# Pavement Analysis and Design

**TE-503/TE-503A**

**Lecture-3**
**23-09-2019**

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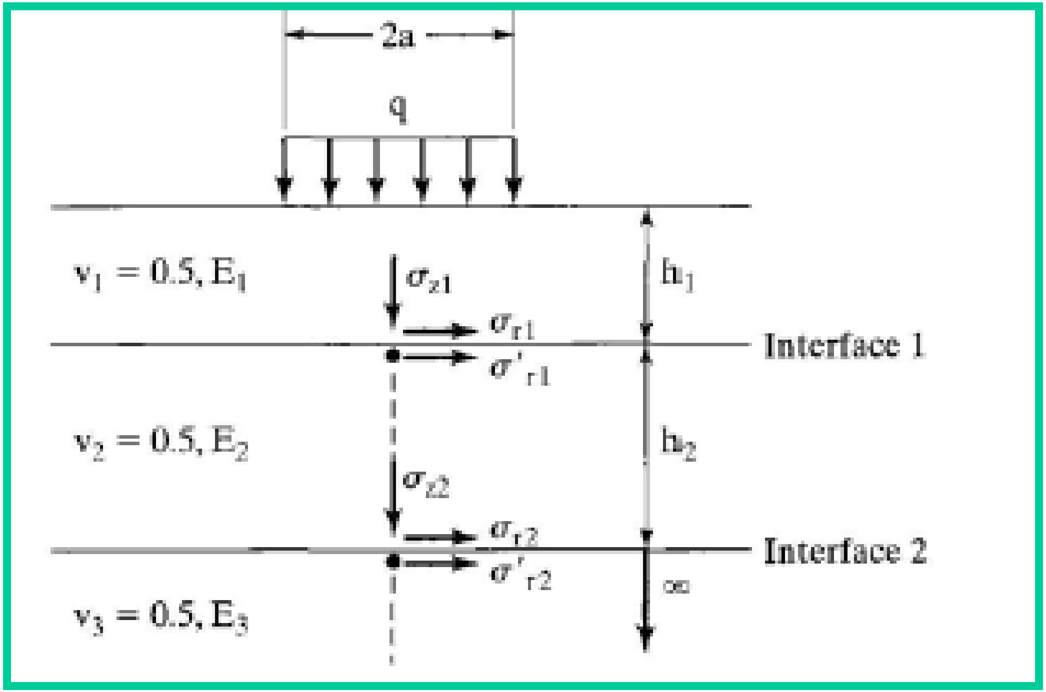
**DTEM**

# Stresses and Strains in Flexible Pavements

## LAYERED SYSTEMS-Three-Layer Systems



# LAYERED SYSTEMS-Three-Layer Systems

## Jones' Tables

**The stresses in a three-layer system depend on the ratios $k_1$, $k_2$, $A$ and $H$ defined as:**

$$k_1 = \frac{E_1}{E_2} \qquad k_2 = \frac{E_2}{E_3}$$

$$A = \frac{a}{h_2} \qquad H = \frac{h_1}{h_2}$$

# LAYERED SYSTEMS-Three-Layer Systems

## Jones' Tables

**The continuity of horizontal displacement at the interface implies that the radial strains at the bottom of one layer are equal to that at the top of the next layer, or,**

$$\sigma_{z1} - \sigma'_{r1} = \frac{\sigma_{z1} - \sigma_{r1}}{k_1}$$

$$\sigma_{z2} - \sigma'_{r2} = \frac{\sigma_{z2} - \sigma_{r2}}{k_2}$$

## LAYERED SYSTEMS-Three-Layer Systems

### Jones' Tables

**The tables presented by Jones consist of four values of $k_1$ and $k_2$ (0.2, 2, 20, and 200), so solutions for intermediate values of $k_1$ and $k_2$ can be obtained by interpolation.**

**The interpolation from the tables requires a large amount of time and effort.**

# LAYERED SYSTEMS-Three-Layer Systems

## Jones' Tables

**Table presents the stress factors for three-layer systems. The sign convention is positive in compression and negative in tension.**

**Four sets of stress factors, ZZ1, ZZ2, ZZ1-RR1 and ZZ2-RR2 are shown. The product of the contact pressure and the stress factors gives the stresses:**

$$\sigma_{z1} = q\,(\mathrm{ZZ1})$$

$$\sigma_{z2} = q\,(\mathrm{ZZ2})$$

$$\sigma_{z1} - \sigma_{r1} = q\,(\mathrm{ZZ1} - \mathrm{RR1})$$

$$\sigma_{z2} - \sigma_{r2} = q\,(\mathrm{ZZ2} - \mathrm{RR2})$$

# Stresses and Strains in Flexible Pavements

## LAYERED SYSTEMS-Three-Layer Systems-Jones' Tables

TABLE 2.3 Stress Factors for Three-Layer Systems

| | | | $k_1 = 2$ | | | | $k_1 = 20$ | | | | $k_1 = 200$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $H$ | $k_2$ | $A$ | ZZ1 | ZZ2 | (ZZ1 − RR1) | (ZZ2 − RR2) | ZZ1 | ZZ2 | (ZZ1 − RR1) | (ZZ2 − RR2) | ZZ1 | ZZ2 | (ZZ1 − RR1) | (ZZ2 − RR2) |
| 0.125 | 2 | 0.1 | 0.42950 | 0.00896 | 0.70622 | 0.01716 | 0.14529 | 0.00810 | 1.81178 | 0.01542 | 0.03481 | 0.00549 | 3.02259 | 0.00969 |
| | | 0.2 | 0.78424 | 0.03493 | 0.97956 | 0.06647 | 0.38799 | 0.03170 | 3.76886 | 0.06003 | 0.11491 | 0.02167 | 8.02452 | 0.03812 |
| | | 0.4 | 0.98044 | 0.12667 | 0.70970 | 0.23531 | 0.78651 | 0.11650 | 5.16717 | 0.21640 | 0.33218 | 0.08229 | 17.64175 | 0.14286 |
| | | 0.8 | 0.99434 | 0.36932 | 0.22319 | 0.63003 | 1.02218 | 0.34941 | 3.43631 | 0.60493 | 0.72695 | 0.27307 | 27.27701 | 0.45208 |
| | | 1.6 | 0.99364 | 0.72113 | −0.19982 | 0.97707 | 0.99060 | 0.69014 | 1.15211 | 0.97146 | 1.00203 | 0.63916 | 23.38638 | 0.90861 |
| | | 3.2 | 0.99922 | 0.96148 | −0.28916 | 0.84030 | 0.99893 | 0.93487 | −0.06894 | 0.88358 | 1.00828 | 0.92560 | 11.87014 | 0.91469 |
| | 20 | 0.1 | 0.43022 | 0.00228 | 0.69332 | 0.03467 | 0.14447 | 0.00182 | 1.80664 | 0.02985 | 0.03336 | 0.00128 | 3.17763 | 0.01980 |
| | | 0.2 | 0.78414 | 0.00899 | 0.92086 | 0.13541 | 0.38469 | 0.00716 | 3.74573 | 0.11697 | 0.10928 | 0.00509 | 8.66097 | 0.07827 |
| | | 0.4 | 0.97493 | 0.03392 | 0.46583 | 0.49523 | 0.77394 | 0.02710 | 5.05489 | 0.43263 | 0.31094 | 0.01972 | 20.12259 | 0.29887 |
| | | 0.8 | 0.97806 | 0.11350 | −0.66535 | 1.49612 | 0.98610 | 0.09061 | 2.92533 | 1.33736 | 0.65934 | 0.07045 | 36.29943 | 1.01694 |
| | | 1.6 | 0.96921 | 0.31263 | −2.82859 | 3.28512 | 0.93712 | 0.24528 | −1.27093 | 2.99215 | 0.87931 | 0.20963 | 49.40857 | 2.64313 |
| | | 3.2 | 0.98591 | 0.68433 | −5.27906 | 5.05952 | 0.96330 | 0.55490 | −7.35384 | 5.06489 | 0.93309 | 0.49938 | 57.84369 | 4.89895 |
| 0.25 | 2 | 0.1 | 0.15524 | 0.00710 | 0.28362 | 0.01353 | 0.04381 | 0.00530 | 0.63215 | 0.00962 | 0.00909 | 0.00259 | 0.96553 | 0.00407 |
| | | 0.2 | 0.42809 | 0.02783 | 0.70225 | 0.05278 | 0.14282 | 0.02091 | 1.83766 | 0.03781 | 0.03269 | 0.01027 | 3.10763 | 0.01611 |
| | | 0.4 | 0.77939 | 0.10306 | 0.96634 | 0.19178 | 0.37882 | 0.07933 | 3.86779 | 0.14159 | 0.10684 | 0.04000 | 8.37852 | 0.06221 |
| | | 0.8 | 0.96703 | 0.31771 | 0.66885 | 0.55211 | 0.75904 | 0.26278 | 5.50796 | 0.44710 | 0.30477 | 0.14513 | 18.95534 | 0.21860 |
| | | 1.6 | 0.98156 | 0.66753 | 0.17331 | 0.95080 | 0.98743 | 0.61673 | 4.24281 | 0.90115 | 0.66786 | 0.42940 | 31.18909 | 0.58553 |
| | | 3.2 | 0.99840 | 0.93798 | −0.05691 | 0.89390 | 1.00064 | 0.91258 | 1.97494 | 0.93254 | 0.98447 | 0.84545 | 28.98500 | 0.89191 |
| | 20 | 0.1 | 0.15436 | 0.00179 | 0.25780 | 0.02728 | 0.04236 | 0.00123 | 0.65003 | 0.01930 | 0.00776 | 0.00065 | 1.08738 | 0.00861 |
| | | 0.2 | 0.42462 | 0.00706 | 0.67115 | 0.10710 | 0.13708 | 0.00488 | 1.90693 | 0.07623 | 0.02741 | 0.00257 | 3.59448 | 0.03421 |
| | | 0.4 | 0.76647 | 0.02697 | 0.84462 | 0.39919 | 0.35716 | 0.01888 | 4.13976 | 0.29072 | 0.08634 | 0.01014 | 10.30923 | 0.13365 |
| | | 0.8 | 0.92757 | 0.09285 | 0.21951 | 1.26565 | 0.68947 | 0.06741 | 6.48948 | 0.98565 | 0.23137 | 0.03844 | 26.41442 | 0.49135 |
| | | 1.6 | 0.91393 | 0.26454 | −1.22411 | 2.94860 | 0.85490 | 0.20115 | 6.95639 | 2.55231 | 0.46835 | 0.13148 | 57.46409 | 1.53833 |
| | | 3.2 | 0.95243 | 0.60754 | −3.04320 | 4.89878 | 0.90325 | 0.48647 | 6.05854 | 4.76234 | 0.71083 | 0.37342 | 99.29034 | 3.60964 |
| 0.5 | 2 | 0.1 | 0.04330 | 0.00465 | 0.08250 | 0.00878 | 0.01122 | 0.00259 | 0.17997 | 0.00440 | 0.00215 | 0.00094 | 0.26620 | 0.00128 |
| | | 0.2 | 0.15325 | 0.01836 | 0.28318 | 0.03454 | 0.04172 | 0.01028 | 0.64779 | 0.01744 | 0.00826 | 0.00373 | 0.98772 | 0.00509 |
| | | 0.4 | 0.42077 | 0.06974 | 0.70119 | 0.12954 | 0.13480 | 0.03998 | 1.89817 | 0.06722 | 0.02946 | 0.01474 | 3.19580 | 0.01996 |
| | | 0.8 | 0.75683 | 0.23256 | 0.96681 | 0.41187 | 0.35175 | 0.14419 | 4.09592 | 0.23476 | 0.09508 | 0.05622 | 8.71973 | 0.07434 |
| | | 1.6 | 0.93447 | 0.56298 | 0.70726 | 0.85930 | 0.70221 | 0.42106 | 6.22002 | 0.62046 | 0.27135 | 0.19358 | 20.15765 | 0.23838 |
| | | 3.2 | 0.98801 | 0.88655 | 0.33878 | 0.96353 | 0.97420 | 0.82256 | 5.41828 | 0.93831 | 0.62399 | 0.52912 | 34.25229 | 0.54931 |
| | 20 | 0.1 | 0.04193 | 0.00117 | 0.08044 | 0.01778 | 0.00990 | 0.00063 | 0.19872 | 0.00911 | 0.00149 | 0.00023 | 0.31847 | 0.00257 |
| | | 0.2 | 0.14808 | 0.00464 | 0.27574 | 0.07027 | 0.03648 | 0.00251 | 0.72264 | 0.03620 | 0.00564 | 0.00094 | 1.19598 | 0.01025 |
| | | 0.4 | 0.40086 | 0.01799 | 0.67174 | 0.26817 | 0.11448 | 0.00988 | 2.19520 | 0.14116 | 0.01911 | 0.00372 | 1.02732 | 0.04047 |
| | | 0.8 | 0.69098 | 0.06476 | 0.86191 | 0.91168 | 0.27934 | 0.03731 | 5.24726 | 0.51585 | 0.05574 | 0.01453 | 12.00885 | 0.15452 |
| | | 1.6 | 0.79338 | 0.19803 | 0.39588 | 2.38377 | 0.50790 | 0.12654 | 10.30212 | 1.59341 | 0.13946 | 0.05399 | 32.77028 | 0.53836 |
| | | 3.2 | 0.85940 | 0.49238 | −0.41078 | 4.47022 | 0.70903 | 0.35807 | 16.38520 | 3.69109 | 0.30247 | 0.18091 | 77.62943 | 1.56409 |

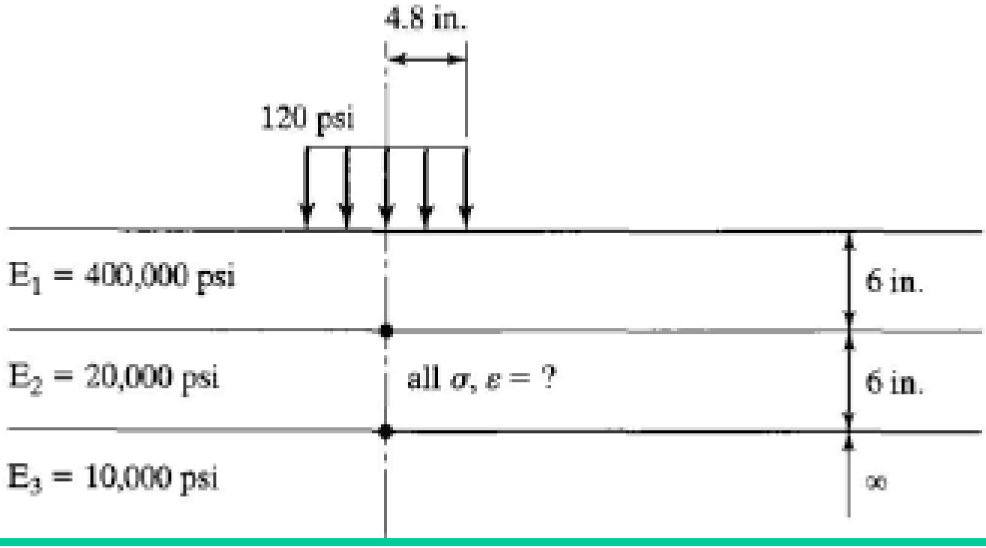
# LAYERED SYSTEMS-Three-Layer Systems-Jones' Tables

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 0.1 | 0.01083 | 0.00241 | 0.02179 | 0.00453 | 0.00263 | 0.00100 | 0.04751 | 0.00160 | 0.00049 | 0.00029 | 0.06883 | 0.00035 |
| | | 0.2 | 0.04176 | 0.00958 | 0.08337 | 0.01797 | 0.01029 | 0.00347 | 0.18481 | 0.00637 | 0.00195 | 0.00116 | 0.26966 | 0.00138 |
| | | 0.4 | 0.14665 | 0.03724 | 0.28491 | 0.06934 | 0.03810 | 0.01565 | 0.66727 | 0.02498 | 0.00746 | 0.00460 | 1.00131 | 0.00545 |
| | | 0.8 | 0.39942 | 0.13401 | 0.71341 | 0.24250 | 0.12173 | 0.05938 | 1.97428 | 0.09268 | 0.02647 | 0.01797 | 3.24971 | 0.02092 |
| | | 1.6 | 0.71032 | 0.38690 | 1.02680 | 0.63631 | 0.31575 | 0.20098 | 4.37407 | 0.29253 | 0.08556 | 0.06671 | 8.92442 | 0.07335 |
| | | 3.2 | 0.92112 | 0.75805 | 0.90482 | 0.97509 | 0.66041 | 0.53398 | 6.97695 | 0.65446 | 0.25186 | 0.22047 | 20.83387 | 0.21288 |
| | 20 | 0.1 | 0.00963 | 0.00061 | 0.02249 | 0.00920 | 0.00193 | 0.00024 | 0.05737 | 0.00322 | 0.00027 | 0.00007 | 0.08469 | 0.00062 |
| | | 0.2 | 0.03697 | 0.00241 | 0.08618 | 0.03654 | 0.00751 | 0.00098 | 0.22418 | 0.01283 | 0.00104 | 0.00028 | 0.33312 | 0.00248 |
| | | 0.4 | 0.12805 | 0.00950 | 0.29640 | 0.14241 | 0.02713 | 0.00387 | 0.82430 | 0.05063 | 0.00384 | 0.00110 | 1.25495 | 0.00985 |
| | | 0.8 | 0.33263 | 0.03578 | 0.76292 | 0.51815 | 0.08027 | 0.01507 | 2.59672 | 0.19267 | 0.01236 | 0.00436 | 4.26100 | 0.03825 |
| | | 1.6 | 0.52721 | 0.12007 | 1.25168 | 1.56503 | 0.17961 | 0.05549 | 6.77014 | 0.66326 | 0.03379 | 0.01683 | 12.91809 | 0.13989 |
| | | 3.2 | 0.65530 | 0.33669 | 1.70723 | 3.51128 | 0.34355 | 0.18344 | 15.23252 | 1.88634 | 0.08859 | 0.06167 | 36.04291 | 0.45544 |
| 2 | 2 | 0.1 | 0.00250 | 0.00100 | 0.00555 | 0.00188 | 0.00059 | 0.00033 | 0.01219 | 0.00051 | 0.00011 | 0.00008 | 0.01737 | 0.00009 |
| | | 0.2 | 0.00991 | 0.00397 | 0.02199 | 0.00750 | 0.00235 | 0.00130 | 0.04843 | 0.00203 | 0.00045 | 0.00033 | 0.06913 | 0.00036 |
| | | 0.4 | 0.03832 | 0.01569 | 0.08465 | 0.02950 | 0.00922 | 0.00518 | 0.18857 | 0.00803 | 0.00179 | 0.00131 | 0.27103 | 0.00142 |
| | | 0.8 | 0.13516 | 0.05974 | 0.29365 | 0.11080 | 0.03412 | 0.02023 | 0.68382 | 0.03093 | 0.00685 | 0.00520 | 1.00808 | 0.00553 |
| | | 1.6 | 0.36644 | 0.20145 | 0.75087 | 0.35515 | 0.10918 | 0.07444 | 2.04134 | 0.10864 | 0.02441 | 0.02003 | 3.27590 | 0.02043 |
| | | 3.2 | 0.67384 | 0.51156 | 1.17294 | 0.77434 | 0.29183 | 0.23852 | 4.60426 | 0.30709 | 0.08061 | 0.07248 | 9.02195 | 0.06638 |
| | 20 | 0.1 | 0.00181 | 0.00025 | 0.00652 | 0.00378 | 0.00033 | 0.00008 | 0.01568 | 0.00094 | 0.00005 | 0.00002 | 0.02160 | 0.00014 |
| | | 0.2 | 0.00716 | 0.00099 | 0.02586 | 0.01507 | 0.00130 | 0.00031 | 0.06236 | 0.00374 | 0.00018 | 0.00007 | 0.08604 | 0.00058 |
| | | 0.4 | 0.02746 | 0.00394 | 0.10017 | 0.05958 | 0.00508 | 0.00123 | 0.24425 | 0.01486 | 0.00071 | 0.00030 | 0.33866 | 0.00229 |
| | | 0.8 | 0.09396 | 0.01535 | 0.35641 | 0.22795 | 0.01782 | 0.00485 | 0.90594 | 0.05789 | 0.00261 | 0.00119 | 1.27835 | 0.00901 |
| | | 1.6 | 0.23065 | 0.05599 | 1.00785 | 0.78347 | 0.05012 | 0.01862 | 2.91994 | 0.21190 | 0.00819 | 0.00467 | 4.35311 | 0.03390 |
| | | 3.2 | 0.37001 | 0.17843 | 2.16033 | 2.13215 | 0.11331 | 0.06728 | 7.95104 | 0.67732 | 0.02341 | 0.01784 | 13.26873 | 0.11666 |
| 4 | 2 | 0.1 | 0.00057 | 0.00034 | 0.00147 | 0.00065 | 0.00013 | 0.00010 | 0.00312 | 0.00015 | 0.00003 | 0.00002 | 0.00437 | 0.00002 |
| | | 0.2 | 0.00228 | 0.00137 | 0.00587 | 0.00260 | 0.00054 | 0.00039 | 0.01245 | 0.00029 | 0.00011 | 0.00009 | 0.01746 | 0.00009 |
| | | 0.4 | 0.00905 | 0.00544 | 0.02324 | 0.01032 | 0.00214 | 0.00154 | 0.04944 | 0.00235 | 0.00042 | 0.00036 | 0.06947 | 0.00036 |
| | | 0.8 | 0.03500 | 0.02135 | 0.08957 | 0.04031 | 0.00837 | 0.00610 | 0.19247 | 0.00924 | 0.00168 | 0.00142 | 0.27221 | 0.00144 |
| | | 1.6 | 0.12354 | 0.07972 | 0.31215 | 0.14735 | 0.03109 | 0.02358 | 0.69749 | 0.03488 | 0.00646 | 0.00560 | 1.01140 | 0.00553 |
| | | 3.2 | 0.34121 | 0.25441 | 0.81908 | 0.43632 | 0.10140 | 0.08444 | 2.09049 | 0.11553 | 0.02332 | 0.02126 | 3.28913 | 0.01951 |
| | 20 | 0.1 | 0.00030 | 0.00008 | 0.00201 | 0.00128 | 0.00005 | 0.00002 | 0.00413 | 0.00025 | 0.00001 | 0.00000 | 0.00545 | 0.00003 |
| | | 0.2 | 0.00119 | 0.00034 | 0.00803 | 0.00510 | 0.00021 | 0.00009 | 0.01651 | 0.00099 | 0.00003 | 0.00002 | 0.02178 | 0.00014 |
| | | 0.4 | 0.00469 | 0.00134 | 0.03191 | 0.02032 | 0.00083 | 0.00035 | 0.06569 | 0.00396 | 0.00013 | 0.00008 | 0.08673 | 0.00054 |
| | | 0.8 | 0.01790 | 0.00532 | 0.12427 | 0.07991 | 0.00321 | 0.00138 | 0.25739 | 0.01565 | 0.00050 | 0.00031 | 0.34131 | 0.00215 |
| | | 1.6 | 0.06045 | 0.02049 | 0.45100 | 0.29991 | 0.01130 | 0.00542 | 0.95622 | 0.05993 | 0.00186 | 0.00124 | 1.28773 | 0.00833 |
| | | 3.2 | 0.14979 | 0.07294 | 1.36427 | 0.97701 | 0.03258 | 0.02061 | 3.10980 | 0.20906 | 0.00612 | 0.00483 | 4.38974 | 0.03010 |

*Source.* After Jones (1962).

## LAYERED SYSTEMS-Three-Layer Systems-Jones' Tables-Numerical Problem

**Given the three-layer system shown in figure, determine all the stresses and strains at the two interfaces on the axis of symmetry.**



# Stresses and Strains in Flexible Pavements

## LAYERED SYSTEMS-Three-Layer Systems-Jones' Tables

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 0.1 | 0.01083 | 0.00241 | 0.02179 | 0.00453 | 0.00263 | 0.00100 | 0.04751 | 0.00160 | 0.00049 | 0.00029 | 0.06883 | 0.00035 |
| | | 0.2 | 0.04176 | 0.00958 | 0.08337 | 0.01797 | 0.01029 | 0.00347 | 0.18481 | 0.00637 | 0.00195 | 0.00116 | 0.26966 | 0.00138 |
| | | 0.4 | 0.14665 | 0.03724 | 0.28491 | 0.06934 | 0.03810 | 0.01565 | 0.66727 | 0.02498 | 0.00746 | 0.00460 | 1.00131 | 0.00545 |
| | | 0.8 | 0.39942 | 0.13401 | 0.71341 | 0.24250 | 0.12173 | 0.05938 | 1.97428 | 0.09268 | 0.02647 | 0.01797 | 3.24971 | 0.02092 |
| | | 1.6 | 0.71032 | 0.38690 | 1.02680 | 0.63631 | 0.31575 | 0.20098 | 4.37407 | 0.29253 | 0.08556 | 0.06671 | 8.92442 | 0.07335 |
| | | 3.2 | 0.92112 | 0.75805 | 0.90482 | 0.97509 | 0.66041 | 0.53398 | 6.97695 | 0.65446 | 0.25186 | 0.22047 | 20.83387 | 0.21288 |
| | 20 | 0.1 | 0.00963 | 0.00061 | 0.02249 | 0.00920 | 0.00193 | 0.00024 | 0.05737 | 0.00322 | 0.00027 | 0.00007 | 0.08469 | 0.00062 |
| | | 0.2 | 0.03697 | 0.00241 | 0.08618 | 0.03654 | 0.00751 | 0.00098 | 0.22418 | 0.01283 | 0.00104 | 0.00028 | 0.33312 | 0.00248 |
| | | 0.4 | 0.12805 | 0.00950 | 0.29640 | 0.14241 | 0.02713 | 0.00387 | 0.82430 | 0.05063 | 0.00384 | 0.00110 | 1.25495 | 0.00985 |
| | | 0.8 | 0.33263 | 0.03578 | 0.76292 | 0.51815 | 0.08027 | 0.01507 | 2.59672 | 0.19267 | 0.01236 | 0.00436 | 4.26100 | 0.03825 |
| | | 1.6 | 0.52721 | 0.12007 | 1.25168 | 1.56503 | 0.17961 | 0.05549 | 6.77014 | 0.66326 | 0.03379 | 0.01683 | 12.91809 | 0.13989 |
| | | 3.2 | 0.65530 | 0.33669 | 1.70723 | 3.51128 | 0.34355 | 0.18344 | 15.23252 | 1.88634 | 0.08859 | 0.06167 | 36.04291 | 0.45544 |
| 2 | 2 | 0.1 | 0.00250 | 0.00100 | 0.00555 | 0.00188 | 0.00059 | 0.00033 | 0.01219 | 0.00051 | 0.00011 | 0.00008 | 0.01737 | 0.00009 |
| | | 0.2 | 0.00991 | 0.00397 | 0.02199 | 0.00750 | 0.00235 | 0.00130 | 0.04843 | 0.00203 | 0.00045 | 0.00033 | 0.06913 | 0.00036 |
| | | 0.4 | 0.03832 | 0.01569 | 0.08465 | 0.02950 | 0.00922 | 0.00518 | 0.18857 | 0.00803 | 0.00179 | 0.00131 | 0.27103 | 0.00142 |
| | | 0.8 | 0.13516 | 0.05974 | 0.29365 | 0.11080 | 0.03412 | 0.02023 | 0.68382 | 0.03093 | 0.00685 | 0.00520 | 1.00808 | 0.00553 |
| | | 1.6 | 0.36644 | 0.20145 | 0.75087 | 0.35515 | 0.10918 | 0.07444 | 2.04134 | 0.10864 | 0.02441 | 0.02003 | 3.27590 | 0.02043 |
| | | 3.2 | 0.67384 | 0.51156 | 1.17294 | 0.77434 | 0.29183 | 0.23852 | 4.60426 | 0.30709 | 0.08061 | 0.07248 | 9.02195 | 0.06638 |
| | 20 | 0.1 | 0.00181 | 0.00025 | 0.00652 | 0.00378 | 0.00033 | 0.00008 | 0.01568 | 0.00094 | 0.00005 | 0.00002 | 0.02160 | 0.00014 |
| | | 0.2 | 0.00716 | 0.00099 | 0.02586 | 0.01507 | 0.00130 | 0.00031 | 0.06236 | 0.00374 | 0.00018 | 0.00007 | 0.08604 | 0.00058 |
| | | 0.4 | 0.02746 | 0.00394 | 0.10017 | 0.05958 | 0.00508 | 0.00123 | 0.24425 | 0.01486 | 0.00071 | 0.00030 | 0.33866 | 0.00229 |
| | | 0.8 | 0.09396 | 0.01535 | 0.35641 | 0.22795 | 0.01782 | 0.00485 | 0.90594 | 0.05789 | 0.00261 | 0.00119 | 1.27835 | 0.00901 |
| | | 1.6 | 0.23065 | 0.05599 | 1.00785 | 0.78347 | 0.05012 | 0.01862 | 2.91994 | 0.21190 | 0.00819 | 0.00467 | 4.35311 | 0.03390 |
| | | 3.2 | 0.37001 | 0.17843 | 2.16033 | 2.13215 | 0.11331 | 0.06728 | 7.95104 | 0.67732 | 0.02341 | 0.01784 | 13.26873 | 0.11666 |
| 4 | 2 | 0.1 | 0.00057 | 0.00034 | 0.00147 | 0.00065 | 0.00013 | 0.00010 | 0.00312 | 0.00015 | 0.00003 | 0.00002 | 0.00437 | 0.00002 |
| | | 0.2 | 0.00228 | 0.00137 | 0.00587 | 0.00260 | 0.00054 | 0.00039 | 0.01245 | 0.00029 | 0.00011 | 0.00009 | 0.01746 | 0.00009 |
| | | 0.4 | 0.00905 | 0.00544 | 0.02324 | 0.01032 | 0.00214 | 0.00154 | 0.04944 | 0.00235 | 0.00042 | 0.00036 | 0.06947 | 0.00036 |
| | | 0.8 | 0.03500 | 0.02135 | 0.08957 | 0.04031 | 0.00837 | 0.00610 | 0.19247 | 0.00924 | 0.00168 | 0.00142 | 0.27221 | 0.00144 |
| | | 1.6 | 0.12354 | 0.07972 | 0.31215 | 0.14735 | 0.03109 | 0.02358 | 0.69749 | 0.03488 | 0.00646 | 0.00560 | 1.01140 | 0.00553 |
| | | 3.2 | 0.34121 | 0.25441 | 0.81908 | 0.43632 | 0.10140 | 0.08444 | 2.09049 | 0.11553 | 0.02332 | 0.02126 | 3.28913 | 0.01951 |
| | 20 | 0.1 | 0.00030 | 0.00008 | 0.00201 | 0.00128 | 0.00005 | 0.00002 | 0.00413 | 0.00025 | 0.00001 | 0.00000 | 0.00545 | 0.00003 |
| | | 0.2 | 0.00119 | 0.00034 | 0.00803 | 0.00510 | 0.00021 | 0.00009 | 0.01651 | 0.00099 | 0.00003 | 0.00002 | 0.02178 | 0.00014 |
| | | 0.4 | 0.00469 | 0.00134 | 0.03191 | 0.02032 | 0.00083 | 0.00035 | 0.06569 | 0.00396 | 0.00013 | 0.00008 | 0.08673 | 0.00054 |
| | | 0.8 | 0.01790 | 0.00532 | 0.12427 | 0.07991 | 0.00321 | 0.00138 | 0.25739 | 0.01565 | 0.00050 | 0.00031 | 0.34131 | 0.00215 |
| | | 1.6 | 0.06045 | 0.02049 | 0.45100 | 0.29991 | 0.01130 | 0.00542 | 0.95622 | 0.05993 | 0.00186 | 0.00124 | 1.28773 | 0.00833 |
| | | 3.2 | 0.14979 | 0.07294 | 1.36427 | 0.97701 | 0.03258 | 0.02061 | 3.10980 | 0.20906 | 0.00612 | 0.00483 | 4.38974 | 0.03010 |

*Source.* After Jones (1962).

## LAYERED SYSTEMS-Three-Layer Systems

### Peattie's Charts

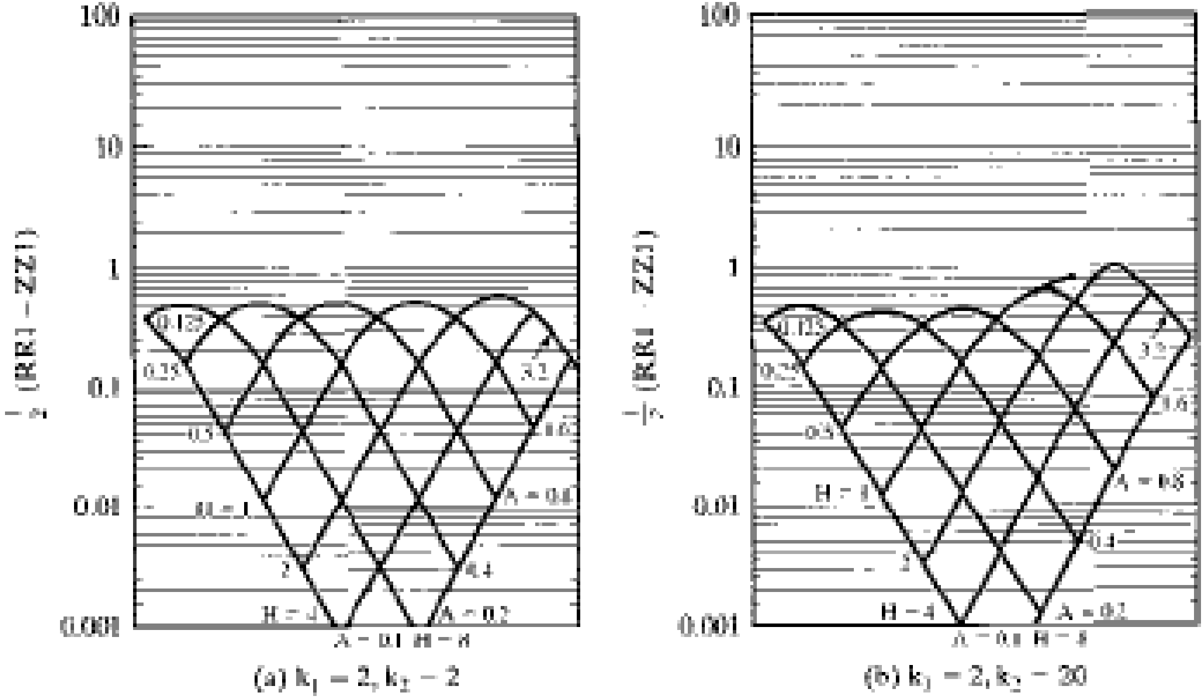
**Peattie (1962) plotted Jones' table in graphical forms. Charts show radial strain factors, (RR1-ZZ1)/2 at the bottom of layer 1. The radial strain can be determined from**

$$\epsilon_r = \frac{q}{E}\left(\frac{\mathrm{RR1} - \mathrm{ZZ1}}{2}\right)$$

**The radial strains at the bottom of layer 1 should be in tension. Although the solutions obtained from the charts are not as accurate as those from the table, the chart has the advantage that interpolation for A and H can be easily done. However, interpolation for $k_1$ and $k_2$ is still cumbersome.**

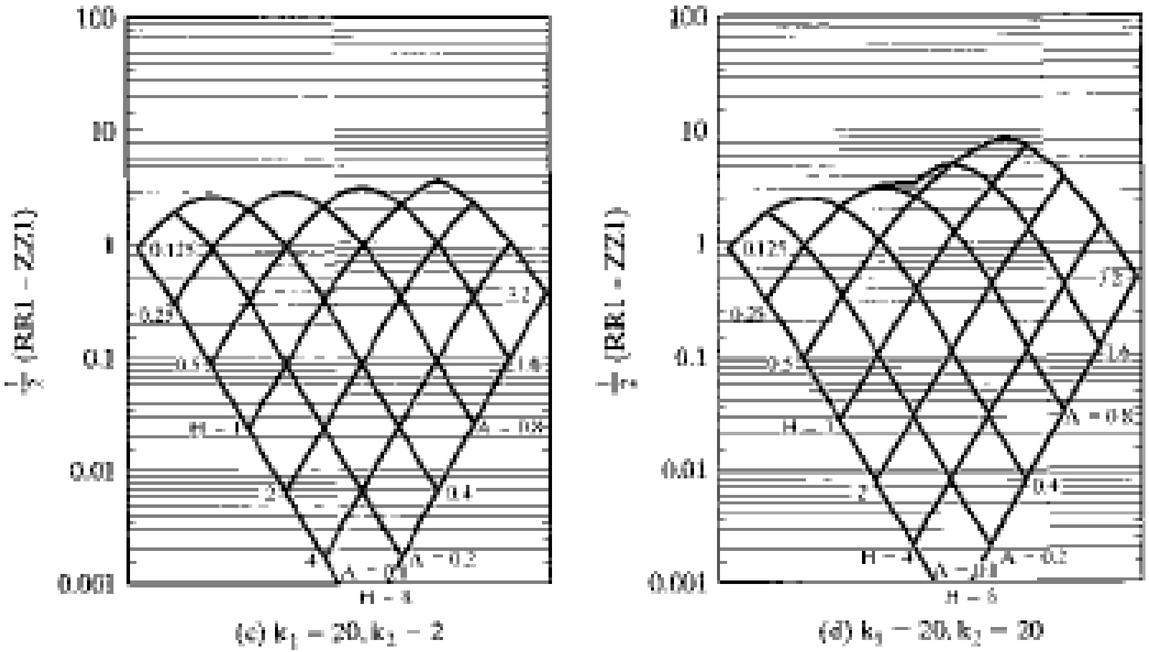
# LAYERED SYSTEMS-Three-Layer Systems

## Peattie's Charts



# LAYERED SYSTEMS-Three-Layer Systems

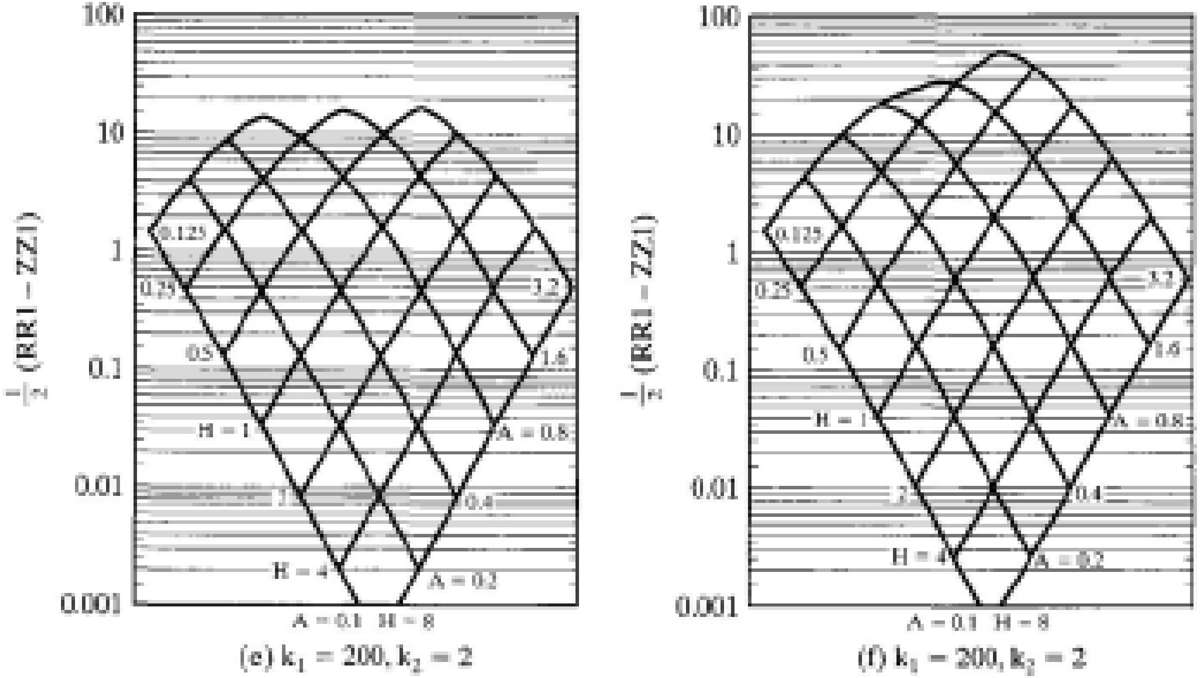
## Peattie's Charts



# Stresses and Strains in Flexible Pavements

## LAYERED SYSTEMS-Three-Layer Systems

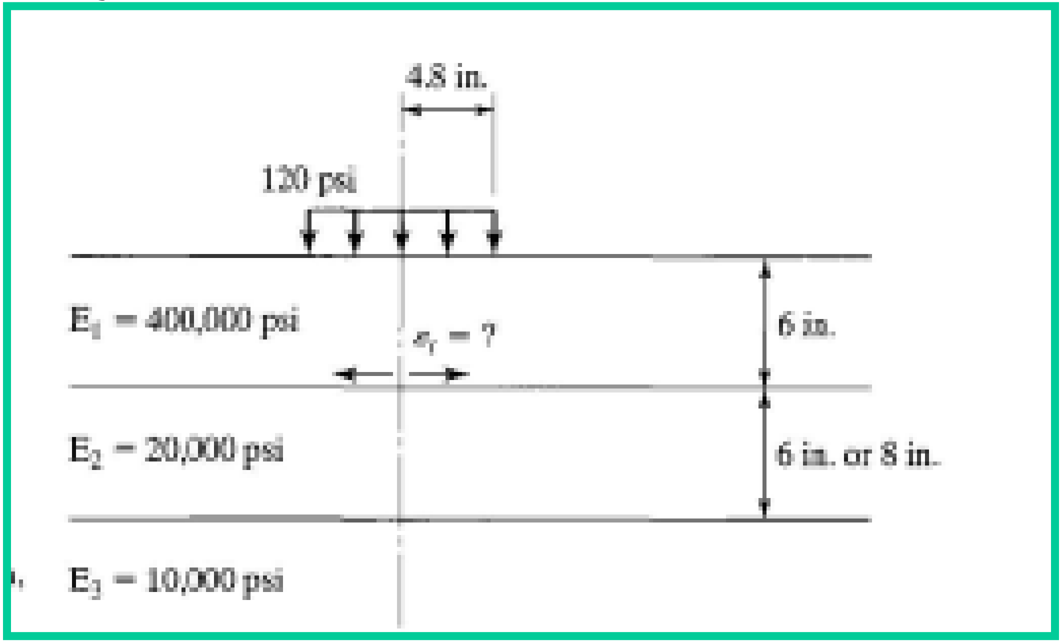
### Peattie's Charts



## LAYERED SYSTEMS-Three-Layer Systems

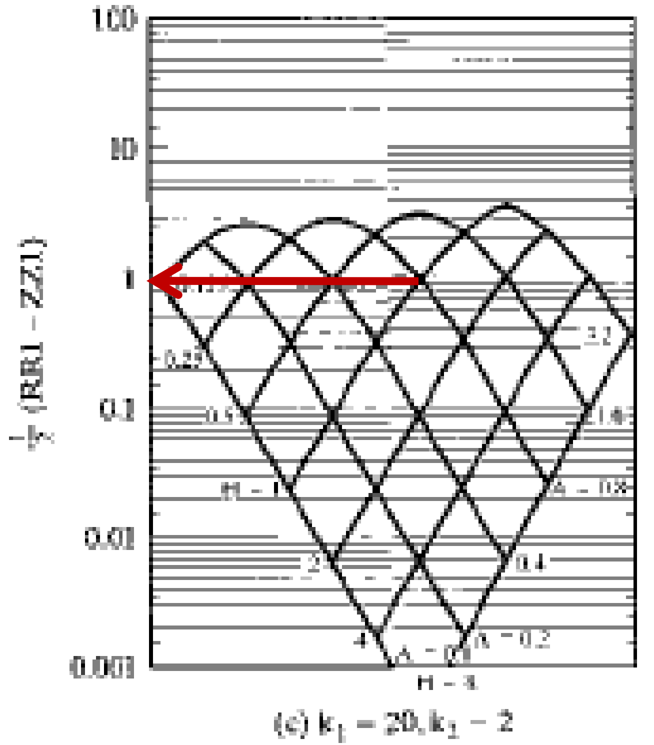
### Peattie's Charts-Numerical Problem

**For the pavement shown, determine the radial strains at the bottom of layer 1.**



# LAYERED SYSTEMS-Three-Layer Systems

## Peattie's Charts-Numerical Problem



## Nonlinear Mass

**Boussinesq's solutions are based on the assumption that the material that constitutes the half-space is linear elastic.**

**It is well known that subgrade soils are not elastic and undergo permanent deformation under stationary loads.**

**However, under the repeated application of moving traffic loads, most of the deformations are recoverable and can be considered elastic.**

**It is therefore possible to select a reasonable elastic modulus corresponding to the speed of moving loads.**

## Nonlinear Mass

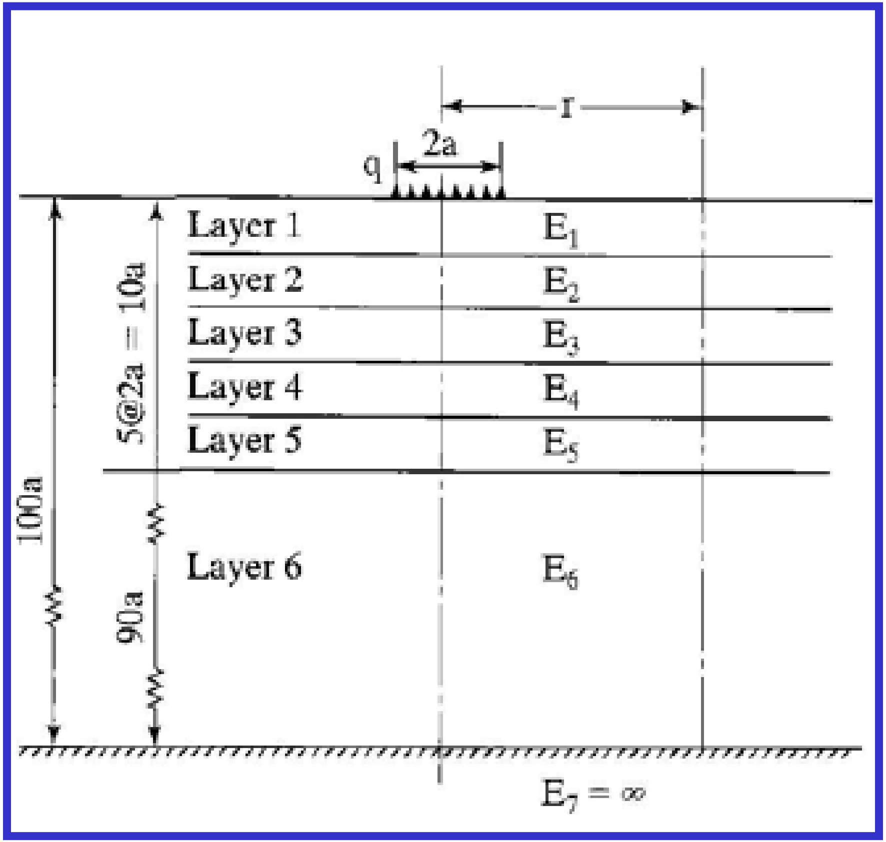
**Linearity implies the applicability of the superposition principle, so the elastic constant must not vary with the state of stresses.**

**In other words, the axial deformation of a linear elastic material under an axial stress should be independent of the confining pressure.**

**This is evidently not true for soils, because their axial deformation depends strongly on the magnitude of confining pressures.**

**Consequently, the effect of nonlinearity on Boussinesq's solution is of practical interest.**

# Nonlinear Mass-Iterative Method



## Nonlinear Mass-Iterative Method

**To show the effect of nonlinearity of granular materials on vertical stresses and deflections, Huang (1968a) divided the half-space into seven layers, as shown in Figure, and applied Burmister's layered theory to determine the stresses at the midheight of each layer. Note that the lowest layer is a rigid base with a very large elastic modulus. After the stresses are obtained, the elastic modulus of each layer is determined from**

$$E = E_0(1 + \beta\theta)$$

# Nonlinear Mass-Iterative Method

$$E = E_0(1 + \beta\theta)$$

**in which**

**$\theta$ is the stress invariant, or the sum of three normal stresses;**

**$E$ is the elastic modulus under the given stress invariant;**

**$E_0$ is the initial elastic modulus, or the modulus when the stress invariant is zero;**

**and $\beta$ is a soil constant indicating the increase in elastic modulus per unit increase in stress invariant.**

**Note that the stress invariant should include both the effects of the applied load and the geostatic stresses; it can be expressed as**

$$\theta = \sigma_z + \sigma_r + \sigma_t + \gamma z(1 + 2K_0)$$

## Nonlinear Mass-Iterative Method

$$\theta = \sigma_z + \sigma_r + \sigma_t + \gamma z(1 + 2K_0)$$

**in which $\sigma_z$, $\sigma_r$ and $\sigma_t$ are the vertical, radial and tangential stresses due to loading;**

**$\gamma$ is the unit weight of soil;**

**$z$ is the distance below ground surface at which the stress invariant is computed; and**

**$K_o$ is the coefficient of earth pressure at rest.**

## Nonlinear Mass-Iterative Method

**The problem can be solved by a method of successive approximations. First, an elastic modulus is assumed for each layer and the stresses are obtained from the layered theory.**

**Given the stresses thus obtained, a new set of moduli is determined from Eq.** $E = E_0(1 + \beta\theta)$ **and a new set of stresses is then computed. The process is repeated until the moduli between two consecutive iterations converge to a specified tolerance.**

## Nonlinear Mass-Iterative Method

**In applying the layered theory for nonlinear analysis, a question immediately arises which radial distance *r* should be used to determine the stresses and the moduli?**

**Huang (1968a) showed that the vertical stresses are not affected significantly by whether the stresses at $r = 0$ or $r = \infty$ are used to determine the elastic modulus, but the vertical displacements are tremendously affected.**

## Nonlinear Mass-Iterative Method

**Huang later used the finite-element method and found that the nonlinear behavior of soils has a large effect on vertical and radial displacements, an intermediate effect on radial and tangential stresses and a very small effect on vertical and shear stresses (Huang, 1969a).**

**Depending on the depth of the point in question, the vertical stresses based on nonlinear theory may be greater or smaller than those based on linear theory and at a certain depth, both theories could yield the same stresses.**

**This may explain why Boussinesq's solutions for vertical stress based on linear theory have been applied to soils with varying degrees of success, even though soils themselves are basically nonlinear.**

## Nonlinear Mass- Approximate Method

**One approximate method to analyze a nonlinear half-space is to divide it into a number of layers and determine the stresses at the midheight of each layer by Boussinesq's equations based on linear theory.**

**From the stresses thus obtained, the elastic modulus *E* for each layer is determined from Eq.**

$$E = E_0(1 + \beta\theta)$$

## Nonlinear Mass- Approximate Method

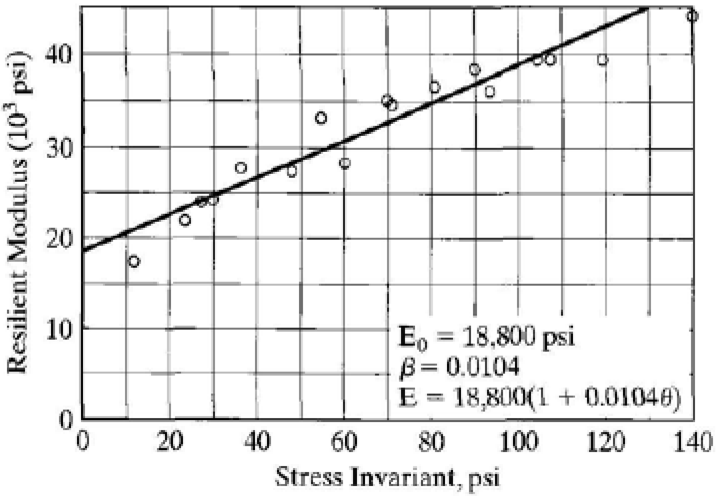
**The deformation of each layer, which is the difference in deflection between the top and bottom of each layer based on the given *E*, can then be obtained.**

**Starting from the rigid base, or a depth far from the surface where the vertical displacement can be assumed zero, the deformations are added to obtain the deflections at various depths.**

## Nonlinear Mass- Approximate Method-Numerical problem

**A circular load having radius 6 in. and contact pressure 80 psi is applied on the surface of a subgrade. The subgrade soil is a sand with the relationship between the elastic modulus and the stress invariant shown in Figure-a. The soil has Poisson ratio 0.3, the unit weight 110 pcf and the coefficient of earth pressure at rest 0.5. The soil is divided into six layers, as shown in Figure-b. Determine the vertical surface displacement at the axis of symmetry.**

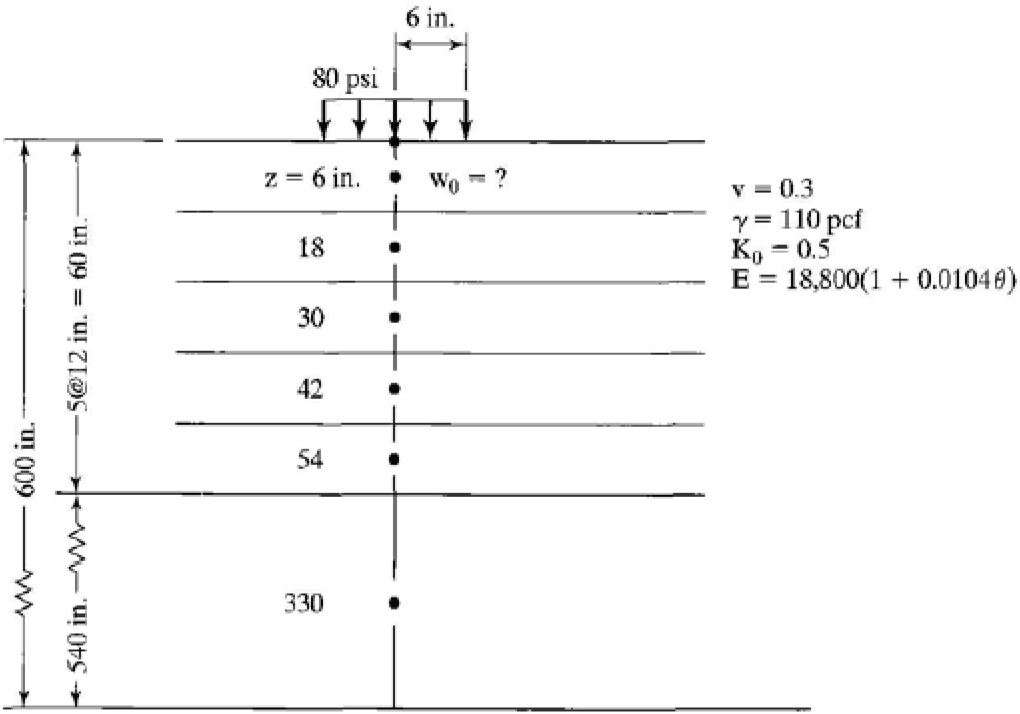
# Nonlinear Mass- Approximate Method-Numerical problem




(a) Resilient Modulus

# Stresses and Strains in Flexible Pavements

# Nonlinear Mass- Approximate Method-Numerical problem




(b) Layer Subdivision

# Nonlinear Mass- Approximate Method-Numerical problem

$$\sigma_z = q\left[1 - \frac{z^3}{(a^2 + z^2)^{1.5}}\right]$$

$$\sigma_r = \frac{q}{2}\left[1 + 2v - \frac{2(1 + v)z}{(a^2 + z^2)^{0.5}} + \frac{z^3}{(a^2 + z^2)^{1.5}}\right]$$

$$\theta = \sigma_z + \sigma_r + \sigma_t + \gamma z(1 + 2K_0)$$

$$E = E_0(1 + \beta\theta)$$

$$w_0 = \frac{2(1 - v^2)qa}{E}$$

$$w = \frac{(1 + v)qa}{E}\left\{\frac{a}{(a^2 + z^2)^{0.5}} + \frac{1 - 2v}{a}\left[(a^2 + z^2)^{0.5} - z)\right]\right\}$$

# Nonlinear Mass- Approximate Method-Numerical problem

| Layer no. | Thickness (in.) | $z$ at mid-height (in.) | $\sigma_z$ (psi) | $\sigma_r$ (psi) | $\theta$ (psi) Loading | $\theta$ (psi) Geostatic | $E$ (psi) | $wE$ (lb/in.) | Deformation (in.) |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | 873.6 | |
| 1 | 12 | 6 | 51.72 | 4.60 | 60.92 | 0.76 | 30,860 | | 0.0174 |
| | | | | | | | | 338.0 | |
| 2 | 12 | 18 | 11.69 | −0.51 | 10.67 | 2.29 | 21,330 | | 0.0073 |
| | | | | | | | | 182.1 | |
| 3 | 12 | 30 | 4.57 | −0.27 | 4.03 | 3.82 | 20,330 | | 0.0029 |
| | | | | | | | | 123.2 | |
| 4 | 12 | 42 | 2.39 | −0.15 | 2.09 | 5.35 | 20,250 | | 0.0015 |
| | | | | | | | | 92.9 | |
| 5 | 12 | 54 | 1.46 | −0.09 | 1.28 | 6.88 | 20,400 | | 0.0009 |
| | | | | | | | | 74.5 | |
| 6 | 540 | 330 | 0.04 | 0.00 | 0.04 | 42.01 | 27,020 | | 0.0025 |
| | | | | | | | | 7.5 | |
| | | | | | | | Total | | 0.0325 |

# Assignment No. 2

**Pavement Analysis and Design by Yang H. Huang**

**Chapter-2**

**Problems 2.1 to 2.7 (Pages 90-93)**

**Date of submission:30-09-2019**