CHAPTER EIGHT

8. PLASTIC ANALYSIS OF STEEL STRUCTURES

8.1. Introduction:

Although the terms Plastic analysis and design normally apply to such procedures for steel structures within the yield flow region, at almost constant stress, however the Idea may also be applied to reinforced concrete structures which are designed to behave elastically in a ductile fashion at ultimate loads near yielding of reinforcement.

The true stress-strain curve for a low grade structural steel is shown in fig. 1 while an idealized one is shown in fig. 2 which forms the basis of Plastic Analysis and Design.

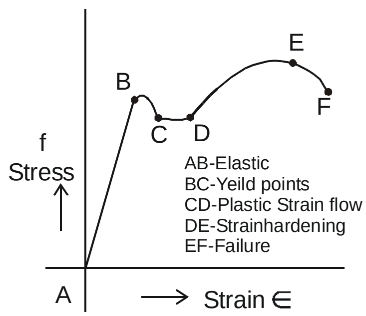


Fig 1:

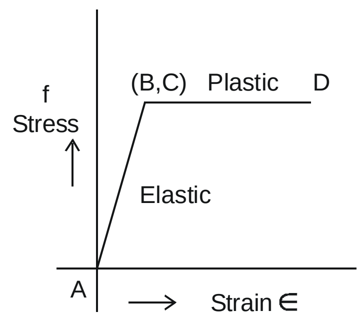


Fig 2:

8.2. Advantages of Plastic Analysis

1. Relatively simpler procedures are involved.
2. Ultimate loads for structures and their components may be determined.
3. Sequence and final mode of failure may be known and the capacity at relevant stages may be determined.

8.3. Assumptions in Plastic bending

1. The material is homogeneous and isotropic.
2. Member Cross-section is symmetrical about the axis at right angles to the axis of bending.
3. Cross-section which were plane before bending remain plane after bending.
4. The value of modulus of Elasticity of the material remains the same in tension as well as in compression.
5. Effects of temperature, fatigue, shear and axial force are neglected.
6. Idealized bi-linear stress-strain curve applies.

8.4. Number of Plastic Hinges

" The number of Plastic Hinges required to convert a structure or a member into a mechanism is one more than the degree of indeterminacy in terms of redundant moments usually. Thus a determinate structure requires only one more plastic hinge to become a mechanism, a stage where it deflects and rotates continuously at constant load and acquires final collapse.

So Mathematically

$$N = n + 1$$

where N = Total number of Plastic hinges required to convert a structure into a mechanism.
and n = degree of indeterminacy of structure in terms of unknown redundant moments.

8.5. Plastic Hinge.

It is that cross-section of a member where bending stresses are equal to yield stresses $\sigma = \sigma y = fy$. It has finite dimensions.

From bending equation $\sigma = \frac{My}{I}$ or $\sigma y = \frac{MpC}{I}$ or $\sigma y = \frac{Mp}{Zp}$ so $Hp = Zp\,\sigma y$

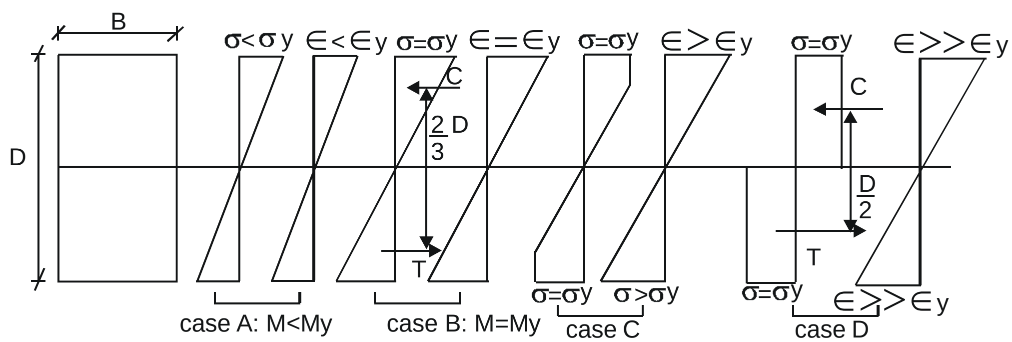
From elastic bending $\frac{\sigma}{y} = \frac{M}{I}$ or $\frac{\sigma I}{y} = M$ where $\frac{I}{y} = Z$

So $M = \sigma Z$ and Z is elastic section modules and is equal to the first moment of area about N.A

$$Z = \int_A ydA.$$

8.6. Plastic moment of a rectangular section.

Consider a simple rectangular beam subject to increasing bending moment at the centre. Various stress-strain stages are encountered as shown below.



Various Stress-strain distributions

Case A - Stresses and strains are within elastic range.
Case B - Stresses and strains at yield levels only at extreme fibers
Case C - Ingress of yielding within depth of section.
Case D - Full plastification of section.
On the onset of yielding $\sigma = \sigma y$ and $M = My = \sigma y.Z.$
On full plastification $\sigma = \sigma y$ and $M = Mp = \sigma y.Zp.$
or $Zp = \int_A yda$ (First moment of area about equal area axis).

All compact sections as defined in AISC manual will develop full plastification under increasing loads realizing Mp. However local buckling of the compression flange before yielding has to be avoided by providing adequate lateral support and by applying width / thickness checks as was done during the coverage of subject of steel structures design.

Case B. Stresses and Strains at yield at extreme fibres only.

Consult corresponding stress and strain blocks.
$M =$ Total compression $\times$ la $=$ Area $\times\ \sigma \times$ la
where Area = Area in compression (from stress block).
σ = Average compression stress.
la = Lever arm i.e. distance b/w total compressive and tensile forces.

So $M = \left(\frac{BD}{2}\right)\left(\frac{\sigma y + o}{2}\right) \cdot \frac{2}{3} D$

In general

$M = Cjd$ or Tjd , where C and T are total compressive and tensile forces respectively which have to be equal for internal force equilibrium.

$$\text{or} \quad M_y = \sigma y \frac{BD^2}{6}, \text{ but } \frac{BD^2}{6} = Z \quad \left[Z = \text{Elastic Section modules} = \frac{I}{C} = \frac{BD^3}{12} \div \frac{D}{2} \right.$$

$$\text{So} \quad M_y = \sigma y . Z. \qquad = \frac{BD^2}{6}$$

Case D: Full plastification, $\sigma = \sigma y$ upto equal area axis.

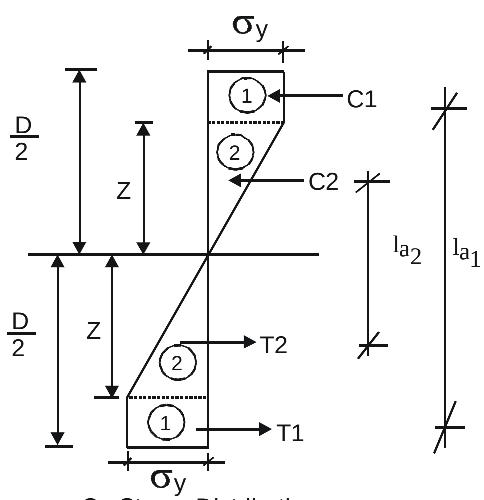
$$M = Cla = \left(B.\frac{D}{2}\right) (\sigma y) \frac{D}{2} \quad \text{where la is lever arm}$$

$$= \sigma y . \frac{BD^2}{4} \quad \text{or} \quad Zp = \frac{BD^2}{4}, \text{ where ZP = Plastic section Modules.}$$

or $Mp = \sigma y . Zp$ or $Zp = \frac{A}{2} [y1 + y2]$ (first moment of areas about equal area axis)

and $y1 + y2 = D/2$ (distance from equal area axis to the centroids of two portions of area.)

Case C: Moment Capacity in Elasto - Plastic range. Extreme fibres have yielded and the yielding ingresses in the section as shown by the stress – distribution.



where
la1 = lever axis b/w C1 and T1
la2 = lever axis b/w C2 and T2
C1 = Av.stress X area of element No.1
C2 = Av-stress acting on element No.2 x area of element 2.

case C : Stress-Distribution

$$M = [C_1 la_1 + C_2 . la_2] \text{ (A)} , \quad la_1 = \left[Z + \frac{\frac{D}{2} - Z}{2} 2 \right] = \frac{D}{2} + Z$$

2

$$C_1 = (\sigma y) \; B \left(\frac{D}{2} - Z\right) \qquad la_2 = \left[\frac{2}{3} \times Z \times 2\right] = \frac{4}{3} \times Z$$

$$C_2 = \left(\frac{\sigma y + o}{2}\right) Z . B = \sigma y \frac{ZB}{2}$$

and so, putting values of C_1 , C_2 la_1 and la_2 in equation A above.

$$M = \sigma y \, . \, B\left(\frac{D}{2} - Z\right)\left(\frac{D}{2} + Z\right) + \sigma y\left(\frac{Z.B}{2}\right) \times \frac{4}{3} Z \quad , \quad \text{Simplifying}$$

$$M = \sigma y \, . \, B\left(\frac{D^2}{4} - Z^2\right) + \frac{2}{3}\sigma y \; BZ^2$$

$$= \sigma y \, . \, B\left(\frac{D^2}{4} - Z^2 + \frac{2}{3} Z^2\right)$$

$$M_r = \sigma y \, . \, B\left(\frac{D^2}{4} - \frac{Z^2}{3}\right) \quad , \quad \text{where } M_r \text{ is moment of resistance.}$$

$$Mp = M_r = \sigma y \, . \, B\left(\frac{3D^2 - 4Z^2}{12}\right) \quad - \quad \text{For rectangular section.}$$

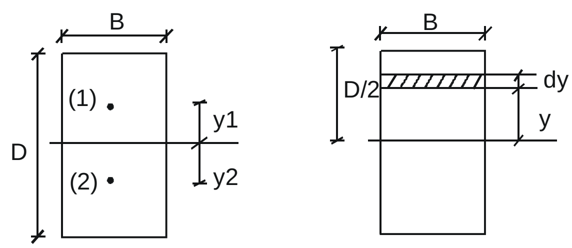
Calculating on similar lines, Plastic moment for various shapes can be calculated.

8.7. Shape Factor (γ)

It is the ratio of full plastic moment Mp to the yield moment My. It depends on the shape of Cross-section for a given material.

$$\text{Shape Factor} = \gamma = \frac{Mp}{My} = \frac{\sigma y \, . \, Zp}{\sigma y \, . \, Z} \quad \text{or} \quad \gamma = \frac{Zp}{Z}$$ (Ratio of Plastic section modulus to Elastic Section Modulus).

8.8. Calculation of Shape Factor for different Sections.



8.8.1 For rectangular section.

$$I = \frac{BD^3}{12}, \quad \frac{I}{C} = Z, \quad C = \frac{D}{2}$$

$$\text{So } Z = \frac{BD^3 \times 2}{12 \times D} = \frac{BD^2}{6}$$

$$Zp = \frac{A}{2}\,[y_1 + y_2] = \frac{BD}{2}\left[\frac{D}{4} + \frac{D}{4}\right]$$

$$= \frac{BD^2}{4}$$

or alternatively, $Zp = \int_A ydA.$

$$= 2\int_o^{D/2} y \, . \, Bdy$$

$$= 2B\int_o^{D/2} ydy.$$

$$\text{or } Zp = \frac{BD^2}{4}$$

$$\gamma = \frac{Zp}{Z} = \frac{BD^2 \times 6}{4 \times BD^2} = \frac{6}{4} = 1.5$$

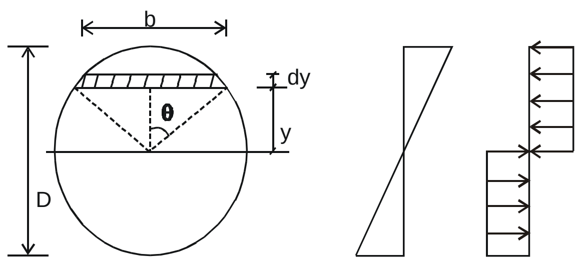
$\gamma = 1.5$ so [Mp is 1.5 times My]

8.8.2 For Circular Cross-section

$$I = \frac{\pi D^4}{64}, \qquad A = \frac{\pi}{4} D^2$$

$$Z = \frac{I}{C} = \frac{\pi D^4}{64} \times \frac{2}{D} = \frac{\pi D^3}{32},$$

$$Zp = \frac{A}{2}[y_1 + y_2]$$



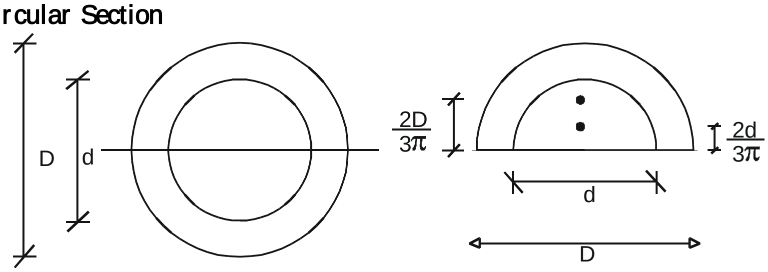
(a) Cross-Section (b) Strain Distribution (c) Stress at full plastification Distribution

$$= \frac{\pi D^2}{8}\left[\frac{2D}{3\pi} + \frac{2D}{3\pi}\right], \qquad r = \frac{D}{2}, \; y1 = \frac{4r}{3\pi} = \frac{4 \times D}{3\pi \times 2} = \frac{2D}{3\pi}$$

$$Zp = \frac{D^3}{6} \qquad \gamma = \frac{Zp}{Z} = \frac{D^3 \times 32}{6 \times \pi D^3} = \frac{32}{6\pi} \cong 1.7$$

γ = 1.7 , [Mp is 1.7 times My]

8.8.3 Hollow Circular Section



$$I = \frac{\pi}{64}(D^4 - d^4)$$

$$\frac{I}{C} = Z_{min} = \frac{\pi}{64}(D^4 - d^4) \cdot \frac{2}{D}$$

$$Z_{min} = \frac{\pi}{32D}(D^4 - d^4)$$

$$Zp = \frac{A}{2}[y1 + y_2] \text{, putting values.}$$

$$= \frac{\pi}{8}(D^2 - d^2)\left[2 \times \frac{2}{3\pi}\frac{(D^3 - d^3)}{(D^2 - d^2)}\right]$$

$$Zp = \left(\frac{D^3 - d^3}{6}\right)$$

putting values $Ay = A_1y_1 + A_2y_2$

$$\frac{\pi}{8}(D^2 - d^2)\, y = \frac{\pi D^2}{8} \cdot \frac{2D}{3\pi} - \frac{\pi d^2}{8} \times \frac{2d}{3\pi}$$

$$\frac{\pi}{8}(D^2 - d^2)\, y = \left(\frac{D^3}{12} - \frac{d^3}{12}\right)$$

$$\gamma = \left(\frac{D^3 - d^3}{6}\right) \times \frac{32D}{(D^4 - d^4)\,\pi} \quad \text{Putting Z and } Z_p \qquad y = \frac{8}{12\pi}\frac{(D^3 - d^3)}{(D^2 - d^2)}$$

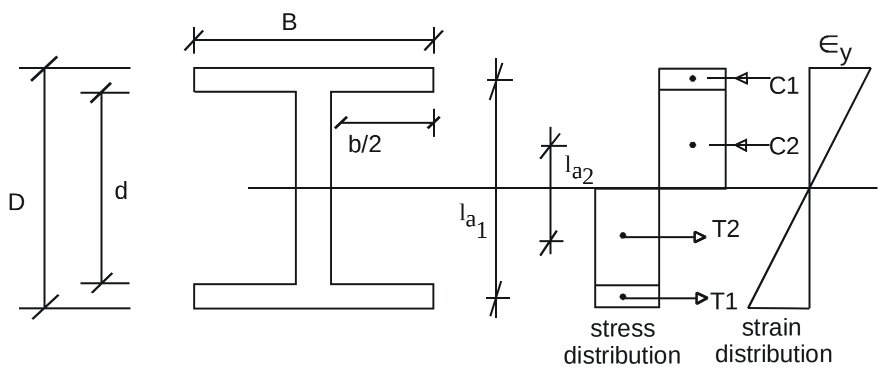
$$\gamma = \frac{32}{6\pi}\frac{D(D^3 - d^3)}{(D^4 - d^4)} \qquad y = \frac{2}{3\pi}\frac{(D^3 - d^3)}{D^2 - d^2}$$

for N-A or equal area axis.

For D = 10”
d = 8”

$$\gamma = 1.403$$

For I - Section:



$$\text{As} = Z = \frac{I}{C} \text{ and } C = \frac{D}{2}$$

$$I = \frac{(BD^3 - bd^3)}{12}, \qquad My = \sigma y \cdot Z = \sigma y \frac{(BD^3 - bd^3)}{6D}, \text{ Putting value of Z from (1)}$$

$$Z = \frac{I}{C} = \frac{(BD^3 - bd^3)}{12}\,\frac{2}{D} \qquad Mp = C_1\, la_1 + C_2\, la_2$$

$$Z = \left[\frac{BD^3 - bd^3}{6D}\right] (1) \qquad la_1 = \left[\frac{d}{2} + \left(\frac{D-d}{2}\right)^{1/2}\right] 2 = (D + d)/2,$$

$$la_2 = \left[\frac{D}{2} - \left(\frac{D-d}{2}\right)\right]^{1/2} \times 2 = \frac{d}{2}$$

$$Mp = \sigma y \cdot B\,\frac{(D-d)}{2}\,\frac{(D+d)}{2} + \sigma y \cdot \frac{d}{2}\,(B-b)\,\frac{d}{2}$$

$$Mp = \sigma y\left[\frac{B}{4}(D^2 - d^2) + \frac{d^2}{4}(B-b)\right]$$

$$\gamma = \frac{Mp}{My} = \frac{\sigma y(BD^2 - bd^2)}{4} \times \frac{BD}{\sigma y(BD^3 - bd^3)} \qquad Mp = \sigma y\left[\frac{BD^2 - bd^2}{4}\right]$$

$$\gamma = \frac{3D}{2}\,\frac{(BD^2 - bd^2)}{(BD^3 - bd^3)} \qquad \text{if} \quad B = 4”$$

b = 3.75”
D = 8” , shape factor γ = 1.160
d = 7.5”

Similarly for T-section, Equilateral Triangle and hollow rectangular section the values of shape-factor are 1.794, 2.343 and 1.29 respectively. For diamond shape its value is 2.0.

8.9. Significance of Shape Factor

Zp is First moment of area about equal area axis.

1. It gives an indication of reserve capacity of a section from on set of yielding at extreme fibres to full plastification.
2. If My is known,, Mp may be calculated.
3. A section with higher shape factor gives a longer warning before collapse.
4. A section with higher shape factor is more ductile and gives greater deflection at collapse.
5. Greater is the γ value, greater is collapse load factor λ_c

8.10. Collapse load of a structure.

Collapse load is found for a structure by investigating various possible collapse mechanisms of a structure under conceivable load systems. For any given mechanism, possible plastic hinge locations are determined by noting the types of loads and support conditions remembering that under increasing loads, the plastic hinges would form in a sequence defined by corresponding elastic moments at the possible plastic hinge locations. **" Collapse loads are usually the applied loads multiplied by collapse load factor λc . λc is defined as the ratio of the collapse load to the working load acting on any structure / element"**. The value of λc may indicate a margin of safety for various collapse mechanisms and steps can be taken in advance to strengthen the weaker structural elements before erection. Benefit of `strength reserve' is obtained due to increased moments of resistance due to plastification. The reserve of strength is large if the section widens out near the vicinity of neutral surface.

8.11. Assumptions made in Plastic Theory.

The plastic analysis is primarily based on following assumptions.

1. For prismatic members,, the value of Mp is independent of magnitude of bending moment.
2. The length of plastic hinge is limited to a point.
3. Material is very ductile and is capable of undergoing large rotations / curvatures at the constant moment without breaking.
4. The presence of axial force and shear force does not change the value of Mp.
5. The structure remains stable until the formation of last plastic hinge and serviceability would not be impaired till such time.
6. Loads acting on structure are assumed to increase in proportion to each other.
7. Continuity of each joint is assumed.

8.12. Fundamental Theorems of Plastic Collapse.

When degree of redundancy increases beyond 2 or 3 in situations where collapse mechanism is not very clear, we try to pick up collapse load with the help of three fundamental theorems.

a. Lower bound theorem or static theorem.

b. Upper bound theorem or kinematic theorem.

c. Uniqueness theorem.

8.12.1 Lower Bound theorem

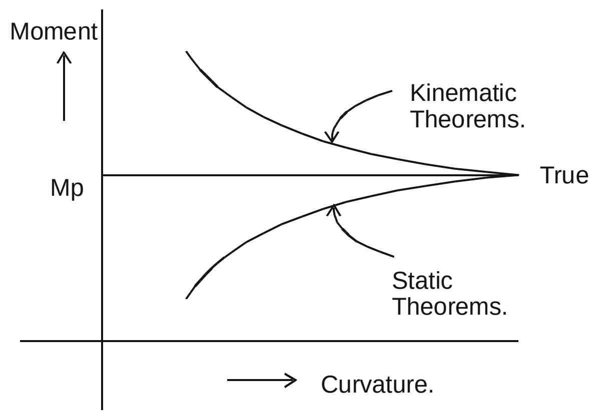
" A Load computed on the basis of bending moment distribution in which moment nowhere exceeds Mp is either equal to or less than the true collapse load".

8.12.2 Upper bound theorem

" A load computed on the basis of an assumed mechanism is either equal to or greater than true collapse load". When several mechanisms are tried, the true collapse load will the smallest of them.

8.12.3. Uniqueness theorem

" A load computed on the basis of bending moment distribution which satisfies both plastic moment and mechanism conditions is true plastic collapse load".



8.13. Methods of analysis

Basically there are two methods of analysis.

a. Equilibrium Method.

b. Mechanism Method.

8.13.1. Equilibrium Method

Normally a free bending moment diagram on simple span due to applied loads is drawn and B.M.D due to reactants is superimposed on this with due regard to their signs leaving the net moment distributed. Then by making the moment values equal to Mp values at the known potential plastic hinge locations, a revised diagram can be drawn. Then by splitting the simple span moment due to applied loads in terms of relevant Mp, the values of collapse load can be determined.

8.13.2. Mechanism Method

In this approach, a mechanism is assumed and plastic hinges are inserted at potential plastic hinge locations. At plastic hinges the corresponding rotations and deflections are computed to write work equations which may be written as follows.

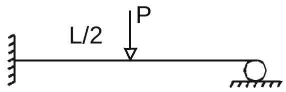
Work done by external loads = Actual loads x Average displacements = Work absorbed at Plastic hinges (internal work done) = Mp. θ

Typically $\Sigma W . \delta = \Sigma Mp . \theta.$

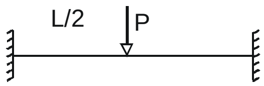
In both methods, the last step is usually to check that $M < Mp$ at all sections.

8.14. Values of Collapse loads for different loaded structures.

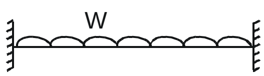
Beam Under loads | **Collapse load Pc or Wc**



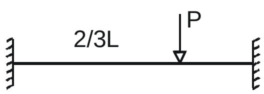
$4\,\frac{Mp}{L}$



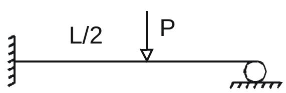
$8\,\frac{Mp}{L}$



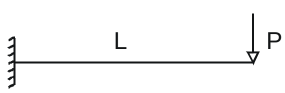
$16\,\frac{Mp}{L^2}$



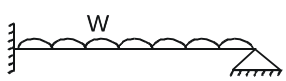
$9\,\frac{Mp}{L}$



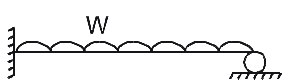
$6\,\frac{Mp}{L}$



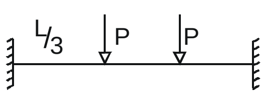
$1\,\frac{Mp}{L}$



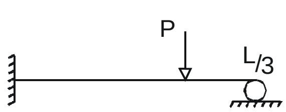
$11.65\,\frac{Mp}{L^2}$



$8\,\frac{Mp}{L^2}$



$6\,\frac{Mp}{L}$



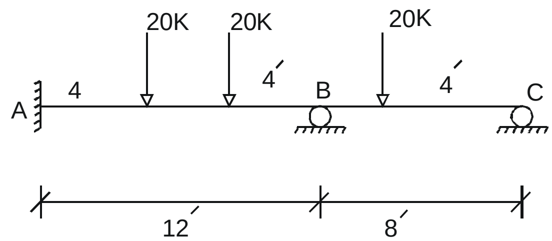
$6\,\frac{Mp}{L}$

8.15. In order to explain the above procedure, Let us solve examples.
Analysis of a Continuous beam by Mechanism Method.

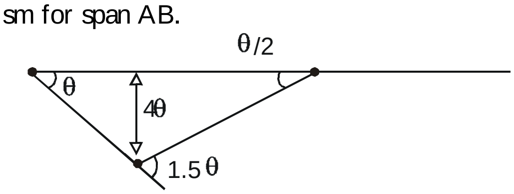
EXAMPLE NO. 1:- Consider the beam loaded as shown. Three independent possible collapse mechanisms along with potential plastic hinge locations are shown.

SOLUTION: degree of indeterminacy in terms
of moments = n = 2 (moments at A and B)

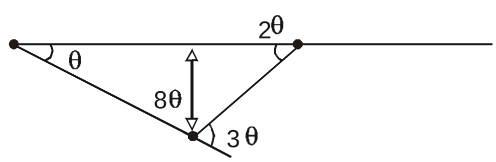
No of Plastic hinges required = 2 + 1 = 3



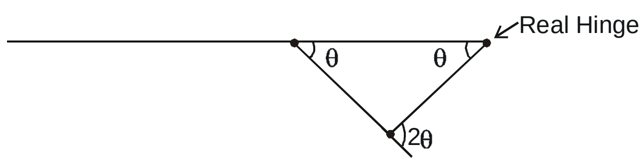
First possible beam mechanism for span AB.



Second possible beam mechanism for span AB.



Possible beam mechanism for span BC



Write work equations for all mechanisms and find corresponding Mp values.

Mechanism (1)

$$20 \times 4\,\theta + 20 \times 2\theta = Mp\,.\,\theta + Mp.1.5\theta + Mp\frac{\theta}{2}$$

$$120\,\theta = 3\,Mp\,\theta$$

$$Mp = 40\text{ K-ft.}$$

Mechanism (2)

$$20 \times 4\,\theta + 20 \times 8\,\theta = Mp\,.\,\theta + Mp.3\theta + Mp\,.\,2\theta$$

$240\,\theta = 6\,Mp\,\theta$

$Mp = 40$ K-ft

Mechanism (3)

$20 \times 4\,\theta = Mp.\,\theta + Mp\,.\,2\,\theta + 0 \times \theta$

$80\,\theta = 3\,Mp\,.\,\theta$

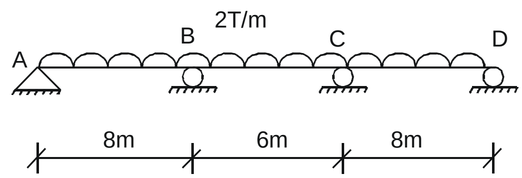
$Mp = 26.67$ K-ft.

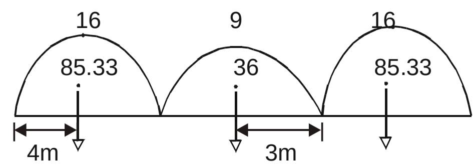
Minimum Collapse load or Max. Mp will be the collapse mechanism

So $Mp = 40$ K-ft. (Corresponding to mechanisms 1 and 2)

8.16. EXAMPLE NO.2:-Find the collapse load for the following continuous beam loaded as shown.

SOLUTION: Do elastic analysis by three moment equation to find Mb and Mc. Apply the equation twice to spans AB and BC and then BC and CD. (In this case, noting symmetry and concluding that Mb = Mc, only one application would yield results).





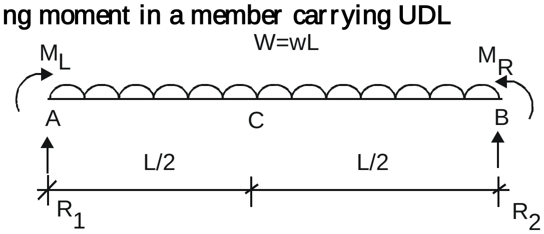
(Simple span B.M.D. due to loads)

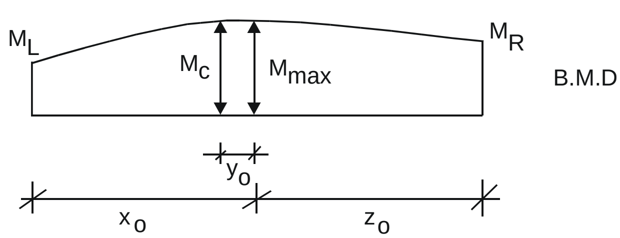
By using three-moment equation

$$\left(\frac{8}{I}\right) Ma + 2Mb\left(\frac{8}{I} + \frac{6}{I}\right) + Mc\left(\frac{6}{I}\right) = \frac{-6 \times 85.33 \times 4}{8} \frac{-6 \times 36 \times 3}{6}$$

$M_a = 0$, $34\,Mb = 364$ So $Mb = Mc = 10.70$ T – m (By symmetry)

8.17. Maximum bending moment in a member carrying UDL





Consider a general frame element subjected to Udl over its span alongwith end moments plot BMD.

After derivation we find the location of maximum moments X_o, Y_o and M_C.

In some books, plastic huge is stated to form in the centre of span. However, the formulae given below are very precise and give correct location of plastic huges due to u.d.l.

Where, ML = Moment at left of element

MR = Moment at right of element

MC = Moment at centre of element

X_o , Z_o , y_o = Location of max. moment from left, right and centre respectively as shown on BMD.

$$y_o = \frac{M_R - M_L}{W_L} = \frac{10.70 - o}{2 \text{ x } 8} = 0.6687 \text{ m} \qquad (1)$$

$$M_C = \frac{W_L^2}{8} + \frac{(M_R - M_L)}{2} = \frac{2(8)^2}{8} + \left(\frac{10.70}{2}\right)$$

$$M_C = 21.35 \text{ T} - \text{m} \qquad (2)$$

$$M_{max} = Mc + \frac{WL \,.\, yo^2}{2L} = 21.35 + \frac{2 \times 8\,(0.6687)^2}{2 \text{ x } 8}$$

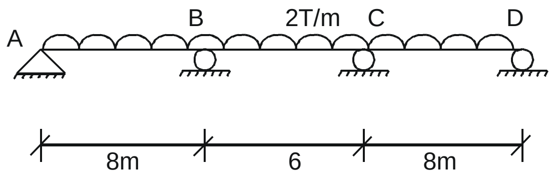
$$M_{max} = 21.79 \text{ T} - \text{m}$$

$$X_o = \frac{4M_C - 3M_R - M_L}{W_L} = \frac{4\,(21.35) - 3\,(10.7) - 0}{2 \times 8} = \text{ at 3.313 m from A and D.}$$

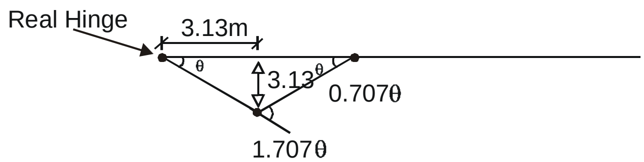
Plastic hinges would form first at a distance X_o = 3.313 m from points A and D and then at points B and C.

Now determine collapse load by mechanism method.

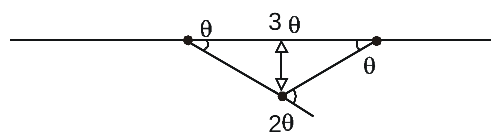
SOLUTION: No internal work is absorbed at real hinges.



First possible collapse mechanism of span AB.



Second possible collapse mechanism of span AB.



For first Mechanism

$$(2 \times 8)\ \frac{3.313\ \theta}{2} \;=\; Mp \times 1.707\ \theta + 0.707\ \theta\ Mp + 0$$

So $\quad Mp \;=\; 10.98\ T - m$

For second Mechanism

$$Mp\ .\ \theta + Mp\ .\ \theta + Mp\ .\ 2\ \theta \;=\; (2 \times 6)\left(\frac{30}{2}\right)$$

$$Mp \;=\; 4.5\ T - m$$

So $\quad Mp = 10.98\ T - m \quad$ or $\quad$ Load factor $\lambda = \dfrac{Mp}{10.98}$

8.18. Types of Collapse

Three types of collapses are possible as described below.

1. Complete collapse
2. Partial collapse
3. Over complete collapse.

8.18.1. Complete Collapse

If in a structure, there are R redundancies and collapse mechanism contains (R + 1) plastic hinges, it is called a complete collapse provided the structure is statically determinate at collapse.

8.18.2. Partial Collapse:

If in a structure, the number of plastic hinges formed at collapse do not render the structure as statically determinate it is called a partial collapse.

8.18.3. Over Complete Collapse

If in a structure there are two or more mechanisms which give the same value of collapse load (or collapse load factor λ_c) then this type of collapse is known as overcomplete collapse.

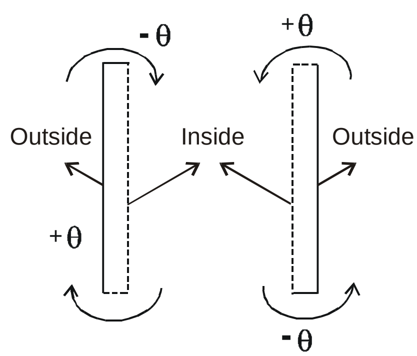
8.19. Analysis of Frames

In portal frames, three types of mechanisms are possible.

1. Beam Mechanisms (due to gravity loads)
2. Sway Mechanisms (due to lateral loads).
3. Combined Mechanisms (both loads).

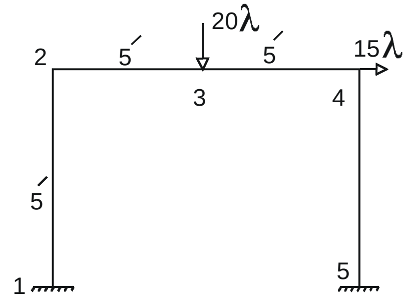
Step 1:
Draw frame in thickness in two lines i.e., solid lines and broken lines. Solid lines are " outside" of frame and broken lines are " inside" of frame.

Step 2: Nodal moments creating compression on out sides are positive or vice-versa.



Step 3: Hinge cancellation at joints occur when rotations of different signs are considered and mechanisms are combined.

EXAMPLE NO. 3:- Analyse the frame shown below
SOLUTION:
1, 2, 3, 4 and 5 are possible plastic Hinge locations. Three independent mechanisms are possible Beam mechanisms, Sway mechanisms and Combined mechanisms are possible.



1. Beam Mechanism

Write work equation (Fig A)

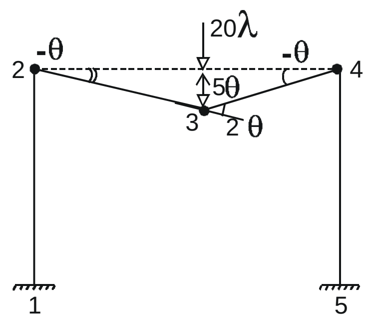
$20\,\lambda.5\,\theta = \quad M2\,(-\,\theta) + \ M3\,(2\,\theta) + \ M4\,(-\,\theta)$

$100\,\lambda \quad = \quad -\,M2 + \ 2M3 - M4$ by taking θ as common above. (1)

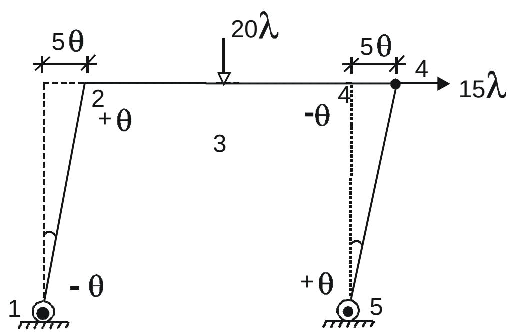
Remember that work is always positive.

putting $M2 \ = \ Mp \qquad M3 \ = \ Mp \qquad M4 \ = \ Mp$ in equation (1), we have

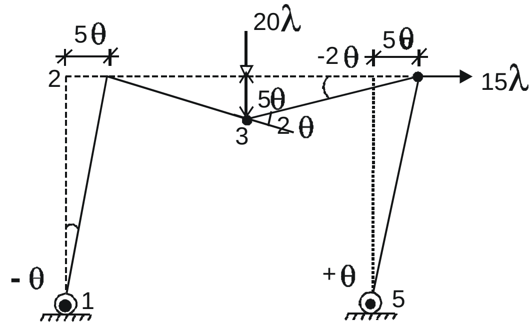
$100\,\lambda \quad = \quad 4\,Mp$ or $[\lambda \ = \ 0.04\,Mp]$



(a) Beam mechanism of element 2-4



(b) Sway Mechanism of Columns



(c) = a + b combined mechanism

2. Sway Mechanism:

$15\lambda . 5\theta = M1(-\theta) + M2\theta + M4(-\theta) + M5(\theta)$

$75\lambda = -M1 + M2 - M4 + M5$ (2)

M1, M2, M4 and M5 are all equal to Mp

$75\lambda = 4\,Mp$ or $[\lambda = 0.053\,Mp]$

3. Combined Mechanism:

$20\lambda . 5\theta + 15\lambda . 5\theta = M1(-\theta) + M2(0) + M3(2\theta) + M4(-2\theta) + M5(\theta)$

$175\lambda = -M1 + 2M3 - 2M4 + M5$ (3)

all these moments are equal to Mp

$175\lambda = 6\,Mp$, $[\lambda = 0.034\,Mp]$ or $Mp = 29.15\lambda$.

Keeping in mind the definition of a true mechanism [one giving highest value of Mp in terms of Pc or lowest value of Pc in terms of Mp or λ]
Combined mechanism is the true collapse mechanism.

So $\lambda c = 0.0343\,Mp$

It will be a complete collapse if the structure is statically determinate and moment anywhere does not exceed Mp value since there are n + 1 plastic hinges in the true collapse mechanism

Note: "Moment checks are normally applied at those plastic hinge positions which are not included in the true collapse mechanism". In the true collapse mechanism which is combined mechanism in this case, moments at points 1, 3, 4 and 5 are equal to Mp, we need to find and check moment value at point 2 only in this case.

The generalized work equations 1 and 2 in terms of moments may be used for the purpose alongwith their signs.

$100\lambda = -M2 + 2M3 - M4$ (1)

$75\lambda = -M1 + M2 - M4 + M5$ (2) Noting that $\lambda = 0.0343\,Mp$

eqn (1) becomes

$100 \times 0.0343\,Mp = -M2 + 2Mp + Mp$ so $M2 = -0.431\,Mp < Mp$ – O.K.

eqn (2) becomes

$75(0.0343\,Mp) = +Mp + M2 + Mp + Mp$ so $M2 = -0.42755\,Mp < Mp$ – O.K.

Net value of M2 = algebraic sum of equations 1 and 2 as combined mechanism is combination of case A and case B.

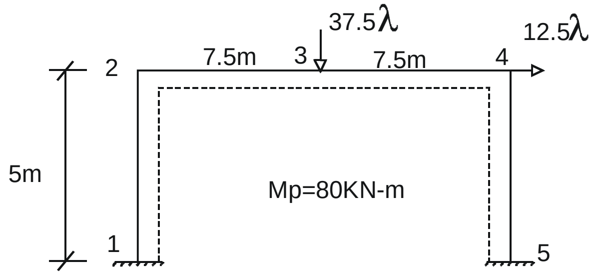
M2 = (– 0.431 – 0.427) Mp = – 0.858 Mp < Mp – O.K.

If at this stage a higher load factor is specified by the designer, there is no need to revise the frame analysis and following formula can be applied to get increased Mp value.

$$(Mp)\ new = \frac{\text{specified new collapse load factor}}{\text{present calculated collapse load factor}} \times (Mp\ Present)$$

8.20. EXAMPLE NO. 4:- Partial or incomplete collapse:

Find collapse load factor for the following loaded frame. Mp is 80 KN-M for all members.



SOLUTION: Draw three possible independent collapse mechanisms. Write work equation and find 1, 2, 3, 4 and 5 possible plastic hinge locations.

1. Beam Mechanism:

$(35.5\,\lambda)\ 7.5\,\theta = -M2\,\theta + M3\,2\,\theta + M4\,(-\theta)$

$281.25\,\lambda = -M2 + 2M3 - M4$ (1)

moment at 2, 3 and 4 is equal to Mp. so

$281.25\,\lambda = 4\,Mp$ (work is always + ve)

or $\lambda = 1.1377$

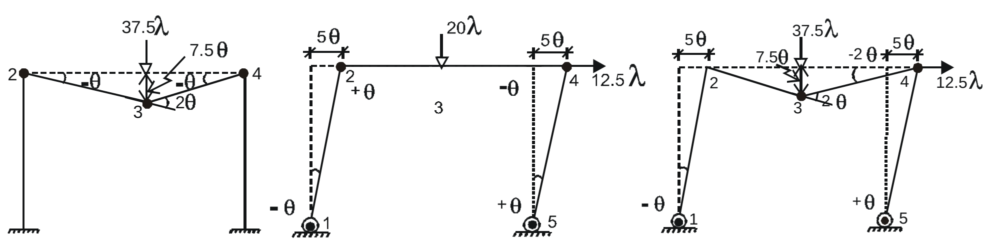
2. Sway Mechanism:

$(12.5\,\lambda)\ 5\,\theta = +\ M1\,(-\theta) + M2\,(\theta) + M4\,(-\theta) + M5\,(\theta)$

$62.5\,\lambda = -M1 + M2 - M4 + M5$ (2), Moment at 1,2,4 and 5 is Mp.

$62.5\,\lambda = 4\,Mp$ or $\lambda = \frac{4}{62.5} \times 80 = 5.12$

$\lambda = 5.12$



(a) Beam mechanism (b) Sway Mechanism (c) Combined mechanism

3. Combined Mechanism:

$(37.5\,\lambda)\ (7.5\theta) + (12.5\,\lambda)\ (5\theta) = M1\,(-\theta) + M2 \times 0 + M3\,(2\theta) + M4\,(-2\theta) + M5\,(\theta)$

$343.75\,\lambda = -M1 + 2M3 - 2M4 + M5$ (3) Moment at 1,3,4 and 5 is Mp

$$343.75\,\lambda = 6\,Mp \quad \text{or} \quad \lambda = \frac{6 \times 80}{343.75} = 1.396$$

$\lambda = 1.396.$

Therefore, according to kinematic theorem, beam mechanism containing 3 Plastic hinges (one less than required) is the collapse mechanism for this frame with 3 redundancies. (N= n+ 1)= 3+ 1= 4 are required.;

Note: In partial or incomplete collapse, only a part of the structure becomes statically determinate.

Check moments at locations (1) and (5) with $\lambda = 1.1377$, M2 , M3 , M4 = Mp substituting is eqn (2).

$62.5\,\lambda = -M1 + M2 - M4 + M5$ or $62.5\,(1.1377) = -M1 + Mp + Mp + M5$

$-88.937 = M5 - M1$ (4) or $M1 - M5 = 88.937$ (4)

Putting same values in eqn (3)

$343.75\,(1.137) = -M1 + 2Mp + 2Mp + M5$

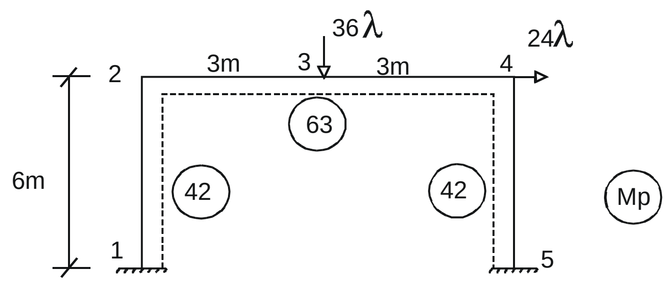
$= -M1 + M5 + 4 \times 80$

$70.84 = M5 - M1$ (5)

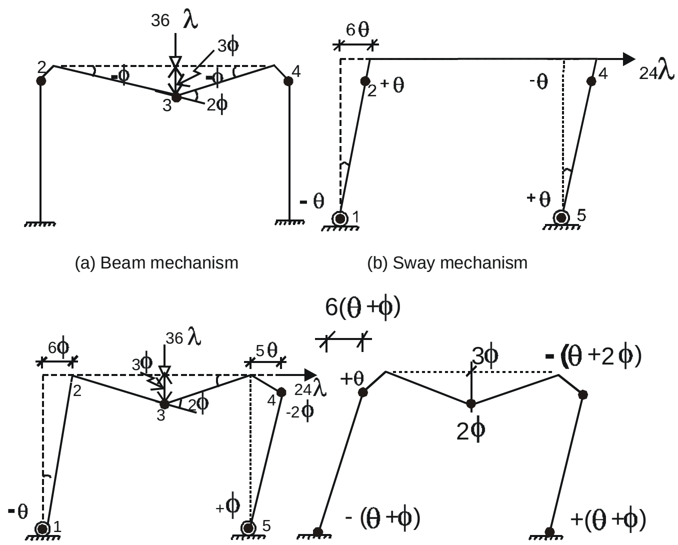
Values of M1 and M5 cannot be found from either of equations (4) and (5) as this is incomplete or partial collapse. Instead of a unique answer on values of M1 and M5 which do not violate yield criteria, different pairs of possible values of M1 and M5 can be obtained satisfying equations 4 and 5. Therefore, according to Uniqueness theorem beam mechanism is the true collapse mechanism. It is a partial collapse case.

8.21. EXAMPLE NO. 5:- Overcomplete collapse

Determine λc for the following loaded frame.



SOLUTION: Sketch possible independent collapse mechanisms. Notice that locations where beam and column meets, plastic huge is formed in weaker member near the joint.



(a) Beam mechanism (b) Sway mechanism

(c) Combined mechanism (a + b) (d) Another Combined mechanism (b+c)

1. Beam Mechanism: Fig A

$(36\lambda)\ 3\phi = -M2\ \phi + M3\ (2\phi) - M4\ \phi$

$108\ \lambda = -M2 + 2M3 - M4$ (1) All are equal to respective Mp. Putting values.

$108\ \lambda = 42 + 2 \times 63 + 42$

$\lambda = 1.944$

2. Sway Mechanism Fig B.

$(24\lambda)\ 6\theta = M1\ (-\theta) + M2\ (\theta) + M4\ (-\theta) + M5(\theta)$

$144\ \lambda = -M1 + M2 - M4 + M5$ (2)

$144\ \lambda = 42 + 42 + 42 + 42$ or $\lambda = 1.166$

3. First Combined Mechanism Fig C

$(24\ \lambda)\ (6\phi) + (36\lambda)\ (3\phi) = M1\ (-\phi) + M2\ (0) + M3\ (2\phi) + M4\ (-2\phi) + M5\ (\phi)$

$252\ \lambda = -M1 + 2M3 - 2M4 + M5$ (3)

$\lambda = \frac{294}{252}$ $\lambda = 1.166$

4. Second Combined Mechanism Fig D

$(36\ \lambda)3\phi + 24\lambda\ (\theta + \phi)6 = M1\ (-\theta - \phi) + M2\ (\theta) + M3\ (2\phi) + M4\ (\theta + 2\phi) + M5\ (\theta + \phi)\ \phi \cong \theta$

$396\ \lambda = -M1 + M2 + 2M3 - 2M4 + 2M5$

$396\ \lambda = 2(42) + 42 + 2(63) + 3 \times 42 + 2 \times 42$

$$\lambda = \frac{462}{396} = 1.166$$

$$\lambda = 1.166.$$

Note: In overcomplete collapse, more than one mechanism give the same value of collapse load factor. Any or both of the collapse mechanisms can contain extra number of plastic hinges than those required for complete collapse. So in this case fig c and d mechanisms give the same value. This was the case of over complete collapse.

Space for notes: