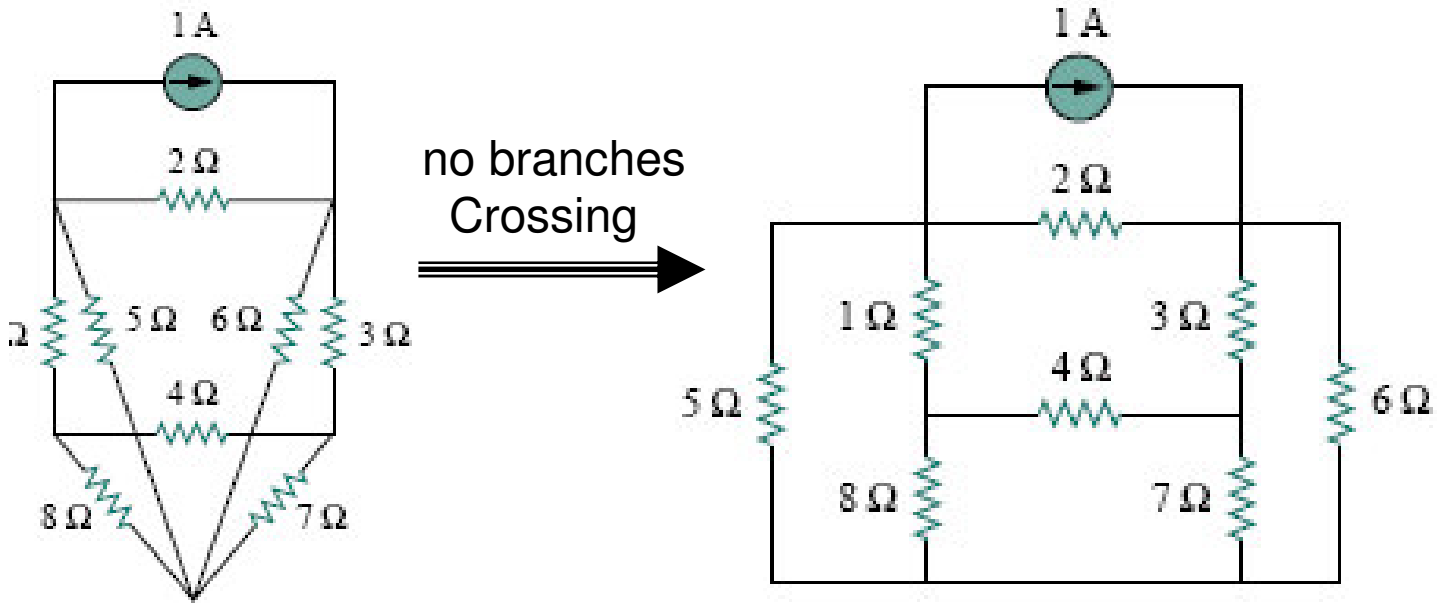
Mesh-Current Method (Loop Analysis)

- Nodal analysis was developed by applying KCL at each non-reference node.
- **Mesh-Current method is developed by applying KVL around meshes in the circuit.**
- **A mesh is a loop which doesn't contain any other loops within it.**
- Loop (mesh) analysis results in a system of linear equations which must be solved for unknown currents.
- Reduces the number of required equations to the number of meshes
- Can be done systematically with little thinking
- As usual, be careful writing mesh equations – follow sign convention.

•Powerful analysis method which applies KVL to find unknown currents.

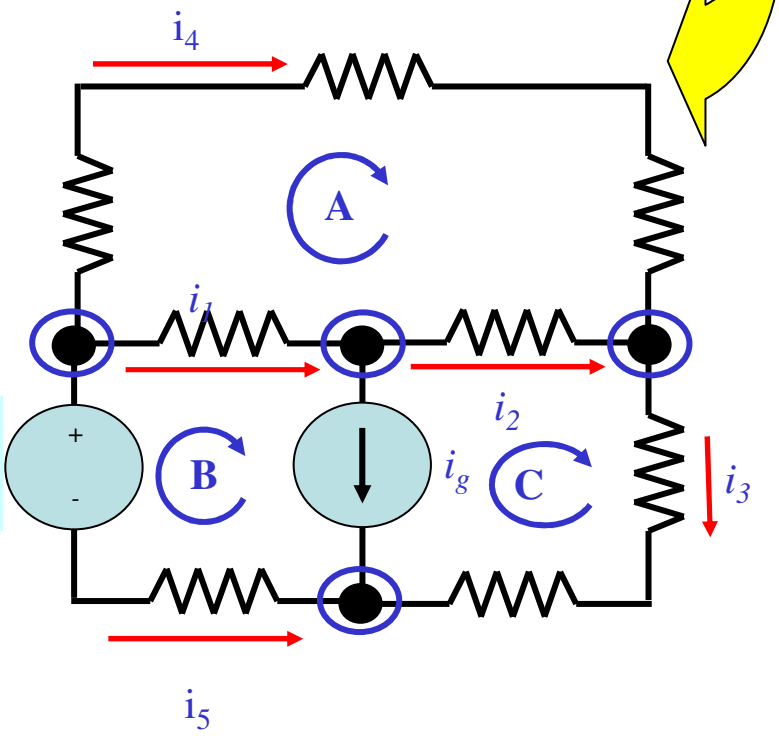
•It is applicable to a circuit with no branches crossing each other.



Definitions

A mesh is a loop which does not contain any other loops within it.

Essential Branch: Path between 2 essential nodes (without crossing other essential nodes).



How many mesh-currents?

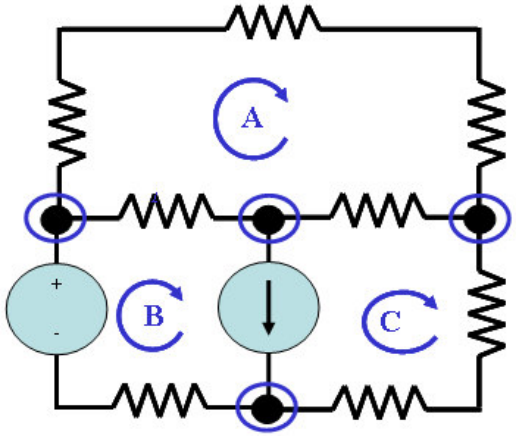
# of essential nodes **Ne = 4**
# of essential branches **Be = 6**

No. of Mesh-currents **M = Be –(Ne-1)**

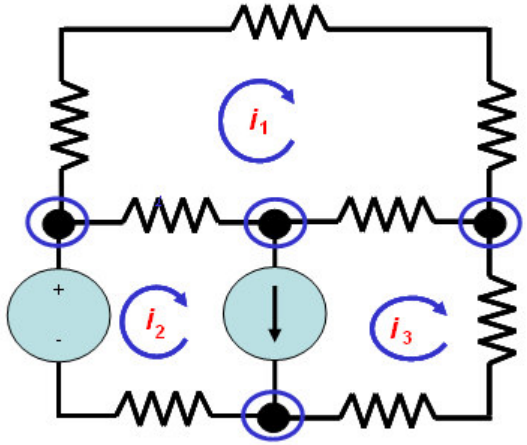
- Enough equations to get unknowns

Steps of Mesh Analysis

Step1
Identify the number of basic meshes.

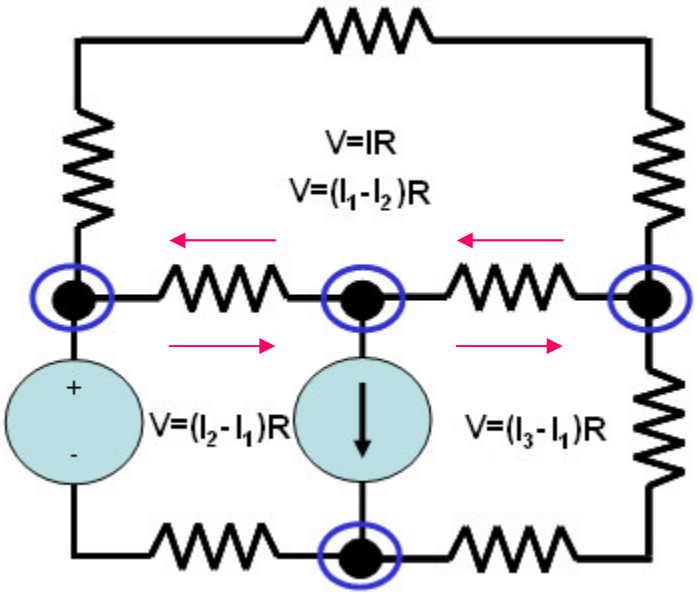


Step 2
Assign a current to each mesh.



Step 3

Apply KVL around each loop to get an equation in terms of the loop currents.



Step 4

Solve the resulting system of linear equations.

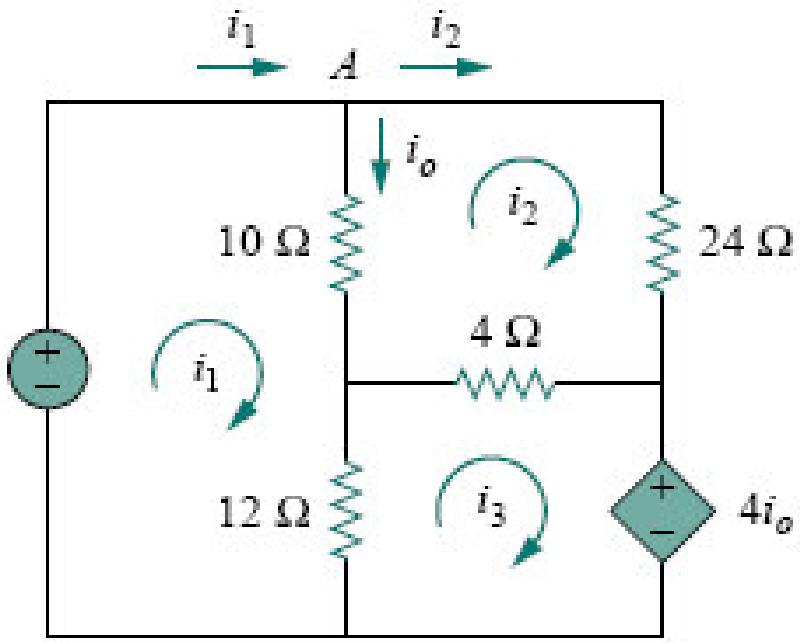
As an example

$V_1 + I_1 R_1 + (I_1 - I_2) R_3 = 0$

$I_2 R_{2+} V_2 + (I_2 - I_1) R_3 = 0$

Example

Use mesh analysis to find the current i_o



Solution

We apply KVL to the three meshes

For mesh 1

$$-24 + 10(i_1 - i_2) + 12(i_1 - i_3) = 0$$

Divide by 2

$$11i_1 - 5i_2 - 6i_3 = 12$$

For mesh 2

$$24i_2 + 4(i_2 - i_3) + 10(i_2 - i_1) = 0$$

Divide by 2

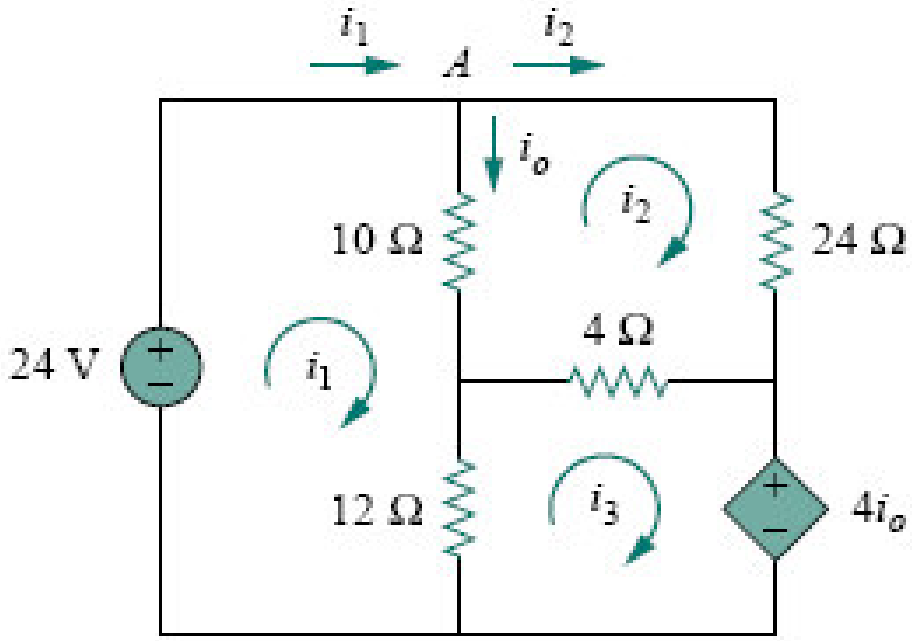
$$-5i_1 + 19i_2 - 2i_3 = 0$$

For mesh 3

$$4i_o + 12(i_3 - i_1) + 4(i_3 - i_2) = 0$$

at node A

$$i_o = i_1 - i_2$$ By Substitution



$$4(i_1 - i_2) + 12(i_3 - i_1) + 4(i_3 - i_2) = 0$$

Divide by 2

$$-i_1 - i_2 + 2i_3 = 0$$

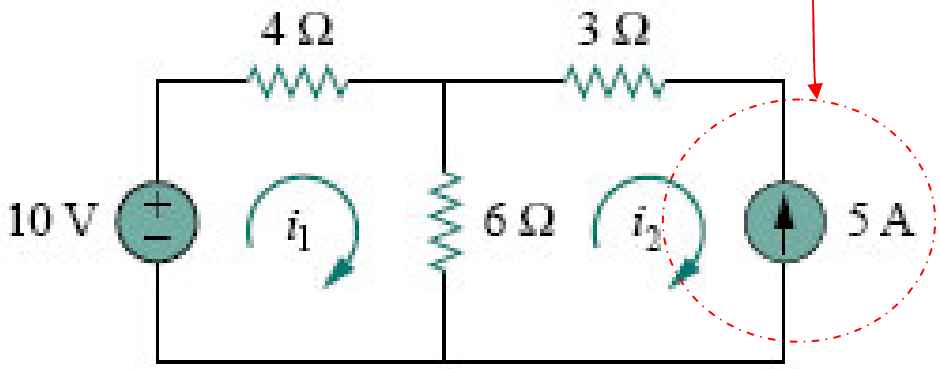
$$i_1 = 2.25 \text{ A},$$
$$i_2 = 0.75 \text{ A}$$
$$i_3 = 1.5 \text{ A}$$

$$\boxed{i_o = i_1 - i_2 = 1.5 \text{ A}.}$$

Cases to be considered for Mesh Analysis

Case I
A current source exists only in one mesh

set $i_2 = -5A$ and write a mesh equation for the other mesh



For Loop 1

$-10 + 4i_1 + 6(i_1 - i_2) = 0$

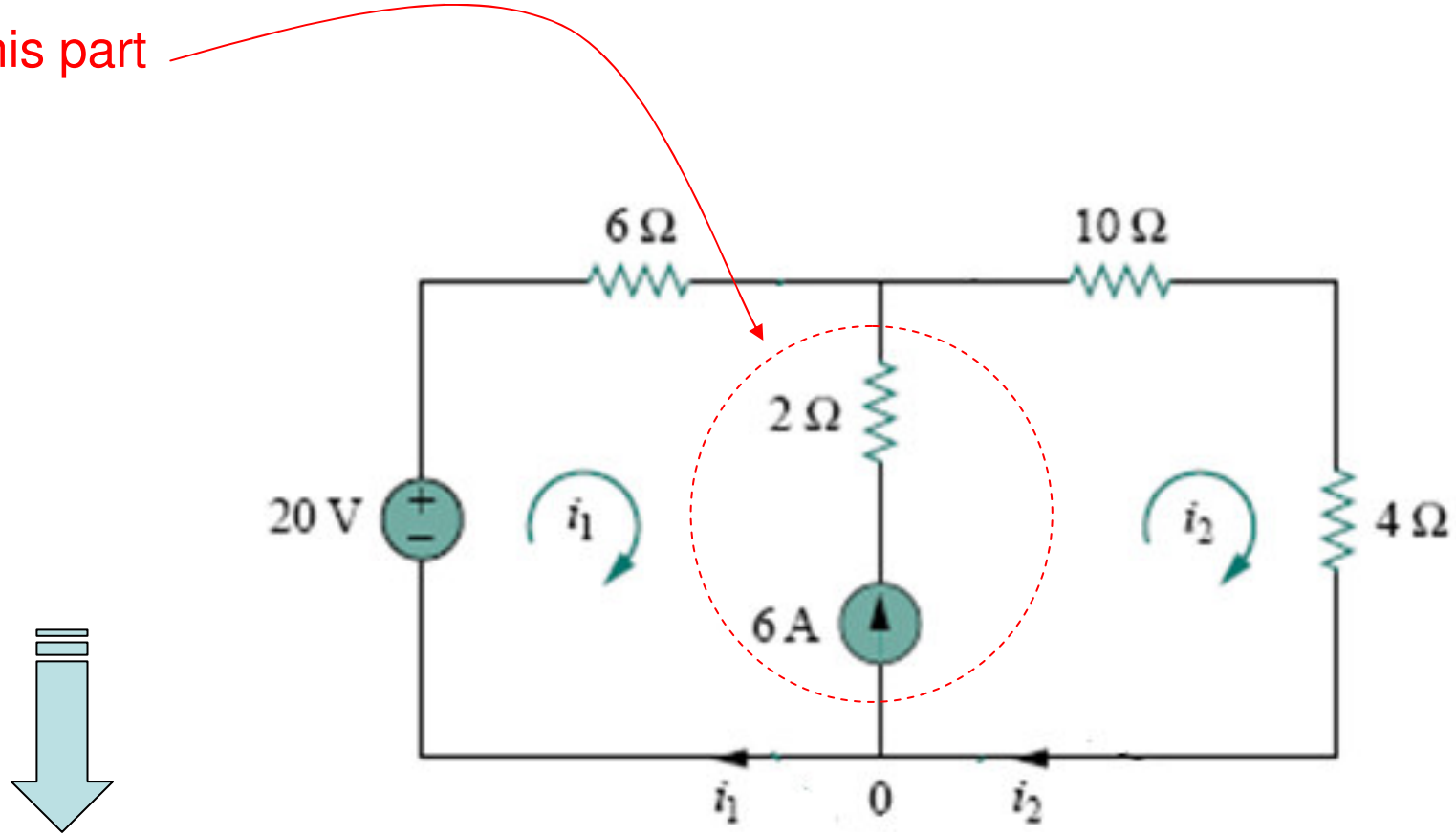
For Loop 2

$i_2 = -5A$

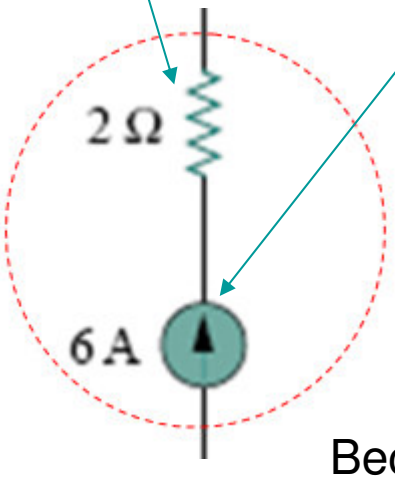
(No need to write a loop equation)

Case II
A current source exists between two meshes

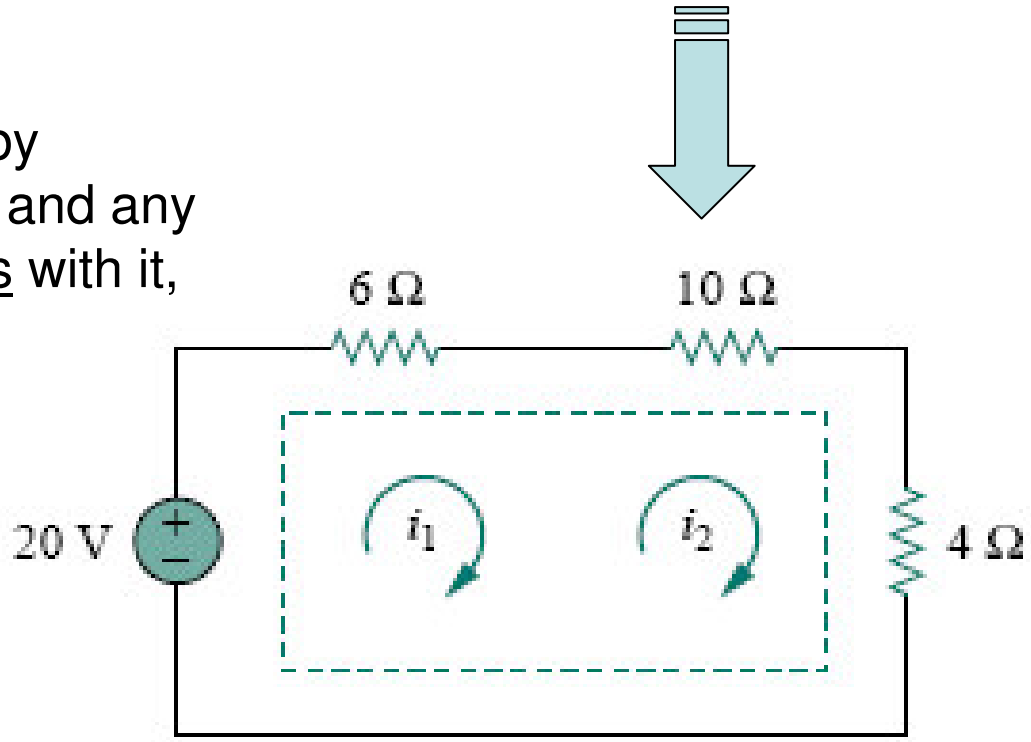
Exclude this part



a **supermesh** was created by excluding the current source and any elements connected in series with it,



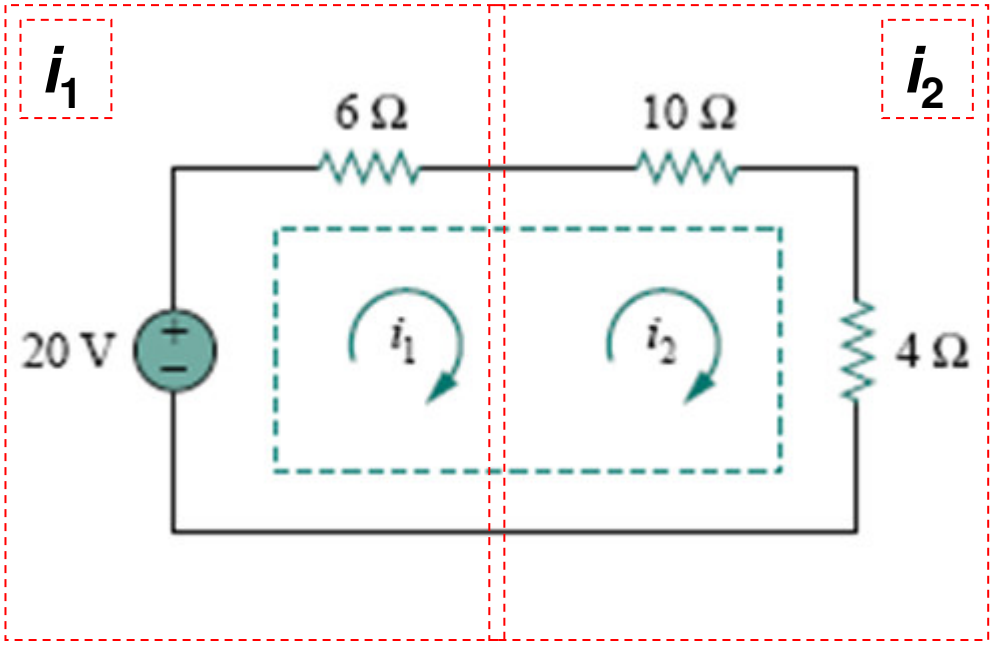
Why???



Because mesh analysis applies KVL—we do not know the voltage across a current source in advance.

A supermesh results when two meshes have a (dependent or independent) current source in common.

Applying KVL to the supermesh



$$-20 + 6i_1 + 10i_2 + 4i_2 = 0$$

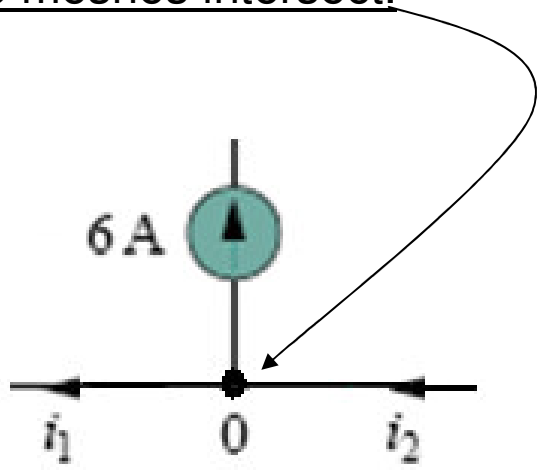
$$6i_1 + 14i_2 = 20$$

Apply KCL to a node in the branch where the two meshes intersect.

$$i_2 = i_1 + 6$$

$$i_1 = -3.2 \text{ A},$$

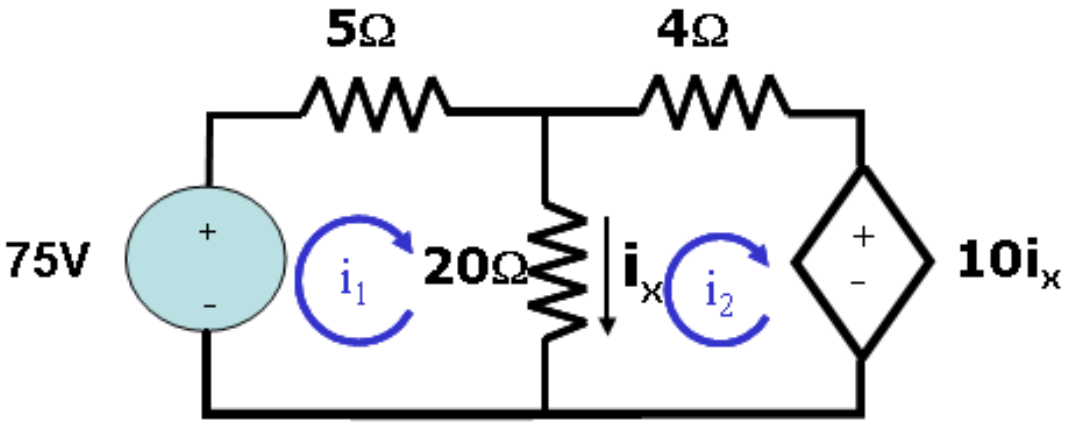
$$i_2 = 2.8 \text{ A}$$



Case III
Mesh with Dependent Sources

Solve for → I_x

$$i_x = i_1 - i_2$$



$$-75 + 5i_1 + 20(i_1 - i_2) = 0$$

$$10i_x + 20(i_2 - i_1) + 4i_2 = 0$$

Solve the two equations

$$-75 + 5i_1 + 20(i_1 - i_2) = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$10(i_1 - i_2) + 20(i_2 - i_1) + 4i_2 = 0 \quad \ldots\ldots\ldots\ldots\ldots (2)$$

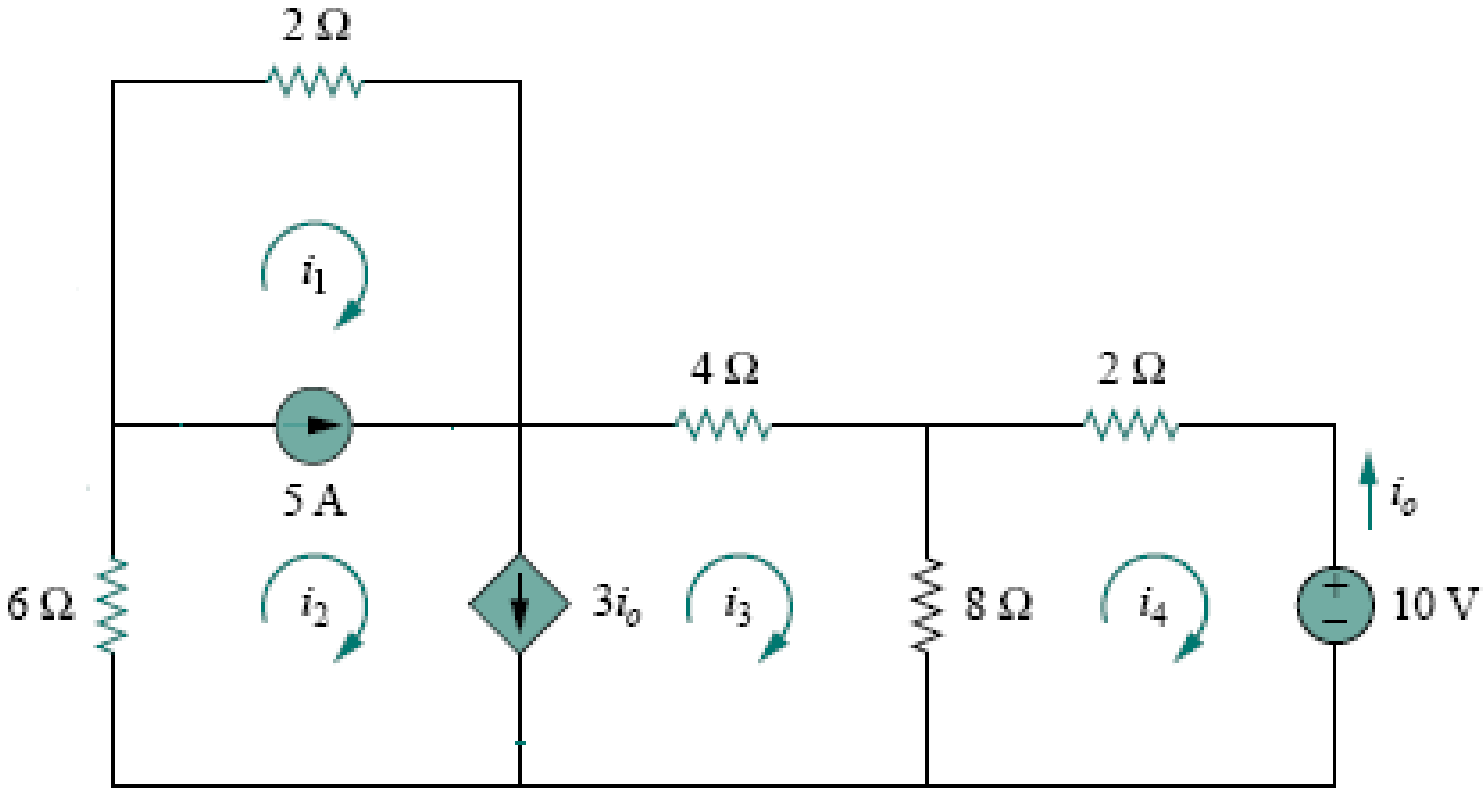
then

$$i_2 = 5A$$

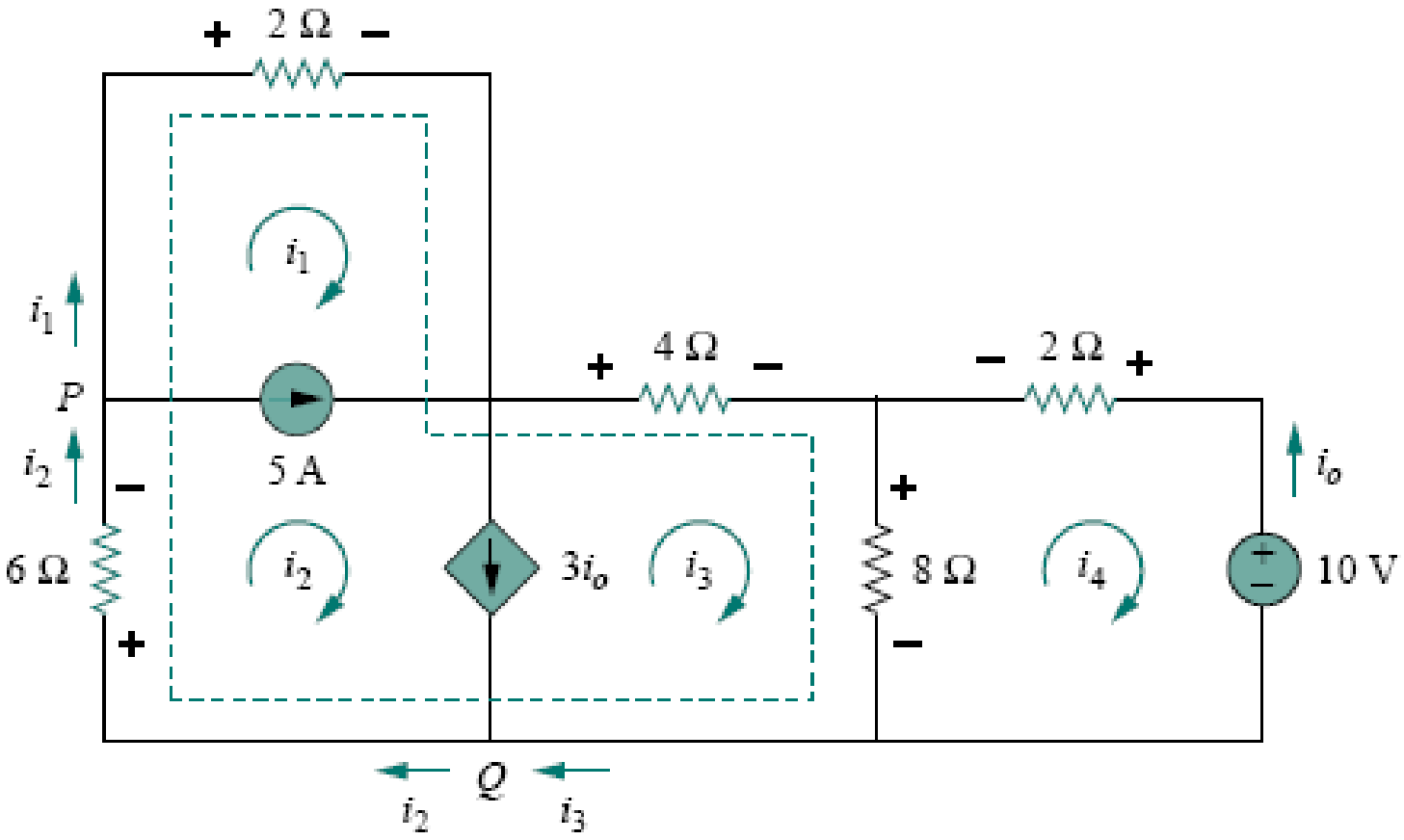
$$i_1 = 7A$$

Example:

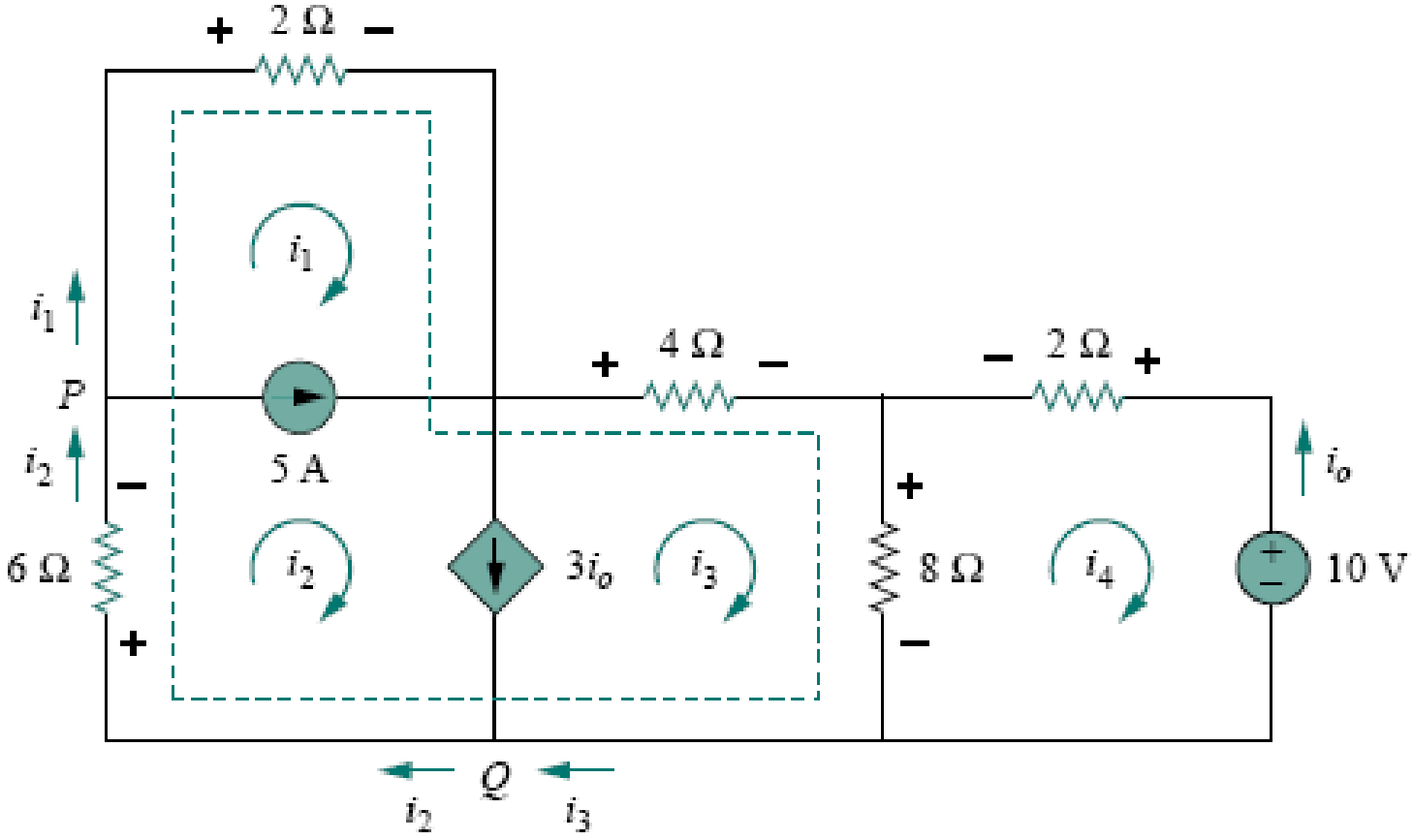
Use the mesh-current method to find i_o



Solution



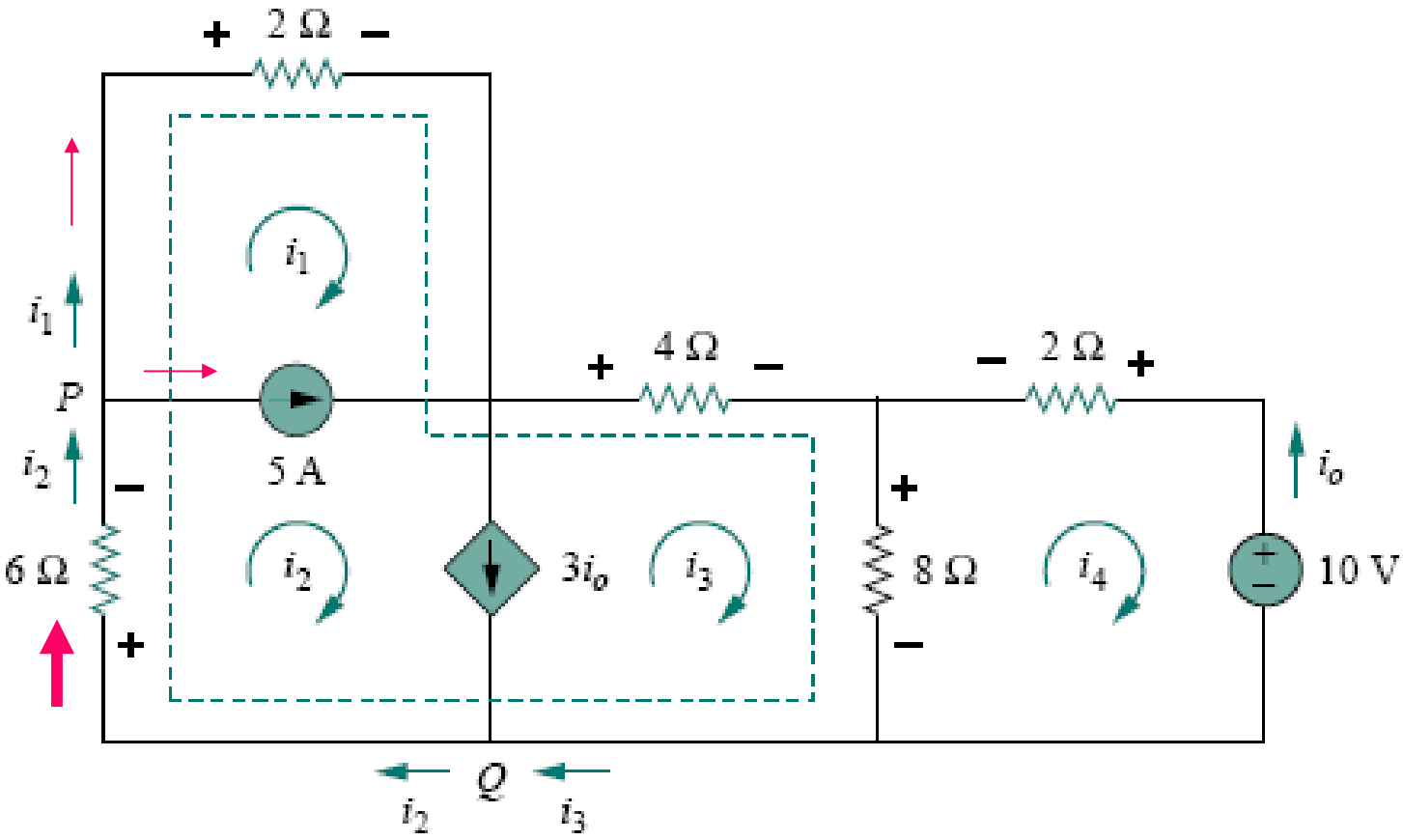
meshes 1 and 2 form a supermesh
meshes 2 and 3 form a supermesh



Applying KVL to the larger supermesh,

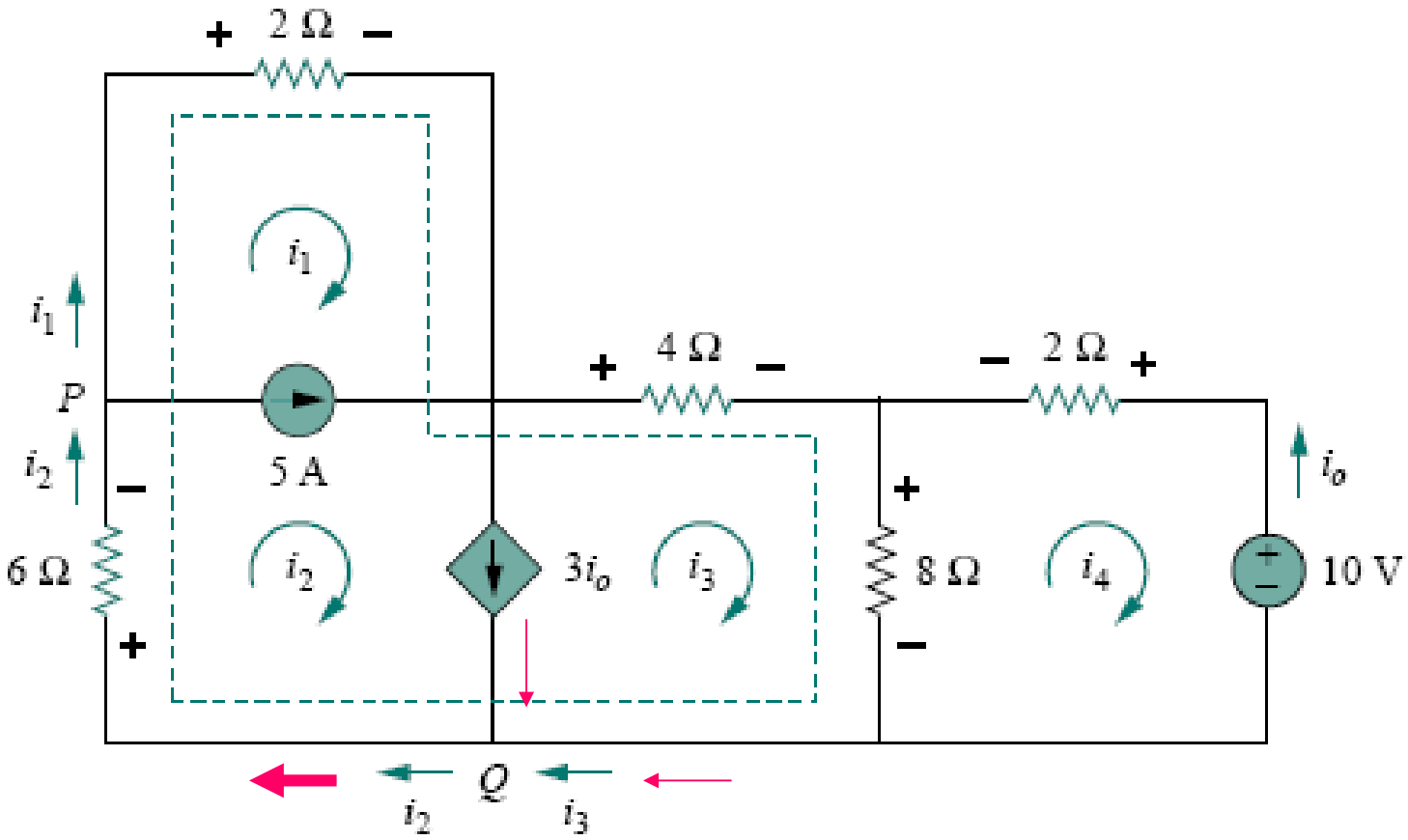
$$2i_1 + 4i_3 + 8(i_3 - i_4) + 6i_2 = 0$$

$$i_1 + 3i_2 + 6i_3 - 4i_4 = 0$$



For the independent current source, we apply KCL to node *P*

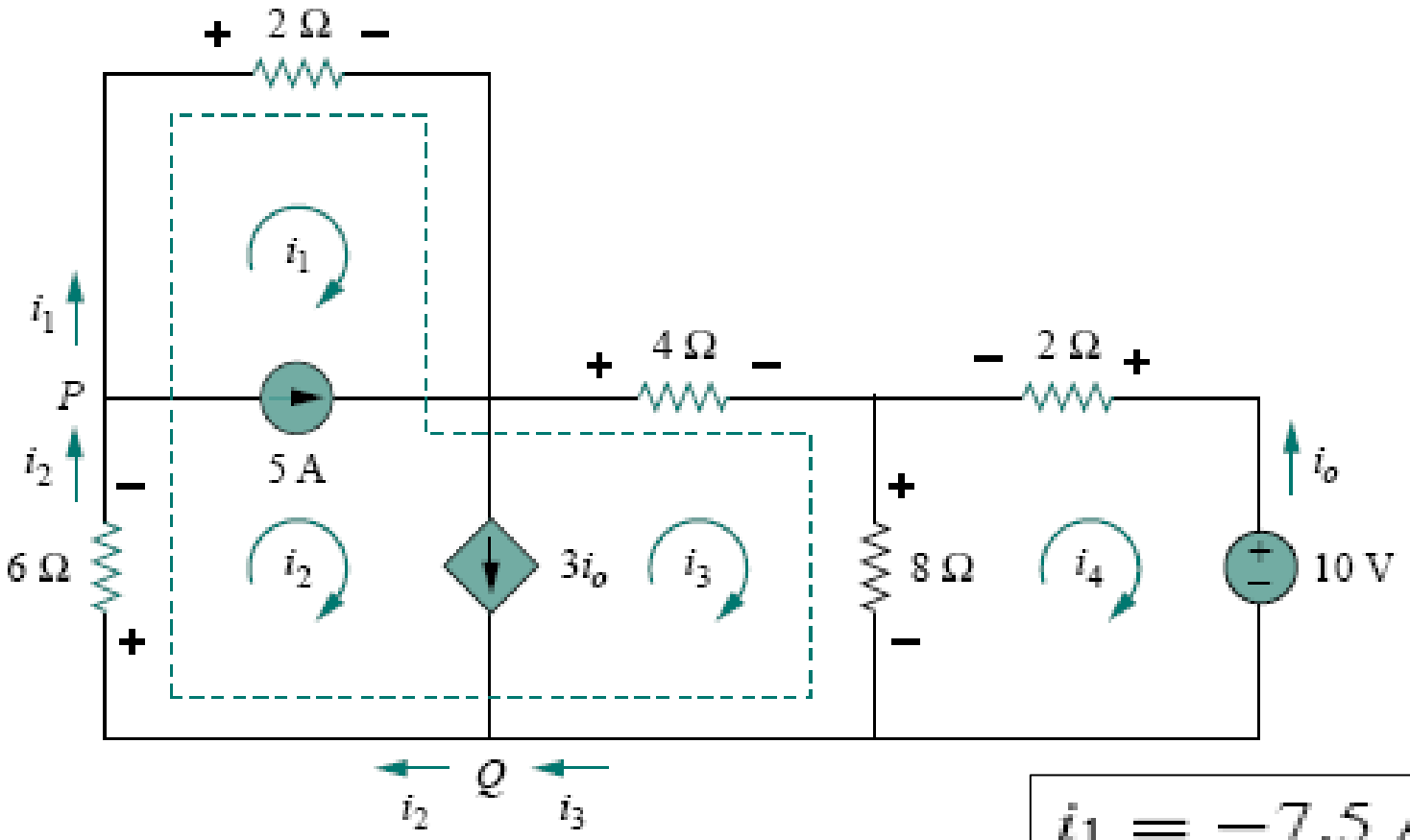
$$i_2 = i_1 + 5$$



For the dependent current source, we apply KCL to node *Q*:

$$i_2 = i_3 + 3i_o$$

$$i_o = -i_4 \longrightarrow i_2 = i_3 - 3i_4$$



$$i_1 = -7.5 \text{ A}$$

$$i_2 = -2.5 \text{ A}$$

$$i_3 = 3.93 \text{ A}$$

$$i_4 = 2.143 \text{ A}$$

Applying KVL in mesh 4,

$$2i_4 + 8(i_4 - i_3) + 10 = 0$$

$$5i_4 - 4i_3 = -5$$