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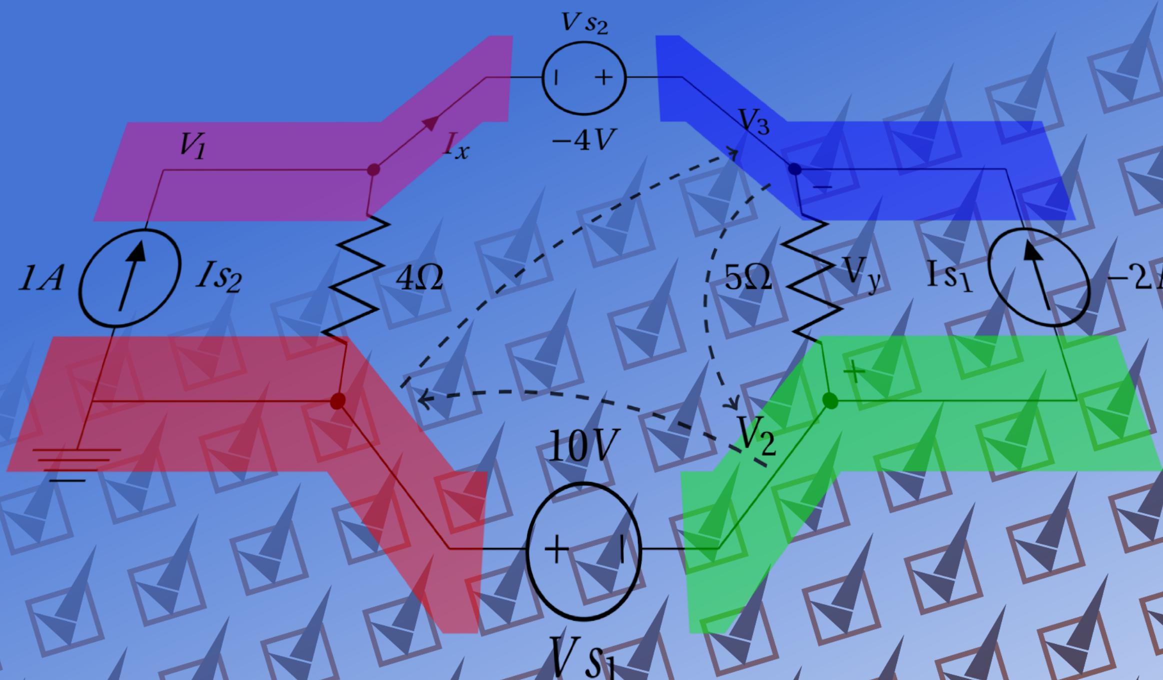
electrical circuits

Nodal Analysis

Dr. Yaz Z. Li



First Edition – June 2010

Nodal Analysis

Yaz Z. Li, PhD

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For almost all problems, their PSpice schematics are provided in our website. These files are created by PSPICE 9.1 Student Version.

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Nodal Analysis

Introduction

Nodal analysis is a systematic method to determine the voltage at each node relative to the reference node by repeatedly applying KCL. In Nodal analysis, also called node-voltage analysis or branch-current method, the voltage between nodes is determined in terms of the branch currents. In this method, a system of equations in which the unknowns are the voltages at the principal nodes of the circuit is set up and solved. The set of equations developed in the nodal analysis in fact represents and describes the circuit. After determining these nodal voltages, the currents in the various branches of the circuit can be easily found.

The nodal analysis starts with selecting one of the nodes as the reference node. Since one of the nodes is selected as the reference node, if there are N nodes in the circuit there will be $N-1$ linearly independent equations in general. The solution of this set of equations yields the $N-1$ unknown node voltages. These node voltages can be used with the Ohm's law to determine all currents in the circuit.

Reference Node

see comments, ask your questions
http://www.solved-problems.com/go/RN/

In circuits, we usually label a node as the reference node also called ground and define the other node voltages with respect to this point. The reference node has a potential of $0V$ by definition. The symbol of reference node is depicted in Fig. 1.

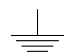

Fig. 1 — Refrence Node Symbol

As mentioned, the selection of the reference node is arbitrary. However, a wise selection can make the solving easier. As a general rule, it is usually chosen to be

Node Voltages

see comments, ask your questions
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The voltage drop from a node to the reference node (ground) is called the node voltage. To keep definition simple, node voltages are usually defined with positive polarities.

Let's find and label node voltages for the circuit shown in Fig. 2:

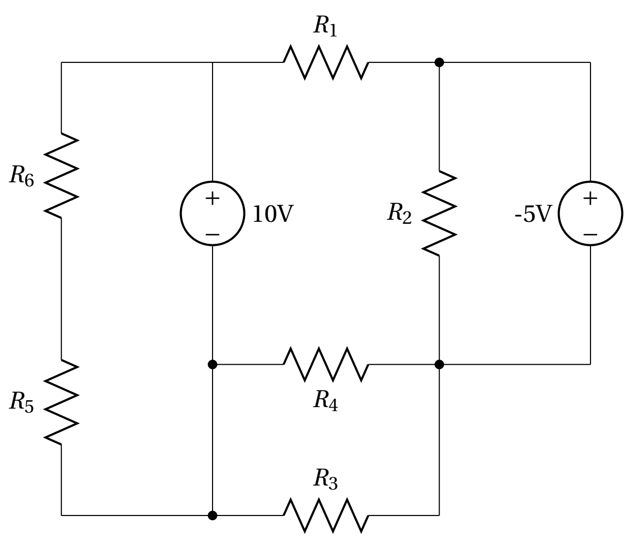


Fig. 2 — Finding Node Voltages

The circuit has 5 nodes as shown in Fig. 3.

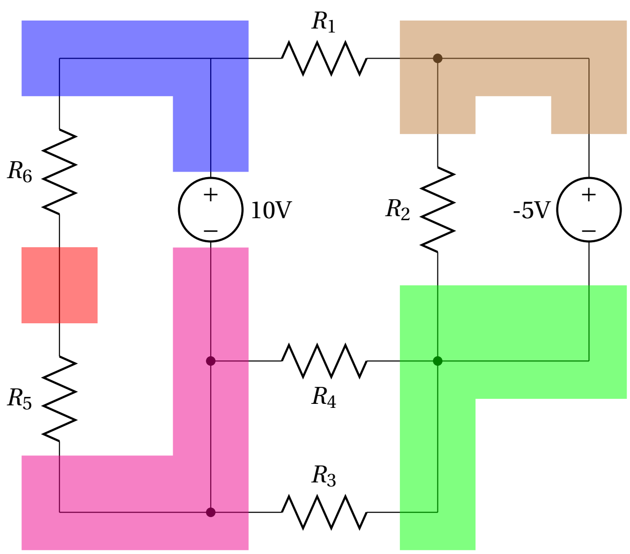


Fig. 3 — All Nodes

Two of the nodes have 4 elements connected to them. These are the best candidates to be reference point.

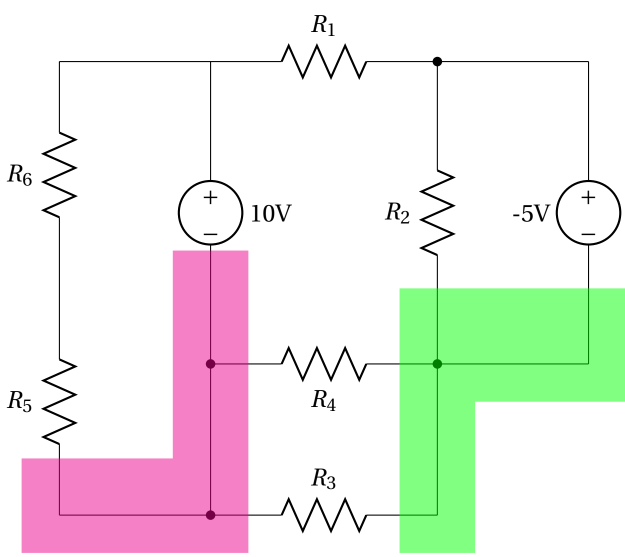
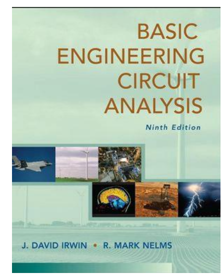


Fig. 4 — Candidates For Reference Node



Let's choose one as the reference node (Fig. 5).

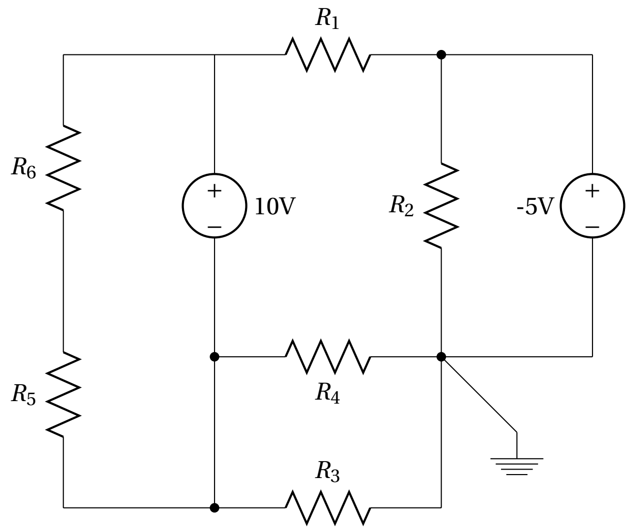


Fig. 5 — The reference Node

Now, we define node voltages for the remaining nodes as depicted in Fig. 6. These node voltages represent the voltage between the node and the reference.

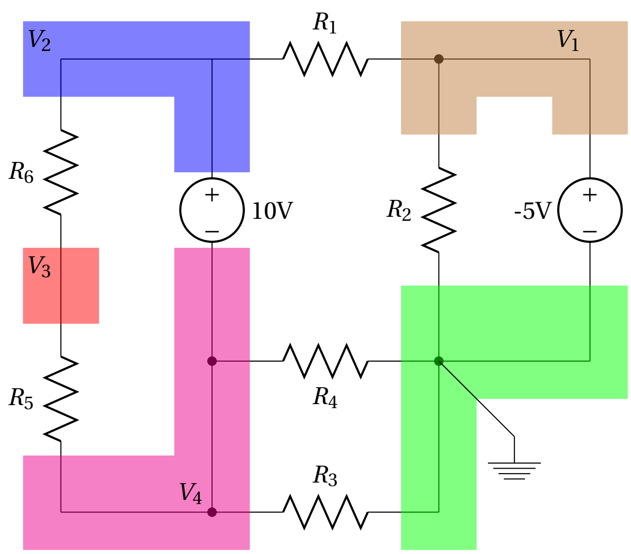


Fig. 6 — Node Voltages

When there is a voltage source between a node and the reference node, the node voltage corresponds exactly to the voltage of the voltage source. In our example, we have two node voltages. The $-5V$ voltage source is placed between the reference and the node labeled as V_1. Therefore, $V_1 = -5V$.

If there is a voltage source between two nodes, the difference between the corresponding node voltages equals to the voltage of the source. In our example, the $10V$ voltage source is located between nodes labeled by V_2 and V_4. Therefore, $V_2 - V_4 = 10V$. It is important to note that voltage of the positive node minus the one of negative node is equal to the voltage of the source. KVL can be used to show this:

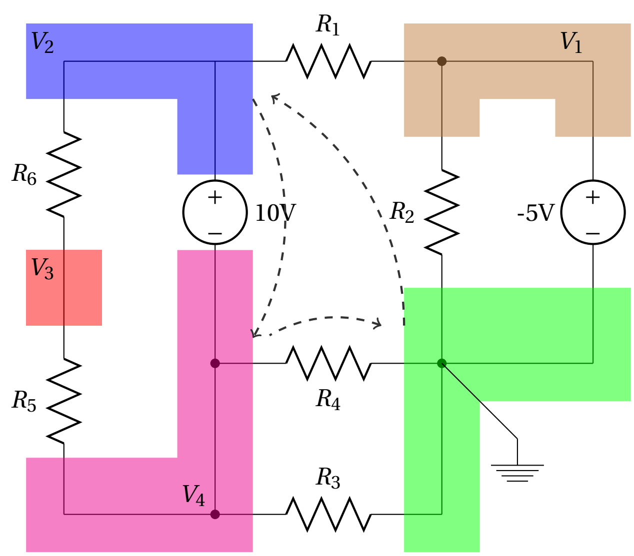


Fig. 7 — Voltage source between two nodes

KVL around the loop: $-V_2 + (+10) + V_4 = 0 \rightarrow V_2 - V_4 = 10V$. Recall that the reference node is always defined to be the negative polarity of all node voltages.

Nodal Analysis Steps

1. Identify all nodes in the circuit. Call the number of nodes N.

2. Select a reference node. Label it with reference (ground) symbol. As a general rule, the reference node is usually chosen to be

   a node with largest number of elements connected to it, or a node which is connected to the greatest number of voltage sources, or a node of symmetry.

3. Assign a variable for each node whose voltage is unknown. If a voltage source is connected between a node and the reference node, the voltage is already known and it is not necessary to assign a variable. If there is a voltage source between two nodes, the difference between the node voltages equals to the voltage of the source. In this case, to cut the number of unknowns assign a variable for one of the nodes and express the voltage of the other one with respect to the assigned variable.

4. If there are dependent sources in the circuit, write down equations that express their values in terms of node voltages.

5. Write down a KCL equation for each node by setting the total current flowing out of the node to zero. Recall that the KCL states that the algebraic sum of all currents entering and exiting a node is equal to zero. It is always a good idea to rearrange these equations into the form $A_1 \times V_1 + A_2 \times V_2 + ... + A_{N-1} \times V_{N-1} = C$ where A_1, A_2, A_{N-1} and C are some constants. If there are voltage sources between two unknown voltages, join the two nodes as a supernode. Note that you should have only one unknown variable for a supernode because the voltage of one the nodes can be expressed with respect to the voltage of the other one. For a supernode, the currents of the two nodes are combined in a single equation, and a new equation for the voltages is formed. For a

circuit with N nodes and M voltage sources N - M - 1 simultaneous linearly independent equations can be written.

Discussion
http://www.solved-problems.com/go/1-11/

Problem 1 - Three-Node Circuit

Solve the circuit shown in Fig. 8 by nodal analysis and find the power of Is_1.

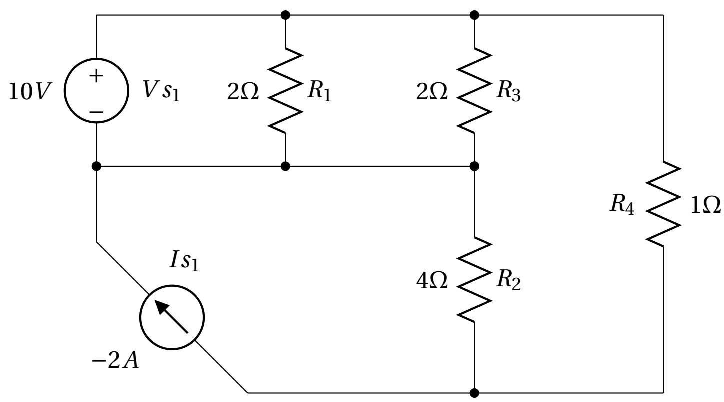


Fig. 8 — Problem 1

Solution

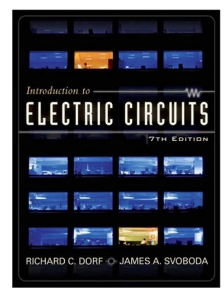
I. Identify all nodes in the circuit. The circuit has 3 nodes as shown in Fig. 9.

II. Select a reference node. Label it with reference (ground) symbol.

Two of the nodes have the largest number of elements. Both are connected to the voltage source. We can choose either of them. By choosing the node that is connected to the negative terminal of the voltage source, the voltage of the other node will be exactly as the voltage of the source.

III. Assign a variable for each node whose voltage is unknown. We label the remaining two nodes as illustrated in Fig. 10 .

IV. If there are dependent sources in the circuit, write down equations that express their values in terms of node voltages. There



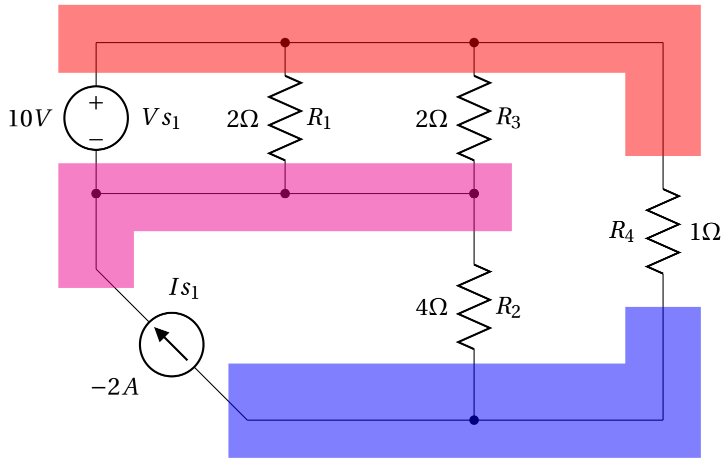


Fig. 9 — Problem 1 - All nodes

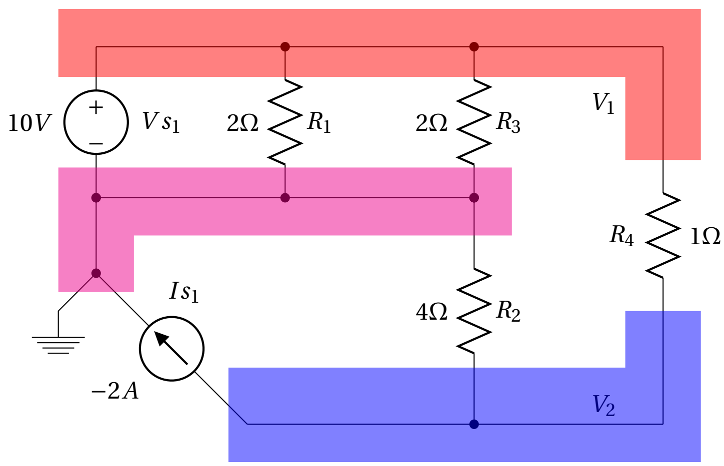


Fig. 10 — Problem 1 - The reference node and node voltages

is no dependent source.

V. Write down a KCL equation for each node. As mentioned before, Node of V_1 is connected to the reference node via a voltage source. Therefore, its voltage can be determined by the voltage of the voltage source and we do not need to write a KCL equation for this node. Writing a KCL equation for nodes which have voltage source is not that useful. The current through a voltage source is not dependent to its voltage and should be completely

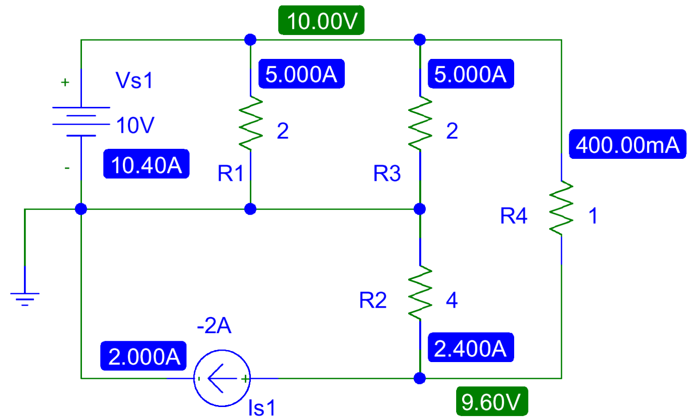


Fig. 11 — Preoblem 1 - PSpice simulation result

determined by other elements of the circuit. V_1 is connected to the positive terminal of the voltage source Vs_1. Therefore, $V_1 = Vs_1$. It is also common to label this type of nodes with the label of the voltage source.

Node V_2: $Is_1 + \frac{V_2}{R_2} + \frac{V_2 - V_1}{R_4} = 0 \rightarrow V_2 = 9.6V$ The circuit is solved. All node voltages are determined and we can find the required value: $P_{Is_1} = Is_1 \times V_2 = -19.2W$, supplying.

PSpice Files

The PSpice simulation result is depicted in Fig. 11. The PSpice schematics can be downloaded from http://www.solved-problems.com/go/NAPSpice/.

Discussion
http://www.solved-problems.com/go/1-10/

Problem 2 - Four-Node Circuit

Use nodal analysis to solve the circuit shown in Fig. 12 and determine V_a.

Solution

I. Identify all nodes in the circuit. There four nodes in the circuit as indicated in Fig. 13.

II. Select a reference node. Label it with reference (ground) symbol. All nodes have three elements connected to. None has

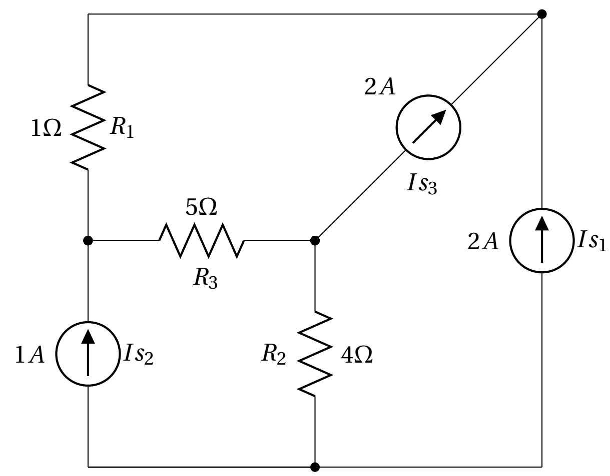


Fig. 12 — Problem 2

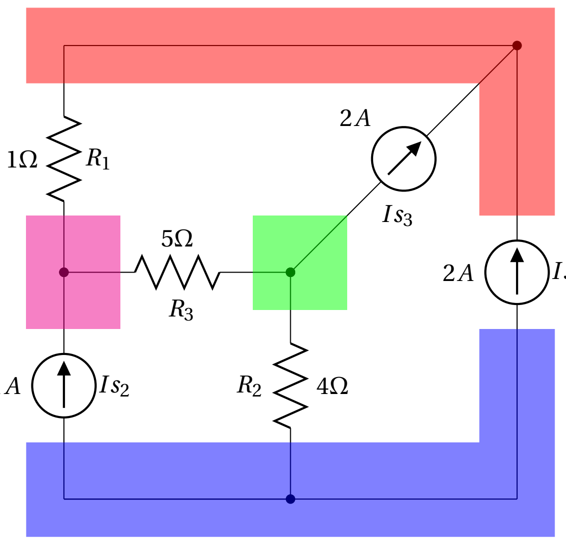


Fig. 13 — Problem 2 - All nodes

a greater advantage over others to be the reference node. We choose the bottom node as the reference node.

III. Assign a variable for each node whose voltage is unknown. Three remaining nodes are labeled as represented in Fig. 14 .

IV. If there are dependent sources in the circuit, write down equations that express their values in terms of node voltages. There

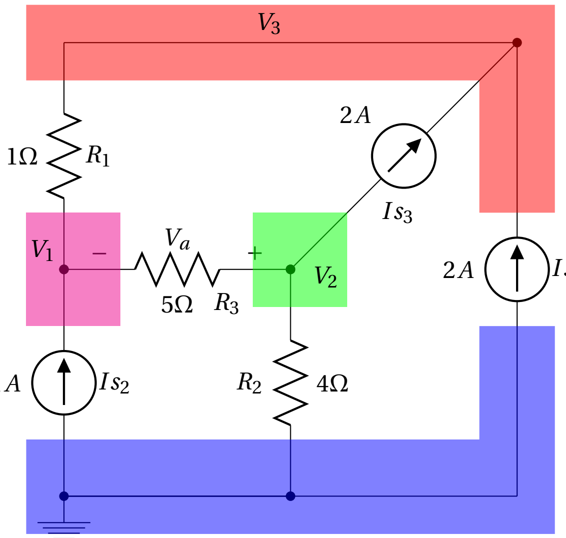


Fig. 14 — Problem 2 - The reference node and node voltages

is no dependent source in this circuit.

V. Write down a KCL equation for each node.

Node of V_1:

$$-Is_2 + \frac{V_1 - V_2}{R_3} + \frac{V_1 - V_3}{R_1} = 0 \rightarrow 6V_1 - V_2 - 5V_3 = 5.$$

(1)

Node of V_2:

$$Is_3 + \frac{V_2 - V_1}{R_3} + \frac{V_2}{R_2} = 0 \rightarrow -4V_1 + 9V_2 = -40.$$

(2) Node of V_3:

$$-Is_1 - Is_3 + \frac{V_3 - V_1}{R_1} = 0 \rightarrow V_3 - V_1 = 4.$$

(3) (1), (2) and (3) imply that $V_1 = 37v$, $V_2 = 12v$ and $V_3 = 41$.

Now, we have all node voltages. The last step is to determine V_a. If we apply KVL in the loop shown in Fig. 15.

$$-V_1 - V_a + V_2 = 0 \rightarrow V_a = -25V.$$

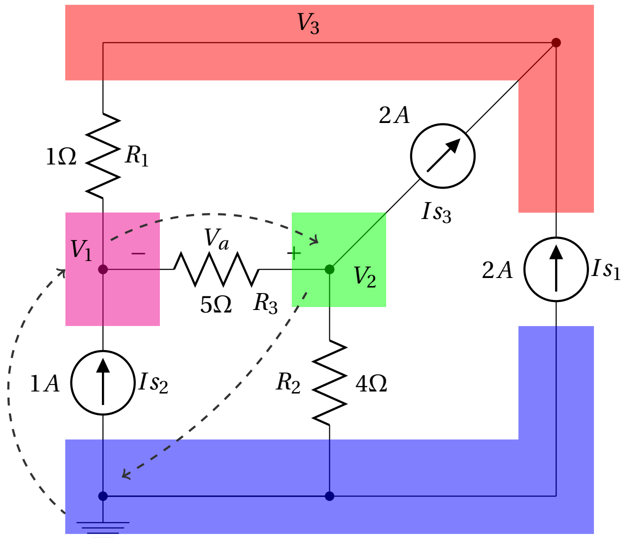


Fig. 15 — Problem 2 - Loop for KVL

The PSpice simulation result is depicted in Fig. 16. The PSpice schematics can be downloaded from http://www.solved-problems.com/go/NAPSpice/.

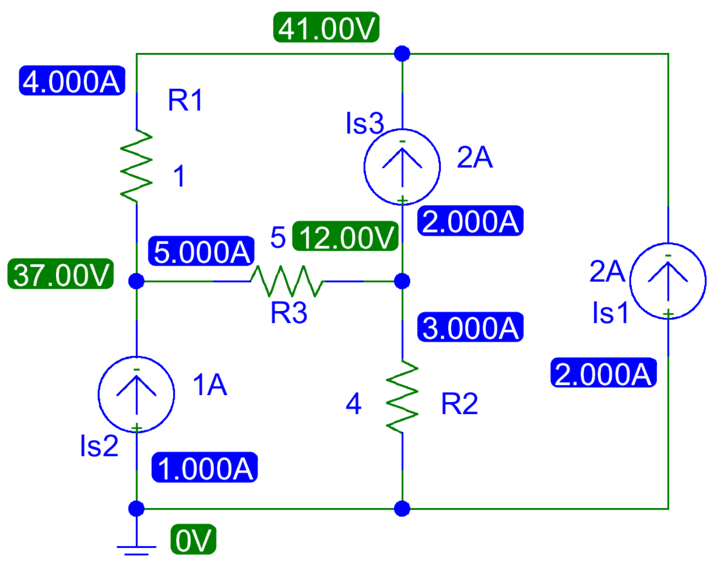


Fig. 16 — Preoblem 2 - PSpice simulation result

Complicated Cases

The nodal analysis method is generally straightforward to apply, but becomes rather difficult in the following cases.

Non-grounded Voltage Sources

Since the current of a voltage source is independent of the voltage, it cannot be used in writing KCL equations. If one node of a voltage source is connected to the reference node, we do not need to know the current passing through the voltage source. The reason is that the voltage of the node can be easily determined by the voltage of the voltage source and there is no need to write KCL equation for the node.

Complicated cases are the ones where a voltage source is located between two non-reference nodes. In these cases, a supernode method should be used. A simple supernode is consist of a source and its nodes. In general, supernodes can have more than one voltage sources. After identifying a supernode, we need to define only one voltage variable for one of the nodes of the supernode and find the voltage of other node(s) with respect to that voltage variable. This equation relates node voltages of the supernode to each other. Then, we should treat a supernode as a node and write a KCL equation for all currents entering and leaving the super node. Now we have one equation and two unknowns (the node voltages). This equation should be added to the set of equations derived for other nodes and the new set of equations should be solved to determine all node voltages.

Discussion
http://www.solved-problems.com/go/1-18/

Problem 3 - Circuit with A Non-grounded Voltage Source

Use nodal analysis to solve the circuit shown in Fig. 3 and determine I_x and V_y.

Solution
I. Identify all nodes in the circuit. There four nodes in the circuit

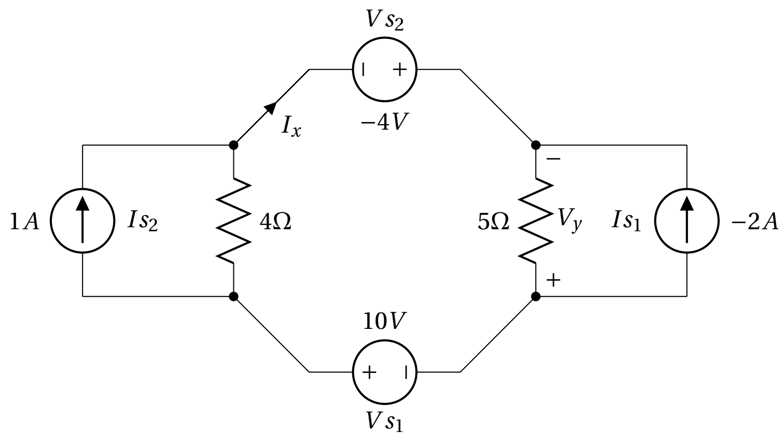


Fig. 17 — Problem 3

as shown in Fig. 18.

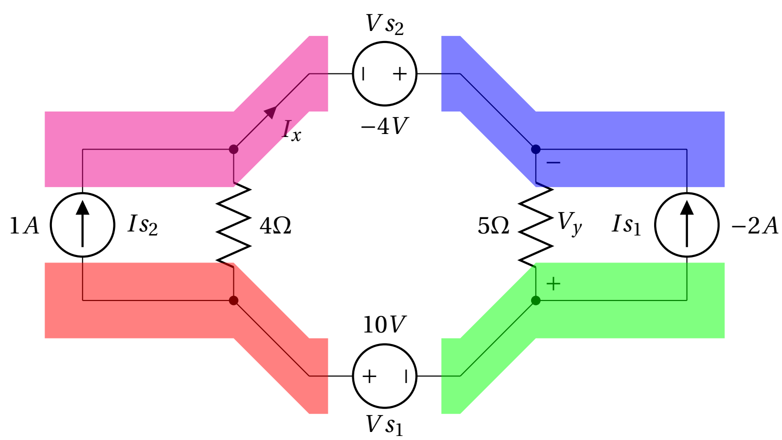


Fig. 18 — Problem 3 - All nodes

II. Select a reference node. Label it with reference (ground) symbol. There is symmetry in this circuit and each node has three elements connected to. One of the elements connected to each node is a voltage source. Thus, any node has no advantage over others in being the reference node. We choose the bottom left node as the reference node.

III. Assign a variable for each node whose voltage is unknown. Three remaining nodes are labeled as illustrated in Fig. 19. With

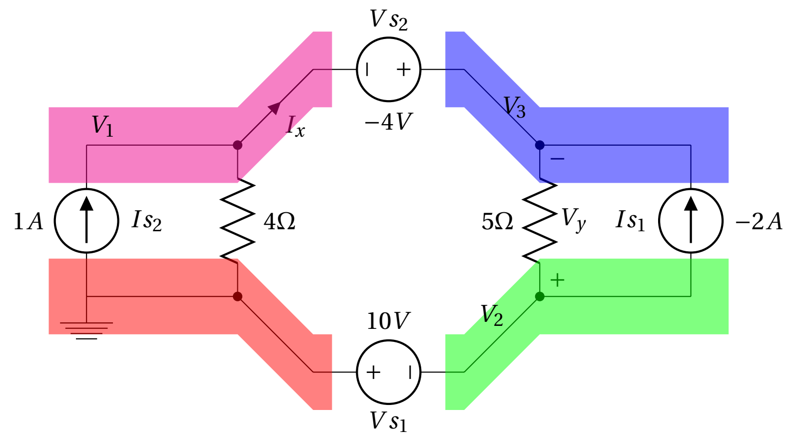


Fig. 19 — Problem 3 -The reference node and node voltages

this selection, we can easily determine V_2. There is a voltage source between node of V_2 and the reference node. The positive terminal of the voltage source is connected to the reference node. Therefore, $V_2 = -Vs_1 = -10V$. We assign V_2 to make referencing to the node easier. One may avoid assigning a label to that node and instead, directly substitute $-10V$ wherever it is required in equations.

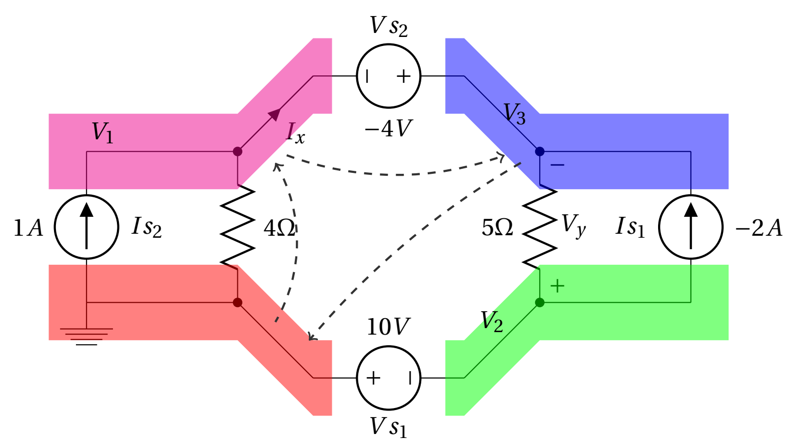


Fig. 20 — Problem 3 - Applying KVL to find supernode voltage

Nodes of V_1 and V_3 are connected to each other by a voltage source. Therefore, they form a supernode. By writing a KVL for the loop depicted in Fig. 20, V_3 can be written in terms of V_1:

$$-V_1 - Vs_2 + V_3 = 0 \rightarrow V_3 = V_1 - 4$$

IV. If there are dependent sources in the circuit, write down equations that express their values in terms of node voltages. There is no dependent source in this circuit.

V. Write down a KCL equation for each node.

We only need to write a KCL equation for the supernode:

$$-Is_2 + \frac{V_1}{R_2} + \frac{V_3 - V_2}{R_1} - Is_1 = 0.$$

Substituting $V_3 = V_1 - 4$ and known variables,

$$V_1 = -\frac{44}{9}V$$

. Therefore,

$$V_3 = V_1 - 4 = -\frac{80}{9}V.$$

The circuits is solved and all node voltages are known now. Now, we can find I_x and V_y from the node voltages.

KCL at node 1:

$$-Is_2 + \frac{V_1}{R_2} + I_x = 0 \rightarrow I_x = \frac{20}{9}A.$$

KVL around the loop shown in the Fig. 21:

$$-V_3 - V_y + V_2 = 0 \rightarrow V_y = -\frac{10}{9}V.$$

The PSpice simulation result is shown in Fig. 22. The PSpice schematics can be downloaded from http://www.solved-problems.com/go/NAPSpice/.

Dependent Current Source

When there is a dependent current source in the circuit, it should be treated as an independent current source but the variable

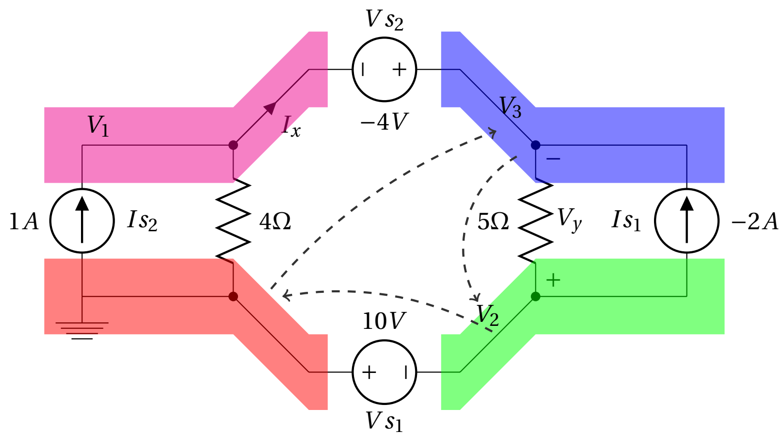


Fig. 21 — Problem 3 - Applying KVL to determine V_y

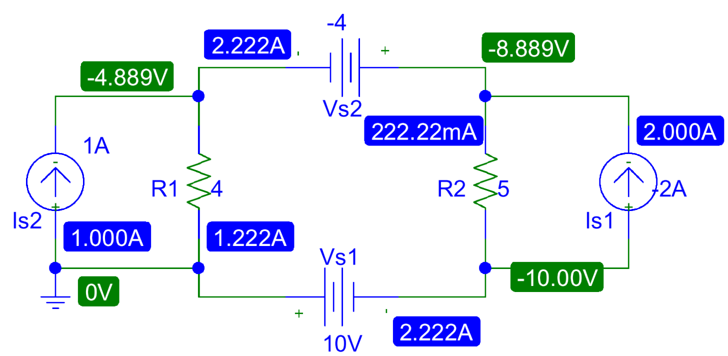


Fig. 22 — Preoblem 2 - PSpice simulation result

which the current source depends on should be expressed in terms of node voltages. For example, if it is current of a resistor, Ohm's law should be used to state the variable in term of the node voltages of the resistor.

Discussion
http://www.solved-problems.com/go/1-19/

Problem 4 - Dependent Current Source

Deploy nodal analysis method to solve the circuit and find the power of the dependent source.

Solution

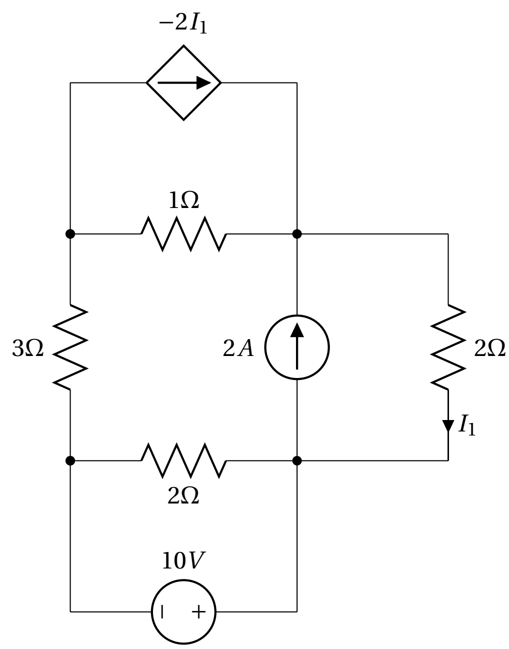


Fig. 23 — Preoblem 4

I. Identify all nodes in the circuit. Call the number of nodes N. The circuit has 4 nodes:

Therefore, $N = 4$.

II. Select a reference node. Label it with reference (ground) symbol.

All nodes have the same number of elements. We prefer to select one of the nodes connected to the voltage source to avoid having to use a supernode.

III. Assign a variable for each node whose voltage is unknown. We label the remaining three nodes as shown above.

IV. If there are dependent sources in the circuit, write down equations that express their values in terms of node voltages. There is one dependent source, which is a current controlled current source. We need to write $-2I_1$ in terms of node voltages. I_1 is the current passing through the 2Ω - resistor. Applying the Ohm's law, $I_1 = \frac{V_2 - V_3}{2\Omega}$. Hence, $-2I_1 = V_3 - V_2$.

V. Write down a KCL equation for each node. Node V_1: $\frac{V_1}{3\Omega} + \frac{V_1 - V_2}{1\Omega} - 2I_1 = 0. \rightarrow 4V_1 - 6V_2 + 3V_3 = 0$ (Eq. 1).

Node V_2: $2I_1 + \frac{V_2 - V_1}{1\Omega} - 2A + \frac{V_2 - V_3}{2\Omega} = 0$. Please note that we avoid using all unknowns except node voltages. Using I_1 in this KCL equation introduces an unnecessary unknown to the equations set. Substituting $2I_1 = V_2 - V_3$ and rearranging results in: $-2V_1 + 5V_2 - 3V_3 = 4$ (Eq. 2).

Node V_3 has a voltage source connected to. Therefore, $V_3 = 10V$. Substituting this in Eq. 1 and Eq.2 leads to

$$\begin{cases} 4V_1 - 6V_2 = -30 \\ -2V_1 + 5V_2 = 34 \end{cases}$$

. By solving the system of equations, $V_1 = 6.75V$ and $V_2 = 9.5$.

Now, we need to find the voltage across the dependent current source and the current passing through it. Lets start with I_1. $I_1 = \frac{V_2 - V_3}{2\Omega} = -0.25A$. Assuming positive terminal placed on the node of V_1, the voltage across the dependent current source is $V_1 - V_2 = -2.75V$. The current flowing through the dependent current source is $-2I_1 = 0.5A$. Therefore the power of the dependent current source is $-2.75 \times 0.5 = -1.375W$. Because the current direction and the voltage polarity is in accordance with the passive sign convention and the power is negative, the dependent current source is supplying power.

The PSpice simulation result is shown in Fig. 26. The PSpice schematics can be downloaded from http://www.solved-problems.com/go/NAPSpice/.

Dependent Voltage Sources

A dependent voltage source can make the solution a bit challenging. The solution follows the same steps mentioned for dependent source with an extra step. After writing super-node KCL equation, the variable that the dependent source depends on should be written in terms of the node voltages.

Discussion
http://www.solved-problems.com/go/1-21/

Problem 5 - Dependent Voltage Source

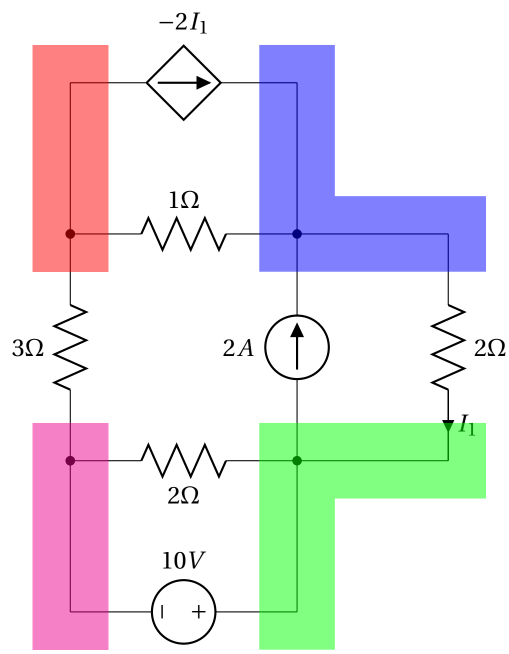


Fig. 24 — Preoblem 4 - All nodes

http://www.solved-problems.com/go/1-21/ Use nodal analysis method to solve the circuit and find the power of the 3Ω- resistor.

Solution

I. Identify all nodes in the circuit. The circuit has 3 nodes as indicated in Fig. 28.

II. Select a reference node. Label it with the reference (ground) symbol.

The node in the middle is connected to 5 nodes and is the node with the largest number of elements connected to it. Therefore, we select it as the reference node of the circuit.

III. Assign a variable for each node whose voltage is unknown. We label the remaining nodes as shown in Fig. 29. V_1 is connected to the reference node through a voltage source. Therefore, it is equal to the voltage of the dependent voltage source:

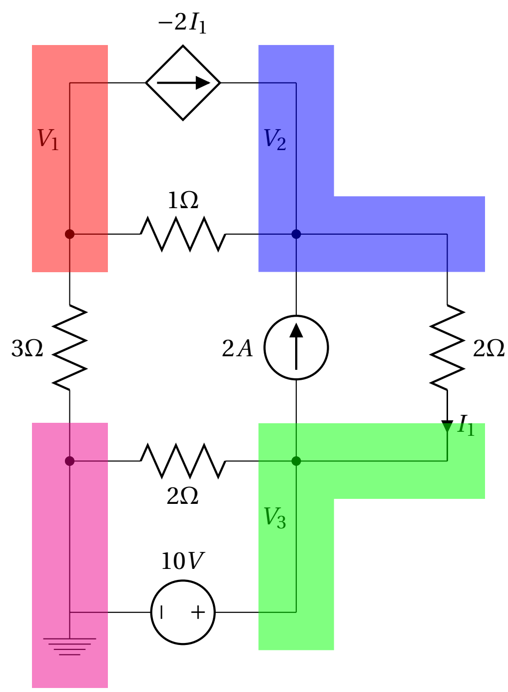


Fig. 25 — Preoblem 4 - The reference node

$V_1 = -6I_x$.

IV. If there are dependent sources in the circuit, write down equations that express their values in terms of node voltages.

The voltage of the dependent voltage source is $-I_x$. We should find this value in terms of the node voltages. I_x is the current of the 5Ω- resistor. The voltage across the resistor is $V_1 - V_2$. You may ask why not $V_2 - V_1$. Well, that is also correct; the voltage across the resistor is either $V_1 - V_2$ or $V_2 - V_1$ depend on which terminal we choose to be the positive one. In this circuit, we are going to use this voltage drop to determine I_x. We prefer to use $V_1 - V_2$ simply because V_1 is the voltage of the terminal that I_x entering from. Therefore, the Ohm's law can be applied in the simple form of $V = R \times I$. By using the voltage drop $V_1 - V_2$, we have

$$I_x = \frac{V_1 - V_2}{5\Omega}.$$

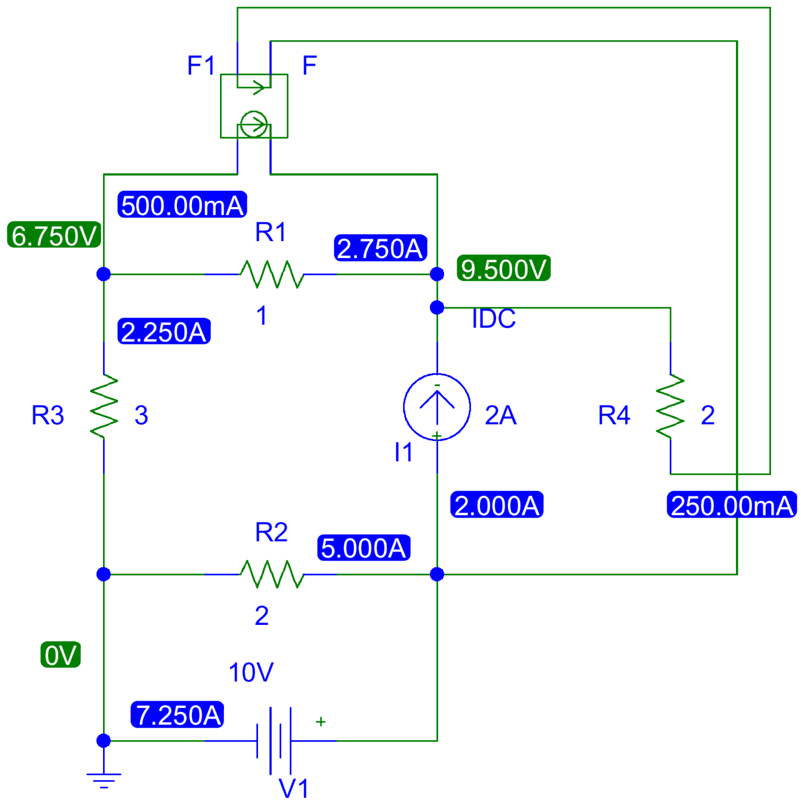


Fig. 26 — Preoblem 4 - PSpice simulation result

V. Write down a KCL equation for each node.

Node of V_1: Because there is a voltage source in this node, there is no advantage in writing a KCL equation for this node. All we need to do is to use the voltage of the dependent voltage source and its relation with other node voltages:

$$\begin{cases} I_x = \frac{V_1 - V_2}{5\Omega} \\ V_1 = -6I_x \end{cases} \rightarrow V_1 = \frac{6}{11} V_2.$$

Node of V_2:

$$\frac{V_2 - V_1}{5\Omega} + \frac{V_2}{3\Omega} - 2A = 0 \rightarrow -3V_1 + 8V_2 = 30.$$

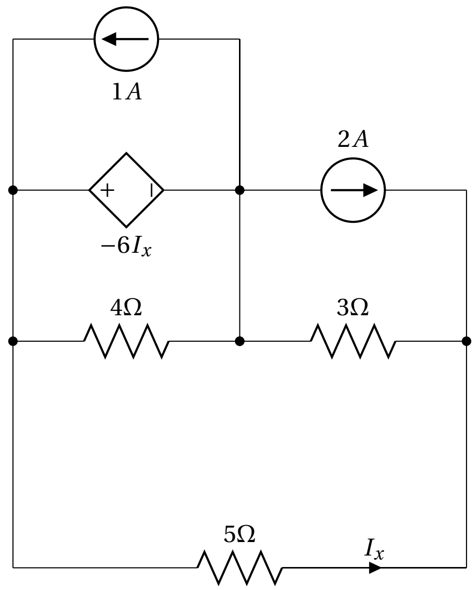


Fig. 27 — Preoblem 5

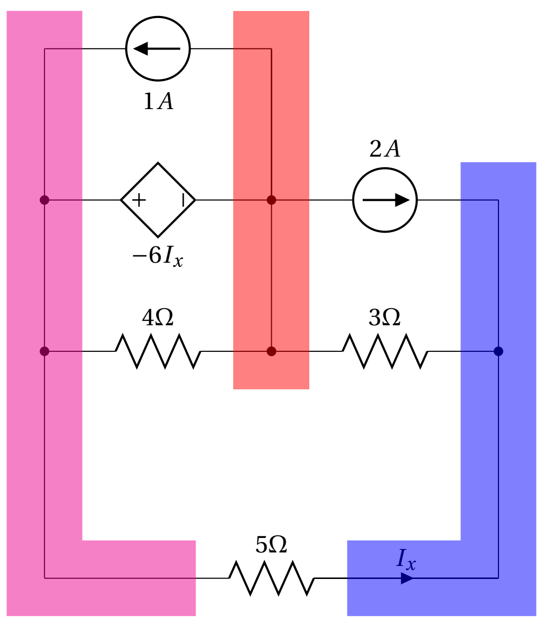


Fig. 28 — Preoblem 5 - All nodes

Substituting $V_1 = \frac{6}{11}V_2$,

$$V_2 = \frac{33}{7}V \rightarrow V_1 = \frac{18}{7}V.$$

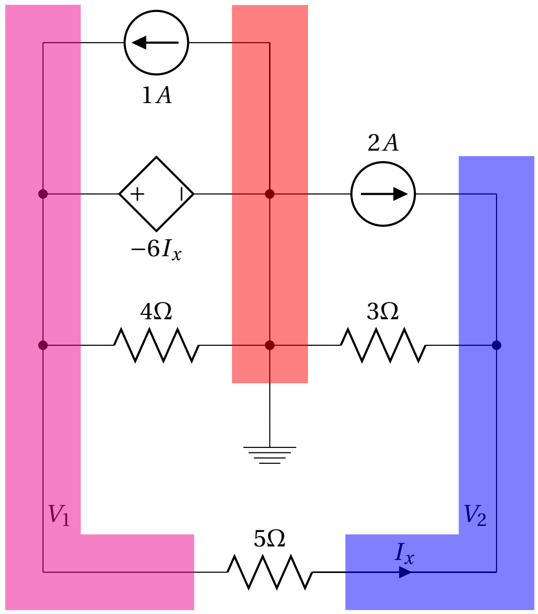


Fig. 29 — Preoblem 5 - The reference node and node voltages

All node voltages are obtained. The power of the 3Ω-resistor is

$$R_{3\Omega} = \frac{V_2^2}{3\Omega} = 7.408W.$$

The PSpice simulation result is indicated in Fig. 30. The PSpice schematics can be downloaded from http://www.solved-problems.com/go/NAPSpice/.

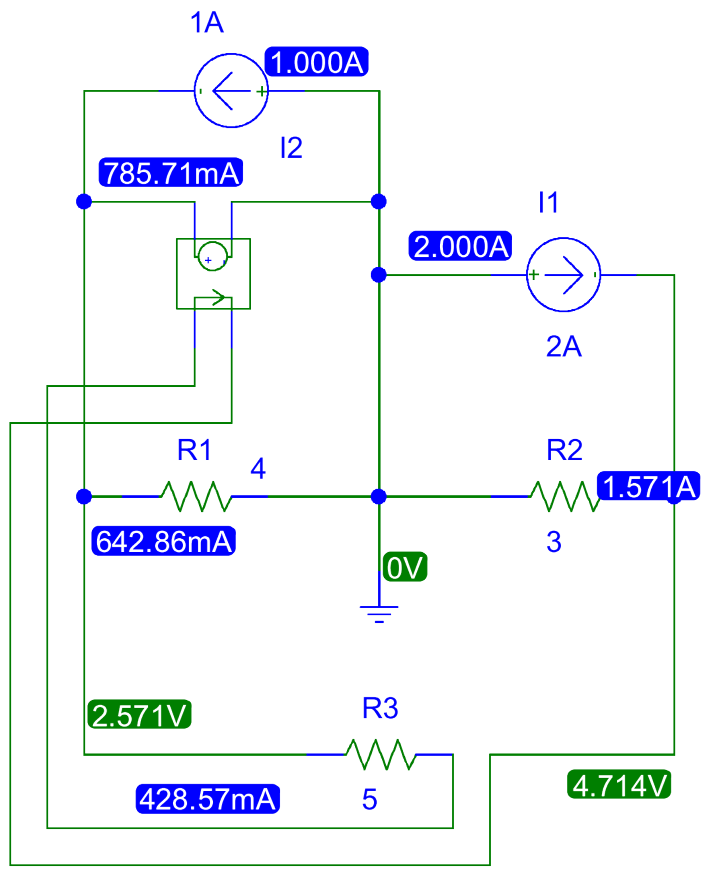


Fig. 30 — Preoblem 5 - PSpice simulation result

More Problems

Problem 6 - 6-Node Circuit

Discussion
http://www.solved-problems.com/go/1-22/

Determine the power of each source after solving the circuit by the nodal analysis.

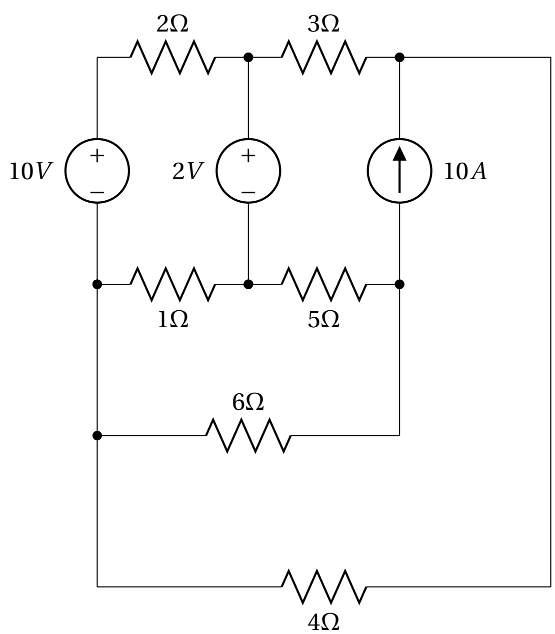
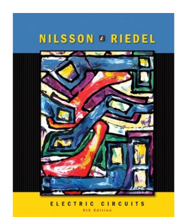


Fig. 31 — Preoblem 6



http://www.solved-problems.com/go/Nilsson/

Electric Circuits (9th Edition) by James W. Nilsson and Susan Riedel

Solution

I. Identify all nodes in the circuit. The circuit has 6 nodes as indicated in Fig. 32.

II. Select a reference node. Label it with the reference (ground) symbol.

The bottom left node is connected to 4 nodes while the other ones are connected to three or less elements. Therefore, we select it as the reference node of the circuit.

III. Assign a variable for each node whose voltage is unknown.

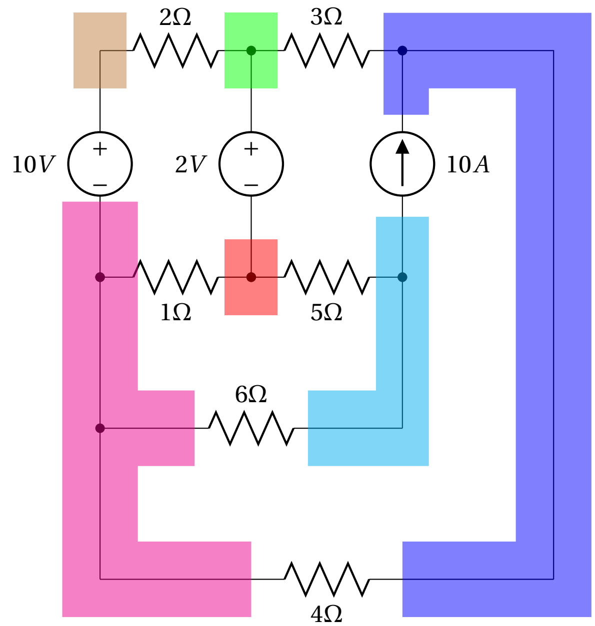


Fig. 32 — Preoblem 6 - All nodes

We label the remaining nodes as shown in Fig. 33. V_1 is connected to the reference node through a voltage source. Therefore, it is equal to the voltage of the voltage source: $V_1 = 10V$.

IV. If there are dependent sources in the circuit, write down equations that express their values in terms of node voltages.

There is no dependent voltage source here.

V. Write down a KCL equation for each node.

Nodes of V_2 and V_3 are connected by a voltage source. Therefore, they form a supernode. The negative terminal of the voltage source is connected to V_3 and the positive terminal is connected to V_2. Thus,

$$V_2 = V_3 + 2.$$

This can also be verified by a KVL around the loop which starts from the reference node, jumps to the node of V_3 with $-V_3$ (the reference is always assumed to be the negative terminal of node voltages), passes through the voltage source by $-2V$ and returns

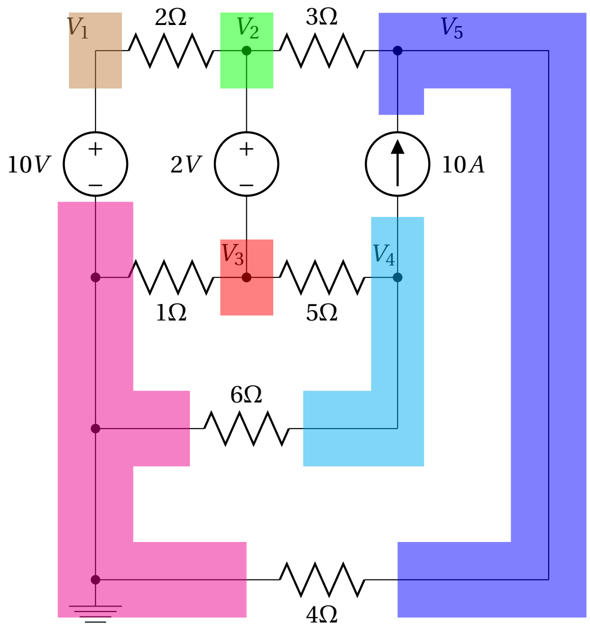


Fig. 33 — Preoblem 6 - The reference node and node voltages

back to the reference node from V_2 as $+V_2$

$$-V_3 - 2V + V_2 = 0 \rightarrow V_2 = V_3 + 2.$$

Supernode of V_2 & V_3:

$$\frac{V_3}{1\Omega} + \frac{V_3 - V_4}{5\Omega} + \frac{V_2 - V_1}{2\Omega} + \frac{V_2 - V_5}{3\Omega} = 0$$
$$\rightarrow V_3 + \frac{V_3}{5} - \frac{V_4}{5} + \frac{V_3 + 2 - 10}{2} + \frac{V_3 + 2 - V_5}{3} = 0$$
$$\rightarrow V_3 + \frac{V_3}{5} - \frac{V_4}{5} + \frac{V_3}{2} - 4 + \frac{V_3}{3} + \frac{2}{3} - \frac{V_5}{3} = 0$$
$$\rightarrow \frac{61}{30} V_3 - \frac{V_4}{5} - \frac{V_5}{3} = \frac{10}{3}$$
$$\rightarrow 61V_3 - 6V_4 - 10V_5 = 100$$

Node of V_4:

$$\frac{V_4}{6\Omega} + \frac{V_4 - V_3}{5\Omega} + 10 = 0$$

$$\rightarrow 5V_4 + 6V_4 - 6V_3 + 300 = 0$$

$$\rightarrow -6V_3 + 11V_4 = -300$$

Node of V_5:

$$\frac{V_5}{4\Omega} + \frac{V_5 - V_2}{3\Omega} - 10 = 0$$

$$\rightarrow 3V_5 + 4V_5 - 4V_2 - 120 = 0$$

$$\rightarrow 7V_5 - 4V_3 - 8 - 120 = 0$$

$$\rightarrow -4V_3 + 7V_5 = 128$$

Hence, we have the following system of equations:

$$\begin{cases} 61V_3 - 6V_4 - 10V_5 = 100 \\ -6V_3 + 11V_4 = -300 \\ -4V_3 + 7V_5 = 128 \end{cases}$$

This system of equations can be solved by any preferred method such as elimination, row reduction, Cramer's rule or other methods. We use the Cramer's rule here:

$$V_3 = \frac{\begin{vmatrix} 100 & -6 & -10 \\ -300 & 11 & 0 \\ 128 & 0 & 7 \end{vmatrix}}{\begin{vmatrix} 61 & -6 & -10 \\ -6 & 11 & 0 \\ -4 & 0 & 7 \end{vmatrix}} = \frac{9180}{4005} = 2.292V$$

$$V_4 = \frac{\begin{vmatrix} 61 & 100 & -10 \\ -6 & -300 & 0 \\ -4 & 128 & 7 \end{vmatrix}}{\begin{vmatrix} 61 & -6 & -10 \\ -6 & 11 & 0 \\ -4 & 0 & 7 \end{vmatrix}} = \frac{-104220}{4005} = -26.022V$$

and

$$V_5 = \frac{\begin{vmatrix} 61 & -6 & 100 \\ -6 & 11 & -300 \\ -4 & 0 & 128 \end{vmatrix}}{\begin{vmatrix} 61 & -6 & -10 \\ -6 & 11 & 0 \\ -4 & 0 & 7 \end{vmatrix}} = \frac{78480}{4005} = 19.595V.$$

Thus,

$$V_2 = V_3 + 2 = 4.292.$$

All node voltages are found. The current of the $10V$ source is the current of the 2Ω resistor, which is $\frac{V_2 - V_1}{2\Omega} = -2.854A$ The current direction shosen such that the current enters from the positive terminal of the voltage source. This is only to comply with the passive sign convention. Now that we have the source current, its power can be easily calculated:
$P_{10V} = 10 \times (-2.854) = -28.54W$ absorbing power

The current of the $2V$ source equals to the summation of the currents of 5Ω and 1Ω resistors. Therefore,

$$I_{2V} = I_{5\Omega} + I_{1\Omega} = \frac{V_3 - V_4}{5\Omega} + \frac{V_3}{1\Omega} = 5.663 + 2.292 = 7.955.$$

Consequently,
$P_{2V} = 2 \times 7.955 = 15.91W$ supplying power.

The voltage across the $10A$ current source is $V_4 - V_5 = -45.617$. Therefore, $P_{10A} = 10 \times (-45.617) = -456.17W$ supplying power.

The PSpice simulation result is indicated in Fig. 34. The PSpice schematics can be downloaded from http://www.solved-problems.com/go/NAPSpice/.

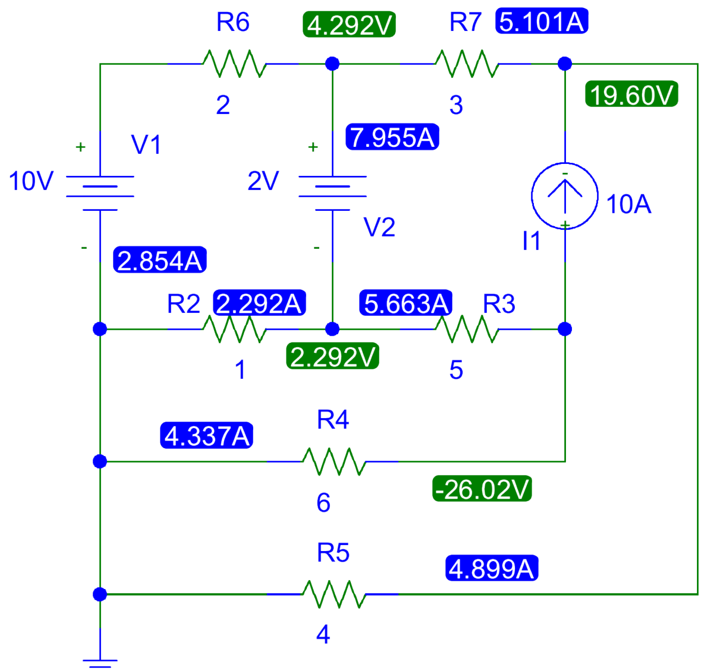


Fig. 34 — Preoblem 6 - PSpice simulation result

Problem 7 - Supernode - Dependent Voltage Source

Discussion
http://www.solved-problems.com/go/1-23/

Determine the power of each source after solving the circuit by the nodal analysis.

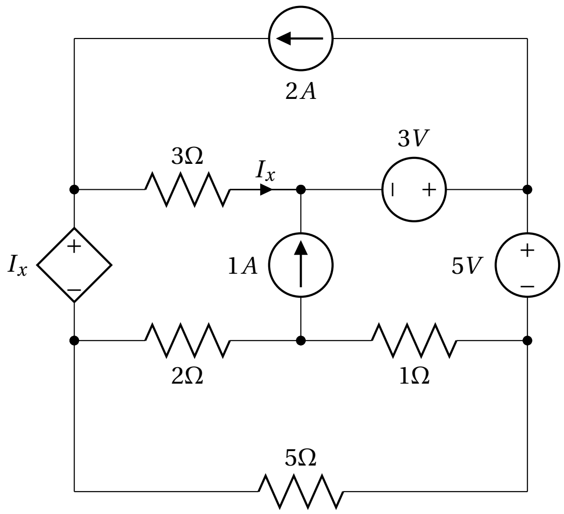


Fig. 35 — Preoblem 7

Answers: $P_{I_x} = 0.497W, P_{1A} = -1.806W, P_{2A} = 4.254W, P_{3V} = -3.87W$, and $P_{5V} = -3.552W$

Solution

I. Identify all nodes in the circuit. The circuit has 6 nodes as indicated in Fig. 36.

II. Select a reference node. Label it with the reference (ground) symbol.

The right top node is connected to two voltage sources and has three elements. All other nodes also have three elements. Hence, we select the right top node because by this selection, we already know the node voltages of two other nodes, i.e. the ones that the reference node is connected to them by voltage sources.

III. Assign a variable for each node whose voltage is unknown. We label the remaining nodes as shown in Fig. 37. Nodes of

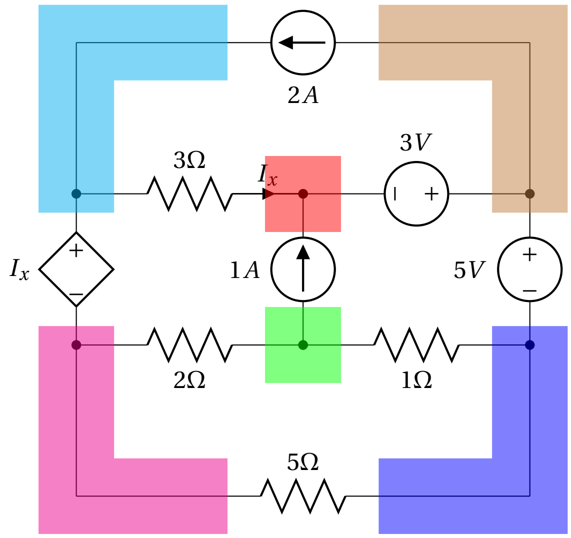


Fig. 36 — Preoblem 7 - All nodes

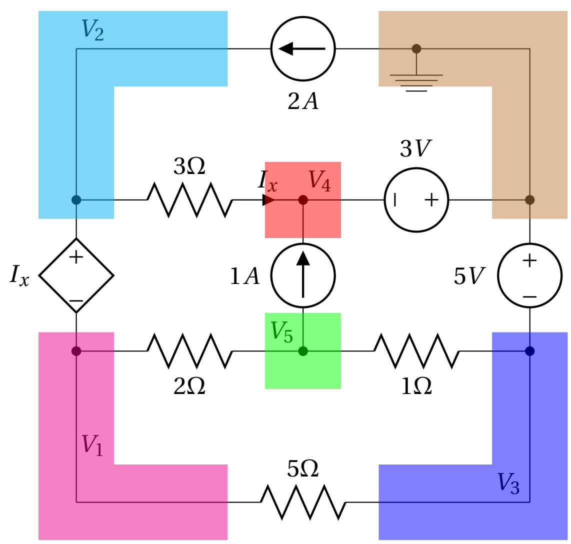


Fig. 37 — Preoblem 7 - The reference node and node voltages

V_3 and V_4 are connected to the reference node through voltage sources. Therefore, V_3 and V_4 can be found easily by the voltages of the voltage sources. For V_3, the negative terminal of the voltage source is connected to the node. Thus, V_3 is equal to minus the source voltage, $V_3 = -5V$. The same argument applies to V_4

and $V_4 = -3V$.

IV. If there are dependent sources in the circuit, write down equations that express their values in terms of node voltages.

The voltage of the dependent voltage source is I_x. We should find this value in terms of the node voltages. I_x is the current of the 3Ω- resistor. The voltage across the resistor is $V_2 - V_4$. We prefer to define $V_{3\Omega}$ as $V_2 - V_4$ instead of $V_4 - V_2$ to comply with passive sign convention. By defining $V_{3\Omega}$ as mentioned, I_x is entering from the positive terminal of $V_{3\Omega}$ and we have $V_{3\Omega} = 3\Omega \times I_x$. Therefore, $I_x = \frac{V_2 - V_4}{3\Omega}$.

$$\rightarrow I_x = \frac{V_2}{3} + 1$$

V. Write down a KCL equation for each node.

Nodes of V_1 and V_2: These two nodes are connected through a voltage source. Therefore, they form a supernode and we can write the voltage of one in terms of the voltage of the other one. Please note that the voltage of the dependent voltage source is I_x and we have

$$V_2 = V_1 + I_x$$

$$\rightarrow V_2 = V_1 + \frac{V_2}{3} + 1 \rightarrow \frac{2}{3}V_2 = V_1 + 1$$

$$\rightarrow V_1 = frac23V_2 - 1$$

KCL for the supernode:

$$\frac{V_1 - V_3}{5\Omega} + \frac{V_1 - V_5}{2\Omega} + \frac{V_2 - V_4}{3\Omega} - 2A = 0$$

$$\rightarrow \frac{V_1 + 5}{5} + \frac{V_1 - V_5}{2} + \frac{V_2 + 3}{3} - 2 = 0$$

$$\rightarrow \frac{V_1}{5} + \frac{V_1 - V_5}{2} + \frac{V_2}{3} = 0$$

$$\rightarrow 21V_1 - 15V_5 + 10V_2 = 0$$

Substituting $V_1 = frac23V_2 - 1$,

$$\rightarrow 8V_2 - 5V_5 = 7$$

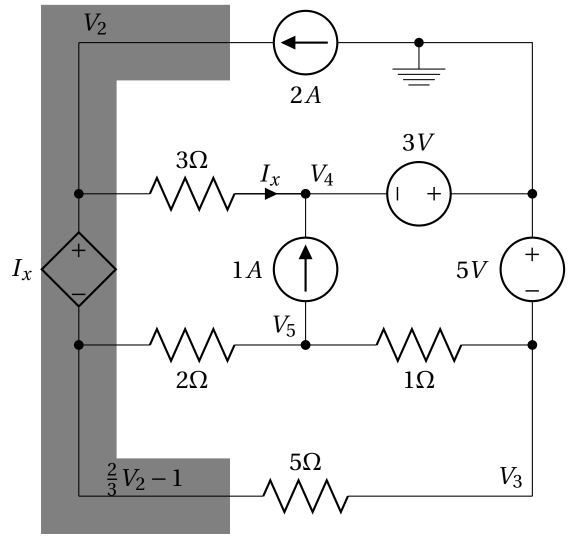


Fig. 38 — Preoblem 7 - The supernode

Node of V_5:

$$\frac{V_5 - V_1}{2\Omega} + \frac{V_5 - V_3}{1\Omega} + 1 = 0$$
$$\rightarrow -V_1 + 3V_5 - 2V_3 + 2 = 0$$

Substituting $V_1 = frac23V_2 - 1$ and $V_3 = -5V$,

$$\rightarrow -2V_2 + 9V_5 = -39$$

Here is the system of equations that we need to solve and obtain V_2 nd V_5:

$$\begin{cases} I : 8V_2 - 5V_5 = 7 \\ II : -2V_2 + 9V_5 = -39 \end{cases}$$

We use elimination to solve this system of equation: $(II) \times 4 + (I)$: $31V_5 = -149 \rightarrow$

$$V_5 = -4.806V$$

$V_2 = \frac{9V_5+39}{2} \rightarrow$

$$V_2 = -2.127V$$

Using $V_1 = frac23V_2 - 1$,

$$V_1 = -2.418V$$

All node voltages are determined. Now, the power of voltage sources can be calculated from the node voltages. For each source, we need to find the voltage across the source as well as the current flowing through it to compute the power.

$2A$ current source: The voltage across the $2A$ current source is equal to V_2. However, the comply with the passive voltage convention, the current should be entering from the positive terminal of the defined voltage as shown in Fig. 39. Therefore, $V_{2A} = -V_2 = 2.127V$.

$P_{2A} = 2A \times V_{2A} = 4.254W$ absorbing power

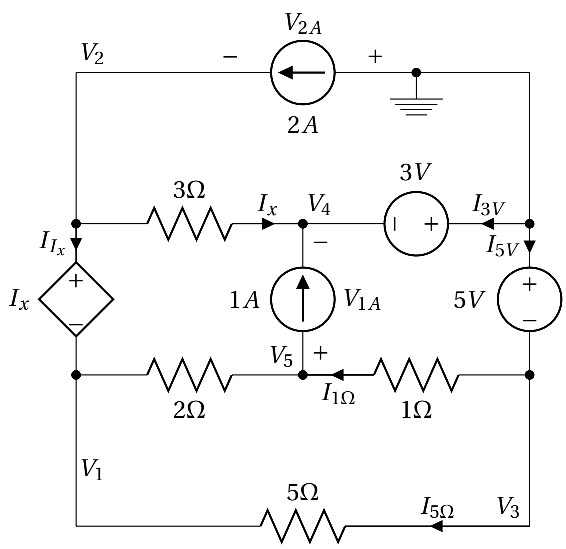


Fig. 39 — Preoblem 7 - Current directions and voltage polarities for sources

$1A$ current source:

To compliant with the passive sign convention, the voltage V_{1A} should be defined with polarity as indicated in Fig. 39. We have $V_{1A} = V_5 - V_4 = -1.806V$. Hence,

$P_{1A} = 1A \times (-1.806V) = -1.806W$ supplying power.

$5V$ voltage source:

I_{5V} should be defined such that it enters from the positive terminal of the source in order to use the voltage of the source in power calculation. Another option is to use V_3 and define the current as entering from the voltage source terminal connected to the node of V_3. We use the first approach here. KCL should be applied in the node of V_3 to determine I_{5V}.

KCL @ Node of V_3:

$$-I_{5V} + I_{1\Omega} + I_{5\Omega} = 0$$
$$\rightarrow I_{5V} = \frac{V_3 - V_5}{1\Omega} + \frac{V_3 - V_1}{5\Omega} = -0.7104A$$

$P_{5V} = 5V \times (-0.7104A) = -3.552W$ supplying power.

$3V$ voltage source:

Likewise, I_{3V} should be defined as shown in Fig. 39 to comply with the passive sign convention. We apply KCL to the reference node to find I_{3V}.

KCL @ the reference node:

$$I_{5V} + I_{3V} + 2A = 0$$
$$\rightarrow I_{3V} = -1.29A$$

$P_{3V} = 3V \times (-1.29A) = -3.87W$ supplying power.

The dependent source: The voltage of the dependent source is I_x and we define its current I_{I_x} with the direction illustrated in Fig. 39. I_{I_x} can be calculated by applying KCL at the node of V_2. The current of the 3Ω resistor is I_x which is equal to $\frac{V_2}{3} + 1 = 0.291A$.

KCL @ Node of V_2:

$$-2A + I_x + I_{I_x} = 0 \rightarrow I_{I_x} = 1.709A$$

$P_{I_x} = I_x \times I_{I_x} = 0.291V \times 1.709A = 0.497W$ absorbing power.

The PSpice simulation result is indicated in Fig. 40. The PSpice schematics can be downloaded from http://www.solved-problems.com/go/NAPSpice/.

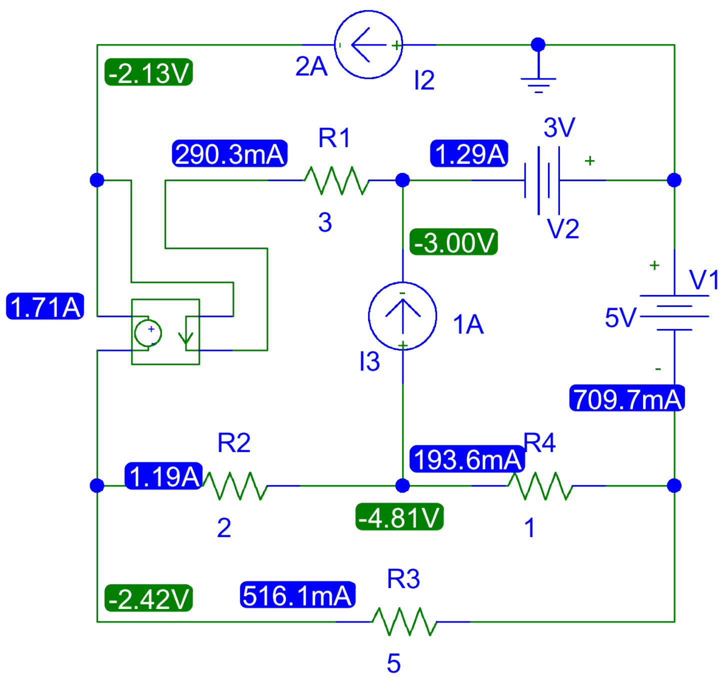


Fig. 40 — Preoblem 7 - PSpice simulation result