NODAL ANALYSIS

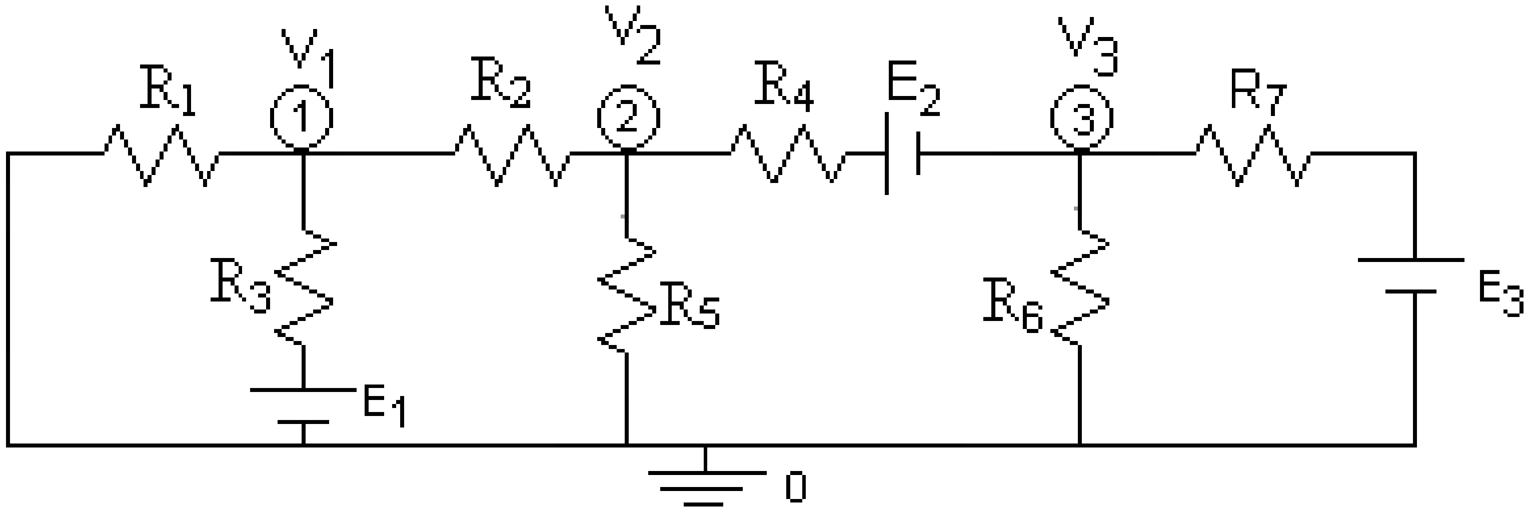
Nodal Analysis Method

It is one of the best circuit analysis methods. It depends on KCL. Label the voltages at every node and apply KCL with the following considerations:-

- **1- Select one of the nodes as node Reference (It is better to be the node with the largest number of branches).**
- **Consider the voltage of the reference node to be zero. Any other node voltage is measured with respect to this reference node voltage**
- **Select the current direction out of the nodes, and if there is a source with known current direction, then this direction could be considered.**

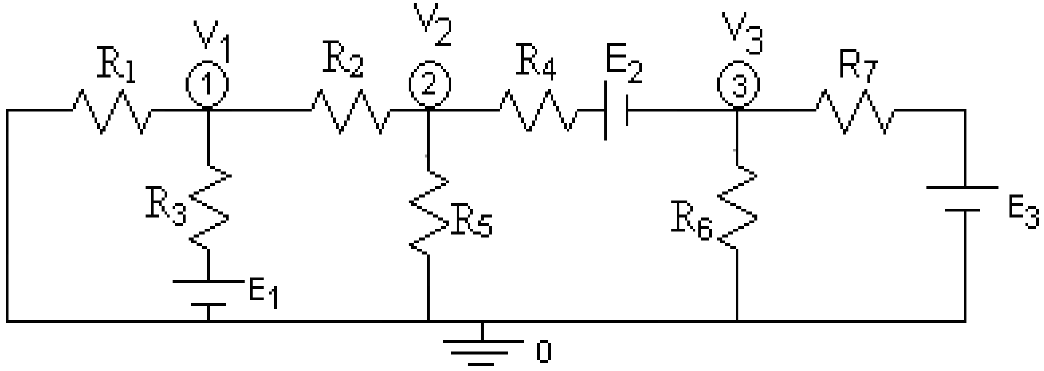
Example of Basic of Nodal analysis

- **Use Nodal analysis to write the equations of this circuit**



Select node 4 as reference node with voltage 0
V_1, V_2, V_3 are voltages of node 1,2, and 3

Example of Basic of Nodal analysis



Apply KCL to node 1:

$$\frac{V_1}{R_1} + \frac{V_1 - V_2}{R_2} + \frac{V_1 - E_1}{R_3} = 0$$

$$\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right]V_1 - \frac{1}{R_2}V_2 = \frac{E_1}{R_3} \quad \text{....(1)}$$

Apply KCL to node 2:

$$\frac{V_2 - V_1}{R_2} + \frac{V_2}{R_5} + \frac{V_2 - E_2 - V_3}{R_4} = 0$$

$$-\frac{1}{R_2}V_1 + \left[\frac{1}{R_2} + \frac{1}{R_5} + \frac{1}{R_4}\right]V_2 - \frac{1}{R_4}V_3 = \frac{E_2}{R_4} \quad \text{....(2)}$$

Apply KCL to node 3:

$$\frac{V_3 - V_2 + E_2}{R_4} + \frac{V_3}{R_6} + \frac{V_3 - E_3}{R_7} = 0$$

$$-\frac{1}{R_4}V_2 + \left[\frac{1}{R_4} + \frac{1}{R_6} + \frac{1}{R_7}\right]V_3 = \frac{E_3}{R_7} - \frac{E_2}{R_4} \quad \text{....(3)}$$

These 3 equations describe the circuit with three unknown values V_1, V_2, and V_3. So if these voltages are found then it is possible to find all the branch currents.

Matrix Form of Nodal Analysis

- These equations can be arranged in form of matrix equation as follows:

$$\left[\frac{1}{R}\right][V] = [I]$$

$$[G][V] = [I]$$

[G] is conductance matrix it contains the reciprocal of the circuit resistances
In details, the above equation can be written as:

$$\begin{bmatrix} \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3} & -\frac{1}{R_2} & 0 \\ -\frac{1}{R_2} & \frac{1}{R_2}+\frac{1}{R_4}+\frac{1}{R_5} & -\frac{1}{R_4} \\ 0 & -\frac{1}{R_4} & \frac{1}{R_4}+\frac{1}{R_6}+\frac{1}{R_7} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} \frac{E_1}{R_3} \\ 0 \\ \frac{E_3}{R_7}-\frac{E_2}{R_4} \end{bmatrix}$$

Matrix Form of Nodal Analysis

Or in conductance form as:

$$\begin{bmatrix} G_1+G_2+G_4 & -G_2 & 0 \\ -G_2 & G_2+G_4+G_5 & -G_4 \\ 0 & -G_4 & G_4+G_6+G_7 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} G_3E_1 \\ 0 \\ G_7E_3-G_4E_2 \end{bmatrix}$$

[G] or [1/R] can be obtained directly from the circuit as follows:

1- G_{ii} the diagonal elements represent the sum of the all resistance inverse's in the branches connected to node "i"

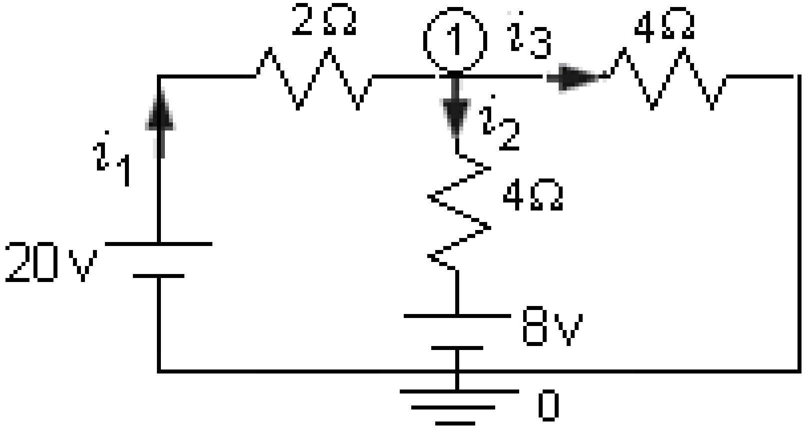
2- G_{ij} or G_{ji} is the sum of the resistance inverse's of the branches connected between node "i" and "j" this element is always negative.

3- [V] vector of the node voltages

4- [I] vector of the node currents from sources: =algebraic sum of the current of sources whose branches are connected to the considered node

Nodal analysis Example1

Use nodal analysis to find all branch currents.



Solution
We have only two nodes, and one of them reference therefore

$$\left[\frac{1}{2}+\frac{1}{4}+\frac{1}{4}\right][V_1]=\left[\frac{8}{4}+\frac{20}{2}\right]$$

thus V_1=12 volt and

$$i_1=\frac{20-12}{2}=4\ Amps$$

$$i_2=\frac{12-8}{4}=1\ Amps$$

$$i_3=\frac{12}{4}=3\ Amps$$

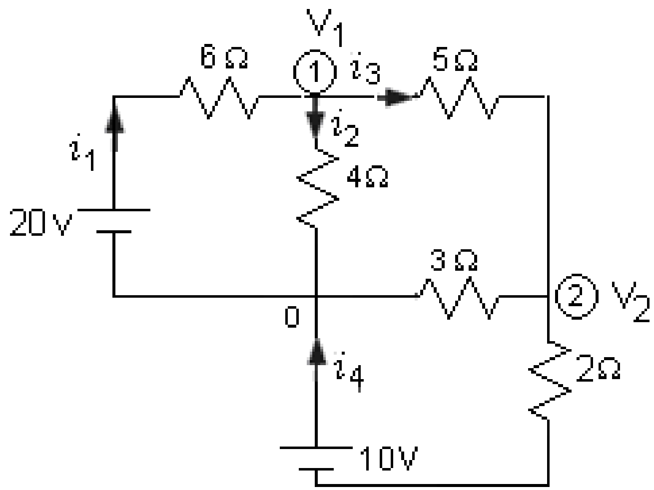
Nodal analysis Example 2

- Find the node voltages and branch currents

Solution:

$$\begin{bmatrix} \frac{1}{6}+\frac{1}{4}+\frac{1}{5} & -\frac{1}{5} \\ -\frac{1}{5} & \frac{1}{5}+\frac{1}{2}+\frac{1}{3} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} \frac{20}{6} \\ -\frac{10}{2} \end{bmatrix}$$

or



$$\begin{bmatrix} 0.62 & -0.2 \\ -0.2 & 1.03 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 3.3 \\ -5 \end{bmatrix}$$

Solve the above equation to obtain

$$0.62V_1 - 0.2V_2 = 3.3$$

$$-0.62V_1 + 1.03V_2 = -5$$

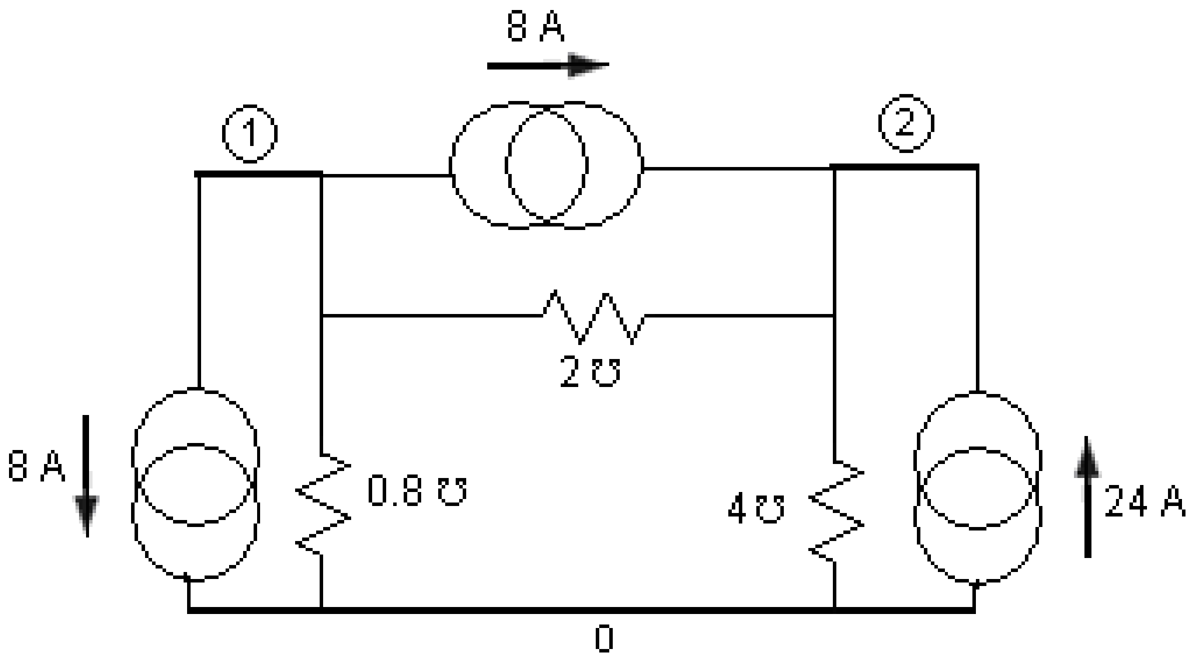
or

V_1=6.6 volt, V_2=-4 volt

Find branch currents??

Nodal analysis Problem

- Find V1 and V2



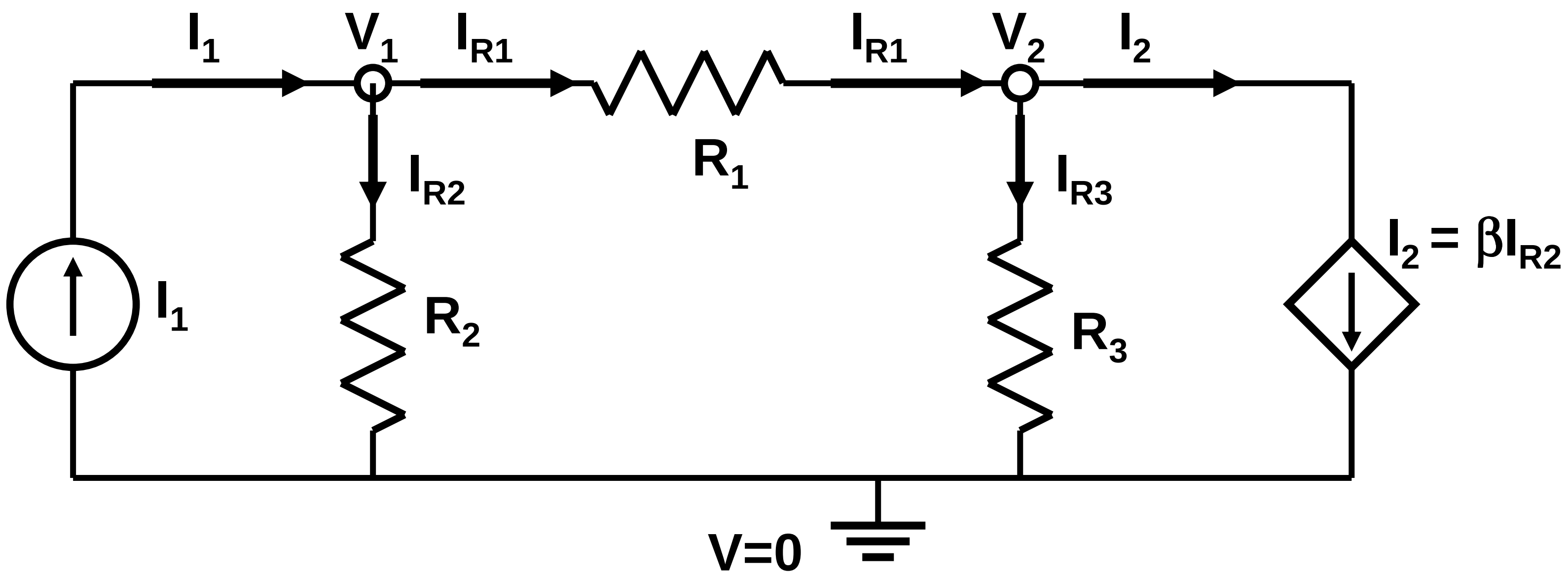
Nodal analysis

Circuits with dependent current sources

- The presence of dependent current sources may destroy the symmetrical form of the nodal equations that define the circuit so KCL should be used directly on the selected node (instate of directly writing the equations)

Circuits with dependent current sources
Example1

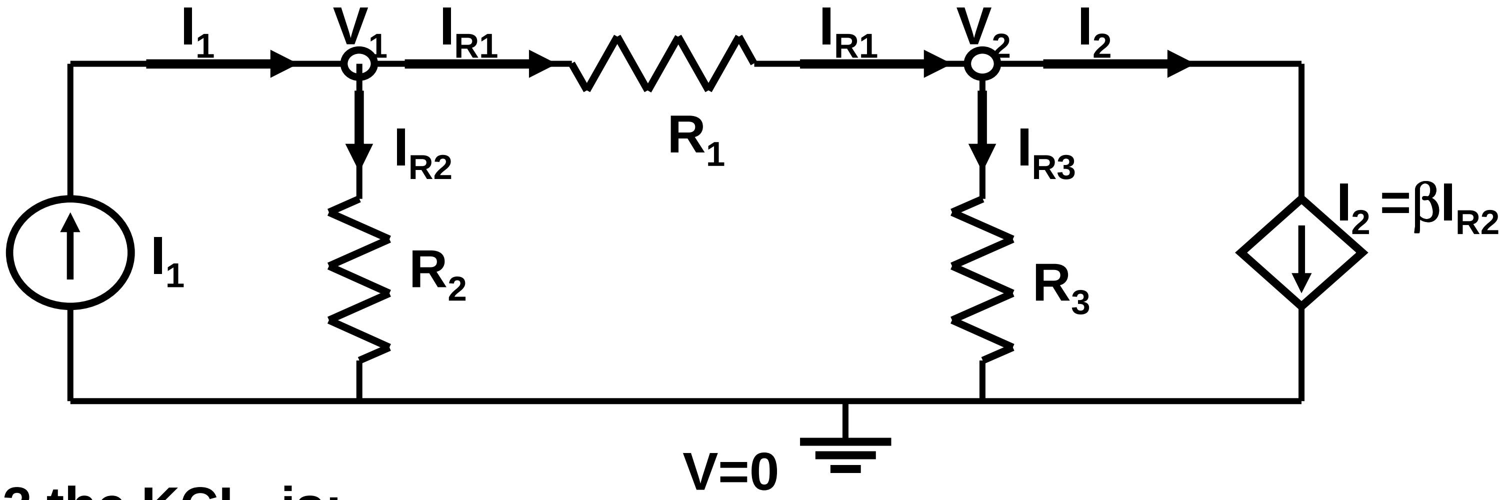
Find the node voltages in the following network and write the matrix form of the equations:



The KCL at node 1 is:

$$(V_1 - V_2) / R_1 + V_1 / R_2 = I_1$$

or $(1 / R_1 + 1 / R_2)\, V_1 - (1 / R_1)\, V_2$



At node 2 the KCL is:

$$V_2 / R_3 + (V_2 - V_1) / R_1 = -\beta I_{R2}$$

Or $-(1/R_1) V_1 + (1/R_1 + 1/R_3) V_2 = -\beta V_1 / R_2$

Or $(1/R_1 + 1/R_2) V_1 - (1/R_1) \quad V_2 = I_1$

$-(1/R_1 - \beta/R_2) V_1 + (1/R_1 + 1/R_3) V_2 = 0$ Or in matrix form as

$$\begin{pmatrix} (1/R_1 + 1/R_2) & -1/R_1 \\ \beta/R_2 - 1/R_1 & (1/R_1 + 1/R_3) \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} I_1 \\ 0 \end{pmatrix}$$

Circuits with dependent current sources
Example1

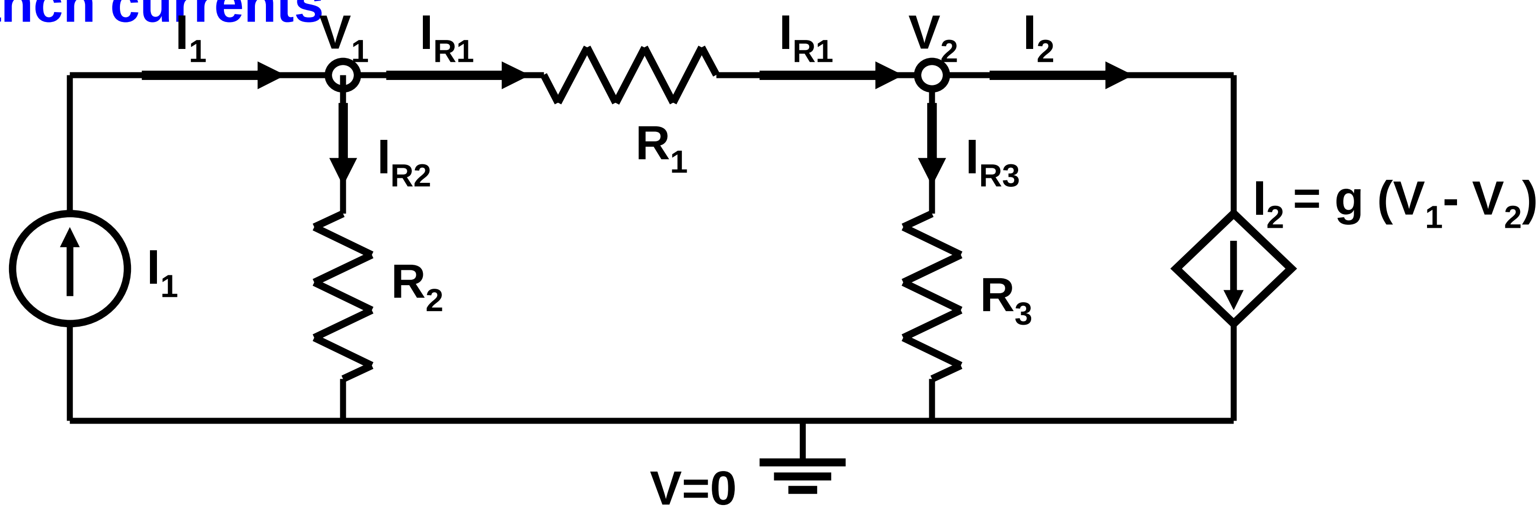
For the previous example if
I_1 = 4mA, β = 5
R_1 = 12 kΩ, R_2 = 6 kΩ and R_3 = 6 kΩ. Find the nodal voltages and branch currents, assuming the given current direction

Answer: V_1 = 8 V , V_2 = - 24 V

$$\begin{pmatrix} (1/R_1 + 1/R_2) & -1/R_1 \\ \beta/R_2 - 1/R_1 & (1/R_1 + 1/R_3) \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} I_1 \\ 0 \end{pmatrix}$$

Problem 2

For the shown circuit, find the voltage at each node and find all The branch currents



answer:

$$\begin{pmatrix} (1/R_1 + 1/R_2) & -1/R_1 \\ g - 1/R_1 & -g + (1/R_1 + 1/R_3) \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} I_1 \\ 0 \end{pmatrix}$$

if $I_1 = 4mA$, $g = 0.1mS$ $R_1 = 12\ k\Omega$, $R_2 = 6\ k\Omega$ and $R_3 = 6\ k\Omega$

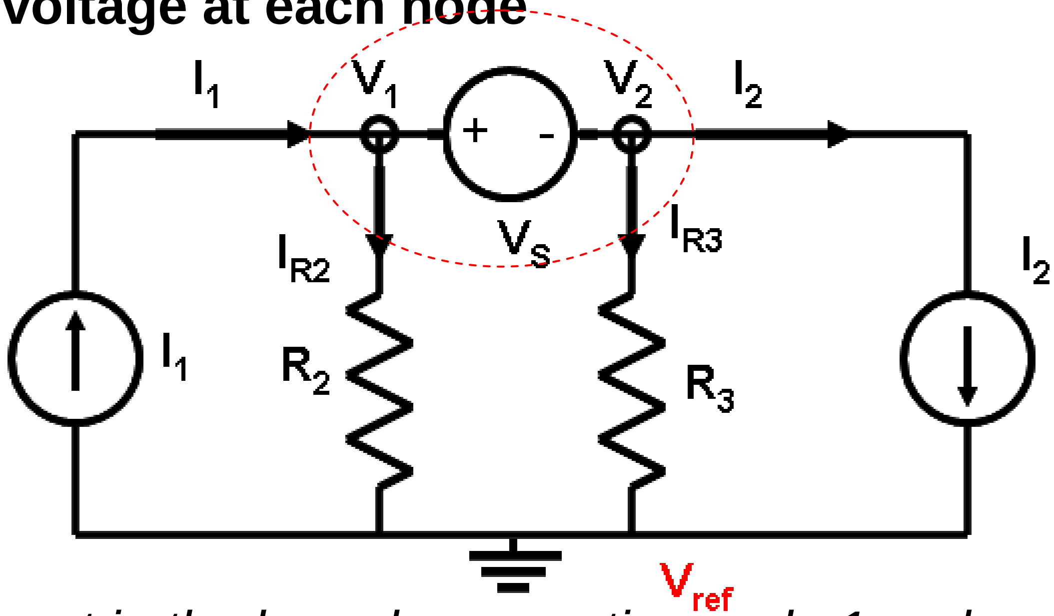
answer: $V_1 = 15.429\ V$ $V_2 = -1.714\ V$

Nodal analysis: circuits containing independent voltage sources- super node

- This section is concerned with the case when a voltage source is connected directly between two unknown nodes and there is no series resistance with the source.
- In this case, a super node is considered, where the two nodes and the voltage source is considered as one node, then KCL is applied to this node. In addition the constrain between the two nodes V_A-V_B =the voltage source is considered one equation, the other nodes in the circuit is treated normally with KCL

Nodal Analysis: Super Node

For the shown circuit find the voltage at each node



Consider the super node enclosed by the dashed circle. The KCL is:

$-I_1 + V_1 / R_2 + V_2 / R_3 + I_2 = 0$, or
$(1/ R_2)V_1 + (1/ R_3)V_2 = I_1 - I_2$ ..(1)

Alternatively, you can assume a current in the branch connecting node 1 and node 2, write the normal node equations and eliminate the assumed current to obtain the same above equation

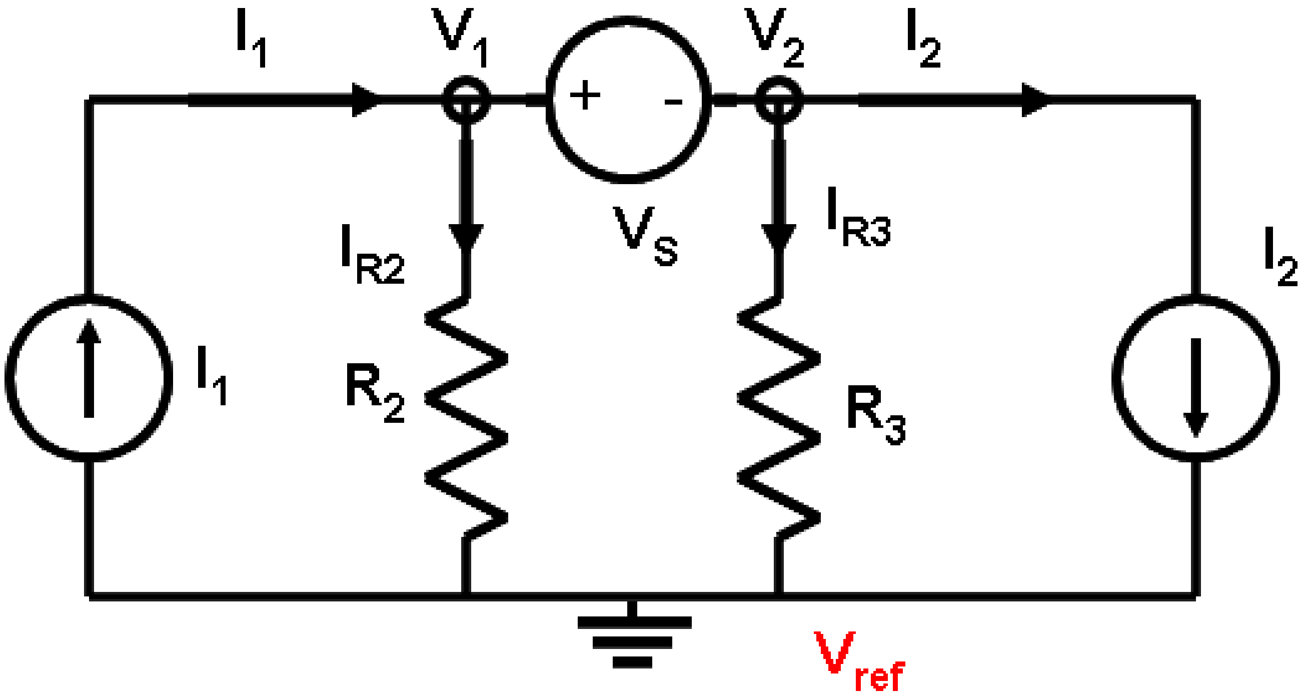
The constrain between the voltage nodes 1 and 2;result in:

$V_1 - V_2 = V_S$ ...(2)

The two equations describe the system in two unknowns, which can be solved to determined

$$\begin{pmatrix} 1/R_2 & 1/R_3 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} I_1 - I_2 \\ V_S \end{pmatrix}$$

Nodal Analysis: Super Node

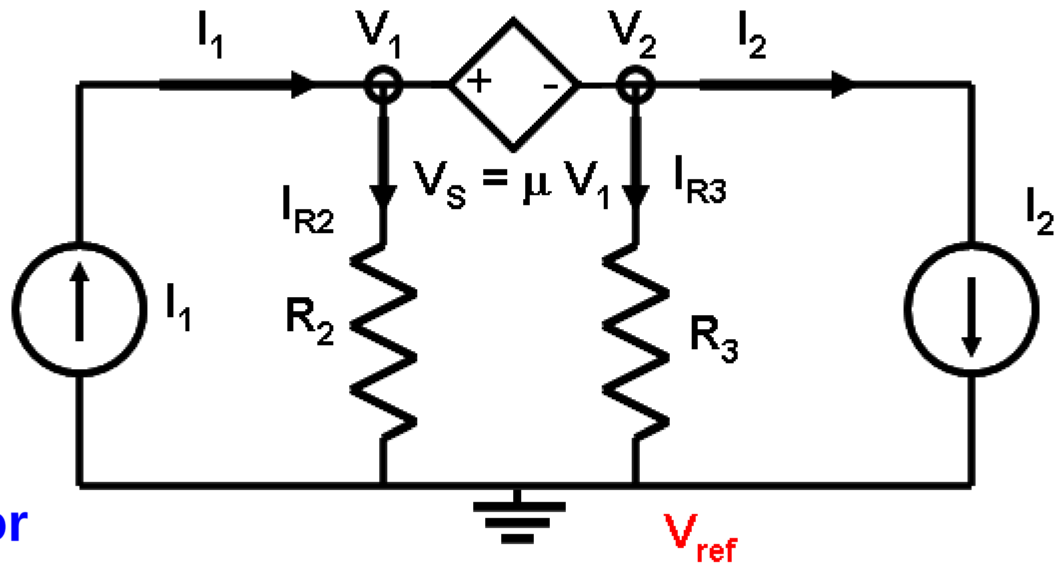


For the shown circuit if I_1 = 6mA, I_2 = 4mA, V_S = 6 V R_2 = 6 kΩ and R_3 = 12 kΩ. Find the node voltages and the current through the resistors R_2 and R_3

answer: V_1 = 16 V,V_2 = 4 V

Nodal analysis: circuits containing dependent voltage sources

Find the node voltages and branch currents in the shown circuit



Consider the super node enclosed by the dashed circle. The KCL is:

$-I_1 + V_1 / R_2 + V_2 / R_3 + I_2 = 0$, or
$(1/ R_2)V_1 + (1/ R_3)V_2 = I_1 - I_2$ ..(1)

The constrain between the voltage nodes 1 and 2;result in:

$V_1 - V_2 = V_S \mu = V_1$ or
$V_1 (\mu - 1) + V_2 = 0$...(2)

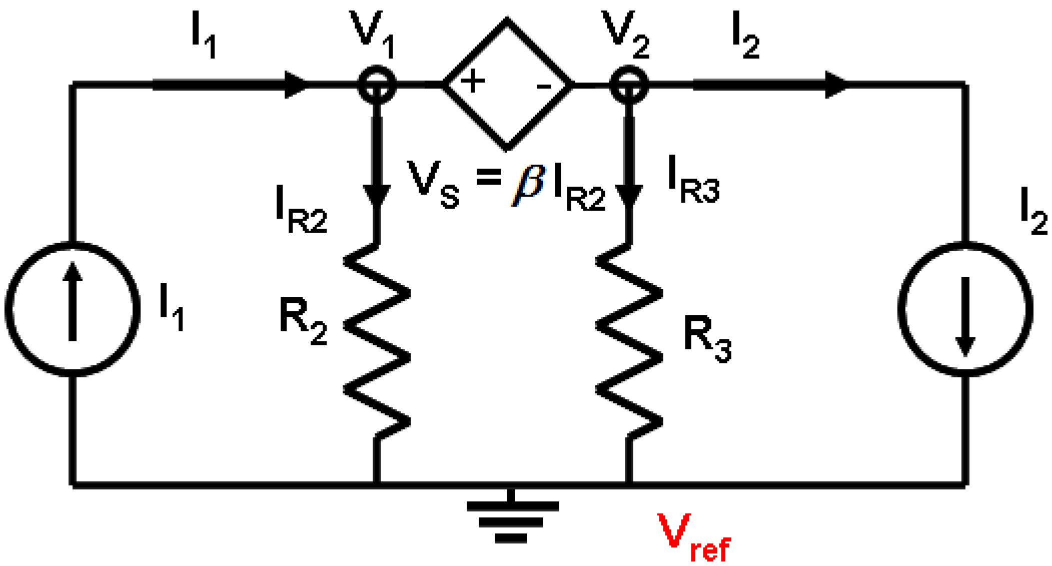
$$\begin{pmatrix} 1 / R_2 & 1 / R_3 \\ \mu - 1 & 1 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} I_1 - I_2 \\ 0 \end{pmatrix}$$

If $I_1 = 4mA$, $I_2 = 2mA$, $\mu = 5$ $R_2 = 6\ k\Omega$ and R3 = $6\ k\Omega$. Find V_1 and V_2.

Answer $V_1 = - 4\ V$, $V_2 = 16$

Nodal analysis: circuits containing dependent voltage sources

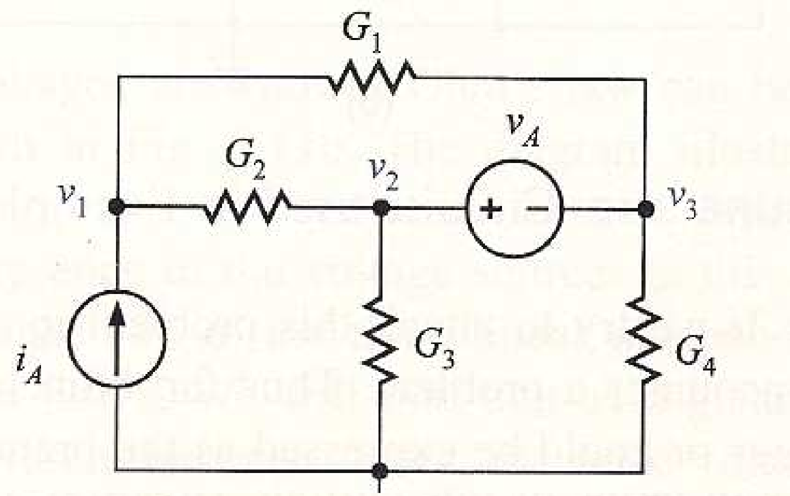
Find the node voltages for the shown circuit



I_1 = 4mA, I_2 = 2mA, β = 5 kΩ ,R_2 = 12 kΩ and R_3 = 6 kΩ
Answer: V_1 = 9 V, V_2 = 7.5 V

Nodal analysis Problems

Find the node voltages



SOLUTION The equation for the node labeled V_1 is

$$(v_1 - v_3)G_1 + (v_1 - v_2)G_2 - i_A = 0$$

The constraint equation for the supernode is

$$v_2 - v_3 = v_A$$

And the KCL equation for the supernode is

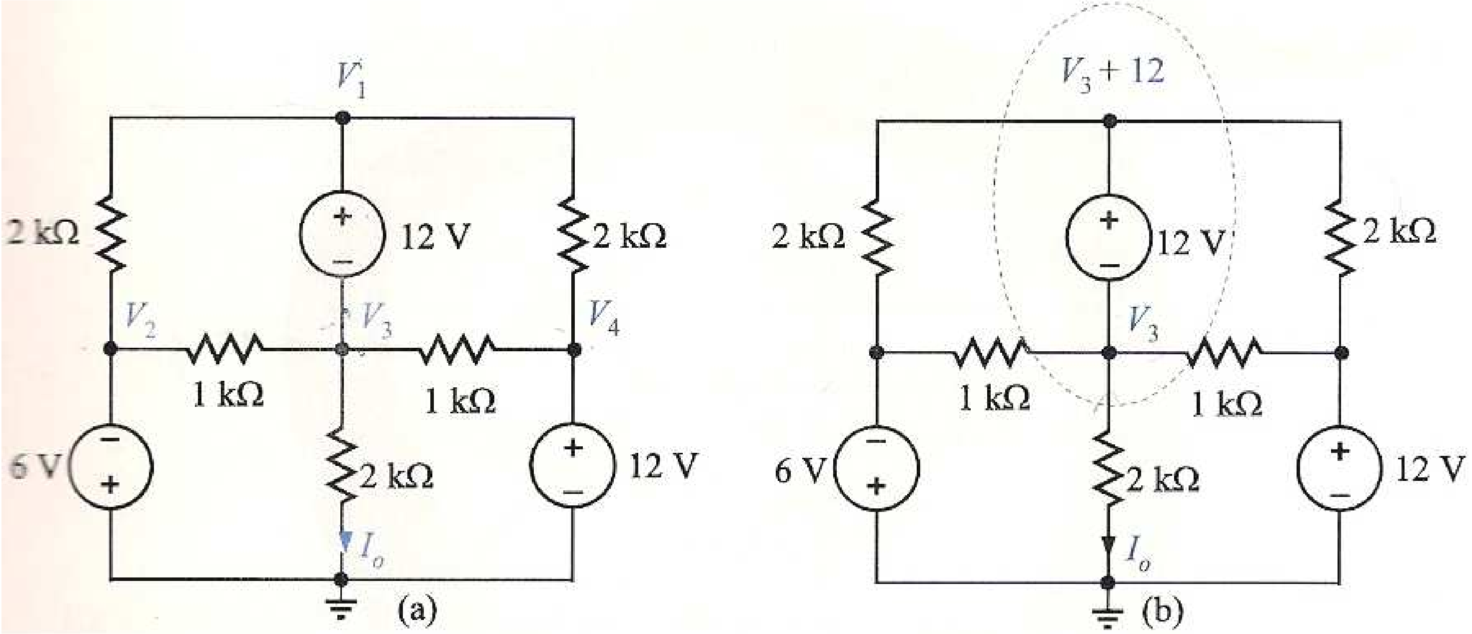
$$(v_2 - v_1)G_2 + v_2 G_3 + (v_3 - v_1)G_1 + v_3 G_4 = 0$$

The three equations will yield the node voltages.

See Basic Engineering Circuit Analysis, Irwin

Nodal analysis

Find the current I_0



and V_4 are known and the node voltages V_1 and V_3 are constrained by the equation

$$V_1 - V_3 = 12$$

Since we want to find the current I_o, V_1 (in the supernode containing V_1 and V_3) is written as $V_3 + 12$. The KCL equation at the supernode is

$$\frac{V_3 + 12 - (-6)}{2k} + \frac{V_3 + 12 - 12}{2k} + \frac{V_3 - (-6)}{1k} + \frac{V_3 - 12}{1k} + \frac{V_3}{2k} = 0$$

Solving the equation for V_3 yields $V_3 = -\frac{6}{7}$ V

I_o can then be computed immediately as $I_o = \frac{-\frac{6}{7}}{2k} = -\frac{3}{7}$ mA

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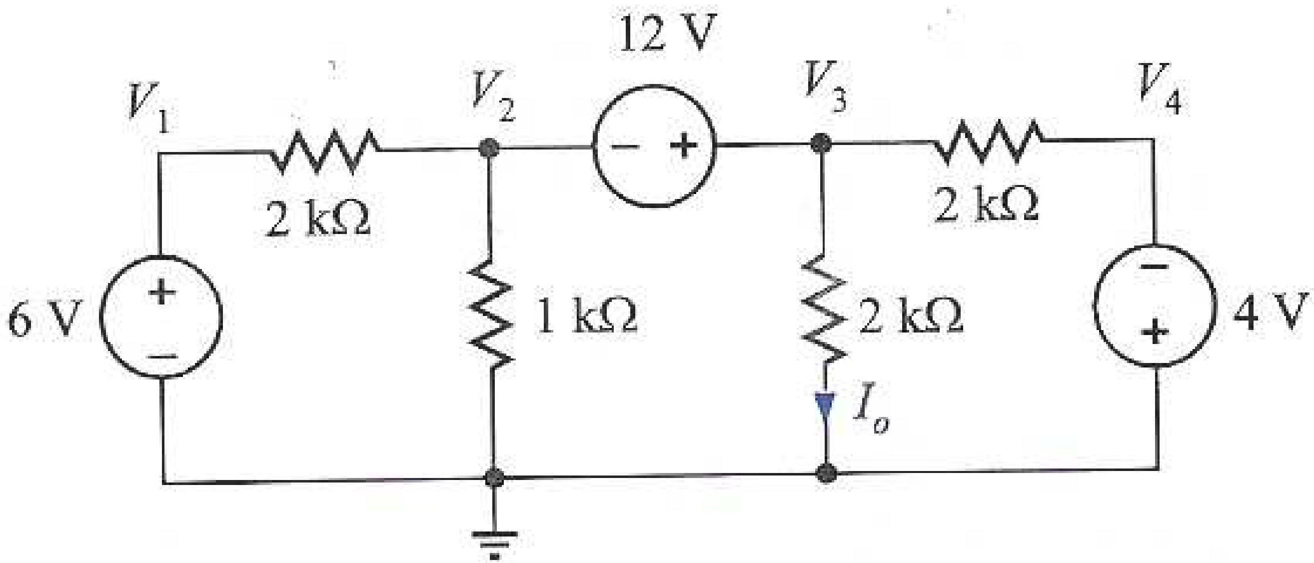
Nodal analysis

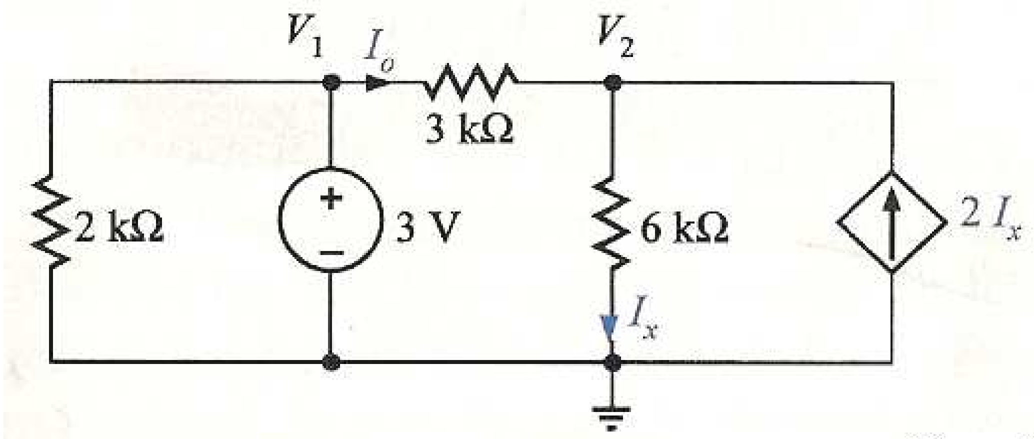


Use nodal method to determine the current I_0.

Answer: I_0=3.8 mA

See Basic Engineering Circuit Analysis, Irwin

Nodal analysis



Use nodal method to determine the current I_0.

$$V_1 = 3 \text{ V}$$

KCL applied to the second nonreference node yields

$$\frac{V_2 - 3}{3\text{k}} + \frac{V_2}{6\text{k}} = 2I_x$$

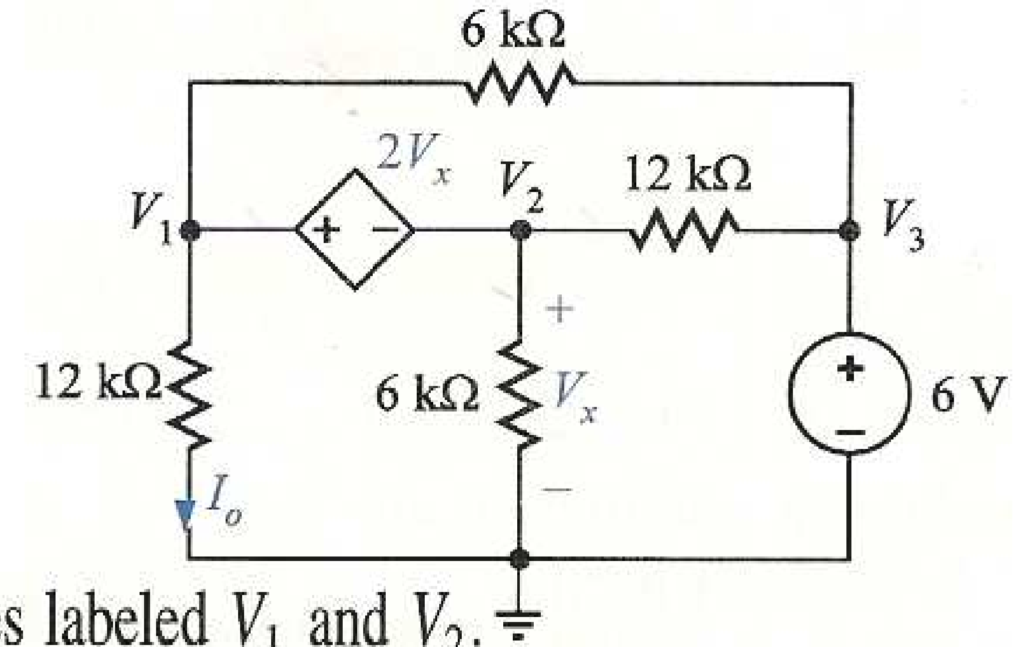
where the controlling equation is

$$I_x = \frac{V_2}{6\text{k}}$$

solving these equations yields $V_2 = 6$ V and, hence,

$$I_o = \frac{3-6}{3\text{k}} = -1 \text{ mA}$$

Nodal analysis



Use nodal method to determine the current I_0.

$V_3 = 6$ V, $V_2 = V_x$,

a supernode exists between the nodes labeled V_1 and V_2.

$$\frac{V_1 - V_3}{6k} + \frac{V_1}{12k} + \frac{V_2}{6k} + \frac{V_2 - V_3}{12k} = 0$$

the constraint for the supernode is $V_1 - V_2 = 2V_x$

The final equation is $V_3 = 6$

Solving these equations we find that $V_1 = \frac{9}{2}$ V

hence, $I_o = \frac{V_1}{12k} = \frac{3}{8}$ mA

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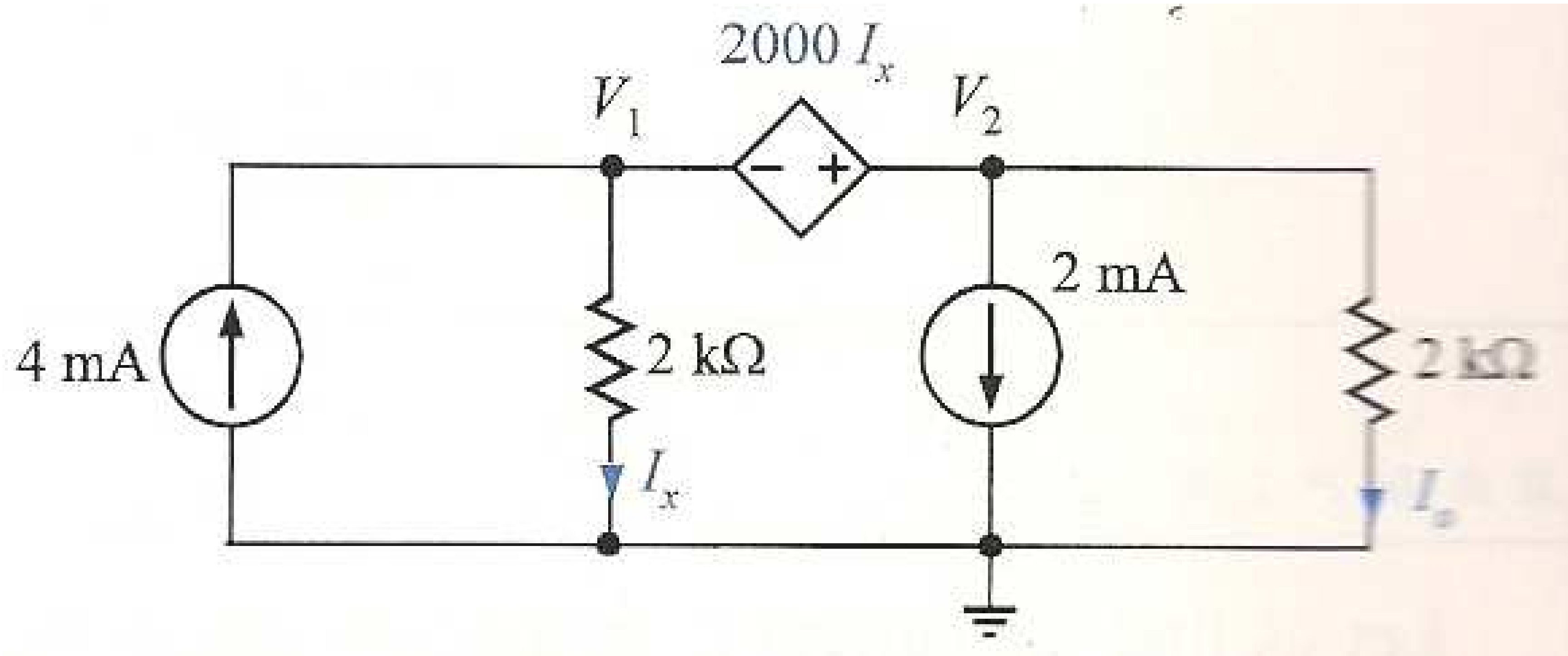
Nodal analysis



Use nodal method to determine the current I_0.

answer=1.333 mA

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