Mohr’s Circle

Academic Resource Center

Introduction

- The transformation equations for plane stress can be represented in graphical form by a plot known as Mohr's Circle.

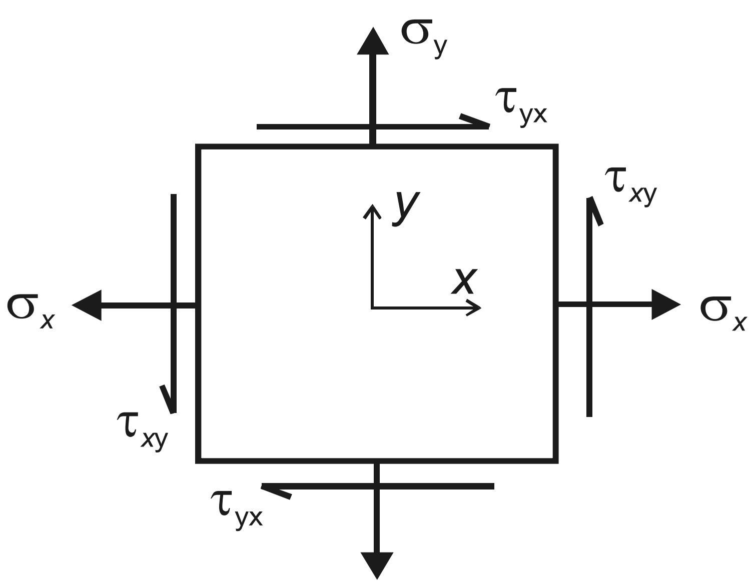
- This graphical representation is extremely useful because it enables you to visualize the relationships between the normal and shear stresses acting on various inclined planes at a point in a stressed body.

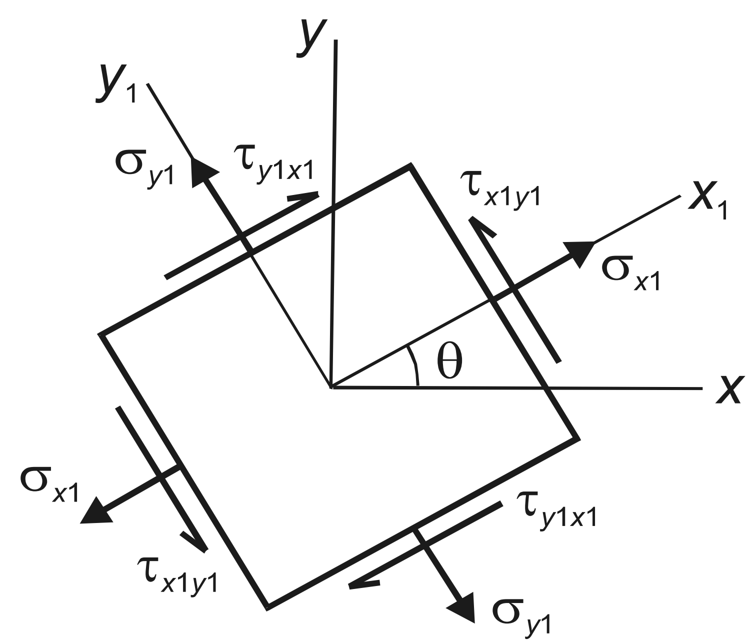
- Using Mohr's Circle you can also calculate principal stresses, maximum shear stresses and stresses on inclined planes.

Stress Transformation Equations

$$s_{x_1} - \frac{s_x + s_y}{2} = \frac{s_x - s_y}{2}\cos 2q + t_{xy}\sin 2q \qquad 1$$

$$t_{x_1 y_1} = -\frac{s_x + s_y}{2}\sin 2q + t_{xy}\cos 2q \qquad 2$$





Derivation of Mohr's Circle

- If we vary θ from 0° to 360°, we will get all possible values of σx_1 and $\tau x_1 y_1$ for a given stress state.
- Eliminate θ by squaring both sides of 1 and 2 equation and adding the two equations together.

$$\left(\sigma_{x1} - \frac{\sigma_x + \sigma_y}{2}\right)^2 + \tau_{x1y1}^{\ 2} = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\ 2}$$

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Derivation of Mohr's Circle (cont'd)

Define σ_{avg} and R

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} \qquad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\ 2}}$$

Substitue for σ_{avg} and R to get

$$\left(\sigma_{x1} - \sigma_{avg}\right)^2 + \tau_{x1y1}^{\ 2} = R^2$$

which is the equation for a **circle** with centre $(\sigma_{avg},0)$ and radius R.

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Mohr’s Circle Equation

- The circle with that equation is called a Mohr’s Circle, named after the German Civil Engineer Otto Mohr. He also developed the graphical technique for drawing the circle in 1882.

$$\left(\sigma_{x1} - \sigma_{avg}\right)^2 + \tau_{x1y1}{}^2 = R^2$$

- The graphical method is a simple & clear approach to an otherwise complicated analysis.

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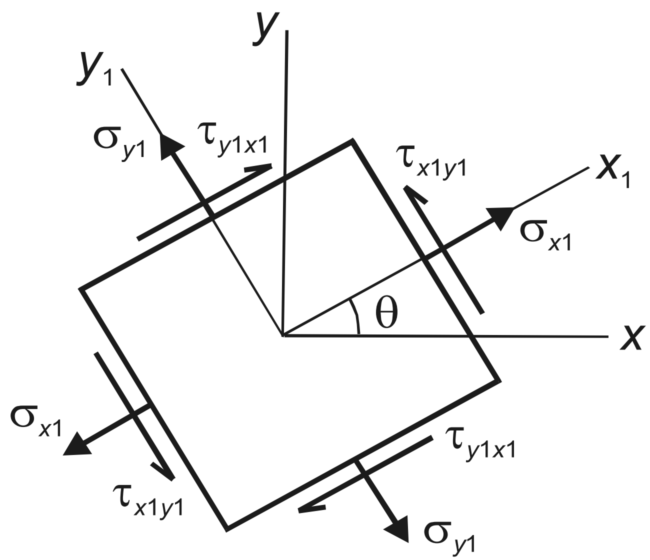
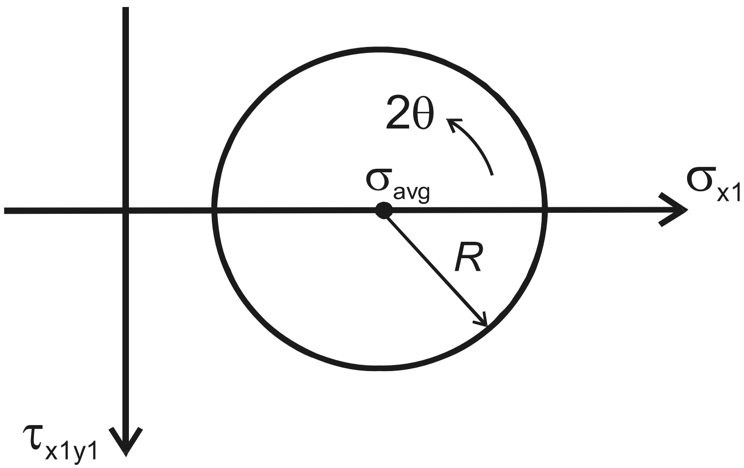
Sign Convention for Mohr's Circle



$$\left(\sigma_{x1} - \sigma_{avg}\right)^2 + \tau_{x1y1}{}^2 = R^2$$



- Shear Stress is plotted as positive downward
- θ on the stress element = 2θ in Mohr's circle

Constructing Mohr's Circle: Procedure

1. Draw a set of coordinate axes with σx_1 as positive to the right and $\tau x_1 y_1$ as positive downward.
2. Locate point *A*, representing the stress conditions on the *x* face of the element by plotting its coordinates $\sigma x_1 = \sigma_x$ and $\tau x_1 y_1 = \tau_{xy}$. Note that point *A* on the circle corresponds to $\theta = 0°$.
3. Locate point *B*, representing the stress conditions on the *y* face of the element by plotting its coordinates $\sigma x_1 = \sigma_y$ and $\tau x_1 y_1 = -\tau_{xy}$. Note that point *B* on the circle corresponds to $\theta = 90°$.

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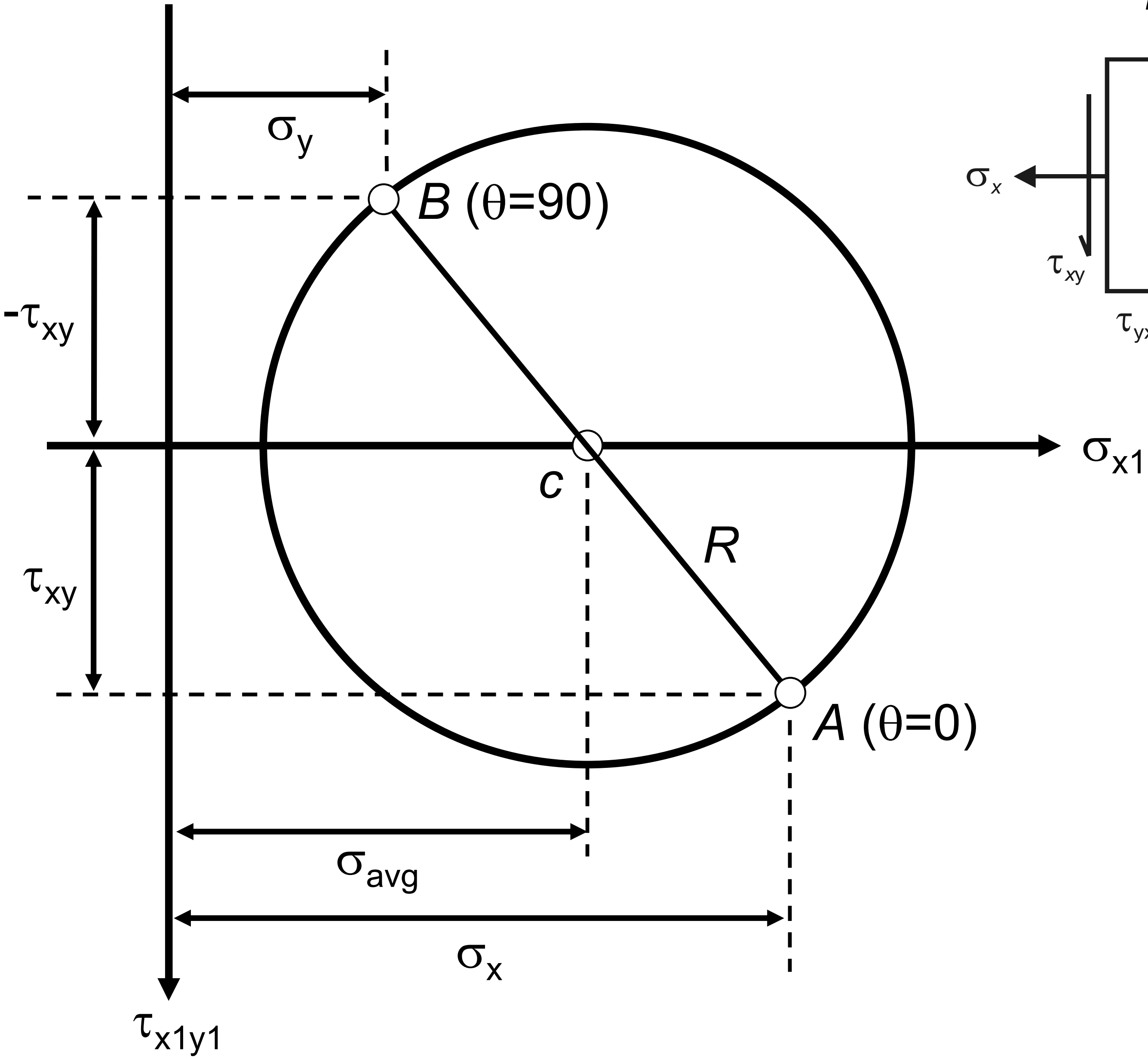
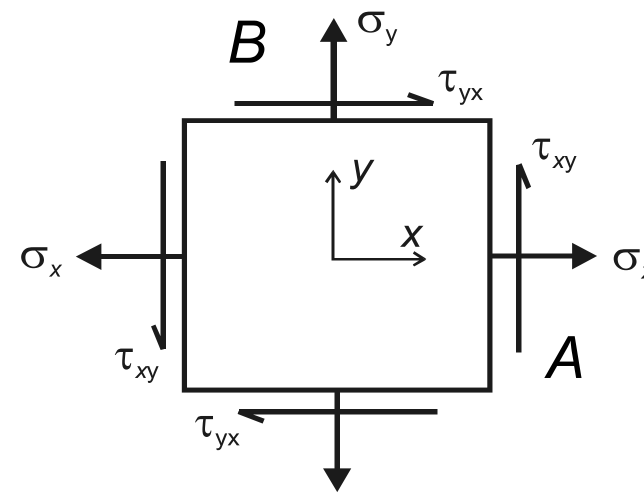
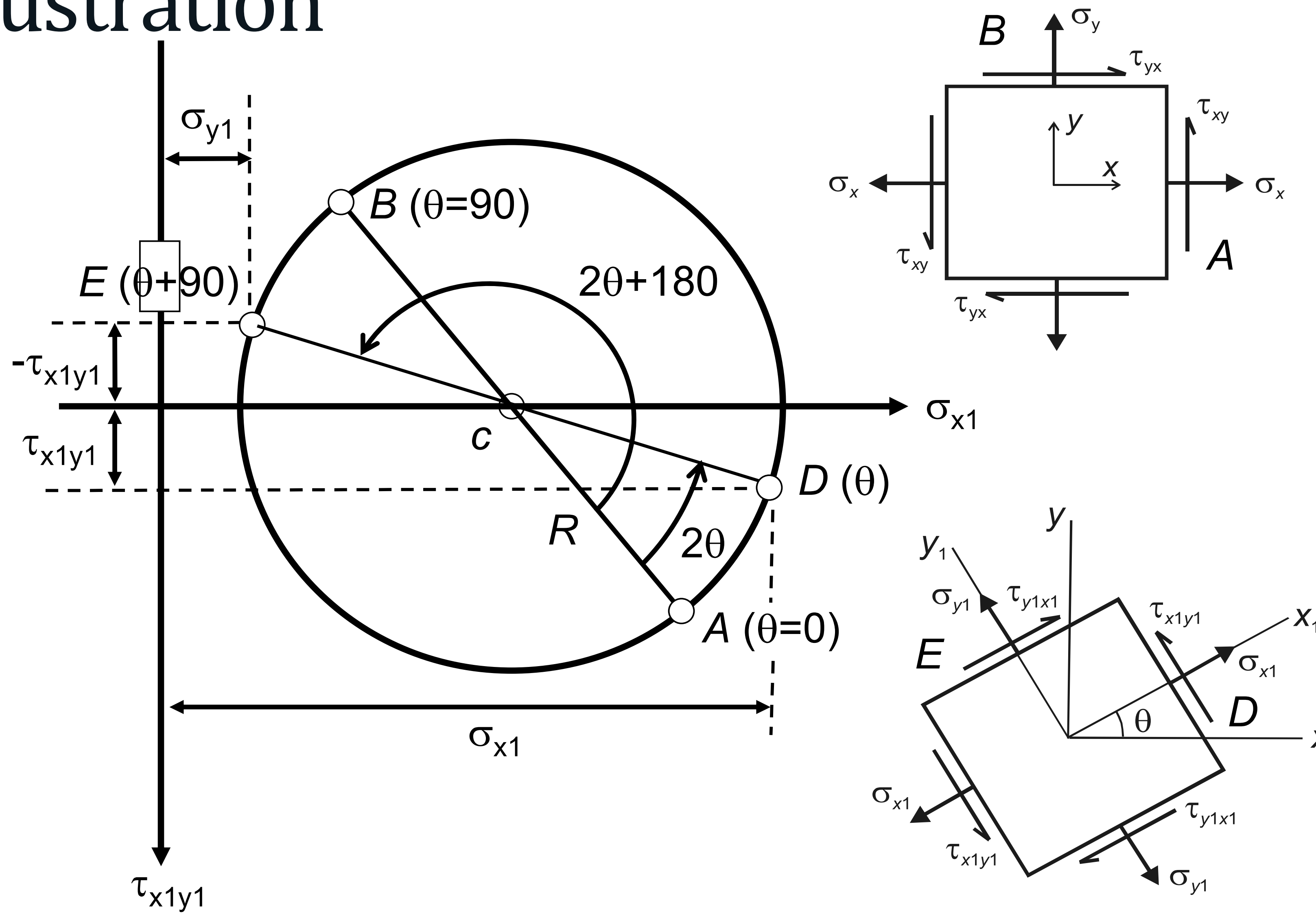

σy
B (θ=90)
-τxy
σx1
c
R
τxy
A (θ=0)
σavg
σx
τx1y1


B
σy
τyx
τxy
y
σx
x
σx
τxy
A
τyx

Procedure (cont'd)

4. Draw a line from point *A* to point *B*, a diagonal of the circle passing through point *c* (center of circle). Points *A* and *B* are at opposite ends of the diameter (and therefore 180° apart on the circle).
5. Using point *c* as the center, draw Mohr's circle through points *A* and *B*. This circle has radius *R*. The center of the circle *c* at the point having coordinates $\sigma x_1 = \sigma_{avg}$ and $\tau x_1 y_1 = 0$.

Stress Transformation: Graphical Illustration



Explanation

- On Mohr's circle, point *A* corresponds to $\theta = 0$. Thus it's the reference point from which angles are measured.
- The angle 2θ locates the point *D* on the circle, which has coordinates σx_1 and $\tau x_1 y_1$. *D* represents the stresses on the x_1 face of the inclined element.
- Point *E*, which is diametrically opposite point *D* is located 180° from *cD*. Thus point *E* gives the stress on the y_1 face of the inclined element.
- Thus, as we rotate the x_1y_1 axes counterclockwise by an angle θ, the point on Mohr's circle corresponding to the x_1 face moves ccw by an angle of 2θ.

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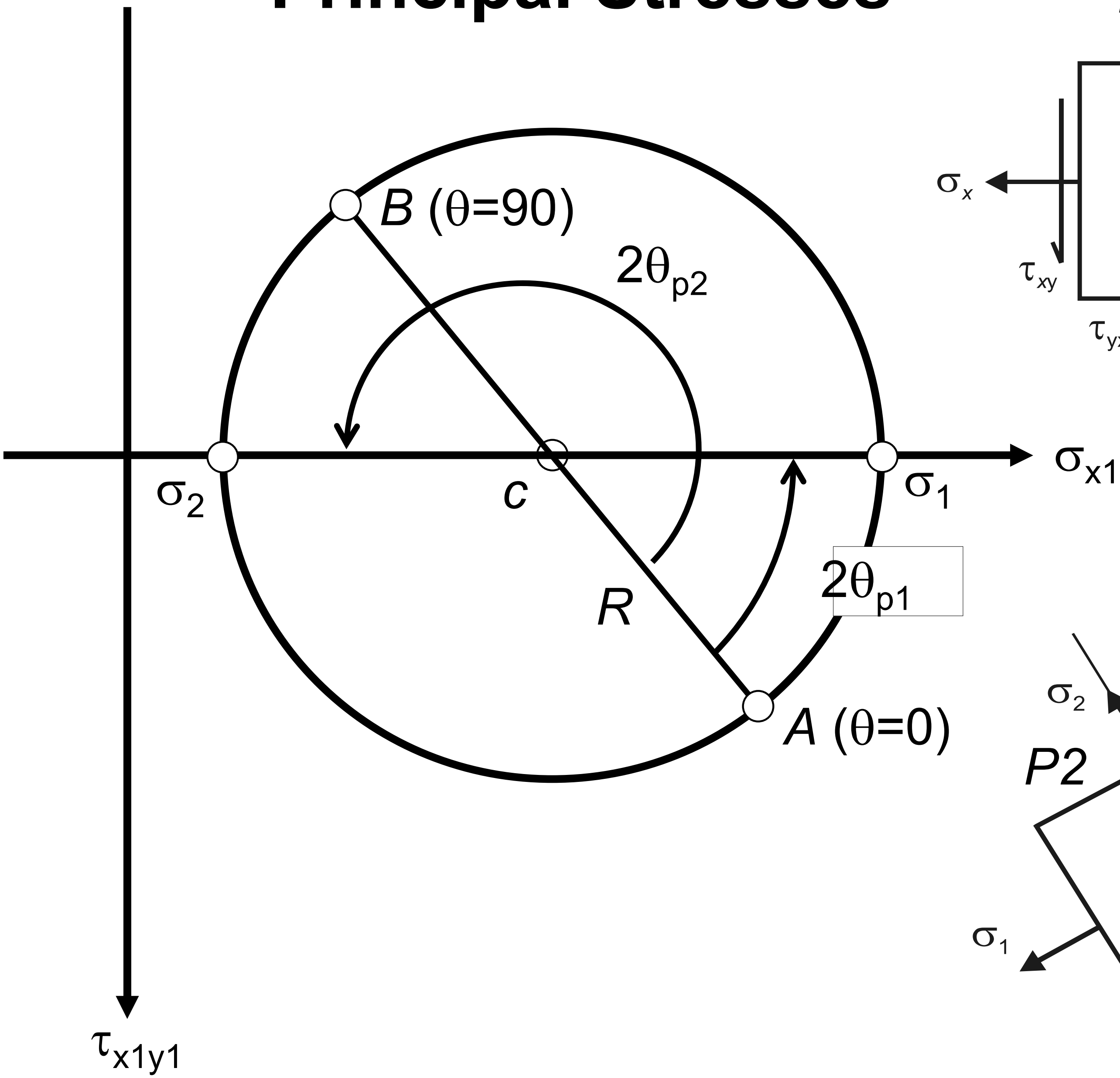
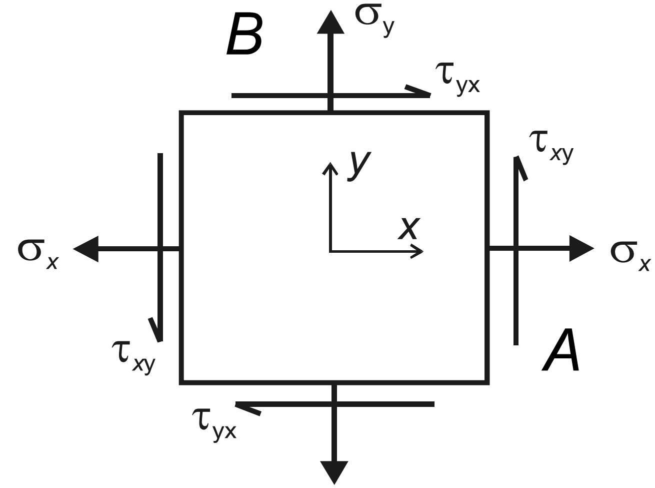
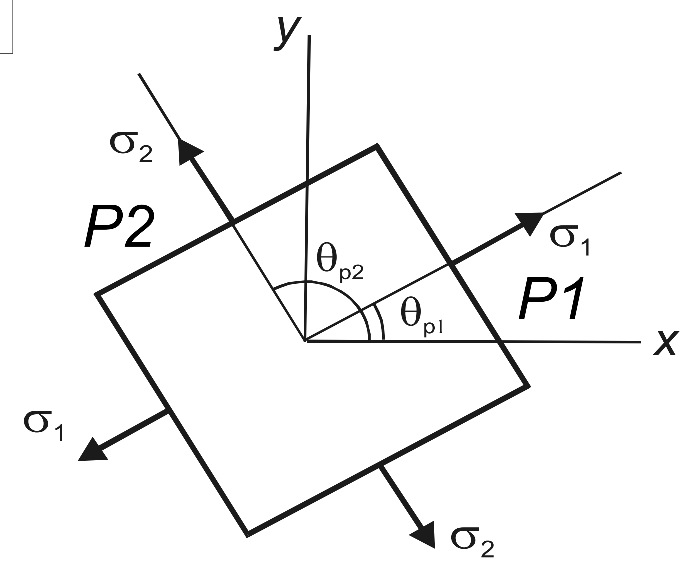
Principal Stresses







Explanation

- Principle stresses are stresses that act on a principle surface. This surface has no shear force components (that means $\tau x_1 y_1 = 0$)
- This can be easily done by rotating A and B to the σx_1 axis.
- σ_1= stress on x_1 surface, σ_2= stress on y_1 surface.
- The object in reality has to be rotated at an angle θ_p to experience no shear stress.

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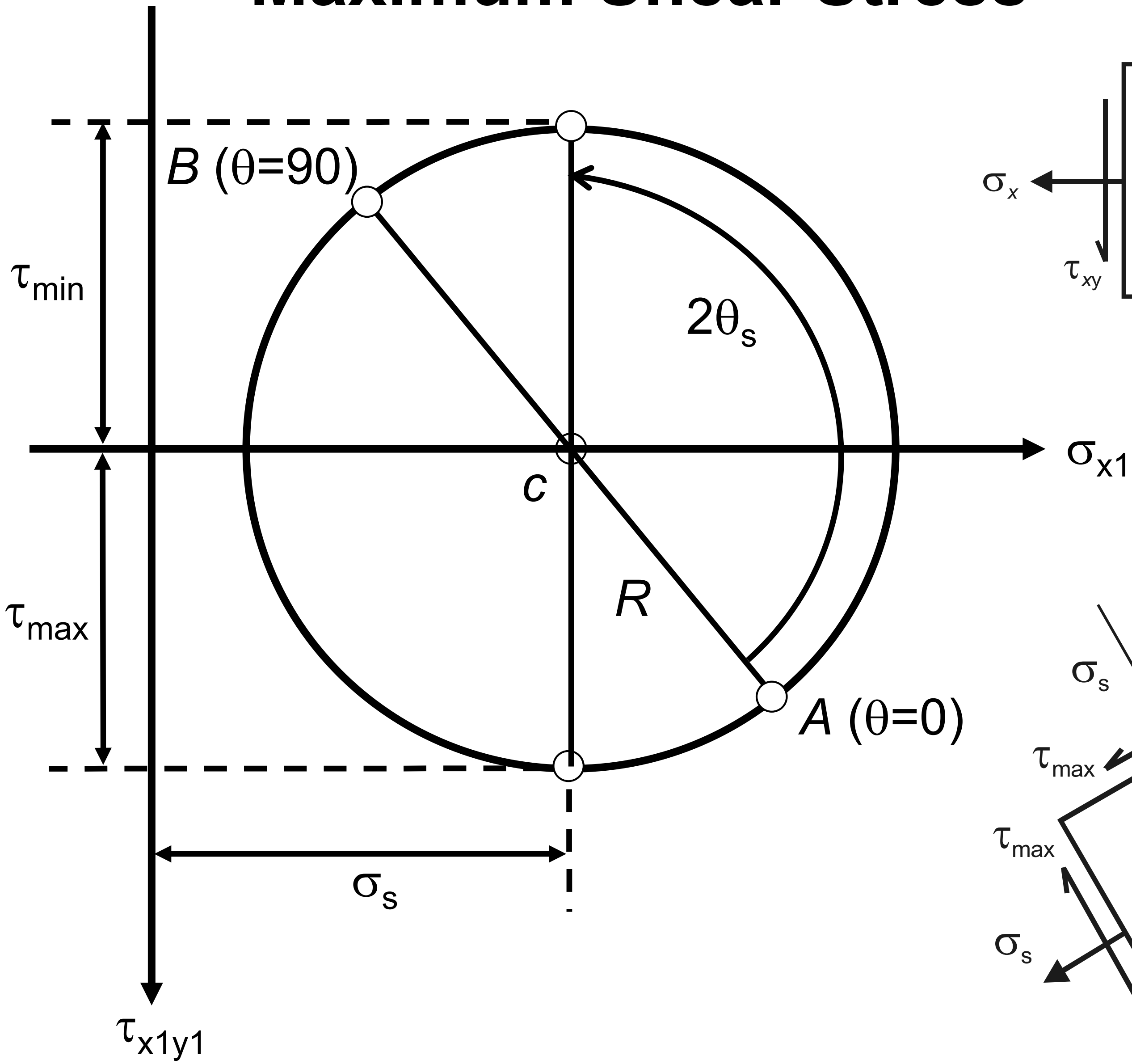
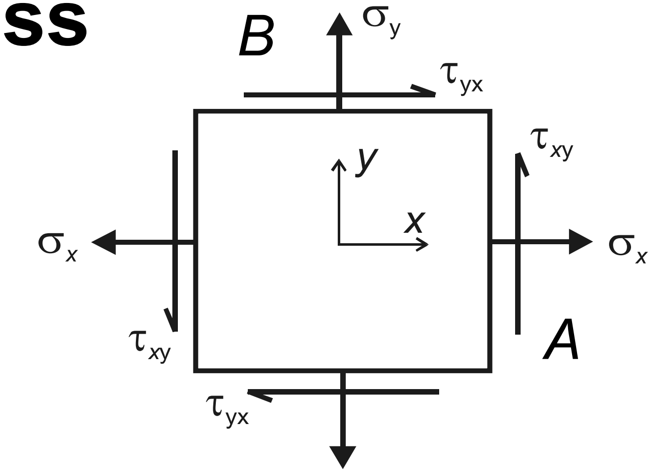
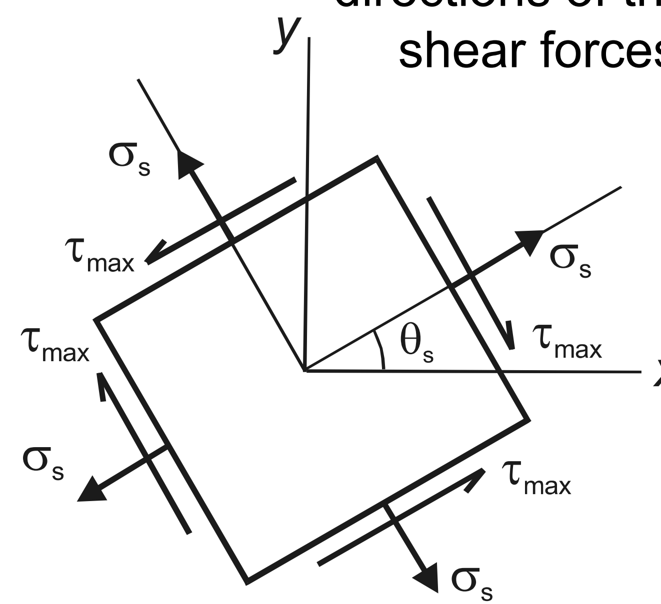
Maximum Shear Stress





Note carefully the directions of the shear forces.



Explanation

- The same method to calculate principle stresses is used to find maximum shear stress.
- Points A and B are rotated to the point of maximum $\tau x_1 y_1$ value. This is the maximum shear stress value $\tau_{max.}$
- Uniform planar stress (σ_s) and shear stress (τ_{max}) will be experienced by both x_1 and y_1 surfaces.
- The object in reality has to be rotated at an angle θ_s to experience maximum shear stress.

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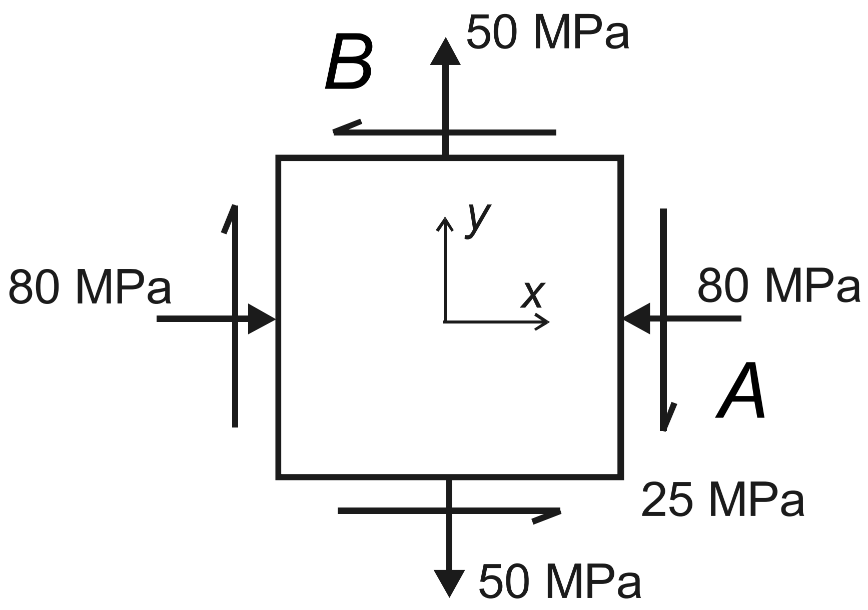
Example 1

Draw the Mohr's Circle of the stress element shown below. Determine the principle stresses and the maximum shear stresses.



What we know:

σ_x = -80 MPa

σ_y = +50 MPa

τ_{xy} = 25 MPa

Coordinates of Points

A: (-80,25)

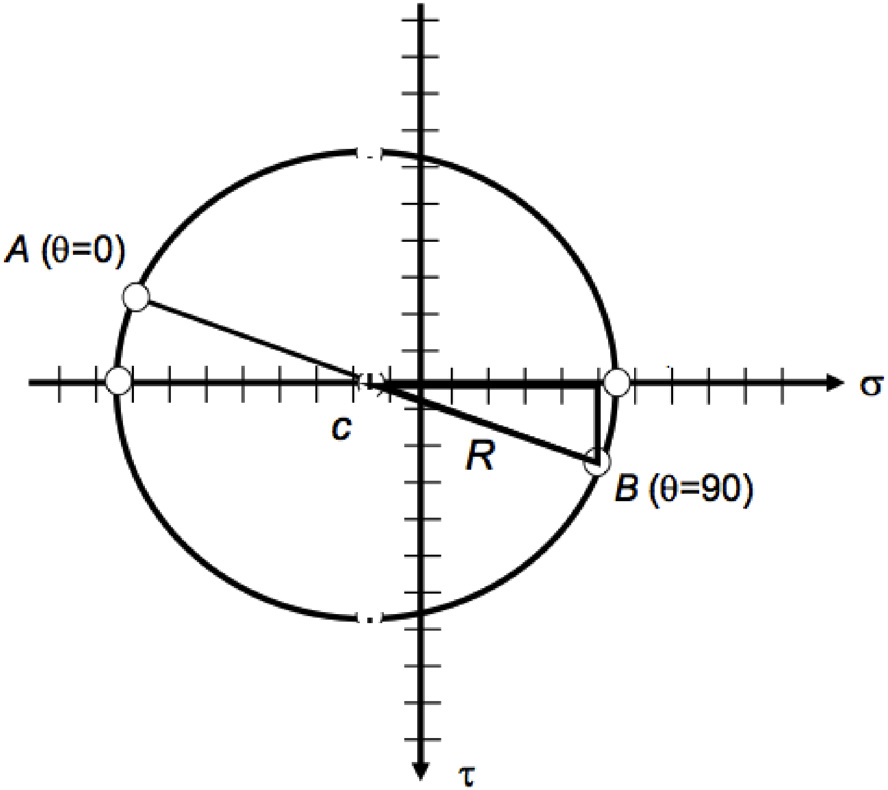
B: (50,-25)

Example 1 (cont'd)

$$c = \sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{-80+50}{2} = -15$$

$$R = \sqrt{(50-(-15))^2 + (25)^2}$$

$$R = \sqrt{65^2 + 25^2} = 69.6$$



Example 1 (cont'd)

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$$\sigma_{1,2} = c \pm R$$

$$\sigma_{1,2} = -15 \pm 69.6$$

$$\sigma_1 = 54.6 \text{ MPa}$$

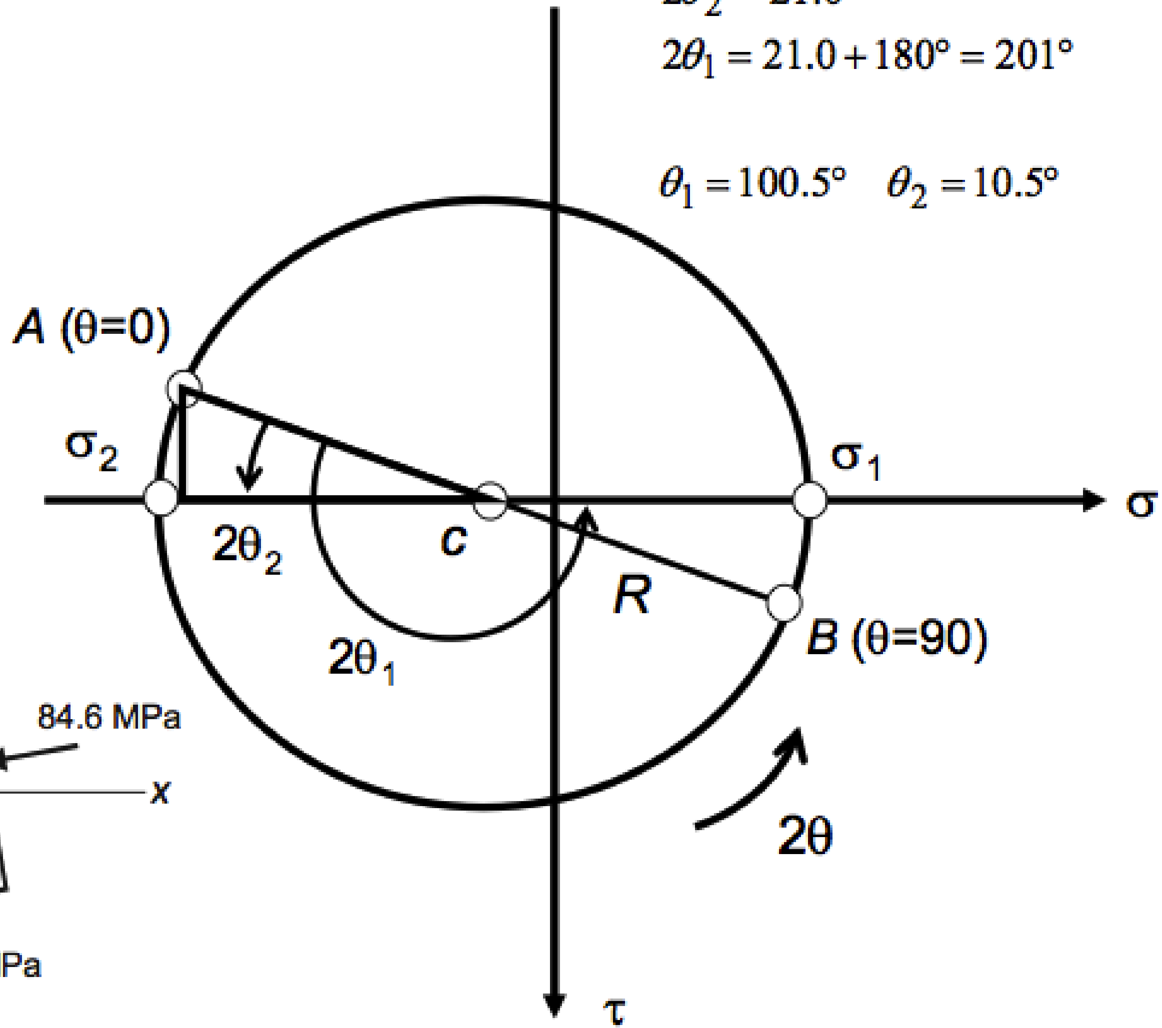
$$\sigma_2 = -84.6 \text{ MPa}$$

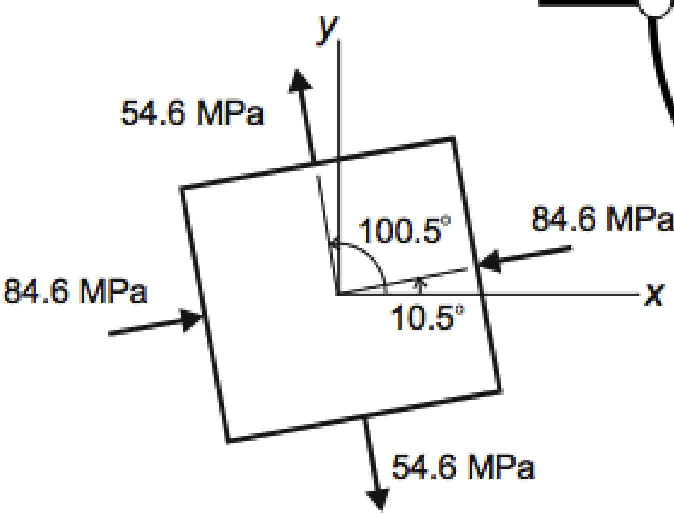
$$\tan 2\theta_2 = \frac{25}{80-15} = 0.3846$$

$$2\theta_2 = 21.0°$$

$$2\theta_1 = 21.0 + 180° = 201°$$

$$\theta_1 = 100.5° \quad \theta_2 = 10.5°$$



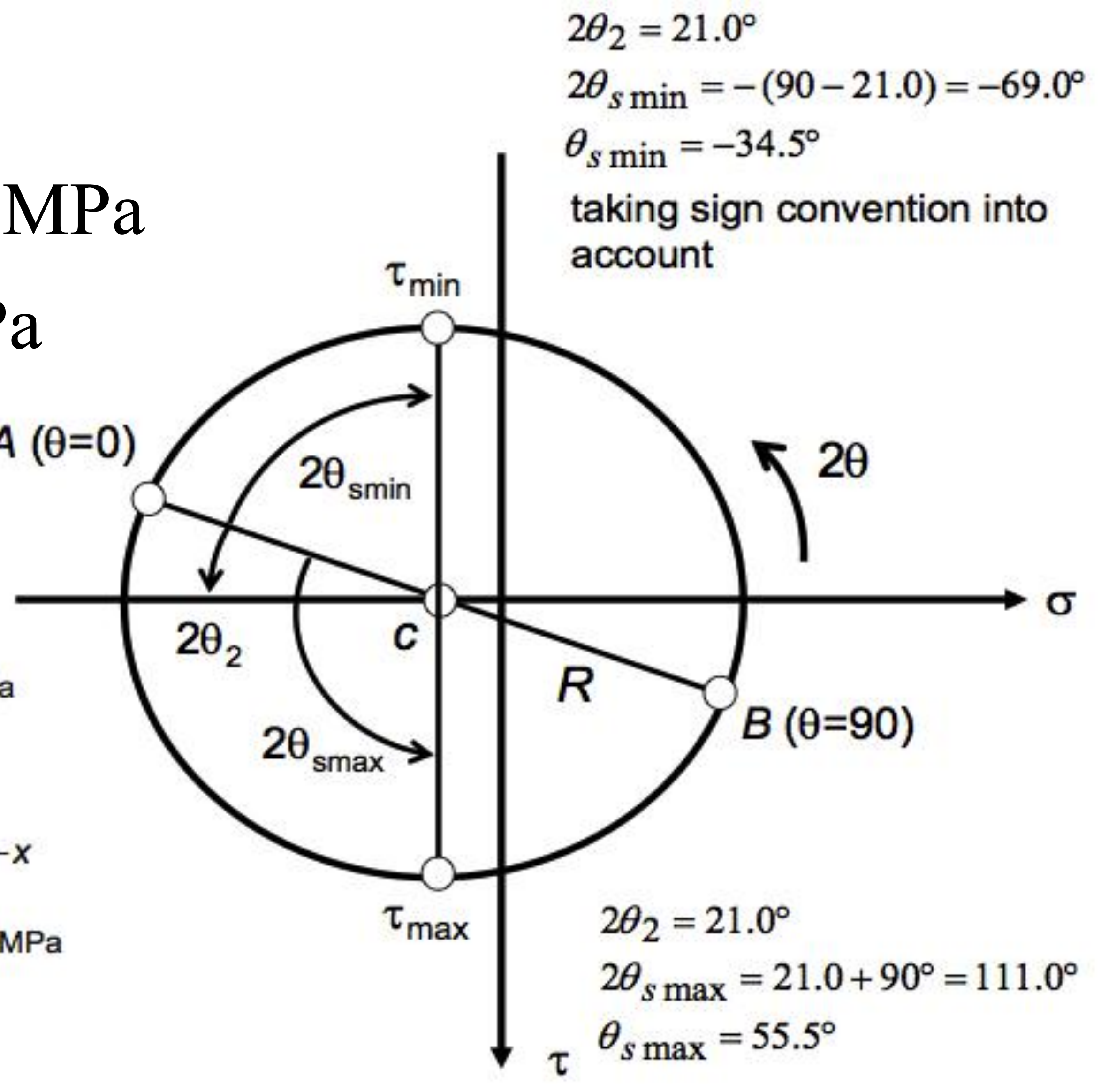


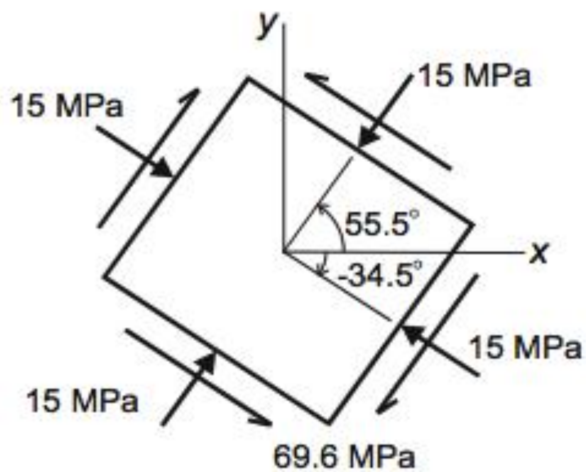
Example 1 (cont'd)

M

$$\tau_{\max} = R = 69.6 \text{ MPa}$$

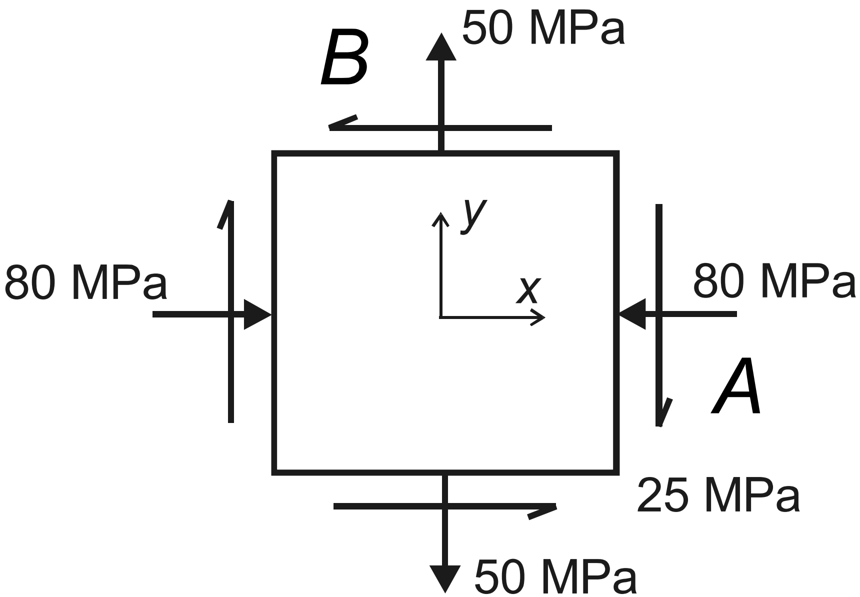
$$\sigma_s = c = -15 \text{ MPa}$$





Example 2

Given the same stress element (shown below), find the stress components when it is inclined at 30° clockwise. Draw the corresponding stress elements.



What we know:

σ_x = -80 MPa

σ_y = +50 MPa

τ_{xy} = 25 MPa

Coordinates of Points

A: (-80,25)

B: (50,-25)

Example 2 (cont'd)

Using stress transformation equation (θ=30°):

$$s_{x_1} - \frac{s_x + s_y}{2} = \frac{s_x - s_y}{2} \cos 2q + t_{xy} \sin 2q$$

$$t_{x_1 y_1} = - \frac{s_x + s_y}{2} \sin 2q + t_{xy} \cos 2q$$

$\sigma_x = -25.8$ MPa $\sigma_y = -4.15$ MPa $\tau_{xy} = 68.8$ MPa

Example 2 (cont'd)

Graphical approach using Mohr's Circle (and trigonometry)