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3 Nodal and Loop Analysis Techniques

LEARNING GOALS

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In Chapter 2 we analyzed the simplest possible circuits, those containing only a single-node pair or a single loop. We found that these circuits can be completely analyzed via a single algebraic equation. In the case of the single-node-pair circuit (i.e., one containing two nodes, one of which is a reference node), once the node voltage is known, we can calculate all the currents. In a single-loop circuit, once the loop current is known, we can calculate all the voltages.

In this chapter we extend our capabilities in a systematic manner so that we can calculate all currents and voltages in circuits that contain multiple nodes and loops. Our analyses are based primarily on two laws with which we are already familiar: Kirchhoff's current law (KCL) and Kirchhoff's voltage law (KVL). In a nodal analysis we employ KCL to determine the node voltages, and in a loop analysis we use KVL to determine the loop currents.

3.1 Nodal analysis

In a nodal analysis the variables in the circuit are selected to be the node voltages. The node voltages are defined with respect to a common point in the circuit. One node is selected as the reference node, and all other node voltages are defined with respect to that node. Quite often this node is the one to which the largest number of branches are connected. It is commonly called *ground* because it is said to be at ground-zero potential, and it sometimes represents the chassis or ground line in a practical circuit.

We will select our variables as being positive with respect to the reference node. If one or more of the node voltages are actually negative with respect to the reference node, the analysis will indicate it.

In order to understand the value of knowing all the node voltages in a network, we consider once again the network in Fig. 2.32, which is redrawn in Fig. 3.1. The voltages, V_S, V_a, V_b, and V_c, are all measured with respect to the bottom node, which is selected as the reference and labeled with the ground symbol ⏚. Therefore, the voltage at node 1 is $V_S = 12$ V with respect to the reference node 5; the voltage at node 2 is $V_a = 3$ V with respect to the reference node 5, and so on. Now note carefully that once these node voltages are known, we can immediately calculate any branch current or the power supplied or absorbed by any element, since we know the voltage across every element in the network. For example, the voltage V_1 across the leftmost 9-kΩ resistor is the difference in potential between the two ends of the resistor; that is,

$$\begin{aligned} V_1 &= V_S - V_a \\ &= 12 - 3 \\ &= 9 \text{ V} \end{aligned}$$

This equation is really nothing more than an application of KVL around the leftmost loop; that is,

$$-V_S + V_1 + V_a = 0$$

In a similar manner, we find that

$$V_3 = V_a - V_b$$

and

$$V_5 = V_b - V_c$$

Then the currents in the resistors are

$$I_1 = \frac{V_1}{9\text{k}} = \frac{V_S - V_a}{9\text{k}}$$

$$I_3 = \frac{V_3}{3\text{k}} = \frac{V_a - V_b}{3\text{k}}$$

$$I_5 = \frac{V_5}{9\text{k}} = \frac{V_b - V_c}{9\text{k}}$$

In addition,

$$I_2 = \frac{V_a - 0}{6\text{k}}$$

$$I_4 = \frac{V_b - 0}{4\text{k}}$$

since the reference node 5 is at zero potential.

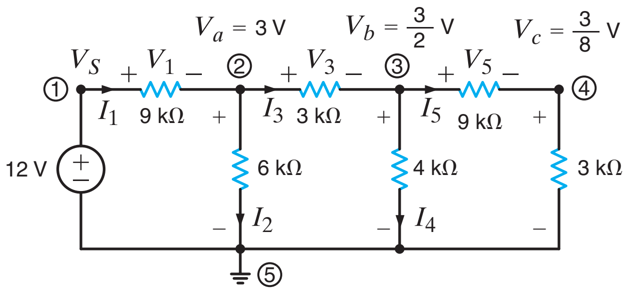


Figure 3.1
Circuit with known node voltages.

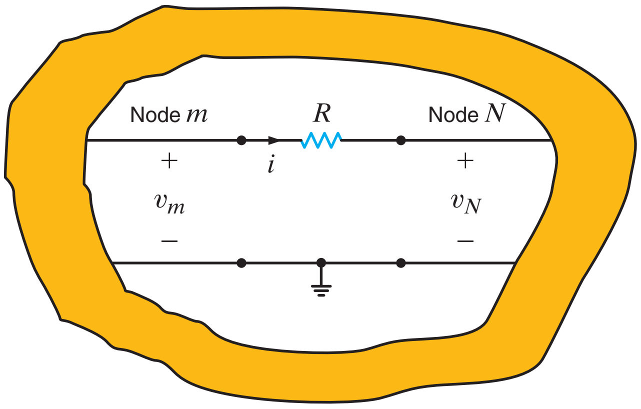


Figure 3.2
Circuit used to illustrate Ohm's law in a multiple-node network.

Thus, as a general rule, if we know the node voltages in a circuit, we can calculate the current through any resistive element using Ohm's law; that is,

$$i = \frac{v_m - v_N}{R} \qquad \textbf{3.1}$$

as illustrated in Fig. 3.2.

Now that we have demonstrated the value of knowing all the node voltages in a network, let us determine the manner in which to calculate them. In a nodal analysis, we employ KCL equations in such a way that the variables contained in these equations are the unknown node voltages of the network. As we have indicated, one of the nodes in an N-node circuit is selected as the reference node, and the voltages at all the remaining $N - 1$ nonreference nodes are measured with respect to this reference node. Using network topology, it can be shown that exactly $N - 1$ linearly independent KCL equations are required to determine the $N - 1$ unknown node voltages. Therefore, theoretically once one of the nodes in an N-node circuit has been selected as the reference node, our task is reduced to identifying the remaining $N - 1$ nonreference nodes and writing one KCL equation at each of them.

In a multiple-node circuit, this process results in a set of $N - 1$ linearly independent simultaneous equations in which the variables are the $N - 1$ unknown node voltages. To help solidify this idea, consider once again Example 2.5. Note that in this circuit only four (i.e., any four) of the five KCL equations, one of which is written for each node in this five-node network, are linearly independent. Furthermore, many of the branch currents in this example (those not contained in a source) can be written in terms of the node voltages as illustrated in Fig. 3.2 and expressed in Eq. (3.1). It is in this manner, as we will illustrate in the sections that follow, that the KCL equations contain the unknown node voltages.

It is instructive to treat nodal analysis by examining several different types of circuits and illustrating the salient features of each. We begin with the simplest case. However, as a prelude to our discussion of the details of nodal analysis, experience indicates that it is worthwhile to digress for a moment to ensure that the concept of node voltage is clearly understood.

At the outset it is important to specify a reference. For example, to state that the voltage at node A is 12 V means nothing unless we provide the reference point; that is, the voltage at node A is 12 V with respect to what? The circuit in Fig. 3.3 illustrates a portion of a network containing three nodes, one of which is the reference node.

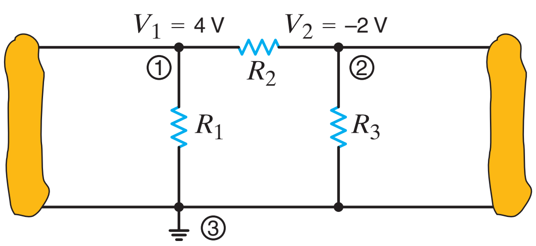


Figure 3.3
An illustration of node voltages.

The voltage $V_1 = 4$ V is the voltage at node 1 with respect to the reference node 3. Similarly, the voltage $V_2 = -2$ V is the voltage at node 2 with respect to node 3. In addition, however, the voltage at node 1 with respect to node 2 is +6 V, and the voltage at node 2 with respect to node 1 is −6 V. Furthermore, since the current will flow from the node of higher potential to the node of lower potential, the current in R_1 is from top to bottom, the current in R_2 is from left to right, and the current in R_3 is from bottom to top.

These concepts have important ramifications in our daily lives. If a man were hanging in midair with one hand on one line and one hand on another and the dc line voltage of each line was exactly the same, the voltage across his heart would be zero and he would be safe. If, however, he let go of one line and let his feet touch the ground, the dc line voltage would then exist from his hand to his foot with his heart in the middle. He would probably be dead the instant his foot hit the ground.

In the town where we live, a young man tried to retrieve his parakeet that had escaped its cage and was outside sitting on a power line. He stood on a metal ladder and with a metal pole reached for the parakeet; when the metal pole touched the power line, the man was killed instantly. Electric power is vital to our standard of living, but it is also very dangerous. The material in this book *does not* qualify you to handle it safely. Therefore, always be extremely careful around electric circuits.

Now as we begin our discussion of nodal analysis, our approach will be to begin with simple cases and proceed in a systematic manner to those that are more challenging. Numerous examples will be the vehicle used to demonstrate each facet of this approach. Finally, at the end of this section, we will outline a strategy for attacking any circuit using nodal analysis.

CIRCUITS CONTAINING ONLY INDEPENDENT CURRENT SOURCES Consider the network shown in Fig. 3.4. Note that this network contains three nodes, and thus we know that exactly $N - 1 = 3 - 1 = 2$ linearly independent KCL equations will be required to determine the $N - 1 = 2$ unknown node voltages. First, we select the bottom node as the reference node, and then the voltage at the two remaining nodes labeled v_1 and v_2 will be measured with respect to this node.

The branch currents are assumed to flow in the directions indicated in the figures. If one or more of the branch currents are actually flowing in a direction opposite to that assumed, the analysis will simply produce a branch current that is negative.

Applying KCL at node 1 yields

$$-i_A + i_1 + i_2 = 0$$

Using Ohm's law $(i = Gv)$ and noting that the reference node is at zero potential, we obtain

$$-i_A + G_1(v_1 - 0) + G_2(v_1 - v_2) = 0$$

or

$$(G_1 + G_2)v_1 - G_2 v_2 = i_A$$

HINT
Employing the passive sign convention.

KCL at node 2 yields

$$-i_2 + i_B + i_3 = 0$$

or

$$-G_2(v_1 - v_2) + i_B + G_3(v_2 - 0) = 0$$

which can be expressed as

$$-G_2 v_1 + (G_2 + G_3)v_2 = -i_B$$

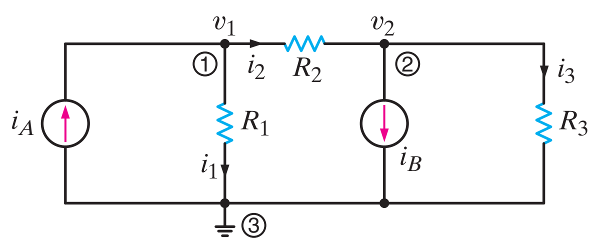


Figure 3.4
A three-node circuit.

Therefore, the two equations for the two unknown node voltages v_1 and v_2 are

$$\begin{aligned}(G_1 + G_2)v_1 - G_2 v_2 &= i_A \\ -G_2 v_1 + (G_2 + G_3)v_2 &= -i_B\end{aligned} \qquad \mathbf{3.2}$$

Note that the analysis has produced two simultaneous equations in the unknowns v_1 and v_2. They can be solved using any convenient technique, and modern calculators and personal computers are very efficient tools for their application.

In what follows, we will demonstrate three techniques for solving linearly independent simultaneous equations: Gaussian elimination, matrix analysis, and the MATLAB mathematical software package. A brief refresher that illustrates the use of both Gaussian elimination and matrix analysis in the solution of these equations is provided in the Problem-Solving Companion for this text. Use of the MATLAB software is straightforward, and we will demonstrate its use as we encounter the application.

The KCL equations at nodes 1 and 2 produced two linearly independent simultaneous equations:

$$-i_A + i_1 + i_2 = 0$$

$$-i_2 + i_B + i_3 = 0$$

The KCL equation for the third node (reference) is

$$+i_A - i_1 - i_B - i_3 = 0$$

Note that if we add the first two equations, we obtain the third. Furthermore, any two of the equations can be used to derive the remaining equation. Therefore, in this $N = 3$ node circuit, only $N - 1 = 2$ of the equations are linearly independent and required to determine the $N - 1 = 2$ unknown node voltages.

Note that a nodal analysis employs KCL in conjunction with Ohm's law. Once the direction of the branch currents has been *assumed*, then Ohm's law, as illustrated by Fig. 3.2 and expressed by Eq. (3.1), is used to express the branch currents in terms of the unknown node voltages. We can assume the currents to be in any direction. However, once we assume a particular direction, we must be very careful to write the currents correctly in terms of the node voltages using Ohm's law.

Example 3.1

Suppose that the network in Fig. 3.4 has the following parameters: $I_A = 1$ mA, $R_1 = 12$ kΩ, $R_2 = 6$ kΩ, $I_B = 4$ mA, and $R_3 = 6$ kΩ. Let us determine all node voltages and branch currents.

SOLUTION For purposes of illustration we will solve this problem using Gaussian elimination, matrix analysis, and MATLAB. Using the parameter values Eq. (3.2) becomes

$$V_1\left[\frac{1}{12\text{k}} + \frac{1}{6\text{k}}\right] - V_2\left[\frac{1}{6\text{k}}\right] = 1 \times 10^{-3}$$

$$-V_1\left[\frac{1}{6\text{k}}\right] + V_2\left[\frac{1}{6\text{k}} + \frac{1}{6\text{k}}\right] = -4 \times 10^{-3}$$

where we employ capital letters because the voltages are constant. The equations can be written as

$$\frac{V_1}{4\text{k}} - \frac{V_2}{6\text{k}} = 1 \times 10^{-3}$$

$$-\frac{V_1}{6\text{k}} + \frac{V_2}{3\text{k}} = -4 \times 10^{-3}$$

Using Gaussian elimination, we solve the first equation for V_1 in terms of V_2:

$$V_1 = V_2\left(\frac{2}{3}\right) + 4$$

This value is then substituted into the second equation to yield

$$\frac{-1}{6\text{k}}\left(\frac{2}{3}V_2 + 4\right) + \frac{V_2}{3\text{k}} = -4 \times 10^{-3}$$

or

$$V_2 = -15 \text{ V}$$

This value for V_2 is now substituted back into the equation for V_1 in terms of V_2, which yields

$$\begin{aligned} V_1 &= \frac{2}{3}V_2 + 4 \\ &= -6 \text{ V} \end{aligned}$$

The circuit equations can also be solved using matrix analysis. The general form of the matrix equation is

$$\mathbf{GV} = \mathbf{I}$$

where in this case

$$\mathbf{G} = \begin{bmatrix} \frac{1}{4\text{k}} & -\frac{1}{6\text{k}} \\ -\frac{1}{6\text{k}} & \frac{1}{3\text{k}} \end{bmatrix}, \mathbf{V} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}, \text{ and } \mathbf{I} = \begin{bmatrix} 1 \times 10^{-3} \\ -4 \times 10^{-3} \end{bmatrix}$$

The solution to the matrix equation is

$$\mathbf{V} = \mathbf{G}^{-1}\mathbf{I}$$

and therefore,

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{4\text{k}} & \frac{-1}{6\text{k}} \\ \frac{-1}{6\text{k}} & \frac{1}{3\text{k}} \end{bmatrix}^{-1} \begin{bmatrix} 1 \times 10^{-3} \\ -4 \times 10^{-3} \end{bmatrix}$$

To calculate the inverse of $\mathbf{G}$, we need the adjoint and the determinant. The adjoint is

$$\text{Adj } \mathbf{G} = \begin{bmatrix} \frac{1}{3\text{k}} & \frac{1}{6\text{k}} \\ \frac{1}{6\text{k}} & \frac{1}{4\text{k}} \end{bmatrix}$$

and the determinant is

$$\begin{aligned} |\mathbf{G}| &= \left(\frac{1}{3\text{k}}\right)\left(\frac{1}{4\text{k}}\right) - \left(\frac{-1}{6\text{k}}\right)\left(\frac{-1}{6\text{k}}\right) \\ &= \frac{1}{18\text{k}^2} \end{aligned}$$

Therefore,

$$\begin{aligned} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} &= 18\text{k}^2 \begin{bmatrix} \frac{1}{3\text{k}} & \frac{1}{6\text{k}} \\ \frac{1}{6\text{k}} & \frac{1}{4\text{k}} \end{bmatrix} \begin{bmatrix} 1 \times 10^{-3} \\ -4 \times 10^{-3} \end{bmatrix} \\ &= 18\text{k}^2 \begin{bmatrix} \frac{1}{3\text{k}^2} - \frac{4}{6\text{k}^2} \\ \frac{1}{6\text{k}^2} - \frac{1}{\text{k}^2} \end{bmatrix} \\ &= \begin{bmatrix} -6 \\ -15 \end{bmatrix} \end{aligned}$$

The MATLAB solution begins with the set of equations expressed in matrix form as

```
G*V=I
```

where the symbol * denotes the multiplication of the voltage vector **V** by the coefficient matrix **G**. Then once the MATLAB software is loaded into the PC, the coefficient matrix **(G)** and the vector **V** can be expressed in MATLAB notation by typing in the rows of the matrix or vector at the prompt >>. Use semicolons to separate rows and spaces to separate columns. Brackets are used to denote vectors or matrices. When the matrix **G** and the vector **I** have been defined, then the solution equation

```
V=inv(G)*I
```

which is also typed in at the prompt >>, will yield the unknown vector **V.**

The matrix equation for our circuit expressed in decimal notation is

$$\begin{bmatrix} 0.00025 & -0.00016666 \\ -0.00016666 & 0.0003333 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0.001 \\ -0.004 \end{bmatrix}$$

If we now input the coefficient matrix **G**, then the vector **I** and finally the equation V = inv(G)*I, the computer screen containing these data and the solution vector **V** appears as follows:

```
>> G = [0.00025 -0.000166666;
-0.000166666 0.00033333]
   G =
       1.0e-003 *
       0.2500      -0.1667
      -0.1667       0.3333
>> I = [0.001 ; -0.004]
   I =
       0.0010
      -0.0040
>> V = inv(G)*I
   V =
      -6.0001
     -15.0002
```

Knowing the node voltages, we can determine all the currents using Ohm's law:

$$I_1 = \frac{V_1}{R_1} = \frac{-6}{12\text{k}} = -\frac{1}{2}\text{ mA}$$

$$I_2 = \frac{V_1 - V_2}{6\text{k}} = \frac{-6-(-15)}{6\text{k}} = \frac{3}{2}\text{ mA}$$

and

$$I_3 = \frac{V_2}{6\text{k}} = \frac{-15}{6\text{k}} = -\frac{5}{2}\text{ mA}$$

Figure 3.5 illustrates the results of all the calculations. Note that KCL is satisfied at every node.

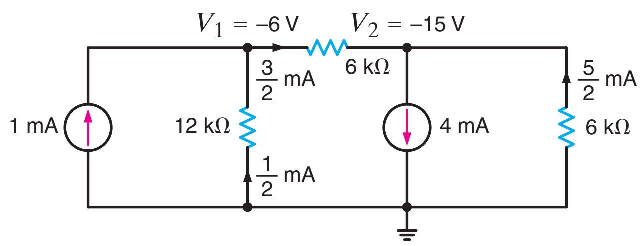


Figure 3.5 *Circuit used in Example 3.1.*

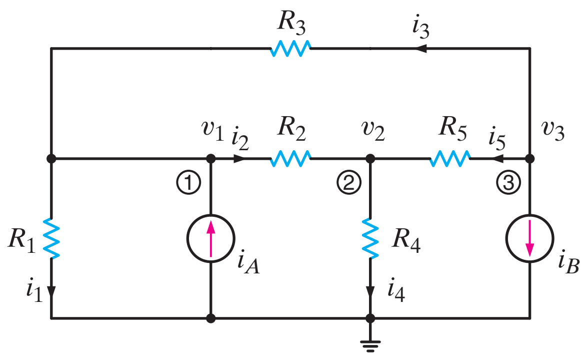


Figure 3.6
A four-node circuit.

Let us now examine the circuit in Fig. 3.6. The current directions are assumed as shown in the figure.

We note that this network has four nodes. The node at the bottom of the circuit is selected as the reference node and labeled with the ground symbol. Since $N = 4$, $N - 1 = 3$ linearly independent KCL equations will be required to determine the three unknown nonreference node voltages labeled v_1, v_2, and v_3.

At node 1, KCL yields

$$i_1 - i_A + i_2 - i_3 = 0$$

or

$$\frac{v_1}{R_1} - i_A + \frac{v_1 - v_2}{R_2} - \frac{v_3 - v_1}{R_3} = 0$$

$$v_1\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) - v_2\frac{1}{R_2} - v_3\frac{1}{R_3} = i_A$$

At node 2, KCL yields

$$-i_2 + i_4 - i_5 = 0$$

or

$$-\frac{v_1 - v_2}{R_2} + \frac{v_2}{R_4} - \frac{v_3 - v_2}{R_5} = 0$$

$$-v_1\frac{1}{R_2} + v_2\left(\frac{1}{R_2} + \frac{1}{R_4} + \frac{1}{R_5}\right) - v_3\frac{1}{R_5} = 0$$

At node 3, the equation is

$$i_3 + i_5 + i_B = 0$$

or

$$\frac{v_3 - v_1}{R_3} + \frac{v_3 - v_2}{R_5} + i_B = 0$$

$$-v_1\frac{1}{R_3} - v_2\frac{1}{R_5} + v_3\left(\frac{1}{R_3} + \frac{1}{R_5}\right) = -i_B$$

Grouping the node equations together, we obtain

$$\begin{aligned} v_1\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) - v_2\frac{1}{R_2} - v_3\frac{1}{R_3} &= i_A \\ -v_1\frac{1}{R_2} + v_2\left(\frac{1}{R_2} + \frac{1}{R_4} + \frac{1}{R_5}\right) - v_3\frac{1}{R_5} &= 0 \\ -v_1\frac{1}{R_3} - v_2\frac{1}{R_5} + v_3\left(\frac{1}{R_3} + \frac{1}{R_5}\right) &= -i_B \end{aligned} \qquad \mathbf{3.3}$$

Note that our analysis has produced three simultaneous equations in the three unknown node voltages v_1, v_2, and v_3. The equations can also be written in matrix form as

$$\begin{bmatrix} \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3} & -\frac{1}{R_2} & -\frac{1}{R_3} \\ -\frac{1}{R_2} & \frac{1}{R_2}+\frac{1}{R_4}+\frac{1}{R_5} & -\frac{1}{R_5} \\ -\frac{1}{R_3} & -\frac{1}{R_5} & \frac{1}{R_3}+\frac{1}{R_5} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} i_A \\ 0 \\ -i_B \end{bmatrix} \tag{3.4}$$

At this point it is important that we note the symmetrical form of the equations that describe the two previous networks. Equations (3.2) and (3.3) exhibit the same type of symmetrical form. The **G** matrix for each network is a symmetrical matrix. This symmetry is not accidental. The node equations for networks containing only resistors and independent current sources can always be written in this symmetrical form. We can take advantage of this fact and learn to write the equations by inspection. Note in the first equation of (3.2) that the coefficient of v_1 is the sum of all the conductances connected to node 1 and the coefficient of v_2 is the negative of the conductances connected between node 1 and node 2. The right-hand side of the equation is the sum of the currents entering node 1 through current sources. This equation is KCL at node 1. In the second equation in (3.2), the coefficient of v_2 is the sum of all the conductances connected to node 2, the coefficient of v_1 is the negative of the conductance connected between node 2 and node 1, and the right-hand side of the equation is the sum of the currents entering node 2 through current sources. This equation is KCL at node 2. Similarly, in the first equation in (3.3) the coefficient of v_1 is the sum of the conductances connected to node 1, the coefficient of v_2 is the negative of the conductance connected between node 1 and node 2, the coefficient of v_3 is the negative of the conductance connected between node 1 and node 3, and the right-hand side of the equation is the sum of the currents entering node 1 through current sources. The other two equations in (3.3) are obtained in a similar manner. In general, if KCL is applied to node j with node voltage v_j, the coefficient of v_j is the sum of all the conductances connected to node j and the coefficients of the other node voltages $\left(\text{e.g., } v_{j-1}, v_{j+1}\right)$ are the negative of the sum of the conductances connected directly between these nodes and node j. The right-hand side of the equation is equal to the sum of the currents entering the node via current sources. Therefore, the left-hand side of the equation represents the sum of the currents leaving node j and the right-hand side of the equation represents the currents entering node j.

Example 3.2

Let us apply what we have just learned to write the equations for the network in Fig. 3.7 by inspection. Then given the following parameters, we will determine the node voltages using MATLAB: $R_1 = R_2 = 2\text{ k}\Omega$, $R_3 = R_4 = 4\text{ k}\Omega$, $R_5 = 1\text{ k}\Omega$, $i_A = 4\text{ mA}$, and $i_B = 2\text{ mA}$.

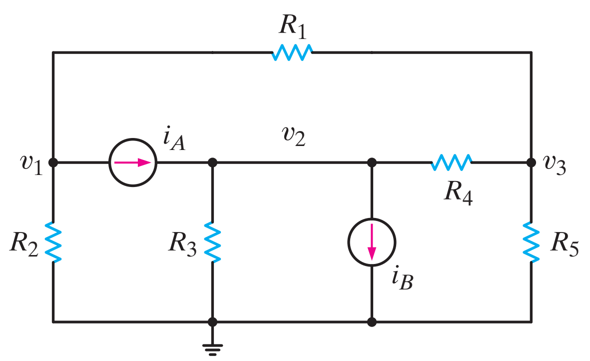


Figure 3.7 *Circuit used in Example 3.2.*

SOLUTION The equations are

$$v_1\left(\frac{1}{R_1}+\frac{1}{R_2}\right) - v_2(0) - v_3\left(\frac{1}{R_1}\right) = -i_A$$

$$-v_1(0) + v_2\left(\frac{1}{R_3}+\frac{1}{R_4}\right) - v_3\left(\frac{1}{R_4}\right) = i_A - i_B$$

$$-v_1\left(\frac{1}{R_1}\right) - v_2\left(\frac{1}{R_4}\right) + v_3\left(\frac{1}{R_1}+\frac{1}{R_4}+\frac{1}{R_5}\right) = 0$$

which can also be written directly in matrix form as

$$\begin{bmatrix} \frac{1}{R_1}+\frac{1}{R_2} & 0 & -\frac{1}{R_1} \\ 0 & \frac{1}{R_3}+\frac{1}{R_4} & -\frac{1}{R_4} \\ -\frac{1}{R_1} & -\frac{1}{R_4} & \frac{1}{R_1}+\frac{1}{R_4}+\frac{1}{R_5} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} -i_A \\ i_A - i_B \\ 0 \end{bmatrix}$$

Both the equations and the **G** matrix exhibit the symmetry that will always be present in circuits that contain only resistors and current sources.

If the component values are now used, the matrix equation becomes

$$\begin{bmatrix} \frac{1}{2\text{k}}+\frac{1}{2\text{k}} & 0 & -\frac{1}{2\text{k}} \\ 0 & \frac{1}{4\text{k}}+\frac{1}{4\text{k}} & -\frac{1}{4\text{k}} \\ -\frac{1}{2\text{k}} & -\frac{1}{4\text{k}} & \frac{1}{2\text{k}}+\frac{1}{4\text{k}}+\frac{1}{1\text{k}} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} -0.004 \\ 0.002 \\ 0 \end{bmatrix}$$

or

$$\begin{bmatrix} 0.001 & 0 & -0.0005 \\ 0 & 0.0005 & -0.00025 \\ -0.0005 & -0.00025 & 0.00175 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} -0.004 \\ 0.002 \\ 0 \end{bmatrix}$$

If we now employ these data with the MATLAB software, the computer screen containing the data and the results of the MATLAB analysis is as shown next.

```
>> G = [0.001 0 -0.0005 ; 0 0.0005 -0.00025 ;
-0.0005 -0.00025 0.00175]

   G =

       0.0010          0      -0.0005
            0     0.0005      -0.0003
      -0.0005    -0.0003       0.0018

>> I = [-0.004 ; 0.002 ; 0]

   I =

      -0.0040
       0.0020
            0

>> V = inv(G)*I

   V =

      -4.3636
       3.6364
      -0.7273
```

LEARNING EXTENSIONS

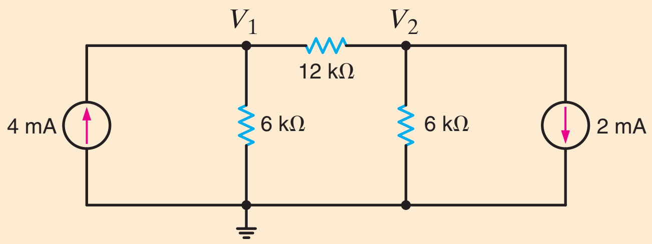
E3.1 Write the node equations for the circuit in Fig. E3.1.

ANSWER:

$$\frac{1}{4\text{k}} V_1 - \frac{1}{12\text{k}} V_2 = 4 \times 10^{-3},$$

$$\frac{-1}{12\text{k}} V_1 + \frac{1}{4\text{k}} V_2 = -2 \times 10^{-3}.$$

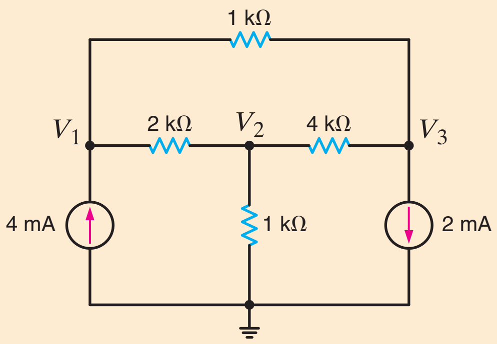
Figure E3.1



E3.2 Find all the node voltages in the network in Fig. E3.2 using MATLAB.

ANSWER: $V_1 = 5.4286$ V, $V_2 = 2.000$ V, $V_3 = 3.1429$ V.

Figure E3.2



CIRCUITS CONTAINING DEPENDENT CURRENT SOURCES The presence of a dependent source may destroy the symmetrical form of the nodal equations that define the circuit. Consider the circuit shown in Fig. 3.8, which contains a current-controlled current source. The KCL equations for the nonreference nodes are

$$\beta i_o + \frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2} = 0$$

and

$$\frac{v_2 - v_1}{R_2} + i_o - i_A = 0$$

where $i_o = v_2/R_3$. Simplifying the equations, we obtain

$$(G_1 + G_2)v_1 - (G_2 - \beta G_3)v_2 = 0$$

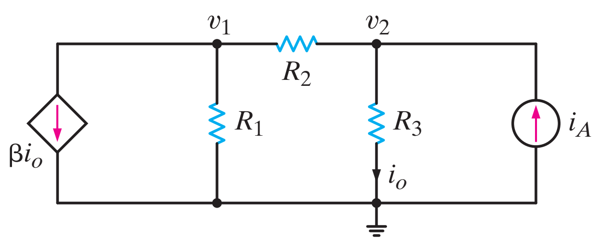
$$-G_2 v_1 + (G_2 + G_3)v_2 = i_A$$

or in matrix form

$$\begin{bmatrix} (G_1 + G_2) & -(G_2 - \beta G_3) \\ -G_2 & (G_2 + G_3) \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ i_A \end{bmatrix}$$

Note that the presence of the dependent source has destroyed the symmetrical nature of the node equations.

Figure 3.8
Circuit with a dependent source.



Example 3.3

Let us determine the node voltages for the network in Fig. 3.8 given the following parameters:

$$\beta = 2 \qquad R_2 = 6\ \text{k}\Omega \qquad i_A = 2\ \text{mA}$$
$$R_1 = 12\ \text{k}\Omega \qquad R_3 = 3\ \text{k}\Omega$$

SOLUTION Using these values with the equations for the network yields

$$\frac{1}{4\text{k}}V_1 + \frac{1}{2\text{k}}V_2 = 0$$
$$-\frac{1}{6\text{k}}V_1 + \frac{1}{2\text{k}}V_2 = 2 \times 10^{-3}$$

Solving these equations using any convenient method yields $V_1 = -24/5$ V and $V_2 = 12/5$ V. We can check these answers by determining the branch currents in the network and then using that information to test KCL at the nodes. For example, the current from top to bottom through R_3 is

$$I_o = \frac{V_2}{R_3} = \frac{12/5}{3\text{k}} = \frac{4}{5\text{k}}\ \text{A}$$

Similarly, the current from right to left through R_2 is

$$I_2 = \frac{V_2 - V_1}{R_2} = \frac{12/5 - (-24/5)}{6\text{k}} = \frac{6}{5\text{k}}\ \text{A}$$

All the results are shown in Fig. 3.9. Note that KCL is satisfied at every node.

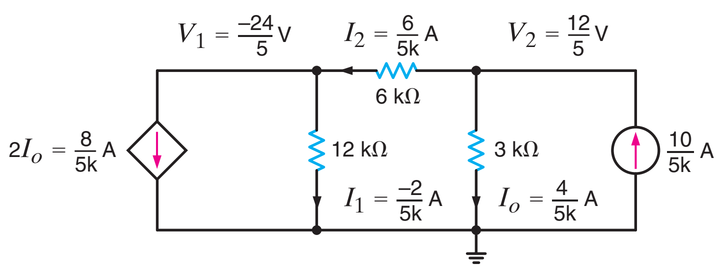


Figure 3.9
Circuit used in Example 3.3.

Example 3.4

Let us determine the set of linearly independent equations that when solved will yield the node voltages in the network in Fig. 3.10. Then given the following component values, we will compute the node voltages using MATLAB: $R_1 = 1\ \text{k}\Omega$, $R_2 = R_3 = 2\ \text{k}\Omega$, $R_4 = 4\ \text{k}\Omega$, $i_A = 2$ mA, $i_B = 4$ mA, and $\alpha = 2$.

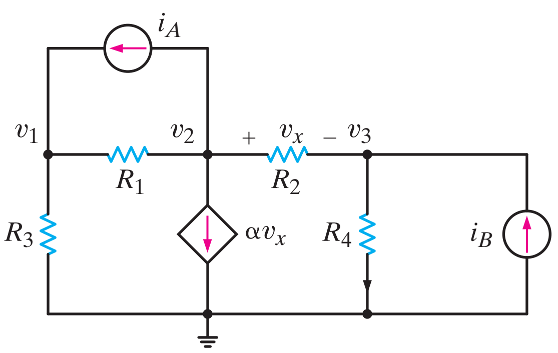


Figure 3.10
Circuit containing a voltage-controlled current source.

SOLUTION Applying KCL at each of the nonreference nodes yields the equations

$$G_3 v_1 + G_1(v_1 - v_2) - i_A = 0$$
$$i_A + G_1(v_2 - v_1) + \alpha v_x + G_2(v_2 - v_3) = 0$$
$$G_2(v_3 - v_2) + G_4 v_3 - i_B = 0$$

where $v_x = v_2 - v_3$. Simplifying these equations, we obtain

$$(G_1 + G_3)v_1 - G_1 v_2 = i_A$$
$$-G_1 v_1 + (G_1 + \alpha + G_2)v_2 - (\alpha + G_2)v_3 = -i_A$$
$$-G_2 v_2 + (G_2 + G_4)v_3 = i_B$$

Given the component values, the equations become

$$\begin{bmatrix} \frac{1}{1\text{k}} + \frac{1}{2\text{k}} & -\frac{1}{\text{k}} & 0 \\ -\frac{1}{\text{k}} & \frac{1}{\text{k}} + 2 + \frac{1}{2\text{k}} & -\left(2 + \frac{1}{2\text{k}}\right) \\ 0 & -\frac{1}{2\text{k}} & \frac{1}{2\text{k}} + \frac{1}{4\text{k}} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0.002 \\ -0.002 \\ 0.004 \end{bmatrix}$$

or

$$\begin{bmatrix} 0.0015 & -0.001 & 0 \\ -0.001 & 2.0015 & -2.0005 \\ 0 & -0.0005 & 0.00075 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0.002 \\ -0.002 \\ 0.004 \end{bmatrix}$$

The MATLAB input and output listings are shown next.

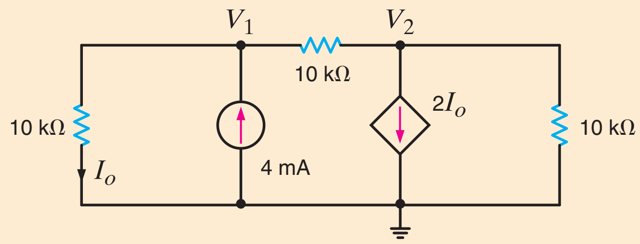
```
>> G = [0.0015 -0.001 0 ; -0.001 2.0015 -2.0005 ;
0 -0.0005 0.00075]

   G =

        0.0015     -0.0010          0
       -0.0010      2.0015    -2.0005
             0     -0.0005     0.0008

>> I = [0.002 ; -0.002 ; 0.004]

   I =

        0.0020
       -0.0020
        0.0040

>> V = inv(G)*I

   V =

        11.9940
        15.9910
        15.9940
```

LEARNING EXTENSIONS

E3.3 Find the node voltages in the circuit in Fig. E3.3.

ANSWER: $V_1 = 16$ V, $V_2 = -8$ V.

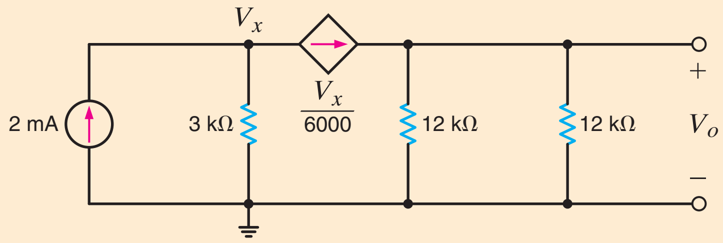
Figure E3.3



E3.4 Find the voltage V_o in the network in Fig. E3.4.

ANSWER: $V_o = 4$ V.

Figure E3.4



CIRCUITS CONTAINING INDEPENDENT VOLTAGE SOURCES As is our practice, in our discussion of this topic we will proceed from the simplest case to those cases that are more complicated. The simplest case is that in which an independent voltage source is connected to the reference node. The following example illustrates this case.

Example 3.5

Consider the circuit shown in Fig. 3.11a. Let us determine all node voltages and branch currents.

SOLUTION This network has three nonreference nodes with labeled node voltages V_1, V_2, and V_3. Based on our previous discussions, we would assume that in order to find all the node voltages we would need to write a KCL equation at each of the nonreference nodes. The resulting three linearly independent simultaneous equations would produce the unknown node voltages. However, note that V_1 and V_3 are known quantities because an independent voltage source is connected directly between the nonreference node and each of these nodes. Therefore, $V_1 = 12$ V and $V_3 = -6$ V. Furthermore, note that the current through the 9-kΩ resistor is $[12 - (-6)]/9\text{k} = 2$ mA from left to right. We do not know V_2 or the current in the remaining resistors. However, since only one node voltage is unknown, a single-node equation will produce it. Applying KCL to this center node yields

$$\frac{V_2 - V_1}{12\text{k}} + \frac{V_2 - 0}{6\text{k}} + \frac{V_2 - V_3}{12\text{k}} = 0$$

or

$$\frac{V_2 - 12}{12\text{k}} + \frac{V_2}{6\text{k}} + \frac{V_2 - (-6)}{12\text{k}} = 0$$

from which we obtain

$$V_2 = \frac{3}{2}\text{ V}$$

Once all the node voltages are known, Ohm's law can be used to find the branch currents shown in Fig. 3.11b. The diagram illustrates that KCL is satisfied at every node.

Note that the presence of the voltage sources in this example has simplified the analysis, since two of the three linear independent equations are $V_1 = 12$ V and $V_3 = -6$ V. We will find that as a general rule, whenever voltage sources are present between nodes, the node voltage equations that describe the network will be simpler.

HINT

Any time an independent voltage source is connected between the reference node and a nonreference node, the nonreference node voltage is known.

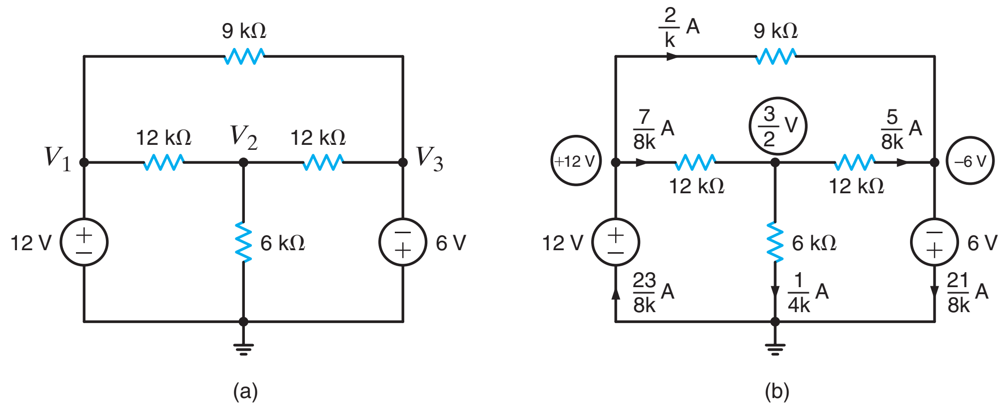


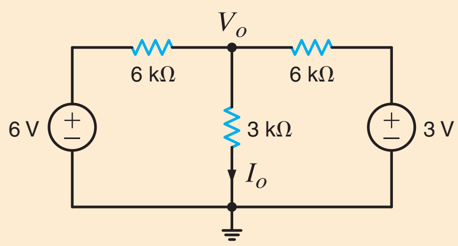
Figure 3.11 *Circuit used in Example 3.5.*

LEARNING EXTENSION

E3.5 Use nodal analysis to find the current I_o in the network in Fig. E3.5.

ANSWER: $I_o = \frac{3}{4}$ mA.

Figure E3.5



Next let us consider the case in which an independent voltage source is connected between two nonreference nodes.

Example 3.6

Suppose we wish to find the currents in the two resistors in the circuit of Fig. 3.12a.

SOLUTION If we try to attack this problem in a brute force manner, we immediately encounter a problem. Thus far, branch currents were either known source values or could be expressed as the branch voltage divided by the branch resistance. However, the branch current through the 6-V source is certainly not known and cannot be directly expressed using Ohm's law. We can, of course, give this current a name and write the KCL equations at the two nonreference nodes in terms of this current. However, this approach is no panacea because this technique will result in *two* linearly independent simultaneous equations in terms of *three* unknowns—that is, the two node voltages and the current in the voltage source.

To solve this dilemma, we recall that $N - 1$ linearly independent equations are required to determine the $N - 1$ nonreference node voltages in an N-node circuit. Since our network has three nodes, we need two linearly independent equations. Now note that if somehow one of the node voltages is known, we immediately know the other; that is, if V_1 is known, then $V_2 = V_1 - 6$. If V_2 is known, then $V_1 = V_2 + 6$. Therefore, the difference in potential between the two nodes is *constrained* by the voltage source and, hence,

$$V_1 - V_2 = 6$$

This constraint equation is one of the two linearly independent equations needed to determine the node voltages.

Next consider the network in Fig. 3.12b, in which the 6-V source is completely enclosed within the dashed surface. The constraint equation governs this dashed portion of the network. The remaining equation is obtained by applying KCL to this dashed surface, which is commonly called a *supernode*. Recall that in Chapter 2 we demonstrated that KCL must hold for a surface, and this technique eliminates the problem of dealing with a current through a voltage source. KCL for the supernode is

$$-6 \times 10^{-3} + \frac{V_1}{6\text{k}} + \frac{V_2}{12\text{k}} + 4 \times 10^{-3} = 0$$

Solving these equations yields $V_1 = 10$ V and $V_2 = 4$ V and, hence, $I_1 = 5/3$ mA and $I_2 = 1/3$ mA. A quick check indicates that KCL is satisfied at every node.

Note that applying KCL at the reference node yields the same equation as shown above. The student may feel that the application of KCL at the reference node saves one from having to deal with supernodes. Recall that we do not apply KCL at any node—even the reference node—that contains an independent voltage source. This idea can be illustrated with the circuit in the next example.

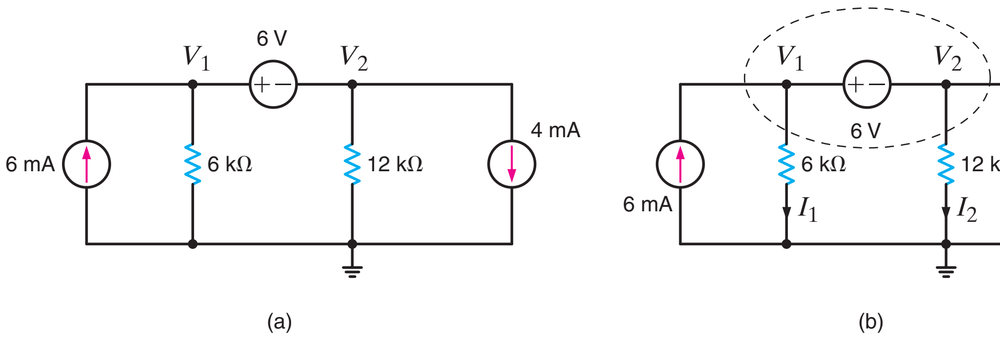


Figure 3.12
Circuits used in Example 3.6.

Example 3.7

Let us determine the current I_o in the network in Fig. 3.13a.

SOLUTION Examining the network, we note that node voltages V_2 and V_4 are known and the node voltages V_1 and V_3 are constrained by the equation

$$V_1 - V_3 = 12$$

The network is redrawn in Fig. 3.13b.

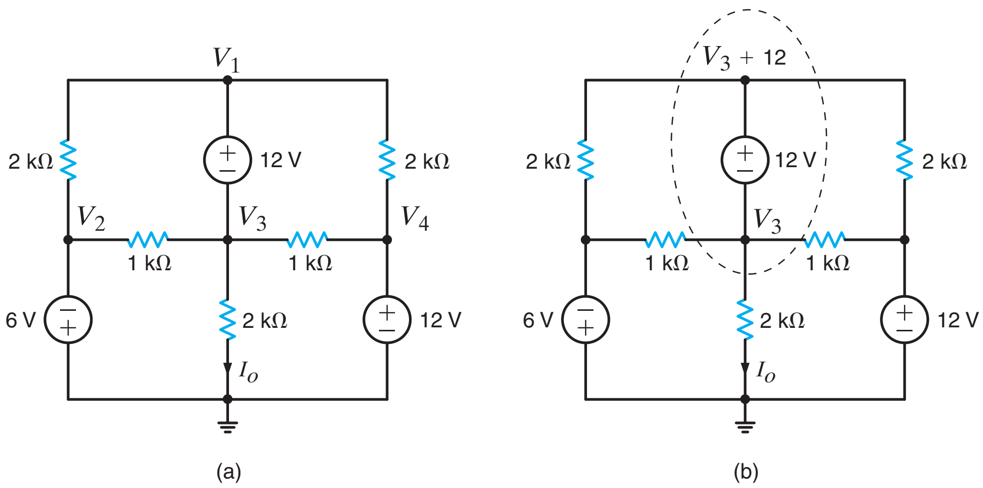


Figure 3.13
Example circuit with supernodes.

Since we want to find the current I_o, V_1 (in the supernode containing V_1 and V_3) is written as $V_3 + 12$. The KCL equation at the supernode is then

$$\frac{V_3 + 12 - (-6)}{2\text{k}} + \frac{V_3 + 12 - 12}{2\text{k}} + \frac{V_3 - (-6)}{1\text{k}} + \frac{V_3 - 12}{1\text{k}} + \frac{V_3}{2\text{k}} = 0$$

Solving the equation for V_3 yields

$$V_3 = -\frac{6}{7}\text{ V}$$

I_o can then be computed immediately as

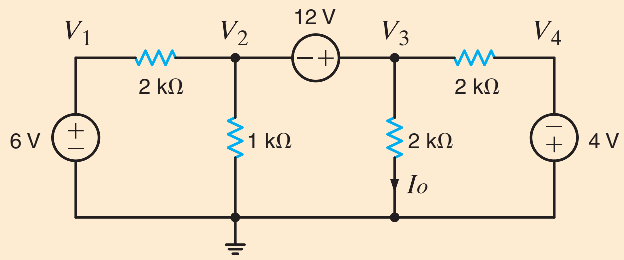
$$I_o = \frac{-\frac{6}{7}}{2\text{k}} = -\frac{3}{7}\text{ mA}$$

LEARNING EXTENSION

E3.6 Use nodal analysis to find I_o in the network in Fig. E3.6.

ANSWER: $I_o = 3.8$ mA.

Figure E3.6



CIRCUITS CONTAINING DEPENDENT VOLTAGE SOURCES As the following examples will indicate, networks containing dependent (controlled) sources are treated in the same manner as described earlier.

Example 3.8

We wish to find I_o in the network in Fig. 3.14.

SOLUTION Since the dependent voltage source is connected between the node labeled V_1 and the reference node,

$$V_1 = 2\text{k}I_x$$

KCL at the node labeled V_2 is

$$\frac{V_2 - V_1}{2\text{k}} - \frac{4}{\text{k}} + \frac{V_2}{1\text{k}} = 0$$

where

$$I_x = \frac{V_2}{1\text{k}}$$

Solving these equations yields $V_2 = 8$ V and $V_1 = 16$ V. Therefore

$$I_o = \frac{V_1 - V_2}{2\text{k}}$$
$$= 4\text{ mA}$$

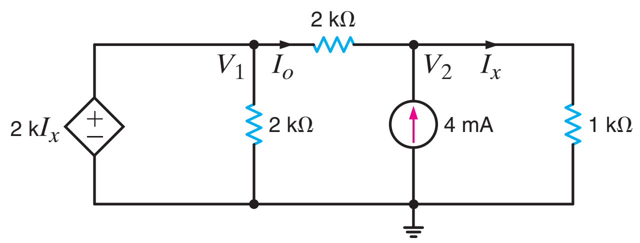


Figure 3.14
Circuit used in Example 3.8.

Example 3.9

Let us find the current I_o in the network in Fig. 3.15.

SOLUTION This circuit contains both an independent voltage source and a voltage-controlled voltage source. Note that $V_3 = 6\text{ V}$, $V_2 = V_x$, and a supernode exists between the nodes labeled V_1 and V_2.

Applying KCL to the supernode, we obtain

$$\frac{V_1 - V_3}{6\text{k}} + \frac{V_1}{12\text{k}} + \frac{V_2}{6\text{k}} + \frac{V_2 - V_3}{12\text{k}} = 0$$

where the constraint equation for the supernode is

$$V_1 - V_2 = 2V_x$$

The final equation is

$$V_3 = 6$$

Solving these equations, we find that

$$V_1 = \frac{9}{2}\text{ V}$$

and, hence,

$$I_o = \frac{V_1}{12\text{k}} = \frac{3}{8}\text{ mA}$$

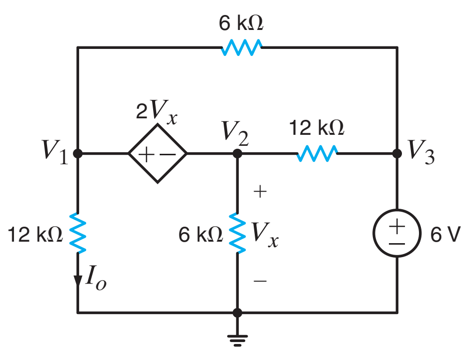


Figure 3.15
Circuit used in Example 3.9.

Finally, let us consider two additional circuits that, for purposes of comparison, we will examine using more than one method.

Example 3.10

Let us find V_o in the network in Fig. 3.16a. Note that the circuit contains two voltage sources, one of which is a controlled source, and two independent current sources. The circuit is redrawn in Fig. 3.16b in order to label the nodes and identify the supernode surrounding the controlled source. Because of the presence of the independent voltage source, the voltage at node 4 is known to be 4 V. We will use this knowledge in writing the node equations for the network.

Since the network has five nodes, four linear independent equations are sufficient to determine all the node voltages. Within the supernode, the defining equation is

$$V_1 - V_2 = 2V_x$$

where

$$V_2 = V_x$$

and thus

$$V_1 = 3V_x$$

Furthermore, we know that one additional equation is

$$V_4 = 4$$

Thus, given these two equations, only two more equations are needed in order to solve for the unknown node voltages. These additional equations result from applying KCL at the supernode and at the node labeled V_3. The equations are

$$-\frac{2}{k} + \frac{V_x}{1k} + \frac{V_x - V_3}{1k} + \frac{3V_x - V_3}{1k} + \frac{3V_x - 4}{1k} = 0$$

$$\frac{V_3 - 3V_x}{1k} + \frac{V_3 - V_x}{1k} = \frac{2}{k}$$

Combining the equations yields the two equations

$$8V_x - 2V_3 = 6$$

$$-4V_x + 2V_3 = 2$$

Solving these equations, we obtain

$$V_x = 2\text{ V} \quad \text{and} \quad V_3 = 5\text{ V}$$

$$V_o = 3V_x - V_3 = 1\text{ V}$$

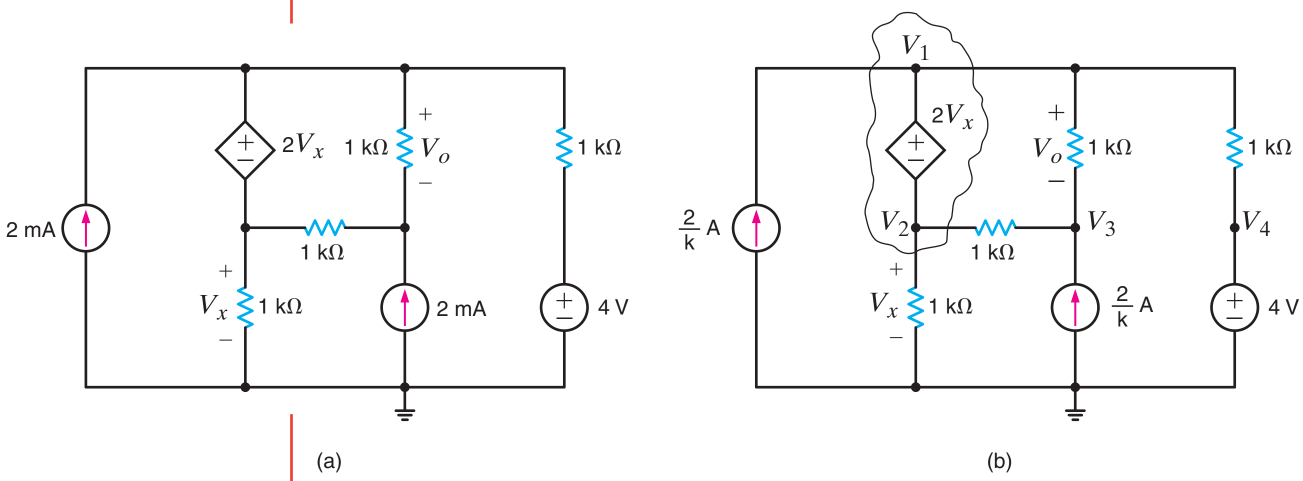


Figure 3.16 *Circuit used in Example 3.10.*

Example 3.11

We wish to find I_o in the network in Fig. 3.17a. Note that this circuit contains three voltage sources, one of which is a controlled source and another is a controlled current source. Because two of the voltage sources are connected to the reference node, one node voltage is known directly and one is specified by the dependent source. Furthermore, the difference in voltage between two nodes is defined by the 6-V independent source.

The network is redrawn in Fig. 3.17b in order to label the nodes and identify the supernode. Since the network has six nodes, five linear independent equations are needed to determine the unknown node voltages.

The two equations for the supernode are

$$V_1 - V_4 = -6$$

$$\frac{V_1 - 12}{1\text{k}} + \frac{V_1 - V_3}{1\text{k}} + 2I_x + \frac{V_4 - V_3}{1\text{k}} + \frac{V_4}{1\text{k}} + \frac{V_4 - V_5}{1\text{k}} = 0$$

The three remaining equations are

$$V_2 = 12$$

$$V_3 = 2V_x$$

$$\frac{V_5 - V_4}{1\text{k}} + \frac{V_5}{1\text{k}} = 2I_x$$

The equations for the control parameters are

$$V_x = V_1 - 12$$

$$I_x = \frac{V_4}{1\text{k}}$$

Combining these equations yields the following set of equations

$$-2V_1 + 5V_4 - V_5 = -36$$

$$V_1 - V_4 = -6$$

$$-3V_4 + 2V_5 = 0$$

Solving these equations by any convenient means yields

$$V_1 = -38 \text{ V}$$

$$V_4 = -32 \text{ V}$$

$$V_5 = -48 \text{ V}$$

Then, since $V_3 = 2V_x$, $V_3 = -100$ V. I_o is -48 mA. The reader is encouraged to verify that KCL is satisfied at every node.

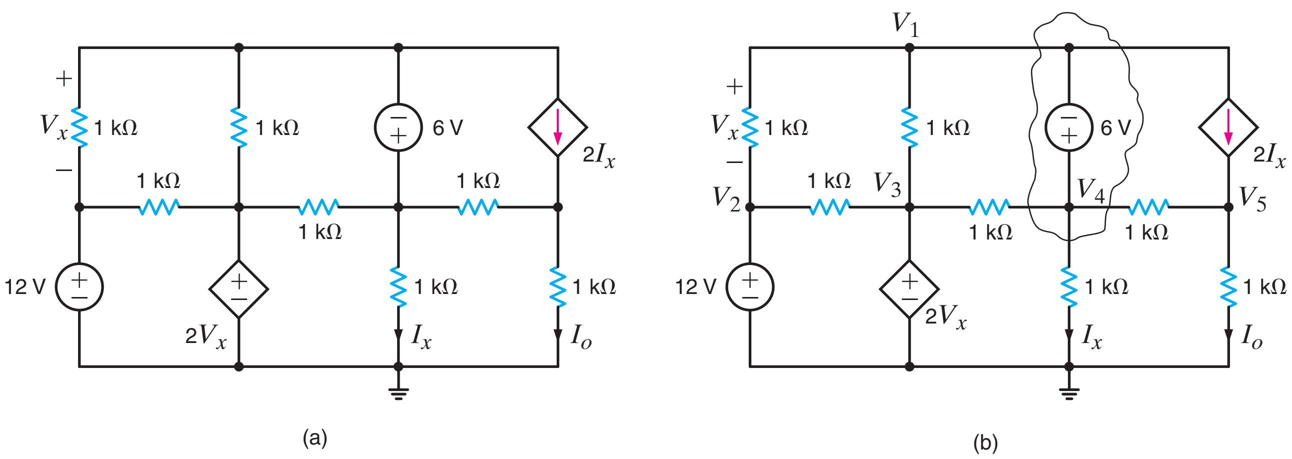


Figure 3.17 Circuit used in Example 3.11.

PROBLEM-SOLVING STRATEGY

Nodal Analysis

Step 1. Select one node in the N-node circuit as the reference node. Assume that the node voltage is zero and measure all node voltages with respect to this node.

Step 2. If only independent current sources are present in the network, write the KCL equations at the $N - 1$ nonreference nodes. If dependent current sources are present, write the KCL equations as is done for networks with only independent current sources; then write the controlling equations for the dependent sources.

Step 3. If voltage sources are present in the network, they may be connected (1) between the reference node and a nonreference node or (2) between two nonreference nodes. In the former case, if the voltage source is an independent source, then the voltage at one of the nonreference nodes is known. If the source is dependent, it is treated as an independent source when writing the KCL equation, but an additional constraint equation is necessary, as described previously.

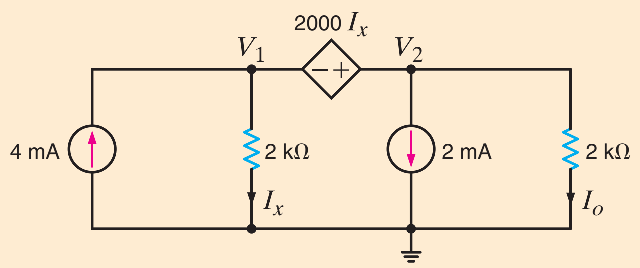
In the latter case, if the source is independent, the voltage between the two nodes is constrained by the value of the voltage source, and an equation describing this constraint represents one of the $N - 1$ linearly independent equations required to determine the N-node voltages. The surface of the network described by the constraint equation (i.e., the source and two connecting nodes) is called a supernode. One of the remaining $N - 1$ linearly independent equations is obtained by applying KCL at this supernode. If the voltage source is dependent, it is treated as an independent source when writing the KCL equations, but an additional constraint equation is necessary, as described previously.

LEARNING EXTENSION

E3.7 Use nodal analysis to find I_o in the circuit in Fig. E3.7. **PSV**

ANSWER: $I_o = \frac{4}{3}$ mA.

Figure E3.7



3.2 Loop Analysis

We found that in a nodal analysis the unknown parameters are the node voltages and KCL was employed to determine them. Once these node voltages have been calculated, all the branch currents in the network can easily be determined using Ohm's law. In contrast to this approach, a loop analysis uses KVL to determine a set of loop currents in the circuit. Once these loop currents are known, Ohm's law can be used to calculate any voltages in the network. Via network topology we can show that, in general, there are exactly $B - N + 1$ linearly independent KVL equations for any network, where B is the number of branches in the circuit and N is the number of nodes. For example, if we once again examine the circuit in Fig. 2.5, we find that there are eight branches and five nodes. Thus, the number

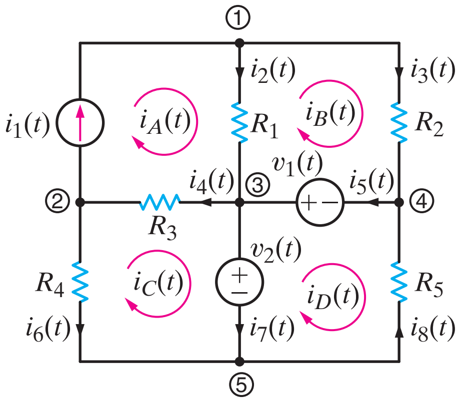


Figure 3.18
Figure 2.5 redrawn with loop currents.

of linearly independent KVL equations necessary to determine all currents in the network is $B - N + 1 = 8 - 5 + 1 = 4$. The network in Fig. 2.5 is redrawn as shown in Fig. 3.18 with 4 loop currents labeled as shown. The branch currents are then determined as

$$
\begin{aligned}
i_1(t) &= i_A(t) \\
i_2(t) &= i_A(t) - i_B(t) \\
i_3(t) &= i_B(t) \\
i_4(t) &= i_A(t) - i_C(t) \\
i_5(t) &= i_B(t) - i_D(t) \\
i_6(t) &= -i_C(t) \\
i_7(t) &= i_C(t) - i_D(t) \\
i_8(t) &= -i_D(t)
\end{aligned}
$$

All the circuits we will examine in this text will be *planar*, which simply means that we can draw the circuit on a sheet of paper in such a way that no conductor crosses another conductor. If a circuit is planar, the loops are more easily identified. For example, recall in Chapter 2 that we found that a single equation was sufficient to determine the current in a circuit containing a single loop. If the circuit contains N independent loops, we will show (and the general topological formula $B - N + 1$ can be used for verification), that N independent simultaneous equations will be required to describe the network.

Our approach to loop analysis will mirror the approach used in nodal analysis (i.e., we will begin with simple cases and systematically proceed to those that are more difficult). Then at the end of this section we will outline a general strategy for employing loop analysis.

CIRCUITS CONTAINING ONLY INDEPENDENT VOLTAGE SOURCES To begin our analysis, consider the circuit shown in Fig. 3.19. We note that this network has seven branches and six nodes, and thus the number of linearly independent KVL equations necessary to determine all currents in the circuit is $B - N + 1 = 7 - 6 + 1 = 2$. Since two linearly independent KVL equations are required, we identify two independent loops, *A-B-E-F-A* and *B-C-D-E-B*. We now define a new set of current variables called *loop currents*, which can be used to find the physical currents in the circuit. Let us assume that current i_1 flows in the first loop and that current i_2 flows in the second loop. Then the branch current flowing from B to E through R_3 is $i_1 - i_2$. The directions of the currents have been assumed. As was the case in the nodal analysis, if the actual currents are not in the direction indicated, the values calculated will be negative.

Applying KVL to the first loop yields

$$+v_1 + v_3 + v_2 - v_{S1} = 0$$

KVL applied to loop 2 yields

$$+v_{S2} + v_4 + v_5 - v_3 = 0$$

where $v_1 = i_1R_1$, $v_2 = i_1R_2$, $v_3 = (i_1 - i_2)R_3$, $v_4 = i_2R_4$, and $v_5 = i_2R_5$.

HINT
The equations employ the passive sign convention.

Substituting these values into the two KVL equations produces the two simultaneous equations required to determine the two loop currents; that is,

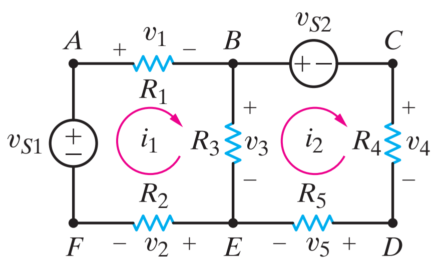
$$i_1(R_1 + R_2 + R_3) - i_2(R_3) = v_{S1}$$
$$-i_1(R_3) + i_2(R_3 + R_4 + R_5) = -v_{S2}$$

or in matrix form

$$\begin{bmatrix} R_1 + R_2 + R_3 & -R_3 \\ -R_3 & R_3 + R_4 + R_5 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} v_{S1} \\ -v_{S2} \end{bmatrix}$$

At this point, it is important to define what is called a *mesh*. A mesh is a special kind of loop that does not contain any loops within it. Therefore, as we traverse the path of a mesh, we do not encircle any circuit elements. For example, the network in Fig. 3.19 contains two meshes defined by the paths *A-B-E-F-A* and *B-C-D-E-B*. The path *A-B-C-D-E-F-A* is a loop, but it is not a mesh. Since the majority of our analysis in this section will involve writing KVL equations for meshes, we will refer to the currents as mesh currents and the analysis as a *mesh analysis*.

Figure 3.19
A two-loop circuit.



Example 3.12

Consider the network in Fig. 3.20a. We wish to find the current I_o.

SOLUTION We will begin the analysis by writing mesh equations. Note that there are no + and – signs on the resistors. However, they are not needed, since we will apply Ohm's law to each resistive element as we write the KVL equations. The equation for the first mesh is

$$-12 + 6\text{k}I_1 + 6\text{k}(I_1 - I_2) = 0$$

The KVL equation for the second mesh is

$$6\text{k}(I_2 - I_1) + 3\text{k}I_2 + 3 = 0$$

where $I_o = I_1 - I_2$.

Solving the two simultaneous equations yields $I_1 = 5/4$ mA and $I_2 = 1/2$ mA. Therefore, $I_o = 3/4$ mA. All the voltages and currents in the network are shown in Fig. 3.20b. Recall from nodal analysis that once the node voltages were determined, we could check our analysis using KCL at the nodes. In this case, we know the branch currents and can use KVL around any closed path to check our results. For example, applying KVL to the outer loop yields

$$-12 + \frac{15}{2} + \frac{3}{2} + 3 = 0$$
$$0 = 0$$

Since we want to calculate the current I_o, we could use loop analysis, as shown in Fig. 3.20c. Note that the loop current I_1 passes through the center leg of the network and, therefore, $I_1 = I_o$. The two loop equations in this case are

$$-12 + 6\text{k}(I_1 + I_2) + 6\text{k}I_1 = 0$$

and

$$-12 + 6\text{k}(I_1 + I_2) + 3\text{k}I_2 + 3 = 0$$

Solving these equations yields $I_1 = 3/4$ mA and $I_2 = 1/2$ mA. Since the current in the 12-V source is $I_1 + I_2 = 5/4$ mA, these results agree with the mesh analysis.

Finally, for purposes of comparison, let us find I_o using nodal analysis. The presence of the two voltage sources would indicate that this is a viable approach. Applying KCL at the top center node, we obtain

$$\frac{V_o - 12}{6\text{k}} + \frac{V_o}{6\text{k}} + \frac{V_o - 3}{3\text{k}} = 0$$

and hence,

$$V_o = \frac{9}{2}\text{ V}$$

and then

$$I_o = \frac{V_o}{6\text{k}} = \frac{3}{4}\text{ mA}$$

Note that in this case we had to solve only one equation instead of two.

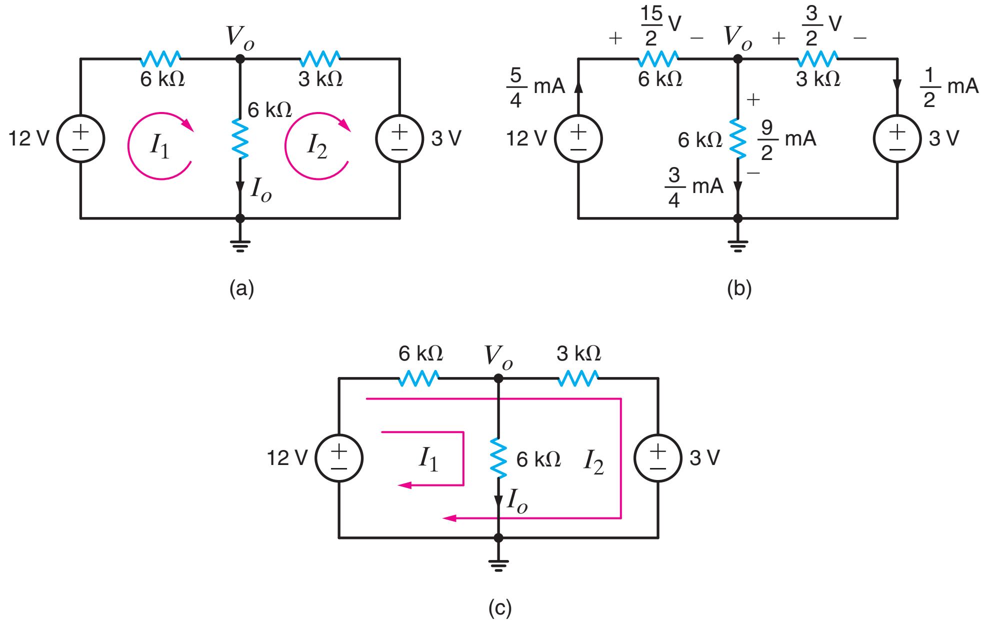


Figure 3.20 *Circuits used in Example 3.12.*

Once again we are compelled to note the symmetrical form of the mesh equations that describe the circuit in Fig. 3.19. Note that the coefficient matrix for this circuit is symmetrical.

Since this symmetry is generally exhibited by networks containing resistors and independent voltage sources, we can learn to write the mesh equations by inspection. In the first equation, the coefficient of i_1 is the sum of the resistances through which mesh current 1 flows, and the coefficient of i_2 is the negative of the sum of the resistances common to mesh current 1 and mesh current 2. The right-hand side of the equation is the algebraic sum of the voltage sources in mesh 1. The sign of the voltage source is positive if it aids the assumed direction of the current flow and negative if it opposes the assumed flow. The first equation is KVL for mesh 1. In the second equation, the coefficient of i_2 is the sum of all the resistances in mesh 2, the coefficient of i_1 is the negative of the sum of the resistances common to mesh 1 and mesh 2, and the right-hand side of the equation is the algebraic sum of the voltage sources in mesh 2. In general, if we assume all of the mesh currents to be in the same direction (clockwise or counterclockwise), then if KVL is applied to mesh j with mesh current i_j, the coefficient of i_j is the sum of the resistances in mesh j and the coefficients of the other mesh currents $\left(\text{e.g., } i_{j-1}, i_{j+1}\right)$ are the negatives of the resistances common to these meshes and mesh j. The right-hand side of the equation is equal to the algebraic sum of the voltage sources in mesh j. These voltage sources have a positive sign if they aid the current flow i_j and a negative sign if they oppose it.

Example 3.13

Let us write the mesh equations by inspection for the network in Fig. 3.21. Then we will use MATLAB to solve for the mesh currents.

SOLUTION The three linearly independent simultaneous equations are

$$(4k + 6k)I_1 - (0)I_2 - (6k)I_3 = -6$$
$$-(0)I_1 + (9k + 3k)I_2 - (3k)I_3 = 6$$
$$-(6k)I_1 - (3k)I_2 + (3k + 6k + 12k)I_3 = 0$$

or in matrix form

$$\begin{bmatrix} 10k & 0 & -6k \\ 0 & 12k & -3k \\ -6k & -3k & 21k \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} -6 \\ 6 \\ 0 \end{bmatrix}$$

Note the symmetrical form of the equations. The general form of the matrix equation is

$$\mathbf{RI} = \mathbf{V}$$

and the solution of this matrix equation is

$$\mathbf{I} = \mathbf{R}^{-1}\mathbf{V}$$

The input/output data for a MATLAB solution are as follows:

```
>> R = [10e3 0 -6e3; 0 12e3 -3e3;
-6e3 -3e3 21e3]
   R =
        10000          0      -6000
            0      12000      -3000
        -6000      -3000      21000
>> V = [-6 ; 6 ; 0]
   V =
       -6
        6
        0
>> I = inv(R)*V
   I =
        1.0e-003 *
         -0.6757
          0.4685
         -0.1261
```

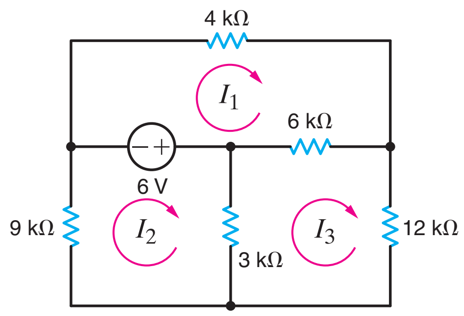


Figure 3.21 *Circuit used in Example 3.13.*

CIRCUITS CONTAINING INDEPENDENT CURRENT SOURCES

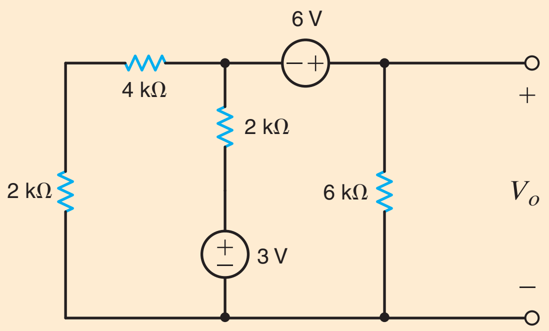
Just as the presence of a voltage source in a network simplified the nodal analysis, the presence of a current source simplifies a loop analysis. The following examples illustrate the point.

LEARNING EXTENSION

E3.8 Use mesh equations to find V_o in the circuit in Fig. E3.8.

ANSWER: $V_o = \dfrac{33}{5}$ V.

Figure E3.8



Example 3.14

Let us find both V_o and V_1 in the circuit in Fig. 3.22.

SOLUTION Although it appears that there are two unknown mesh currents, the current I_1 goes directly through the current source and, therefore, I_1 is constrained to be 2 mA. Hence, only the current I_2 is unknown. KVL for the rightmost mesh is

$$2\text{k}(I_2 - I_1) - 2 + 6kI_2 = 0$$

And, of course,

$$I_1 = 2 \times 10^{-3}$$

These equations can be written as

$$-2\text{k}I_1 + 8\text{k}I_2 = 2$$
$$I_1 = 2/\text{k}$$

The input/output data for a MATLAB solution are as follows:

```
>> R = [-2000 8000; 1 0]

   R =

      -2000      8000
          1         0

>> V = [2 ; 0.002]

   V =

        2.0000
        0.0020

>> I = inv(R)*V

   I =

        0.0020
        0.0008
```

```
>> format long
>> I
   I =
       0.00200000000000
       0.00075000000000
```

Note carefully that the first solution for I_2 contains a single digit in the last decimal place. We are naturally led to question whether a number has been rounded off to this value. If we type "format long," MATLAB will provide the answer using 15 digits. Thus, instead of 0.008, the more accurate answer is 0.0075. And hence,

$$V_o = 6\text{k}I_2 = \frac{9}{2}\text{ V}$$

To obtain V_1 we apply KVL around any closed path. If we use the outer loop, the KVL equation is

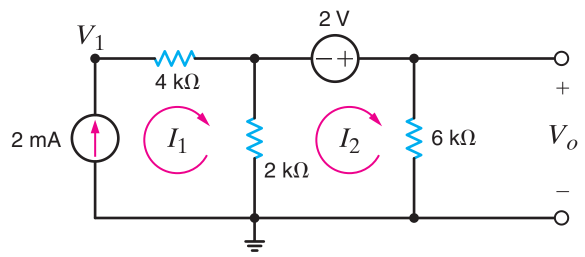
$$-V_1 + 4\text{k}I_1 - 2 + 6\text{k}I_2 = 0$$

And therefore,

$$V_1 = \frac{21}{2}\text{ V}$$

Note that since the current I_1 is known, the 4-kΩ resistor did not enter the equation in finding V_o. However, it appears in every loop containing the current source and, thus, is used in finding V_1.

Figure 3.22 *Circuit used in Example 3.14.*



Example 3.15

We wish to find V_o in the network in Fig. 3.23.

SOLUTION Since the currents I_1 and I_2 pass directly through a current source, two of the three required equations are

$$I_1 = 4 \times 10^{-3}$$
$$I_2 = -2 \times 10^{-3}$$

The third equation is KVL for the mesh containing the voltage source; that is,

$$4\text{k}(I_3 - I_2) + 2\text{k}(I_3 - I_1) + 6\text{k}I_3 - 3 = 0$$

These equations yield

$$I_3 = \frac{1}{4}\text{ mA}$$

and hence,

$$V_o = 6\text{k}I_3 - 3 = \frac{-3}{2}\text{ V}$$

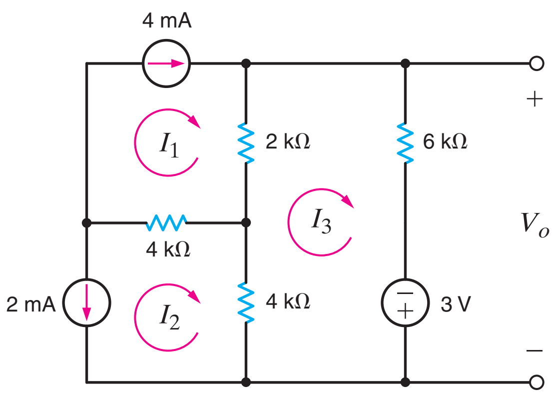


Figure 3.23
Circuit used in Example 3.15.

What we have demonstrated in the previous example is the general approach for dealing with independent current sources when writing KVL equations; that is, use one loop through each current source. The number of "window panes" in the network tells us how many equations we need. Additional KVL equations are written to cover the remaining circuit elements in the network. The following example illustrates this approach.

Example 3.16

Let us find I_o in the network in Fig. 3.24a.

SOLUTION First, we select two loop currents I_1 and I_2 such that I_1 passes directly through the 2-mA source, and I_2 passes directly through the 4-mA source, as shown in Fig. 3.24b. Therefore, two of our three linearly independent equations are

$$I_1 = 2 \times 10^{-3}$$
$$I_2 = 4 \times 10^{-3}$$

The remaining loop current I_3 must pass through the circuit elements not covered by the two previous equations and cannot, of course, pass through the current sources. The path for this remaining loop current can be obtained by open-circuiting the current sources, as shown in Fig. 3.24c. When all currents are labeled on the original circuit, the KVL equation for this last loop, as shown in Fig. 3.24d, is

$$-6 + 1\text{k}I_3 + 2\text{k}(I_2 + I_3) + 2\text{k}(I_3 + I_2 - I_1) + 1\text{k}(I_3 - I_1) = 0$$

Solving the equations yields

$$I_3 = \frac{-2}{3}\text{ mA}$$

and therefore,

$$I_o = I_1 - I_2 - I_3 = \frac{-4}{3}\text{ mA}$$

Next consider the supermesh technique. In this case the three mesh currents are specified as shown in Fig. 3.24e, and since the voltage across the 4-mA current source is unknown, it is assumed to be V_x. The mesh currents constrained by the current sources are

$$I_1 = 2 \times 10^{-3}$$
$$I_2 - I_3 = 4 \times 10^{-3}$$

The KVL equations for meshes 2 and 3, respectively, are

$$2\text{k}I_2 + 2\text{k}(I_2 - I_1) - V_x = 0$$
$$-6 + 1\text{k}I_3 + V_x + 1\text{k}(I_3 - I_1) = 0$$

HINT

In this case the 4-mA current source is located on the boundary between two meshes. Thus, we will demonstrate two techniques for dealing with this type of situation. One is a special loop technique, and the other is known as the supermesh approach.

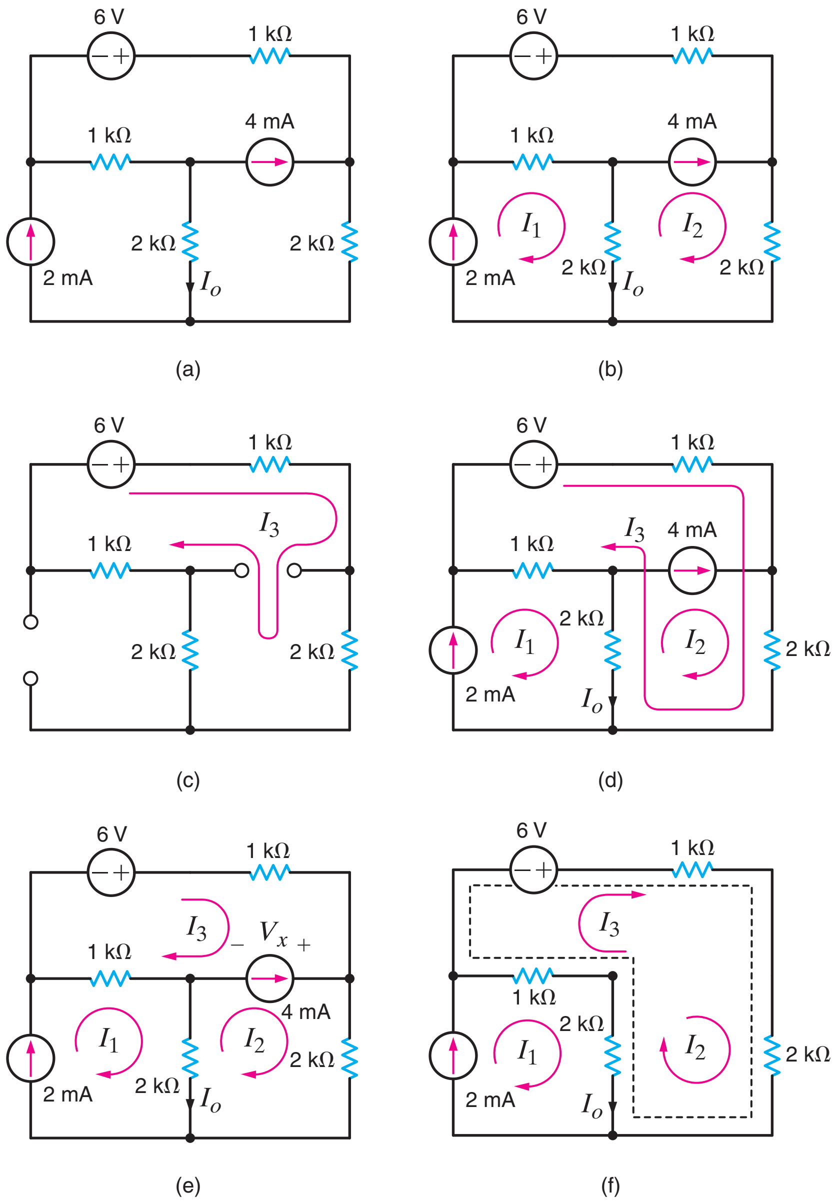


Figure 3.24 *Circuits used in Example 3.16.*

Adding the last two equations yields

$$-6 + 1\mathrm{k}I_3 + 2\mathrm{k}I_2 + 2\mathrm{k}(I_2 - I_1) + 1\mathrm{k}(I_3 - I_1) = 0$$

Note that the unknown voltage V_x has been eliminated. The two constraint equations, together with this latter equation, yield the desired result.

The purpose of the supermesh approach is to avoid introducing the unknown voltage V_x. The supermesh is created by mentally removing the 4-mA current source, as shown in Fig. 3.24f. Then writing the KVL equation around the dotted path, which defines the supermesh, using the original mesh currents as shown in Fig. 3.20e, yields

$$-6 + 1\mathrm{k}I_3 + 2\mathrm{k}I_2 + 2\mathrm{k}(I_2 - I_1) + 1\mathrm{k}(I_3 - I_1) = 0$$

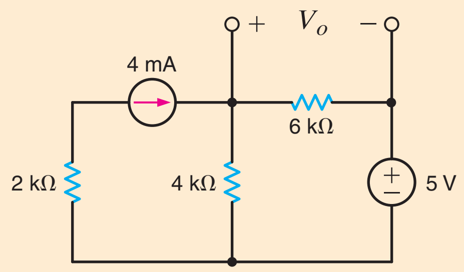
Note that this supermesh equation is the same as that obtained earlier by introducing the voltage V_x.

LEARNING EXTENSIONS

E3.9 Find V_o in the network in Fig. E3.9.

ANSWER: $V_o = \frac{33}{5}$ V.

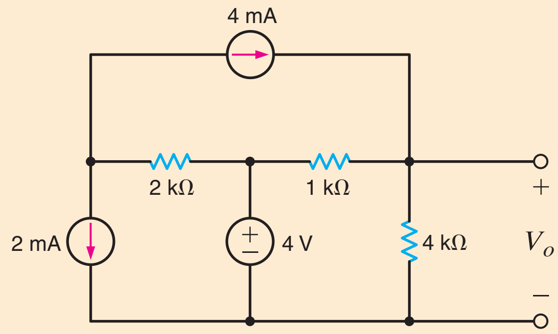
Figure E3.9



E3.10 Find V_o in the network in Fig. E3.10.

ANSWER: $V_o = \frac{32}{5}$ V.

Figure E3.10



CIRCUITS CONTAINING DEPENDENT SOURCES We deal with circuits containing dependent sources just as we have in the past. First, we treat the dependent source as though it were an independent source when writing the KVL equations. Then we write the controlling equation for the dependent source. The following examples illustrate the point.

Example 3.17

Let us find V_o in the circuit in Fig. 3.25, which contains a voltage-controlled voltage source.

SOLUTION The equation for the loop currents shown in the figure are

$$-2V_x + 2k(I_1 + I_2) + 4kI_1 = 0$$
$$-2V_x + 2k(I_1 + I_2) - 3 + 6kI_2 = 0$$

where

$$V_x = 4kI_1$$

These equations can be combined to produce

$$-2kI_1 + 2kI_2 = 0$$
$$-6kI_1 + 8kI_2 = 3$$

The input/output data for a MATLAB solution are

```
>> R = [-2000 2000; -6000 8000]
   R =
        -2000        2000
        -6000        8000
```

```
>> V = [0; 3]
   V =
        0
        3
>> I = inv(R)*V
   I =
        0.00150000000000
        0.00150000000000
```

and therefore,

$$V_o = 6kI_2 = 9 \text{ V}$$

For comparison, we will also solve the problem using nodal analysis. The presence of the voltage sources indicates that this method could be simpler. Treating the 3-V source and its connecting nodes as a supernode and writing the KCL equation for this supernode yields

$$\frac{V_x - 2V_x}{2\text{k}} + \frac{V_x}{4\text{k}} + \frac{V_x + 3}{6\text{k}} = 0$$

where

$$V_o = V_x + 3$$

These equations also yield $V_o = 9$ V.

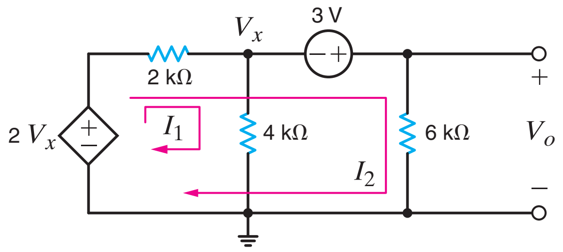


Figure 3.25 *Circuit used in Example 3.17.*

Example 3.18

Let us find V_o in the circuit in Fig. 3.26, which contains a voltage-controlled current source.

SOLUTION The currents I_1 and I_2 are drawn through the current sources. Therefore, two of the equations needed are

$$I_1 = \frac{V_x}{2000}$$
$$I_2 = 2 \times 10^{-3}$$

The KVL equation for the third mesh is

$$-3 + 2\text{k}(I_3 - I_1) + 6\text{k}I_3 = 0$$

where

$$V_x = 4\text{k}\,(I_1 - I_2)$$

Combining these equations yields

$$-I_1 + 2I_2 = 0$$
$$I_2 = 2/\text{k}$$
$$-2\text{k}I_2 + 8\text{k}I_3 = 3$$

The MATLAB solution for these equations is

```
>> R = [-1 2 0; 0 1 0; -2000 0 8000]
   R =

           -1      2          0
            0      1          0
        -2000      0       8000

>> V = [0; 0.002; 3]
   V =

                          0
        0.00200000000000
        3.00000000000000

>> I = inv(R)*V
   I =

        0.00400000000000
        0.00200000000000
        0.00137500000000
```

And hence, $V_o = 8.25$ V

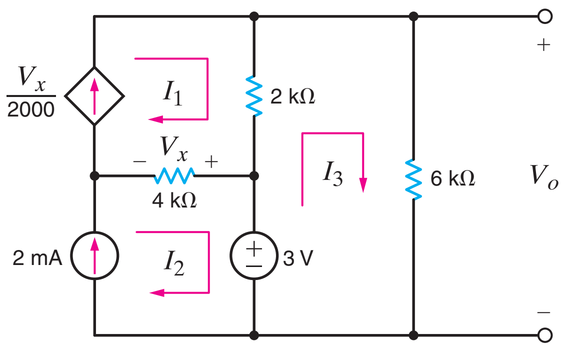


Figure 3.26
Circuit used in Example 3.18.

Example 3.19

The network in Fig. 3.27 contains both a current-controlled voltage source and a voltage-controlled current source. Let us use MATLAB to determine the loop currents.

SOLUTION The equations for the loop currents shown in the figure are

$$I_1 = \frac{4}{k}$$
$$I_2 = \frac{V_x}{2k}$$
$$-1kI_x + 2k(I_3 - I_1) + 1k(I_3 - I_4) = 0$$
$$1k(I_4 - I_3) + 1k(I_4 - I_2) + 12 = 0$$

where

$$V_x = 2k(I_3 - I_1)$$
$$I_x = I_4 - I_2$$

Combining these equations yields

$$I_1 = \frac{4}{k}$$
$$I_1 + I_2 - I_3 = 0$$
$$1kI_2 + 3kI_3 - 2kI_4 = 8$$
$$1kI_2 + 1kI_3 - 2kI_4 = 12$$

In matrix form the equations are

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & -1 & 0 \\ 0 & 1k & 3k & -2k \\ 0 & 1k & 1k & -2k \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} \frac{4}{k} \\ 0 \\ 8 \\ 12 \end{bmatrix}$$

The input and output data for the MATLAB solution are as follows:

```
>> R = [1 0 0 0 ; 1 1 -1 0;   0 1000 3000 -2000;
0 1000 1000 -2000]

   R =

       1        0         0         0
       1        1        -1         0
       0     1000      3000     -2000
       0     1000      1000     -2000

>> V = [0.004; 0; 8; 12]

   V =

        0.0040
             0
        8.0000
       12.0000

>> I = inv(R)*V

   I =

       0.0040
      -0.0060
      -0.0020
      -0.0100
```

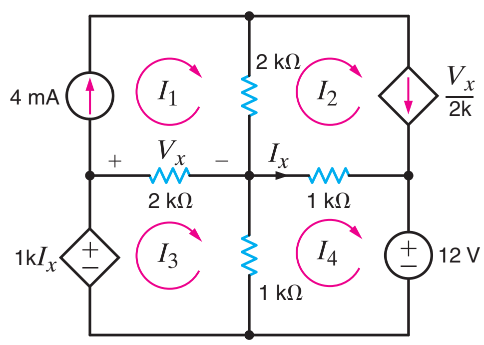


Figure 3.27 *Circuit used in Example 3.19.*

Example 3.20

At this point we will again examine the circuit in Example 3.10 and analyze it using loop equations. Recall that because the network has two voltage sources, the nodal analysis was somewhat simplified. In a similar manner, the presence of the current sources should simplify a loop analysis.

Clearly, the network has four loops, and thus four linearly independent equations are required to determine the loop currents. The network is redrawn in Fig. 3.28 where the loop currents are specified. Note that we have drawn one current through each of the independent current sources. This choice of currents simplifies the analysis since two of the four equations are

$$I_1 = 2/\text{k}$$
$$I_3 = -2/\text{k}$$

The two remaining KVL equations for loop currents I_2 and I_4 are

$$-2V_x + 1\text{k}I_2 + (I_2 - I_3)1\text{k} = 0$$
$$(I_4 + I_3 - I_1)1\text{k} - 2V_x + 1\text{k}I_4 + 4 = 0$$

where

$$V_x = 1\text{k}(I_1 - I_3 - I_4)$$

Substituting the equations for I_1 and I_3 into the two KVL equations yields

$$2\text{k}I_2 + 2\text{k}I_4 = 6$$
$$4\text{k}I_4 = 8$$

Solving these equations for I_2 and I_4, we obtain

$$I_4 = 2\text{ mA}$$
$$I_2 = 1\text{ mA}$$

and thus

$$V_o = 1\text{V}$$

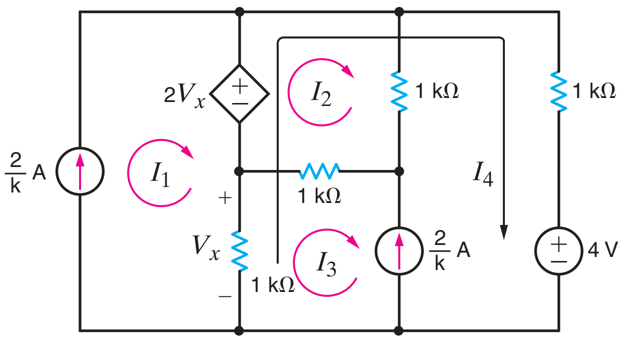


Figure 3.28
Circuit used in Example 3.20.

Example 3.21

Let us once again consider Example 3.11. In this case we will examine the network using loop analysis. Although there are four sources, two of which are dependent, only one of them is a current source. Thus, from the outset we expect that a loop analysis will be more difficult than a nodal analysis. Clearly, the circuit contains six loops. Thus, six linearly independent equations are needed to solve for all the unknown currents.

The network is redrawn in Fig. 3.29 where the loops are specified. The six KVL equations that describe the network are

$$1\text{k}I_1 + 1\text{k}(I_1 - I_2) + 1\text{k}(I_1 - I_4) = 0$$
$$1\text{k}(I_2 - I_1) - 6 + 1\text{k}(I_2 - I_5) = 0$$
$$I_3 = 2I_x$$

$$-12 + 1\text{k}(I_4 - I_1) + 2V_x = 0$$
$$-2V_x + 1\text{k}(I_5 - I_2) + 1\text{k}(I_5 - I_o) = 0$$
$$1\text{k}(I_o - I_5) + 1\text{k}(I_o - I_3) + 1\text{k}I_o = 0$$

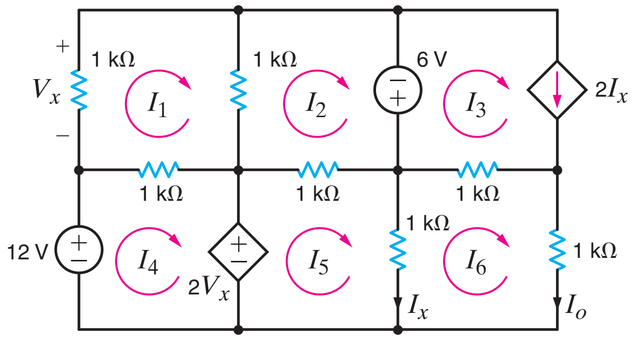


Figure 3.29 *Circuit used in Example 3.21.*

And the control variables for the two dependent sources are

$$V_x = -1\text{k}I_1$$
$$I_x = I_5 - I_o$$

Substituting the control parameters into the six KVL equations yields

$$\begin{array}{cccccccl}
3I_1 & -I_2 & 0 & -I_4 & 0 & 0 & = 0 \\
-I_1 & +2I_2 & 0 & 0 & -I_5 & 0 & = 6/\text{k} \\
0 & 0 & I_3 & 0 & -2I_5 & +2I_o & = 0 \\
-3I_1 & 0 & 0 & +I_4 & 0 & 0 & = 12/\text{k} \\
2I_1 & -I_2 & 0 & 0 & +2I_5 & -I_o & = 0 \\
0 & 0 & 0 & 0 & -3I_5 & +5I_o & = 0
\end{array}$$

which can be written in matrix form as

$$\begin{bmatrix} 3 & -1 & 0 & -1 & 0 & 0 \\ -1 & 2 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & -2 & 2 \\ -3 & 0 & 0 & 1 & 0 & 0 \\ 2 & -1 & 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 0 & -3 & 5 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_o \end{bmatrix} = \begin{bmatrix} 0 \\ 6/\text{k} \\ 0 \\ 12/\text{k} \\ 0 \\ 0 \end{bmatrix}$$

Although these six linearly independent simultaneous equations can be solved by any convenient method, we will employ a MATLAB solution. As the results listed below indicate, the current I_o is −48 mA.

```
>> R = [3 -1 0 -1 0 0 ; -1 2 0 0 -1 0 ; 0 0 1 0 -2 2 ;
-3 0 0 1 0 0 ; 2 -1 0 0 2 -1 ; 0 0 0 0 -3 5]

   R =

       3    -1     0    -1     0     0
      -1     2     0     0    -1     0
       0     0     1     0    -2     2
      -3     0     0     1     0     0
       2    -1     0     0     2    -1
       0     0     0     0    -3     5

>> V = [0; 0.006; 0; 0.012; 0; 0]
```

```
V =

         0
    0.0060
         0
    0.0120
         0
         0

>> I = inv(R)*V

I =

    0.0500
   -0.0120
   -0.0640
    0.1620
   -0.0800
   -0.0480

>>
```

As a final point, it is very important to examine the circuit carefully before selecting an analysis approach. One method could be much simpler than another, and a little time invested up front may save a lot of time in the long run.

PROBLEM-SOLVING STRATEGY

Loop Analysis

Step 1. One loop current is assigned to each independent loop in a circuit that contains N independent loops.

Step 2. If only independent voltage sources are present in the network, write the N linearly independent KVL equations, one for each loop. If dependent voltage sources are present, write the KVL equation as is done for circuits with only independent voltage sources; then write the controlling equations for the dependent sources.

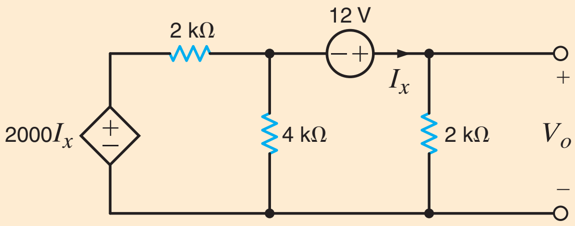
Step 3. If current sources are present in the network, either of two techniques can be used. In the first case, one loop current is selected to pass through one of the current sources. This is done for each current source in the network. The remaining loop currents ($N -$ the number of current sources) are determined by open-circuiting the current sources in the network and using this modified network to select them. Once all these currents are defined in the original network, the N loop equations can be written. The second approach is similar to the first with the exception that if two mesh currents pass through a particular current source, a supermesh is formed around this source. The two required equations for the meshes containing this source are the constraint equations for the two mesh currents that pass through the source and the supermesh equation. As indicated earlier, if dependent current sources are present, the controlling equations for these sources are also necessary.

LEARNING EXTENSIONS

E3.11 Use mesh analysis to find V_o in the circuit in Fig. E3.11. **PSV**

ANSWER: $V_o = 12$ V.

Figure E3.11



E3.12 Use loop analysis to solve the network in Example 3.5 and compare the time and effort involved in the two solution techniques.

E3.13 Use nodal analysis to solve the circuit in Example 3.15 and compare the time and effort involved in the two solution strategies.

3.3 Application Example

Application Example 3.22

A conceptual circuit for manually setting the speed of a dc electric motor is shown in Figure 3.30a. The resistors R_1 and R_2 are inside a component called a potentiometer, or pot, which is nothing more than an adjustable resistor, for example, a volume control. Turning the knob changes the ratio $\alpha = R_2/(R_1 + R_2)$, but the total resistance, $R_{pot} = R_1 + R_2$, is unchanged. In this way the pot forms a voltage divider that sets the voltage V_{speed}. The power amplifier output, V_M, is four times V_{speed}. Power amplifiers can output the high currents needed to drive the motor. Finally, the dc motor speed is proportional to V_M, that is, the speed in rpm is some constant k times V. Without knowing the details of the power amplifier, can we analyze this system? In particular, can we develop a relationship between rpm and α?

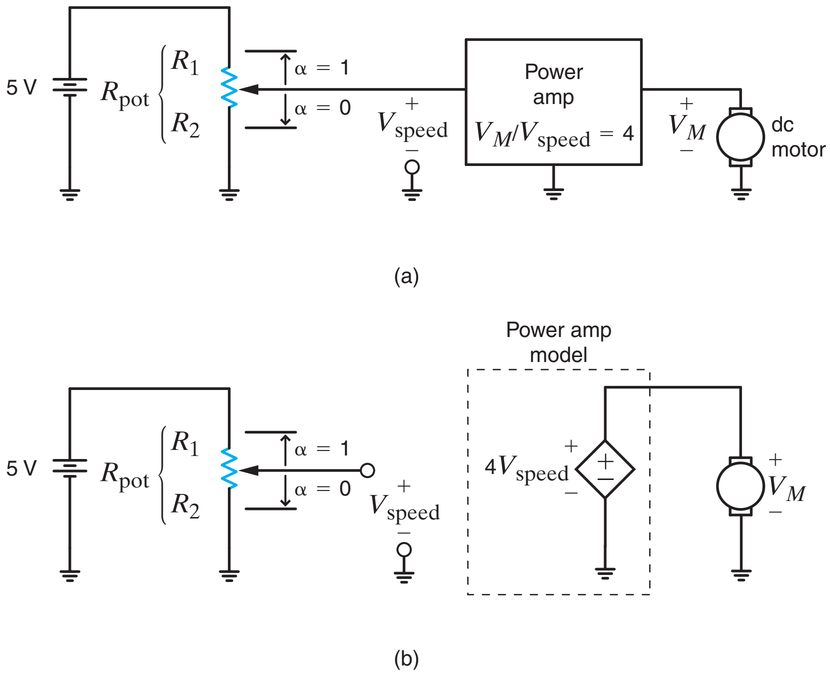


Figure 3.30 (a) A simple dc motor driver and (b) the circuit model used to analyze it.

SOLUTION Since the power amplifier output voltage is proportional to its input, we can model the amplifier as a simple dependent source. The resulting circuit diagram is shown in Fig. 3.30b. Now we can easily develop a relationship between motor speed and the pot position, α. The equations that govern the operation of the motor, power amplifier, and the voltage divider are

$$\text{speed (rpm)} = K_M V_M$$
$$V_M = 4V_{\text{speed}}$$
$$V_{\text{speed}} = 5\frac{R_2}{R_1 + R_2} = 5\left[\frac{R_2}{R_{\text{pot}}}\right] = 5\alpha$$
$$R_2 = \alpha R_{\text{pot}} \qquad R_1 = (1 - \alpha)R_{\text{pot}}$$

Combining these relationships to eliminate V_{speed} yields a relationship between motor speed and α, that is, rpm $= 20\alpha$. If, for example, the motor constant K_M is 50 rpm/V, then

$$\text{rpm} = 1000\alpha$$

This relationship specifies that the motor speed is proportional to the pot knob position. Since the maximum value of α is 1, the motor speed ranges from 0 to 1000 rpm.

Note that in our model, the power amplifier, modeled by the dependent source, can deliver *any* current the motor requires. Of course, this is not possible, but it does demonstrate some of the tradeoffs we experience in modeling. By choosing a simple model, we were able to develop the required relationship quickly. However, other characteristics of an actual power amplifier have been omitted in this model.

3.4 Design Example

Design Example 3.23

An 8-volt source is to be used in conjunction with two standard resistors to design a voltage divider that will output 5 V when connected to a 100-μA load. While keeping the consumed power as low as possible, we wish to minimize the error between the actual output and the required 5 volts.

SOLUTION The divider can be modeled as shown in Fig. 3.31. Applying KCL at the output node yields the equation

$$\frac{V_S - V_o}{R_1} = \frac{V_o}{R_2} + I_o$$

Using the specified parameters for the input voltage, desired output voltage, and the current source, we obtain

$$R_1 = \frac{3R_2}{5 + (100\mu)R_2}$$

By trial and error, we find that excellent values for the two standard resistors are $R_1 = 10\,\text{k}\Omega$ and $R_2 = 27\,\text{k}\Omega$. Large resistor values are used to minimize power consumption. With this selection of resistors the output voltage is 5.11 V, which is a percent error of only 2.15%.

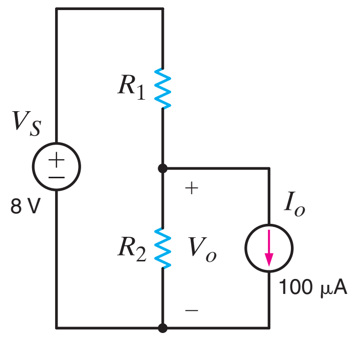


Figure 3.31
A simple voltage-divider circuit with a 100-μA load.

SUMMARY

Nodal analysis for an N-node circuit

- Select one node in the N-node circuit as the reference node. Assume that the node voltage is zero and measure all node voltages with respect to this node.
- If only independent current sources are present in the network, write the KCL equations at the $N - 1$ nonreference nodes. If dependent current sources are present, write the KCL equations as is done for networks with only independent current sources; then write the controlling equations for the dependent sources.
- If voltage sources are present in the network, they may be connected (1) between the reference node and a nonreference node or (2) between two nonreference nodes. In the former case, if the voltage source is an independent source, then the voltage at one of the nonreference nodes is known. If the source is dependent, it is treated as an independent source when writing the KCL equations, but an additional constraint equation is necessary.

  In the latter case, if the source is independent, the voltage between the two nodes is constrained by the value of the voltage source and an equation describing this constraint represents one of the $N - 1$ linearly independent equations required to determine the N-node voltages. The surface of the network described by the constraint equation (i.e., the source and two connecting nodes) is called a supernode. One of the remaining $N - 1$ linearly independent equations is obtained by applying KCL at this supernode. If the voltage source is dependent, it is treated as an independent source when writing the KCL equations, but an additional constraint equation is necessary.

Loop analysis for an N-loop circuit

- One loop current is assigned to each independent loop in a circuit that contains N independent loops.
- If only independent voltage sources are present in the network, write the N linearly independent KVL equations, one for each loop. If dependent voltage sources are present, write the KVL equations as is done for circuits with only independent voltage sources; then write the controlling equations for the dependent sources.
- If current sources are present in the network, either of two techniques can be used. In the first case, one loop current is selected to pass through one of the current sources. This is done for each current source in the network. The remaining loop currents ($N -$ the number of current sources) are determined by open-circuiting the current sources in the network and using this modified network to select them. Once all these currents are defined in the original network, the N-loop equations can be written. The second approach is similar to the first with the exception that if two mesh currents pass through a particular current source, a supermesh is formed around this source. The two required equations for the meshes containing this source are the constraint equations for the two mesh currents that pass through the source and the supermesh equation. If dependent current sources are present, the controlling equations for these sources are also necessary.

PROBLEMS

PSV **CS** **both available on the web at: http://www.justask4u.com/irwin**

SECTION 3.1

3.1 Find I_o in the circuit in Fig. P3.1 using nodal analysis. **CS**

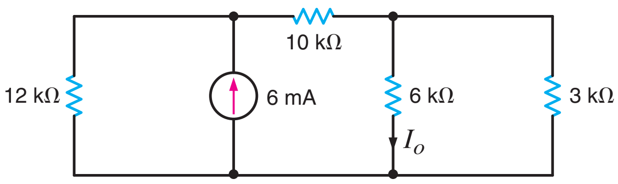


Figure P3.1

3.2 Use nodal analysis to find V_1 in the circuit in Fig. P3.2.

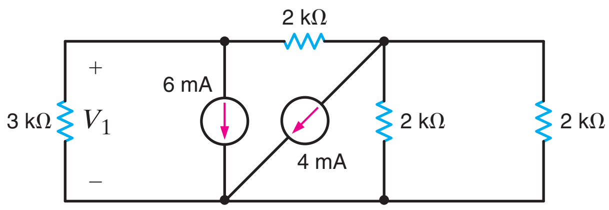


Figure P3.2

3.3 Use nodal analysis to find both V_1 and V_o in the circuit in Fig. P3.3. PSV

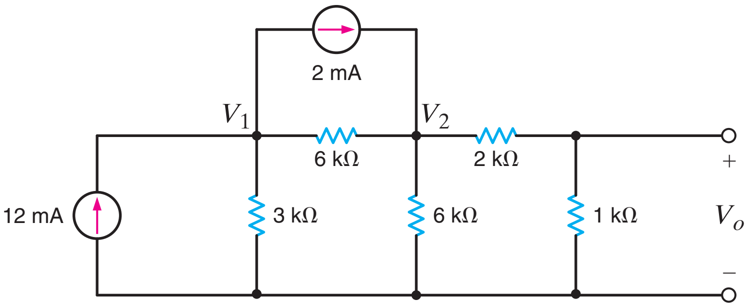


Figure P3.3

3.4 Find V_1 and V_2 in the circuit in Fig. P3.4 using nodal analysis. Then solve the problem using MATLAB and compare your answers.

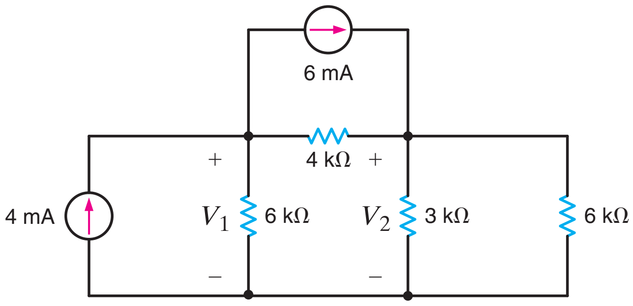


Figure P3.4

3.5 Find I_o in the circuit in Fig. P3.5 using nodal analysis. CS

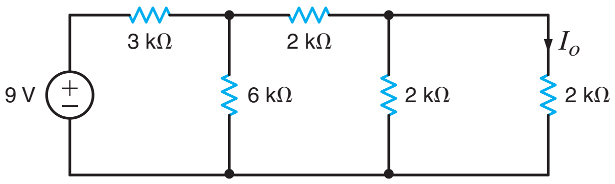


Figure P3.5

3.6 Find I_o in the network in Fig. P3.6 using nodal analysis.

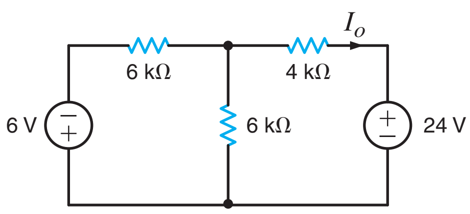


Figure P3.6

3.7 Find V_o in the network in Fig. P3.7 using nodal analysis. CS

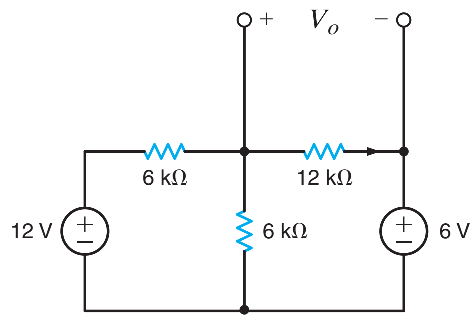


Figure P3.7

3.8 Find V_o in the circuit in Fig. P3.8 using nodal analysis. PSV

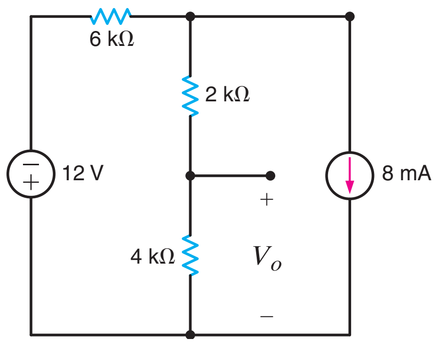


Figure P3.8

3.9 Use nodal analysis to find V_o in the circuit in Fig. P3.9.

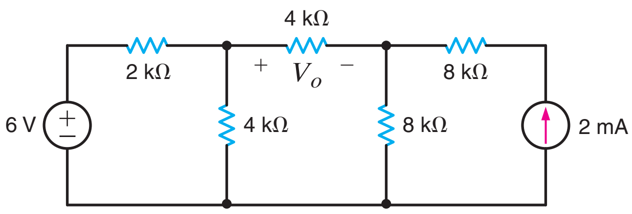


Figure P3.9

3.10 Find I_o in the circuit in Fig. P3.10 using nodal analysis.

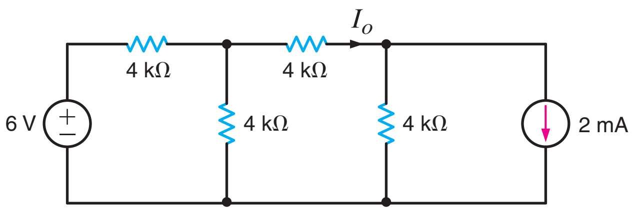


Figure P3.10

3.11 Use nodal analysis to find V_o in the network in Fig. P3.11. Then solve the problem using MATLAB and compare your answers.

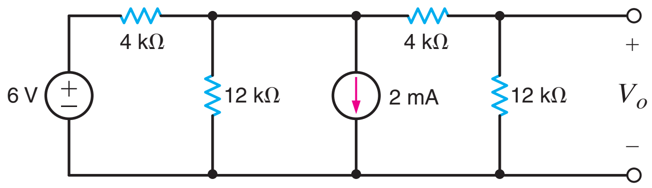


Figure P3.11

3.12 Use nodal analysis to find V_o in the circuit in Fig. P3.12.

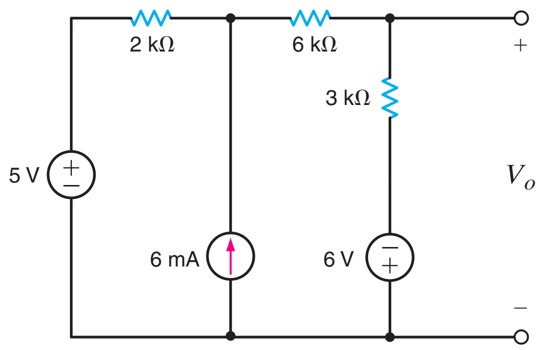


Figure P3.12

3.13 Use nodal analysis to find V_o in the circuit in Fig. P3.13.

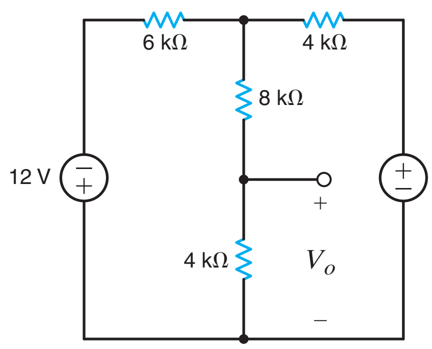


Figure P3.13

3.14 Find I_o in the network in Fig. P3.14. **cs**

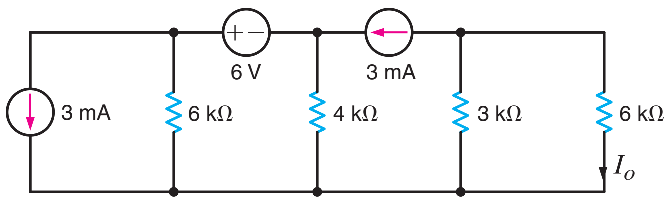


Figure P3.14

3.15 Find I_1 in the network in Fig. P3.15. **cs**

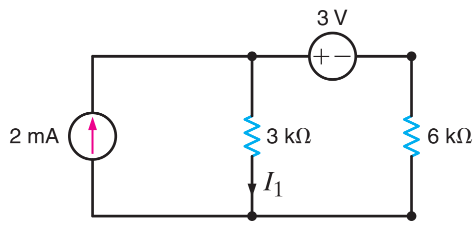


Figure P3.15

3.16 Find I_o in the network in Fig. P3.16.

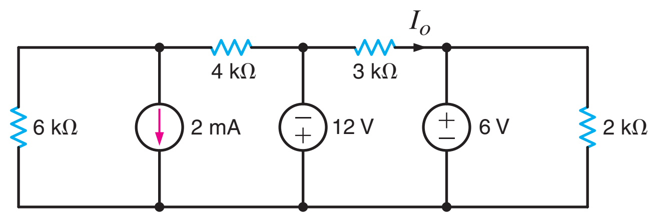


Figure P3.16

3.17 Use nodal analysis to find V_x and V_y in the circuit in Fig. P3.17.

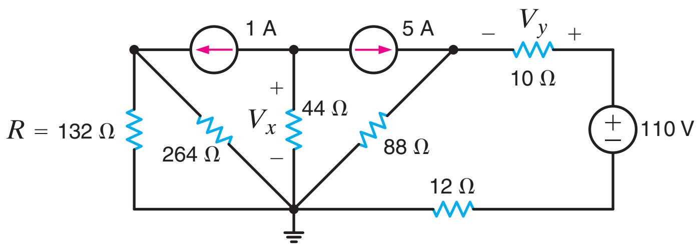


Figure P3.17

3.18 For the network in Fig P3.17, explain why the resistor R plays no role in determining V_x and V_y.

3.19 Use nodal analysis to find V_o in the network in Fig P3.19.

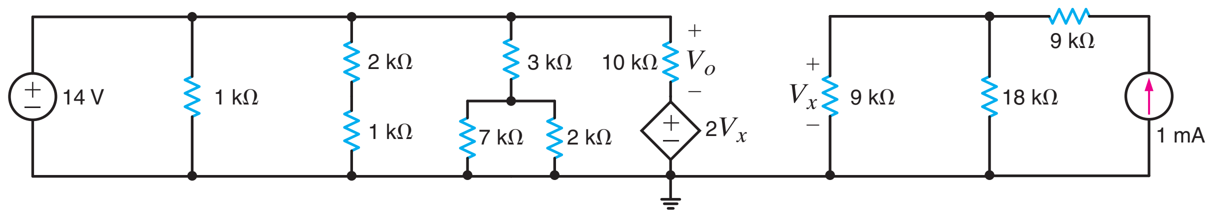


Figure P3.19

3.20 Use nodal analysis to find V_A and V_B in the network in Fig. P3.20. Simplify the analysis by making an insightful choice for the reference node.

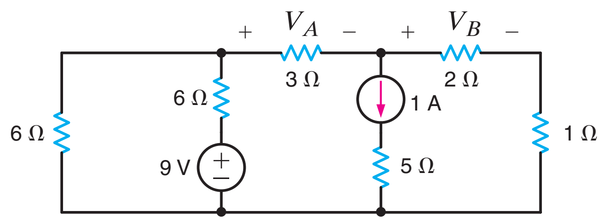


Figure P3.20

3.21 Find I_o in the circuit in Fig. P3.21.

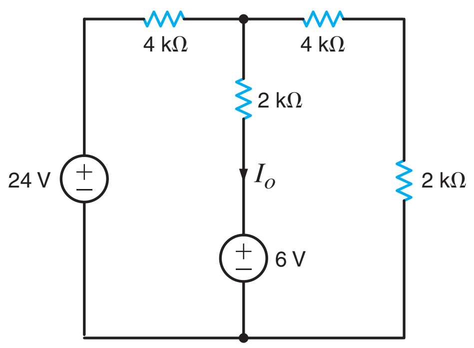


Figure P3.21

3.22 Use nodal analysis to find I_o and I_S in the circuit in Fig. P3.22.

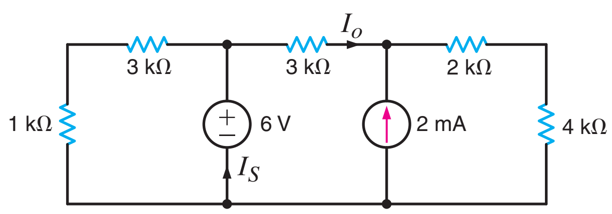


Figure P3.22

3.23 Use nodal analysis to find V_o in the network in Fig. P3.23.

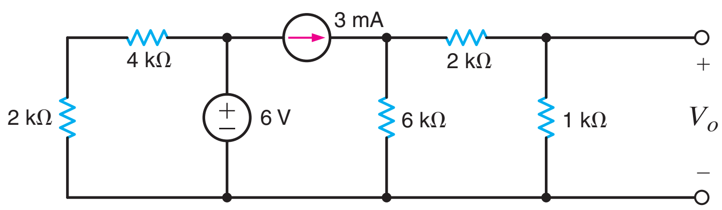


Figure P3.23

3.24 Use nodal analysis to find I_o in the circuit in Fig. P3.24.

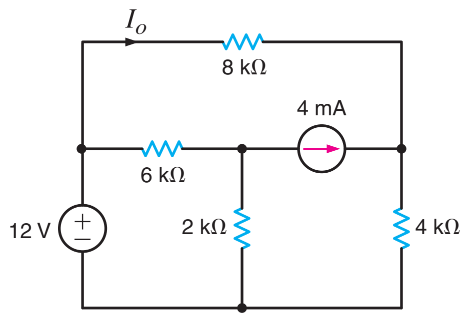


Figure P3.24

3.25 Find I_o in the network in Fig. P3.25 using nodal analysis.

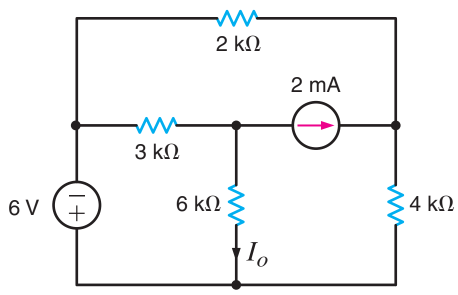


Figure P3.25

3.26 Use nodal analysis to find I_o in the network in Fig. P3.26.

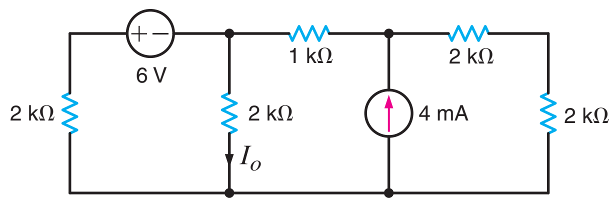


Figure P3.26

3.27 Use nodal analysis to find V_o in the network in Fig. P3.27. Then solve this problem using MATLAB and compare your answers. cs

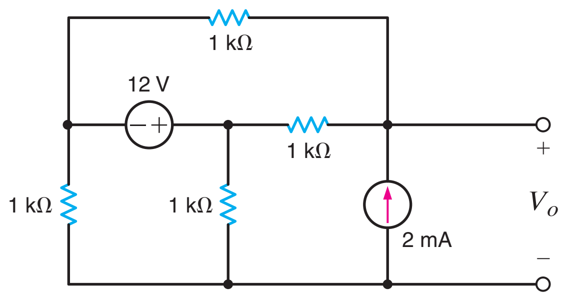


Figure P3.27

3.28 Find V_o in the circuit in Fig. P3.28 using nodal analysis.
PSV

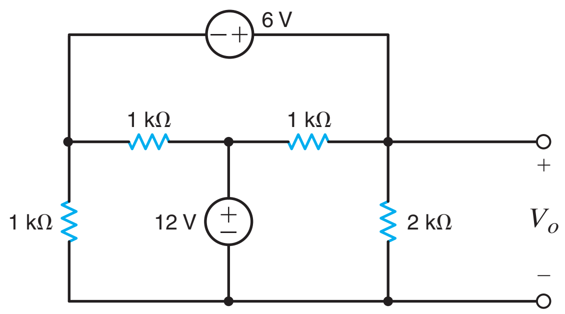


Figure P3.28

3.29 Use nodal analysis to find V_o in the circuit in Fig. P3.29.
CS

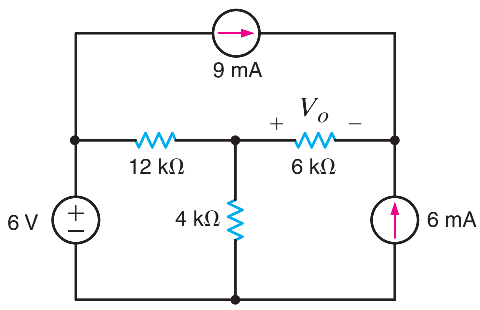


Figure P3.29

3.30 Use nodal analysis to find V_o in the circuit in Fig. P3.30.

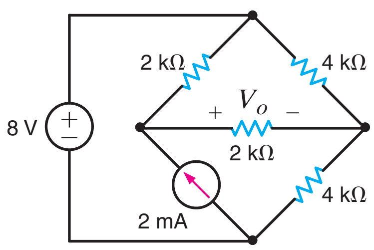


Figure P3.30

3.31 Use nodal analysis to find V_o in the circuit in Fig. P3.31.

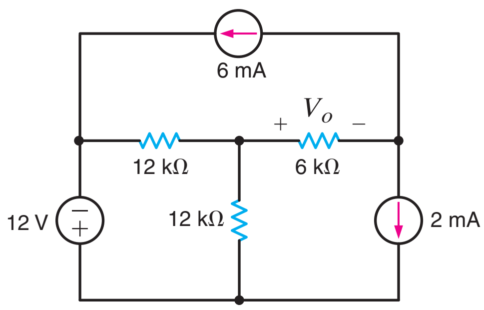


Figure P3.31

3.32 Find V_o in the network in Fig. P3.32 using nodal analysis.
CS

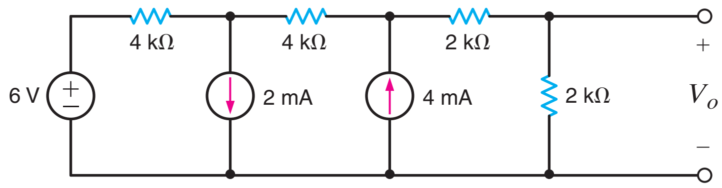


Figure P3.32

3.33 Use nodal analysis to find V_o in the network in Fig. P3.33.

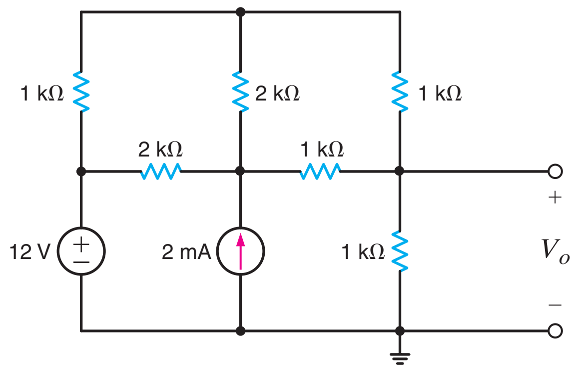


Figure P3.33

3.34 Find V_o in the circuit in Fig. P3.34 using nodal analysis.

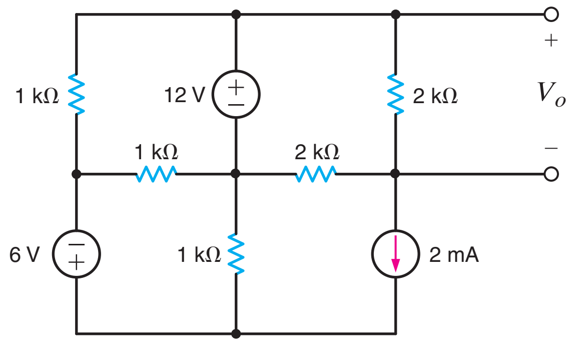


Figure P3.34

3.35 Use nodal analysis to find V_o in the network in Fig. P3.35.

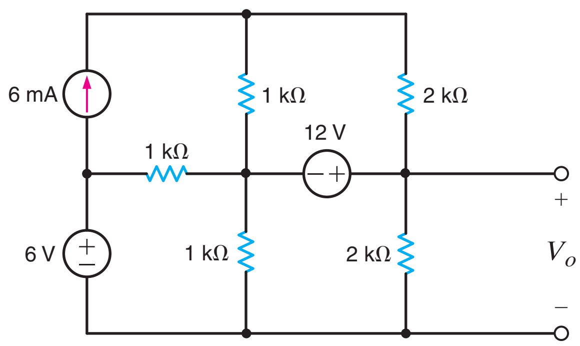


Figure P3.35

3.36 Use MATLAB to find the node voltages in the network in Fig. P3.36. CS

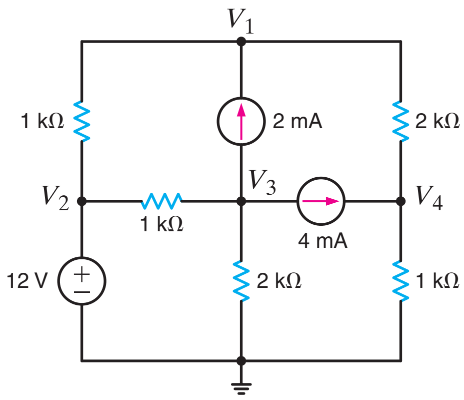


Figure P3.36

3.37 Determine V_o in the network in Fig. P3.37 using nodal analysis.

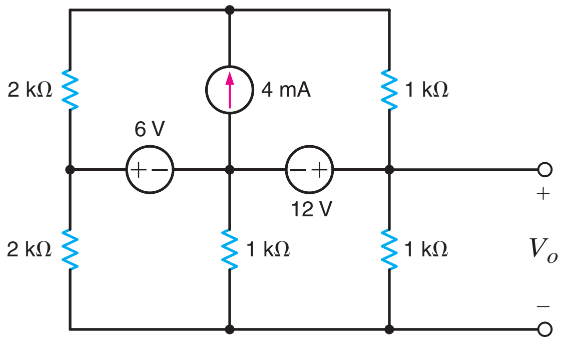


Figure P3.37

3.38 Find V_o in the circuit in Fig. P3.38.

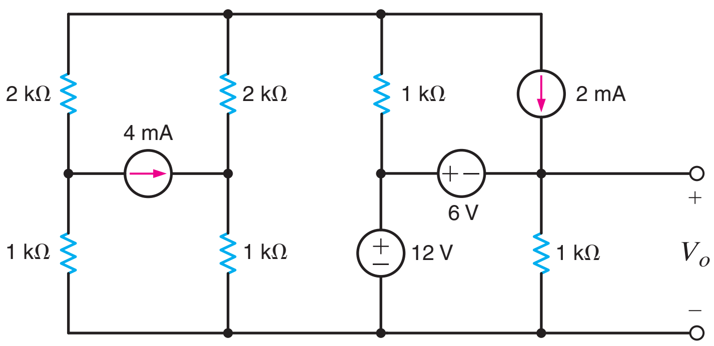


Figure P3.38

3.39 Find V_o in the network in Fig. P3.39.

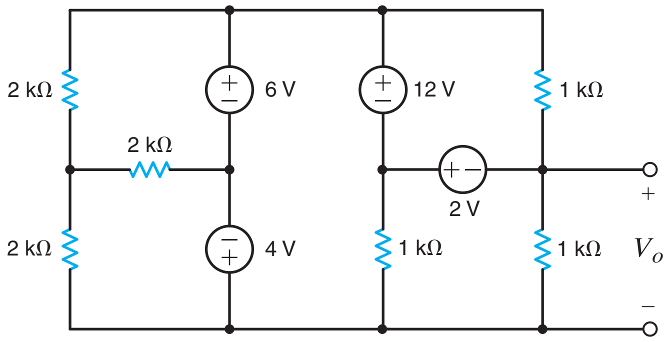


Figure P3.39

3.40 Use nodal analysis to find V_o in the circuit in Fig. P3.40.

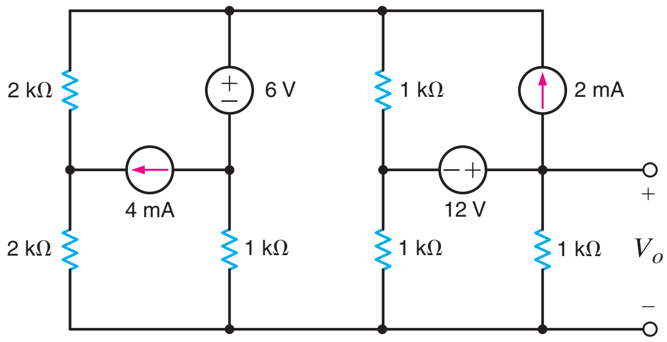


Figure P3.40

3.41 Determine V_o in the network in Fig. P3.41.

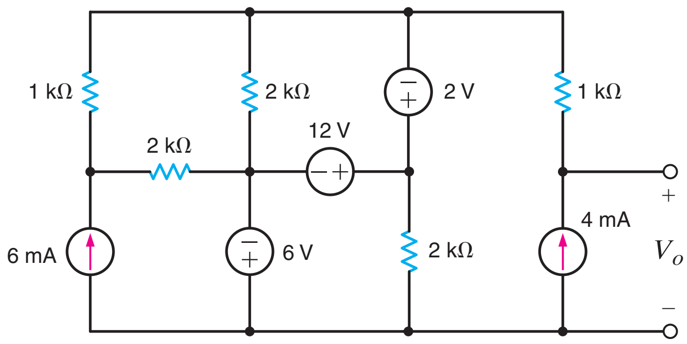


Figure P3.41

3.42 Find V_o in the circuit in Fig. P3.42.

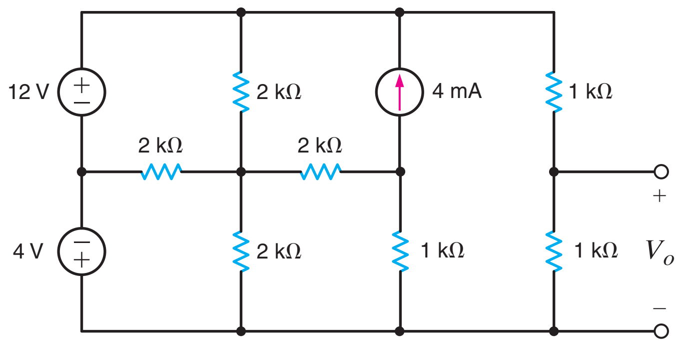


Figure P3.42

3.43 Find I_o in the circuit in Fig. P3.43 using nodal analysis.

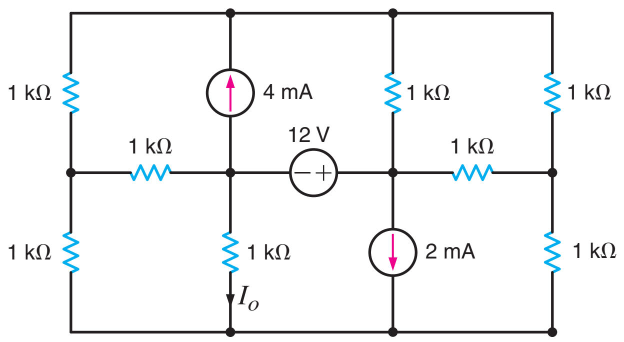


Figure P3.43

3.44 Use nodal analysis to find V_o in Fig. P3.44.

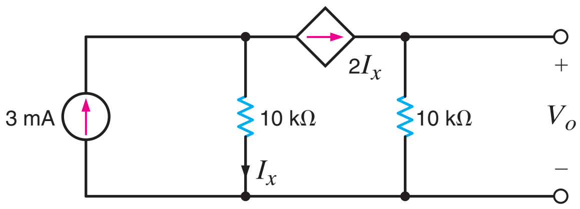


Figure P3.44

3.45 Find V_o in the circuit in Fig. P3.45 using nodal analysis.

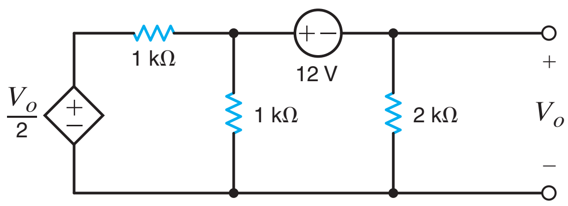


Figure P3.45

3.46 Find V_o in the circuit in Fig. P3.46 using nodal analysis. Then solve the problem using MATLAB and compare your answers. **CS**

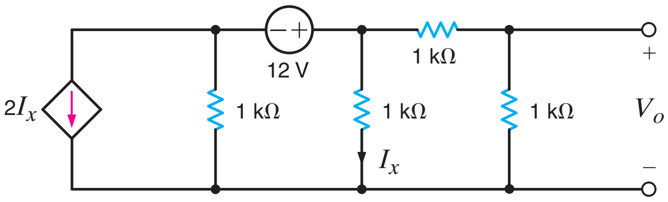


Figure P3.46

3.47 Find I_o in the network in Fig. P3.47. **PSV**

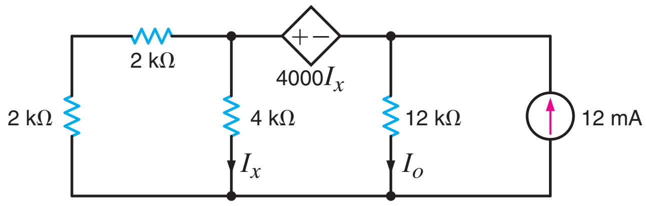


Figure P3.47

3.48 Find I_o in the circuit in Fig. P3.48 using nodal analysis.

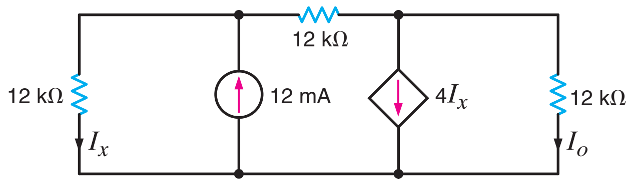


Figure P3.48

3.49 Find V_o in the network in Fig. P3.49 using nodal analysis. **CS**

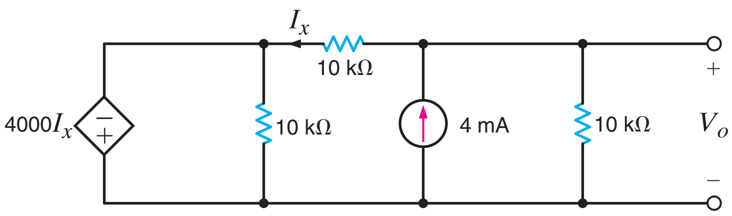


Figure P3.49

3.50 Find V_o in the circuit in Fig. P3.50. **PSV**

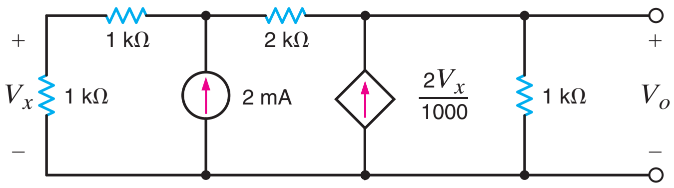


Figure P3.50

3.51 Use nodal analysis to find V_o in the circuit in Fig. P3.51. In addition, find all branch currents and check your answers using KCL at every node.

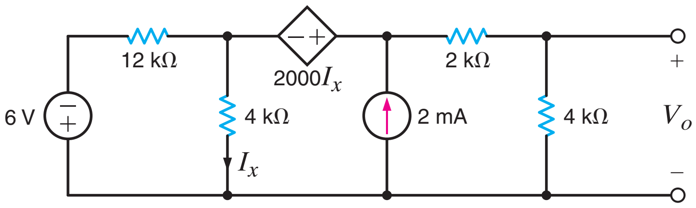


Figure P3.51

3.52 Find the power supplied by the 2-A current source in the network in Fig. P3.52 using nodal analysis.

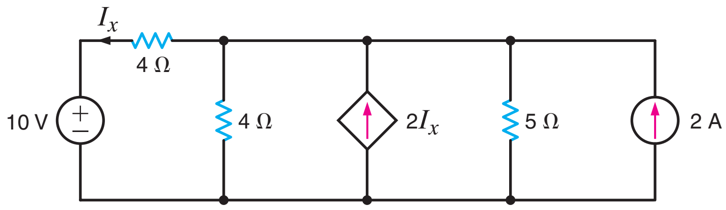


Figure P3.52

3.53 Use nodal equations for the circuit in Fig. P3.53 to determine V_o.

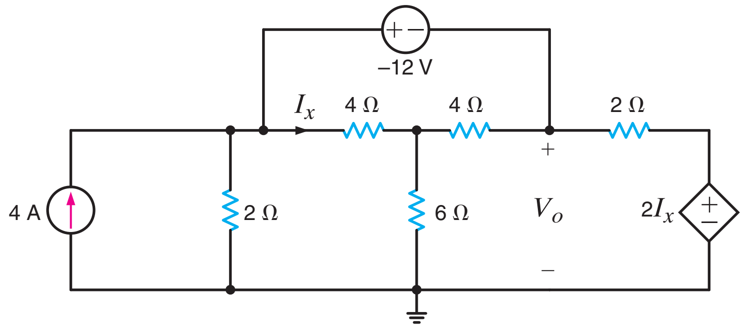


Figure P3.53

3.54 Determine V_o in the network in Fig. P3.54 using nodal analysis.

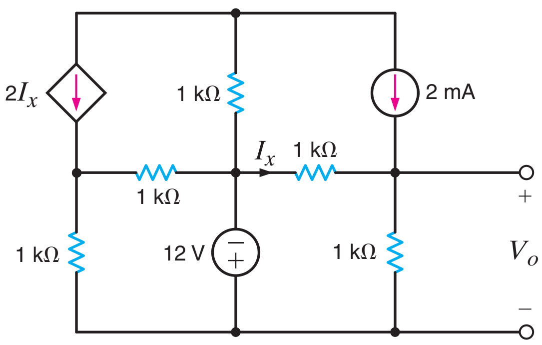


Figure P3.54

3.55 Calculate V_o in the circuit in Fig. P3.55 using nodal analysis.

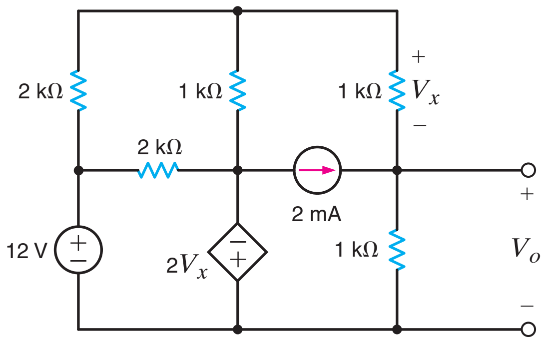


Figure P3.55

3.56 Using nodal analysis, find V_o in the network in Fig. P3.56.

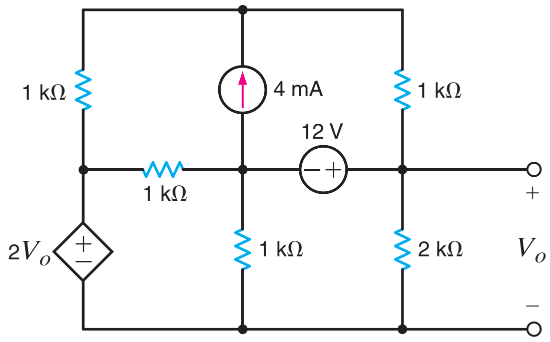


Figure P3.56

3.57 Use nodal analysis to find V_o in the circuit in Fig. P3.57.

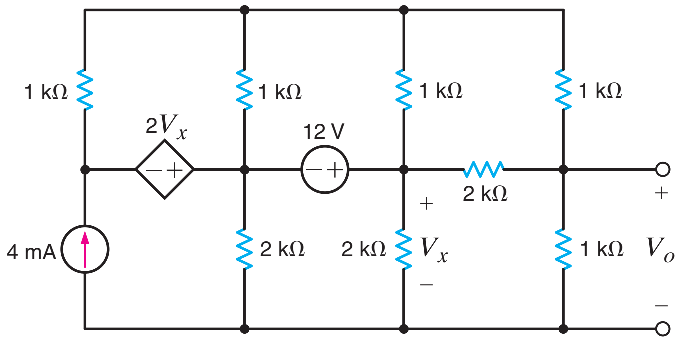


Figure P3.57

3.58 Use nodal analysis to determine I_o in the circuit in Fig. P3.58.

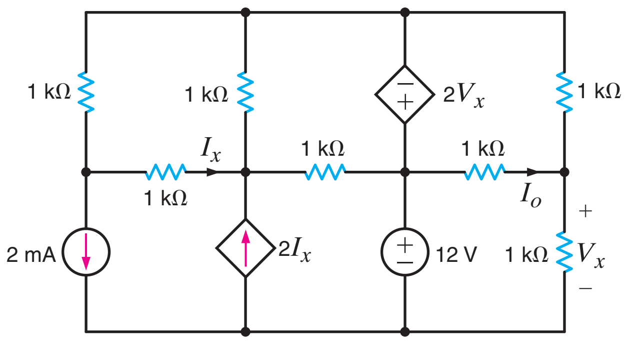


Figure P3.58

3.59 Find I_o in the network in Fig. P3.59 using nodal analysis.

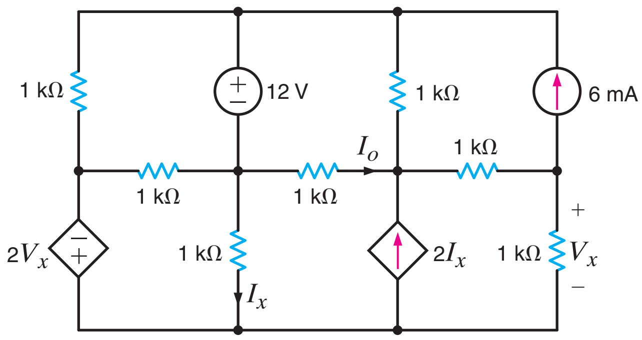


Figure P3.59

3.60 Given the network in Fig. P3.60, we wish to determine the power dissipated in the resistor R_3.

(a) Is mesh or nodal analysis the most efficient approach? Why?

(b) For a nodal analysis, comment on the advantages of selecting node 1 as the reference node. Repeat for nodes 2, 3, and 4.

(c) Based on your results in (b), write the node equations.

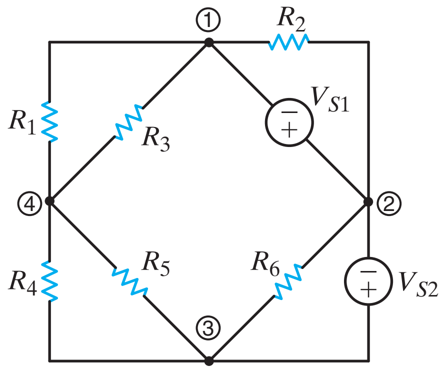


Figure P3.60

3.61 In the circuit in Fig. P3.61, use Gaussian elimination to determine V_o.

(a) Would mesh or nodal analysis be the most efficient approach? Why?

(b) If mesh analysis is used, are any supermeshes required? Write the mesh equations. If nodal analysis is used, are any supernodes required? If so, how many? What is the best location for the reference node and why? Write the node equations.

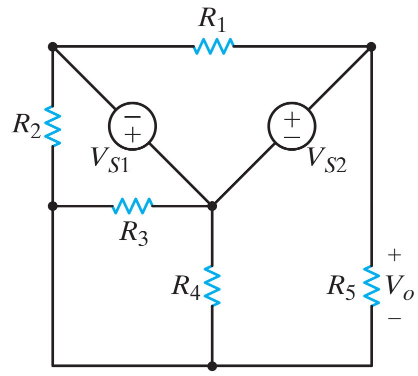


Figure P3.61

SECTION 3.2

3.62 Use mesh equations to find V_o in the circuit in Fig. P3.62.

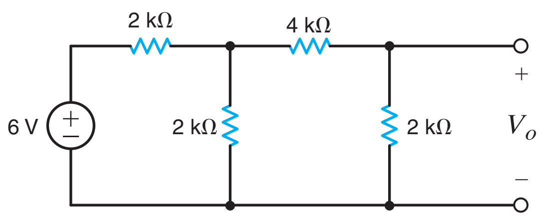


Figure P3.62

3.63 Find V_o in the network in Fig. P3.63 using mesh equations. **PSV**

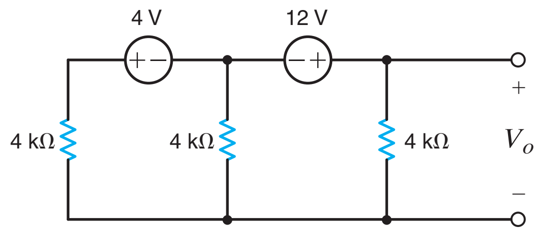


Figure P3.63

3.64 Use mesh analysis to find V_o in the circuit in Fig. P3.64.

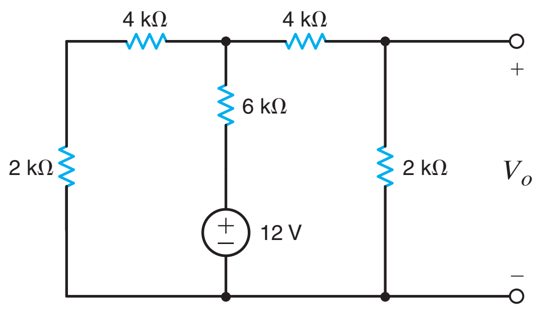


Figure P3.64

3.65 Use mesh analysis to find V_o in the circuit in Fig. P3.65.

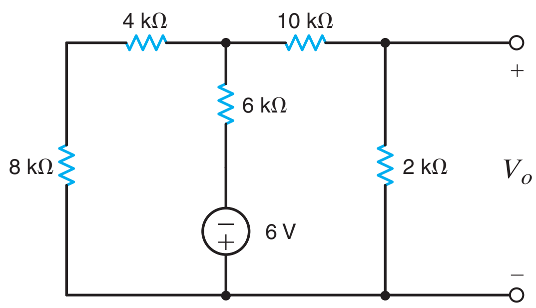


Figure P3.65

3.66 Use mesh analysis to find V_o in the network in Fig. P3.66. CS

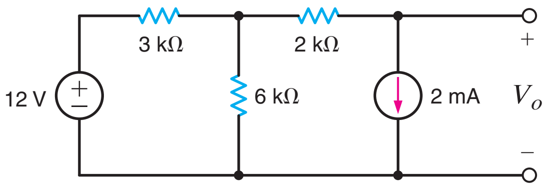


Figure P3.66

3.67 Use loop analysis to find V_o in the circuit in Fig. P3.67.

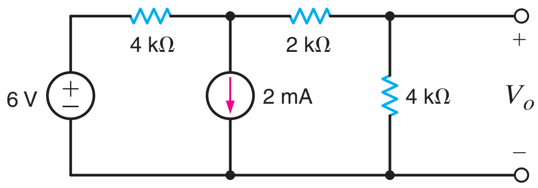


Figure P3.67

3.68 Use loop analysis to find V_o in the network in Fig. P3.68. PSV

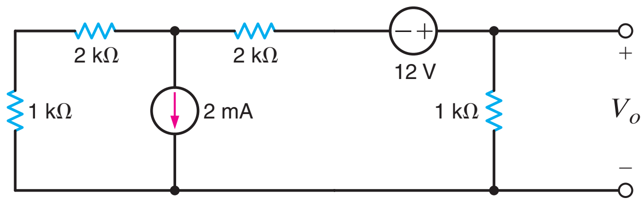


Figure P3.68

3.69 Find I_o in the network in Fig. P3.69 using mesh analysis. CS

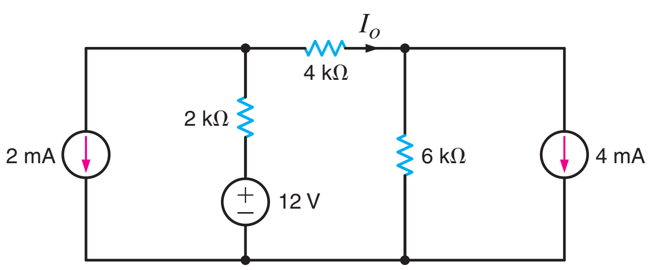


Figure P3.69

3.70 Use both nodal analysis and mesh analysis to find I_o in the circuit in Fig. P3.70.

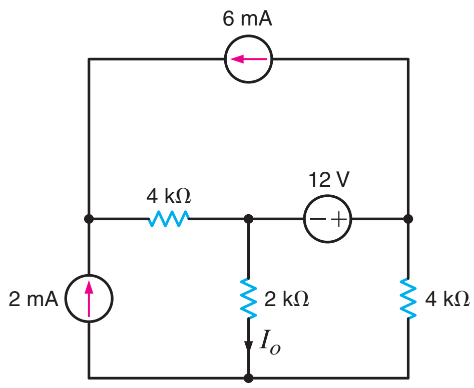


Figure P3.70

3.71 Find I_o in the network in Fig. P3.71 using loop analysis. Then solve the problem using MATLAB and compare your answers. CS

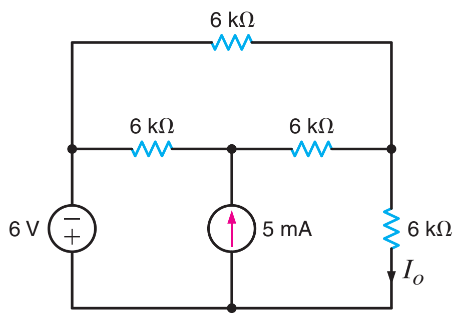


Figure P3.71

3.72 Find V_o in the network in Fig. P3.72 using both mesh and nodal analysis. PSV

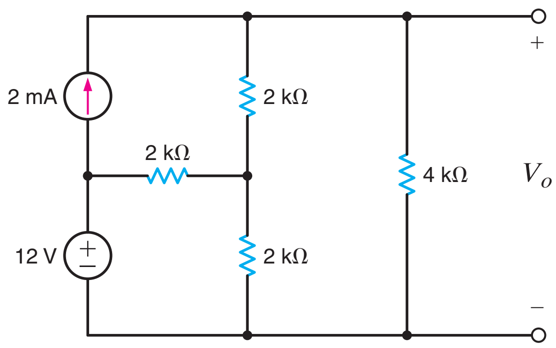


Figure P3.72

3.73 Use loop analysis to find I_o in the network in Fig. P3.73. CS

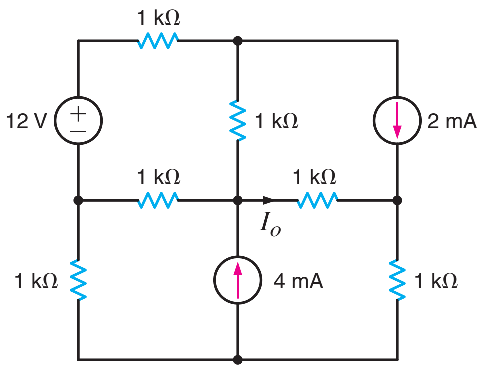


Figure P3.73

3.74 Find I_o in the circuit in Fig. P3.74. CS

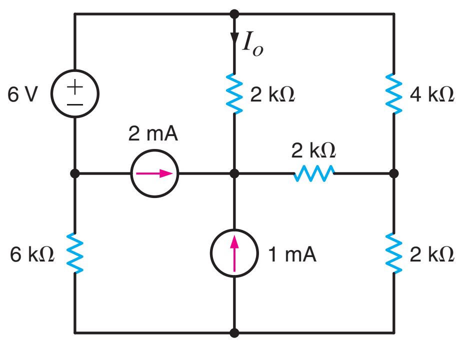


Figure P3.74

3.75 Solve Problem 3.33 using loop analysis.

3.76 Solve Problem 3.34 using loop analysis.

3.77 Solve Problem 3.35 using loop analysis.

3.78 Solve Problem 3.37 using loop analysis.

3.79 Solve Problem 3.40 using loop analysis.

3.80 Solve Problem 3.43 using loop analysis.

3.81 Use MATLAB to find the mesh currents in the network in Fig. P3.81. CS

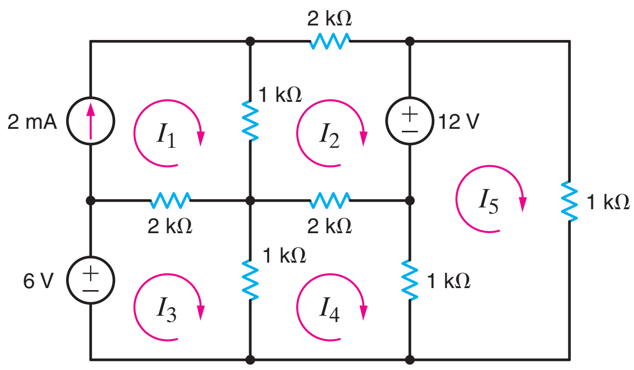


Figure P3.81

3.82 Write mesh equations for the circuit in Fig. P3.82 using the assigned currents.

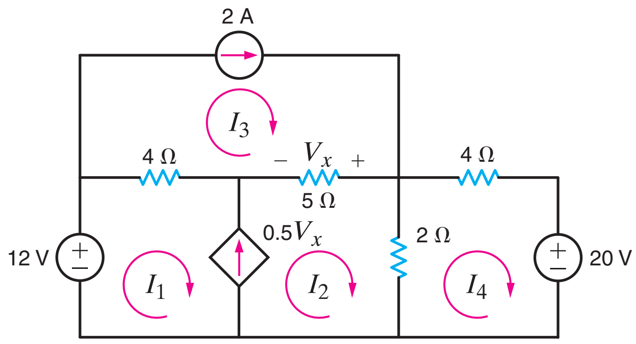


Figure P3.82

3.83 Use mesh analysis to find V_o in the circuit in Fig. P3.83. PSV

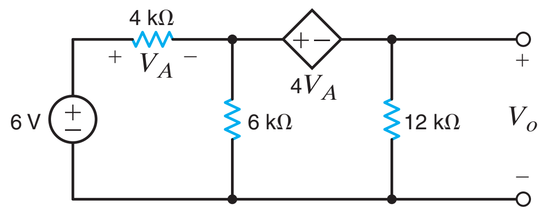


Figure P3.83

3.84 Find V_o in the circuit in Fig. P3.84 using mesh analysis. CS

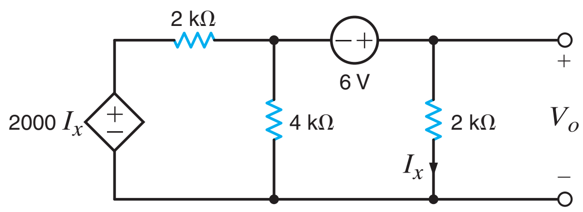


Figure P3.84

3.85 Use loop analysis to find V_o in the network in Fig. P3.85.

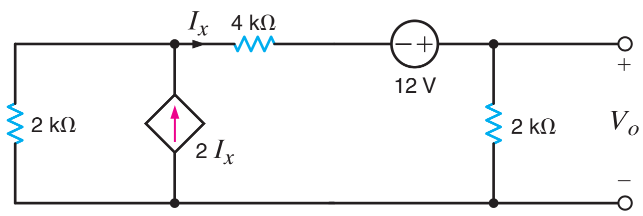


Figure P3.85

3.86 Use loop analysis to find V_o in the circuit in Fig. P3.86.

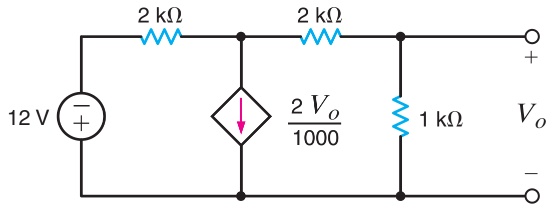


Figure P3.86

3.87 Use both nodal analysis and mesh analysis to find V_o in the circuit in Fig. P3.87.

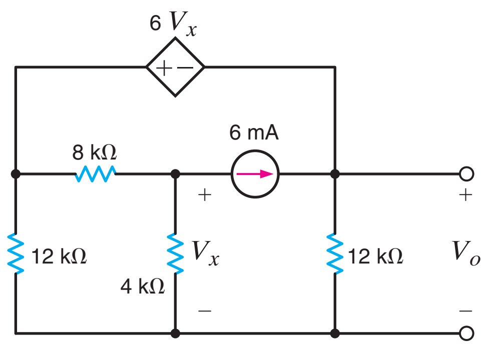


Figure P3.87

3.88 Using mesh analysis, find V_o in the circuit in Fig. P3.88.

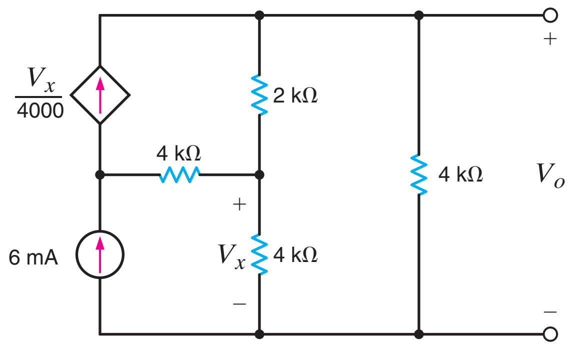


Figure P3.88

3.89 Find V_o in the network in Fig. P3.89. PSV

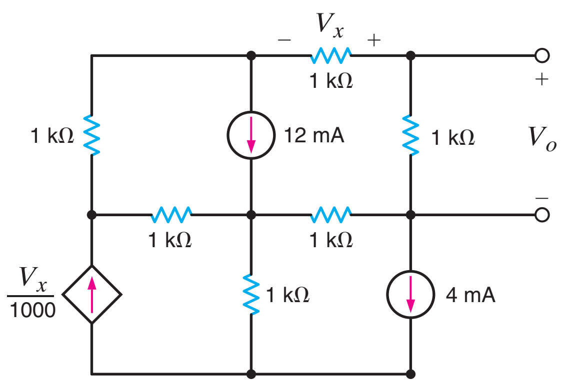


Figure P3.89

3.90 Solve Problem 3.54 using loop analysis.

3.91 Solve Problem 3.55 using loop analysis.

3.92 Solve Problem 3.56 using loop analysis.

3.93 Solve Problem 3.57 using loop analysis.

3.94 Solve Problem 3.58 using loop analysis.

3.95 Solve Problem 3.59 using loop analysis.

3.96 Use mesh analysis to determine the power delivered by the independent 3-V source in the network in Fig. P3.96.

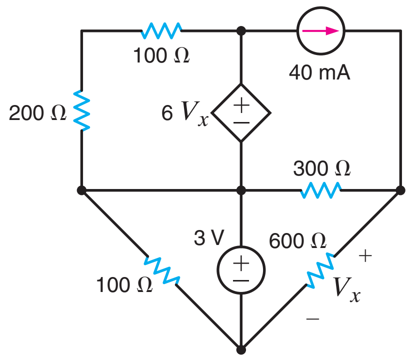


Figure P3.96

3.97 Use mesh analysis to find the power delivered by the current-controlled voltage source in the circuit in Fig. P3.97.

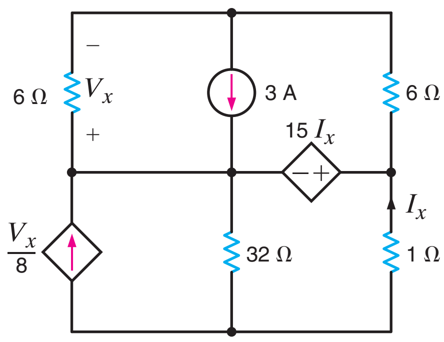


Figure P3.97

TYPICAL PROBLEMS FOUND ON THE FE EXAM

3FE-1 Find V_o in the circuit in Fig. 3PFE-1. CS

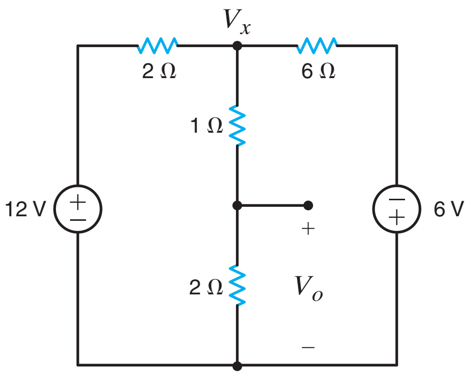


Figure 3PFE-1

3FE-2 Determine the power dissipated in the 6-ohm resistor in the network in Fig. 3PFE-2.

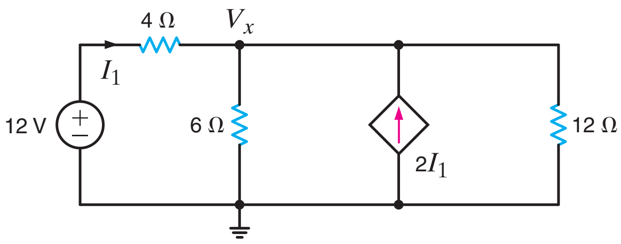


Figure 3PFE-2

3FE-3 Find the current I_x in the 4-ohm resistor in the circuit in Fig. 3PFE-3. CS

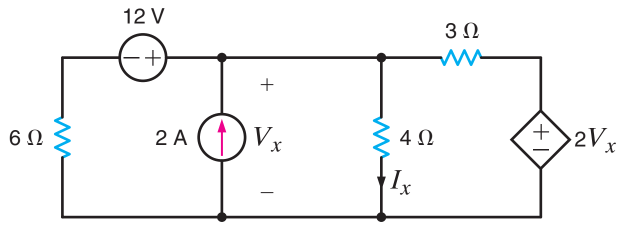


Figure 3PFE-3

3FE-4 Determine the voltage V_o in the circuit in Fig. 3PFE-4.

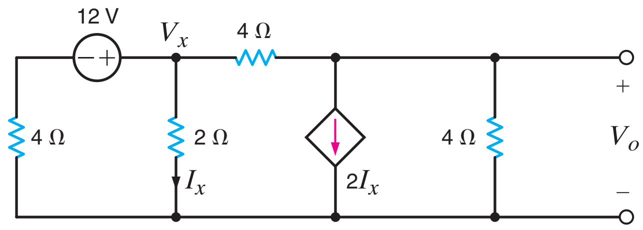


Figure 3PFE-4