Pavement Analysis and Design

TE-503/ TE-503 A

Lecture-4
30-09-2019

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DTEM

STRESSES DUE TO CURLING

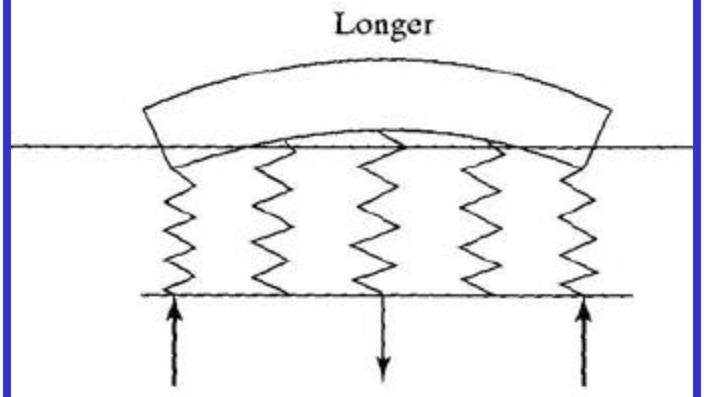
During the day, when the temperature on the top of the slab is greater than that at the bottom, the top tends to expand with respect to the neutral axis, while the bottom tends to contract. However, the weight of the slab restrains it from expansion and contraction; thus, compressive stresses are induced at the top, tensile stresses at the bottom.

At night, when the temperature on the top of the slab is lower than that at the bottom, the top tends to contract with respect to the bottom; thus, tensile stresses are induced at the top and compressive stresses at the bottom.

STRESSES DUE TO CURLING

Another explanation of curling stress can be made in terms of the theory of a plate on a Winkler, or liquid, foundation.

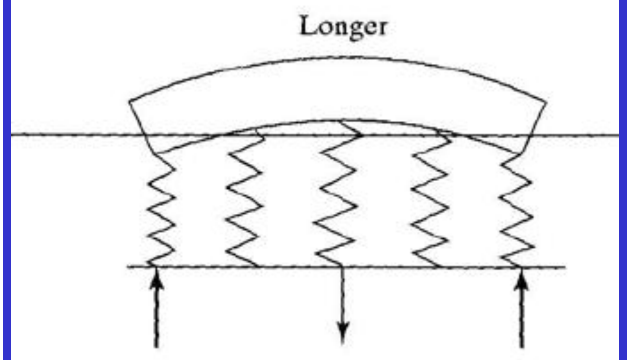
A Winkler foundation is characterized by a series of springs attached to the plate, as shown in Figure.



(a) Downward Curling

STRESSES DUE TO CURLING

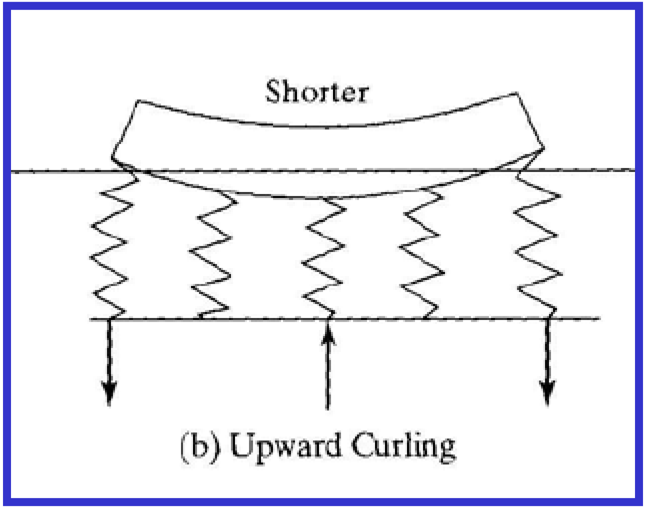
When the temperature on the top is greater than that at the bottom, the top is longer than the bottom and the slab curls downward. The springs at the outside edge are in compression and push the slab up, while the springs in the interior are in tension and pull the slab down. As a result, the top of the slab is in compression and the bottom is in tension.



(a) Downward Curling

STRESSES DUE TO CURLING

When the temperature on the top is lower than that at the bottom, the slab curls up-ward. The exterior springs pull the slab down while the interior springs push the slab up, thus resulting in tension at the top and compression at the bottom.



Bending of Infinite Plate

The difference between a beam and a plate is that the beam is stressed in only one direction, the plate in two directions. For stresses in two directions, the strain ε_x, in the *x* direction can be determined by the generalized Hooke's law,

$$\epsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} \quad \text{..................Eq.1}$$

in which *E* is the elastic modulus of concrete. The first term on the right side of Eq.1 indicates the strain in the *x* direction caused by stress in the *x* direction; the second term indicates the strain in the *x* direction caused by stress in the y direction.

Bending of Infinite Plate

Similarly,

$$\epsilon_y = \frac{\sigma_y}{E} - \nu\frac{\sigma_x}{E} \qquad \text{..................Eq.2}$$

When the plate is bent in the *x* direction, ε_y , should be equal to 0 because the plate is so wide and well restrained that no strain should ever occur unless near the very edge. Setting Eq.2 to 0 yields:

$$\sigma_y = \nu\sigma_x \qquad \text{..................Eq.3}$$

Substituting Eq.3 into Eq.1 and solving for ε_x gives

$$\sigma_x = \frac{E\epsilon_x}{1 - \nu^2} \qquad \text{..................Eq.4}$$

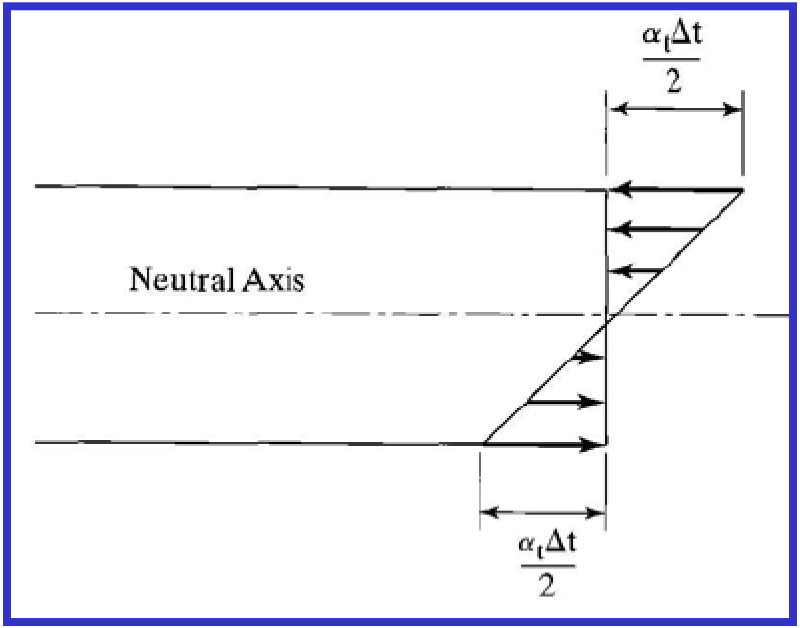
Eq.4 indicates the stress in the bending direction, Eq.3 the stress in the direction perpendicular to bending.

Bending of Infinite Plate

When bending occurs in both the *x* and *y* directions, as is the case for temperature curling, the stresses in both directions must be superimposed to obtain the total stress.

The maximum stress in an infinite slab due to temperature curling can be obtained by assuming that the slab is completely restrained in both *x* and *y* directions.

Bending of Infinite Plate



Bending of Infinite Plate

Let Δt be the temperature differential between the top and the bottom of the slab and α_t be the coefficient of thermal expansion of concrete. If the slab is free to move and the temperature at the top is greater than that at the bottom, the top will expand by a strain of $\alpha_t \Delta t/2$ and the bottom will contract by the same strain, as shown in Figure. If the slab is completely restrained and prevented from moving, a compressive strain will result at the top and a tensile strain at the bottom. The maximum strain is

$$\epsilon_x = \epsilon_y = \frac{\alpha_t \Delta t}{2} \qquad \text{..................Eq.5}$$

Bending of Infinite Plate

From Eq.4, the stress in the *x* direction due to bending in the *x* direction is

$$\sigma_x = \frac{E\alpha_t \Delta t}{2(1 - v^2)}$$ **...................Eq.6**

Because Eq.6 is also the stress in the *y* direction due to bending in the *y* direction, from Eq.3, the stress in the *x* direction due to bending in the *y* direction is

$$\sigma_x = \frac{vE\alpha_t \Delta t}{2(1 - v^2)}$$ **...................Eq.7**

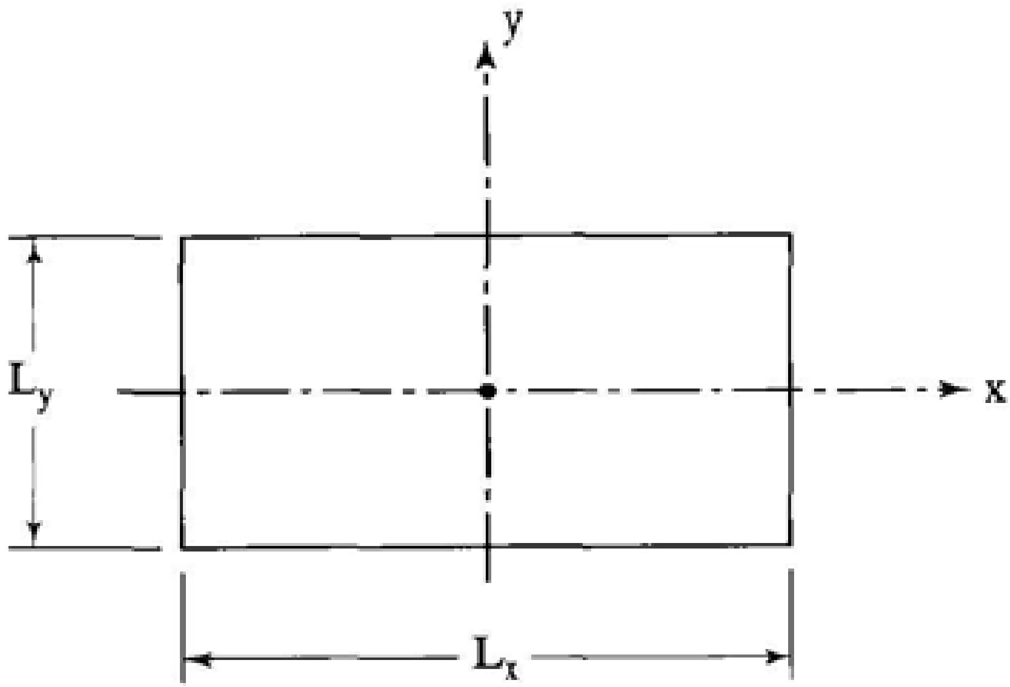
The total stress is the sum of Eqs.6 and 7:

$$\sigma_x = \frac{E\alpha_t \Delta t}{2(1 - v^2)}(1 + v) = \frac{E\alpha_t \Delta t}{2(1 - v)}$$ **............Eq.8**

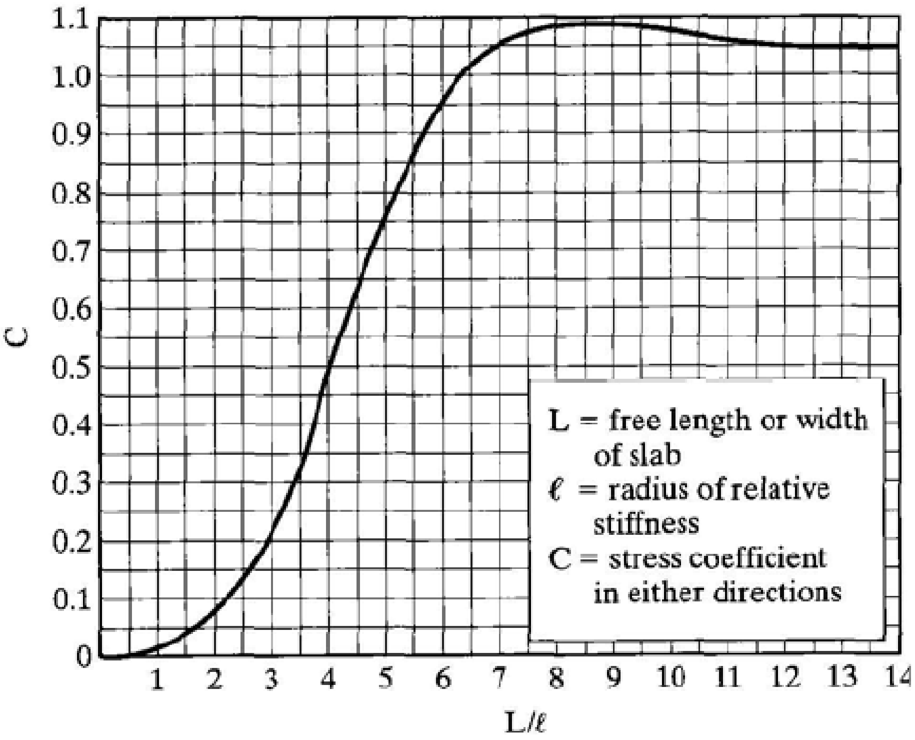
Bending of Infinite Plate

The preceding analysis is based on the assumption that the temperature distribution is linear throughout the depth of the slab. This is an approximation, because the actual temperature distribution is nonlinear.

Curling Stresses in Finite Slab



Curling Stresses in Finite Slab



Curling Stresses in Finite Slab

Figure shows a finite slab with lengths L_x in the x direction and L_y in the y direction. The total stress in the x direction can be expressed as:

$$\sigma_x = \frac{C_x E \alpha_t \Delta t}{2(1-v^2)} + \frac{C_y v E \alpha_t \Delta t}{2(1-v^2)} = \frac{E \alpha_t \Delta t}{2(1-v^2)}(C_x + vC_y) \quad \text{.........Eq.9a}$$

in which C_x and C_y are correction factors for a finite slab. The first term in Eq.9a is the stress due to bending in the x direction and the second term is the stress due to bending in the y direction. Similarly, the stress in the y direction is

$$\sigma_y = \frac{E \alpha_t \Delta t}{2(1-v^2)}(C_y + vC_x) \quad \text{............Eq.9b}$$

Curling Stresses in Finite Slab

Using Westergaard's analysis, Bradbury (1938) developed a simple chart for determining C_x and C_y, as shown in Figure. The correction factor C_x depends on L_x /e and the correction factor C_y depends on L_y /e where *e* is the radius of relative stiffness, defined as:

$$\ell = \left[\frac{Eh^3}{12(1-\nu^2)k}\right]^{0.25} \qquad \text{.........Eq.10}$$

in which *E* is the modulus of elasticity of concrete, *h* is the thickness of the slab, *υ* is Poisson ratio of concrete, and *k* is the modulus of subgrade reaction.

Curling Stresses in Finite Slab

Here modulus of 4 x 10^6 psi and a Poisson ratio of 0.15 are assumed for the concrete. Equation 9 gives the maximum interior stress at the center of a slab. The edge stress at the midspan of the slab can be determined by:

$$\sigma = \frac{CE\alpha_t \Delta t}{2} \qquad \text{.........Eq.11}$$

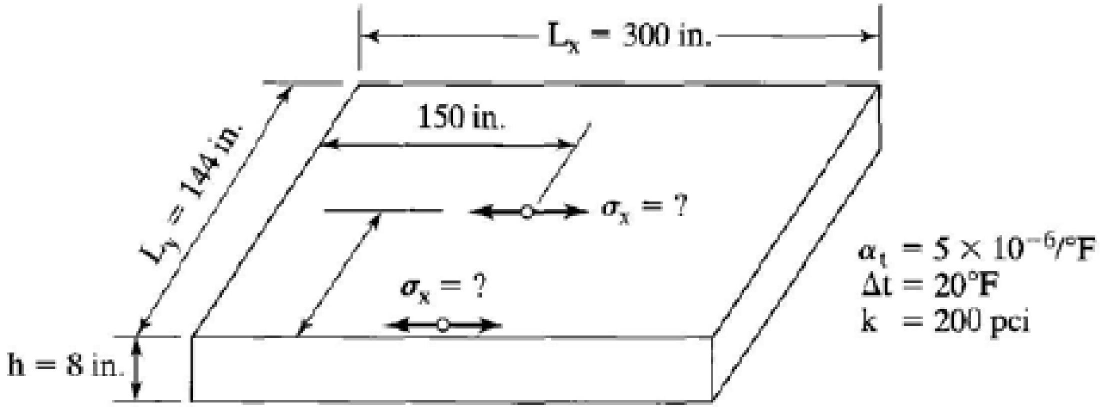
in which σ may be σ_x or σ_y depending on whether *C* is C_x or C_y.

Note that Eq.11 is the same as Eq.9 when the Poisson ratio at the edge is taken as 0.

It can be seen from Figure that the correction factor *C* increases as the ratio *L/e* increases.

Curling Stresses in Finite Slab-Numerical Problem

Figure shows a concrete slab, *25 ft* long, *12 ft* wide and *8 in.* thick, subjected to a temperature differential of *20°F*. Assuming that *k* = *200 pci* and α_t = 5*x10^{-6} in./in./°F*, determine the maximum curling stress in the interior and at the edge of the slab.



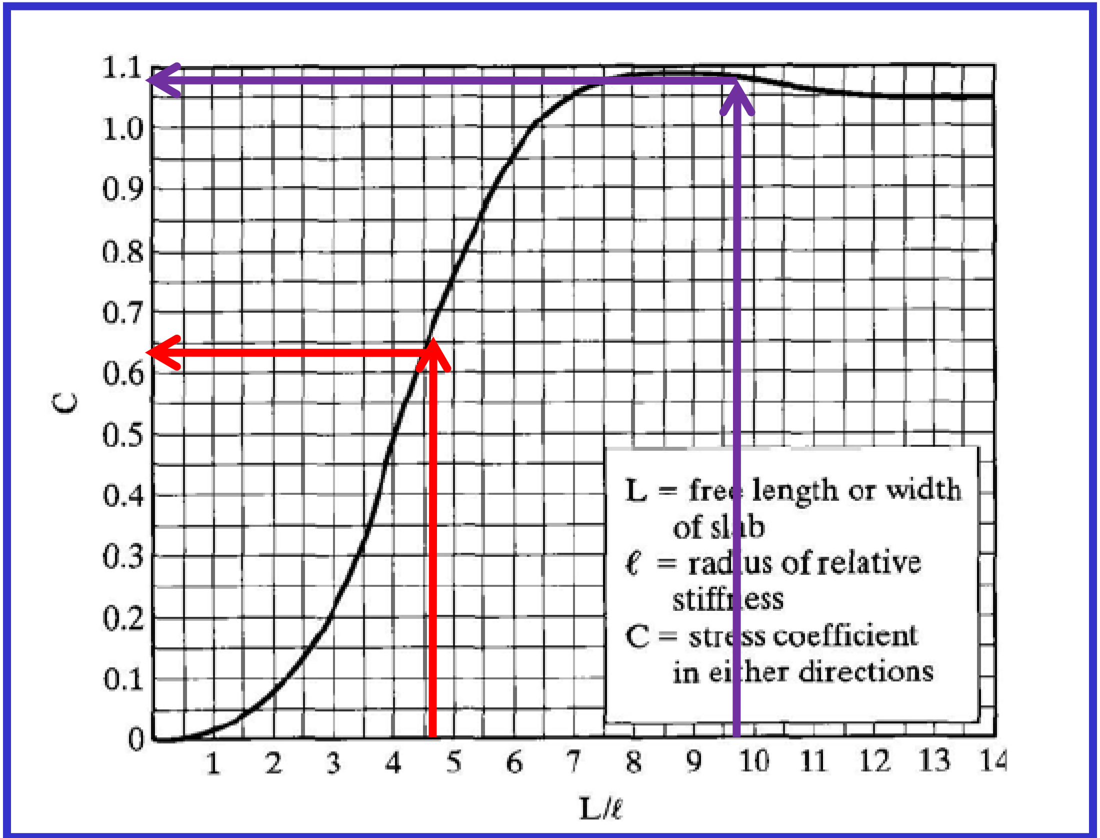
Curling Stresses in Finite Slab-Numerical Problem

$$\ell = \left[\frac{Eh^3}{12(1 - v^2)k}\right]^{0.25}$$

$$\sigma_x = \frac{C_x E \alpha_t \Delta t}{2(1 - v^2)} + \frac{C_y v E \alpha_t \Delta t}{2(1 - v^2)} = \frac{E \alpha_t \Delta t}{2(1 - v^2)}(C_x + vC_y)$$

$$\sigma = \frac{C E \alpha_t \Delta t}{2}$$

Curling Stresses in Finite Slab-Numerical Problem



Curling Stresses in Finite Slab-Temperature Differentials

Curling stresses in concrete pavements vary with the temperature differentials between the top and bottom of a slab.

Unless actual field measurements are made, it is reasonable to assume a maximum temperature gradient of 2.5 to 3.5°F per inch of slab (0.055 to 0.077°C/mm) during the day and about half the above values at night.

Curling Stresses in Finite Slab-Combined Stresses

Even though curling stresses can be quite large and can cause concrete to crack when combined with loading stresses, they are usually not considered in the thickness design for the following reasons:

1. Joints and steel are used to relieve and take care of curling stresses. Curling stresses are relieved when the concrete cracks. Minute cracks will not affect the load-carrying capacity of pavements as long as the load transfer across cracks can be maintained.

2. When the fatigue principle is used for design, it is not practical to combine loading and curling stresses. A pavement might be subjected to millions of load repetitions during the design period, but the number of stress reversals due to curling is quite limited.

Curling Stresses in Finite Slab-Combined Stresses

3. Curling stresses may be added to or subtracted from loading stresses to obtain the combined stresses. If the design is governed by the edge stress, curling stresses should be added to loading stresses during the day but subtracted from the loading stresses at night. Due to this compensative effect and the fact that a large number of heavy trucks are driven at night, it may not be critical if curling stresses are ignored.

Stresses and Deflections due to Loading

Three methods can be used to determine the stresses and deflections in concrete pavements:

closed-form formulas, influence charts and finite-element computer programs.

The formulas originally developed by Westergaard can be applied only to a single-wheel load with a circular, semicircular, elliptical or semielliptical contact area.

The influence charts developed by Pickett and Ray (1951) can be applied to multiple-wheel loads of any configuration. Both methods are applicable only to a large slab on a liquid foundation.

If the loads are applied to multiple slabs on a liquid, solid or layer foundation with load transfer across the joints, the finite-element method should be used.

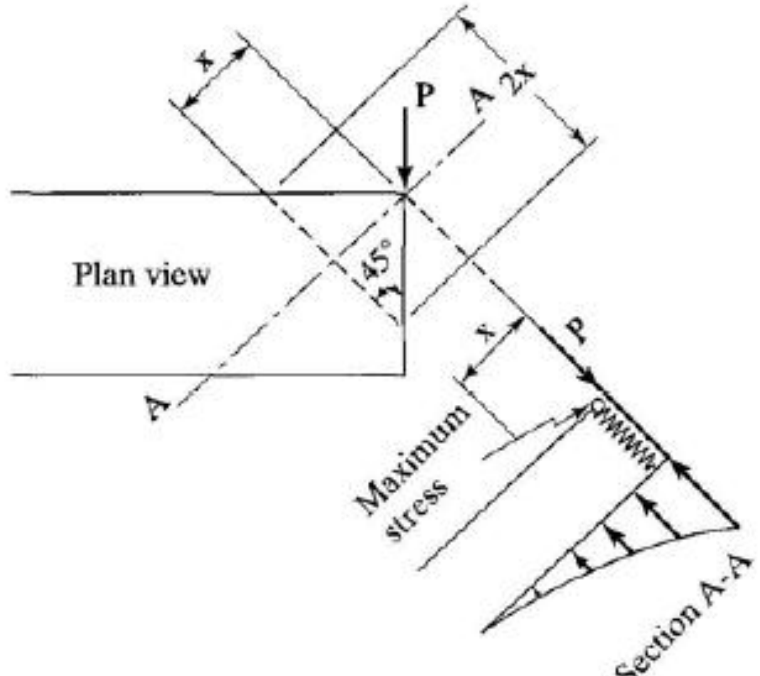
Stresses and Deflections due to Loading

Closed-Form Formulas

These formulas are applicable only to a very large slab with a single-wheel load applied near the corner, in the interior of a slab at a considerable distance from any edge and near the edge far from any corner.

Stresses and Deflections due to Loading

Closed-Form Formulas-Corner Loading



(a) Concentrated Load

Stresses and Deflections due to Loading

Closed-Form Formulas-Corner Loading

The Goldbeck (1919) and Older (1924) formula is the earliest one for use in concrete pavement design.

The formula is based on a concentrated load P applied at the slab corner, as shown in Figure.

When a load is applied at the corner, the stress in the slab is symmetrical with respect to the diagonal.

For a cross section at a distance *x* from the corner, the bending moment is *Px* and the width of section is *2x*.

Stresses and Deflections due to Loading

Closed-Form Formulas-Corner Loading-Concentrated Load

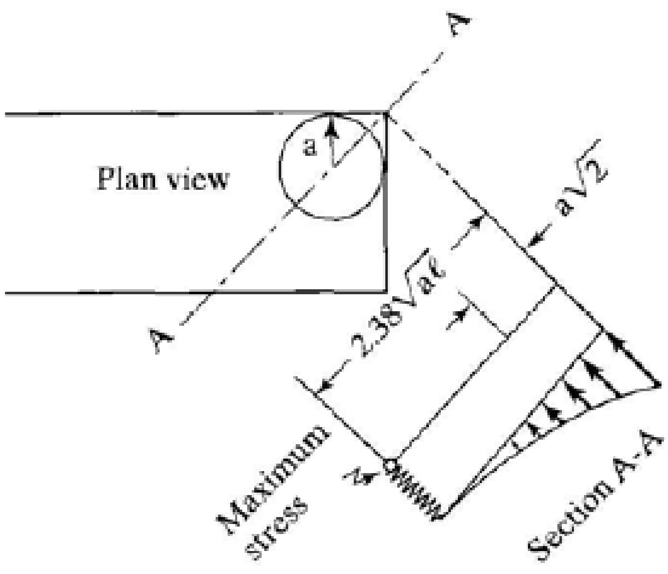
When the subgrade support is neglected and the slab is considered as a cantilever beam, the tensile stress on top of the slab is:

$$\sigma_c = \frac{Px}{\frac{1}{6}(2x)\,h^2} = \frac{3P}{h^2}$$

in which σ_c is the stress due to corner loading, *P* is the concentrated load and *h* is the thickness of the slab. Note that σ_c does not depend on *x*. In other words, every cross section, no matter how far from the corner, will have the same stress. If the load is really a concentrated load applied at the very corner, above Eq. is an exact solution, because, at a cross section near to the load, with *x* approaching 0, the subgrade reaction is very small and can be neglected.

Stresses and Deflections due to Loading

Closed-Form Formulas-Corner Loading-Circular Load



(b) Circular Load

Stresses and Deflections due to Loading

Closed-Form Formulas-Corner Loading-Circular Load

Figure shows a circular load applied near the corner of a slab. Because the section of maximum stress is not near the corner, the total subgrade reactive force is quite large and cannot be neglected. Westergaard (1926) applied a method of successive approximations and obtained the formulas

$$\sigma_c = \frac{3P}{h^2}\left[1 - \left(\frac{a\sqrt{2}}{\ell}\right)^{0.6}\right]$$

$$\Delta_c = \frac{P}{k\ell^2}\left[1.1 - 0.88\left(\frac{a\sqrt{2}}{\ell}\right)\right]$$

in which Δ_c is the corner deflection, *e* is the radius of relative stiffness, *a* is the contact radius and *k* is the modulus of subgrade reaction.

Stresses and Deflections due to Loading

Closed-Form Formulas-Corner Loading-Circular Load

Westergaard also found that the maximum moment occurs at a distance of $2.38\sqrt{a\ell}$ from the corner.

For a concentrated load with a=0, Eqs.

$$\sigma_c = \frac{3P}{h^2}\left[1 - \left(\frac{a\sqrt{2}}{\ell}\right)^{0.6}\right]$$

$$\sigma_c = \frac{Px}{\frac{1}{6}(2x)\,h^2} = \frac{3P}{h^2}$$

are identical.

Stresses and Deflections due to Loading

Closed-Form Formulas-Corner Loading-Circular Load

Ioannides *et al. (1985)* applied the finite-element method to evaluate Westergaard's solutions. They suggested the use of the relationships:

$$\sigma_c = \frac{3P}{h^2}\left[1 - \left(\frac{c}{\ell}\right)^{0.72}\right]$$

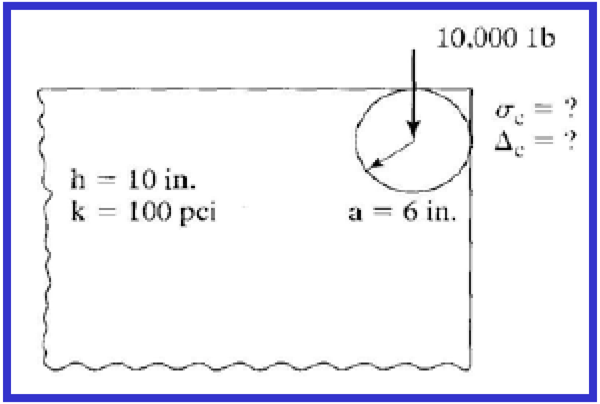
$$\Delta_c = \frac{P}{k\ell^2}\left[1.205 - 0.69\left(\frac{c}{\ell}\right)\right]$$

in which *c* is the side length of a square contact area. They found that the maximum moment occurs at a distance of $1.80c^{0.32}\ \ell^{0.59}$ from the corner. If a load is applied over a circular area, the value of *c* must be selected so that the square and the circle have the same contact area: $c = 1.772a$

Stresses and Deflections due to Loading

Closed-Form Formulas-Corner Loading-Circular Load-Numerical Problem

Figure shows a concrete slab subjected to a corner loading. Given *k=100 pci*, *h=10 in.*, *a = 6 in. and P=10,000lb*, determine the maximum stress and deflection due to corner loading by Westergaard formula and Ioannides *et al.* formula.



Stresses and Deflections due to Loading

Closed-Form Formulas-Corner Loading-Circular Load-Numerical Problem

Westergaard formula

$$\sigma_c = \frac{3P}{h^2}\left[1 - \left(\frac{a\sqrt{2}}{\ell}\right)^{0.6}\right]$$

$$\ell = \left[\frac{Eh^3}{12(1 - \nu^2)k}\right]^{0.25}$$

$$\Delta_c = \frac{P}{k\ell^2}\left[1.1 - 0.88\left(\frac{a\sqrt{2}}{\ell}\right)\right]$$

Ioannides *et al.* formula

c = 1 .772a

$$\sigma_c = \frac{3P}{h^2}\left[1 - \left(\frac{c}{\ell}\right)^{0.72}\right]$$

$$\Delta_c = \frac{P}{k\ell^2}\left[1.205 - 0.69\left(\frac{c}{\ell}\right)\right]$$

Stresses and Deflections due to Loading

Closed-Form Formulas-Interior Loading-Circular Load

The earliest formula developed by Westergaard (1926) for the stress in the interior of a slab under a circular loaded area of radius *a* is:

$$\sigma_i = \frac{3(1+v)P}{2\pi h^2}\left(\ln\frac{\ell}{b} + 0.6159\right)$$

in which *e* is the radius of relative stiffness and

$$b = a \qquad \text{when } a \geq 1.724h$$

$$b = \sqrt{1.6a^2 + h^2} - 0.675h \quad \text{when } a < 1.724h$$

Stresses and Deflections due to Loading

Closed-Form Formulas-Interior Loading-Circular Load

For a Poisson ratio of 0.15 and in terms of base-10 logarithms, Eq. can be written as:

$$\sigma_i = \frac{0.316P}{h^2}\left[4\log\left(\frac{\ell}{b}\right) + 1.069\right]$$

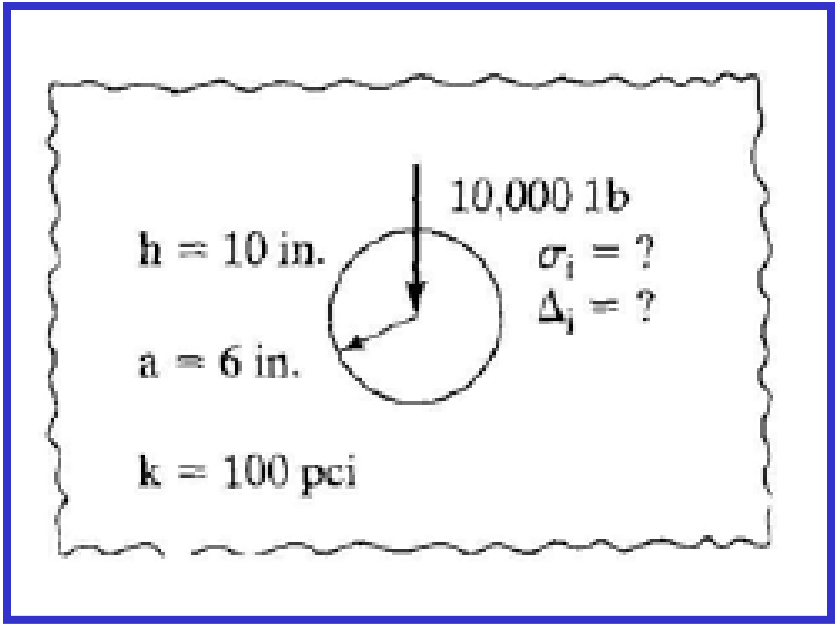
The deflection equation due to interior loading (Westergaard, 1939) is:

$$\Delta_i = \frac{P}{8k\ell^2}\left\{1 + \frac{1}{2\pi}\left[\ln\left(\frac{a}{2\ell}\right) - 0.673\right]\left(\frac{a}{\ell}\right)^2\right\}$$

Stresses and Deflections due to Loading

Closed-Form Formulas-Interior Loading-Circular Load-Numerical Problem

For the loading shown in Figure, determine the maximum stress and deflection due to interior loading.



Stresses and Deflections due to Loading

Closed-Form Formulas-Interior Loading-Circular Load-Numerical Problem

$$\ell = \left[\frac{Eh^3}{12(1 - \nu^2)k}\right]^{0.25}$$

$$b = a \qquad \text{when } a \geq 1.724h$$

$$b = \sqrt{1.6a^2 + h^2} - 0.675h \quad \text{when } a < 1.724h$$

$$\sigma_i = \frac{0.316P}{h^2}\left[4\log\left(\frac{\ell}{b}\right) + 1.069\right]$$

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Stresses and Deflections due to Loading

Closed-Form Formulas-Edge Loading-Circular Load

The stress due to edge loading was presented by Westergaard (1926, 1933, 1948) in several different papers. In his 1948 paper, he presented generalized solutions for maximum stress and deflection produced by elliptical and semielliptical areas placed at the slab edge. Setting the length of both major and minor semiaxes of the ellipse to the contact radius *a* leads to the corresponding solutions for a circular or semicircular loaded area. In the case of a semicircle, its straight edge is in line with the edge of the slab. The results obtained from these new formulas differ significantly from those of the previous formulas.

Stresses and Deflections due to Loading

Closed-Form Formulas-Edge Loading-Circular Load

According to Ioannides *et al. (1985)*, the following equations are the correct ones to use:

$$\underset{\text{(circle)}}{\sigma_e} = \frac{3(1+\nu)P}{\pi(3+\nu)h^2}\left[\ln\left(\frac{Eh^3}{100ka^4}\right) + 1.84 - \frac{4\nu}{3} + \frac{1-\nu}{2} + \frac{1.18(1+2\nu)a}{\ell}\right]$$

$$\underset{\text{(semicircle)}}{\sigma_e} = \frac{3(1+\nu)P}{\pi(3+\nu)h^2}\left[\ln\left(\frac{Eh^3}{100ka^4}\right) + 3.84 - \frac{4\nu}{3} + \frac{(1+2\nu)a}{2\ell}\right]$$

$$\underset{\text{(circle)}}{\Delta_e} = \frac{\sqrt{2+1.2\nu}P}{\sqrt{Eh^3k}}\left[1 - \frac{(0.76+0.4\nu)a}{\ell}\right]$$

$$\underset{\text{(semicircle)}}{\Delta_e} = \frac{\sqrt{2+1.2\nu}P}{\sqrt{Eh^3k}}\left[1 - \frac{(0.323+0.17\nu)a}{\ell}\right]$$

Stresses and Deflections due to Loading

Closed-Form Formulas-Edge Loading-Circular Load

For Poissons ratio of 0.15, the above equations can be written as:

$$\underset{(\text{circle})}{\sigma_e} = \frac{0.803P}{h^2}\left[4\log\left(\frac{\ell}{a}\right) + 0.666\left(\frac{a}{\ell}\right) - 0.034\right]$$

$$\underset{(\text{semicircle})}{\sigma_e} = \frac{0.803P}{h^2}\left[4\log\left(\frac{\ell}{a}\right) + 0.282\left(\frac{a}{\ell}\right) + 0.650\right]$$

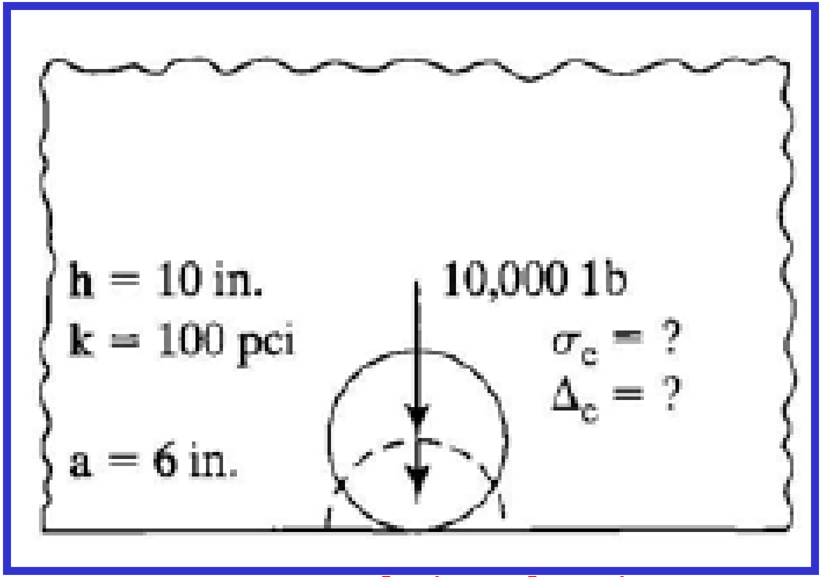
$$\underset{(\text{circle})}{\Delta_e} = \frac{0.431P}{k\ell^2}\left[1 - 0.82\left(\frac{a}{\ell}\right)\right]$$

$$\underset{(\text{semicircle})}{\Delta_e} = \frac{0.431P}{k\ell^2}\left[1 - 0.349\left(\frac{a}{\ell}\right)\right]$$

Stresses and Deflections due to Loading

Closed-Form Formulas-Edge Loading-Circular Load-Numerical Problem

For the load shown in shown in Figure, determine the maximum stress and deflection under both circular and semicircular loaded areas.



Stresses and Deflections due to Loading

Closed-Form Formulas-Edge Loading-Circular Load-Numerical Problem

$$\underset{\text{(circle)}}{\sigma_e} = \frac{0.803P}{h^2}\left[4\log\left(\frac{\ell}{a}\right) + 0.666\left(\frac{a}{\ell}\right) - 0.034\right]$$

$$\underset{\text{(semicircle)}}{\sigma_e} = \frac{0.803P}{h^2}\left[4\log\left(\frac{\ell}{a}\right) + 0.282\left(\frac{a}{\ell}\right) + 0.650\right]$$

$$\underset{\text{(circle)}}{\Delta_e} = \frac{0.431P}{k\ell^2}\left[1 - 0.82\left(\frac{a}{\ell}\right)\right]$$

$$\underset{\text{(semicircle)}}{\Delta_e} = \frac{0.431P}{k\ell^2}\left[1 - 0.349\left(\frac{a}{\ell}\right)\right]$$

Stresses and Deflections due to Loading

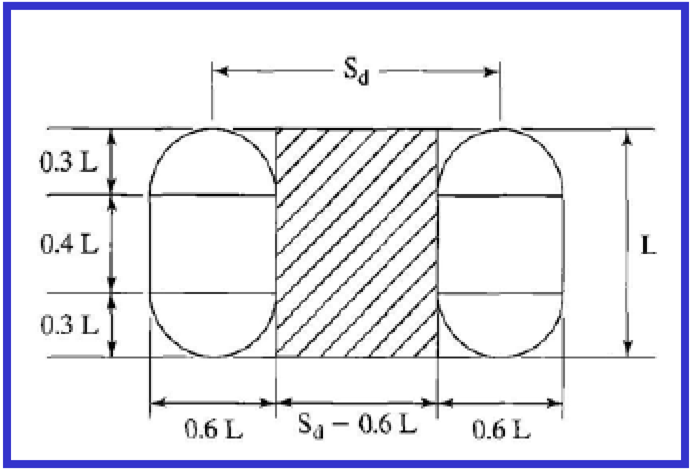
Closed-Form Formulas-Dual Tyres

With the exception of Eqs. for a semicircular loaded area, all of the closed-form formulas presented so far are based on a circular loaded area. When a load is applied over a set of dual tyres, it is necessary to convert it into a circular area, so that the equations based on a circular loaded area can be applied. If the total load is the same but the contact area of the circle is equal to that of the duals, as has been frequently assumed for flexible pavements, the resulting stresses and deflection will be too large. Therefore, for a given total load, a much larger circular area should be used for rigid pavements.

Stresses and Deflections due to Loading

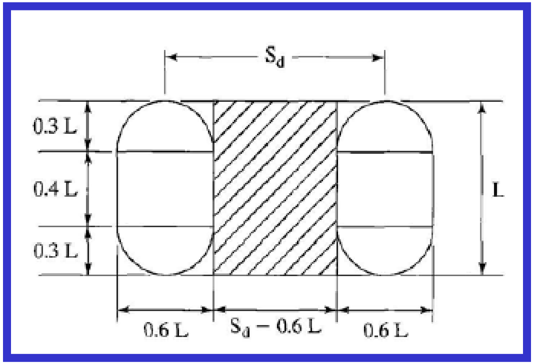
Closed-Form Formulas-Dual Tyres

Figure shows a set of dual tyres. It has been found that satisfactory results can be obtained if the circle has an area equal to the contact area of the duals plus the area between the duals, as indicated by the hatched area shown in the figure.



Stresses and Deflections due to Loading

Closed-Form Formulas-Dual Tyres



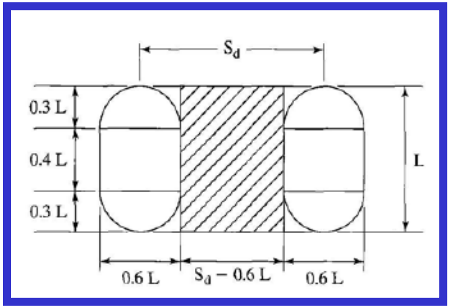
If P_d is the load on one tyre and q is the contact pressure, the area of each tyre is:

$$\frac{P_d}{q} = \pi(0.3L)^2 + (0.4L)(0.6L) = 0.5227L^2$$

$$L = \sqrt{\frac{P_d}{0.5227q}}$$

Stresses and Deflections due to Loading

Closed-Form Formulas-Dual Tyres



The area of an equivalent circle is:

$$\pi a^2 = 2 \times 0.5227 L^2 + (S_d - 0.6L)L = 0.4454 L^2 + S_d L$$

Substituting value of *L*

$$\pi a^2 = \frac{0.8521 P_d}{q} + S_d \sqrt{\frac{P_d}{0.5227 q}}$$

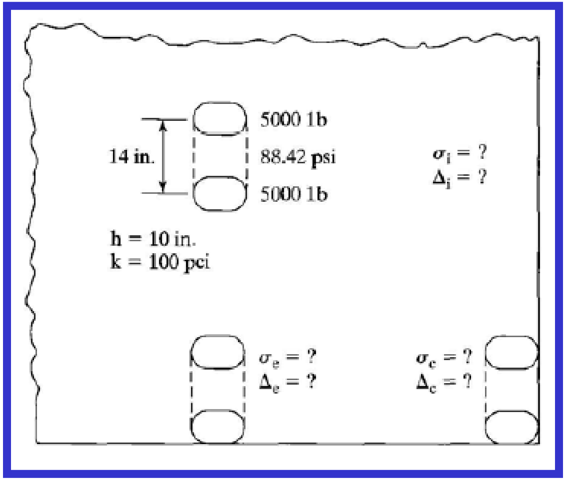
So the radius of contact area is

$$a = \sqrt{\frac{0.8521 P_d}{q\pi} + \frac{S_d}{\pi}\left(\frac{P_d}{0.5227 q}\right)^{1/2}}$$

Stresses and Deflections due to Loading

Closed-Form Formulas-Dual Tyres-Numerical Problem

Using Westergaard's formulas, determine the maximum stresses and deflections if the 10,000-lb load is applied on a set of duals spaced at 14 in. on centers, as shown in Figure, instead of over a 6 in. circular area.



Stresses and Deflections due to Loading

Closed-Form Formulas-Dual Tyres-Numerical Problem

$$a = \sqrt{\frac{0.8521 P_d}{q\pi} + \frac{S_d}{\pi}\left(\frac{P_d}{0.5227q}\right)^{1/2}}$$

$$\ell = \left[\frac{Eh^3}{12(1 - v^2)k}\right]^{0.25}$$

$$\sigma_c = \frac{3P}{h^2}\left[1 - \left(\frac{a\sqrt{2}}{\ell}\right)^{0.6}\right]$$

$$\Delta_c = \frac{P}{k\ell^2}\left[1.1 - 0.88\left(\frac{a\sqrt{2}}{\ell}\right)\right]$$

Stresses and Deflections due to Loading

Closed-Form Formulas-Dual Tyres-Numerical Problem

$$b = a \qquad \text{when } a \geq 1.724h$$

$$b = \sqrt{1.6a^2 + h^2} - 0.675h \quad \text{when } a < 1.724h$$

$$\sigma_i = \frac{0.316P}{h^2}\left[4\log\left(\frac{\ell}{b}\right) + 1.069\right]$$

$$\Delta_i = \frac{P}{8k\ell^2}\left\{1 + \frac{1}{2\pi}\left[\ln\left(\frac{a}{2\ell}\right) - 0.673\right]\left(\frac{a}{\ell}\right)^2\right\}$$

Stresses and Deflections due to Loading

Closed-Form Formulas-Dual Tyres-Numerical Problem

$$\underset{(\text{circle})}{\sigma_e} = \frac{0.803P}{h^2}\left[4\log\left(\frac{\ell}{a}\right) + 0.666\left(\frac{a}{\ell}\right) - 0.034\right]$$

$$\underset{(\text{circle})}{\Delta_e} = \frac{0.431P}{k\ell^2}\left[1 - 0.82\left(\frac{a}{\ell}\right)\right]$$

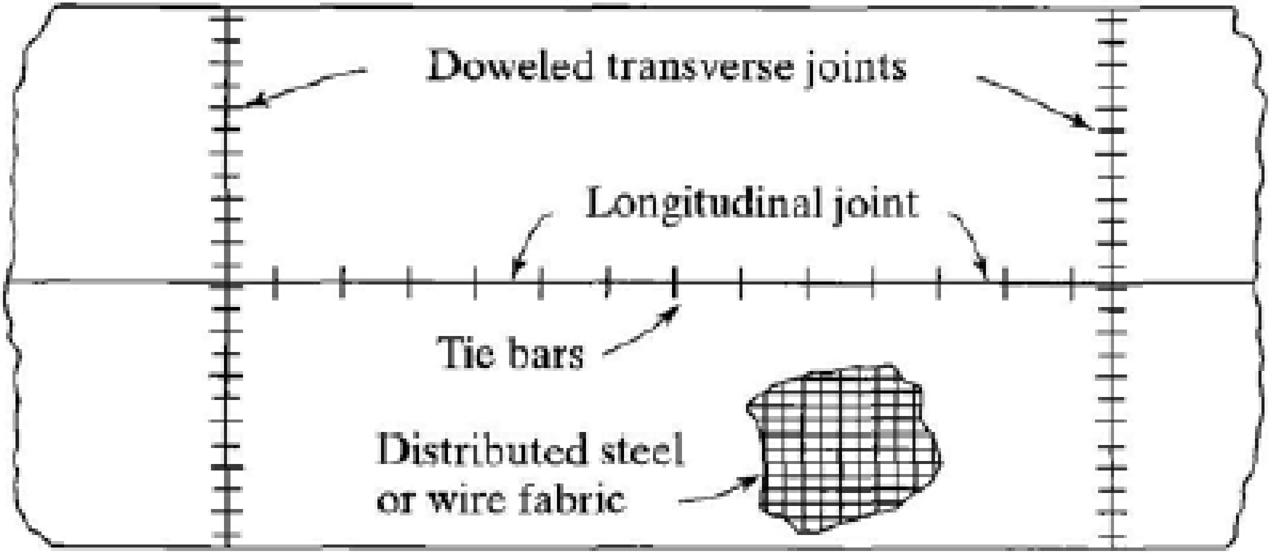
STRESSES DUE TO FRICTION

The friction between a concrete slab and its foundation causes tensile stresses in the concrete, in the steel reinforcements, if any, and in the tie bars.

For plain concrete pavements, the spacing between contraction joints must be so chosen that the stresses due to friction will not cause the concrete to crack.

For longer joint spacings, steel reinforcements must be provided to take care of the stresses caused by friction. The number of tie bars required is also controlled by the friction. Figure shows the arrangement of joints and steel in concrete pavements.

STRESSES DUE TO FRICTION



STRESSES DUE TO FRICTION-Effect of Volume Change on Concrete

The volume change caused by the variation of temperature and moisture has two important effects on concrete.

First, it induces tensile stresses and causes the concrete to crack.

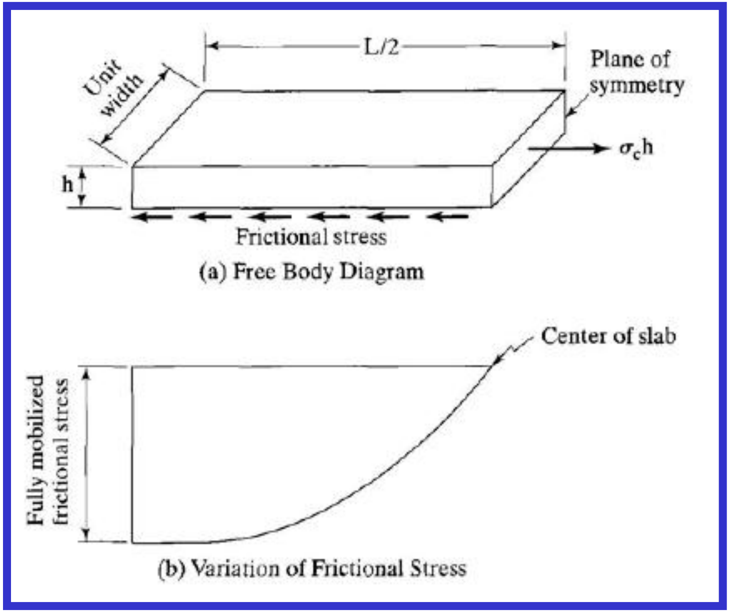
Second, it causes the joint to open and decreases the efficiency of load transfer.

STRESSES DUE TO FRICTION-Effect of Volume Change on Concrete

Concrete Stress

Figure shows a concrete pavement subject to a decrease in temperature. Due to symmetry, the slab tends to move from both ends toward the center, but the subgrade prevents it from moving; thus, frictional stresses are developed between the slab and the subgrade.

The amount of friction depends on the relative movement, being zero at the center where no movement occurs and maximum at some distance from the center where the movement is fully mobilized, as shown in Figure.



STRESSES DUE TO FRICTION-Effect of Volume Change on Concrete

Concrete Stress

For practical purposes, an average coefficient of friction f_a may be assumed. The tensile stress in the concrete is greatest at the center and can be determined by equating the frictional force per unit width of slab, $\gamma_c h L f_a /2$, to the tensile force $\sigma_c h$, as shown in Figure:

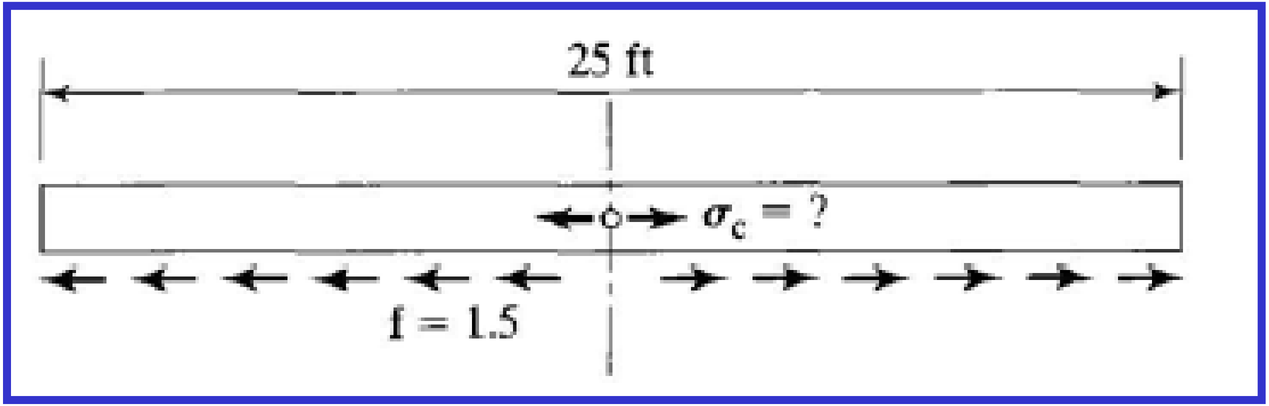
$$\sigma_c = \gamma_c L f_a /2$$

in which σ_c is the stress in the concrete, γ_c is the unit weight of the concrete, L is the length of the slab, and f_a is the average coefficient of friction between slab and subgrade, usually taken as 1.5. Equation implies that the stress in the concrete due to friction is independent of the slab thickness.

STRESSES DUE TO FRICTION-Effect of Volume Change on Concrete

Concrete Stress-Numerical problem

Given a concrete pavement with a joint spacing of 25 ft and a coefficient of friction of 1.5, as shown. Determine the stress in concrete due to friction.



STRESSES DUE TO FRICTION-Effect of Volume Change on Concrete

Concrete Stress-Numerical problem

$\gamma_c = 150\ pcf = 150/12^3 = 0.0868\ pci$

$L = 25ft = 25x12 = 300\ in.$

$f_a = 1.5$

$\sigma_c = \gamma_c L f_a /2 = 0.0868x300x1.5/2 = 19.5\ psi$

STRESSES DUE TO FRICTION

Joint Opening

The spacing of joints in plain concrete pavements depends more on the shrinkage characteristics of the concrete rather than on the stress in the concrete.

Longer joint spacings cause the joint to open wider and decrease the efficiency of load transfer. The opening of a joint can be computed approximately by (Darter and Barenberg, 1977):

$$\Delta L = C\, L\, (\alpha_t \Delta T + \varepsilon)$$

STRESSES DUE TO FRICTION-Joint Opening $\Delta L = C\,L\,(\alpha_t \Delta T + \varepsilon)$

in which ΔL is the joint opening caused by temperature change and drying shrinkage of concrete;

α_t is the coefficient of thermal expansion of concrete, generally 5 to 6 x10^{-6} /°F (9 to 10.8x10^{-6}/°C);

ε is the drying shrinkage coefficient of concrete, approximately 0.5 to 2.5x10^{-4};

L is the joint spacing or slab length;

ΔT is the temperature range, which is the temperature at placement minus the lowest mean monthly temperature; and

C is the adjustment factor due to slab-subbase friction, 0.65 for stabilized base and 0.8 for granular subbase.

STRESSES DUE TO FRICTION-Joint Opening-Numerical problem

Given ΔT = 60°F, α_t = 5.5x10^{-6}/°F, ε = 1.0x10^{-4} , C = 0.65 and the allowable joint openings for undoweled and doweled joints are 0.05 and 0.25 in. respectively, determine the maximum allowable joint spacing.

$$\Delta L = C\,L\,(\alpha_t \Delta T + \varepsilon)$$

$$L = \Delta L / C\,(\alpha_t \Delta T + \varepsilon)$$

For undoweled joints:

$L=\Delta L/C\,(\alpha_t \Delta T+ \varepsilon)=0.05/0.65(5.5\text{x}10^{-6}\text{x}60+ 1.0\text{x}10^{-4})=178.9\ in=14.9ft$

For doweled joints:

$L=\Delta L/C\,(\alpha_t \Delta T+ \varepsilon)=0.25/0.65(5.5\text{x}10^{-6}\text{x}60+ 1.0\text{x}10^{-4})=892.9in=74.4ft$

STRESSES DUE TO FRICTION-Steel stress

Steel is used in concrete pavements as reinforcements, tie bars and dowel bars.

The design of longitudinal and transverse reinforcements and of the tie bars across longitudinal joints is based on the stresses due to friction.

STRESSES DUE TO FRICTION-Steel stress

Reinforcements

Wire fabric or bar mats may be used in concrete slabs for control of temperature cracking. These reinforcements do not increase the structural capacity of the slab but are used for two purposes:

- **To increase the joint spacing and**
- **To tie the cracked concrete together and maintain load transfers through aggregate interlock.**

STRESSES DUE TO FRICTION-Steel stress

Reinforcements

$$\sigma_c = \gamma_c L f_a / 2$$

When steel reinforcements are used, it is assumed that all tensile stresses are taken by the steel alone, so $\sigma_c h$ must be replaced by $A_s f_s$ and above equation becomes:

$$A_s = \gamma_c h L f_a / 2 f_s$$

in which A_s is the area of steel required per unit width and f_s is the allowable stress in steel. This equation indicates that the amount of steel required is proportional to the length of slab.

STRESSES DUE TO FRICTION-Steel stress

Reinforcements

The steel is usually placed at the mid depth of the slab and discontinued at the joint.

The amount of steel obtained from above equation is at the center of the slab and can be reduced toward the end.

However, in actual practice the same amount of steel is used throughout the length of the slab.

STRESSES DUE TO FRICTION-Steel stress

Reinforcements

Table 4.1 gives the allowable stress for different types and grades of steel. The allowable stress is generally taken as two-thirds of the yield strength.

TABLE 4.1 Yield Strength and Allowable Stress for Steel

Type and grade of steel	Yield strength (psi)	Allowable stress (psi)
Billet steel, intermediate grade	40,000	27,000
Rail steel or hard grade of billet steel	50,000	33,000
Rail steel, special grade	60,000	40,000
Billet steel, 60,000 psi minimum yield	60,000	40,000
Cold drawn wire (smooth)	65,000	43,000
Cold drawn wire (deformed)	70,000	46,000

Note. 1 psi = 6.9 kPa.

STRESSES DUE TO FRICTION-Steel stress

Reinforcements

TABLE 4.2 Weights and Dimensions of Standard Reinforcing Bars

Bar size designation	Weight (lb/ft)	Nominal dimensions, round sections: Diameter (in.)	Cross-sectional area (in.^{2})	Perimeter (in.)
No. 3	0.376	0.375	0.11	1.178
No. 4	0.668	0.500	0.20	1.571
No. 5	1.043	0.625	0.31	1.963
No. 6	1.502	0.750	0.44	2.356
No. 7	2.044	0.875	0.60	2.749
No. 8	2.670	1.000	0.79	3.142
No. 9	3.400	1.128	1.00	3.544
No. 10	4.303	1.270	1.27	3.990
No. 11	5.313	1.410	1.56	4.430

Note. 1 in. = 25.4 mm, 1 lb = 4.45 N, 1 ft = 0.305 m.

STRESSES DUE TO FRICTION-Steel stress

Reinforcements

TABLE 4.3 Weights and Dimensions of Welded Wire Fabric

Wire size no.		Diameter	Weight	Cross-sectional area (in.^{2}/ft) center-to-center spacing (in.)						
Smooth	Deformed	(in.)	lb/ft	2	3	4	6	8	10	12
W31	D31	0.628	1.054	1.86	1.24	.93	.62	.465	.372	.31
W30	D30	0.618	1.020	1.80	1.20	.90	.60	.45	.36	.30
W28	D28	0.597	.952	1.68	1.12	.84	.56	.42	.336	.28
W26	D26	0.575	.934	1.56	1.04	.78	.52	.39	.312	.26
W24	D24	0.553	.816	1.44	.96	.72	.48	.36	.288	.24
W22	D22	0.529	.748	1.32	.88	.66	.44	.33	.264	.22
W20	D20	0.504	.680	1.20	.80	.60	.40	.30	.24	.20
W18	D18	0.478	.612	1.08	.72	.54	.36	.27	.216	.18
W16	D16	0.451	.544	.96	.64	.48	.32	.24	.192	.16
W14	D14	0.422	.476	.84	.56	.42	.28	.21	.168	.14
W12	D12	0.390	.408	.72	.48	.36	.24	.18	.144	.12
W11	D11	0.374	.374	.66	.44	.33	.22	.165	.132	.11

STRESSES DUE TO FRICTION-Steel stress-Reinforcements

TABLE 4.3 Weights and Dimensions of Welded Wire Fabric

Wire size no. Smooth	Wire size no. Deformed	Diameter (in.)	Weight lb/ft	Cross-sectional area (in.^{2}/ft) center-to-center spacing (in.) 2	3	4	6	8	10	12
W10.5		0.366	.357	.63	.42	.315	.21	.157	.126	.105
W10	D10	0.356	.340	.60	.40	.30	.20	.15	.12	.10
W9.5		0.348	.323	.57	.38	.285	.19	.142	.114	.095
W9	D9	0.338	.306	.54	.36	.27	.18	.135	.108	.09
W8.5		0.329	.289	.51	.34	.255	.17	.127	.102	.085
W8	D8	0.319	.272	.48	.32	.24	.16	.12	.096	.08
W7.5		0.309	.255	.45	.30	.225	.15	.112	.09	.075
W7	D7	0.298	.238	.42	.28	.21	.14	.105	.084	.07
W6.5		0.288	.221	.39	.26	.195	.13	.097	.078	.065
W6	D6	0.276	.204	.36	.24	.18	.12	.09	.072	.06
W5.5		0.264	.187	.33	.22	.165	.11	.082	.066	.055
W5	D5	0.252	.170	.30	.20	.15	.10	.075	.06	.05
W4.5		0.240	.153	.27	.18	.135	.09	.067	.054	.045
W4	D4	0.225	.136	.24	.16	.12	.08	.06	.048	.04

Note. Wire sizes other than those listed above may be produced provided the quantity required is sufficient to justify manufacture. 1 in. = 25.4 mm, 1 lb = 4.45 N, 1 ft = 0.305 m.

Source. After WRI (1975).

STRESSES DUE TO FRICTION-Steel stress

Reinforcements

Table 4.2 shows the weight and dimensions of reinforcing bars and Table 4.3 shows those of welded wire fabric.

Welded wire fabric is prefabricated reinforcement consisting of parallel series of high-strength, cold-drawn wires welded together in square or rectangular grids. The spacings and sizes of wires are identified by "style." A typical style designation is 6x12-W8xW6, in which the spacing of longitudinal wires is 6 in. (152 mm), the spacing of transverse wires is 12 in. (305 mm), the size of longitudinal wire is W8 with a cross-sectional area of 0.08 in^2. (51.6 mm^2) and the size of transverse wires is W6 with a cross sectional area of 0.06 in^2. (38.7 mm^2).

The typical style with deformed welded wire fabric is 6x12-D8xD6.

STRESSES DUE TO FRICTION-Steel stress-Reinforcements

The following standard practices on wire sizes, spacings, laps and clearances are recommended by the Wire Reinforcement Institute (WRI, 1975):

1. Because the fabric is subjected to bending stresses as well as tensile stresses at cracks, neither the longitudinal nor the transverse wires should be less than W4 or D4 .

2. To provide generous opening between wires to permit placement and vibration of concrete, the minimum spacing between wires should not be less than 4 in. (102 mm). The maximum spacing should not be greater than 12 in. (305 mm) between longitudinal wires and 24 in. (610 mm) between transverse wires.

STRESSES DUE TO FRICTION-Steel stress

Reinforcements

The following standard practices on wire sizes, spacings, laps and clearances are recommended by the Wire Reinforcement Institute (WRI, 1975):

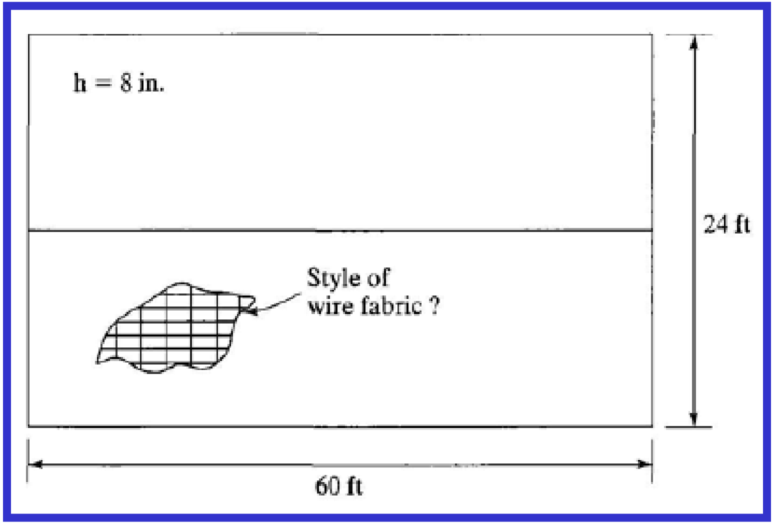
3. Because the dimensions of a concrete slab are usually greater than those of the welded wire fabric, the fabric should be installed with end and side laps. The end lap should be about 30 times the longitudinal wire diameter but not less than 12 in. (305 mm). The side laps should be about 20 times the transverse wire diameter but not less than 6 in. (152 mm).

4. The fabric should extend to about 2 in. (51 mm) but not more than 6 in. (152 mm) from the slab edges. The depth from the top of slab should not be less than 2.5 in. (64 mm) or more than mid depth.

STRESSES DUE TO FRICTION-Steel stress

Reinforcements-Numerical problem

Determine the wire fabric required for a two-lane concrete pavement, 8 in. thick, 60 ft long and 24 ft wide, with a longitudinal joint at the center, as shown.



STRESSES DUE TO FRICTION-Steel stress-Reinforcements-Numerical problem

$\gamma_c = 150$ *pcf* $= 150/12^3 = 0.0868$ *pci*

L=60ft =60x12=720in.

W=24ft=24x12=288in.

h = 8in.

$f_a = 1.5$

$f_s = 43{,}000$ *psi*

Longitudinal steel:

$A_s = \gamma_c h L f_a / 2 f_s = 0.0868 x 8 x 720 x 1.5/(2x43{,}000)$

$= 0.00872\ in^2/in. = 0.105\ in^2/ft.$

Transverse steel:

$A_s = \gamma_c h L f_a / 2 f_s = 0.0868 x 8 x 288 x 1.5/(2x43{,}000)$

$= 0.00349\ in^2/in. = 0.042\ in^2/ft.$

From Table 4.3 use 6x12-W5.5xW4.5 with steel area 0.11 in^2/ft. for longitudinal wires and 0.045 in^2/ft. for transverse wires.

STRESSES DUE TO FRICTION-Steel stress-Reinforcements

TABLE 4.3 Weights and Dimensions of Welded Wire Fabric

Wire size no.				Cross-sectional area ($in.^2/ft$) center-to-center spacing (in.)						
Smooth	Deformed	Diameter (in.)	Weight lb/ft	2	3	4	6	8	10	12
W10.5		0.366	.357	.63	.42	.315	.21	.157	.126	.105
W10	D10	0.356	.340	.60	.40	.30	.20	.15	.12	.10
W9.5		0.348	.323	.57	.38	.285	.19	.142	.114	.095
W9	D9	0.338	.306	.54	.36	.27	.18	.135	.108	.09
W8.5		0.329	.289	.51	.34	.255	.17	.127	.102	.085
W8	D8	0.319	.272	.48	.32	.24	.16	.12	.096	.08
W7.5		0.309	.255	.45	.30	.225	.15	.112	.09	.075
W7	D7	0.298	.238	.42	.28	.21	.14	.105	.084	.07
W6.5		0.288	.221	.39	.26	.195	.13	.097	.078	.065
W6	D6	0.276	.204	.36	.24	.18	.12	.09	.072	.06
W5.5		0.264	.187	.33	.22	.165	.11	.082	.066	.055
W5	D5	0.252	.170	.30	.20	.15	.10	.075	.06	.05
W4.5		0.240	.153	.27	.18	.135	.09	.067	.054	.045
W4	D4	0.225	.136	.24	.16	.12	.08	.06	.048	.04

Note. Wire sizes other than those listed above may be produced provided the quantity required is sufficient to justify manufacture. 1 in. = 25.4 mm, 1 lb = 4.45 N, 1 ft = 0.305 m.

Source. After WRI (1975).