UNIFIED HANDOUT
MATERIALS AND STRUCTURES - #M-6
Fall, 2008

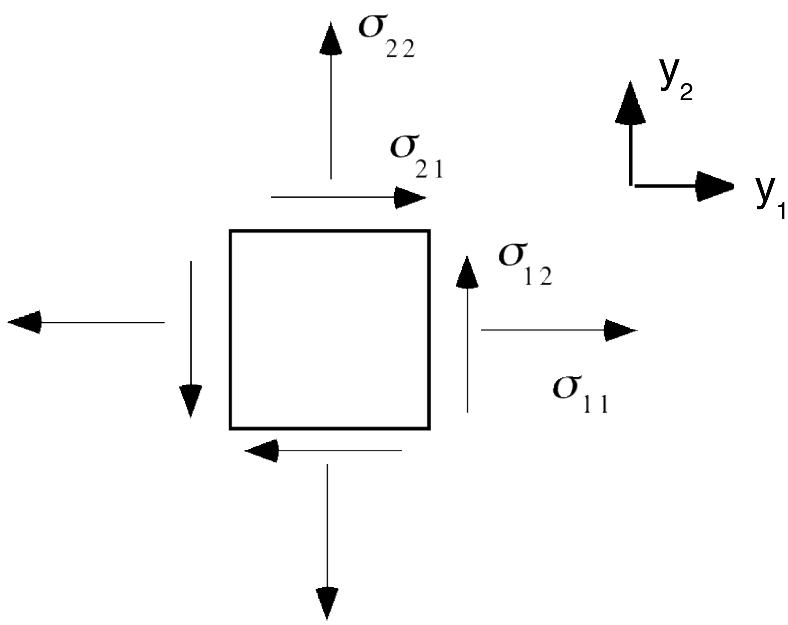
Mohr's Circle

Mohr's circle is a geometric representation of the 2-D transformation of stresses and is very useful to perform quick and efficient estimations, checks of more extensive work, and other such uses.

(Note: a similar formulation can be used for tensorial strain)

CONSTRUCTION:

Given the following state of stress:



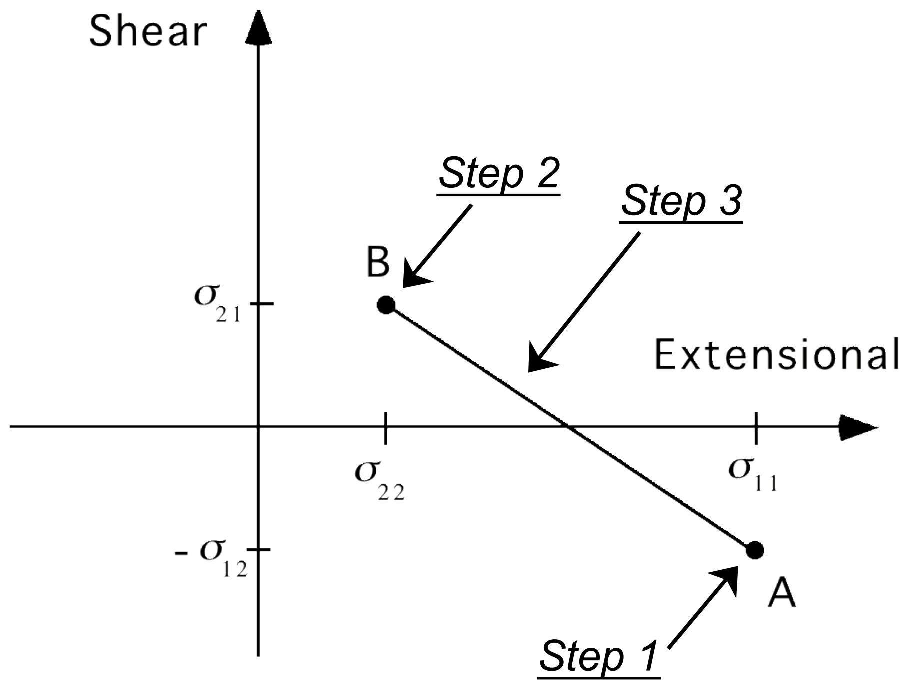
with the definition (*by Mohr*) of positive and negative shear:

"Positive shear would cause a clockwise rotation of the infinitesimal element about the element center."

Thus, from the illustration above, σ_{12} is plotted negative *on Mohr's circle*, and σ_{21} is plotted positive *on Mohr's circle*.

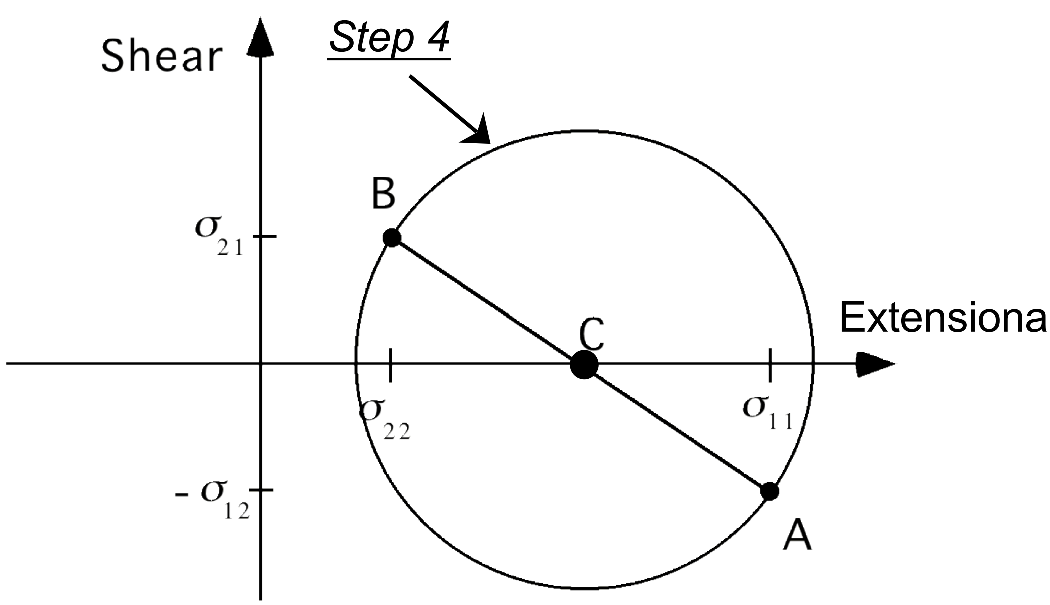
• Begin the construction by doing the following:

1. Plot σ_{11}, - σ_{12} as point A
2. Plot σ_{22}, σ_{21} as point B
3. Connect A and B



• Then complete the circle by doing Step 4:

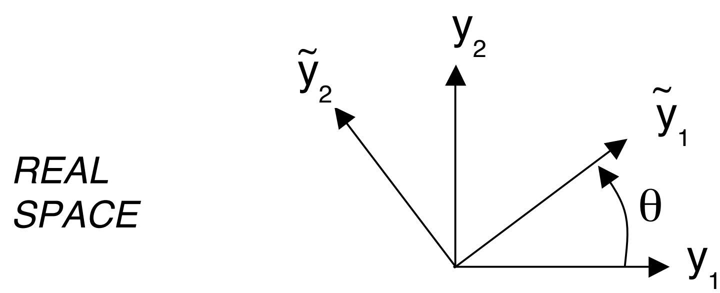
4. Draw a circle of diameter of the line AB about the point where the line AB crosses the horizontal axis (denote this as point C)



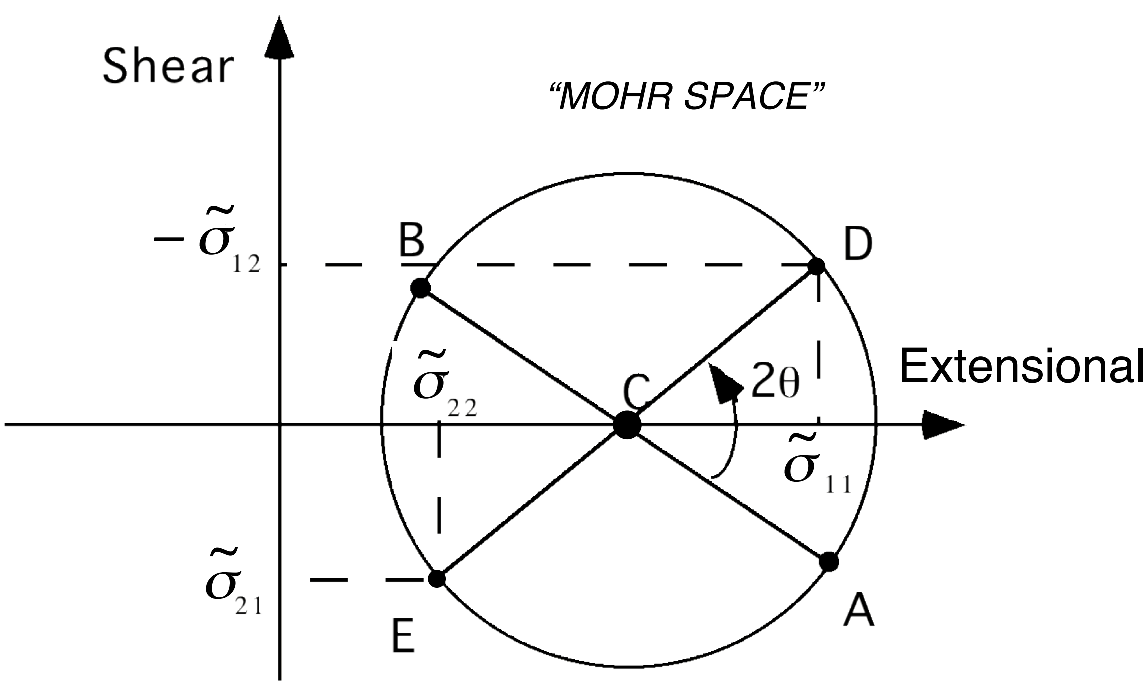
USE OF THE CONSTRUCTION:

• To read off stresses for a rotated system:

1. Note that the vertical axis is the shear stress axis and the horizontal axis is the extensional stress axis.
2. Positive rotations are measured counterclockwise as referenced to the original system and thus to the line AB.



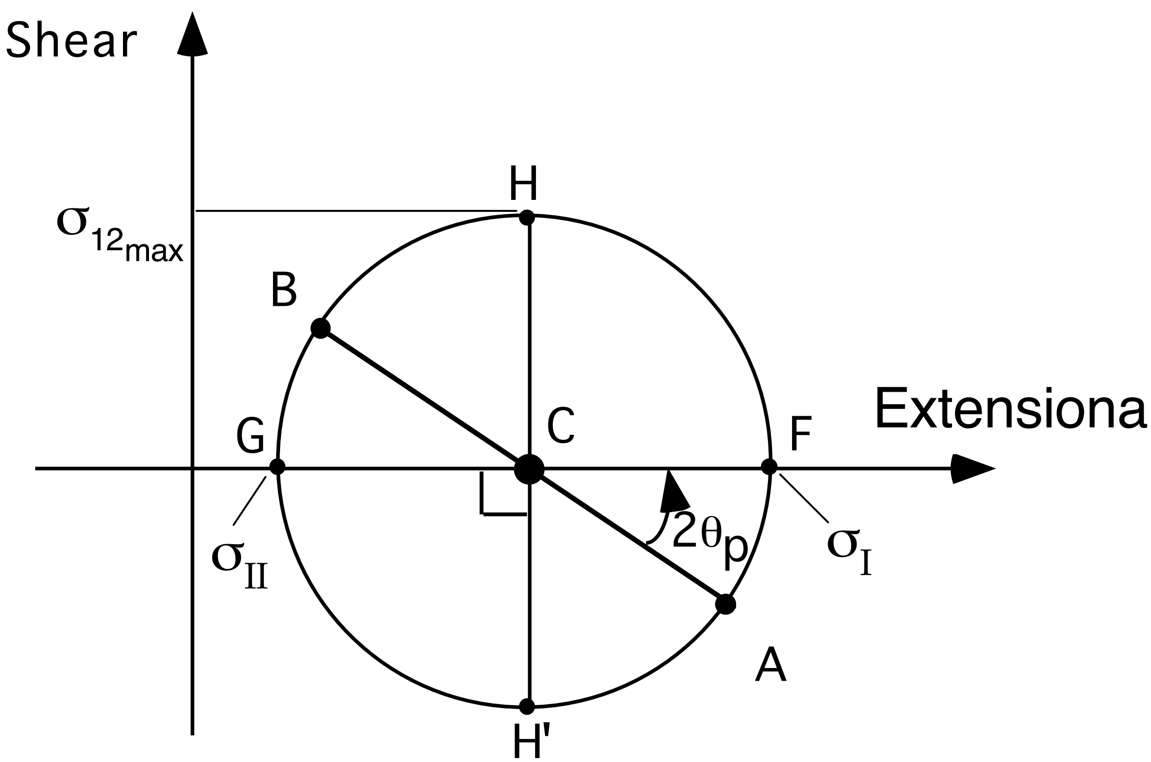
3. Rotate line AB about point C by the angle 2θ where θ is the angle between the unrotated and rotated systems.
4. The points D and E where the rotated line intersects the circle are used to read off the stresses in the rotated system. The vertical location of D is - $\tilde{\sigma}_{12}$; the horizontal location of D is $\tilde{\sigma}_{11}$. The vertical location of E is $\tilde{\sigma}_{21}$, the horizontal location of E is $\tilde{\sigma}_{22}$ (*Recall Mohr definition with regard to negative/positive sense of shear stress on Mohr's circle*).



• We can immediately see the following:

5. The principal stresses, σ_I and σ_{II}, are defined by the points F and G (along the horizontal axis where $\tilde{\sigma}_{12} = 0$). The rotation angle to the principal axis is θ_p which is 1/2 the angle from the line AB to the horizontal line FG.

6. The maximum shear stress is defined by the points H and H' which are the endpoints of the vertical line. The line is orthogonal to the principal stress line and thus the maximum shear stress acts along a plane 45° (= 90°/2) from the principal stress system.



Full two-dimensional stress transformation equations

(θ as on p.3 figure):

$$\tilde{\sigma}_{11} = cos^2\theta\,\sigma_{11} + sin^2\theta\,\sigma_{22} + 2\,sin\theta\,cos\theta\,\sigma_{12}$$

$$\tilde{\sigma}_{22} = sin^2\theta\,\sigma_{11} + cos^2\theta\,\sigma_{22} - 2\,sin\theta\,cos\theta\,\sigma_{12}$$

$$\tilde{\sigma}_{12} = -\sin\theta\,cos\theta\,\sigma_{11} + sin\theta\,cos\theta\,\sigma_{22} + \left(\cos^2\theta - sin^2\theta\right)\sigma_{12}$$

Note: θ is *not* the direction cosine angle in the tensor transformation relations,

e.g.: $\tilde{\sigma}_{\alpha\beta} = \ell_{\tilde{\alpha}\theta}\,\ell_{\tilde{\beta}\tau}\,\sigma_{\theta\tau}$

Rather, recall: $\ell_{\tilde{m}n}$ = cosine of angle from $\tilde{y}_m$ to y_n

by convention, angle is measured positive counterclockwise (+ CCW).

For the situation developed here for Mohr's circle, the direction cosines are:

$$\ell_{\tilde{1}1} = \cos\left(360^\circ - \theta\right) = \cos(\theta)$$

$$\ell_{\tilde{2}1} = \cos\left(270^\circ - \theta\right) = -\sin(\theta)$$

$$\ell_{\tilde{1}2} = \cos\left(90^\circ - \theta\right) = \sin(\theta)$$

$$\ell_{\tilde{2}2} = \cos\left(360^\circ - \theta\right) = \cos(\theta)$$

but this does yield the same set of operating equations!