ECE 211 Workshop: Nodal and Loop Analysis

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Agenda

- Background: KCL and KVL.
- Nodal Analysis:
  - Independent Sources and relating problems,
  - Dependent Sources and relating problems.
- Loop (Mesh Analysis):
  - Independent Sources and relating problems,
  - Dependent Sources and relating problems.
- Practice Problems and solutions.

KCL and KVL Review

Rule: Algebraic sum of electrical current that merge in a common node of a circuit is zero.

$$\Sigma I_{in} = \Sigma I_{out}$$

Rule: The sum of voltages around a closed loop circuit is equal to zero.

$$\sum_{k=1}^{n} V_k = 0$$

KCL and KVL Example

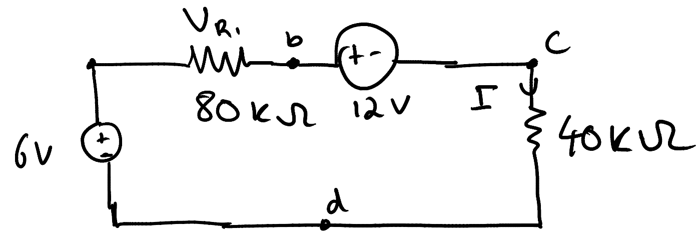
- Find I and Vbd in the following circuit?
- <u>Solution:</u>

  Using KCL we know that only 1 current *I* flows in the loop.

  Then we apply Ohm's law to find the current *I*.

  Lastly, we use KVL in the single loop to evaluate the voltage Vbd.

  We therefore see how KCL and KVL can used as simple analysis tools.



Using Ohm's law

We find

$I = \frac{V_T}{R_T}$, where $V_T = (12-6)$

$= -6\,V$

taking 'I' going out of src as +ve.

$R_T = 80k + 40k = 120k\Omega$

$I = \frac{-6}{120k} = -0.05\,mA$

Now using KVL to the loop.

$V_{bd} = 6 + V_{R_1}$

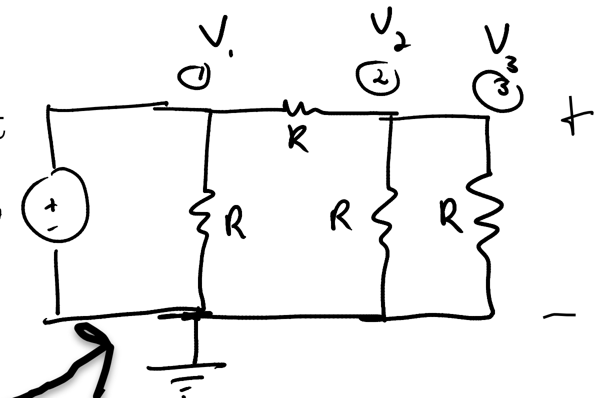
$= 6 + 80k(0.05m) = \underline{\underline{10\,V}}$.

Nodal Analysis

- Nodal Analysis of electronic circuits is based on assigning Nodal voltages at various nodes of the circuit with respect to a reference and then finding these nodal voltages to analyze the circuit.

  Simple representation of Nodal Voltages shown below:

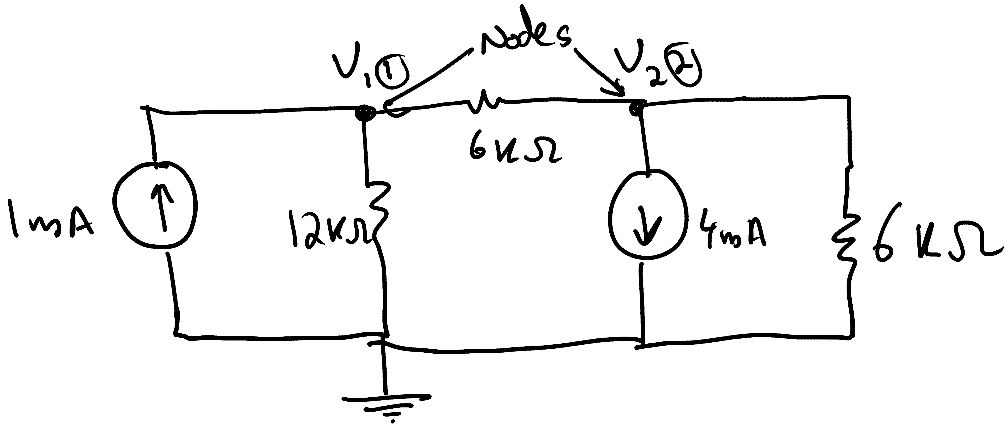
As shown in Figure, a *node* is a point in a circuit where two or more wires meet. At these nodes one can assign a nodal voltage with respect to the reference ground shown.



NODAL ANALYSIS: INDEPENDENT SOURCES ONLY

- First we find the number of KCL equations (These are used to find the nodal voltages). ***N -1 = n,*** here N = number of equations, n = number of nodes.
- Then we write the KCL equations for the nodes and solve them to find the respected nodal voltages.
- Once we have these nodal voltages, we can use them to further analyze the circuit.
- **SuperNode:** Two Nodes with a independent Voltage source between them is a Super node and one forms a KVL equation for it.

Example1 (Circuit with Ind. Current Sources):



Find the Nodal Voltages in the circuit?

Nodal Analysis: Independent Sources Only

In this example we write the KCL equations at the nodes as Shown, then solve them to find The respected nodal voltages.

Using KCL:
write Nodal Equations for nodes ① & ②

Assume current leading out of node as +ve.

$$-1m + \frac{V_1}{12k} + \frac{V_1 - V_2}{6k} = 0 \longrightarrow ①$$

$$4m + \frac{V_2 - V_1}{6k} + \frac{V_2}{6k} = 0 \longrightarrow ②$$

KCL Eqn.

Solving for V_1 & V_2

$$-12 + V_1 + 2V_1 - 2V_2 = 0$$

$$3V_1 - 2V_2 = 12 \longrightarrow ①a$$

$$24 + 2V_2 - V_1 = 0 \longrightarrow ②a$$

Equate 1a & 2a

$$V_1 = 24 + 2V_2 \quad \& \quad 3V_1 - 2V_2 = 12$$

So

$$3(24) + 6V_2 - 2V_2 = 12$$

$$4V_2 = -60$$

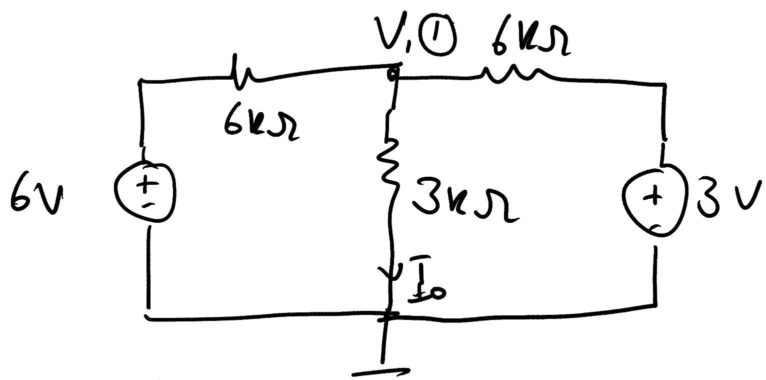
$$\underline{\underline{V_2 = -15\ V}}$$

So $V_1 = 24 - 30$

$$\underline{\underline{= -6\ V}}$$

NODAL ANALYSIS: INDEPENDENT SOURCES ONLY

Example2 (Ind. Voltage Sources Only):



For this Problem, we first make the main KCL equation at the only node 1. Current is taken to be coming out of the node as positive. We solve this equation to find The ***nodal*** voltage V1. Once this is determined, I_o is simply found by using Ohm's law at the sole resistor of 3KOhm.

$$\frac{V_1 - 6}{6k} + \frac{V_1}{3k} + \frac{V_1 - 3}{6k} = 0$$

$$V_1 - 6 + 2V_1 + V_1 - 3 = 0$$

$$4V_1 = 9$$

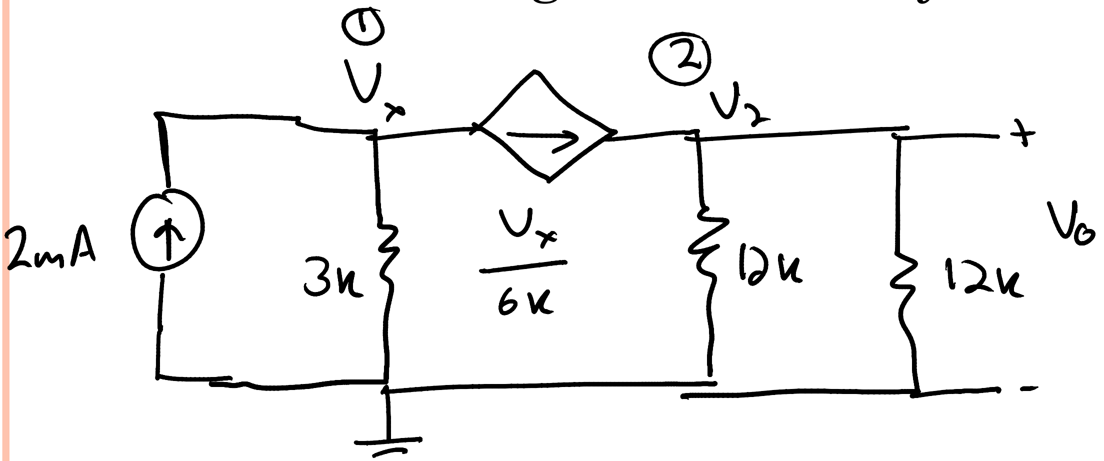
$$\boxed{V_1 = \frac{9}{4}\ V}$$

So Now

$$I_o = \frac{V_1}{3k} = \frac{\cancel{9}^{3}/4}{\cancel{3}k} = \frac{3}{4}\ mA$$

Nodal Analysis: Dependent Sources

- Find V_o using Nodal Analysis?



For this Circuit, We need to Overlook the node with dependent Source and form equations round it. We use KCL at Nodes 1 and 2 and derive the equations based on Current flow.

At Node 1 taking passive sign convention

KCL Eqn

$$-2m + \frac{V_x}{3k} + \frac{V_2}{12k} + \frac{V_2}{12k} = 0 \quad \text{—(1)}$$

At node 2

$$\frac{V_2}{12k} + \frac{V_2}{12k} - 2m + \frac{V_x}{3k} = 0 \quad \text{—(2)}$$

Nodal Analysis: Dependent Sources

Controlling Equation at Node 1

$$2m = \frac{V_x}{6k} + \frac{V_x}{3k} \quad \text{—— } (3)$$

$$12 = V_x + 2V_x \qquad V_x = 4\ V$$

After setting up the Nodal Eqn. For dependent Circuits, on has To make the Controlling eqn. This equation is made using the Dependent source. Using the Controlling equation we solve For the nodal voltages.

Now put in Eqn (2)

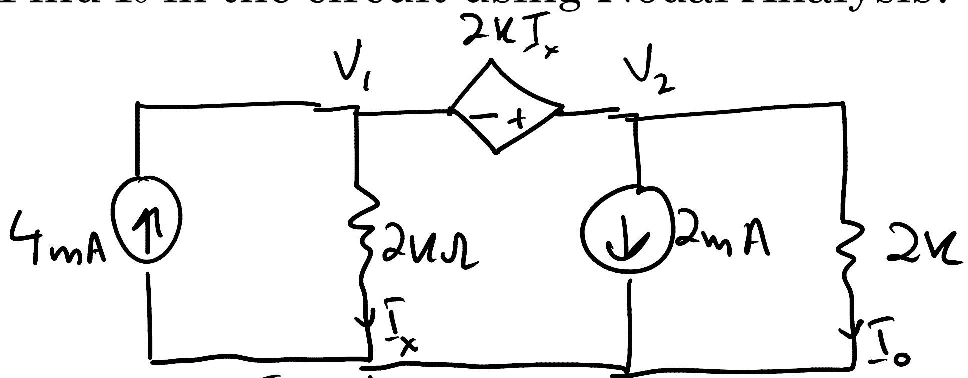
$$\frac{V_2}{12k} + \frac{V_2}{12k} - 2m + \frac{4}{3k} = 0$$

$$V_2 - 12 + 8 = 0$$

$$V_2 = 4\ V \qquad \text{As } \underline{V_0 = V_2 = 4\ V}$$

NODAL ANALYSIS: DEPENDENT SOURCES

Find I_o in the circuit using Nodal Analysis?



Using KCL at nodes 2 and Forming a controlling equation At node 1 we can simplify the Problem into simple equations. Current entering the node is Summed at the node to form The equations.

Forming KCL Eqn at node 2

$$\frac{V_2}{2k} + 2m + \frac{V_1}{2k} - 4m = 0 \quad \text{—— (1)}$$

Controlling Eqn

$$V_1 + 2K I_x = V_2 \quad \text{—— (2)}$$

$$\& \quad I_x = \frac{V_1}{2k} \quad \text{—— (3)}$$

$$V_1 + 2k\left(\frac{V_1}{2k}\right) = V_2 \quad \therefore \quad \boxed{2V_1 = V_2} \quad \text{—— (4)}$$

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NODAL ANALYSIS: DEPENDENT SOURCES

Put (4) in (1)

$$\frac{V_2}{2k} + 2m + \frac{V_2/2}{2k} - 4m = 0$$

$$2V_2 + 8 + V_2 - 16 = 0$$

$$3V_2 = 8$$

$$\boxed{V_2 = \frac{8}{3}\ V}$$

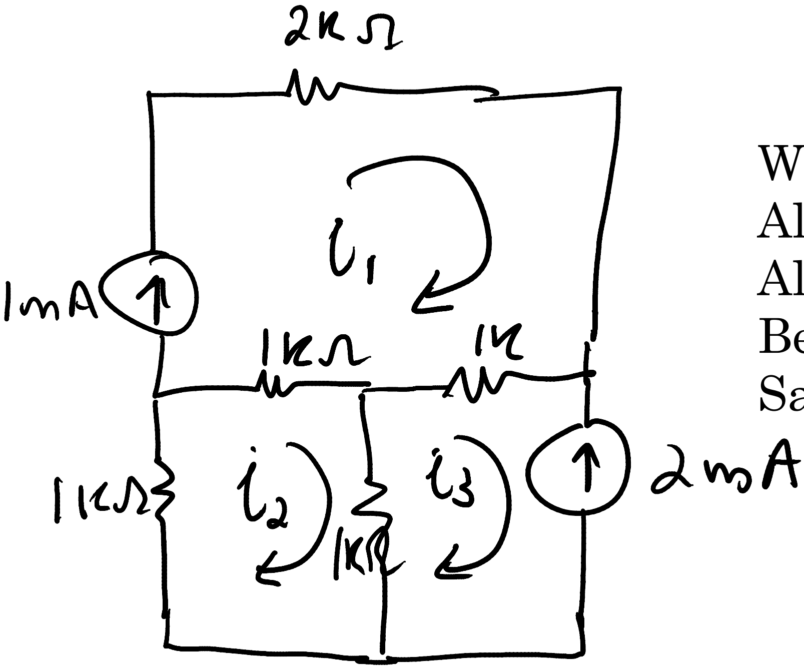
So

$$I_o = \frac{V_2}{2k} = \frac{8/3}{2k} = \boxed{\frac{4}{3}\ mA}$$

Simplify the circuit to Obtain nodal voltage.

Mesh Analysis

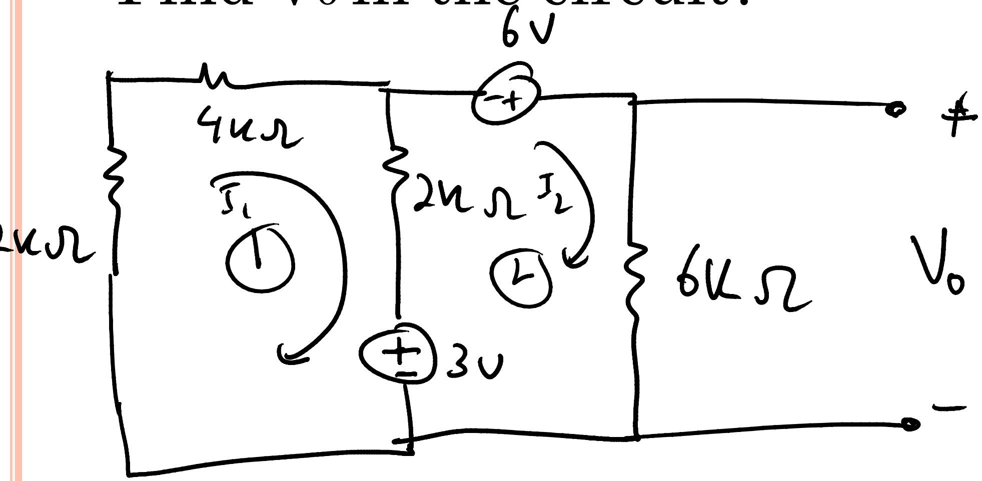
- Mesh Analysis involves solving electronic circuits via finding mesh or loop currents of the circuit. This is done by forming KVL equations for respected loops and solving the equations to find individual mesh currents.



We simply assume clockwise current flow in
All the loops and find them to analyze the circuit.
Also any independent current source in a loop
Becomes the loop current as current in series is
Same.

MESH ANALYSIS: INDEPENDENT SOURCES

Find V_o in the circuit?



Using KVL at loops 1 and 2, we form KVL equations using the current and Components in the loops in terms of The loop currents.
Important thing to look at it the Subtraction of the opposing loop Current in the shared section of the Loop.

loop ①

$$6k I_1 + (I_1 - I_2)2k + 3 = 0 \quad \text{——} \ (1)$$

Loop / Mesh ②

$$-6 + 6k I_2 + (I_2 - I_1)2k - 3 = 0 \quad \text{——} \ (2)$$

Mesh Analysis: Independent Sources

Simplifying

$$-2kI_2 + 8k\cancel{I_1} = -3$$

$$4\times(8kI_2 - 2k\cancel{I_1} = 9)$$

$$30kI_2 = 33$$

$$I_2 = \frac{11}{10}\,mA$$

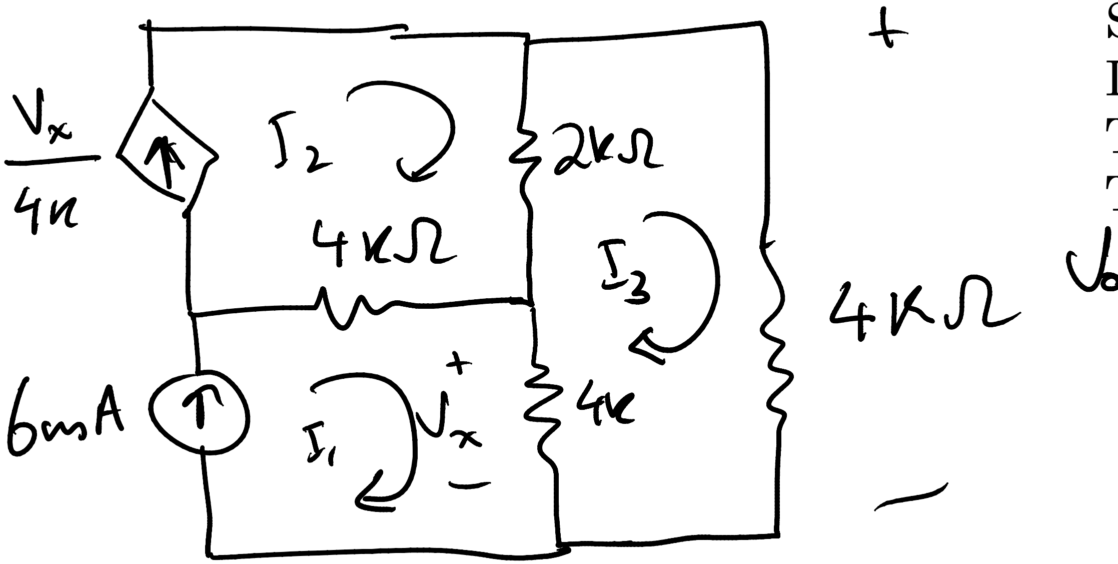
The mesh equations are solved Simultaneously and the required loop Current is found. Then we use this loop Current to find Vo in across the resistor.

Now once we found I_2

$$V_o = 6k \times I_2 = \cancel{6k}^{3} \times \frac{11\,m}{\cancel{10}_{5}} = \boxed{\frac{33}{5}\,V}$$

MESH ANALYSIS: DEPENDENT SOURCES

Find Vo in the circuit using Mesh Analysis?



Simplify the circuit using the Independent sources by assigning Them to the mesh currents for The specific loop.

$I_1 = 6mA$ (primary current in loop 1) —— (1)

$I_2 = \frac{V_x}{4k}$ where —— (2)

$V_x = 4k(I_1 - I_3)$ —— (3)

Mesh Analysis: Dependent Sources

Obj. Find I_3 using Mesh Analysis

Form KVL in Main loop 3.

then $\boxed{V_o = 4k(I_3)}$ — (4)

KVL

$$4k(I_3) + 4k(I_3 - \underset{I_1}{6m}) + 2k(I_3 - I_2) = 0 \quad \text{— (5)}$$

So $I_2 = \frac{\cancel{4k}}{\cancel{4k}}(I_1 - I_3)$ — (6)

Simplify Using Vx and I2.

Now put (6) in (5)
we get.

$$4k(I_3) + 4k(I_3 - 6m) + 2k(I_3 + I_3 - I_1) = 0$$

$$12kI_3 - 24 - 2k(6m) = 0$$

$$12kI_3 = 36$$

$$I_3 = \underline{3mA}$$

Hence $V_o = 4k \times 3m$
$= \underline{\underline{12V}}$

Practice Problems

Find V_o in the circuit in Fig. P3.28 using nodal analysis.

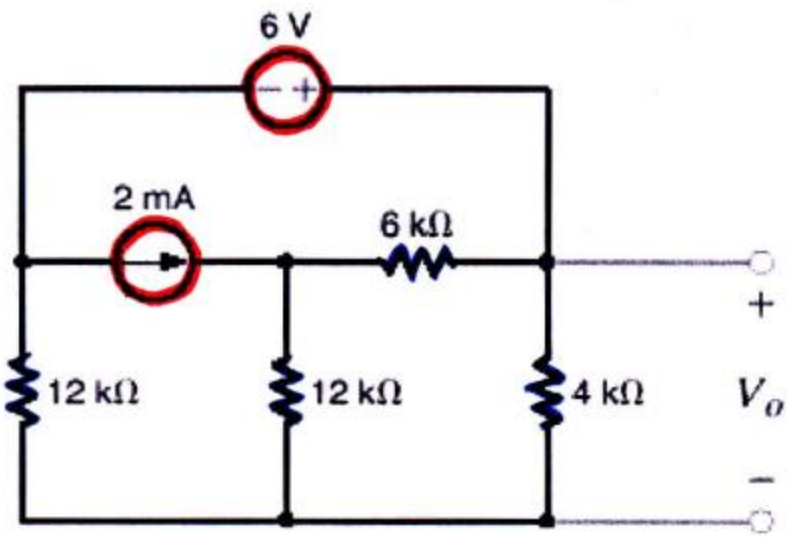


Figure P3.28

Practice Problems

Use nodal analysis to find V_o in the circuit in Fig. P3.29.

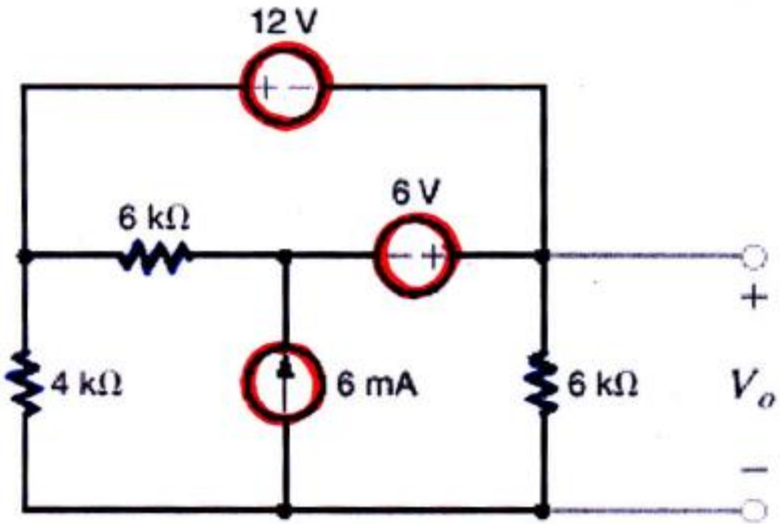


Figure P3.29

Practice Problems

Find V_o in the circuit in Fig. P3.36 using nodal analysis.

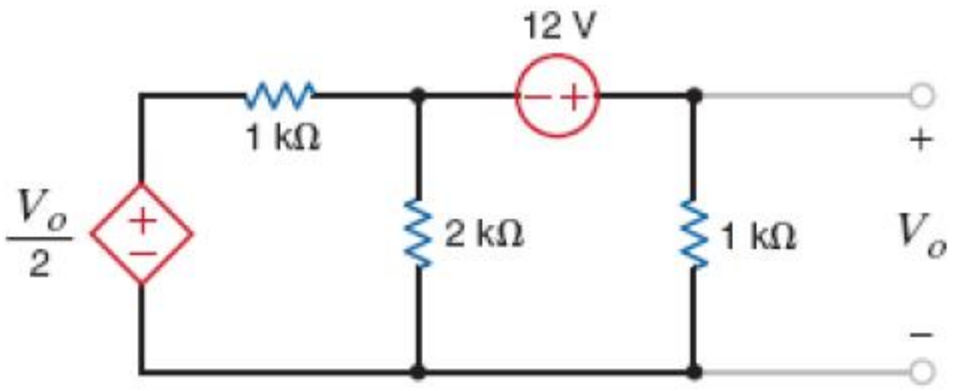


Figure P3.36

Practice Problems

Use nodal analysis to find V_1, V_2, V_3, and V_4 in the circuit in Fig. P3.44.

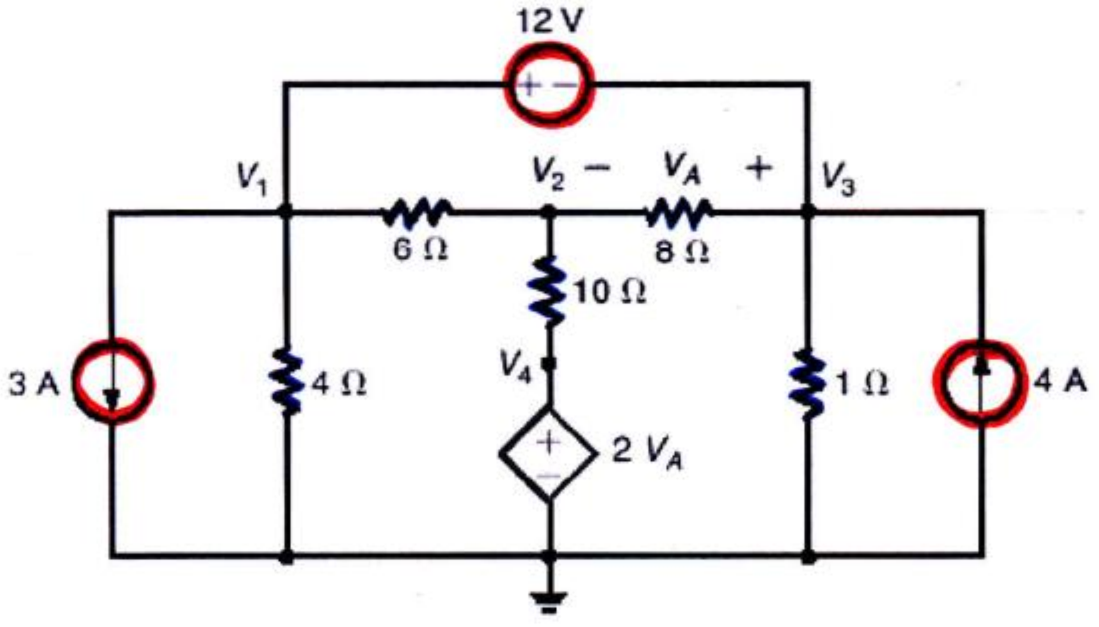


Figure P3.44

Practice Problems

Use mesh analysis to find V_o in the circuit in Fig. P3.47.

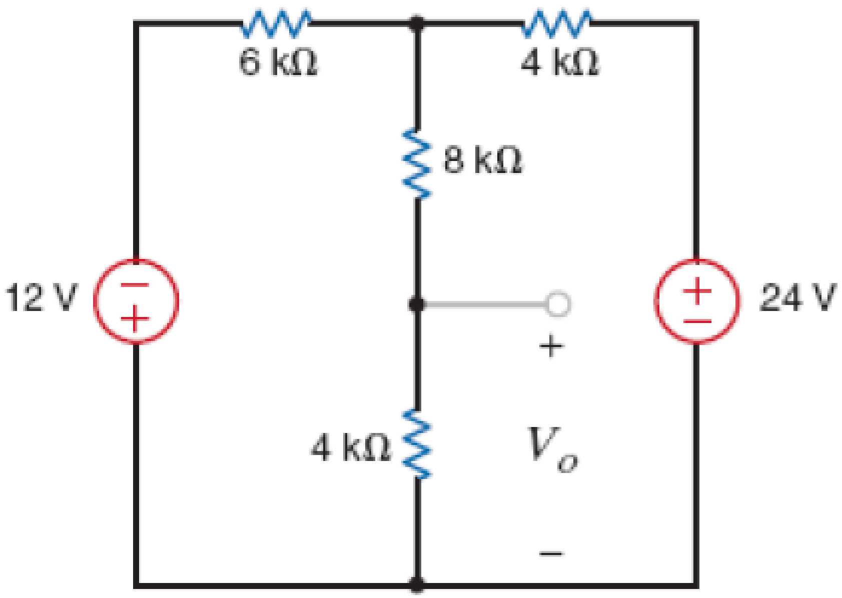


Figure P3.47

Practice Problems

Use mesh analysis to find V_o in the circuit in Fig. P3.84.

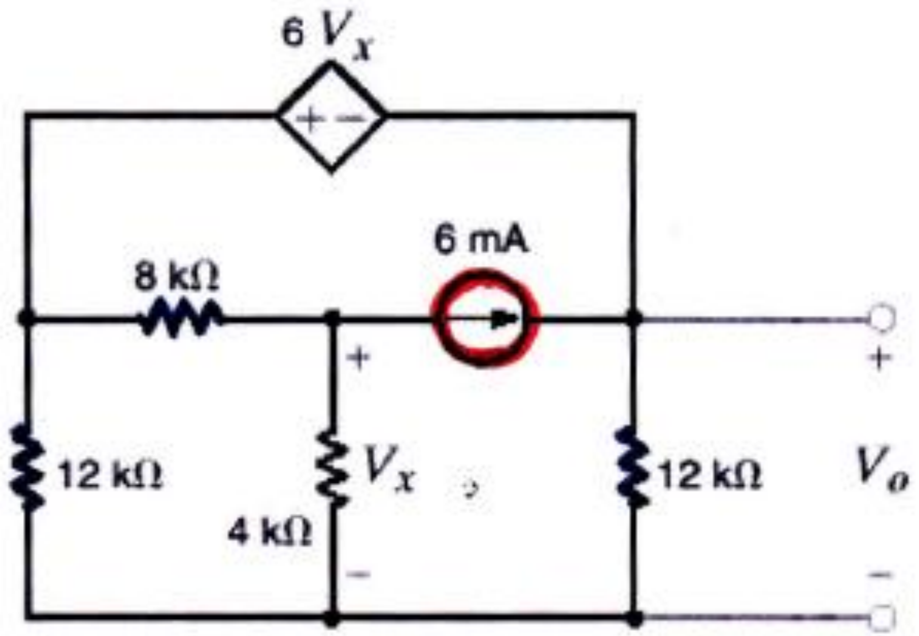


Figure P3.84

Practice Problems

Write mesh equations for the circuit in Fig. P3.88 using the assigned currents.

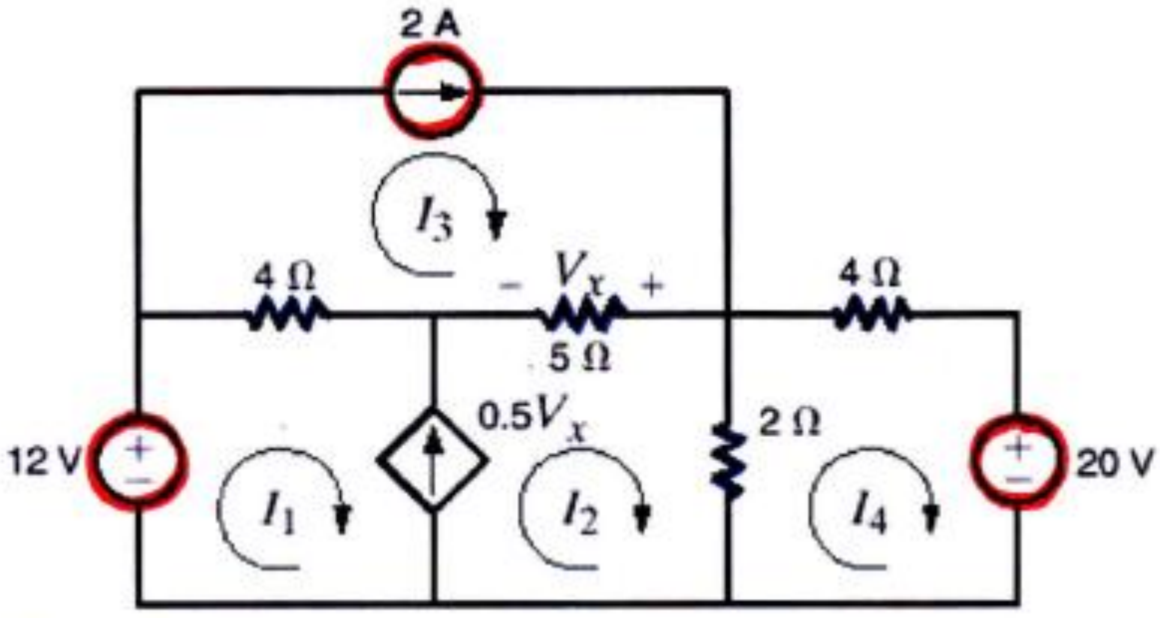


Figure P3.88

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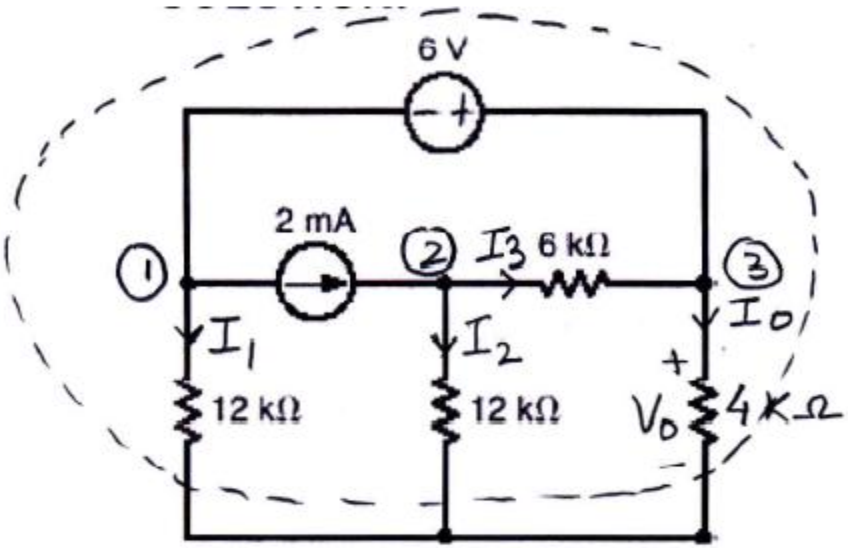
- END!!

Solutions to Practice Problems



KCL at ② : $2m = I_2 + I_3$

$$2m = \frac{V_2}{12K} + \frac{V_2 - V_3}{6K}$$

$$V_2 + 2V_2 - 2V_3 = 24$$

$$\boxed{3V_2 - 2V_3 = 24}$$

Solutions to Practice Problems

KCL at supernode: $I_1 + I_2 + I_0 = 0$

$$\frac{V_1}{12K} + \frac{V_2}{12K} + \frac{V_3}{4K} = 0$$

$$\boxed{V_1 + V_2 + 3V_3 = 0}$$

$$V_3 - V_1 = 6$$

$$\boxed{-V_1 + V_3 = 6}$$

$V_1 = -6.43$ V

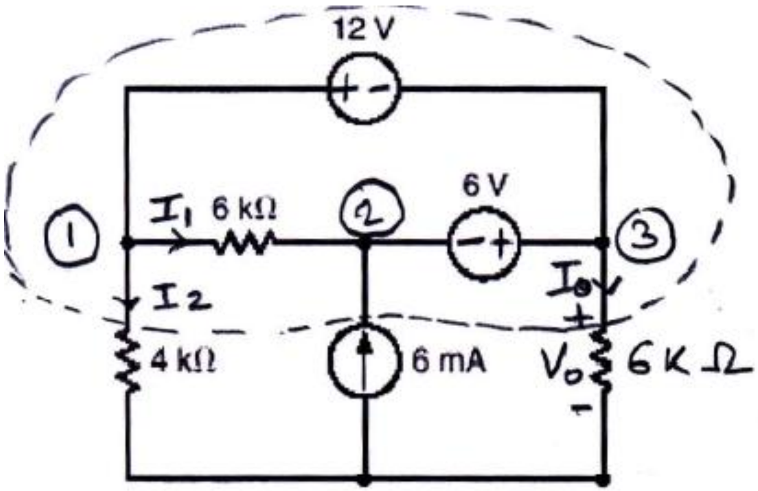
$V_2 = 7.71$ V

$V_3 = -0.43$ V

$V_0 = V_3 = -0.43$ V

$V_0 = -0.43$ V

SOLUTIONS TO PRACTICE PROBLEMS



KCL at supernode : $6m = I_2 + I_o$

$$\frac{V_1}{4K} + \frac{V_3}{6K} = 6m$$

Solutions to Practice Problems

$$\boxed{3V_1 + 2V_3 = 72}$$

$$\boxed{V_1 - V_3 = 12}$$

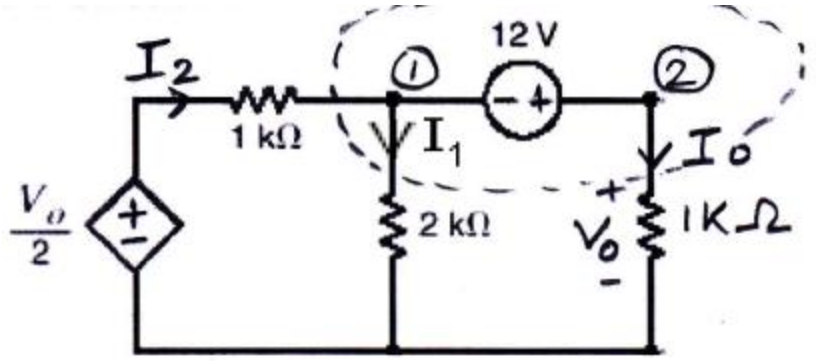
$$3V_1 + 2V_3 = 72$$
$$V_1 - V_3 = 12$$

$$V_1 = 19.2 \text{ V}$$
$$V_3 = 7.2 \text{ V}$$

$$V_o = V_3 = 7.2 \text{ V}$$

$$V_o = 7.2 \text{ V}$$

Solutions to Practice Problems



KCL at supernode : $I_2 = I_1 + I_0$

$$\frac{\frac{V_0}{2} - V_1}{1K} = \frac{V_1}{2K} + \frac{V_2}{1K}$$

$$V_0 = V_2$$

$$\frac{\frac{V_2}{2} - V_1}{1K} = \frac{V_1}{2K} + \frac{V_2}{1K}$$

Solutions to Practice Problems

$$V_2 - 2V_1 = V_1 + 2V_2$$

$$\boxed{3V_1 + V_2 = 0}$$

$$V_2 - V_1 = 12$$

$$\boxed{-V_1 + V_2 = 12}$$

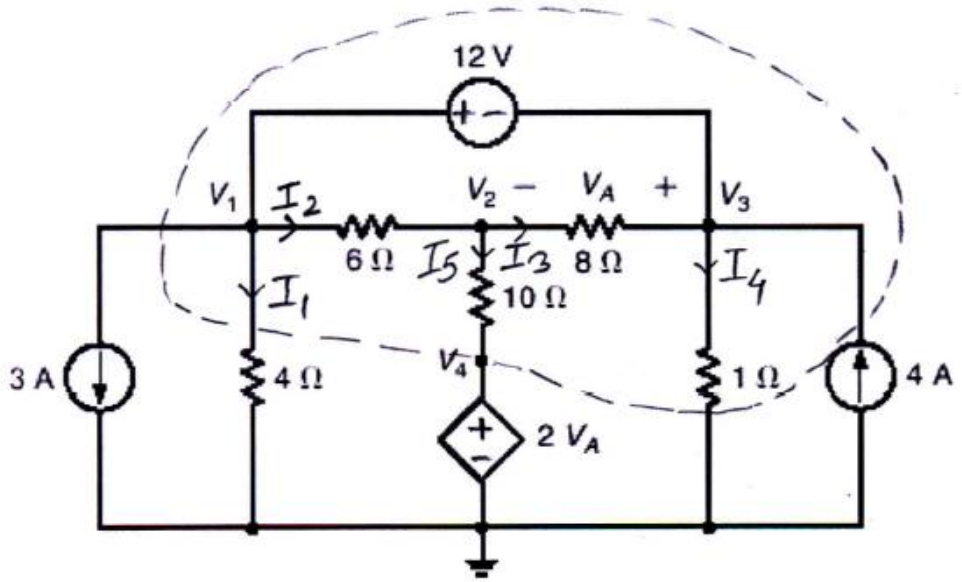
$$3V_1 + V_2 = 0$$
$$-V_1 + V_2 = 12$$

$$V_1 = -3V \qquad V_2 = 9\ V$$

$$V_o = V_2 = 9\ V$$

$$V_o = 9\ V$$

Solutions to Practice Problems



KCL at (2): $I_2 = I_5 + I_3$

$$\frac{V_1 - V_2}{6} = \frac{V_2 - V_4}{10} + \frac{V_2 - V_3}{8}$$

$$5V_1 - 5V_2 = 3V_2 - 3V_4 + 3.75V_2 - 3.75V_3$$

Solutions to Practice Problems

$$\boxed{5V_1 - 11.75V_2 + 3.75V_3 + 3V_4 = 0}$$

KCL at supernode: $3 + I_1 + I_5 + I_4 = 4$

$$\frac{V_1}{4} + \frac{V_2 - V_4}{10} + \frac{V_3}{1} = 1$$

$$5V_1 + 2V_2 - 2V_4 + 20V_3 = 20$$

$$\boxed{5V_1 + 2V_2 + 20V_3 - 2V_4 = 20}$$

$$V_1 - V_3 = 12$$

$$V_4 = 2V_A$$

$$V_A = V_3 - V_2$$

$$V_4 = 2(V_3 - V_2)$$

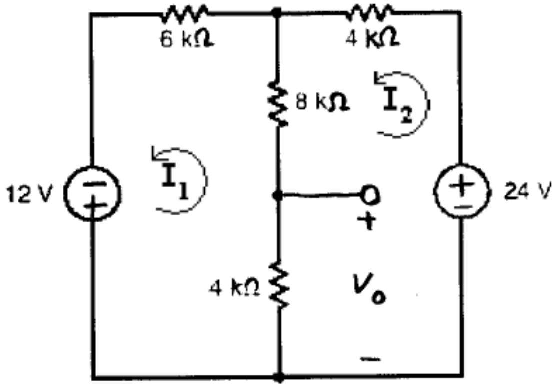
$$\boxed{-2V_2 + 2V_3 - V_4 = 0}$$

Solutions to Practice Problems

$$5V_1 - 11.75V_2 + 3.75V_3 + 3V_4 = 0$$
$$5V_1 + 2V_2 + 20V_3 - 2V_4 = 20$$
$$V_1 + 0V_2 - V_3 + 0V_4 = 12$$
$$0V_1 - 2V_2 + 2V_3 - V_4 = 0$$

$$V_1 = 9.68\ V$$
$$V_2 = 1.45\ V$$
$$V_3 = -2.32\ V$$
$$V_4 = -7.54\ V$$

Solutions to Practice Problems



I_1 & I_2 are two loop currents (mA)

KVL in the first loop:

$$12 = 4(I_1 - I_2) + 8(I_1 - I_2) + 6I_1$$

$$\Rightarrow 12 = 18I_1 - 12I_2$$

$$\Rightarrow 2 = 3I_1 - 2I_2 \quad \text{—— (1)}$$

KVL in the second loop:

$$24 = 4I_2 + 8(I_2 - I_1) + 4(I_2 - I_1)$$

Solutions to Practice Problems

$$\Rightarrow \quad 24 = 16 I_2 - 12 I_1$$

$$6 = 4 I_2 - 3 I_1 \quad — \; 2$$

From equation 1 and 2

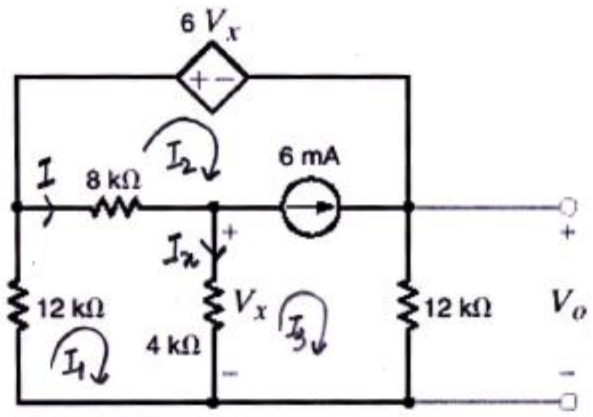
$$I_1 = 10/3 \text{ mA} \quad , \quad I_2 = 4 \text{ mA}$$

$$V_0 = 4 \times (I_2 - I_1)$$

$$\Rightarrow V_0 = 4 \times \left(4 - \frac{10}{3}\right) \text{ Volts}$$

$$\Rightarrow V_0 = 8/3 = 2.667 \text{ Volts}$$

Solutions to Practice Problems



KCL : $I_1 = I + I_2$

$I = I_1 - I_2$

KCL : $I = 6m + I_x$

$I_x = I_1 - I_2 - 6m$

KVL lower left loop:

$12k\,I_1 + 8k\,I + 4k\,I_x = 0$

$12k\,I_1 + 8k(I_1 - I_2) + 4k(I_1 - I_2 - 6m) = 0$ $\boxed{24k\,I_1 - 12k\,I_2 = 24}$

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Solutions to Practice Problems

KVL outer loop: $12k\,I_1 + 6V_x + 12k\,I_3 = 0$

$V_x = 4k\,I_x = 4k(I_1 - I_2 - 6m)$

$V_x = 4k\,I_1 - 4k\,I_2 - 24$

$12k\,I_1 + 6[4k\,I_1 - 4k\,I_2 - 24] + 12k\,I_3 = 0$

$36k\,I_1 - 24k\,I_2 + 12k\,I_3 = 144$

KCL: $I_x + I_3 = I_1$

$I_3 = I_1 - [I_1 - I_2 - 6m]$

$I_3 = I_2 + 6m$

$36k\,I_1 - 24k\,I_2 + 12k(I_2 + 6m) = 144$

$\boxed{36k\,I_1 - 12k\,I_2 = 72}$

$24k\,I_1 - 12k\,I_2 = 24$

$36k\,I_1 - 12k\,I_2 = 72$

$I_1 = 4mA$

$I_2 = 6mA$

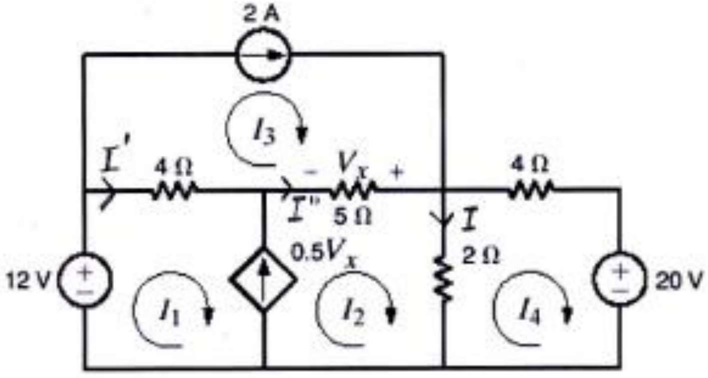
$I_3 = 6m + 6m$

$I_3 = 12mA$

$V_0 = 12k(I_3) = 12k(12m)$

$V_0 = 144V$

Solutions to Practice Problems



KCL: $I_1 = I' + I_3$

$I' = I_1 - I_3$

KCL: $I + I_4 = I_2$

$I = I_2 - I_4$

KCL: $I'' + I_3 = I + I_4$

$I'' = -I_3 + I_4 + I_2 - I_4$

$I'' = I_2 - I_3$

KCL: $I_2 = 0.5V_x + I_1$

$-I_1 + I_2 - 0.5V_x = 0$

$V_x = -I''(5) = -5(I_2 - I_3)$

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Solutions to Practice Problems

$V_x = -5I_2 + 5I_3$

$-I_1 + I_2 - 0.5(-5I_2 + 5I_3) = 0$

$\boxed{-I_1 + 3.5I_2 - 2.5I_3 = 0}$

$I_3 = 2A$

KVL: $4I_4 + 20 + 2(-I) = 0$

$4I_4 - 2(I_2 - I_4) = -20$

$\boxed{-2I_2 + 6I_4 = -20}$

KVL: $12 = 4I' + 5I'' + 4I_4 + 20$

$4(I_1 - I_3) + 5(I_2 - I_3) + 4I_4 = -8$

$4I_1 + 5I_2 - 9I_3 + 4I_4 = -8$

$\boxed{4I_1 + 5I_2 + 4I_4 = 10}$

$-I_1 + 3.5I_2 - 2.5I_3 = 0$

$-2I_2 + 6I_4 = -20$

$4I_1 + 5I_2 + 4I_4 = 10$

$I_3 = 2A$

$-I_1 + 3.5I_2 = 5$

$-2I_2 + 6I_4 = -20$

$4I_1 + 5I_2 + 4I_4 = 10$

$I_1 = 2.46A$

$I_2 = 2.13A$

$I_4 = -2.62A$