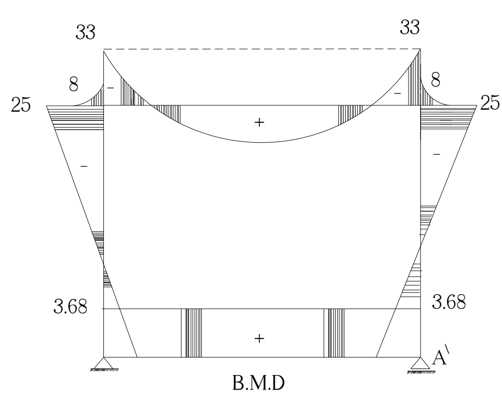
- Slope deflection equations:

$M_{AA} = (2\theta_A - \theta_A) = \theta_A$

$M_{AB} = (2\theta_A - \theta_B) \times 1.6$

$M_{BA} = (2\theta_B - \theta_A) \times \theta_A$

$M_{BB} = 42.67 + (2\theta_B - \theta_B)$



Hence:

$\theta_A + 3.2\theta_A + 1.6V_B = 0$

$4.2\theta_A + 1.6\theta_B = 0 \dots. (1)$

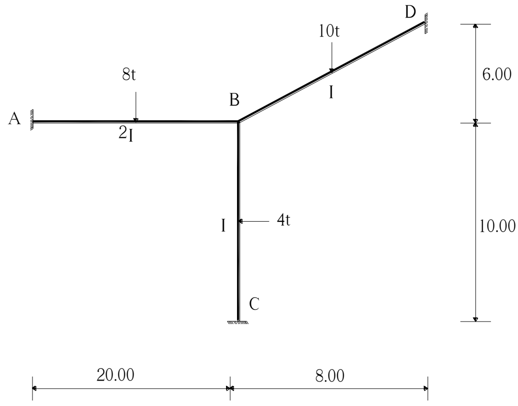
$3.2\theta_B + 1.6\theta_A + \theta_B - 42.67 + 8 = 0$

$4.2\theta_B + 1.6\theta_A = 34.67\dots (2)$

By Solving 1 & 2 $\theta_A = -3.68$, $\theta_B = 9.66$

Hence $M_{AA} = -3.68$, $M_{AB} = 3.68$ t.m

$M_{BA} = 25$ $M_{BB} = 33$



Example 6:

- Draw B.M.D for the given structure.

Solution:- once statically indeterminate.

1- Fixed end moments

$$MF_{AB} = -\frac{8 \times 20}{8} = -20 \text{ t.m}$$

$$MF_{BA} = -\frac{8 \times 20}{8} = -20 \text{ t.m}$$

$$MF_{BC} = -\frac{4 \times 10}{8} = -5 \text{ t.m}$$

$$MF_{CB} = -\frac{10 \times 8}{8} = -10 \text{ t.m}$$

$$MF_{DB} = 10 \text{ t.m}$$

2- From Static:- $\sum M_B = o$

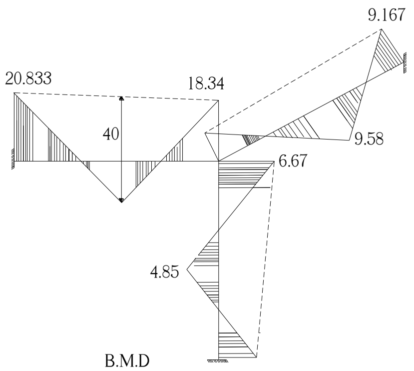
$M_{BA} + M_{BC} + M_{BD} = 0$

$M_{BA} = MF_{BA} + (2\theta_B)$

$M_{BA} = 20 + 2\theta_B$ ……………….. (1)

$M_{BC} = - 5 + 2\theta_B$ ……………….. (2)

$M_{BD} = - 10 + 2\theta_B$ ………………. (3)



Hence: $5 + 6\theta_B = o$

θ_B = - o.833

Hence:

M_{BA} = 18.34 t.m , MBC = -6.67, MBD = -11.67 t.m

$M_{AB} = - 20$ = - 20.833 t.m

$M_{CB} = 5 + \theta B$ = - 4.167 t.m

$M_{DB} = 10 + \theta B$ = 9.167 t.m

<u>Example 7:</u>

Draw B.M.D for the shown frame

Solution:

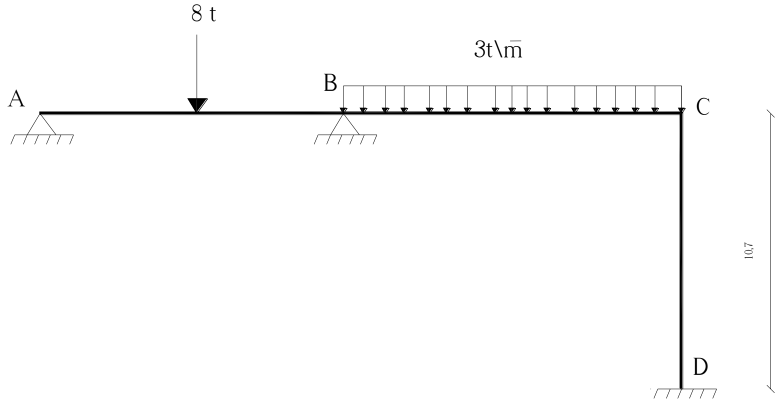
" 3 time statically ind." θ_A , θ_B , & θ_C

1- **Fixed end moments:**

$MF_{AB} = -10$

$MF_{BA} = +10$

$MF_{BC} = -25$



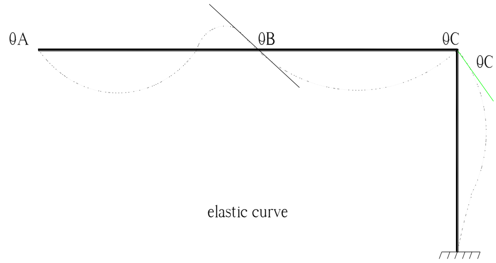
$MF_{CD} = MF_{DC} = \text{zero}$

2- **Relative Stiffness** 1:1:1

$M_{AB} = 0$ ……………………….(1)

$M_{BA} + M_{BC} = 0$ ……………………….(2)

$M_{CB} + M_{CD} = 0$ ……………………….(3)



Equs.

$M_{AB} = -10 + (2\theta_A + \theta_B)$

$M_{BA} = 10 + (2\theta_B + \theta_A)$

$M_{BC} = -25 + 2\theta_B + \theta_C$

$M_{CB} = 25 + 2\theta_C + \theta_B$

$M_{CD} = 2\theta_C$

$M_{DC} = \theta_C$

From 1,2 & 3

$2\theta_A + \theta_B = 10$ ....(1)

$4\theta_B + \theta_A + \theta_C = 15$ ....(2)

$4\theta_C + \theta_B = -25$ ....(3)

By solving the three eqns. hence;

$\theta_A = 2.5$ $\theta_B = 5$ $\theta_C = -7.5$

Substitute in eqns of moments hence;

$M_{AB} = -10 + 5 =$ zero (o.k)

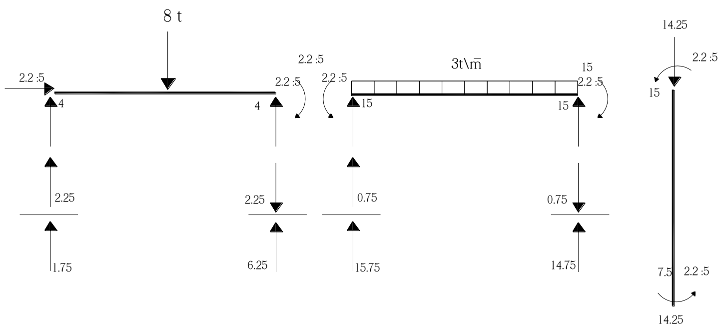
$M_{BA} = 10 + 10 + 2.5 = 22.5$ t.m

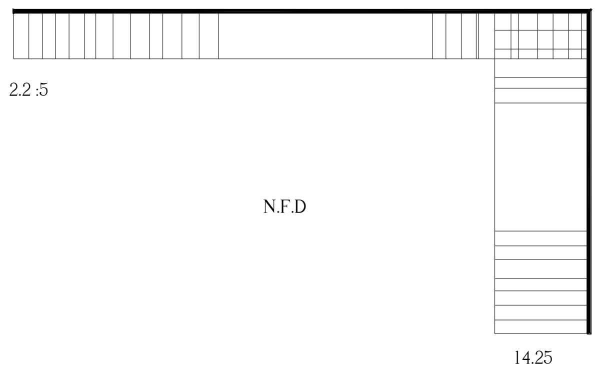
$M_{BC} = -25 + 10 - 7.5 = -22.5$ t.m

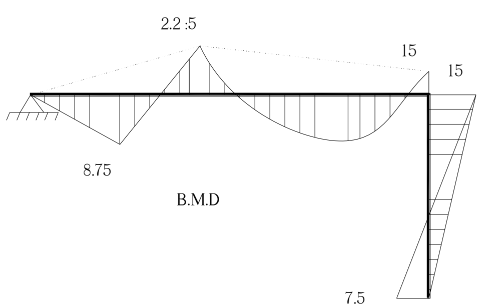
$M_{CB} = 25 - 15 + 5 = 15$ t.m

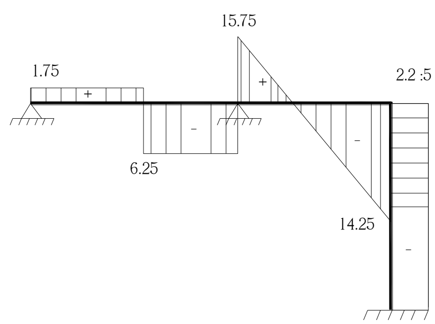
$M_{CD} \qquad = -15 \quad t.m$

$M_{DC} \qquad = -7.5 \quad t.m$









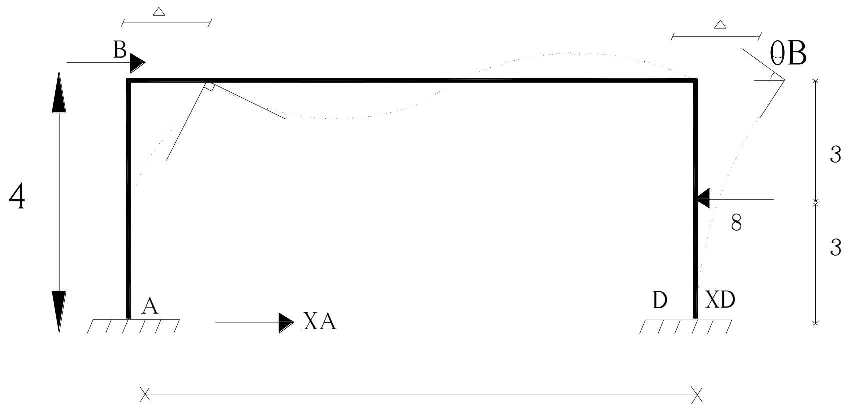
3-6 **<u>Frames with Translation</u>**

Examples to frames with a single degree of freedom in translation.

Example 8:

Draw B.M.D for the shown frame.

1- **Unknowns**:   θB , θC , Δ



2- **Relative stiffness**

$K_{AB} : K_{BA} : K_{CD}$

$$\frac{1}{4} : \frac{2}{8} : \frac{1.5}{6}$$

1: 1 : 1

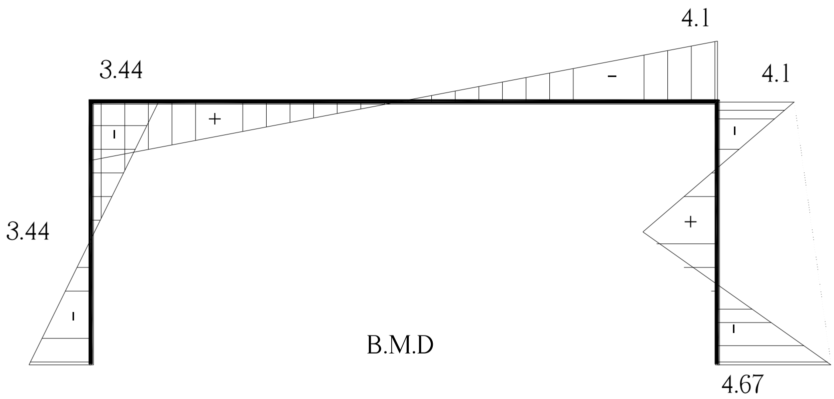
3- **Fixed end moments**

$MF_{AB} = o$ $M_{BA} = o$

$MF_{BC} = MCB = zero$

$MF_{CD} = -6$ t.m

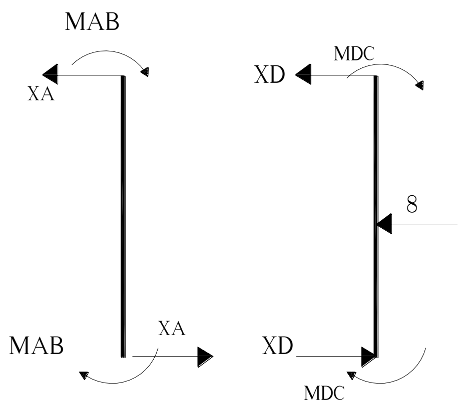
$MF_{DC} = +6$ t.m



4- From Statics the equilibrium eqns

$M_{BA} + M_{BC} = 0$ …………………(1)

$M_{CB} + M_{CD} = 0$ …………………(2)



5- Shear equation (In direction of X, $\sum \times = o$)

$6 + X_A + X_D - 8 \qquad = o$

$$6 + \frac{M_{BA} + M_{AB}}{4} + \frac{M_{CD} + M_{DC}}{6} - 4 = 0 \quad (3)$$

$$hence \quad X_A + \frac{MBA + MAB}{4} \; and \; xD = \frac{MCD + MDC}{6} + 4$$

6- Slope deflection eqns:

$$M_{BA} - 0 + 1\left(2\theta_B - 3\,\frac{\Delta}{4}\right),\; M_{AB} = 0 + 1\left(\theta_B - 3.\,\frac{\Delta}{4}\right)$$

$M_{BC} = 0 + 1\ (2\theta_B + \theta_C)$

<u>Hence:</u> $4\theta_B \ - 0.75\Delta + \theta_C = 0$ (1)

$M_{CB} = 0 + 1\ (2\theta_C + \theta_B)$

$M_{CD} = -6 + 1\ \ (2\theta_C - 3\ \frac{\Delta}{6})$,

$M_{DC} = +6 + 1\ (\theta_C\text{-}3\ \frac{\Delta}{4})$

<u>Hence:</u>

$4\ \theta_C + \theta_B - \frac{1}{2}\Delta\ = 6$ (2)

$$2+\frac{(2\theta_B-.75\Delta)+(1\theta_B-.75\Delta)}{4}+\frac{(-6+2\theta_C-\Delta)+(6+1\theta_C-\ \ \Delta}{6}=0$$

$2 + 0.75\ \theta_B - .375\ \Delta + \frac{1}{2}\theta_C - 0.1667\Delta = o$

$\theta_B + .67\ \theta_C - 072\Delta = -2.66$ (3)

Subtract (3) from (2)

$3.33\ \theta\frac{1}{2} + 0.288\ \Delta = 8.33$

$\theta_C - 0.067\ \Delta = 2.6$ (4)

Subtract (1) from (2) × (4)

$15\ \theta_C - 1.25\ \Delta \qquad = 24$

$\theta_C - 0.08\ \Delta \qquad = 1.6$ (5)

From (4) & (5) $0.147\Delta = 1$

$\Delta \ \ = 6.80$

$\theta_C = 2.149$

$\theta_B \ = 0.799$

M_{BA} = - 3.5 t.m ,M_{AB} = - 4.301 t,m, M_{BC} = 3.79

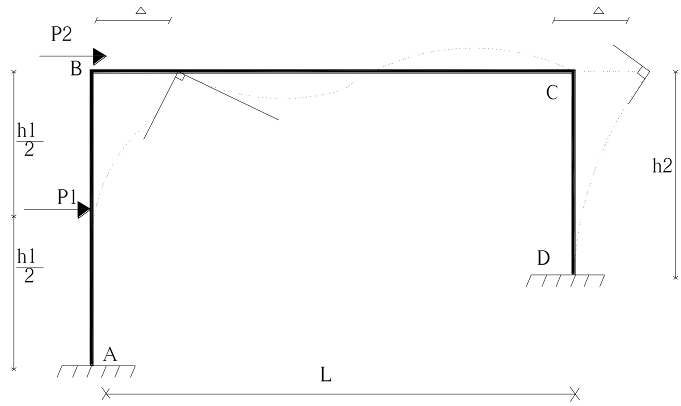
M_{CB} = 5.1 t.m ,M_{CD} = - 5.1 t.m , M_{DC} = 4.744

Example 9:-

Write the shear & condition eqns for the following frame.

Solution:-

Three unknowns: θB , θC, Δ



Condition equations:

$M_{BA} + M_{BC} = o$ (1)

$M_{CB} + M_{CD} = o$ (2)

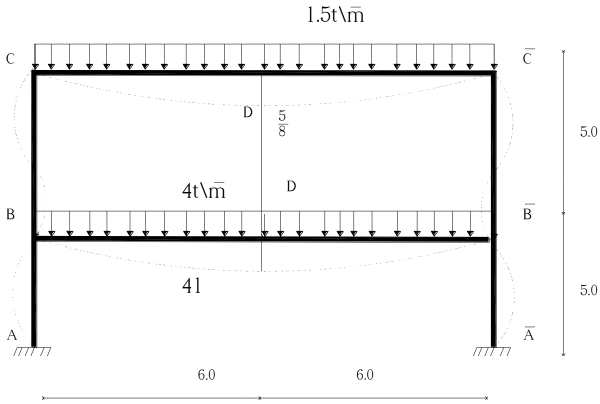
Shear eqn.

$X_A + X_B + P1 + P2 = o$

$$\left(-\frac{P1}{2}+\frac{M_{AB}+M_{BA}}{h1}\right) + \left(\frac{M_{CD}+M_{DC}}{h2}\right) P1 + P2) = o \qquad (3)$$

Example 10:

Find the B.M.D for the shown structure.



Solution:-

$\theta_D = \theta_E = o$

$\theta_C = - \theta_{\bar{C}}$

$\theta_B = - \theta_{\bar{B}}$

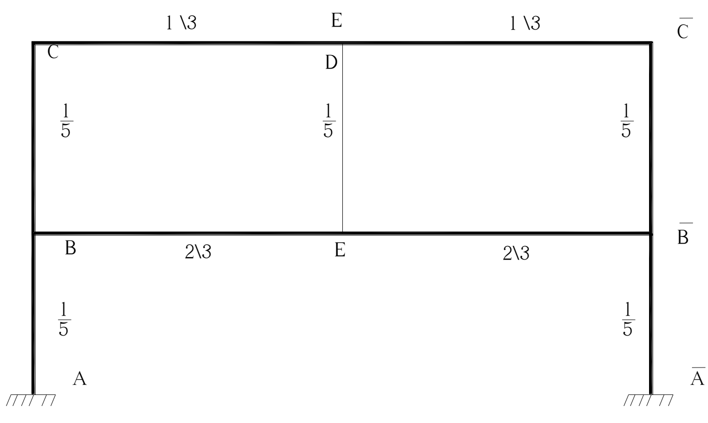
1- **Unknown displacements are:** θB , θC , Δ

2- **Relative Stiffness:**

AB : BE : BC : CD : ED

$$\frac{1}{5} : \frac{2}{3} : \frac{1}{5} : \frac{1}{3} : \frac{1}{3}$$

$$3 : 10 : 3 : 5 : 5$$



3- **Fixed end moment:-**

MF_{BE} = - $\frac{4 \times 36}{12} = -12$ t.m

MF_{EB} = + 12 t.m

MF_{CD} = $\frac{1.5 \times 36}{12} = -4.5$ t.m

MF_{DC} = + 4.5

4- Equilibrium equations:-

1- $M_{CD} + M_{CB} = o$

2- $M_{BC} + M_{BA} + M_{BE} = o$

3- Shear condition:$(33 – 16.5) + \frac{M_{CD} + M_{DC}}{6} + \frac{M_{DE} + M_{ED}}{6}$

$M_{CD} = - 4.5 + 5 (2\theta_C + \theta_D – 3R)$

$M_{CB} = 0 \;\; 3 (2\theta_C + \theta_B)$

$M_{BC} = 0 + 3 (2\theta_B + \theta_C)$

$M_{BA} = 0 + 3 (2\theta_B)$

$M_{BE} = - 12 + 10 (2\theta_B – 3R)$

Hence

$- 4.5 + 10\, \theta_C – 15\, R + 6\, \theta_C + 3\, \theta_B = 0$

$16\theta_C + 3\theta_B – 15R – 4.5 = 0$ (1)

And

$16\theta_B + 3\theta_C + 6\theta_B – 12 + \theta_B - 3\theta_R = 0$

$3\theta_C + 32\theta_B – 30R – 12 = 0$ (2)

and

$16.5(\frac{15\theta_C - 30R}{6} + \frac{30 \;\; \theta_B - 60R}{6}) = 0$

$2.5\ \theta_C + 5\theta_C + 17R + 16.5 \qquad = 0 \qquad (3)$

by solving equation 1,2 & 3 get

$M_{AB} = +\ 6.66$ t.m

$M_{BA} = +\ 13.32$ t.m

$M_{BC} = +\ 19.0$ t.m

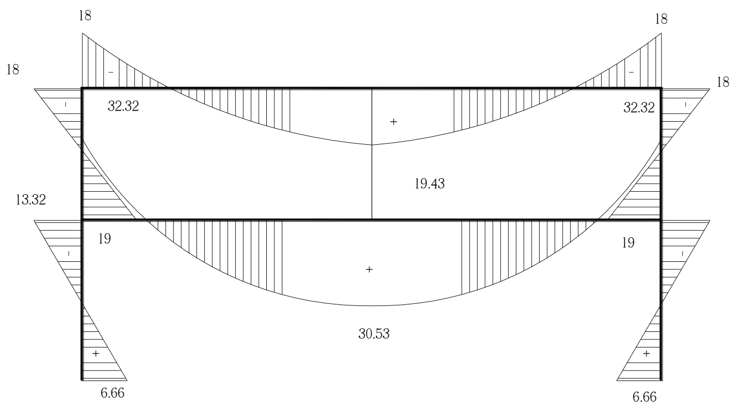
$M_{CB} = +\ 18$ t.m

$M_{BE} = -\ 32.32$ t.m

$M_{EB} = -\ 30.53$ t.m

$M_{CD} = -\ 18$ t.m

$M_{DC} = -\ 18.43$ t.m



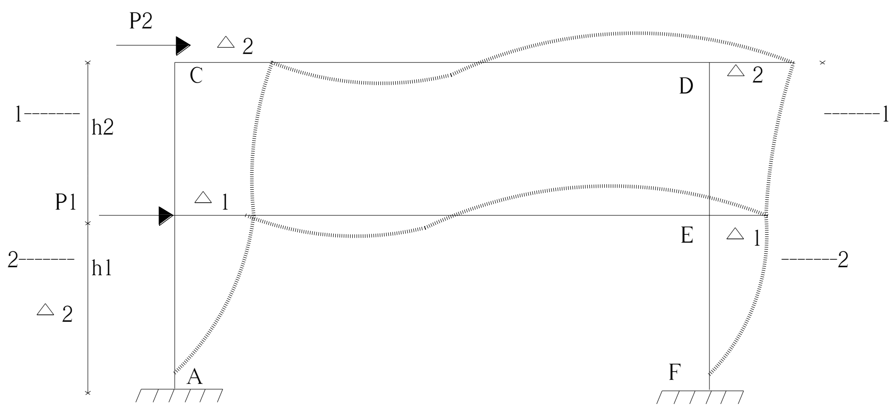
3-7 **Frame with multiple degree of freedom in translation.**

Example 11:

Write the shown equations and condition eqns for the given frame.

Solution

Unknowns: θB , θC , θD , θE , $\Delta 1$, $\Delta 1$



Condition eqns

$$M_{BE} + M_{BA} + M_{BC} = 0 \quad (1)$$

$$M_{CB} + M_{CD} = 0 \quad (2)$$

$$M_{DC} + M_{DE} = 0 \quad (3)$$

$$M_{EB} + M_{EF} + M_{ED} = 0 \quad (4)$$

Shear eqns :

Equilibrium of the two stories.

At sec (1) – (1) :-

(Level CD)

$$P_2 + X_c + X_E = 0$$

$$P_2 + \frac{M_{CB} + M_{BC}}{h2} + \frac{M_{DE} + M_{ED}}{h2} = 0$$

At sec. (2) – (2):-

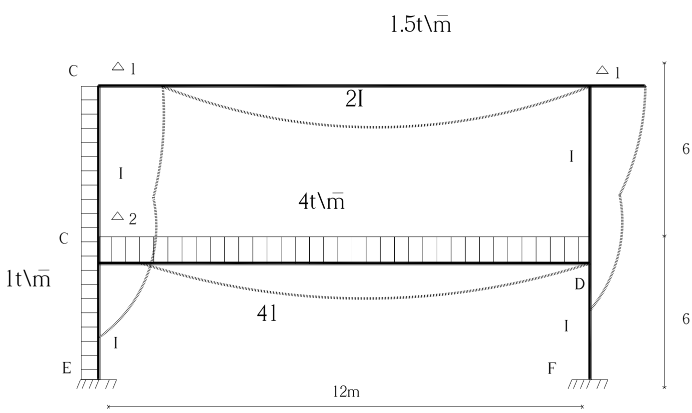
(Level BE) or $\sum x = 0$

$$P_1 + P_2 + x_A + x_F = 0$$

$$P_1 + P_2 + \frac{M_{BA} + M_{AB}}{h1} + \frac{M_{EF} + M_{FE}}{h1} = 0$$

Example 12:-

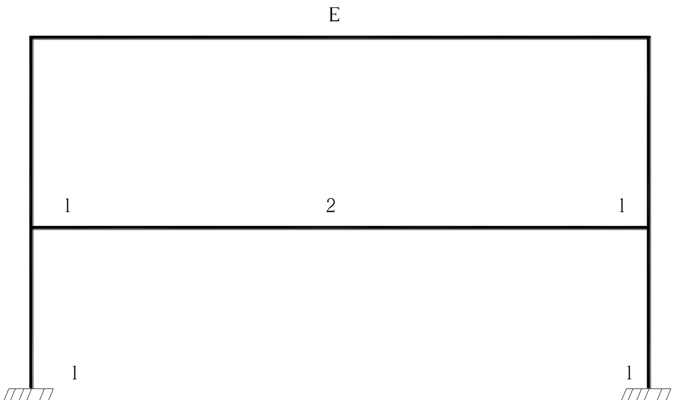
Draw B.M.D for the given structure.



S

olution:-

1- Relative Stiffness:-



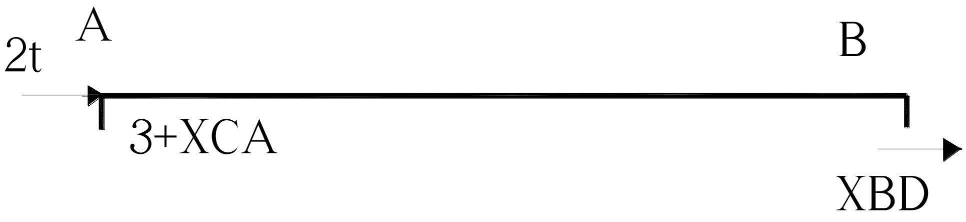
2- <u>Equilibrium equations:-</u>

$M_{AB} + M_{AC} = 0 \quad (1)$

$M_{BA} + M_{BD} = 0 \quad (2)$

$M_{CA} + M_{CD} + M_{CE} = 0 \quad (3)$

$M_{DB} + M_{DF} + M_{DC} = 0 \quad (4)$



$\Sigma x = 0$ at Level A-B

$$2 + (6-3) + \frac{M_{AC} + M_{CA}}{6} + \frac{M_{BD} + M_{DB}}{6} = 0 \quad (5)$$

$\Sigma x = 0$ at Level CD

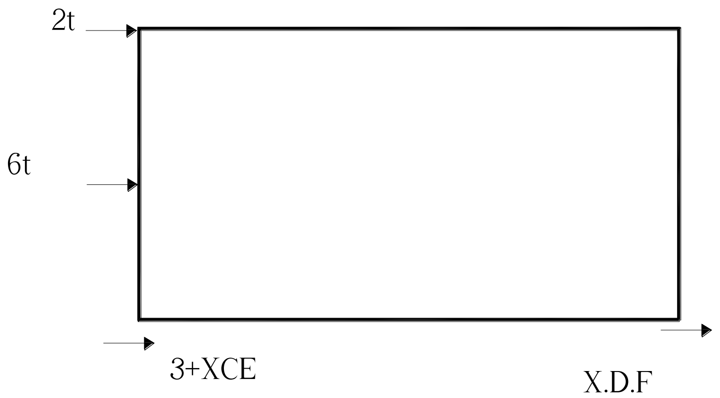
$$11 + \frac{M_{CE} + M_{EC}}{6} + \frac{M_{DF} + M_{FD}}{6} = 0 \quad (6)$$

M_{AB} = - 8 + 1 ($2\theta_A + \theta_B$)

M_{AC} = 3 + ($2\theta_A + \theta_C – 3R_1$)

M_{AC} = - 3 + ($2\theta_C + \theta_A – 3R_1$)

M_{CA} = 16 + ($2\theta_B + \theta_A$)



M_{BD} = 0 + ($2\theta_B + \theta_D – 3R_1$)

M_{DB} = 0 + ($2\theta_D + \theta_B$ - $3R_1$)

M_{DF} = 0 + ($2\theta_D$ + 0 - $3R_2$)

M_{FD} = 0 + (θ_D - $3R_2$)

M_{CD} = - 48 + 2 ($2\theta_C + \theta_C$)

M_{CD} = + 48 + 2 ($2\theta_D + \theta_C$)

M_{CE} = - 8 + ($2\theta_C$ - $3R_2$)

M_{EC} = + (θ_C - $3R_2$)

3- <u>Fixed end moment:-</u>

$$MF_{AB} = -\frac{9\times4\times8\times4}{12\times12} = -8\,t.m$$

$$MF_{BA} = -\frac{9\times8^2\times4}{12^2} = +16\,t.m$$

$$MF_{AC} = \frac{1\times6^2}{12} = +3\,t.m$$

$MF_{CA} = \frac{1\times6^2}{12} = -3$ t.m

$MF_{CD} = \frac{4\times12^2}{12} = -48$ t.m

$MF_{DC} = +48$ t.m

4- <u>Unknown displacement:</u>

θA , θB , θC , θD , Δ_1 , Δ_2

by Solving the six equations one can get;

M_{AB} = - 3.84 t.m

M_{BA} = + 18.39 t.m

M_{AC} = 3.84 t.m

M_{CA} = + 7.29 t.m

M_{BD} = - 18.39 t.m

M_{DB} = - 22.97 t.m

M_{CD} = - 11.15 t.m

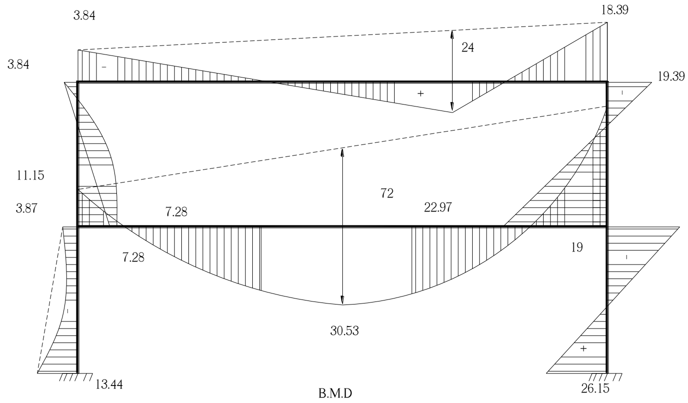
M_{DC} = - 53.44 t.m

M_{CE} = 3.87 t.m

M_{EC} = - 13.44 t.m

M_{DF} = - 30.47 t.m

M_{FD} = - 26.15 t.m



<u>Example (13)</u>:-

Write the shear equations & equilibrium equations for the shown frame.

Solution:

Shear eqns:

$X_{CE} + X_{BA} + P_1 \quad = 0$ ……………………….....(1)

$$\frac{M_{EC} + M_{CE}}{h_1} + \frac{M_{AB} + M_{BA}}{h_1 + h_2} + P_1 = o$$

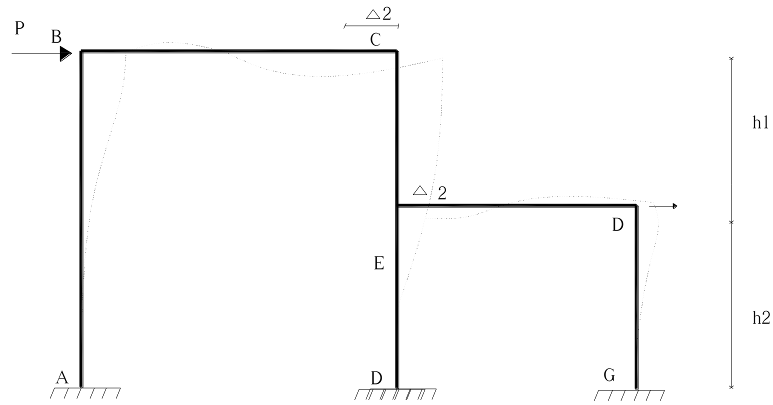
$x_D + x_G + x_E + P_2 = 0$ --- (2)

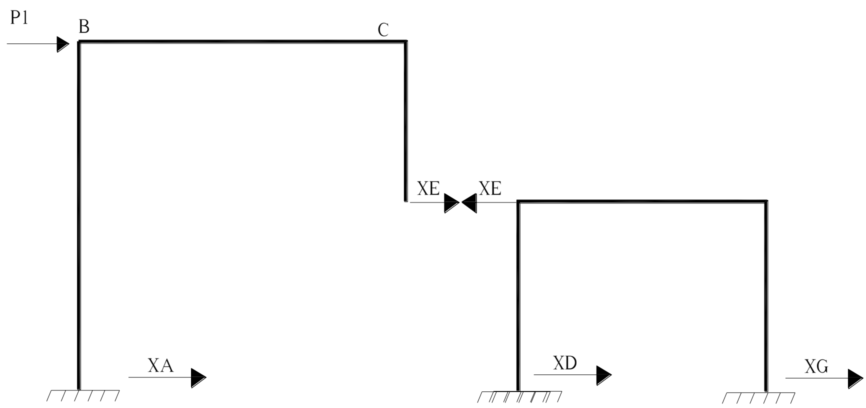
$$\frac{M_{DE} + M_{ED}}{h_2} + \frac{M_{GF} + M_{FG}}{h_2} - \frac{M_{EC} + M_{CE}}{h_1} + P_2 = 0$$

Or:

$$X_A + X_D + X_G + P_1 + P_2 = 0$$

$$\frac{M_{AB}}{h_1 + h_2} + \frac{M_{DE} + M_{ED}}{h_2} + \frac{M_{GF} + M_{FG}}{h_2} + P_1 + P_2 = 0$$





<u>Example 14:-</u>

a- Write the equations of equilibrium including the shear equations for the frame.

b- Write the slope deflection equations in matrix for members CE & GH.

c- By using the slope - deflection method; sketch elastic curve.

d- Sketch your expected B.M.D

Solution:-

($\textbf{Unknowns} = \theta_C\ ,\ \theta_D\ ,\ \theta_E\ ,\ \theta_F\ ,\ \theta_G\ ,\ \theta_A+\theta_K\ ,\ \theta_L\ ,\ \Delta_1\ ,$

$\Delta_2\ ,\ \Delta_3\ ,\ \Delta_4$

Relative stiffness: 1 : 1

a- equilibrium equations

$M_{KL} + M_{KG} = 0$ (1)

$M_{LK} + M_{LH} = 0$ (2)

$M_{GK} + M_{GH} + M_{GE} = 0$ (3)

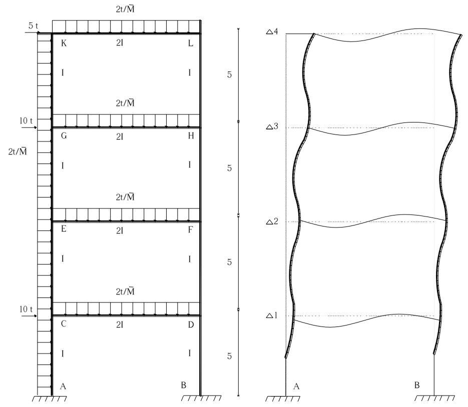
$M_{HG} + M_{HL} + M_{HF} = 0$ (4)

$M_{EG} + M_{EC} + M_{FF} = 0$ (5)

$M_{FE} + M_{FD} + M_{FH} = 0$ (6)

$M_{CE} + M_{CD} + M_{CA} = 0$ (7)

$M_{DC} + M_{DB} + M_{DF} = 0$ (8)



<u>Shear equations:-</u>

a- at Level GH

$5 + 10 + (X_G - 5) + X_H = 0$ (9)

Where:

$$X_G = \frac{M_{GK} + M_{KG}}{5}$$

$$X_H = \frac{M_{HL} + M_{LH}}{5}$$

b- at Level EF

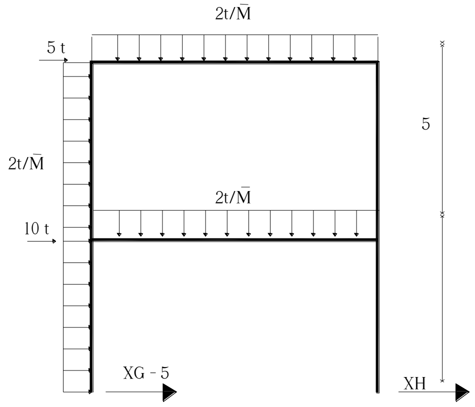
$$5 + 10 + 20 + (X_E - 5) + X_F = 0$$

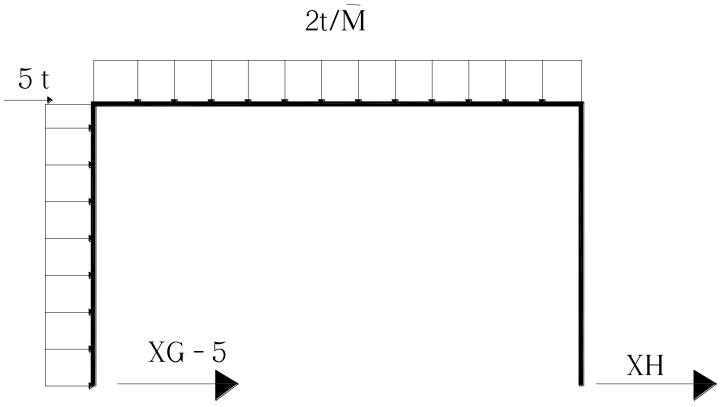
$$30 + X_E + X_F = 0 \qquad (10)$$

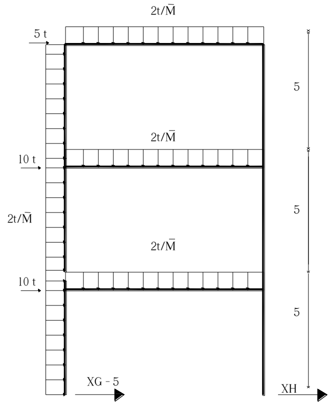
Where:

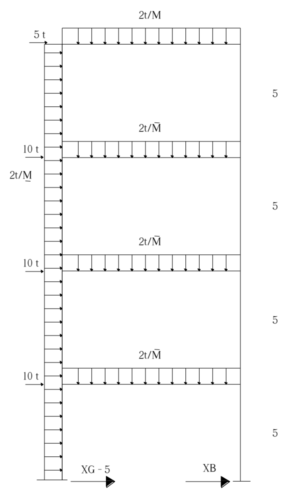
$$X_E = \frac{M_{EG} + M_{GE}}{5}$$

$$X_F = \frac{M_{FH} + M_{HF}}{5}$$


2t/M̄
5 t
2t/M̄
2t/M̄
10 t
5
XG - 5
XH


2t/M̄
5 t
XG - 5
XH





c- at Level CD

$5 + 10 + 10 + 30 + (X_C - 5) + X_D = 0$

$50 + X_C + X_D = 0$ ………(11)

Where:

$$X_C = \frac{M_{CE} + M_{EC}}{5}$$

$$X_D = \frac{M_{DF} + M_{FD}}{5}$$

d- at Sec AB:-

$5 + 10 + 10 + 10 + 40 + (X_A - 5) + X_B = 0$

$$70 + X_A + X_B = 0 \quad .....(12)$$

$$X_A = \frac{M_{AC} + M_{CA}}{5}$$

$$X_B = \frac{M_{CE} + M_{EC}}{5}$$

3-8 Slope deflection eqns in matrix form:

1- Member CE

$$M_{CD} = MF_{CE} + \frac{2EI}{5}\left(2\theta_C + \theta_E - 3\,\frac{\Delta 2 - \Delta 1}{5}\right)$$

$$M_{EC} = MF_{EC} + \frac{2EI}{5}\left(2\theta_E + \theta_C - 3\,\frac{\Delta 2 - \Delta 1}{5}\right)$$

Where:

$$MF_{CE} = -\frac{2\times 5^2}{12} = -4.16 \quad t.m$$

$$MF_{EC} = +4.16 \quad t.m$$

In Matrix form:

$$\begin{vmatrix} MCE \\ \\ MEC \end{vmatrix} = \begin{vmatrix} -4.16 \\ \\ 4.16 \end{vmatrix} + \frac{2EI}{5}\begin{vmatrix} 2 & 1 & -3 \\ \\ 1 & 2 & -3 \end{vmatrix} \begin{vmatrix} \theta_C \\ \theta_E \\ R_2 \end{vmatrix}$$

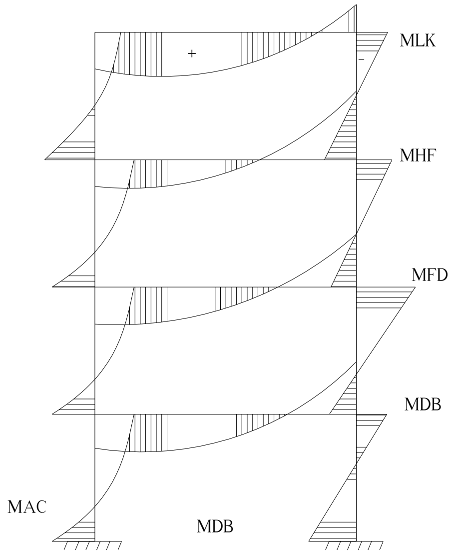
Where:

$$R_2 = \frac{\Delta_2 - \Delta_1}{5}$$

2- member GH

$$\begin{vmatrix} M_{GH} \\ \\ M_{HG} \end{vmatrix} = \begin{vmatrix} -16.67 \\ \\ +16.67 \end{vmatrix} + \frac{26I}{5} \begin{vmatrix} 2 & 1 \\ \\ 1 & 2 \end{vmatrix} \begin{vmatrix} \theta_G \\ \\ \theta_H \end{vmatrix}$$

d- B.M.D



<u>Example 15:-</u>

By using slope deflection method;

1- Draw B.M.D for the shown frame.

2- Sketch elastic curve.

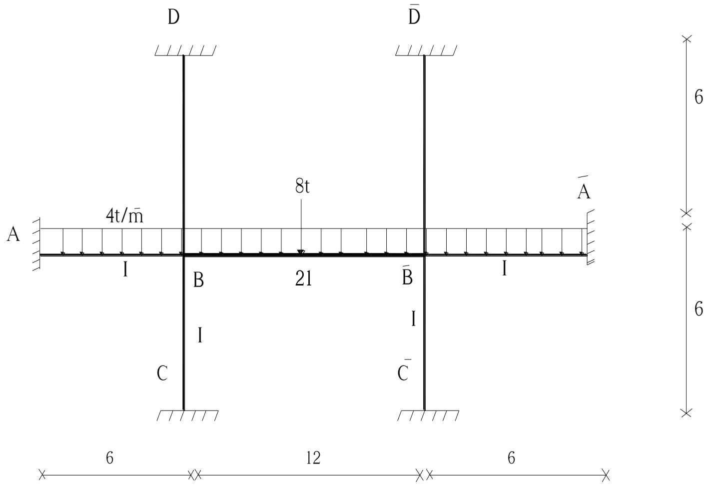
Solution:

1- Relative stiffness 1: 1

2- unknowns: $\theta_B = -\theta_B$ (From symmetry)

3- Equilibrium eqns

$M_{BA} + M_{BC} + M_{BD} + M_{BB} = o$ (1)



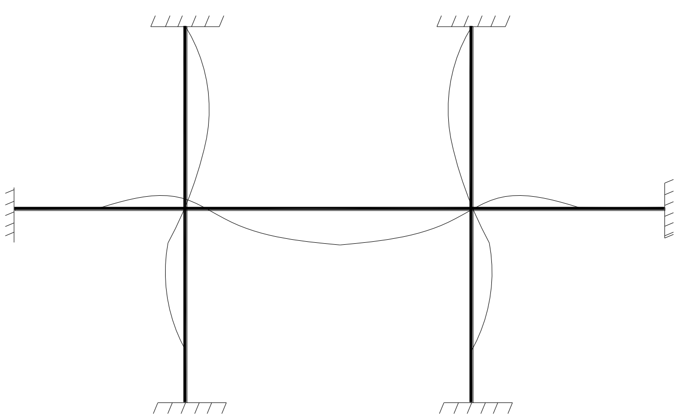
4-Fixed end moments

$$MF_{AB} = \frac{4\times 6_2}{12} = -12 \text{ t.m}$$

$$MF_{BA} = \quad = +12$$

$$MF_{BC} = MF_{CB} = MF_{BD} = MF_{DE} = o$$

$$MF_{B\bar{B}} = -\frac{2\times 12^2}{12} - \frac{8\times 12}{8} = -36 \text{ t.m}$$



4- Slope deflection eqns

$$M_{AB} = -12 + (\theta B)$$

$$M_{BA} = 12 + 2\theta B$$

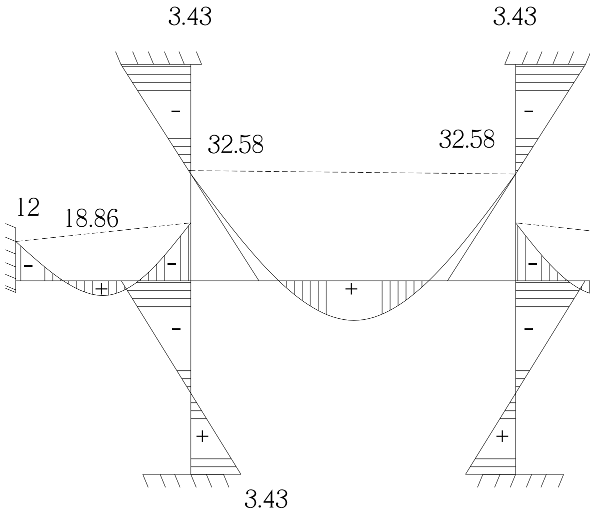
M_{BC} = 2θB

M_{BD} = 2θB

M_{BB} = - 36 + θB

M_{CB} = θB

M_{DB} = θB



From eqn (1)

$$(12 + 2\theta_B) + (2\theta_B) + (2\theta_B) + (-36 + \theta_B) = 0$$

$$7\theta_B - 24 = 0$$

$$\theta_B = 3.4286$$

hence

M_{AB} = - 8.57 t.m

M_{BA} = 18.86 t.m

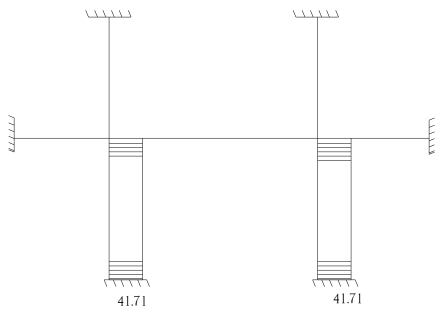
M_{BC} = 6.86 t.m

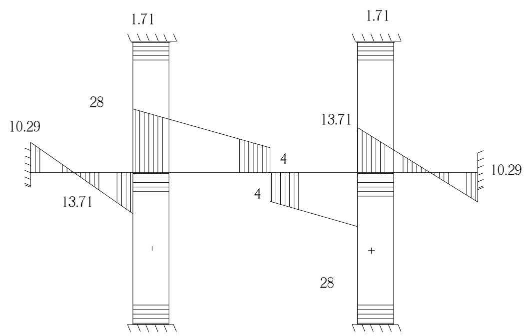
M_{BB} = - 32.58 t.m

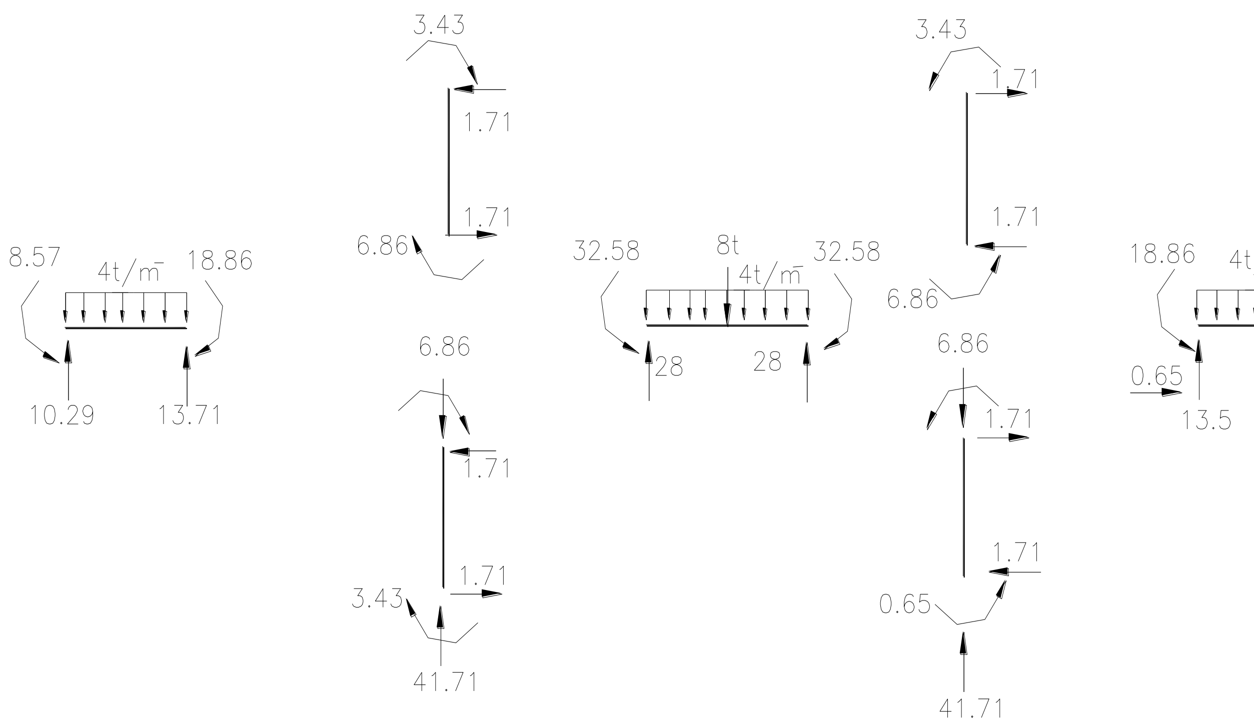
M_{CB} = 3.428 t.m

$M_{DB} =$ 3.428 t.m

$M_{BD} =$ 6.86

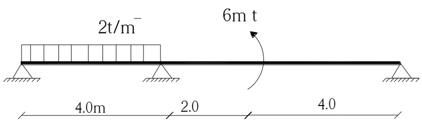


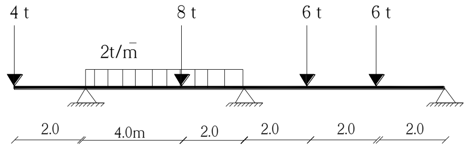


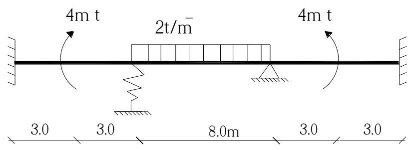


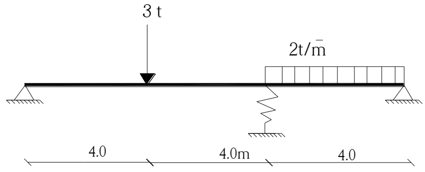
The Free Body Diagram to find the S. F. & N. F.
<u>SHEET (3)</u>

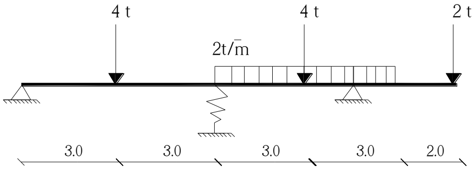
1) Draw S.F.D. and B.M.D. for the statically indeterminate beams shown in figs. From 1 to 10.

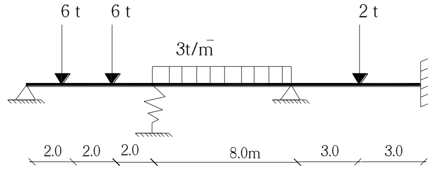


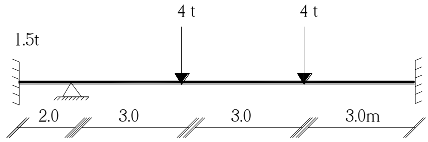


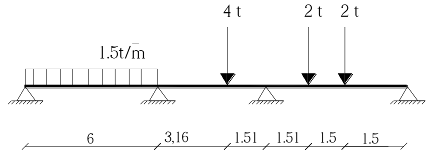


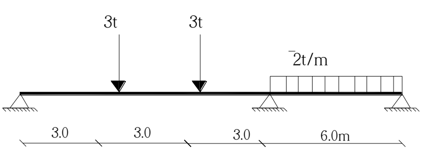


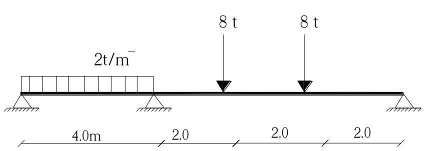












2) Draw N.F.D., S.F.D. for the statically indeterminate frames shown in figs. 11 to 17. Using matrix approach 1.