FE Exam Review for
Structural Analysis
Prof. V. Saouma

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- Structural Analysis is part of the afternoon exam.
- In the afternoon, you are to answer 60 questions, and Structural Analysis is about 10% of the test content (or about 6 questions).
- Each question is worth 2 points.
- You are expected to know:
    1. Structural analysis of statically determinate beams, trusses and frames.
    2. Deflection analysis of statically determinate beams, trusses and frames.
    3. Stability analysis of beams, trusses and frames.
    4. Column analysis (e.g. buckling, boundary conditions).
    5. Loads and load paths (e.g. dead, live, moving).
    6. Elementary statically indeterminate structures.
- The only page in the "Supplied-Reference Handbook" related to Structural Analysis (shown in the next page).
- Make sure that you know how to make best use of it, as it contains:
    1. Reminder of what do we mean by "Moving Loads".
    2. Beam-Stiffness and moment carryover: to use for the analysis of statically indeterminate beams (unlikely that you get a SI frame).
    3. Equations for the calculations of the deflections of trusses and beams using the virtual work method. Careful it is the virtual force/moment time the actual displacement (FL/AE for trusses, and M/EI for beams).
    4. Member fixed end actions for uniform and concentrated load.
- I strongly recommend that you also memorize: $\Delta = \frac{5wL^4}{384\,EI}$ for the maximum deflection of a uniformly loaded, simply supported beam.
- Careful with the SI units, GPa is 10^9 Pa or 10^9 N/m^2 Many problems use the SI system.
- In most cases, you will be dealing with round numbers, which greatly simplify your calculations.
- Do not be tricked in believing that all triangles are 3-4-5.

STRUCTURAL ANALYSIS

## Influence Lines for Beams and Trusses

An influence line shows the variation of an effect (reaction, shear and moment in beams, bar force in a truss) caused by moving a unit load across the structure. An influence line is used to determine the position of a moveable set of loads that causes the maximum value of the effect.

Moving Concentrated Load Sets
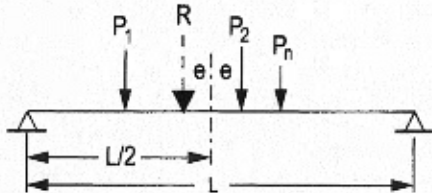
The **absolute maximum moment** produced in a beam by a set of "n" moving loads occurs when the resultant "R" of the load set and an adjacent load are equal distance from the centerline of the beam. In general, two possible load set positions must be considered, one for each adjacent load.

Beam Stiffness and Moment Carryover
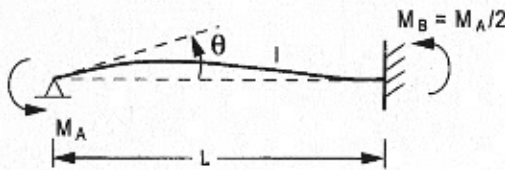
$$\theta = \frac{M\,L}{4\,EI} \quad \Rightarrow \quad M = \left(\frac{4\,EI}{L}\right)\theta = k_{AB}\,\theta$$

k_{AB} = stiffness $M_B = M_A/2$ = carryover

Truss Deflection by Unit Load Method

The displacement of a truss joint caused by external effects (truss loads, member temperature change, member misfit) is found by applying a unit load at the point that corresponds to the desired displacement.

$$\Delta_{joint} = \sum_{i=1}^{members} f_i (\Delta L)_i$$

where: Δ_{joint} = joint displacement at point of application of unit load (+ in direction of unit load)

f_i = force in member "i" caused by unit load (+ tension)

$(\Delta L)_i$ = change in length caused by external effect (+ for increase in member length):

$= \left(\dfrac{FL}{AE}\right)_i$ for bar force F caused by external load

$= \alpha L_i (\Delta T)_i$ for temperature change in member (α = coefficient of thermal expansion)

$=$ member misfit

L, A = member length and cross-sectional area

E = member elastic modulus

Frame Deflection by Unit Load Method

The displacement of any point on a frame caused by external loads is found by applying a unit load at that point that corresponds to the desired displacement:

$$\Delta = \sum_{i=1}^{members} \int_{x=0}^{x=L_i} \frac{m_i M_i}{EI_i} dx$$

where: Δ = displacement at point of application of unit load (+ in direction of unit load)

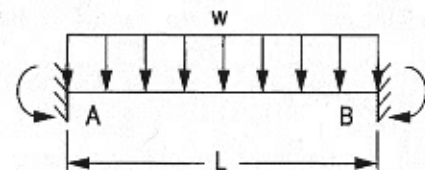m_i = moment equation in member "i" caused by the unit load

M_i = moment equation in member "i" caused by loads applied to frame
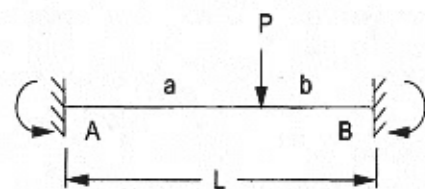
L_i = length of member "i"

I_i = moment of inertia of member "i"

If either the real loads or the unit load cause no moment in a member, that member can be omitted from the summation.

Member Fixed-End Moments (magnitudes)

$$FEM_{AB} = \quad FEM_{BA} = \quad \frac{wL^2}{12}$$

$$FEM_{AB} = \frac{Pab^2}{L^2} \qquad FEM_{BA} = \frac{Pa^2 b}{L^2}$$

From the PVC, the low point is located at

$$\frac{L}{2} + 44 \text{ m} = \frac{800 \text{ m}}{2} + 44 \text{ m}$$
$$= 444 \text{ m}$$

Determine the tangent offset, y, at the low point.

$$y = \frac{(G_2 - G_1)\, x^2}{2L}$$
$$= \frac{(0.016 - (-0.02))(444 \text{ m})^2}{(2)(800 \text{ m})}$$
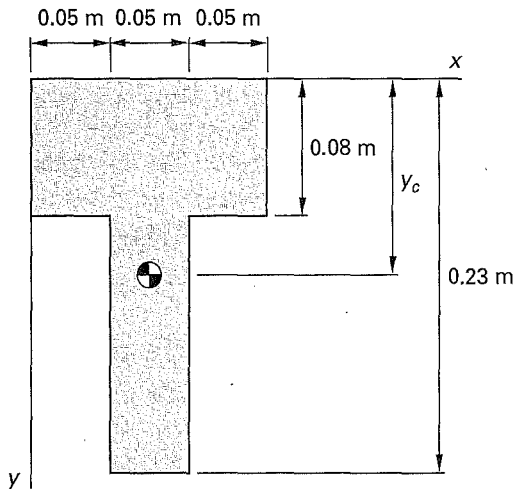$$= 4.44 \text{ m}$$

$$\text{elev}_{\text{low point}} = 742.12 \text{ m} + 4.44 \text{ m}$$
$$= 746.56 \text{ m} \quad (747 \text{ m})$$

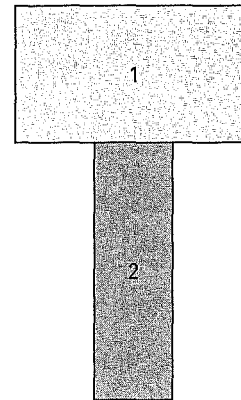The answer is B.

STRUCTURAL ANALYSIS

Problem 38

The y-coordinate (measured from the top) of the centroid for the T-shaped beam is

(A) 0.08 m
(B) 0.16 m
(C) 0.30 m
(D) 0.38 m

Solution

Divide the beam into two shapes as shown

$$A_1 = (0.08 \text{ m})(0.15 \text{ m})$$
$$= 0.012 \text{ m}^2$$
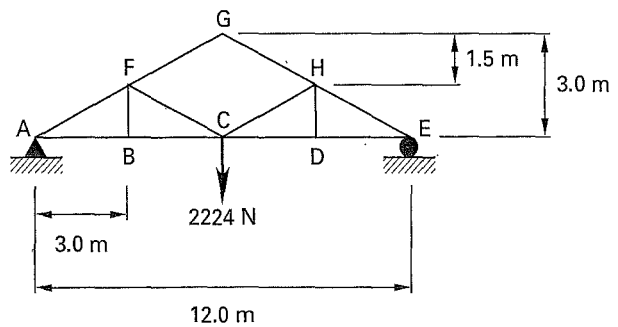$$A_2 = (0.05 \text{ m})(0.15 \text{ m})$$
$$= 0.0075 \text{ m}^2$$

The distance from the top of section 2 to the centroid is

$$y_c = \frac{\sum A_i y_{c,i}}{\sum A_i}$$

$$= \frac{\left(\left((0.012 \text{ m}^2)\left(\frac{0.08 \text{ m}}{2}\right) \right) + \left((0.0075 \text{ m}^2)\left(\frac{0.15 \text{ m}}{2} + 0.08 \text{ m}\right) \right) \right)}{0.012 \text{ m}^2 + 0.0075 \text{ m}^2}$$

$$= 0.08 \text{ m}$$

The answer is A.

Problem 39

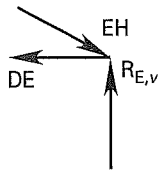The force in member HE from the truss shown is most nearly

(A) 1110 N compression
(B) 1110 N tension
(C) 2490 N compression
(D) 2490 N tension

Solution

Sum moments about A to find the vertical reaction at E.

$$\sum M_A = 0$$

$$= \frac{R_{E,v}}{d_{EA}}$$

$$R_{E,v} = \frac{(2224 \text{ N})(6 \text{ m})}{12 \text{ m}}$$

$$= 1112 \text{ N}$$

The free-body diagram about point E is

Summing forces in the horizontal direction gives the horizontal reaction at E as 0 N.

The sum of forces in the vertical direction is

$$F_{EH,v} - R_{E,v} = 0 \text{ lbf}$$

$$= 1112 \text{ N compression}$$

The horizontal component of the force in member EH is
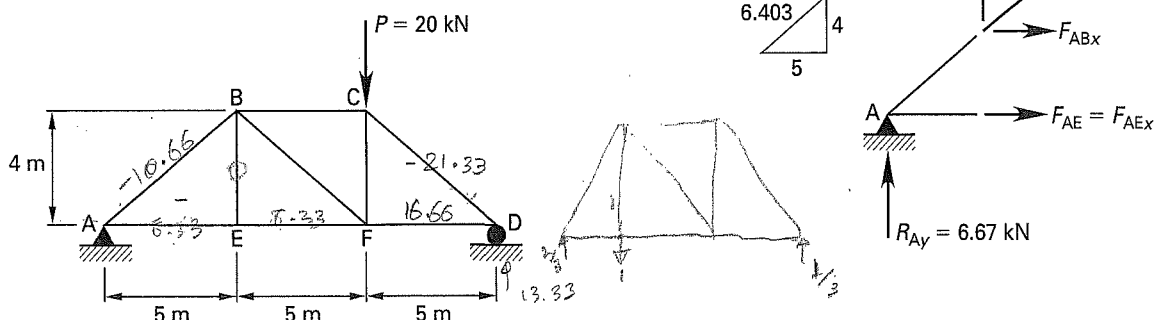
$$F_{EH,h} = (2)(1112 \text{ N})$$

$$= 2224 \text{ N}$$

The resultant force in member EH is

$$F_{EH} = \sqrt{(F_{EH,v})^2 + (F_{EH,h})^2}$$

$$= \sqrt{(1112 \text{ N})^2 + (2224 \text{ N})^2}$$

$$= 2487 \text{ N compression} \quad (2490 \text{ N compression})$$

The answer is C.

Problems 40–42 are based on the following information and illustration.

A plane truss is loaded as shown.

Problem 40

The magnitude of the vertical reaction force at support A is most nearly

(A) 3.3 kN
(B) 6.7 kN
(C) 10 kN
(D) 16 kN

Solution

Since support D is a roller support, the horizontal reaction force, R_{Ax}, is 0 kN. To find the vertical reaction at support A, R_{Ay}, a free-body diagram is drawn of the entire truss and moments are summed about support D.

$$R_{Ay}(15 \text{ m}) - (20 \text{ kN})(5 \text{ m}) = 0 \text{ kN}$$

$$R_{Ay} = \frac{(20 \text{ kN})(5 \text{ m})}{15 \text{ m}}$$

$$= 6.67 \text{ kN} \quad (6.7 \text{ kN})$$

The answer is B.

Problem 41

The magnitude of the compressive force in member AB is most nearly

(A) 4.2 kN
(B) 6.7 kN
(C) 8.5 kN
(D) 11 kN

Solution

From Sol. 40, the reaction at support A is 6.67 kN. Next, a free-body diagram is drawn for support A with member forces and their force components. Using the Pythagorean theorem, the relative magnitudes of each force and each force's horizontal and vertical components can be found. (In this case, 6.403, 5, and 4 are the relative magnitudes of the member AB force and the forces's horizontal and vertical components, respectively.)

For equilibrium, all forces on a free body must sum to 0 kN. Summation of vertical forces gives

$$F_{ABy} + R_{Ay} = 0 \text{ kN}$$

This can be rearranged to give

$$F_{ABy} = -R_{Ay} = -6.67 \text{ kN}$$

Recall that joints in trusses are frictionless, so no bending moments exist.

The force and its components are proportional to the geometric lengths of the triangle sides.

$$\begin{aligned} F_{AB} &= \left(\frac{6.403 \text{ m}}{4 \text{ m}}\right) F_{ABy} \\ &= \left(\frac{6.403 \text{ m}}{4 \text{ m}}\right)(-6.67 \text{ kN}) \\ &= -10.68 \text{ kN} \quad (-11 \text{ kN}) \end{aligned}$$

The answer is negative. This means that the calculated force is in the opposite direction to the assumed force direction on the free-body diagram. Since the member AB force was assumed to apply tension on the free-body diagram, the negative answer means that member AB is in compression.

The answer is D.

Problem 42

The magnitude of the force in member EF is most nearly

(A) 4.2 kN
(B) 5.3 kN
(C) 6.7 kN
(D) 8.3 kN

Solution

From Sol. 41, F_{ABy} is -6.67 kN. The horizontal component of the member AB force is

$$\begin{aligned} F_{ABx} &= \left(\frac{5 \text{ m}}{4 \text{ m}}\right) F_{ABy} \\ &= \left(\frac{5 \text{ m}}{4 \text{ m}}\right)(-6.667 \text{ kN}) \\ &= -8.33 \text{ kN} \end{aligned}$$

For equilibrium at support A, the sum of the horizontal forces must be equal to 0 kN. The force in member AE is

$$\begin{aligned} F_{AE} &= F_{AEx} \\ &= -F_{ABx} \\ &= -(-8.33 \text{ kN}) \\ &= 8.33 \text{ kN} \end{aligned}$$

The positive sign in the calculated member AE force means that the assumed direction of the force on the free-body diagram, which indicates tension, is in the same direction as the calculated force.

A free-body diagram of joint E will show that the vertical member BE is unable to sustain any horizontal force. Therefore, the force in member EF is the same as the force in member AE.

$$\begin{aligned} F_{EF} &= F_{AE} \\ &= 8.33 \text{ kN} \quad (8.3 \text{ kN}) \end{aligned}$$

The answer is D.

Problem 43

If the truss members are made of steel and the cross-sectional area of each member is 1000 mm², the magnitude of the vertical deflection at joint E is most nearly
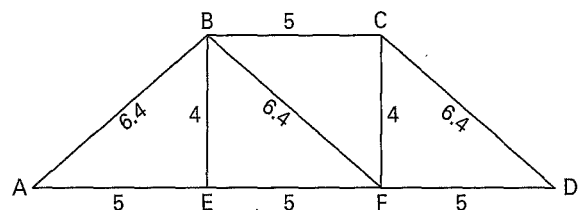
(A) 0.70 mm
(B) 1.1 mm
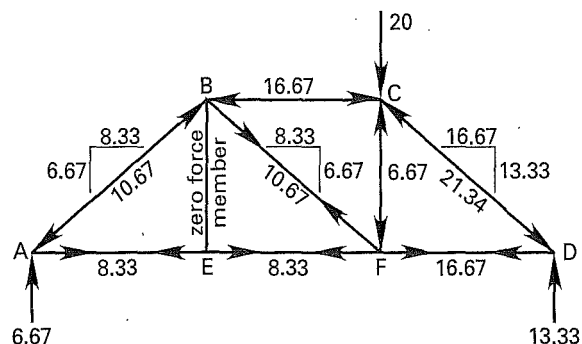(C) 1.6 mm
(D) 2.8 mm

Solution

Use the principle of virtual work (unit load method).

The actual forces in each member can be determined by applying the equations of equilibrium to each truss joint and are shown as follows. (Some round-off error exists in these calculated numbers.)
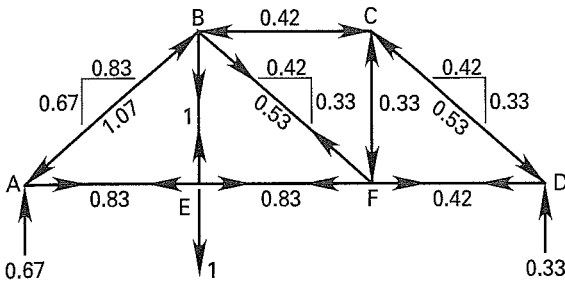
Truss member lengths in meters are as follows.

The actual forces in kilonewtons are shown.

Application of a vertical unit load at joint E results in the virtual member forces in kilonewtons as follows. (Some round off error exists in these calculated numbers.)

It is recommended that a table be used to keep all variables organized. Use negative values to represent compression.

| member	F_P (kN)	F_Q (kN)	L (m)	$F_Q F_P L$ (kN^2·m)
AB	−10.67	−1.07	6.4	73.1
BC	−16.67	−0.42	5.0	35.0
CD	−21.34	−0.53	6.4	72.4
AE	+8.33	+0.83	5.0	34.6
EF	+8.33	+0.83	5.0	34.6
FD	+16.67	+0.42	5.0	35.0
BE	+0.00	+1.00	4.0	0.0
CF	−6.67	+0.33	4.0	−8.8
BF	+10.67	−0.53	6.4	−36.2
				$\sum = 239.7$

The modulus of elasticity of steel is $E = 2.1 \times 10^{11}$ Pa. Since the area and modulus of elasticity are the same for all truss members, their product, AE, is common to all members and can be taken outside of the summation for simplification. Therefore, the vertical deflection at point E can be found by

$$\Delta_E = \sum F_Q \delta L = \sum F_Q \frac{F_P L}{AE}$$

$$= \left(\frac{1}{AE}\right) \sum F_Q F_P L$$

$$= \left(\frac{239.7 \text{ kN}^2 \cdot \text{m}}{(1000 \text{ mm}^2)\left(\dfrac{1 \text{ m}}{1000 \text{ mm}}\right)^2 \\ \times (2.1 \times 10^{11} \text{ Pa})\left(\dfrac{1 \text{ kN}}{1000 \text{ N}}\right)} \right)$$

$$\times \left(1000 \ \frac{\text{mm}}{\text{m}}\right)$$

$$= 1.14 \text{ mm} \quad (1.1 \text{ mm})$$

Since the unit load in the virtual force system was downward and the answer is positive in sign, the actual deflection is also downward.

The answer is B.

Problems 44 and 45 are based on the following illustration.
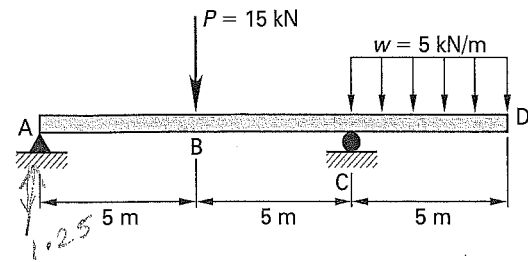
Problem 44

The magnitude of the vertical reaction force at support A is most nearly

(A) 1.3 kN
(B) 5.0 kN
(C) 13 kN
(D) 20 kN

Solution

Since support C is a roller support, there is no horizontal reaction force at that point. The vertical reaction force at support A, R_{Ay}, can be found by converting the uniformly distributed load, w, into a resultant point load.

$$W = wL = \left(5 \ \frac{\text{kN}}{\text{m}}\right)(5 \text{ m})$$

$$= 25 \text{ kN}$$

This resultant load is located at the centroid of the uniformly distributed load.
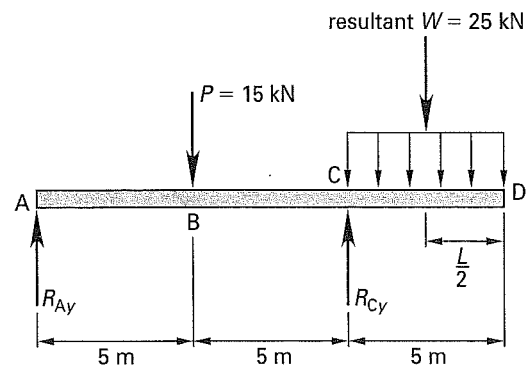
The support A vertical reaction can then be found by summing moments about support C. This results in a vertical reaction at support A of

$$R_{Ay}(10 \text{ m}) - (15 \text{ kN})(5 \text{ m})$$
$$+ (25 \text{ kN})(2.5 \text{ m}) = 0 \text{ kN·m}$$

$$R_{Ay} = \frac{(15 \text{ kN})(5 \text{ m}) - (25 \text{ kN})(2.5 \text{ m})}{10 \text{ m}}$$

$$= 1.25 \text{ kN} \quad (1.3 \text{ kN})$$

Since the answer is positive in sign, the direction of the calculated reaction is the same as that of the assumed reaction; that is, the direction of the reaction is upward.
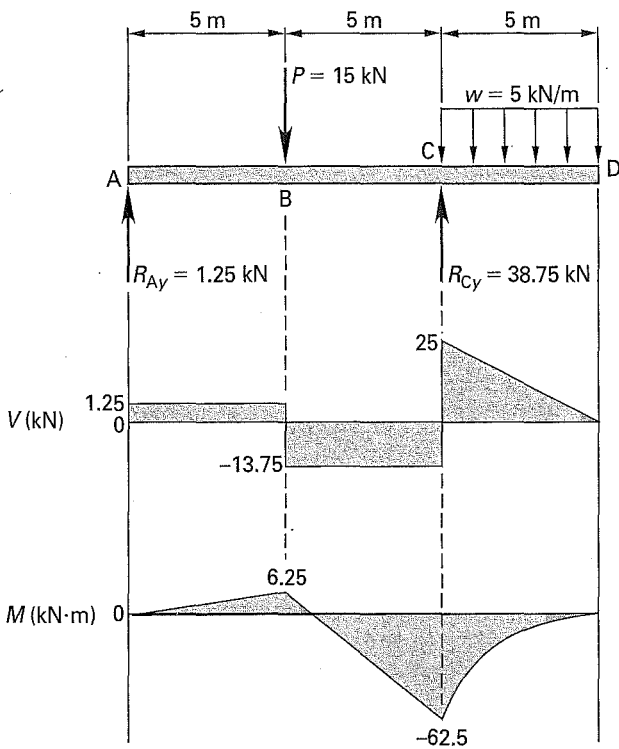
The answer is A.

Problem 45

The magnitude of the maximum vertical shear in the beam is most nearly

    (A) 1.3 kN
    (B) 14 kN
    (C) 25 kN
    (D) 39 kN

Solution

One way to determine the answer to this problem is to construct a shear diagram. The change in shear is the area under the applied loading. Although a moment diagram is not required for this problem, it follows that the change in moment is the area under the shear diagram, so a moment diagram is usually included.

As can be seen, the maximum value of vertical shear is 25 kN at support C.
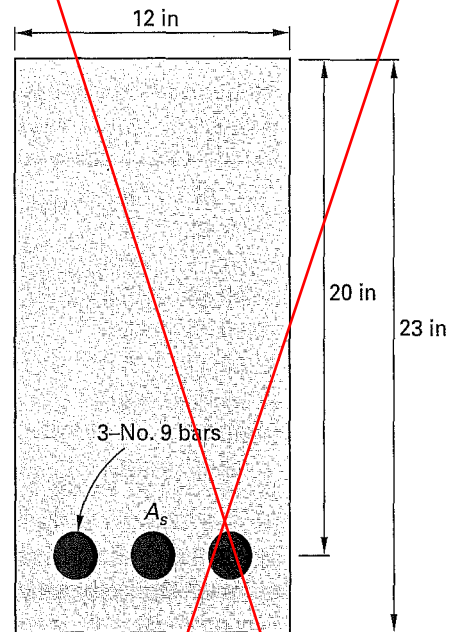
The answer is C.

STRUCTURAL DESIGN

Problems 46 and 47 are based on the following information and illustration.

The cross section of a reinforced concrete beam with tension reinforcement is shown. Assume that the beam is underreinforced.

$$f'_c = 3000 \text{ lbf/in}^2$$
$$f_y = 40,000 \text{ lbf/in}^2$$
$$A_s = 3 \text{ in}^2 \quad [\text{three no. 9 bars}]$$

Problem 46

In accordance with American Concrete Institute (ACI) strength design, the allowable moment capacity of the beam is most nearly

    (A) 160 ft-kips
    (B) 180 ft-kips
    (C) 200 ft-kips
    (D) 210 ft-kips

Solution

$$\rho = \frac{A_s}{bd} = \frac{3 \text{ in}^2}{(12 \text{ in})(20 \text{ in})}$$
$$= 0.0125$$

In an actual design and analysis situation, a check should always be made to see that the actual reinforcing steel ratio falls between the allowable maximum and allowable minimum steel ratios, even though this check is not required to solve this specific problem.

The minimum required length of spiral transition between the curve and road is most nearly

(A) 28 ft
(B) 36 ft
(C) 44 ft
(D) 72 ft

36. The design requirements for a section of highway with a 1.5% grade are as follows.

design speed = 80 km/h
coefficient of friction = 0.35
driver reaction time = 2.0 s
driver eye height = 1.2 m
object (to be avoided) height = 0.2 m

The downhill design braking distance for this highway is most nearly

(A) 45 m
(B) 75 m
(C) 100 m
(D) 120 m

37. A crest on a section of highway consists of a vertical curve with a 1500 m radius and a positive 1% grade followed by a negative 3% grade. The design requirements are as follows.

design speed = 80 km/h
driver eye height = 1.2 m
object (to be avoided) height = 0.2 m
stopping sight distance = 300 m

The minimum required length of vertical curve needed to satisfy the design stopping sight distance is most nearly

(A) 680 m
(B) 700 m
(C) 760 m
(D) 840 m

38. A superpave design mixture for a highway with ESALs < 10^7 has a nominal maximum aggregate size of 19 mm. The mixture has been tested and has the following characteristics:

air voids = 4.0%
VMA = 13.2%
VFA = 70%
dust-to-asphalt ratio = 0.97
at N = 8 gyrations, G_{mm} = 87.1%
at N = 174 gyrations, G_{mm} = 97.5%

Do these characteristics satisfy their corresponding superpave requirements?

(A) Yes, all the parameters are within an acceptable range.
(B) No, the VMA is excessive.
(C) No, the dust-to-asphalt ratio is too high.
(D) No, G_{mm} at N_{max} is too high.

39. A road leading to a stone quarry is traveled by 40 trucks, with each truck making an average of 10 trips per day. When fully loaded, each truck consists of a front single axle transmitting a force of 10,000 lbf and two rear tandem axles, each axle transmitting a force of 20,000 lbf. The load equivalency factor for the front single axle is 0.0877. The load equivalency factor for each rear tandem axle is 0.1206.

The 18,000 lbf equivalent single axle load (ESAL) for the truck traffic on this road for 5 yr is most nearly

(A) 0.33 ESAL
(B) 130 ESAL
(C) 48,000 ESAL
(D) 240,000 ESAL

Problems 40 and 41 are based on the following illustration.
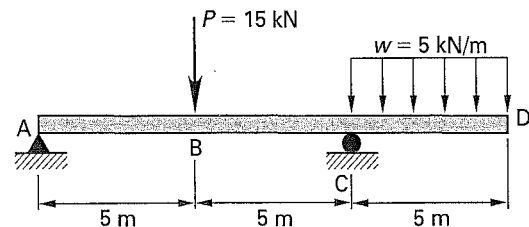
**40.** The magnitude of the maximum bending moment in the beam is most nearly

(A) 6.3 kN·m
(B) 14 kN·m
(C) 25 kN·m
(D) 63 kN·m

41. If the beam is made entirely of steel and the whole beam has a moment of inertia about the axis of bending of 2.0×10^8 mm^4, the magnitude of the vertical deflection at point D is most nearly

(A) 0.20 mm
(B) 2.3 mm
(C) 23 mm
(D) 50 mm

Problems 42 and 43 are based on the following information and illustration.

A truck is facing in its intended direction of travel along the beam as shown.
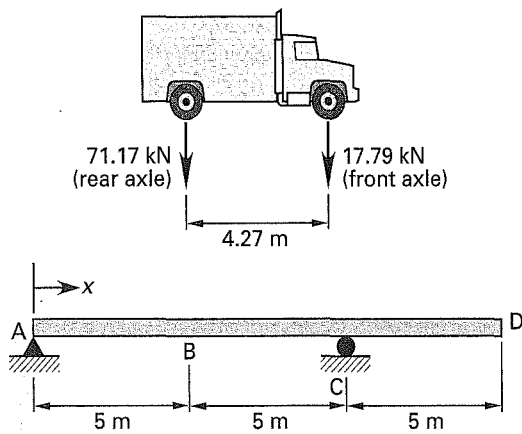
42. For a truck traveling in the direction shown, the maximum vertical live load shear at support C is most nearly

(A) 27 kN
(B) 76 kN
(C) 100 kN
(D) 110 kN

43. For a truck traveling in the direction shown, the maximum live load bending moment at support C is most nearly

(A) 80 kN·m
(B) 90 kN·m
(C) 140 kN·m
(D) 360 kN·m

Problems 44 and 45 are based on the following information and illustration.

A plane truss span is shown. The roadway behaves as simply supported beam spans between the supporting lower chord truss joints. By convention, positive forces are tensile forces, and negative forces are compressive forces.
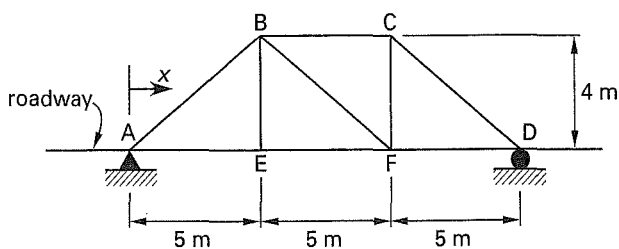
**44.** In the x-direction as shown, the maximum influence line ordinate for tensile force in member BF is most nearly

(A) 0.18 kN/kN
(B) 0.36 kN/kN
(C) 0.53 kN/kN
(D) 0.71 kN/kN

45. In the x-direction as shown, the maximum influence line ordinate for compressive force in member BF is most nearly

(A) −0.71 kN/kN
(B) −0.53 kN/kN
(C) −0.36 kN/kN
(D) −0.18 kN/kN

Problems 46 and 47 are based on the following information and illustration.

The cross sections of two short, concentrically loaded reinforced concrete columns are shown.

round spiral column
(cross section)
(Prob. 46)

square tied column
(cross section)
(Prob. 47)
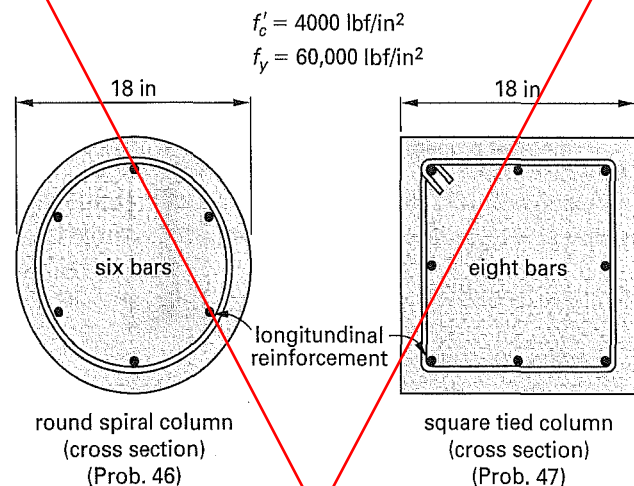
46. For the short round spiral column, the applied axial dead load is 150 kips, and the applied axial live load is 350 kips. Assuming that the longitudinal reinforcing bars are all the same size, the minimum required size of each longitudinal reinforcing bar is

(A) no. 7
(B) no. 8
(C) no. 9
(D) no. 10

47. For the short square tied column, the applied axial dead load is 150 kips, and the applied axial live load is 250 kips. Assuming that the longitudinal reinforcing

37. There are two equations to check.

First, where stopping sight distance, S, is less than the curve length, L:

$$L = \frac{AS^2}{(100\%)\left(\sqrt{2H_1} + \sqrt{2H_2}\right)^2}$$

$$= \frac{\left(1\% - (-3\%)\right)(300 \text{ m})^2}{(100\%)\left(\sqrt{(2)(1.2 \text{ m})} + \sqrt{(2)(0.2 \text{ m})}\right)^2}$$

$$= 756.4 \text{ m}$$

Second, where stopping sight distance is greater than the curve length:

$$L = 2S - \frac{(200\%)\left(\sqrt{H_1} + \sqrt{H_2}\right)^2}{A}$$

$$= (2)(300 \text{ m}) - \frac{(200\%)\left(\sqrt{1.2 \text{ m}} + \sqrt{0.2 \text{ m}}\right)^2}{1\% - (-3\%)}$$

$$= 481.0 \text{ m}$$

Since the previous two equations show that the stopping sight distance is less than the curve length, the minimum required vertical curve length is

$$L = 756.4 \text{ m} \quad (760 \text{ m})$$

The answer is C.

38. This mixture would be designated as a 19 mm superpave mixture. The limits for such a mixture are

$$\text{air voids} = 4.0\%$$
$$\text{minimum VMA} = 13\%$$
$$\text{VFA} = 65\text{--}75\%$$
$$\text{dust-to-asphalt ratio} = 0.6\text{--}1.2$$
$$\text{at } N_{\text{init}} = 8 \text{ gyrations, maximum } G_{\text{mm}} = 89\%$$
$$\text{at } N_{\text{max}} = 174 \text{ gyrations, maximum } G_{\text{mm}} = 98\%$$

All parameters in this mixture are within superpave specifications.

The answer is A.

39. The total ESAL per truck for each trip is

$$\text{ESAL}_{\text{truck}} = (1 \text{ single axle})(0.0877)$$
$$+ (2 \text{ tandem axles})(0.1206)$$
$$= 0.3289 \text{ ESAL/truck-trip}$$

The total daily ESAL for 40 trucks, each making 10 trips a day, is

$$\text{ESAL}_{\text{day}} = (40 \text{ trucks})\left(10 \frac{\text{trips}}{\text{day}}\right)$$
$$\times \left(0.3289 \frac{\text{ESAL}}{\text{truck-trip}}\right)$$
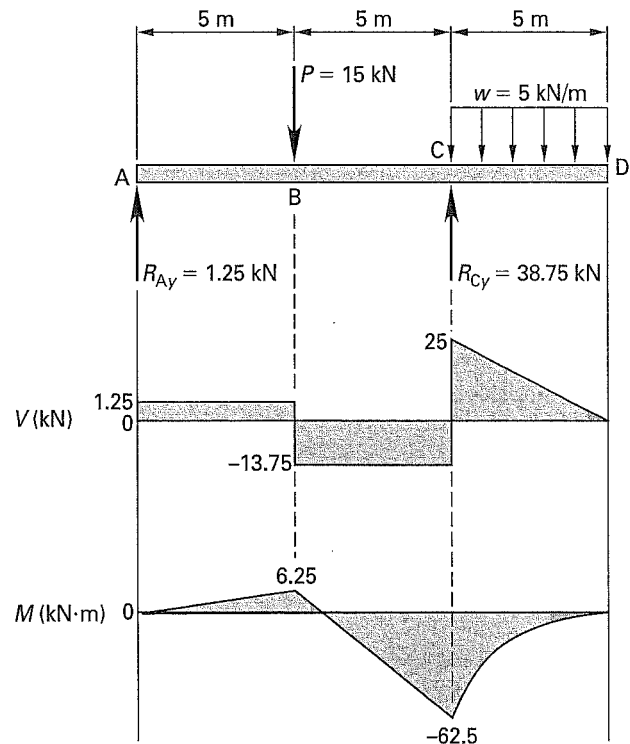$$= 131.56 \text{ ESAL/day}$$

For 5 yr, the total ESAL is

$$\text{ESAL}_{5 \text{ yr}} = (5 \text{ yr})\left(365 \frac{\text{days}}{\text{yr}}\right)\left(131.56 \frac{\text{ESAL}}{\text{day}}\right)$$

$$= 240{,}097 \text{ ESAL} \quad (240{,}000 \text{ ESAL})$$

The answer is D.

40. Construct shear and moment diagrams. The change in shear is the area under the applied loading, and the change in moment is the area under the shear diagram.
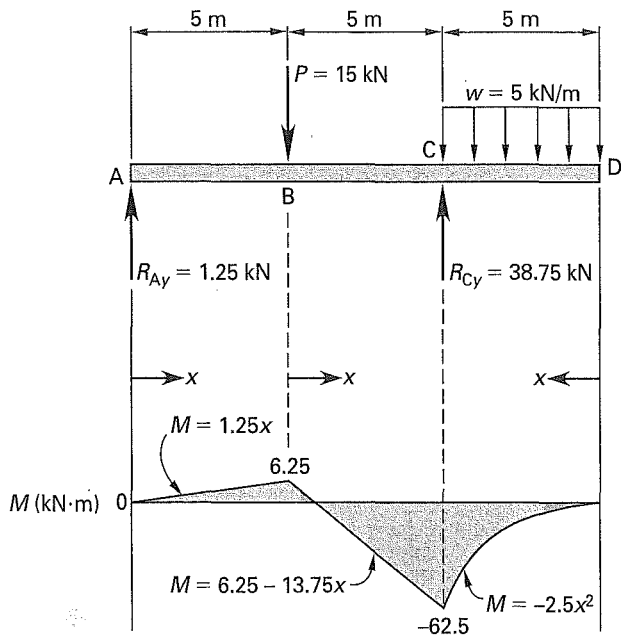

The largest magnitude of bending moment is -62.5 kN·m (63 kN·m) at support C.
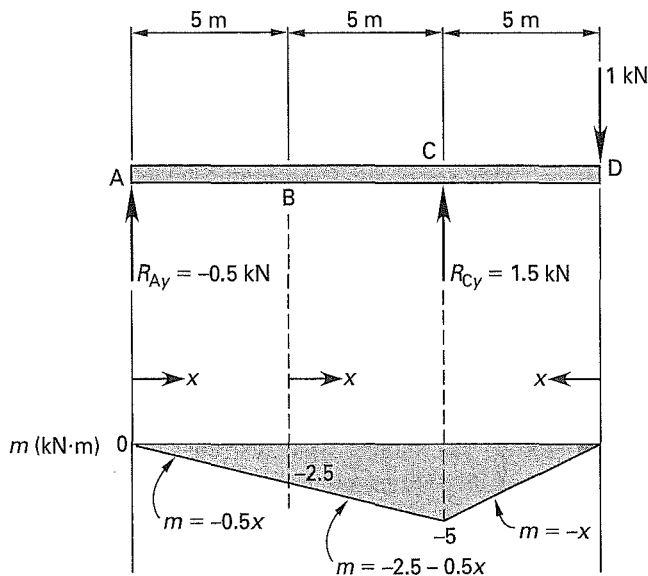
The answer is D.

41. Use the principle of virtual work to find the vertical deflection at point D.

$$\Delta_D = \sum \left(\int m \left(\frac{M}{EI} \right) dx \right)$$

The moment functions in the directions indicated by the local x-coordinate for each beam segment under the actual loading are shown on the following moment diagram.

The modulus of elasticity of steel is $E = 2.1 \times 10^{11}$ Pa.

From the principle of virtual work, the vertical deflection at point D is

$$\Delta_{\mathrm{D}} = \int_0^5 \frac{(-0.5x)(1.25x)}{EI}\, dx$$

$$+ \int_0^5 \frac{(-2.5 - 0.5x)(6.25 - 13.75x)}{EI}\, dx$$

$$+ \int_0^5 \frac{(-x)(-2.5x^2)}{EI}\, dx$$

$$= \frac{1}{EI} \left(\begin{array}{l} \left.\dfrac{-0.625x^3}{3}\right|_0^5 + \left. -15.625x \right|_0^5 \\[2mm] + \left.\dfrac{31.25x^2}{2}\right|_0^5 + \left.\dfrac{6.875x^3}{3}\right|_0^5 \\[2mm] + \left.\dfrac{2.5x^4}{4}\right|_0^5 \end{array} \right)$$

$$= \frac{\left(\begin{array}{l} -26.042 \text{ kN}^2\cdot\text{m}^3 - 78.125 \text{ kN}^2\cdot\text{m}^3 \\ + 390.625 \text{ kN}^2\cdot\text{m}^3 \\ + 286.458 \text{ kN}^2\cdot\text{m}^3 \\ + 390.625 \text{ kN}^2\cdot\text{m}^3 \end{array}\right)}{\left(2.1 \times 10^{11} \text{ Pa}\right)\left(\dfrac{1 \text{ kPa}}{1000 \text{ Pa}}\right)}$$
$$\times \left(2.0 \times 10^8 \text{ mm}^4\right)\left(\dfrac{1 \text{ m}}{1000 \text{ mm}}\right)^4$$
$$\times \left(1000 \dfrac{\text{mm}}{\text{m}}\right)$$
$$= 22.9 \text{ mm} \quad (23 \text{ mm})$$

Since the unit load in the virtual force system was downward and the answer is positive in sign, the actual deflection is also downward.

The answer is C.

The moment functions in the directions indicated by the local x-coordinate for each beam segment under the virtual unit loading at point D are shown on the following moment diagram.

A table summarizes the moment functions as follows.

function	region A-B $0 \le x \le 5$	region B-C $0 \le x \le 5$	region C-D $0 \le x \le 5$
M	$1.25x$	$6.25 - 13.75x$	$-2.5x^2$
m	$-0.5x$	$-2.5 - 0.5x$	$-x$
EI	EI	EI	EI

42. Construct and use an influence line for vertical shear at support C. The influence line is constructed by plotting the change in response on a free-body diagram of a section of beam at support C as a unit load travels across the structure.
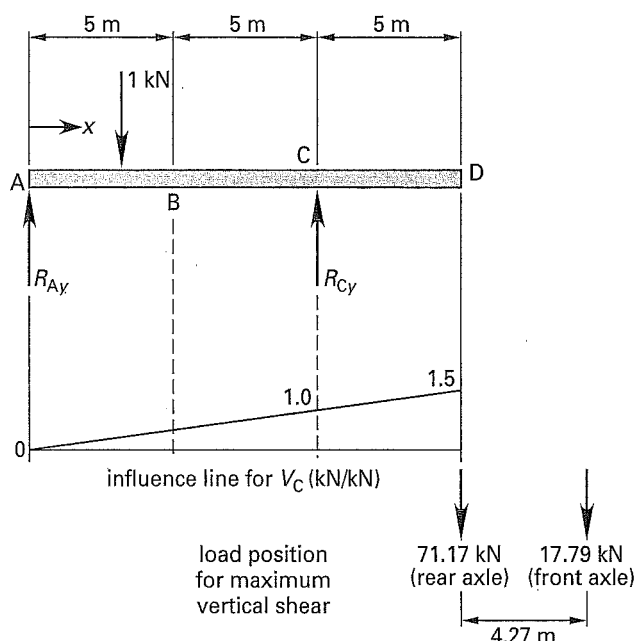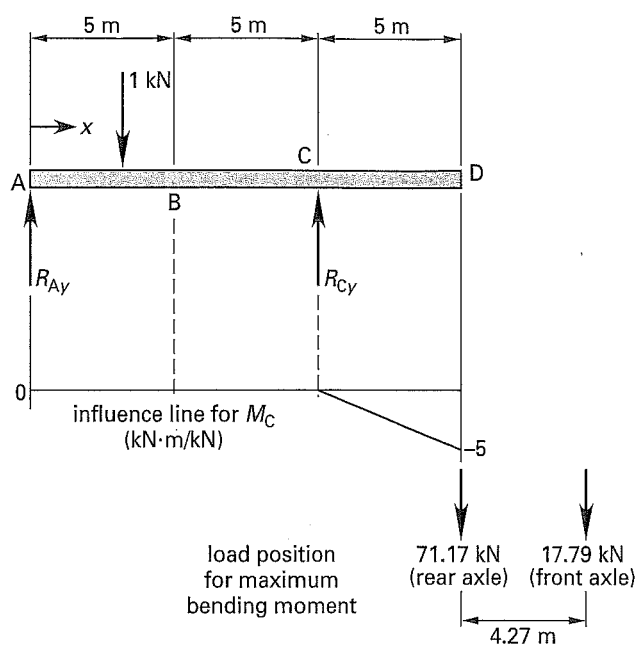

The load position shown results in the maximum response in the beam at support C for the specified direction of travel. The maximum vertical shear can be found by superposition as

$$V_{C,max} = \left(1.5 \; \frac{kN}{kN}\right)(71.17 \text{ kN})$$

$$= 106.8 \text{ kN} \quad (110 \text{ kN})$$

This problem asked for the magnitude of maximum vertical shear at support C for a truck *traveling in the direction shown*. If this problem had asked for the magnitude of maximum vertical shear at support C *for the given truck axle configuration*, the axle load positions would have to be switched around and both axle loads placed on the influence line with the heavier axle load on the larger influence line ordinate and the lighter axle load on the smaller influence line ordinate. This would result in a larger numerical answer.

The answer is D.

43. Construct and use an influence line for bending moment at support C. The influence line is constructed by plotting the change in response on a free-body diagram of a section of beam at support C as a unit load travels across the structure.


The load position shown gives the maximum response in the beam at support C for the specified direction of travel. The maximum bending moment can be found by superposition as

$$M_{C,max} = \left|\left(-5 \; \frac{kN\cdot m}{kN}\right)(71.17 \text{ kN})\right|$$

$$= 355.8 \text{ kN·m} \quad (360 \text{ kN·m})$$

This problem asked for the magnitude of maximum bending moment at support C for a truck *traveling in the direction shown*. If this problem had asked for the magnitude of maximum bending moment at support C *for the given truck axle configuration*, the axle load positions would have to be switched around and both axle loads placed on the influence line with the heavier axle load on the larger influence line ordinate and the lighter axle load on the smaller influence line ordinate. This would result in a larger numerical answer.

The answer is D.

44. Construct an influence line for the force in member BF. The influence line is constructed by plotting the change in response in member BF as a unit load travels across the structure.

influence line for F_{BF} (kN/kN)
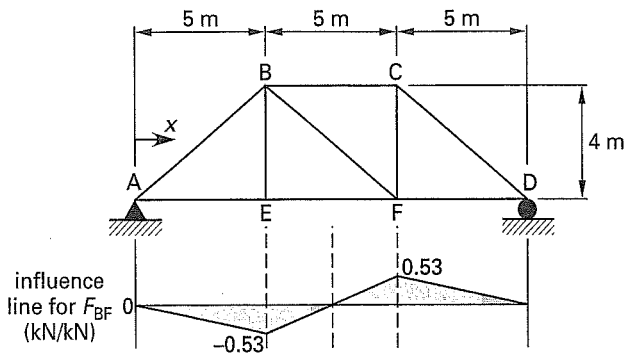
When moving the unit load across a truss structure, this load must be distributed to the two joints. It is stated that the roadway acts as simple beam spans between truss joints. Therefore, when a load is placed between truss joints, the two joints adjoining the beam span act as simple beam supports and the magnitudes of the loads applied to the two truss joints are the same as for the calculated reaction forces of this beam.

The maximum ordinate for tensile force in member BF is 0.53 kN/kN.

The answer is C.

45. From the influence line in Sol. 44, the maximum ordinate for compressive force in member BF is -0.53 kN/kN.

The answer is B.

46. Determine the amount of reinforcing steel required by the minimum required reinforcement ratio, ρ_g, of 0.01. The minimum area of reinforcing steel required is

$$A_s = \rho_g A_g$$
$$= (0.01)\left(\frac{\pi(18 \text{ in})^2}{4}\right)$$
$$= 2.54 \text{ in}^2$$

Determine the required amount of reinforcing steel based on the factored axial load, P_u.

$$P_u = 1.2P_{\text{dead}} + 1.6P_{\text{live}}$$
$$= (1.2)(150 \text{ kips}) + (1.6)(350 \text{ kips})$$
$$= 740 \text{ kips}$$

The nominal axial compressive load capacity is given by

$$P_n = 0.85P_o$$
$$= (0.85)(0.85f'_c A_{\text{concrete}} + f_y A_s)$$
$$= (0.85)(0.85f'_c(A_g - A_s) + f_y A_s)$$

It is required that $\phi P_n \geq P_u$. For axial compression with spiral reinforcement, $\phi = 0.70$. Setting $\phi P_n = P_u$ and solving for the area of longitudinal reinforcing steel gives

$$A_s = \frac{\frac{P_u}{0.85\phi} - 0.85f'_c A_g}{f_y - 0.85f'_c}$$

$$= \frac{\frac{(740 \text{ kips})\left(1000 \frac{\text{lbf}}{\text{kip}}\right)}{(0.85)(0.70)}}{60,000 \frac{\text{lbf}}{\text{in}^2} - (0.85)\left(4000 \frac{\text{lbf}}{\text{in}^2}\right)}$$
$$= \frac{-(0.85)\left(4000 \frac{\text{lbf}}{\text{in}^2}\right)\left(\frac{\pi(18 \text{ in})^2}{4}\right)}{60,000 \frac{\text{lbf}}{\text{in}^2} - (0.85)\left(4000 \frac{\text{lbf}}{\text{in}^2}\right)}$$

$$= 6.69 \text{ in}^2$$

$A_s = 6.69 \text{ in}^2$, required for the given applied axial compressive loads, is greater than $A_s = 2.54 \text{ in}^2$ based on the minimum allowed reinforcement ratio, $\rho_g = 0.01$. The minimum required area of reinforcement is $A_s = 6.69 \text{ in}^2$.

The column has six longitudinal reinforcing bars. The required area of each longitudinal reinforcing bar is

$$A = \frac{A_s}{n_{\text{bars}}} = \frac{6.69 \text{ in}^2}{6 \text{ bars}}$$
$$= 1.11 \text{ in}^2/\text{bar}$$

A bar area of 1.11 in² is satisfied by a no. 10 bar, which has a nominal area of 1.27 in².

In an actual design/analysis situation, a check should also be made to see that the actual longitudinal reinforcement ratio does not exceed the maximum allowable ratio of 0.08.

The answer is D.

47. Determine the amount of reinforcing steel required by the minimum required reinforcement ratio, ρ_g, of 0.01. The minimum area of reinforcing steel required is

$$A_s = \rho_g A_g$$
$$= (0.01)(18 \text{ in})^2$$
$$= 3.24 \text{ in}^2$$

Determine the required amount of reinforcing steel based on the factored axial load, P_u.

$$P_u = 1.2P_{\text{dead}} + 1.6P_{\text{live}}$$
$$= (1.2)(150 \text{ kips}) + (1.6)(250 \text{ kips})$$
$$= 580 \text{ kips}$$

The traffic flow relationship is given by $q = kv$, in which q is the traffic volume in veh/hr. The maximum traffic volume for this road is most nearly

(A) 760 veh/hr
(B) 880 veh/hr
(C) 900 veh/hr
(D) 960 veh/hr

38. The stopping sight distance is 430 ft for a design speed of 50 mph on a section of highway. The grades for this highway section are −1% followed by 3%. The required length of vertical curve needed to satisfy the AASHTO stopping sight distance for this design speed is most nearly

(A) 270 ft
(B) 380 ft
(C) 410 ft
(D) 450 ft

39. A one-lane rural road has a 10° curve extending for 230 m along its centerline. The road is 5 m wide with 3 m wide shoulders. The design speed for this road is 75 km/h.

The superelevation needed so that side friction is not needed is most nearly

(A) 0.00050
(B) 0.034
(C) 1.1
(D) 1.9

40. The worn surface course of a high-volume pavement is being replaced with a design requiring a total structural number of 6.6. The engineer has decided to replace 6 in of the surface with recycled-in-place asphalt concrete having a surface course strength coefficient of 0.42, leaving in place 3 in of sound original pavement having a strength coefficient of 0.3. Under the original pavement are a 10 in cement-treated base having a strength coefficient of 0.20, and an 8 in sandy gravel subbase. What is the minimum strength coefficient for the subbase?

(A) 0.05
(B) 0.10
(C) 0.15
(D) 0.20

Problems 41 and 42 are based on the following information and illustration.

The beam shown is loaded with two 1000 N point loads. The separation is maintained at 2 m, but the loads may be moved to any location on the beam.
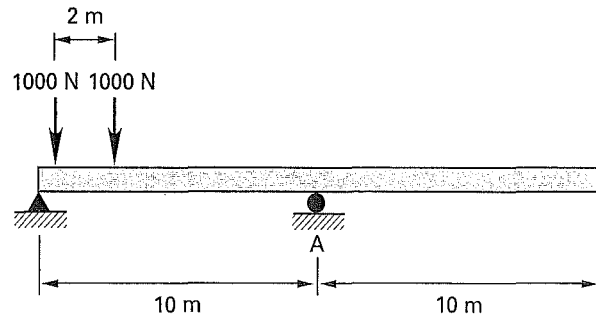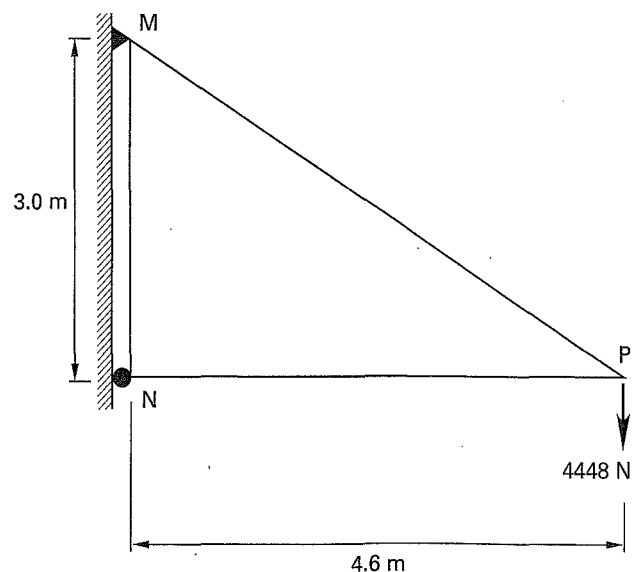

41. The maximum value for shear at support A is most nearly

(A) 2000 N
(B) 2800 N
(C) 3000 N
(D) 3800 N

42. The maximum value for moment at support A is most nearly
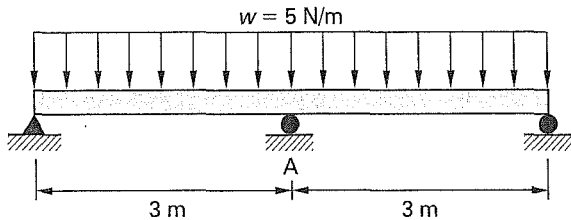
(A) 1.8 kN·m
(B) 8.0 kN·m
(C) 10 kN·m
(D) 18 kN·m

43. A triangular pin-connected truss carries a load of 4448 N as shown. Each member has the same modulus of elasticity and cross-sectional area.

The truss member properties are $E = 200 \times 10^6$ kPa and $A = 2580.6$ mm^2. The vertical deflection at point P is most nearly

(A) 0.25 mm
(B) 0.48 mm
(C) 0.51 mm
(D) 0.75 mm

Problems 44–46 are based on the following illustration.

44. If the reaction at support A is 18.75 N, the reaction at each of the outer supports is most nearly

(A) 5.6 N
(B) 7.2 N
(C) 11 N
(D) 14 N

**45.** The maximum value of vertical shear at any point along the beam is most nearly

(A) 4.7 N
(B) 9.4 N
(C) 14 N
(D) 18 N

46. The maximum value of moment at any point along the beam is most nearly

(A) 1.2 N·m
(B) 3.1 N·m
(C) 4.6 N·m
(D) 5.7 N·m

Problems 47–49 are based on the following information and illustration.

The span length and cross section of a reinforced concrete beam are shown. The beam is underreinforced. The concrete and reinforcing steel properties are $f'_c = 3000$ lbf/in^2, $f_y = 40{,}000$ lbf/in^2, and $A_s = 3$ in^2.
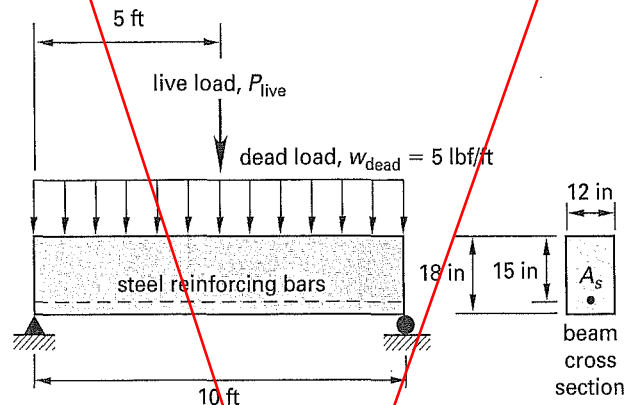
47. Neglecting beam self-weight and based only on the allowable moment capacity of the beam as determined using American Concrete Institute (ACI) strength design specifications, the maximum allowable live load is most nearly

(A) 23,000 lbf
(B) 29,000 lbf
(C) 35,000 lbf
(D) 50,000 lbf

48. The beam supports a concentrated live load of 50,000 lbf. Neglect beam self-weight. The minimum amount of shear reinforcement required for a center-to-center stirrup spacing of 12 in under ACI strength design specifications is most nearly

(A) 0.18 in^2
(B) 0.36 in^2
(C) 0.67 in^2
(D) 0.78 in^2

49. The balanced reinforcing steel ratio for this beam in accordance with ACI specifications is most nearly

(A) 0.037
(B) 0.043
(C) 0.051
(D) 0.058

The density is 33.125 veh/mi for maximum traffic volume. Substitution into the traffic flow relationship gives the maximum traffic volume.

$$q = 53k - 0.8k^2$$
$$= \left(53 \ \frac{\text{mi}}{\text{hr}}\right)\left(33.125 \ \frac{\text{veh}}{\text{mi}}\right)$$
$$- \left(0.8 \ \frac{\text{mi}^2}{\text{veh·hr}}\right)\left(33.125 \ \frac{\text{veh}}{\text{mi}}\right)^2$$
$$= 878 \text{ veh/hr} \quad (880 \text{ veh/hr})$$

The answer is B.

38. Since there is a negative grade preceding a positive grade, this is a sag vertical curve. Using the sag vertical curve equations from the civil engineering section of the NCEES Handbook, the algebraic difference between grades is
$$A = |-1\% - 3\%|$$
$$= 4\%$$

Where the stopping sight distance, S, is less than the vertical curve length, L,

$$L = \frac{AS^2}{400 + 3.5S}$$
$$= \frac{(4)(430 \text{ ft})^2}{400 + (3.5)(430 \text{ ft})}$$
$$= 388.2 \text{ ft}$$

Where the stopping sight distance is greater than the curve length,

$$L = 2S - \frac{400 + 3.5S}{A}$$
$$= (2)(430 \text{ ft}) - \frac{400 + (3.5)(430 \text{ ft})}{4}$$
$$= 383.8 \text{ ft}$$

From the two values for length of curve, it can be seen that the stopping sight distance is greater than the curve length. Therefore, the required vertical length of curve is
$$L = 383.8 \text{ ft} \quad (380 \text{ ft})$$

The answer is B.

39. The radius of curvature is
$$R = \frac{s}{\phi} = \frac{230 \text{ m}}{(10°)\left(\frac{\pi \text{ rad}}{180°}\right)}$$
$$= 1318 \text{ m}$$

For a side-friction factor of $f = 0$, the superelevation is given by

$$e + f = e = \frac{v^2}{gR}$$
$$= \frac{\left(\left(75 \ \frac{\text{km}}{\text{h}}\right)\left(1000 \ \frac{\text{m}}{\text{km}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right)\right)^2}{\left(9.81 \ \frac{\text{m}}{\text{s}^2}\right)(1318 \text{ m})}$$
$$= 0.0336 \quad (0.034)$$

The answer is B.

40. The structural number is the sum of products of the layer depths (thicknesses) and strength coefficients.

$$\text{SN} = a_{\text{recycle}}D_{\text{recycle}} + a_{\text{original surface}}D_{\text{original surface}}$$
$$+ a_{\text{base}}D_{\text{base}} + a_{\text{subbase}}D_{\text{subbase}}$$
$$6.6 = (0.42)(6 \text{ in}) + (0.3)(3 \text{ in}) + (0.2)(10 \text{ in})$$
$$+ a_{\text{subbase}}(8 \text{ in})$$
$$a_{\text{subbase}} = 0.15$$

The answer is C.

41. Draw an influence line for shear at support A with loads positioned as shown for maximum shear value. For example, for shear at support A, using basic beam statics,


influence line for shear at support A

loads placed for maximum shear value at support A
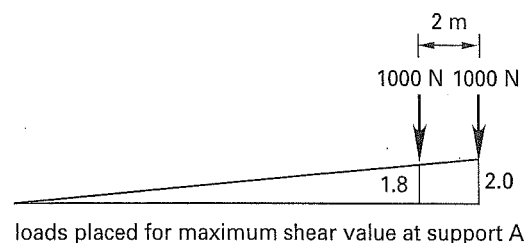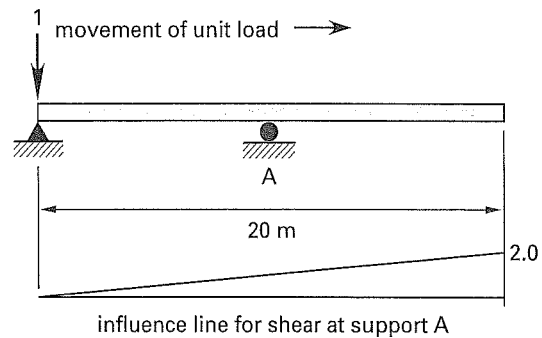
The maximum value for shear at support A is

$$V_A = (1000 \text{ N})(1.8) + (1000 \text{ N})(2.0)$$
$$= 3800 \text{ N}$$

The answer is D.

42. An influence line for bending moment at support A can be drawn with loads positioned as shown for maximum moment value. As in finding the influence line for shear, a unit load is moved across the beam and the variation in bending moment at point A is graphed.

influence line for moment at support A

loads placed for maximum value at support A
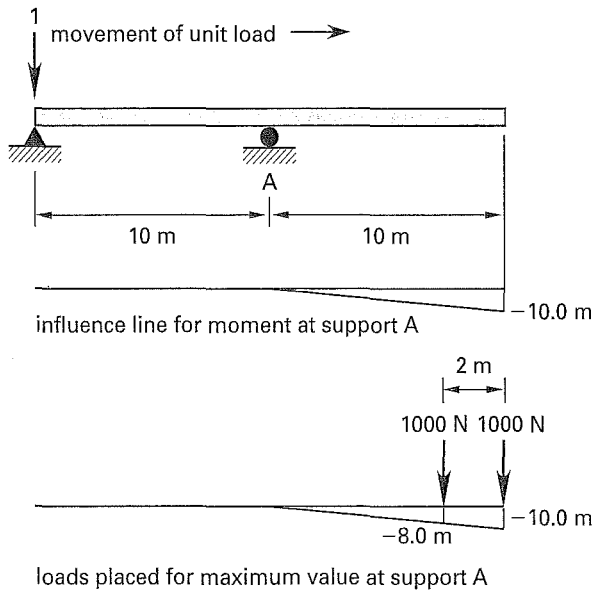
The maximum value for moment at support A (disregard the sign, since the maximum value is wanted) is

$$M_A = (1000\ \text{N})(8.0\ \text{m}) + (1000\ \text{N})(10.0\ \text{m})$$
$$= 18\,000\ \text{N·m} \quad (18\ \text{kN·m})$$

The answer is D.

43. The principle of virtual work can be used to find the deflection at point P.

Choose the positive directions as upward and to the right. Choose positive moments as clockwise.

$$\sum M_N = 0\ \text{N}$$
$$= R_{M_x}(3.0\ \text{m}) + (4448\ \text{N})(4.6\ \text{m})$$
$$R_{M_x} = 6820\ \text{N} \quad \text{[to the left]}$$
$$\sum M_M = 0\ \text{N}$$
$$= -R_{N_x}(3.0\ \text{m}) + (4448\ \text{N})(4.6\ \text{m})$$
$$R_{N_x} = 6820\ \text{N} \quad \text{[to the right]}$$
$$\sum F_y = 0\ \text{N}$$
$$= R_{M_y} - 4448\ \text{N}$$
$$R_{M_y} = 4448\ \text{N} \quad \text{[upward]}$$

By inspection, the x-component of force in member MP is the same magnitude and opposite direction as the reaction at point M.

$$\text{MP} = R_M = \sqrt{R_{M_x}^2 + R_{M_y}^2}$$
$$= \sqrt{(6820\ \text{N})^2 + (4448\ \text{N})^2}$$
$$= 8142\ \text{N}$$
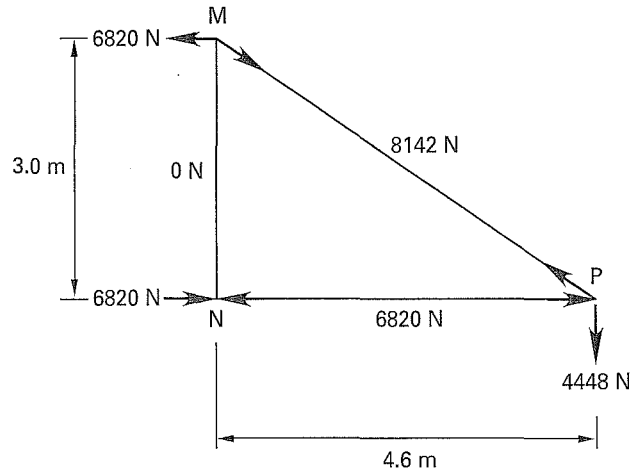
Applying a unit load at point P produces the following virtual forces.
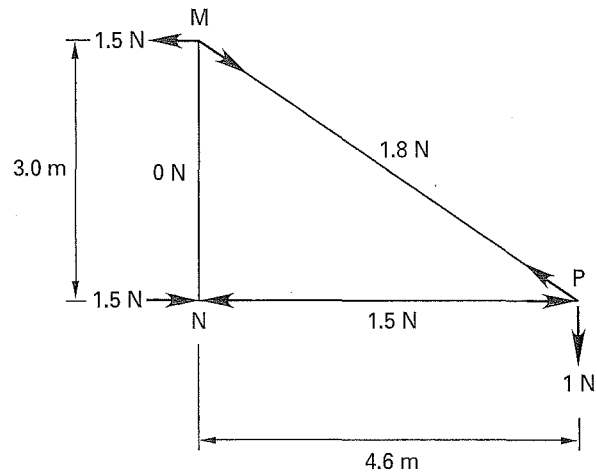
The following table summarizes the actual and virtual forces.

member	F_Q (N)	F_P (virtual force)	L (m)	$F_Q F_P L$ (N·m)
MP	8142	1.8	5.49	80 459
NP	6820	1.5	4.6	47 058
MN	0	0	3.0	0
			total =	$\overline{127\,517}$

Calculate the vertical deflection at point P.

$$\Delta_P = \sum F_Q \delta L = \sum F_Q \frac{F_P L}{EA}$$

$$= \frac{1}{EA} \sum F_Q F_P L$$

$$= \left(\cfrac{1}{\left(\left(200 \times 10^6 \text{ kPa} \right) \left(1000 \cfrac{\text{Pa}}{\text{kPa}} \right) \right) \atop \times (2580.6 \text{ mm}^2) \left(\cfrac{1 \text{ m}}{1000 \text{ mm}} \right)^2 } \right)$$

$$\times (127\,517 \text{ N·m}) \left(1000 \frac{\text{mm}}{\text{m}} \right)$$

$$\doteq 0.25 \text{ mm}$$

The answer is A.

44. From the laws of equilibrium, each reaction at an outer support is

$$R_{\text{outer}} = \tfrac{1}{2}(wL - 18.75 \text{ N})$$

$$= \left(\frac{1}{2} \right) \left(\left(5 \frac{\text{N}}{\text{m}} \right) (6 \text{ m}) - 18.75 \text{ N} \right)$$

$$= 5.6 \text{ N}$$

The answer is A.

45. The maximum shear can be determined from a shear diagram. (Although not necessary for this problem, a moment diagram has also been constructed, and this is used for solving the next problem.)

The change in shear is the area under the load diagram up to that point. Up to the center of the beam, point A,

$$V = 5.6 \text{ N} - \left(5 \frac{\text{N}}{\text{m}} \right) (3 \text{ m})$$

$$= -9.4 \text{ N}$$

At point A,
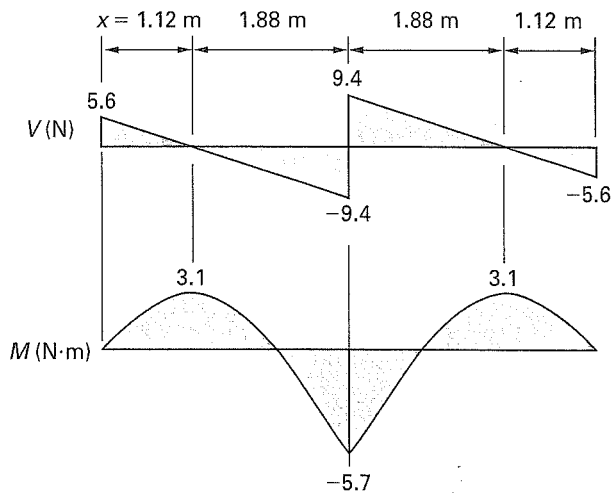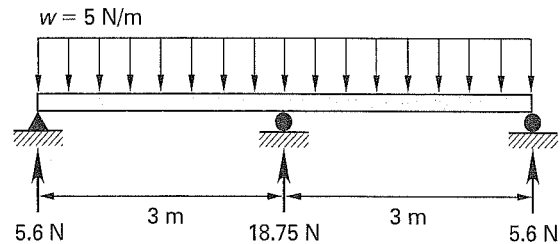
$$V = -9.4 \text{ N} + 18.75 \text{ N}$$

$$= 9.35 \text{ N} \quad (9.4 \text{ N})$$

The moment is equal to the area under the shear diagram.

$$M = \int V \, dx$$

The change in moment is given by the area under the shear diagram up to that point.

$$M_x = M_{x=1.12 \text{ m}}$$

$$= \left(\frac{1}{2} \right) (5.6 \text{ N})(1.12 \text{ m})$$

$$= 3.1 \text{ N·m}$$

$$M_{x=3 \text{ m}} = M_{x=1.12 \text{ m}} + \left(\frac{1}{2} \right) (-9.4 \text{ N})(1.88 \text{ m})$$

$$= 3.1 \text{ N·m} - 8.8 \text{ N·m}$$

$$= -5.7 \text{ N·m}$$

From the shear diagram, the maximum value for shear occurs at support A.

$$V_{\text{max}} = 9.4 \text{ N}$$

The answer is B.

46. From the moment diagram in Sol. 45, the value for maximum moment occurs at support A.

$$M_{\text{max}} = |-5.7 \text{ N·m}|$$

$$= 5.7 \text{ N·m}$$

The answer is D.

42. In using chlorine for disinfection
    a. concentration of HOCl is pH-dependent
    b. [HOCl] equals [OCl$^-$] at pH 7
    c. effectiveness increases with time of contact
    d. total elimination of bacteria is possible

43. A 61-cm-diameter circular concrete sewer is set on a slope of 0.001. The flow when it is half-full is
    a. $\frac{1}{2}$ of flow at Q_{full}
    b. 0.08 m^3/s
    c. 0.20 m^3/s
    d. 0.28 m^3/s

44. A rigid bar BCD is supported by a hinged connection at support B and by a cable AC as shown. The reaction force at support B is
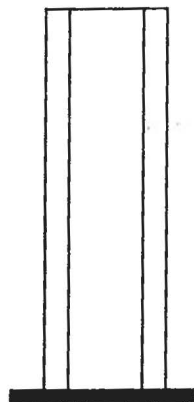
Practice Exam III
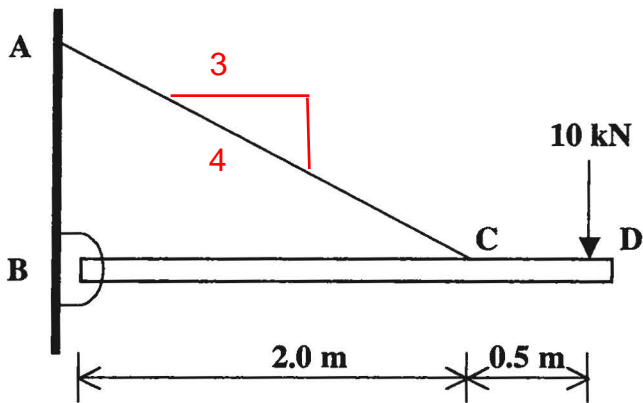
a. 2.50 kN
b. 14.20 kN
c. 9.70 kN
d. 14.42 kN

45. A simply supported beam AB is subjected to a uniformly distributed load and a concentrated load as shown. The midspan deflection is
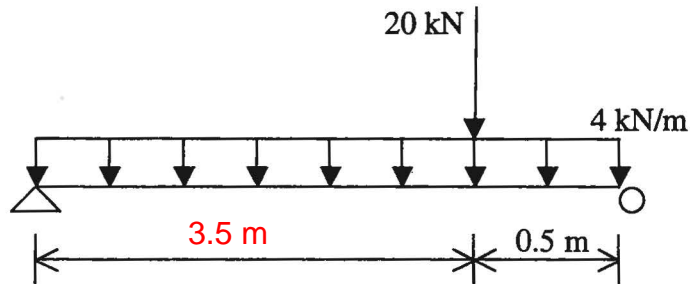

$$E = 200 \text{ GPa}$$
$$I = 3 \times 10^6 \text{ mm}^4$$

a. 16 mm
b. 38 mm
c. 22 mm
d. 8 mm

46. A steel column (W21 × 93) has the bottom end supported rigidly so as to prevent both rotation and translation. The top of the column is free to translate and rotate in the plane of the diagram but pinned (rotation allowed, translation prevented) in the perpendicular direction. The length of the column is 32 ft. The critical buckling load as given by Euler's theory is

a. 13.4 kips
b. 36.7 kips
c. 366 kips
d. 1002 kips
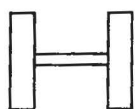
47. The three-hinged arch ABC carries a uniformly distributed load from a horizontal deck as shown. The value of the horizontal thrust at the supports is
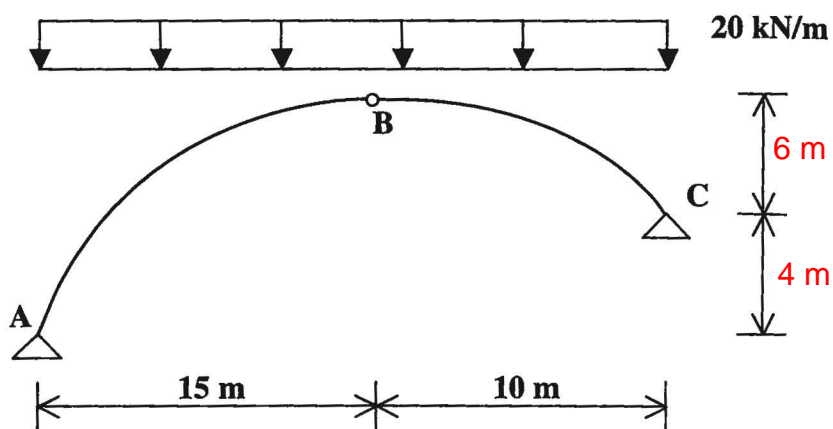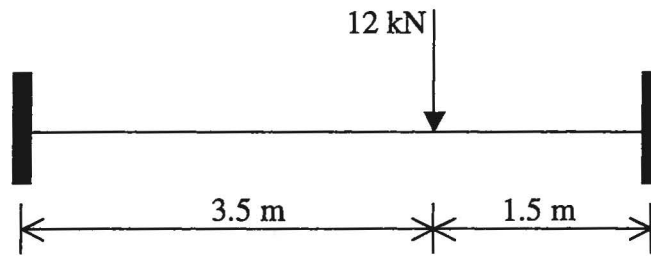
a. 282 kN
b. 197 kN
c. 218 kN
d. 252 kN

48. Given the truss shown, the vertical deflection at B is
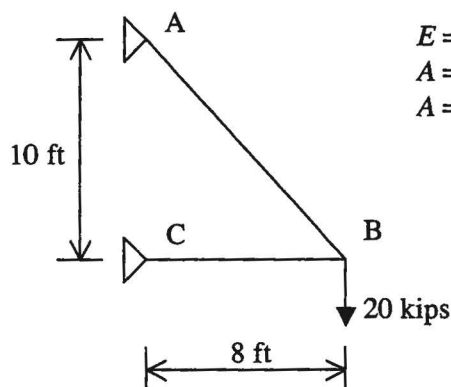
$E = 29,000$ ksi for both members
$A = 2$ in^2 for AB
$A = 3$ in^2 for BC

a. 1.2 in
b. 0.05 in
c. 0.2 in
d. 0.1 in

49. The maximum bending moment for the beam shown is

12 kN

3.5 m   1.5 m

a. 12.60 kNm
b. 8.82 kNm
c. 9.41 kNm
d. 10.25 kNm

50. A client requires a new facility quickly to replace one destroyed by a hurricane. He is not certain exactly what it should include. The best type of contract to be used in this situation is
a. unit price
b. lump sum
c. cost plus fixed fee
d. surety

51. What is the height (ft) of a conical spoil pile, if 100 ft bank cu yd of common earth with an angle of repose of 32° and a 12% swell is deposited?
a. 9.6
b. 19.2
c. 10.4
d. 33.3

**52.** Determine the volume of fill for a whole station (100 ft) of cut with the end areas shown below.

$A_1 = 120$ ft² $A_m = 150$ ft² $A_2 = 100$ ft²
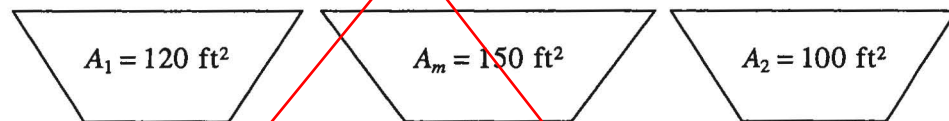
a. 820 yd³
b. 456.8 yd³
c. 506.2 yd³
d. 569.4 yd³

53. ABC Construction Company is the low bidder at $500,000 on a building project. The company has provided the typical bid bond. After award of the contract, the company decides to withdraw its bid. The second lowest bid was $520,000. How much is the surety company required to reimburse the client for the failure of ABC to sign?
a. $500,000
b. $50,000
c. $20,000
d. $5000

44. d. The free body diagram for BCD is shown. Note that a common mistake could be assuming that cable AC forms a 3-4-5 triangle when it doesn't. The vertical and horizontal components of T are $0.66T$ and $0.75T$, respectively.
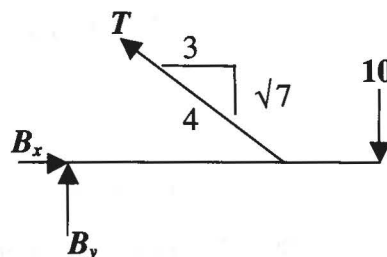
$$\sum M_B = 0.66T \times 2 - 10 \times 2.5 = 0 \Rightarrow T = 18.94$$

$$\sum F_x = B_x - 0.75 \times 18.94 = 0 \Rightarrow B_x = 14.20$$

$$\sum F_y = B_y + 0.66 \times 18.94 - 10 = 0 \Rightarrow B_y = -2.50$$

$$R_B = \sqrt{B_x^2 + B_y^2} = \sqrt{14.2^2 + 2.5^2} = 14.42$$

45. b. $EI = \left(2 \times 10^8 \dfrac{kN}{m^2}\right) \times (3 \times 10^{-6} m^4) = 600 \text{ kNm}^2$

Deflection at midspan ($x = 2$ m) due to point load is given by

$$\delta = \frac{Pb}{6LEI}[-x^3 + (L^2 - b^2)x] = \frac{20 \times 0.5}{6 \times 4 \times 600}[-2^3 + (4^2 - 0.5^2) \times 2] = 0.016$$

Deflection at midspan ($x = 2$ m) due to distributed load is given by

$$\delta = \frac{5wL^4}{384EI} = \frac{5 \times 4 \times 4^4}{384 \times 600} = 0.022$$

Total midspan deflection = 0.038 m = 38 mm

46. c. Buckling about the strong (x) axis:

$K = 2.0; \ L = 32 \text{ ft}; \ r_x = 8.70 \text{ in}; \ KL/r = 88.3$

Buckling about the weak (y) axis:

$K = 0.7; \ L = 32 \text{ ft}; \ r_y = 1.84 \text{ in}; \ KL/r = 146.1$

$$P_E = \frac{\pi^2 EA}{\left(\dfrac{KL}{r}\right)^2} = \frac{\pi^2 \times 29,000 \times 27.3}{146.1^2} = 366 \text{ kip}$$

47. b. There are four external reactions—A_x, A_y, C_x, and C_y—and two internal hinge forces—B_x and B_y—to be determined. Recognize that A_x and C_x are equal and opposite. Solving for either A_x or C_x is sufficient.

Taking moments about A (entire structure):

$$\sum M_A = 500 \times 12.5 + 4C_x - 25C_y = 0$$

Taking moments about B (right half of structure):

$$\sum M_{B,right} = 200 \times 5 - 6C_x - 10C_y = 0$$

Solving these equations, we get $C_x = -197.4$ kips.
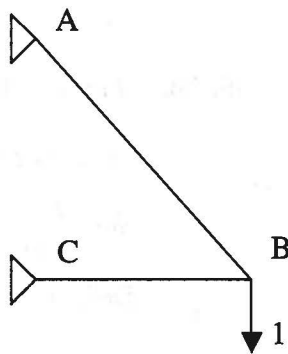
48. d. Step 1: Using method of joints at joint B:

$$F_{AB} = +25.6 \text{ k (T)}$$
$$F_{BC} = -16.0 \text{ k (C)}$$

Step 2: Calculate the member forces due to virtual load (note: in this case, the real load and the virtual load look similar, so we can use scaling):

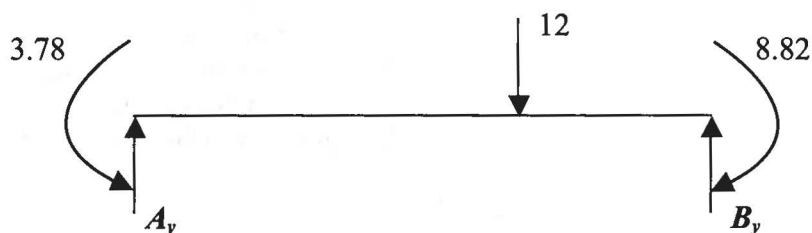$$f_{AB} = +1.28 \text{ (T)}$$
$$f_{BC} = -0.80 \text{ (C)}$$

Step 3:

$$\Delta = \sum \left(\frac{FfL}{AE}\right) = \frac{25.6 \times 1.28 \times (12.81 \times 12)}{2 \times 29,000} + \frac{-16 \times -0.8 \times (8 \times 12)}{3 \times 29,000} = 0.10 \text{ in}$$

49. b. Using the fixed end moment load cases in the *FE Supplied-Reference Handbook* with the following data:

$$P = 12 \text{ kN}; \ a = 3.5 \text{ m}; \ b = 1.5 \text{ m}; \ L = 5 \text{ m}$$

$$FEM_{AB} = \frac{Pab^2}{L^2} = \frac{12 \times 3.5 \times 1.5^2}{5^2} = 3.78 \text{ kNm}$$
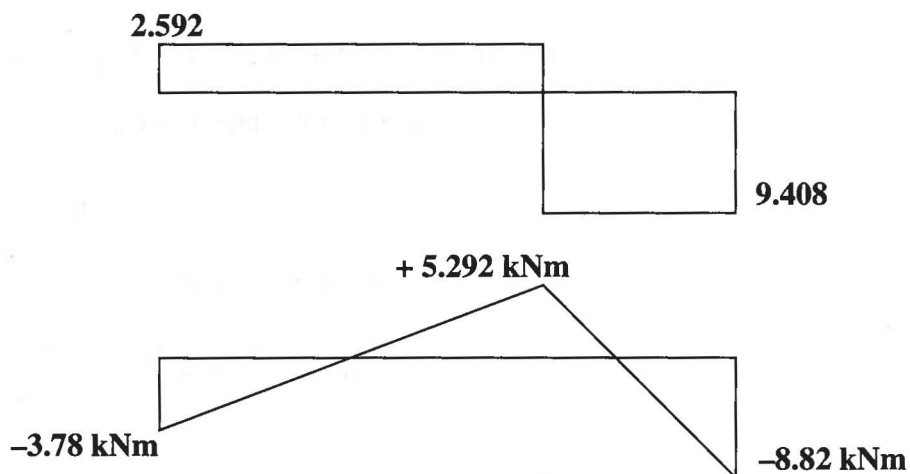
$$FEM_{BA} = \frac{Pa^2b}{L^2} = \frac{12 \times 3.5^2 \times 1.5}{5^2} = 8.82 \text{ kNm}$$

Taking moments about A,

$$\sum M_A = +3.78 - 12 \times 3.5 - 8.82 + 5B_y = 0 \Rightarrow B_y = 9.408 \text{ and } A_y = 2.592$$

The shear diagram and bending moment diagrams are:

50. **c.** Cost plus fixed fee. Choices a and b require a detailed design; choice d is a bonding company.

51. **c.** Loose cu ft = 100 cu yd × 1.12 × 27 cu ft/cu yd = 3024 cu ft
Base diameter (B) = (7.64 × 3024 / tan 32°)$^{1/3}$ = 33.3 ft
Height = (33.3/2) (tan 32°) = 10.4 ft

52. **d.** This problem can be solved using the prismoidal method. Volume = (1 cu yd / 27 cu ft) 100 ft [120 sq ft + (4 × 150 sq ft) + 100 sq ft] / 6 = 506.2 cu yd

53. **c.** The client can only recover the loss incurred by the default, which is the difference between the low bid and the second lowest bid: $20,000.

54. **b.** Total cost = $50,000 + $5000 = $55,000
Unit cost = $55,000 / 200 = $275.00/ft Note that the unit price must include the contractor's profit/markup.

Determinacy and Stability I

<div style="text-align: right; border: 1px solid red; display: inline-block;">TRUSSES</div>

- Trusses are statically determinate when all the bar forces can be determined from the equations of statics alone. Otherwise the truss is **statically indeterminate**.
- A truss may be statically/externally determinate or indeterminate with respect to the reactions (more than 3 or 6 reactions in 2D or 3D problems respectively).
- A truss may be internally determinate or indeterminate.
- If we refer to j as the number of joints, R the number of reactions and m the number of members, then we would have a total of $m + R$ unknowns and $2j$ (or $3j$) equations of statics (2D or 3D at each joint). If we do not have enough equations of statics then the problem is indeterminate, if we have too many equations then the truss is unstable.

	2D	3D
Static Indeterminacy		
External	$R > 3$	$R > 6$
Internal	$m + R > 2j$	$m + R > 3j$
Unstable	$m + R < 2j$	$m + R < 3j$

- This method should be used when all member forces must be determined.
- In truss analysis, there is no sign convention. A member is **assumed** to be under tension (or compression). If after analysis, the force is found to be negative, then this would imply that the wrong assumption was made, and that the member should have been under compression (or tension).
- On a **free body diagram**, the internal forces are represented by arrow acting on the joints and not as end forces on the element itself. That is for tension, the arrow is pointing away from the joint, and for compression toward the joint.
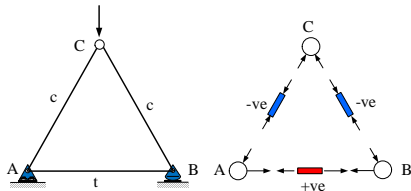
Example; Method of Joints I
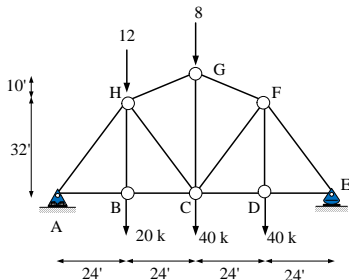
1. $R = 3$, $m = 13$, $2j = 16$, and $m + R = 2j \checkmark$

2. We compute the reactions

$$\left(+ \curvearrowleft \right) \Sigma M_z^E = 0; \quad \Rightarrow \quad (20 + 12)(3)(24) + (40 + 8)(2)(24) + (40)(24) - R_{A_y}(4)(24) = 0$$
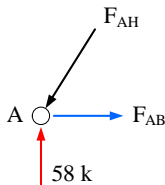
$$\Rightarrow \quad R_{A_y} = \boxed{58 \ \mathbf{k} \uparrow}$$

$$\left(+ \downarrow \right) \Sigma F_y = 0; \quad \Rightarrow \quad 20 + 12 + 40 + 8 + 40 - 58 - R_{E_y} = 0$$

$$\Rightarrow \quad R_{E_y} = \boxed{62 \ \mathbf{k} \uparrow}$$

③ Consider each joint separately:

Node A: Clearly *AH* is under compression, and *AB* under tension.

$$F_{AH}$$

$$A \quad \longrightarrow \quad F_{AB}$$

$$58 \text{ k}$$

$$\left(+ \uparrow \right) \Sigma F_y = 0; \quad \Rightarrow \quad -F_{AH_y} + 58 = 0$$

$$F_{AH} = \frac{l}{l_y}(F_{AH_y})$$

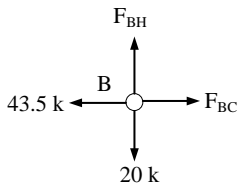$$l_y = 32; l = \sqrt{32^2 + 24^2} = 40$$

$$\Rightarrow \quad F_{AH} = \frac{40}{32}(58) = \boxed{72.5 \text{ k Compression}}$$

$$\left(+ \rightarrow \right) \Sigma F_x = 0; \quad \Rightarrow \quad -F_{AH_x} + F_{AB} = 0$$

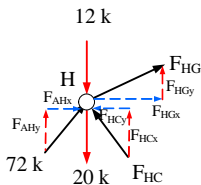$$F_{AB} = \frac{L_x}{L_y}(F_{AH_y}) = \frac{24}{32}(58) = \boxed{43.5 \text{ k Tension}}$$

Node B:

Example; Method of Joints III

$$\left(+\rightarrow\right)\Sigma F_x = 0; \quad \Rightarrow \quad F_{BC} \quad = \quad \boxed{43.5 \text{ k Tension}}$$

$$\left(+\uparrow\right)\Sigma F_y = 0; \quad \Rightarrow \quad F_{BH} \quad = \quad \boxed{20 \text{ k Tension}}$$

Node H:

Example; Method of Joints IV

$$
\left(+ \rightarrow \right) \Sigma F_x = 0; \quad \Rightarrow \quad F_{AH_x} - F_{HC_x} + F_{HG_x} = 0
$$
$$
43.5 - \frac{24}{\sqrt{24^2+32^2}}(F_{HC}) + \frac{24}{\sqrt{24^2+10^2}}(F_{HG}) = 0
$$
$$
\left(+ \uparrow \right) \Sigma F_y = 0; \quad \Rightarrow \quad F_{AH_y} + F_{HC_y} - 12 + F_{HG_y} - 20 = 0
$$
$$
58 + \frac{32}{\sqrt{24^2+32^2}}(F_{HC}) - 12 + \frac{10}{\sqrt{24^2+10^2}}(F_{HG}) - 20 = 0
$$

This can be most conveniently written as

$$
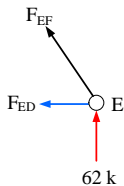\left[\begin{array}{cc} 0.6 & -0.921 \\ -0.8 & -0.385 \end{array} \right] \left\{ \begin{array}{c} F_{HC} \\ F_{HG} \end{array} \right\} = \left\{ \begin{array}{c} 43.5 \\ 26.0 \end{array} \right\}
$$

Solving we obtain $F_{HC} = -7.5$ and $F_{HG} = -52$, thus we made an erroneous assumption in the free body diagram of node H, and the final answer is

$$
F_{HC} = \boxed{7.5 \text{ k Tension}}
$$
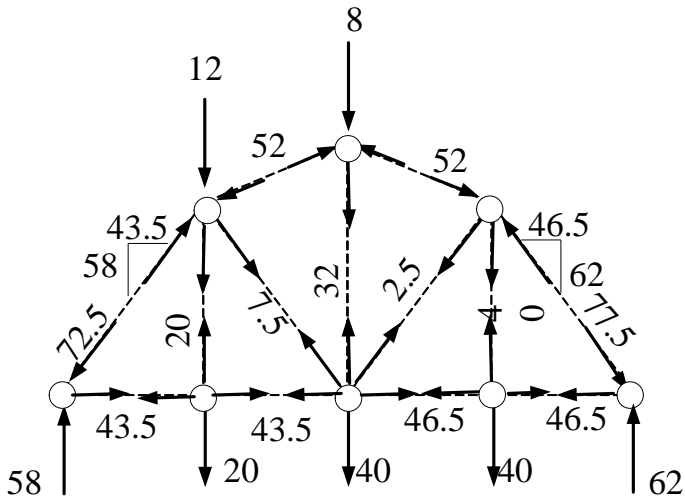$$
F_{HG} = \boxed{52 \text{ k Compression}}
$$

Node E:

Example; Method of Joints V

$$\Sigma F_y = 0; \quad \Rightarrow \quad F_{EF_y} = 62 \quad \Rightarrow F_{EF} = \frac{\sqrt{24^2 + 32^2}}{32}(62) \quad = \boxed{77.5 \text{ k C}}$$

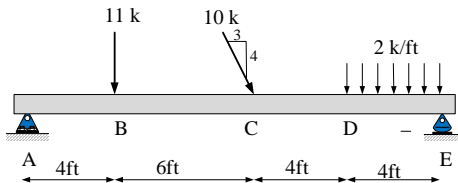$$\Sigma F_x = 0; \quad \Rightarrow \quad F_{ED} = F_{EF_x} \quad \Rightarrow F_{ED} = \frac{24}{32}(F_{EF_y}) = \frac{24}{32}(62) \quad = \boxed{46.5 \text{ k T}}$$

The results of this analysis are summarized below

4 We could check our calculations by verifying equilibrium of forces at a node not previously used, such as *D*

SHEER AND MOMENT DIAGRAMS

- Reactions are determined from the equilibrium equations
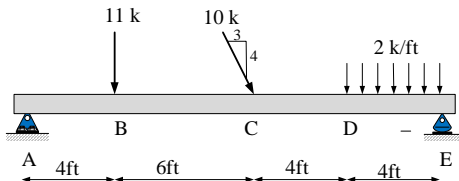
$$(+ \rightarrow) \Sigma F_x = 0; \quad \Rightarrow \quad -A_x + 6 = 0 \Rightarrow A_x = 6 \text{ k}$$
$$(+ \circlearrowleft) \Sigma M_A = 0; \quad \Rightarrow \quad (11)(4) + (8)(10) + (4)(2)(14 + 2) - E_y(18) = 0$$
$$\Rightarrow R_{E_y} = 14 \text{ k}$$
$$(+ \uparrow) \Sigma F_y = 0; \quad \Rightarrow \quad A_y - 11 - 8 - (4)(2) + 14 = 0 \Rightarrow A_y = 13 \text{ k}$$
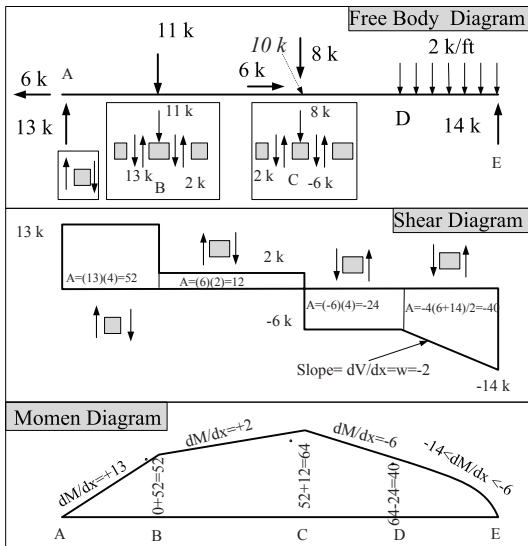
Example Beam; 2

- Shear are determined next.
    1. At *A* the shear is equal to the reaction and is positive.
    2. At *B* the shear drops (negative load) by 11 k to 2 k.
    3. At *C* it drops again by 8 k to -6 k.
    4. It stays constant up to *D* and then it decreases (constant negative slope since the load is uniform and negative) by 2 k per linear foot up to -14 k.
    5. As a check, -14 k is also the reaction previously determined at *F*.
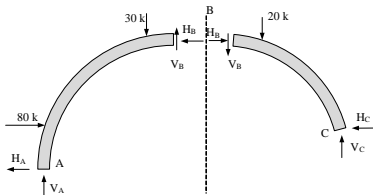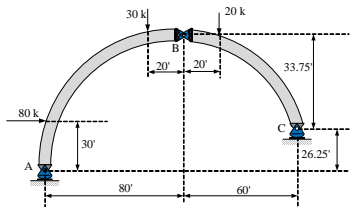- Moment is determined last:
    1. The moment at *A* is zero (hinge support).
    2. The change in moment between *A* and *B* is equal to the area under the corresponding shear diagram, or $\Delta M_{B-A} = (13)(4) = 52$.
    3. etc...

Free Body Diagram

11 k

10 k   8 k

6 k   A   6 k   2 k/ft

13 k   11 k   8 k   D   14 k

13 k   B   2 k   2 k   C   -6 k   E

Shear Diagram

13 k

A=(13)(4)=52   2 k

A=(6)(2)=12

-6 k   A=(-6)(4)=-24   A=-4(6+14)/2=-40

Slope= dV/dx=w=-2

-14 k

Momen Diagram

dM/dx=+2   dM/dx=-6   -14<dM/dx<-6

dM/dx=+13   0+52=52   52+12=64   64-24=40

A   B   C   D   E

Three Hinged Arch; Point Loads I

ARCHES

- Four unknowns, three equations of equilibrium, one equation of condition $\Rightarrow$ statically determinate.

$$\left(+\circlearrowleft\right)\ \Sigma M_z^C\ =\ 0;\quad (R_{Ay})(140) + (80)(3.75) - (30)(80) - (20)(40) + R_{Ax}(26.25)\ =\ 0$$
$$\Rightarrow 140R_{Ay} + 26.25R_{Ax}\ =\ 2.900$$

$$\left(+\rightarrow\right)\ \Sigma F_x\ =\ 0;\quad 80 - R_{Ax} - R_{Cx}\ =\ 0$$

$$\left(+\uparrow\right)\ \Sigma F_y\ =\ 0;\quad R_{Ay} + R_{Cy} - 30 - 20\ =\ 0$$

$$\left(+\circlearrowleft\right)\ \Sigma M_z^B\ =\ 0;\quad (R_{ax})(60) - (80)(30) - (30)(20) + (R_{Ay})(80)\ =\ 0$$
$$\Rightarrow 80R_{Ay} + 60R_{Ax}\ =\ 3,000$$

(22)

● Solving those four equations simultaneously we have:

$$
\begin{bmatrix}
140 & 26.25 & 0 & 0 \\
0 & 1 & 0 & 1 \\
1 & 0 & 1 & 0 \\
80 & 60 & 0 & 0
\end{bmatrix}
\begin{Bmatrix}
R_{Ay} \\
R_{Ax} \\
R_{Cy} \\
R_{Cx}
\end{Bmatrix}
=
\begin{Bmatrix}
2,900 \\
80 \\
50 \\
3,000
\end{Bmatrix}
\Rightarrow
\begin{Bmatrix}
R_{Ay} \\
R_{Ax} \\
R_{Cy} \\
R_{Cx}
\end{Bmatrix}
=
\begin{Bmatrix}
15.1 \text{ k} \\
29.8 \text{ k} \\
34.9 \text{ k} \\
50.2 \text{ k}
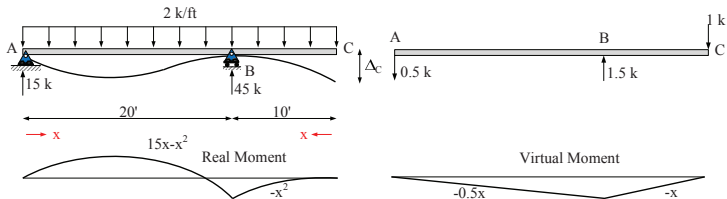\end{Bmatrix}
\tag{23}
$$

● We can check our results by considering the summation with respect to B from the right:

$$
\left(+ \curvearrowleft \right) \Sigma M_z^B = 0; \; -(20)(20) - (50.2)(33.75) + (34.9)(60) = 0 \checkmark
\tag{24}
$$

BEAM DEFLECTION, VIRTUAL FORCE

Determine the deflection at point C. $E = 29,000$ ksi, $I = 100$ in^4.

Element	$x = 0$	M	$\delta \bar{M}$
AB	A	$15x - x^2$	$-0.5x$
BC	C	$-x^2$	$-x$

Example: Beam II

Applying the principle of virtual work, we obtain

$$
\begin{aligned}
\underbrace{\Delta_C \delta \overline{P}}_{\delta \overline{W}^*} &= \underbrace{\int_0^L \delta \overline{M}(x) \frac{M(x)}{EI_z} \mathrm{d}x}_{\delta \overline{U}^*} \\
(1)\Delta_C &= \int_0^{20} (-0.5x)\frac{(15x - x^2)}{EI}\mathrm{d}x + \int_0^{10} (-x)\frac{-x^2}{EI}\mathrm{d}x \\
&= \frac{2,500}{EI} \\
\Delta_C &= \frac{(2,500)\,\mathrm{k} - \mathrm{ft}^3(1,728)\,\mathrm{in}^3/\,\mathrm{ft}^3}{(29,000)\,\mathrm{ksi}(100)\,\mathrm{in}^4} \\
&= \boxed{1.49 \text{ in}}
\end{aligned}
$$

Truss; Simple I

Determine the deflection at node 2 for the truss. TRUSS-DEFLECTION; Virtual force
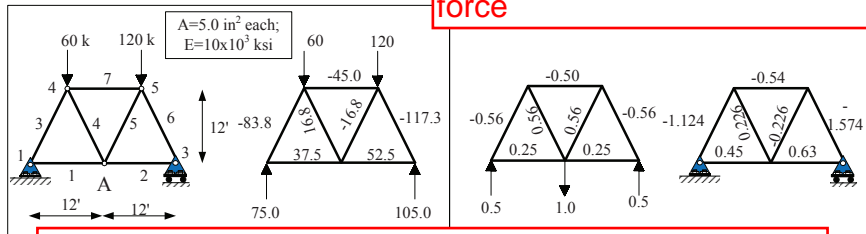
You will not get something as complex as this truss

Member	$\delta \overline{P}^{(e)}$ kips	$P^{(e)}$, kips	L, ft	A, in^2	E, ksi	$\delta \overline{P}^{(e)} \frac{P^{(e)} L}{AE}$
1	+0.25	+37.5	12	5.0	10×10^3	$+22.5 \times 10^{-4}$
2	+0.25	+52.5	12	5.0	10×10^3	$+31.5 \times 10^{-4}$
3	-0.56	-83.8	13.42	5.0	10×10^3	$+125.9 \times 10^{-4}$
4	+0.56	+16.8	13.42	5.0	10×10^3	$+25.3 \times 10^{-4}$
5	+0.56	-16.8	13.42	5.0	10×10^3	-25.3×10^{-4}
6	-0.56	-117.3	13.42	5.0	10×10^3	$+176.6 \times 10^{-4}$
7	-0.50	-45.0	12	5.0	10×10^3	$+54.0 \times 10^{-4}$
						$+410.5 \times 10^{-4}$

The deflection is thus given by

$$
\begin{aligned}
\delta \overline{P} \Delta &= \sum_{1}^{7} \delta \overline{P}^{(e)} \frac{PL}{AE} \\
\Delta &= (410.5 \times 10^{-4})(12 \text{ in/ ft}) = \boxed{0.493 \text{ in}}
\end{aligned}
$$

∎

Truss with initial camber I

It is desired to provide 3 in. of camber at the center of the truss shown below

by fabricating the endposts and top chord members additionally long. How much s
length of each endpost and each panel of the top chord be increased?
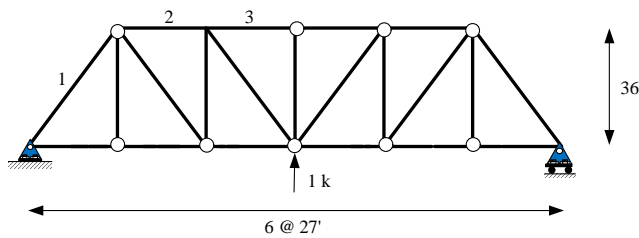
You may get something along those lines, or with temperature load

- Assume that each endpost and each section of top chord is increased 0.1 in.

Member	$\delta \overline{P}_{int}^{(e)}$	ΔL	$\delta \overline{P}_{int}^{(e)} \Delta L$
1	+0.625	+0.1	+0.0625
2	+0.750	+0.1	+0.0750
3	+1.125	+0.1	+0.1125
			+0.2500

Thus,

$$(2)(0.250) = 0.50 \text{ in}$$

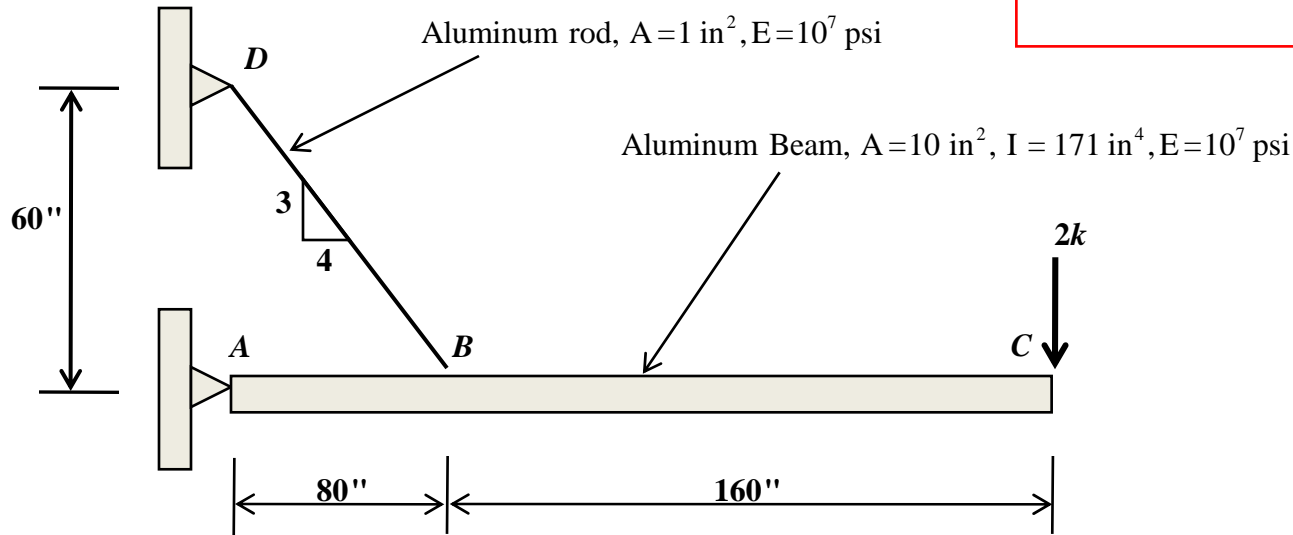- Since the structure is linear and elastic, the required increase of length for each section will be

$$\left(\frac{3.0}{0.50}\right)(0.1) = 0.60 \text{ in}$$

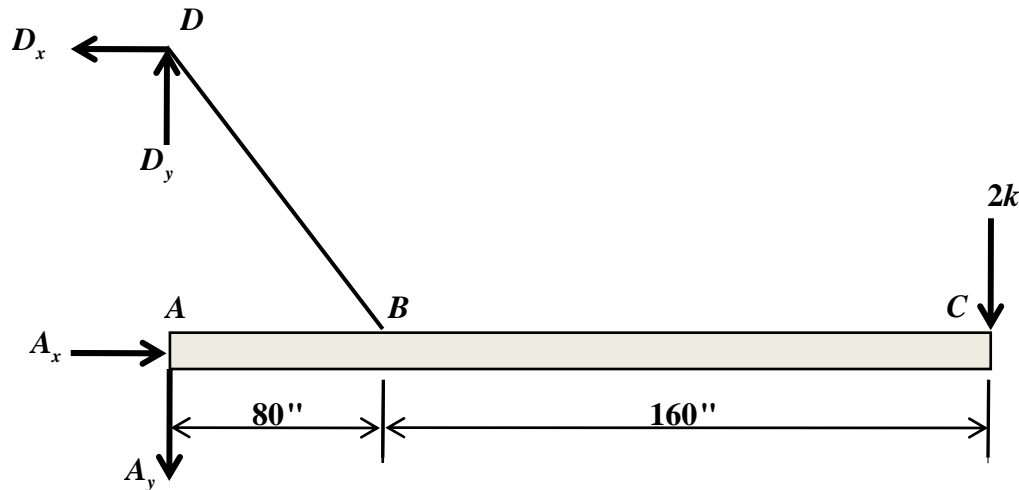- If we use the practical value of 0.625 in., the theoretical camber will be

$$\frac{(6.25)(0.50)}{0.1} = \boxed{3.125 \text{ in}}$$

Solve for the vertical displacement at C of the following structure.

Aluminum rod, $A = 1$ in^2, $E = 10^7$ psi

Aluminum Beam, $A = 10$ in^2, $I = 171$ in^4, $E = 10^7$ psi

D

3
4

60"

A   B   C   2k

80"   160"

Step 1. Draw free body diagram and calculate all forces due to real load and virtual load .

D_x D

D_y

A_x A   B   C   2k

A_y

80"   160"

$(\text{CCW} +ve)\Sigma M_A = 0$

$D_x(60) - 2(240) = 0$

$D_x = 8k$
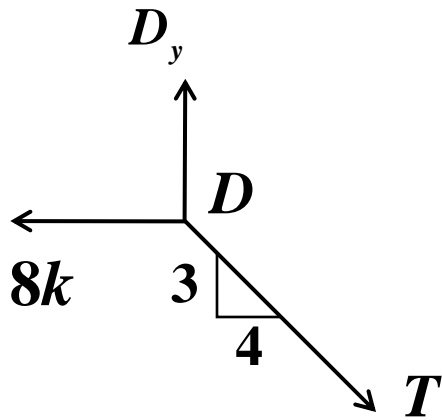
$\xrightarrow{+}\Sigma F_x = 0$

$-D_x + A_x = 0$

$A_x = 8k$

$+\uparrow \Sigma F_y = 0$

$-A_y + D_y - 2 = 0 \qquad (1)$

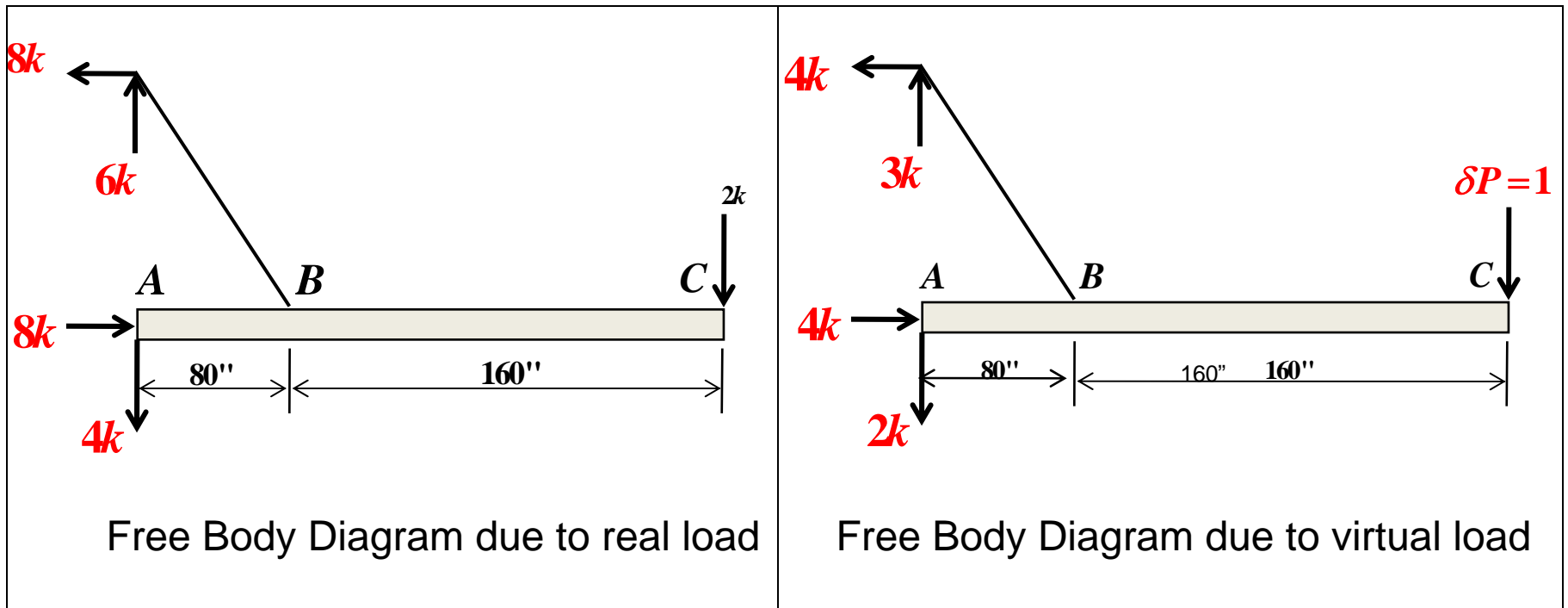$$\frac{3}{5}T = D_y \qquad (2)$$

$$\frac{4}{5}T = D_x = 8k$$

$$T = 10k$$

From Eq.(2)

$D_y = 6k$

From *Eq.*(1)

$A_y = 4k$

Likewise, we can get the reaction due to unit virtual load at point C by similar manner.

Free Body Diagram due to real load

Free Body Diagram due to virtual load

Step 2. Determine forces in cable and beam for each elements due to real load and virtual load
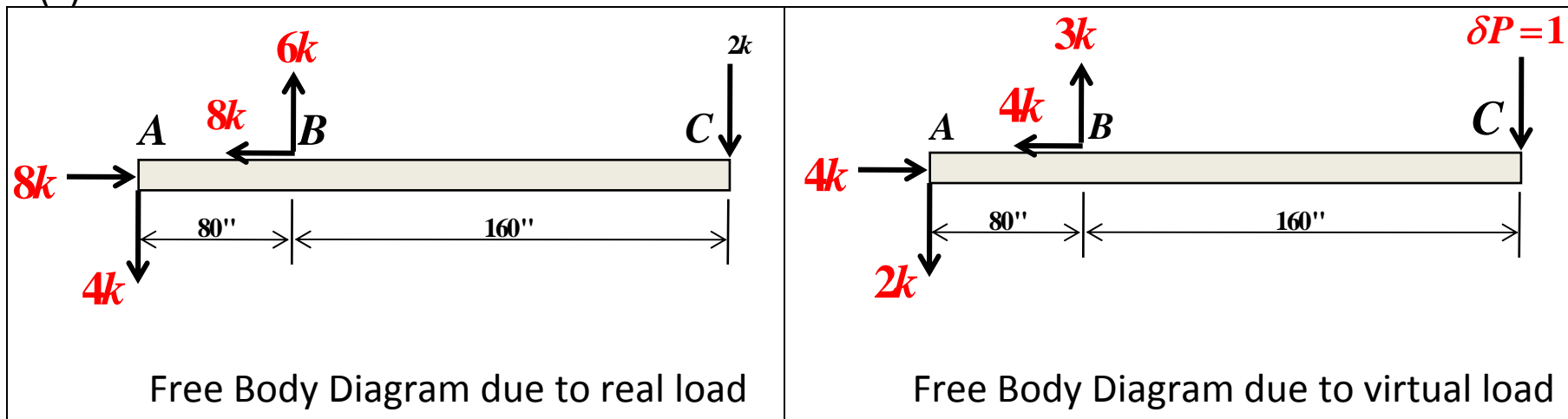
(i) Cable.

Tension is constant along the cable length.

> You may not get something as complex for deflection; however make sure that you understand this.

Element	x=0	P	δP
DB	D	10	5

(ii) Beam.

Free Body Diagram due to real load

Free Body Diagram due to virtual load

Element	x=0	P	δP	M	δM
AB	A	-8	-4	-4x	-2x
BC	B	0	0	2x-320	x-160

From (i) and (ii)

$$\underbrace{\Delta_C \delta P}_{\delta w^*} = \underbrace{\int\limits_L \delta P \frac{P}{AE} dx + \int\limits_L \delta M \frac{M}{EI} dx}_{\delta u^*}$$

$$\Delta_C(1) = \int\limits_0^{100} \frac{(5)(10)}{A_c E} dx + \int\limits_0^{80} \frac{(-4)(-8)}{A_b E} dx$$

$$+ \int\limits_0^{80} \frac{(-2x)(-4x)}{EI_b} + \int\limits_0^{160} \frac{(x-160)(2x-320)}{EI_b} dx$$

$$= \int\limits_0^{100} \frac{(5)(10)}{(1 in^2)(10^7 \ psi)} dx + \int\limits_0^{80} \frac{(-4)(-8)}{(10 in^2)(10^7 \ psi)} dx$$

$$+ \int\limits_0^{80} \frac{(-2x)(-4x)}{(10^7 \ psi)(171 in^4)} + \int\limits_0^{160} \frac{(x-160)(2x-320)}{(10^7 \ psi)(171 in^4)} dx$$
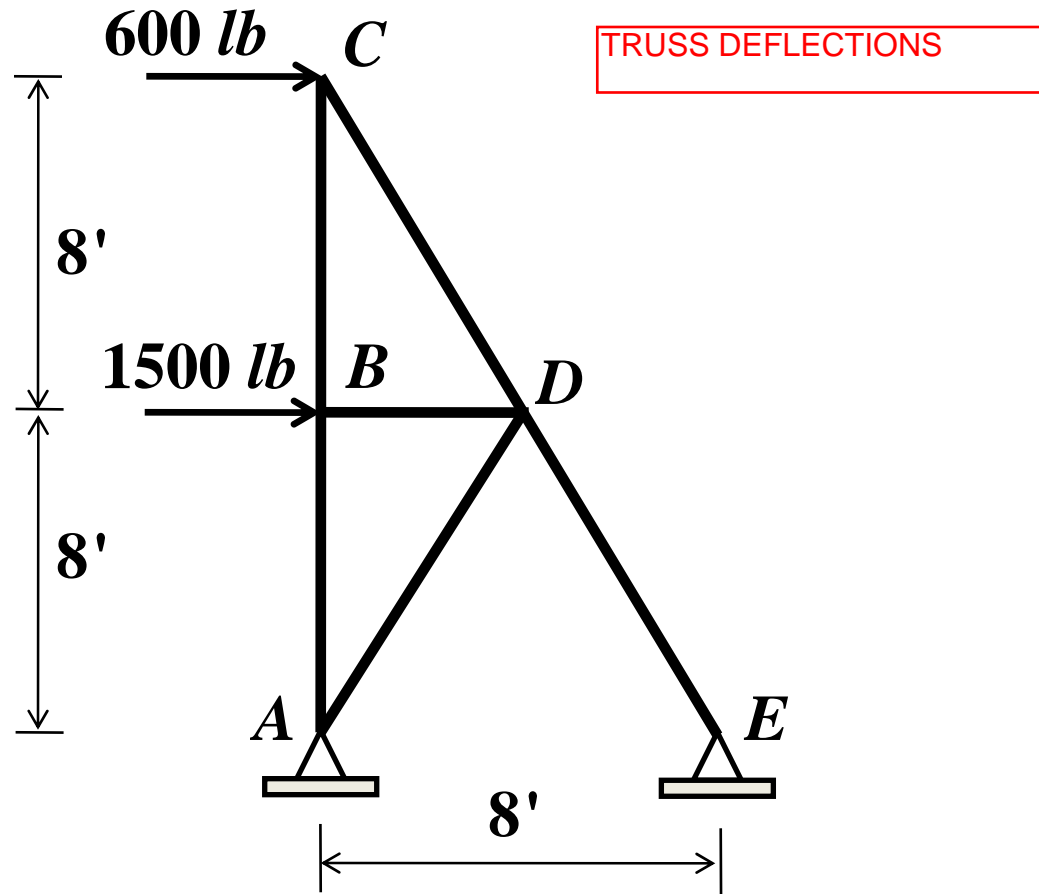
$$\Delta_C = 0.0005 + 0.0000256 + 0.0007984 + 0.001597$$

$$= \underbrace{0.0005256}_{Axial\ Contribution} + \underbrace{0.0023954}_{Flexure\ Contribution}$$

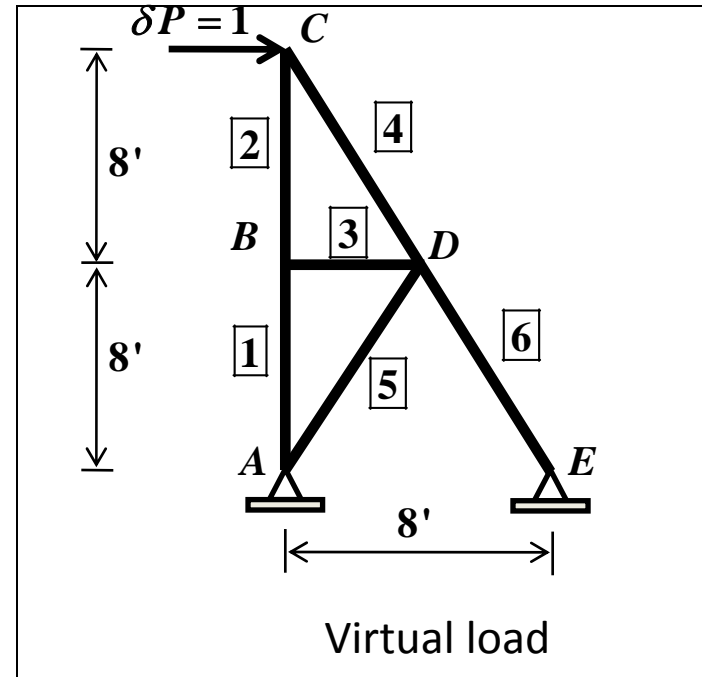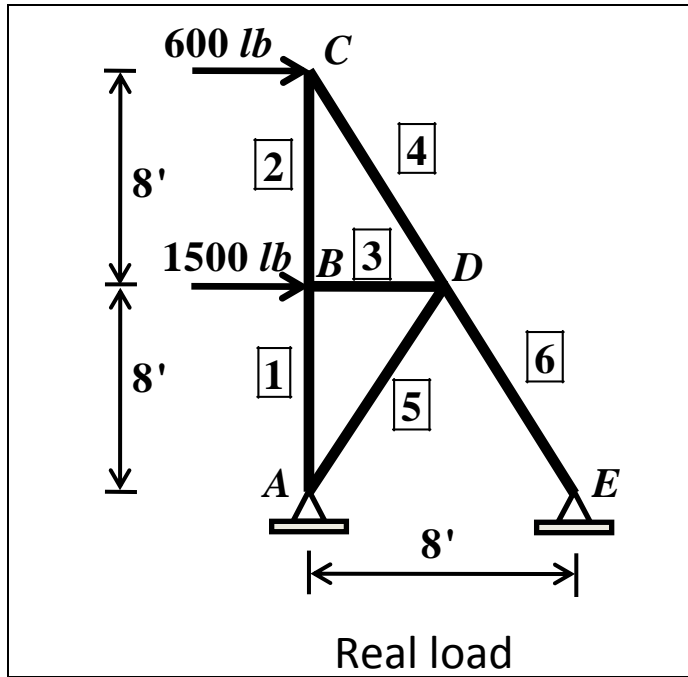$$\Delta_C = 0.0029 \ in \, (\downarrow)$$

For the truss shown below:
   a. Determine the horizontal deflection at C, Assume AE constant.

TRUSS DEFLECTIONS

600 lb   C

8'

1500 lb   B   D

8'

A   E

8'

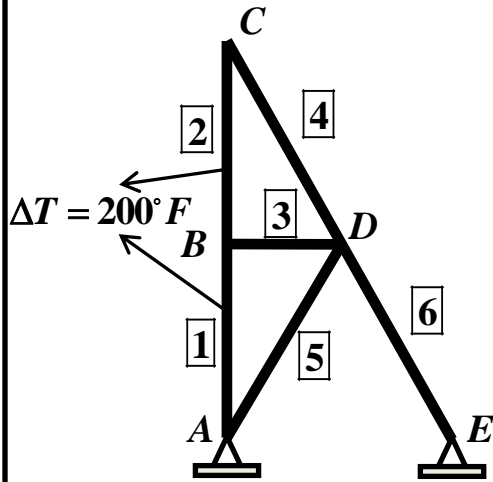## (a). Determine the horizontal deflection at C. Assume AE constant.

Real load

Virtual load

You will not get such a complex truss, however the procedure is very important to understand

Member	δP(lb)	P(lb)	L(in)	AE	δP.(PL/AE)
1	2	1,200	96	AE	230,400/AE
2	2	1,200	96	AE	230,400/AE
3	0	-1,500	48	AE	0
4	-2.24	-1,341.64	107.33	AE	322,560/AE
5	0	1,677.05	107.33	AE	0
6	-2.24	-3,018.69	107.33	AE	725,759.6/AE

$$\sum = \frac{1,509,119 \, lb - in}{AE}$$

$$\Delta C_h = \frac{1.51 \times 10^6 \, lb - in}{AE} \quad (\rightarrow)$$

Member	L(in)	α(1/°F)	ΔT(°F)	ΔL=αΔTL(in)	δP	δP.ΔL
1	96	10^{-6}	200	0.0192	2	0.0384
2	96	10^{-6}	200	0.0192	2	0.0384
3	48	10^{-6}	0	0	0	0
4	107.33	10^{-6}	0	0	-2.24	0
5	107.33	10^{-6}	0	0	0	0
6	107.33	10^{-6}	0	0	-2.24	0

$$\sum = 0.0768$$

$$\Delta C_h = 0.0768 \; in \; (\rightarrow)$$

You will not get such a complex truss, however the procedure is very important to understand

c. Remove the loads and determine the horizontal displacement of C if member CD is fabricated 0.5 in too short.

Fabricated 0.5 in too short.
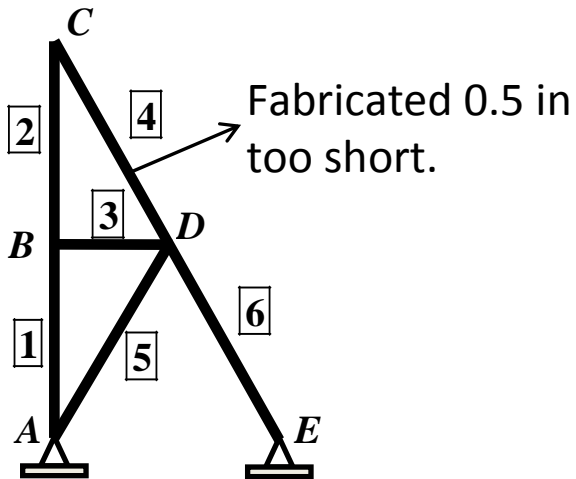
Member	L(in)	ΔL(in)	δP	δP.ΔL
1	96	0	2	0
2	96	0	2	0
3	48	0	0	0
4	107.33	-0.5	-2.24	1.12
5	107.33	0	0	0
6	107.33	0	-2.24	0

$$\sum = 1.12$$

You will not get such a complex truss, however the procedure is very important to understand

$$\Delta C_h = 1.12 \ in \ (\rightarrow)$$