CHAPTER TEN

10. INFLUENCE LINES

This is also another very useful technique in classical structural analysis. Influence lines are plotted for various structural effects like axial forces, reactions, shear forces, moments and thrust etc. As structural members are designed for maximum effects, ILD' s help engineer decide the regions to be loaded with live load to produce a maxima at a given section.

> " An influence line is a graphical representation of variation of a particular structural effect at a given section for all load positions on its span."

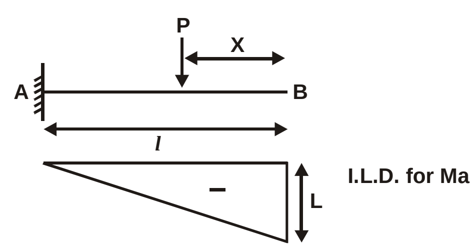
Two methods, viz, static method and virtual displacement method are used for the construction of ILD' s. Mostly it is the later method which is prefered. All structures in general and Railway and Highway bridges in particular are frequently subjected to various types of moving loads. As influence lines describe variation at a particular section for all load positions on span, the effects of moving loads can be calculated very easily. It must be remembered that a system of moving loads moves as a unit. For Railway bridges standard cooper' s E-60 and E-72 loadings are used whereas for highway bridges AASHTO lane loadings and truck loadings or sometimes tank loadings are used. When dealing with calculations regarding moving loads the problem is how to place the system so as to produce maximum effects at a given section. Sometimes mathematical criteria are used for the live load purpose and sometimes simple inspection is made. In each case influence lines help us simplify the things.

10.1. Statical Method of Constructing Influence Lines

In this method, a load may be placed at several positions within span/(s) and a mathematical expression for a particular structural effects at a section is set-up. By placing limits of X (the distance), the shape and ordinates of influence lines (called influence co-efficients also) can be determined.

For example consider the cantilever loaded below and let moment at fixed end A be represented by its influence line.

For a generalized load position as defined by distance X in the diagram, moment at A is.

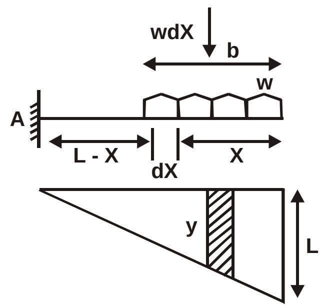


$$Ma = -P(L - X) \qquad 0 < X < L$$

Minus sign with P shows a negative moment at A for all load positions (consider sign convention for moments)

For X = 0 (load at point B) moment at A is – PL. Influence co-efficient is L at B. If X = L load is at A so moment at A is zero. Influence co-efficent is zero. In between A and B, moment at A varies linearly, joining the points, ILD for Ma is obtained. Now even if several loads are placed on the cantilever, Ma is simply the sum of all loads when multiplied by corresponding ordinates.

If a cantilever supports a ud.l, the above I.L.D for Ma is applicable. Consider a strip of width dX located at a distance X from free end,



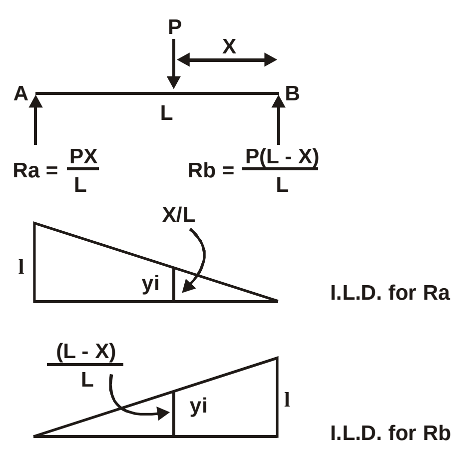
$$Ma = \int_o^b WydX = w\int_o^b ydX$$

Where $\int_o^b ydX$ is area of I.L.D between limits zero to b.

10.2. Influence Lines for beam Reactions:

ILD' s for reactions in case of simple beams and compound beams (determinate beams resting over several supports) can be drawn by using the already described procedure. Consider a simple beam with a single load sitting at any moment of time as shown

From statics it can be shown that



$$Ra = \frac{PX}{L} \text{ and } Rb = P\left(\frac{L-X}{L}\right) \quad 0 < X < L$$

When $X = 0$ (load at B); $Ra = 0$ and $Rb = P$ (by putting limits in above expressions)

When $X = L$ (load at A); $Ra = P$ and $Rb = 0$ (by putting limits in above expressions)

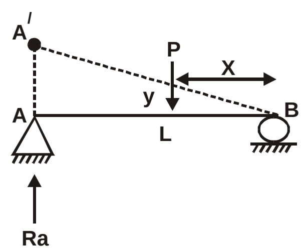
Instead of maximum co-efficients equal to P it is costomary to have them equal to 1 so that these could be evaluated by the product of loads and respective ordinates and these diagrams become valid for several loads. So algebraically

$Ra = \sum Pi\ yi$

$Rb = \sum Pi\ yi$

10.3. Principal of Virtual Displacements:

Consider a simple beam under the action of load P as shown. Ra can be found by virtual displacements by imagining that support at A has been removed and beam is under the action of load P and Ra. Under the action of Ra, beam is displaced as $A^{/}B$. The virtual work equation is



$Ra \times AA^{/} - Py = 0$ (Force $\times$ displacement)

So $Ra = \dfrac{Py}{AA^{/}}$ where y is the displacement due to Ra under P.

If $AA^{/} = 1$, $Ra = Py$ A result already obtained.

This procedure of drawing ILDs' is more useful for the complicated cases.

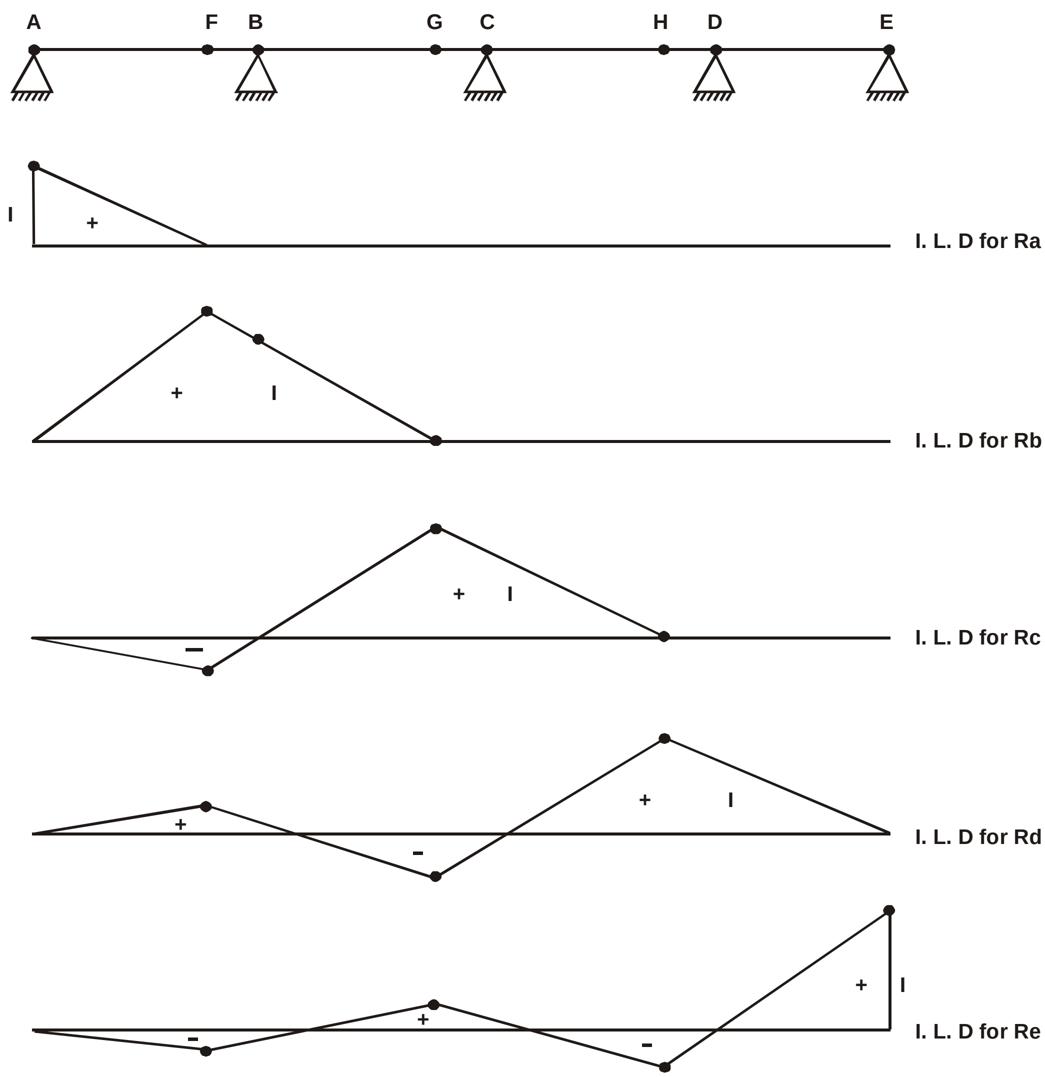
10.4. Reactions for Compound Beams:

A beam resting over several supports which has been made determinate by the availability of inserted hinges at suitable points is called a compound beam. The following Rules must be kept in mind while constructing ILD's for such cases.

1. Points of I.L.D corresponding to supports should show zero displacement except where virtual displacement is given (in case of reactions).
2. Portion of the beam between hinges which are straight before virtual displacements should remain straight after virtual displacement.
3. If a beam is continuous over two consecutive support and there is a hinge after these two supports, that portion of beam behaves a unit in case the virtual displacement is given elsewhere.
4. Portions of beam between pins which is straight before virtual displacement, shall remain

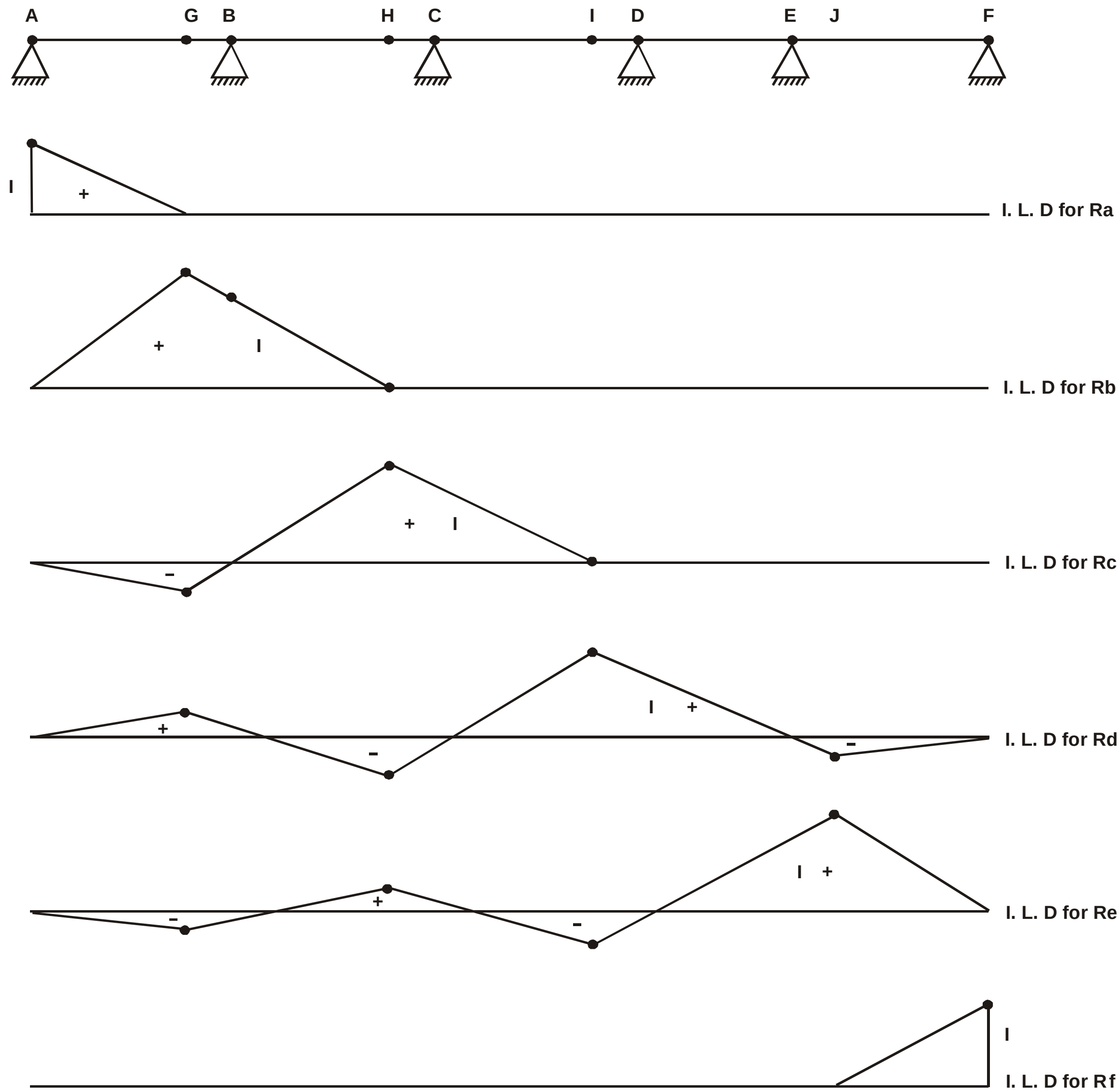
straight after virtual displacement.

Considering these guidelines given, draw influence lines for reactions for the following beam.



If positive areas of above diagrams are loaded, upward reactions at corresponding support will occur or vice-versa.

Construct Influence lines for reactions for the following compound beam by virtual displacements.



Evaluation of maximum upward and down reaction due to concentrated loads and udl can be done by using the basic principles described already.

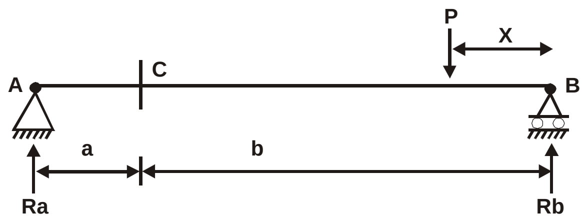
If several moving loads, from right to left direction, approach left hand support of a simple beam, the left reaction continues to increase and becomes maximum till leading wheel is at the left support. This corresponding first maxima will decrease immediately if the load falls off and leaves the span from left upon further advance, reaction at left support will start increasing and will become maximum again when second wheel is at the left support. So there will be as many maxima as is the number of loads.

Evaluation of reactions due to live load udl is rather simple as the span portion required to be loaded for maximum upward and downward support reactions are obvious by the simple inspection. Of course positive areas if loaded will give maximum upward reactions and vice-versa.

10.5. Influence Lines for Shear Force:

In structural analysis, normally we develop the methods by considering simple cases and some generalized conclusions are drawn which can then be applied to more complicated cases. So consider the following simple beam wherein a moving load (right to left) occupies the position shown at any instant of time.

Using left-up and write-down as sign convention for positive shear force.



For all load positions to right of point C, the shear force for at C (Vc) is equal to + Ra.

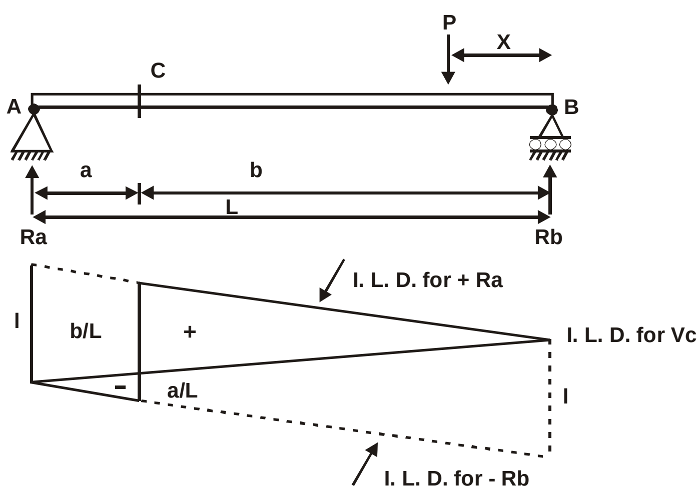
Vc = Ra

It means that for load position between point B and C, the Shape of ILD for SF at C will be the same as the shape of ILD for + Ra.

For all load positions to left of point C, the shear force at C (Vc) is equal to – Rb.

Vc = – Rb

It means that for load position between point A and C, the shape of ILD for SF at C will the same as shape of ILD for –Rb. Knowing that positive ILD is drawn above the reference line and negative ILD is drawn below the reference line, we obtain the ILD for Vc as shown below with the help of ILD's for reactions (+ Ra1 – Rb)



Mathematically

$$Ra = \frac{PX}{L} \qquad 0 < X < L$$

$$Rb = P\frac{(L - X)}{L} \qquad 0 < X < L$$

At X= 0, load is at B and Vc is zero. At x= b, load is at C and Vc = + Ra = $\frac{Pb}{L}$ or $\frac{b}{L}$ if P= 1.

The ordinates $\frac{a}{L}$ and $\frac{b}{L}$ can be obtained by using similar triangles. Now inspect the ILD for Vc. For a right to left advance of load system, Vc keeps on increasing till the "leading load is at the section", when leading load just crosses the section, Vc drops by the magnitude of load and this process continues. So we can write that for maximum SF at a section, "the load should be at that section". This is the first criterion of calculation of Vmax. Now the question comes to mind that which load among the moving load system should be placed at the section? To address this question, we have noted, that change in SF at a section, ΔV, is equal to change in Ra (ΔRa) minus the load leaving the Section. (Pn)

So, $\quad \Delta V = \Delta Ra - Pn$

If W is sum of all the loads on the span L before advance of *a*, it can be shown that

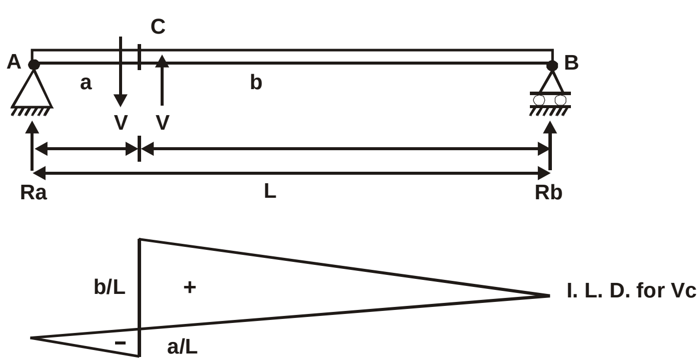
$$\Delta Ra = \frac{Wa}{L}$$

So, $\quad \Delta V = \frac{Wa}{L} - Pn$

Any load which reverses this expression, should be brought back and placed at that section to realize the maximum SF at that section. So a change in the sign of above expression can be regarded as the second criterion for maximum shear force at a section.

It can also be shown that loads entering or leaving the span as a result of any particular advance do not affect the above expression very significantly.

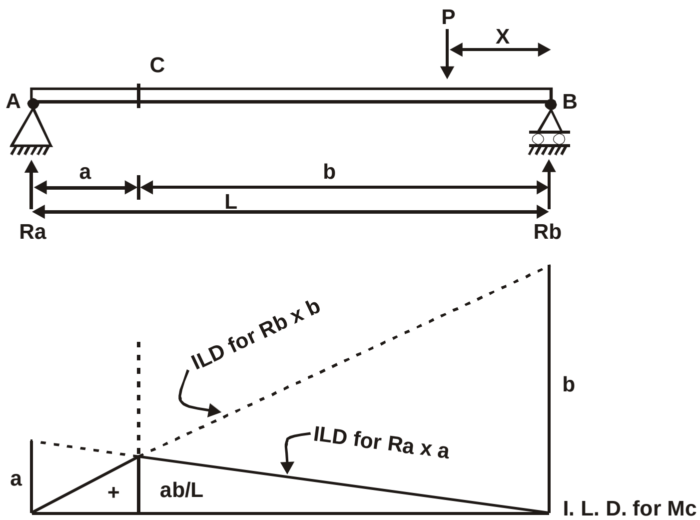
The above method is called the statical method. The same shape of ILD for Vc can be obtained by virtual displacement method also.



Now imagine that resistance to vertical displacement at C has been destroyed (imagine a sort of cut at the section) and the vertical shear force as shown (opposite to sign convention for positive shear force). The area enclosed between the original position before virtual displacement and the deformed position after virtual displacement is the ILD for Vc.

10.6. Influence Line Diagrams for Bending Moment:

Again we consider the simple beam under the action of a simple moving load as shown. Let it be required to construct ILD for Mc.



If the load is between points B and C.

$$Mc = Ra \times a = \frac{PX}{L} \times a \qquad 0 < X < b$$

at $X = 0$; load at B, $Mc = 0$.

If $X = b$;

$$Mc = \frac{Pab}{L}\left(= \frac{ab}{L} \text{ if } P = 1\right)$$

It means that for portion BC, the shape of ILD for Mc is the same as the shape of ILD for Ra multiplied by distance *a*.

If the load is between points A and C

$$Mc = Rb \times b = \frac{P(L - X)}{L} \times b \qquad b < X < L$$

At $X = b$, load is at C; , $Mc = Rb \times b$

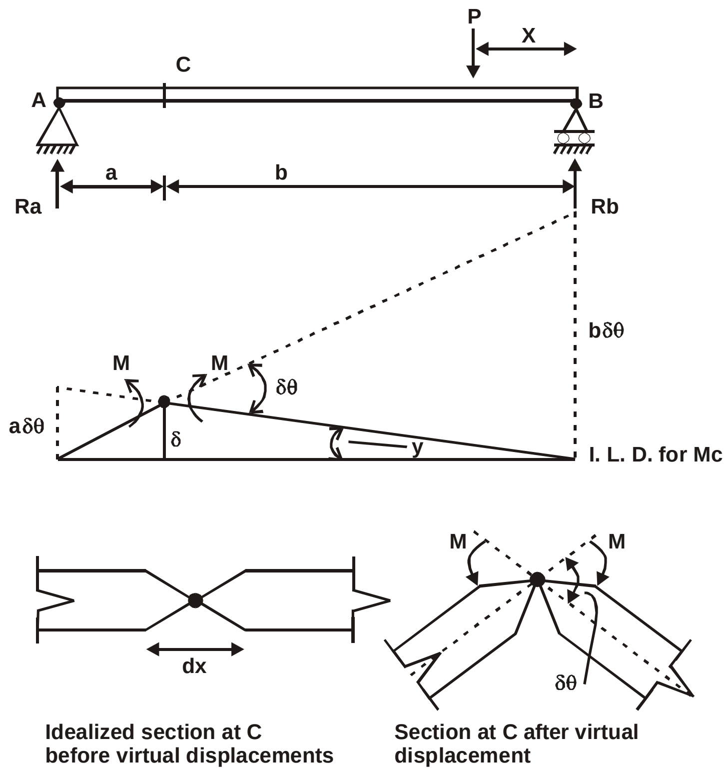
So $$Mc = \frac{Pab}{L}\left(= \frac{ab}{L} \text{ if } P = 1\right)$$

It means that for portion AC, the shape of ILD for Mc is the same as the shape of ILD for Rb multiplied by b.

At X = L;

Load at A; Mc = 0

The same shape of ILD for Mc can be obtained by virtual displacements also.



The virtual work equation is

work done by loads = work done by the moments.

$P \times y = M \times \delta\theta.$

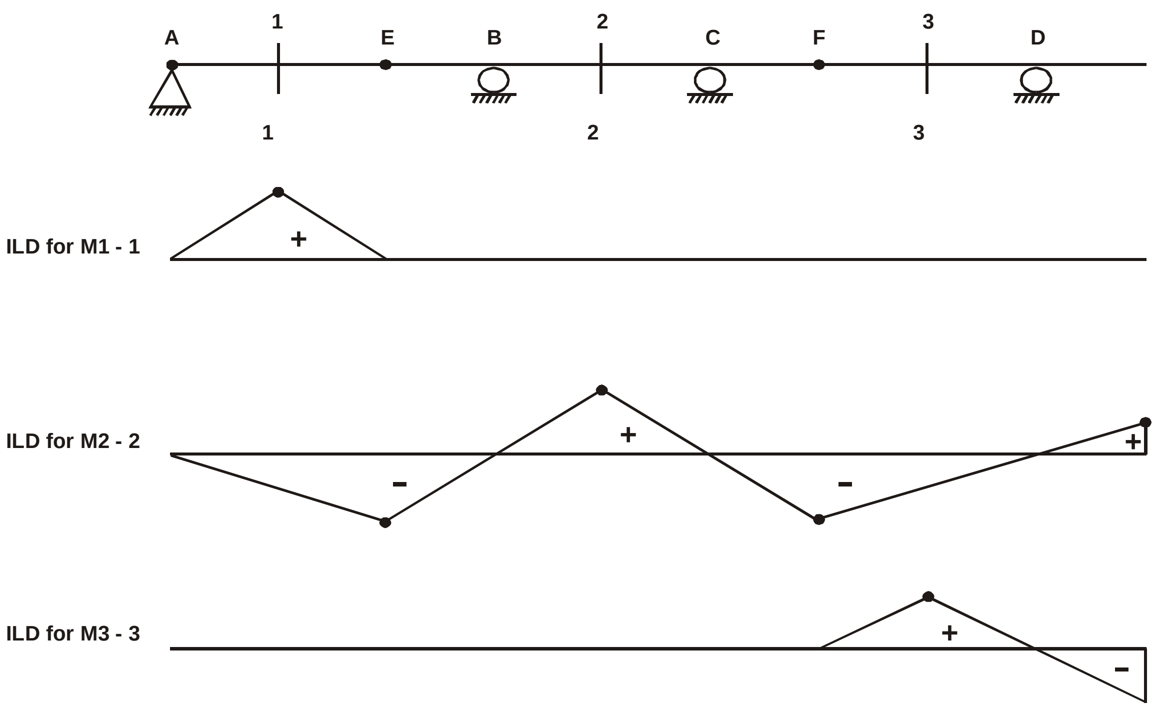
Or $M = \dfrac{Py}{\delta\theta}$

So, if $\delta\theta = 1$; the moment at Section C for a single load system will be load multiplied by corresponding influence ordinate (influence co-efficient) while constructing ILD's by virtual displacements, loads are not considered. Now construct ILD for Mc by virtual displacements.

At Section C, we imagine that the beam resistance to moments which produce rotations has been destroyed while resistance to shear and axial loads is intact. This situation is obtained by considering that at Section C; there is a sort of hinge (one degree of freedom system). On this hinge the moments are applied on two sides of hinge as shown alone. The segments of beam rotate and the displaced beam position is ILD for Mc.

The one-degree of freedom system such as a hinge is further explained in diagrams shown which illustrate the movement. This procedure can now be applied to more complicated cases where statical approach may be laborious.

The method of virtual displacements can be applied to more complicated cases like compound beams etc., by considering the basic ideas established in this chapter.



10.7. Evaluation of M_{max} at a Section

In case of a simple beam supporting a moving load system, the maximum moment at a section is obtained when

1. One of the loads is at the section.

2. In case of several moving loads, that load shall be placed at the Section, for producing maximum moment at that Section, which reverses the average loading on two portions of span adjacent to Section.

$$\text{Average loading on any portion} = \frac{\text{sum of all loads on that portion}}{\text{length of portion}}$$

10.8. Absolute Maximum bending Moment

In case of a series of moving loads traverse on a beam, the absolute maximum bending moment occurs near the mid span under the adjusted position of that load which gave us maximum bending moment at mid span. Procedure is as follows:

1. Apply the criteria of maximum bending moment at mind span to find the load which is to be placed at mid span.

2. For this position of loads find the position of resultant of all loads on span.

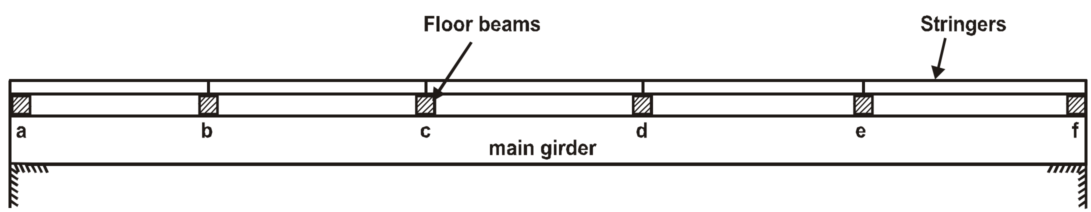
3. Move the system slightly so that mid-span is bisected by the resultant of all loads on span and the load which gives us maximum bending moment at mid-span.

4. Find absolute maximum bending moment. It will occur under displaced position of that load which gave us maximum bending moment at mid-span.

Considering that invariably loads would be magnified for design purpose and appreciating that the numerical difference between the values of maximum mid-span bending moment and absolute maximum bending moment is insignificant, evaluation of absolute maximum bending moment for a given moving load system appears to be of theoretical interest only. How interested students can evaluate it for only moving load system by considering the above four points and guidelines contained in this chapter.

10.9. Girders with Floor beams (Panelled girders)

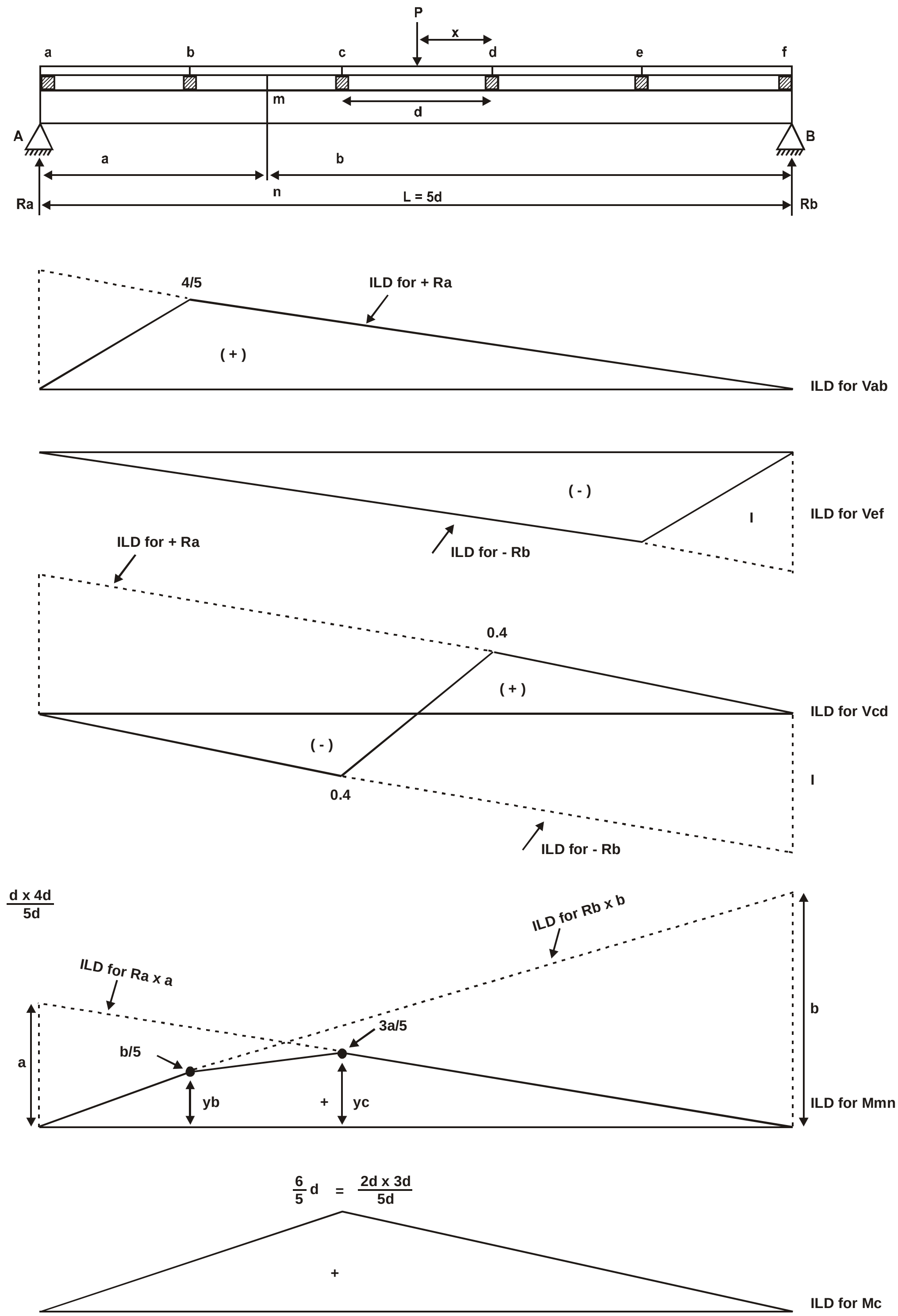
Normally in bridge construction, moving loads are hardly applied to the main girders directly but instead following arrangement is used for the load transfer.



The moving load system comes on the stringers which transfer it to the main girder through floor beams in form of concentrated loads (Reactions of floor beams). So main girder is subjected to concentrated loads only. For large spans the main girder may be of steel, poured in-situ reinforced concrete or pre-stressed concrete. Points a, b, c, … F are called panel points and the distance between any two panel points is called a panel.

With the above mentioned load-transfer mechanisms, it can be easily seen that ILD' s for main reactions remain same as that for a simple beam as discussed already.

As there will be no load on the main girder except floor beam reactions, it is stated that for a given load position, the shear force within a panel remains constant so we can talk of shear force in panels rather that shear force at a section (panel and becomes *a* section). Let us now construct ILD' s for shear force for various panels of girder already shown.



A five panels main girder is shown for which various ILD' s have been sketched.

10.10. ILD For Vab (ILD for shear in end panel)

If a load P is placed at a distance X from panel point b, then reactions at panel points a and b will be $\frac{PX}{d}$ and $\frac{P(d-X)}{d}$ respectively.

P_a = Panel point load at a or reaction of floor beam at a = $\frac{Px}{d}$, $0 < X < d$

P_b = Panel poiint load at b or reaction of floor beam at b = $\frac{P(d-X)}{d}$ $0 < X < d$.

if X = 0, load P will be at b, then $P_a = 0$ and $P_b = P$

if X = d, load P will be at a, then $P_a = P$ and $P_b = 0$ So, $V_{ab} \equiv 0$

In between a and b, shear force varies linearly.

Now inspect the shape of ILD for V_{ab}, it resembles with the shape of ILD for moment at point b considering the panelled girder as a simple beam. So to evaluate $(V_{ab})_{max}$, criteria of max bending moment at a section b (reversal of average loading expression) will be applied.

10.11. ILD for V_{ef} (ILD for shear in other end panel)

The construction of ILD for V_{ef} is same as that for V_{ab} and same arguments apply. Inspecting this diagram, it is clear that the shape resembles with ILD for bending moment at e if panelled girder was treated as a simple beam. So to evaluate $(V_{ef})_{max}$, the criteria for maximum bending at point e shall be applied.

10.12. ILD for V_{ed} (ILD for shear in intermediate panel)

Considering the load P on panel cd acting at a distance X from panel point d.

P_d = Panel point load at d or floor-beam reaction at d = $\frac{P(d-X)}{d}$, $0 < X < d$.

P_c = Panel point load at c or floor-beam reaction at c = $\frac{P(X)}{d}$, $0 < X < d$.

If load is to right of d; $V_{cd} = +$ Ra So, ILD for V_{cd} for this region will be the same as that for Ra. If load is to left of C, $V_{cd} = -$ Rb. So for this region shape of ILD for V_{cd} will be the same as the shape of ILD for – Rb. Now third possibility is load actinig on span CD itself as shown.

Inspecting the expressions for panel point loads at d and c stated above, we observe that the shear Vcd within the panel varies linearly. So joining the ordinates under points C and D by a straight line will complete ILD for V_{cd}.

10.13. Evaluation of $(V_{cd})_{max}$ (Maximum shear force in intermediate panel)

If a moving load is advanced at point d in a direction from right to left, considering $W^{/}$ is resultant of all loads on span CD, the following criteria can be easily developed as a consequence of variation of shear force is panel CD due to an advance.

$$\frac{W}{L} > \frac{W^{/}}{d}$$

Any load which reverses the above criteria shall give $(V_{cd})_{max}$.

10.14. ILD for M_{mn}

Section mn is located within panel bc. Same technique can be applied for constructing ILD for M_{mn}. If load P is to right of panel point C.

$$M_{mn} = R_a \times a.$$

It means that if load is between points c and f, the shape of ILD for M_{mn} will be the same as shape of ILD for Ra multiplied by a. If load P is to left of panel point b, then.

$$M_{mn} = R_b \times b.$$

It means that if load is between points a and b, then shape of ILD for M_{mn} will be the same as shape of ILD for Rb multiplied by b. Now consider load within panel bc with P acting at a distance X from c.

$$P_b = \frac{PX}{d} \text{ and } P_c = \frac{P(d - X)}{d} \qquad 0 < X < d.$$

$$\text{then } M_{mn} = P_b y_b + P_c y_c = \frac{PX}{d} y_b + \frac{P(d - X)}{d} y_c \qquad 0 < X < d.$$

So between the panel, the moment varies linearly. Therefore joing the ordinates of ILD for M_{mn} at b and c by a straight line, we complete the ILD for M_{mn}.

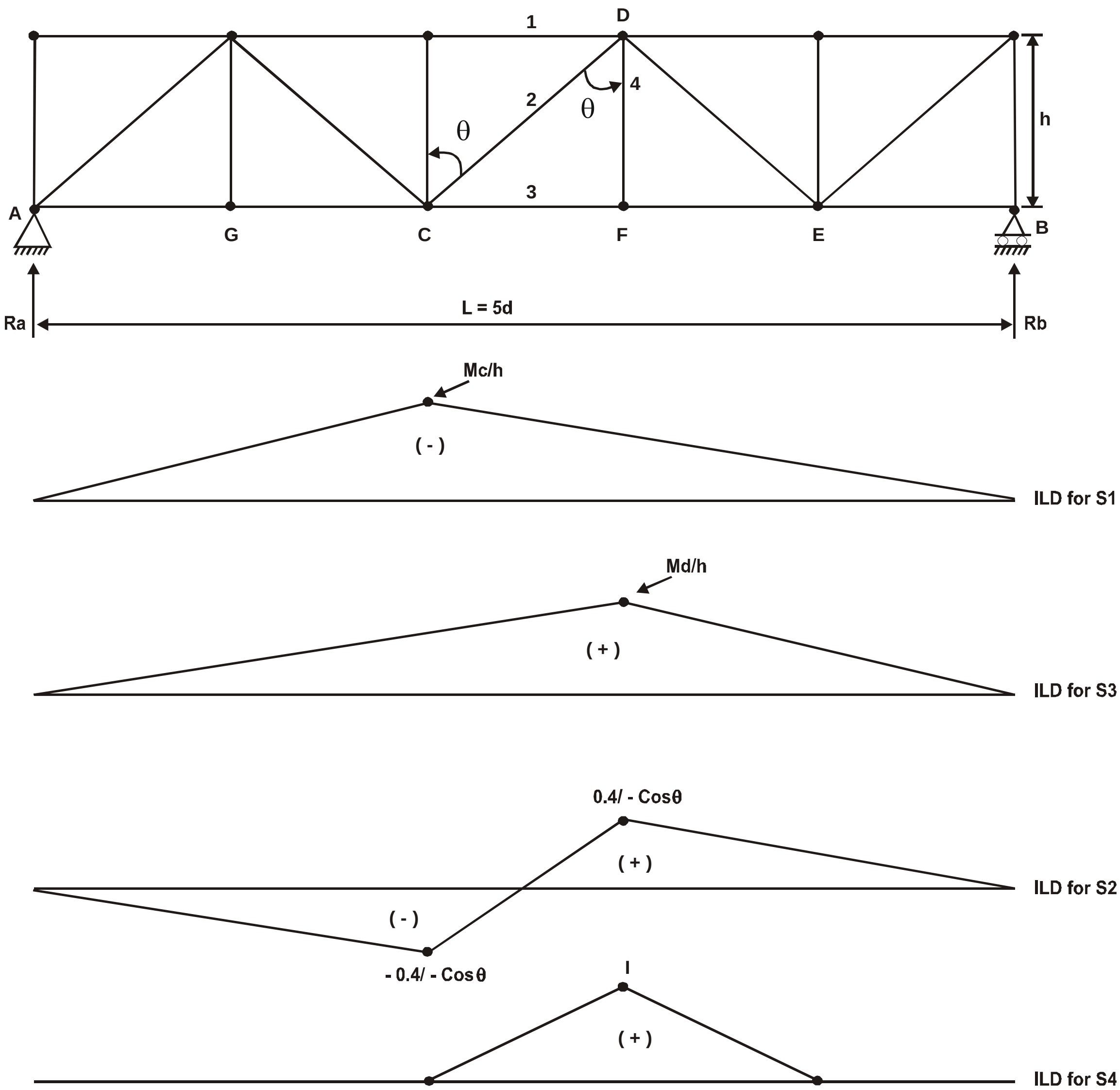
Now it is understood that SF is generally maximum near the support while moment is generally maximum near the mid-span. So ILD for M_{mn} can also be used to evaluate corresponding maxima. If criteria of maximum bending moment is applied at a section corresponding to bigger ordinate, then $(M_{mn})_{max}$ can be calculated for a moving load system.

10.15. ILD for Mc

At the panel points, the load is directly transmitted to the main girder and the panel girder behaves as a simple beam at the panel points. So ILD for Mc will be drawn considering the girder as a simple beam.

10.16. Influence Lines for axial forces in Truss Members:

As before, let us consider a simple case of parallel chord truss carrying loads at its lower chord. The conclusions obtained are general and can be extended to non-parallel chord trusses.



ILD for S_1

When a moving load system traverses the bottom chord of this trussed bridge, it is known that forces in top chord members will be compressive in nature while that in bottom chord will be tensile in nature. The forces in chord members are a function of moment divided by truss height. For a chord member take " moment at the point where other two members completing the same triangle meet divided by height of truss." This has already been established in this book when discussing method of moments and shears. So applying this S_1 is a compressive force, so assigned a negative sign, equal to moment at C divided by the height of truss. So considering the truss as a simple beam, draw an ILD for Mc and divide it by the height of Truss. $(S_1)_{max}$ can be evalutated by applying the criteria of maximum bending moment (Average loadings) at point C considering the truss as a simple beam.

ILD for S_3

It is a tensile force equal to moment at D divided by height of Truss. $(S_3)_{max}$ can be evalauted by applying the criteria of maximum bending moment at point D.

ILD for S_2

It is known that axial force in an inclined member is $\frac{\pm V}{\pm \cos\theta}$. Minus before $\cos\theta$ shall be taken if the angle " between inclined member and vertical" is counterclockwise. Now if the load is right of D, SF applicable to member 2 is + Ra. So corresponding portion of ILD for + Ra is taken. This is divided by $-\cos\theta$. If the load is to left of C, SF applicable to member 2 is – Rb. So corresponding portion of ILD for – Rb is taken. This is again divided by $-\cos\theta$. In between the panel SF varies linearly so we can join the corresponding points.

The shape of ILD for S_2 resembles with the shape of ILD for intermediate panel shear in a panelled girder. So $(S_2)_{max}$ can be evaluated by applying the criteria of maximum intermediate panel shear.

ILD for S_4

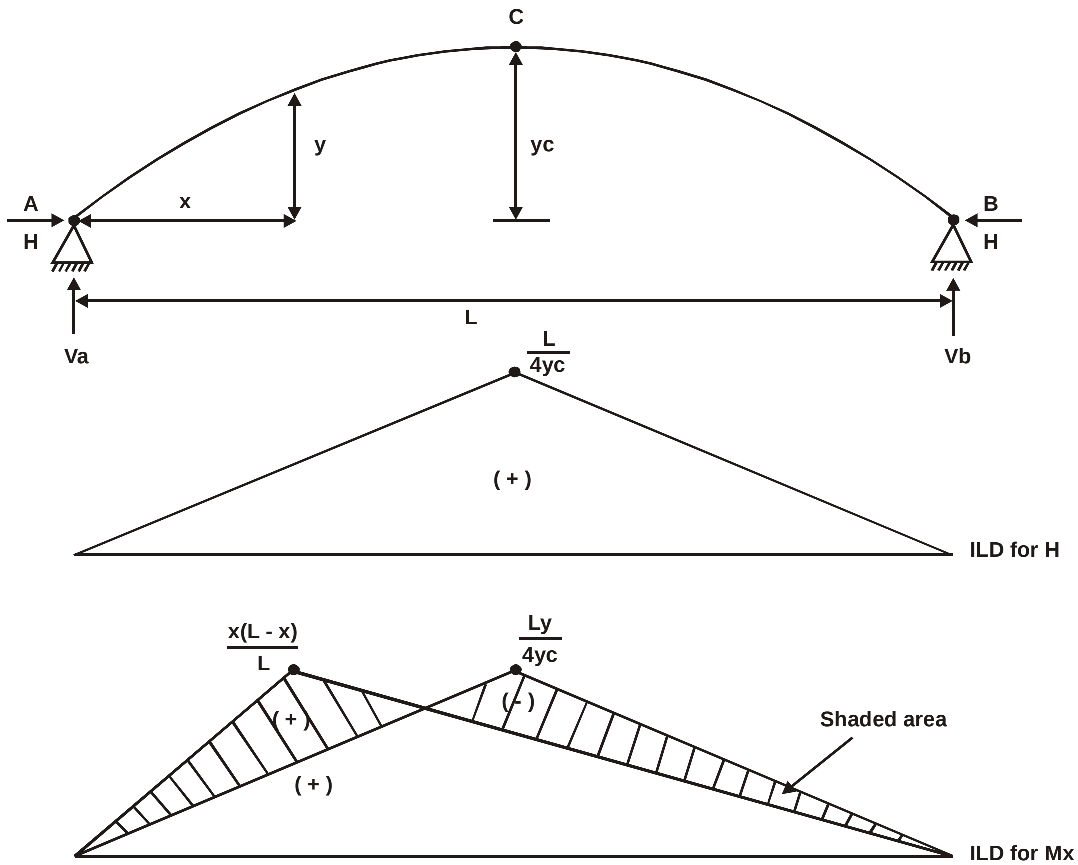
If the load is at E or right of E, Force in member 4 is zero and if load is at or to left of point C, again the force in member 4 is zero. If the load is at F, the same will be the tensile force in member. Using these boundary conditions, ILD for S_4 is constructed. Now inspect its shape. It resembles with the shape of ILD for moment at F (or D) in an equivalent simple beam of span CE. So $(S_4)_{max}$ can be evaluated by applying the criteria of maximum bending moment (average loading criteria) at F (or D).

10.17. Influence lines for moment and horizontal thrust in a three hinged arch.

We know that $H = \frac{\mu c}{yc}$ and

$Mx = \mu x - Hy.$

Where y will be the rise of arch at a distance X from origin (usually a support).



Influence line for any structural effect can be drawn by following the formula for that structural effect.

10.17.1. ILD for horizontal thrust H

Horizontal thrust H is developed at the springings (supports) of an arch. Examine the formula for H $\left(H = \frac{\mu c}{yc}\right)$. So ILD for H will be obtained if ILD for moment at centre is drawn, considering the arch to be a simple bam, and is then divided by yc. The peak ordinate of ILD for H will be $\frac{L}{\mu yc}$. $(H)_{max}$ due to a moving load system can be obtained by applynig the criteria of maximum bending moment at the centre.

10.17.2. ILD for Moment in the arch

From the Eddy' s theorem we know that bending moment in the arch at a distance x from support is

$$M_x = \mu_x - Hy$$

where μ_x = simple span bending moment at a distance X.

So as a first step, we construct ILD for simple span bending moment at a distance X. Then we subtract the ILD for Hy. The net area between these two diagrams is the ILD for moment in the arch as shown.

10.18. Standard Leadings

For the design of Railway bridges standard Cooper' s E-60 and E-72 loadings consisting of two locomotives each weighing 213 tons on 18 axles each followed by infinite udl representing compartments is considered. Structural affects obtained for a E loading can be used to get the same for another E loading by simply multiplying them with the ratio of E loadings.

Original E-60 or E-72 loadings are in kip-ft. system as follows:

15 30 30 30 30 | 4 of 19.5 | 15 | 4 of 30 | 4 of 19.5 | 3/ft

↓ 8′ ↓ 5′ ↓ 5′ ↓ 5′ ↓ 9′ ↓ 5 ↓ 6 ↓ 5 ↓ 8′ ↓ 8′ ↓ 5′ ↓ 5′ ↓ 5′ ↓ 9′ ↓ 5′ ↓ 6′ ↓ 5′ ↓ 5′

Above wheel loads are in kips per rail or tonnes per track. (1 Ton = 2 Kips ; small ton)

Converting E-72 loading in SI Units we have IK = 5 KN approximately.

80 KN | 4 of 160 KN | 4 of 104 KN | 80

↓ 2.44 ↓ 1.52 ↓ 1.52 ↓ 1.52 ↓ 1.52 ↓ 2.74 ↓ 1.52 ↓ 1.83 ↓ 1.52 ↓ 2.44 ↓ 2.44

4 of 104 KN | 4 of 104 KN | 53 KN/m

↓ 1.52 ↓ 1.52 ↓ 1.52 ↓ 2.42 ↓ 1.52 ↓ 1.83 ↓ 1.52 ↓ 1.52

Cooper' s E-72 loading in SI-units is shown above and E-60 below:

66.75 | 4 of 133.5 KN | 4 of 86.77 KN | 66.75

↓ 2.44 ↓ 1.52 ↓ 1.52 ↓ 1.52 ↓ 2.74 ↓ 1.52 ↓ 1.83 ↓ 1.52 ↓ 2.44 ↓ 2.44

4 of 133.5 KN | 4 of 86.77 KN | 43.8 KN/m

↓ 1.52 ↓ 1.52 ↓ 1.52 ↓ 2.74 ↓ 1.52 ↓ 1.83 ↓ 1.52 ↓ 1.52

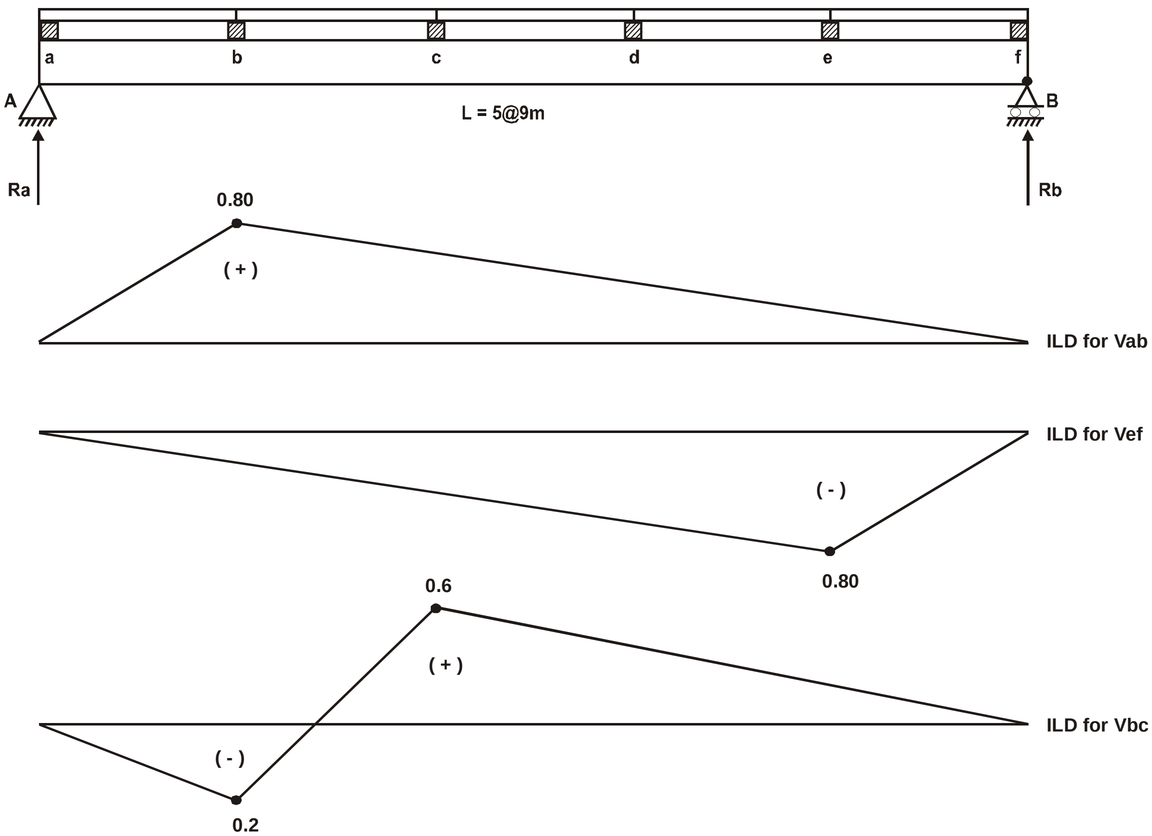
Distance between loads is in meters.

For highway bridges AASHTO HS24-44 loading is internationally considered and it consists of a Tractor and Semi-trailer with three axles carrying 0.2W, 0.4W and 0.4W respectively. These loads when converted into kips are 16k, 32k and 32k. Standard AASHTO lane loading is probably 100 lbs/ft^2.

However, in our country, due to circumstances 70 ton tank loading or Truck-train loading described in Pakistan Highway code can be used.

We shall use railway loadings only. Let us solve some typical problems now.

Example No.1: In a girder with Floor beams having five equal panels of length 9 meters each. Determine (a) Maximum positive and negative end panel shears. (b) Maximum Shear in the first intermediate panel from left hand end. The live load is Coopers E-72 loading.

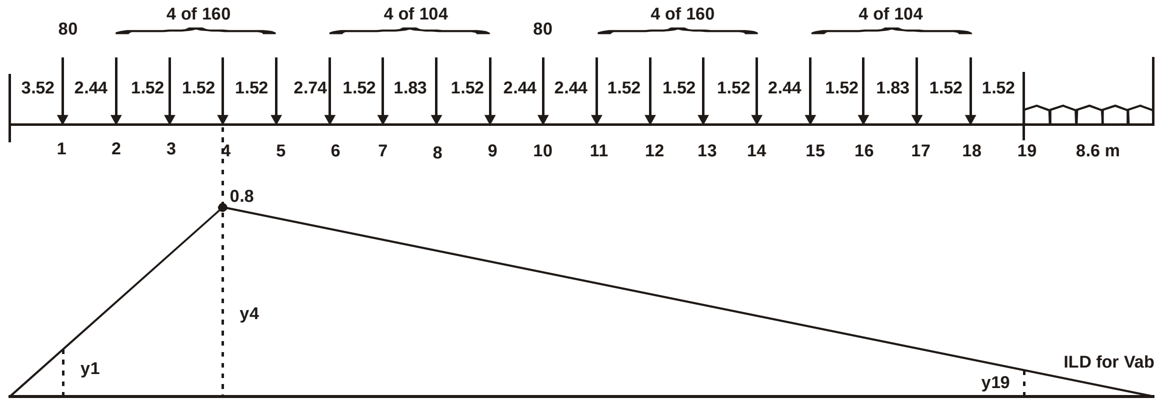


SOLUTION: 1. Maximum positive End Panel Shears $(V_{ab})_{max}$

Advance loads at section B and use criteria $\frac{W^{/}}{d} < \frac{W}{L}$

Portion ab		Portion bf	
$\frac{80}{9}$	<	$\frac{2498.87}{45}$	after 1st advance.
$\frac{240}{9}$	<	$\frac{2338.87}{45}$	after 2nd advance
$\frac{400}{9}$	<	$\frac{2178.87}{45}$	after 3rd advance
$\frac{560}{9}$	>	$\frac{2018.87}{45}$	after 4th advance.

It means that once 3rd load of 160 KN crosses point b, the criterion is reversed so for maximum end panel shear, 3rd load of 160 KN should be placed at point b. Now place the system of loads accordingly and compute corresponding ordinates.



Ordinates Under Loads:

$y_1 = 0.3128$ $y_2 = 0.5297$ $y_3 = 0.6648$

$y_4 = 0.80$ $y_5 = 0.766$ $y_6 = 0.7053$

$y_7 = 0.6715$ $y_8 = 0.6308$ $y_9 = 0.597$

$y_{10} = 0.5428$ $y_{11} = 0.488$ $y_{12} = 0.4548$

$y_{13} = 0.421$ $y_{14} = 0.387$ $y_{15} = 0.333$

$y_{16} = 0.299$ $y_{17} = 0.2586$ $y_{18} = 0.2248$

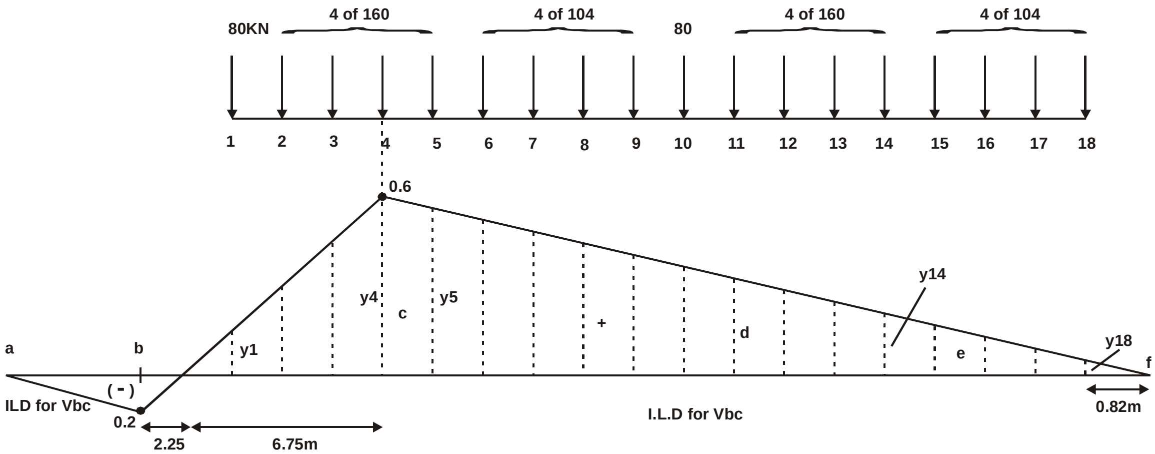
$y_{19} = 0.191$

$$
\begin{aligned}
(Vab)_{max} = {} & 80 \times 0.3128 + 160\,(0.5297 + 0.6648 + 0.8 + 0.766) \\
& + 104\,(0.7053 + 0.6715 + 0.6308 + 0.597) \\
& + 80 \times 0.5428 + 160\,(0.488 + 0.4548 + 0.421 + 0.387) \\
& + 104\,(0.333 + 0.299 + 0.2586 + 0.2248) + \frac{1}{2} \times 8.6 \times 0.191 \times 53 \\
= {} & 25.024 + 441.68 + 271.62 + 43.42 + 280.128 + 116 + 43.52 \\
= {} & 1221.4 \text{ KN.}
\end{aligned}
$$

Similarly $(Vef)_{max} = -1145$ KN (Do the Process yourself)

We have to observe a similar Process for evaluation of $(Vef)_{max}$ as was used for $(Vab)_{max}$. The loads will be advanced at point e and average loadings on portions ae and ef will be compared. The load which produces reversal after advance should be brought back and placed at section e for $(V_{ef})_{max}$.

Evaluation of $(V_{bc})_{max}$



Once loads are advanced from right to left at C, the following criteria shall be used to evaluate maximum intermediate panel shear $(V_{bc})_{max}$

$$\frac{W}{L} > \frac{W'}{d}$$

Portion bc		portion cf	
$\frac{80}{9}$	<	$\frac{2064}{45}$	after 1st advance.
$\frac{240}{9}$	<	$\frac{2168}{45}$	after 2nd advance
$\frac{400}{9}$	<	$\frac{2272}{45}$	after 3rd advance
$\frac{560}{9}$	>	$\frac{2315.46}{45}$	after 4th advance.

So maximum positive SF in panel bc will be obtained when 3rd wheel of 160 KN is placed at point c. Now place loads as shown above and determine corresponding ordinates of ILD. Multiply loads and ordinates by giving due care to signs of ILD, we obtain $(V_{bc})_{max}$.

Now from similar triangles, influence co-efficients $y_1, \ldots\ldots y_{18}$ are:

$y_1 = 0.113$	$y_2 = 0.33$	$y_3 = 0.465$
$y_4 = 0.6$	$y_5 = 0.566$	$y_6 = 0.505$
$y_7 = 0.472$	$y_8 = 0.431$	$y_9 = 0.397$
$y_{10} = 0.343$	$y_{11} = 0.289$	$y_{12} = 0.255$
$y_{13} = 0.221$	$y_{14} = 0.187$	$y_{15} = 0.126$

$y_{16} = 0.093$ $y_{17} = 0.052$ $y_{18} = 0.018$

So, $(Vbc)_{max} = 80 \times 0.113 + 160 (0.33 + 465 + 0.6 + 0.566)$

$+ 104 (0.505 + 0.472 + 0.431 + 0.397) + 80 \times 0.343$

$+ 160 (0.289 + 0.255 + 0.221 + 0.187)$

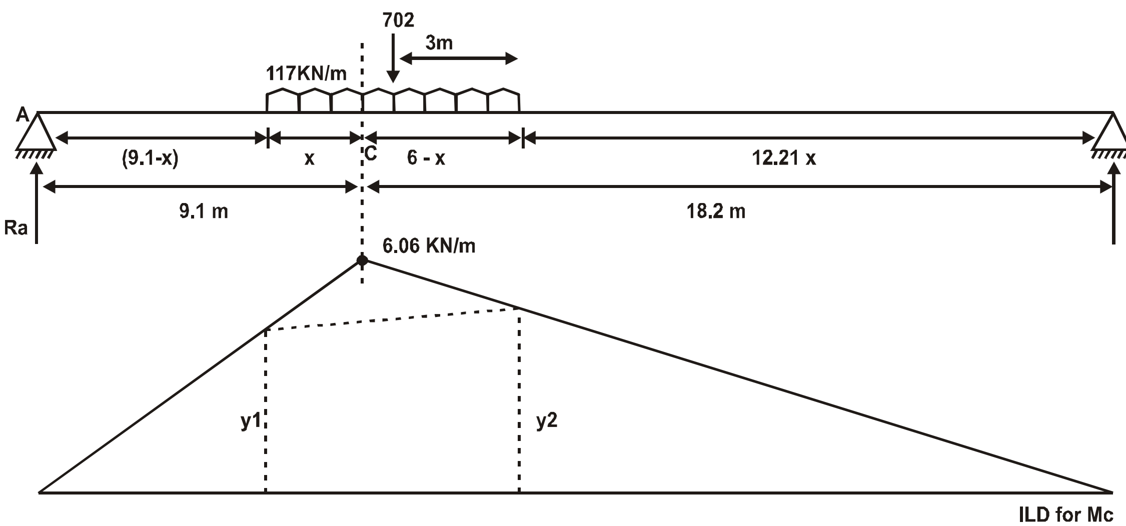
$+ 104 (0.126 + 0.093 + 0.052 + 0.018)$

$(Vbc)_{max} = 720.34$ KN

EXAMPLE NO.2: Determine the maximum bending moment at a cross-section 9.1m from left hand for a beam of span 27.3m. The moving live load is 117 KN/m having a length of 6m.

SOLUTION:

Sketch ILD for moment at the indicated section.



Now let us assume that the given position of Udl gives us$(Mc)_{max}$ at a distance X from C as shown. Determine Ra for this position

$\sum Mb = 0$

$Ra \times 27.3 = 702 (3 + 12.2 + X)$

$Ra = 390.84 + 25.71\, X$

Moment at C $= Mc = Ra \times 9.1 - \frac{117\, X^2}{2}$

$$Mc = (390.84 + 25.71)\, 9.1 - \frac{117\, X^2}{2}$$

Simplify

$$Mc = 3556.64 + 233.96\, X - 58.5\, X^2$$

If BM at C is maximum, then

$$\frac{d\, Mc}{dX} = Vc = 0$$

$$233.96\, X - 2 \times 58.5\, X = 0$$

$$X = 2m$$

Now compute y1 and y2 from similar triangles of ILD

$$\frac{18.2}{27.3} = \frac{y_1}{7.1} \rightarrow y_1 = 4.733\ m$$

$$\frac{9.1}{27.3} = \frac{y_2}{14.2} \rightarrow y_2 = 4.733\ m$$

So $(Mc)_{max}$ = ud.l × area of ILD Under UDL

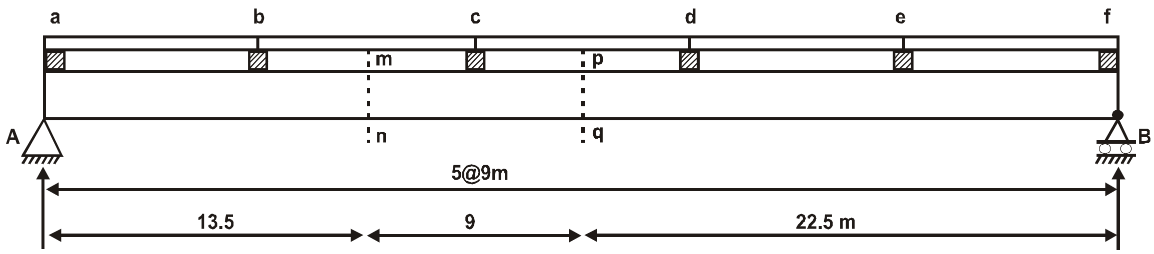
$$= 117\left(6 \times 4.733 + \frac{1}{2} \times 6 \times 1.327\right)$$

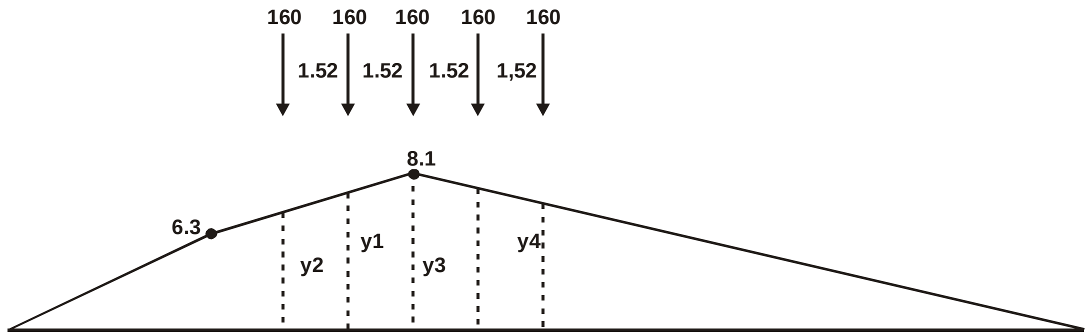
$$= 3788.3\ \text{KN-m}$$

EXAMPLE NO. 3:

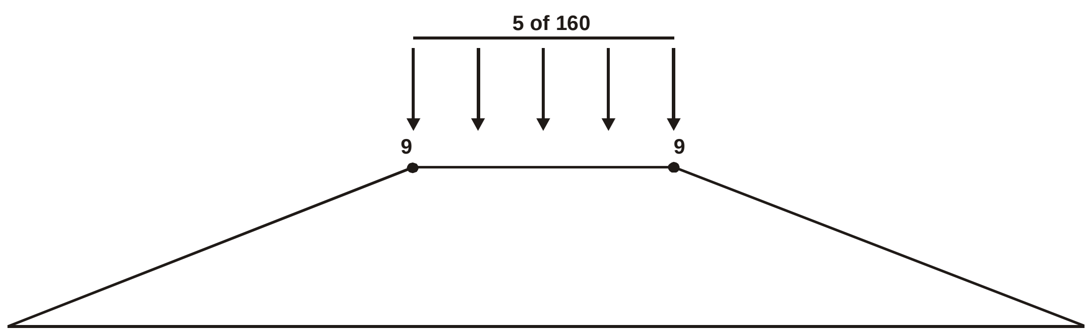
Calculate maximum bending moment at Section mn and pq of a five panel bridge. Each panel is of 9m.

Five loads of 160 KN each spaced at 1.52m travel from right to left.





ILD for Mmn



ILD for Mpq

Evaluation of $(Mmn)_{max}$

It is recommended that criteria of maximum bending moment be applied at maximum ordinate of 8.1 corresponding to Panel point C. Now comparing average loadings on portion ac and cf, we find that 3rd load reverses the criterion as it crosses. So it must be placed at point C. Determine ordinates

$$\frac{8.1}{27} = \frac{y_3}{25.48} \rightarrow y_3 = 7.644, \quad y_4 = 7.188, \qquad y_1 = 6.3 + 1.496 = 7.796$$

$$y_2 = 6.3 + 1.192 = 7.492$$

$$(M_{mn})_{max} = 160\,(7.492 + 7.796 + 8.1 + 7.644 + 7.188) = 38.22 \times 160$$

$$= 6115.2 \text{ KN-m}$$

The reader is also suggested to calculate $(Mmn)_{max}$. by coinciding the resultant of moving load system with the maximum ordinate. Place the loads accordingly. Compute influence co-efficients and multiply loads with respective ordinates to compute $(Mmn)_{max}$. Compare this value with the previous one.

$(Mpq)_{max}$

As ILD for Mpq is symmetrical about centre-line (mid span), Arrange the loads such that the resultant falls on mid-span. All five loads shall be accommodated and will have an ordinate of 9.

$$(M_{pq})_{max} = 160\,(9 + 9 + 9 + 9 + 9) = 7200 \text{ KN-m}$$

Important:

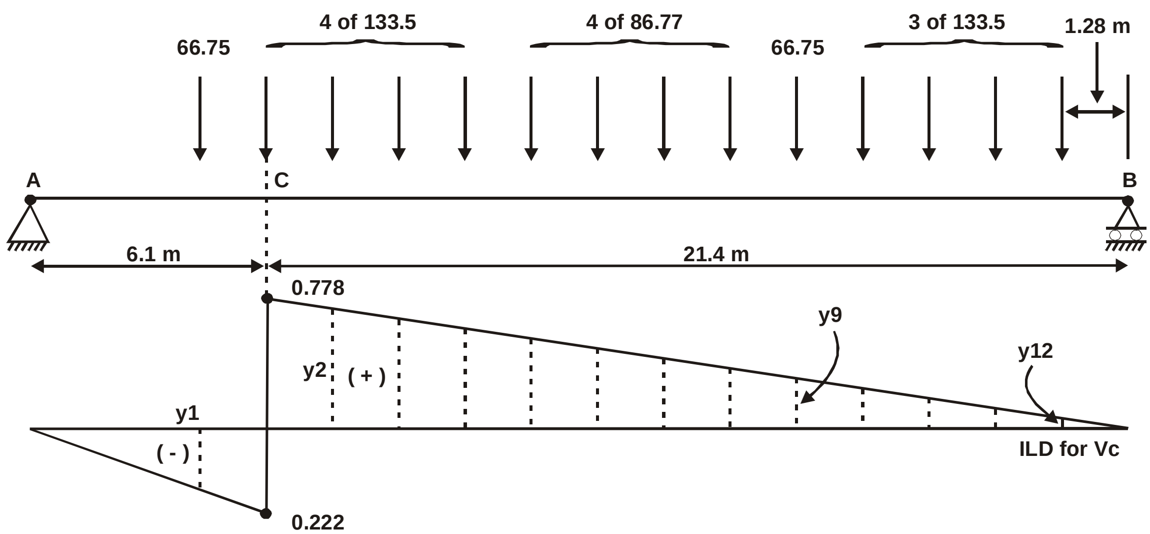
The instructor is advised to work with lesser number of loads, usually five to seven, in the class and Establish the procedure. The students can then be given assignments involvinig standard trains etc., for clarification of their concepts.

EXAMPLE NO. 4:

A simple beam has a clear span of 27.5 m. Construct ILD for SF at a section 6.1m from left support. How should Coopers-E-60 loading be placed to calculate maximum shear force at this section?

SOLUTION:

Draw ILD for Vc. Advance the loads at section C. We shall show the load position required for $(Vc)_{max}$ only.



Computing influence co-efficients y_1……y_{12} from similar triangles.

$y_1 = -0.133$, $y_2 = 0.722$, $y_3 = 0.667$,

$y_4 = 0.612$, $y_5 = 0.512$ $y_6 = 0.4566$,

$y_7 = 0.3901$, $y_8 = 0.335$, $y_9 = 0.246$,

$y_{10} = 0.157$ $y_{11} = 0.10$, $y_{12} = 0.0466$

In order to have $(Vc)_{max}$, at least one load should be at C. To decide which load should be placed at C, reversal in the sign of following equation is sought.

$$\Delta V = \frac{Wa}{L} - Pn$$

W = Sum of all the loads on span before advance.

a = any particular Advance

L = Span

Pn = magnitude of Load crossing the section due to an advance.

— For the first advance

$$\Delta V = \frac{1281.58 \times 2.44}{27.5} - 66.75 = +46.96 \text{ KN.}$$

It shows that SF at C has increased due to 1st advance.

— For second advance.

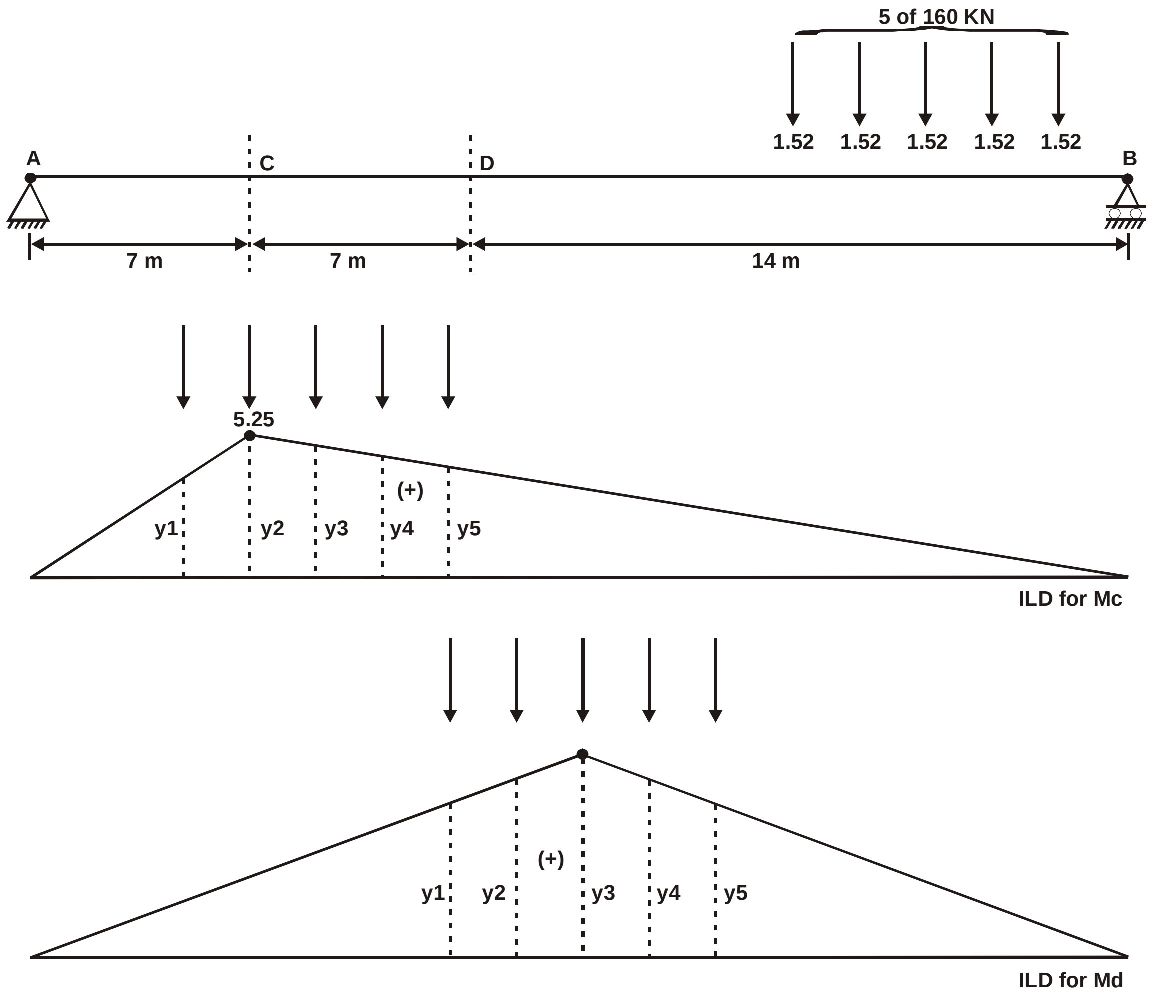
$$\Delta V = \frac{1415 \times 1.524}{27.5} - 133.5 = -55.08 \text{ KN.}$$

It shows that if second advance at C is made, Vc decreases. So for $(Vc)_{max}$, position before 2nd advance (after 1st advance) is required. For this position above influence co-efficients have been computed.

$$(Vc)_{max} = 66.75(-0.133) + 133.5(0.778 + 0.722 + 0.667 + 0.612)$$
$$+ 86.77(0.512 + 0.4566 + 0.3901 + 0.335)$$
$$+ 66.75(0.246) + 133.5(0.157 + 0.1 + 0.046)$$
$$= 567.37 \text{ KN}$$

EXAMPLE NO. 5:- Calculate the maximum bending moment at the points C and D if five loads of

160 KN each spaced at 1.52 m cross-the bean from right to left.



MC$_{Max}$

Line-up all loads upto point C (theoretically slightly to right of C). Give advances at point C and compare average loading in portion AC and BC due to various advances.

Portion Ac		Portion Bc	
$\frac{160}{7}$	$<$	$\frac{4 \times 160}{21}$	after 1st advance.
$\frac{2 \times 160}{7}$	$>$	$\frac{3 \times 160}{21}$	after 2nd advance.

So, as the second load of 160 KN crosses ponit C, reversal is obtained. So for (Mc)$_{max}$, this load should be brought back and placed at C (position before 2nd advance or after 1st advance). Compute influence co-efficients.

$y_1 = 4.11$, $y_2 = 5.25$, $y_3 = 4.87$

$y_4 = 4.49$, $y_5 = 4.11$

$$(M_c)_{max} = 160\,(4.11 + 5.25 + 4.87 + 4.49 + 4.11) = 3652.8 \text{ KN-m}$$

$(M_d)_{max}$

This section is mid span of beam. Clearly applying the criteria of maximum bending moment at D (comparing Average loadings on AB and BD), we get

Span AD		Span BD	
$\frac{160}{14}$	<	$\frac{4 \times 160}{14}$	after 1st advance
$\frac{2 \times 160}{14}$	<	$\frac{3 \times 160}{14}$	after 2nd advance
$\frac{3 \times 160}{14}$	>	$\frac{2 \times 160}{14}$	after 3rd advance.

So position before 3rd advance (or after 2nd advance) will give us $(M_d)_{max}$. Place the loads accordingly and compute influence co-efficients.

$y_1 = y_5 = 5.48$ $y_2 = y_4 = 6.24$ $y_3 = 7$

$$\text{So, } (M_d)_{max} = 160\,(5.48 + 6.24 + 7 + 6.24 + 5.48)$$

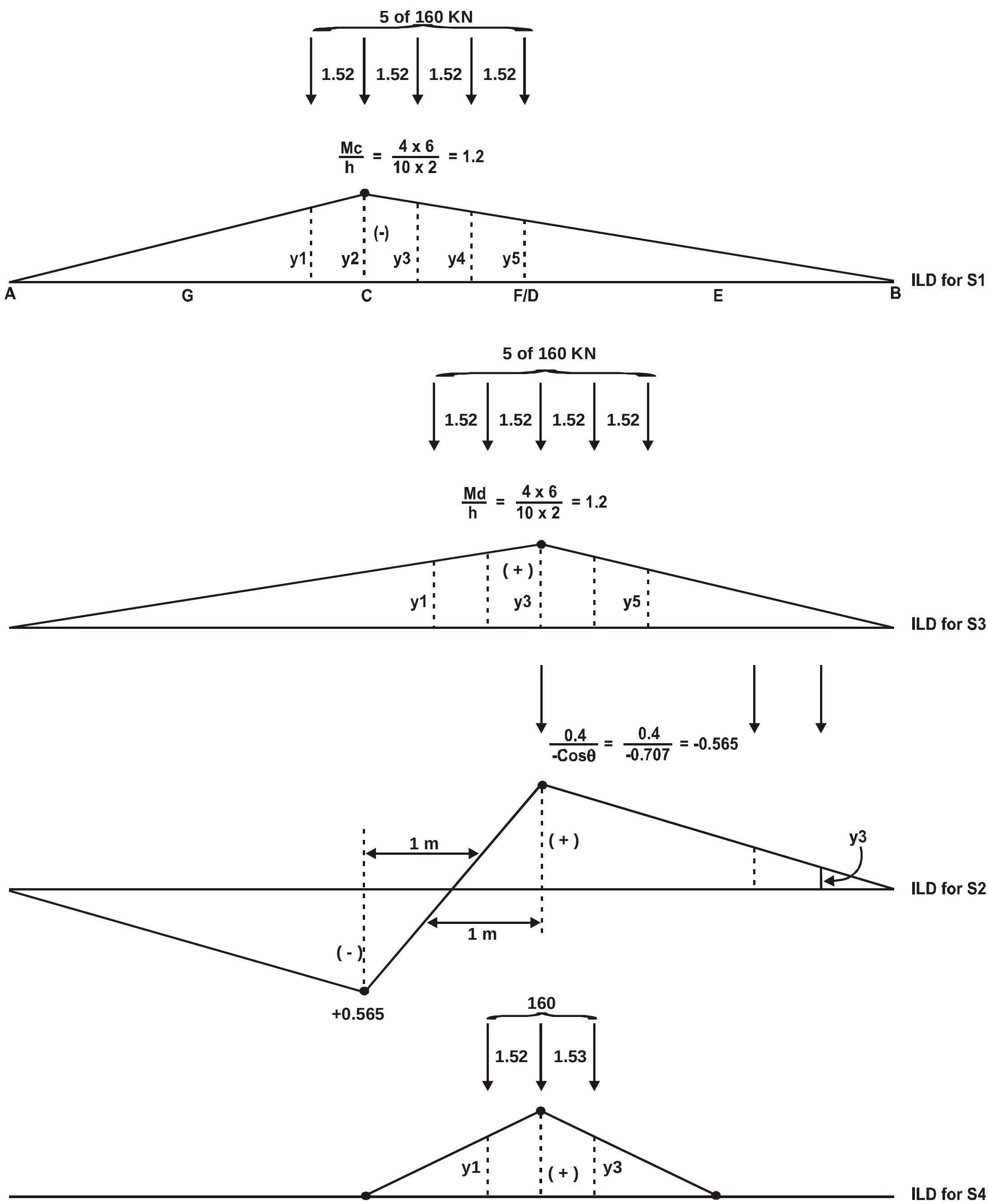
$$= 4870.4 \text{ KN-m}$$

EXAMPLE NO.6:

Calculate maximum axial forces induced in members 1, 2, 3 and 4 of truss already shown if five loads of 150 KN each spaced at 1.52m corsses at the bottom chord from right to left. Take h = 2m and span = 5d = 10 meters.

SOLUTION:

The corresponding ILD's for S_1.....S_4 have already been plotted. Now we will use those diagrams to calculate maxima. See the Truss of article 9.16.



$S1_{max}$.

The shape of ILD for S1 resembles with the shape of ILD for Mc in an equivalent simple beam. So giving advances at C (now forget the truss and play with ILD' s only). Apply the criterion for maximum moment at C.

Portion Ac		Portion Bc	
$\frac{160}{4}$	$<$	$\frac{4 \times 160}{6}$	after 1st advance.
$\frac{2 \times 160}{4}$	$=$	$\frac{3 \times 160}{6}$	after 2nd advance.

Considering equality as a reversal, $S1_{max}$ will be obtained for position before second advance (or after 1st advance). Place loads accordingly and compute influence co-efficients.

$y_1 = .744, \quad y_2 = 1.2 \quad y_3 = 0.896$

$y_4 = 0.592 \quad y_5 = 0.288$

So, $S1_{max} = 160\,(0.744 + 1.2 + 0.896 + 0.592 + 0.288)$

$= -595.2$ KN (It is a compressive force)

$S3_{max}$

Inspect the shape of ILD for S3. It resembles with the shape of ILD for moment at D considering the truss to be a simple beam. So apply the criterion of maximum moment at D.

Portion AD		Portion BD	
$\frac{160}{6}$	$<$	$\frac{3 \times 160}{4}$	(last load not on span) after 1st advance.
$\frac{2 \times 160}{6}$	$<$	$\frac{3 \times 160}{4}$	After 2nd advance.
$\frac{3 \times 160}{6}$	$=$	$\frac{2 \times 160}{4}$	After 3rd Advance.

So for $S3_{max}$, position before 3rd advance is valid (After second advance). Place the loads accordingly and compute influence co-efficients.

$y_1 = 0.592, \quad y_2 = 0.893, \quad y_3 = 1.2,$

$y_4 = 0.744, \quad y_5 = 0.288$

$(S3)_{max} = 160\,(0.592 + 0.893 + 1.2 + 0.744 + 0.288)$

$= 594.72$ KN (It is a tensile force).

$S2_{max}$

Inspect the shape of ILD for S2. It resembles with the shape of ILD for as shear force in a intermediate panel of a panelled girder. So for evaluating $S2_{max}$, we apply the criterion of maximum intermediate panel shear. Advance is made at D or F.

$$\frac{W'}{d} < \frac{W}{L}$$

$$\frac{160}{2} = \frac{5 \times 160}{10} \qquad \text{after 1st advance.}$$

So for $S2_{max}$, the leading load should be placed at maximum ordinate, only three loads will be acting on portion BD.

$y_1 = -0.565$ $y_2 = -0.3503$ $y_3 = -0.1356$

$(S2)_{max} = 160(-0.565 - 0.3503 - 0.1356)$

$= -168.144$ KN (It is a compressive force)

$S1_{max}$

$y_1 = y_3 = 0.24$ $y_1 = 1$

$S1_{max} = 160(0.24 + 1 + 0.24)$

$= 236.8$ KN (It is a tensile force)

10.19. Influence Lines for Statically Indeterminate Structures:

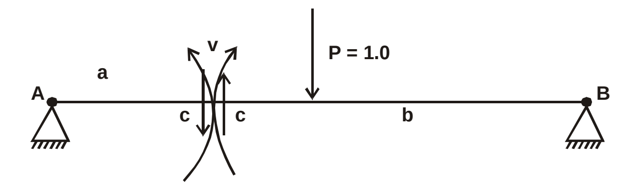
The same procedure can be adopted for constructing ILDs' for indeterminate structures. However, compatibility and redundants have to be considered as demonstrated earlier.

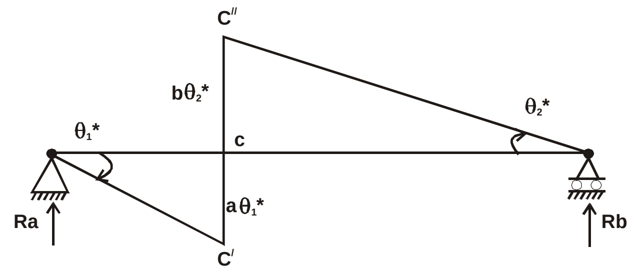
INFLUENCE LINE DIAGRAM FOR INDETERMINATE BEAMS (By method of virtual displacement)

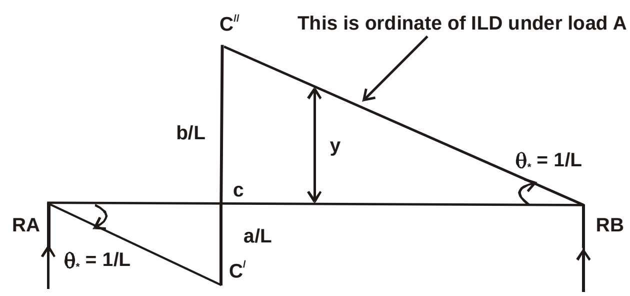
Influence line diagram for Shear.

In virtual work for shear the B.M. does not do any work only shear force does the work.

Case 1: Let us investigate ILD at a section of a simple beam. The section is at a distance a from A and at b from B support. This has already been done.







By Virtual Work:

Both the lines are parallel therefore, its work done by Moment is equal to zero.

$$\theta^*_1 = \theta^*_2 = \theta$$

$$Va\,\theta^* + Vb\,\theta^*$$

Virtual Work:
(Virtual displacement)

(i) total displacement equal to 1 unit.

$$a\theta^* + b\theta^* = 1$$

(ii) total B.M. equal to zero.

$$V(a\theta^* + b\theta^*) - M\theta^* + M\theta^* - Py^* = 0 \qquad \text{put } a\theta + b\theta = 1$$

$$V(1^*) - Py^* = 0$$

If we take $P = 1$

$$V = y^*$$

Or $\theta = \dfrac{1}{L}$

Case 2: I.L.D for bending moment at the same section. Write work equation and equate to zero.

$$M\theta^*_1 + M\theta^*_2 - Va\,\theta^*_1 + Vb\,\theta^*_2 - Py^* = 0$$

or $\quad M(\theta^*_1 + \theta^*_2) - 0 - Py^* = 0$

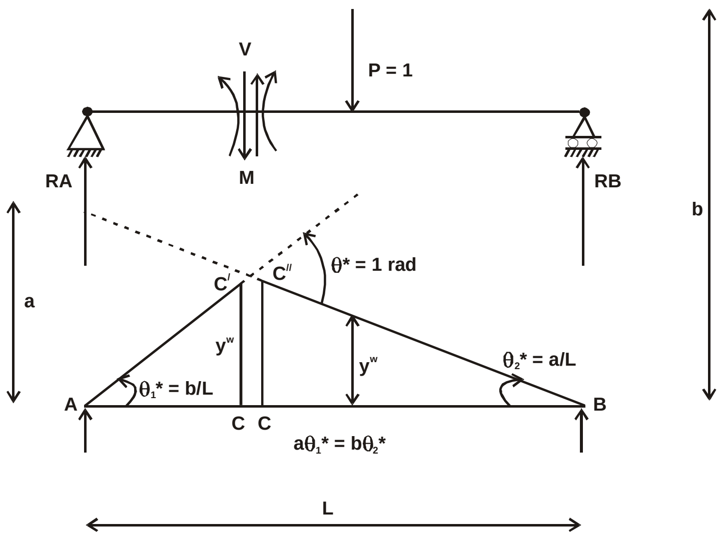
$\quad M(\theta^*) = Py^* \quad$ or $\quad M = \frac{Py}{\theta}$. If P = 1 and θ = 1 radian.

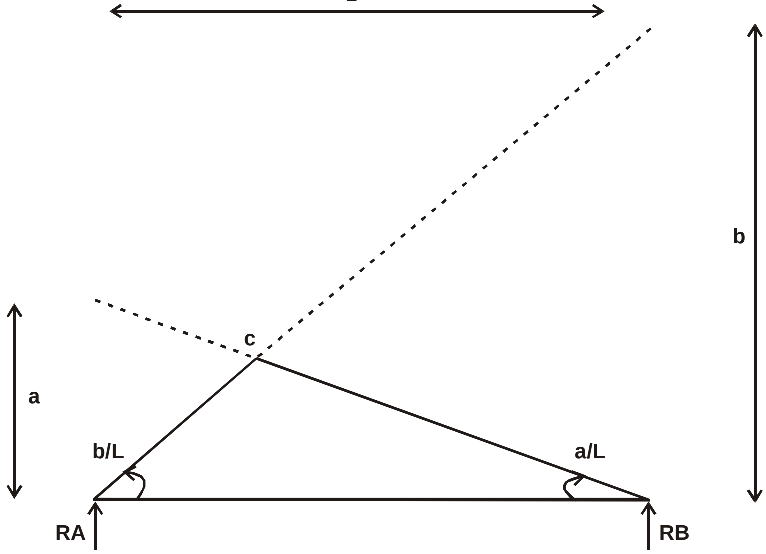
than $\quad M = y^*$

So $\quad a\theta^*_1 = b\theta^*_2 \qquad$ Or $\qquad \theta^*_1 + \theta^*_2 = 1$

$\Rightarrow \quad \theta^*_1 + \frac{a}{b}\theta^*_1 = 1 \qquad \Rightarrow \qquad \theta^*_1 = \frac{b}{L}$

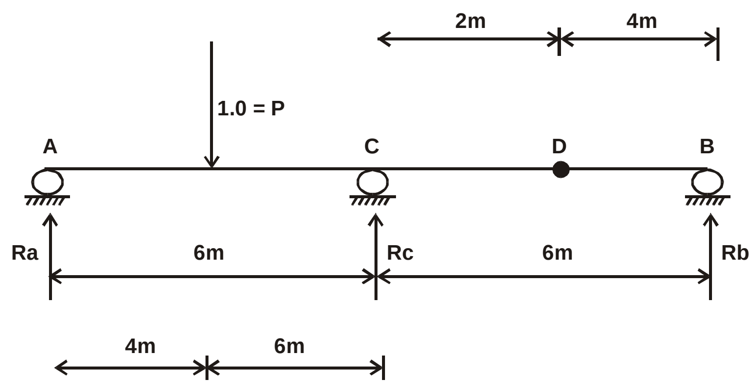
$\quad \theta^*_2 = \frac{a}{L}$



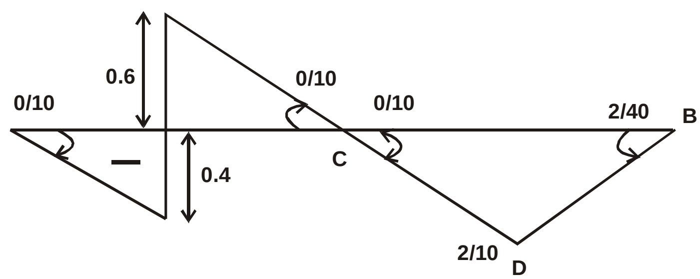


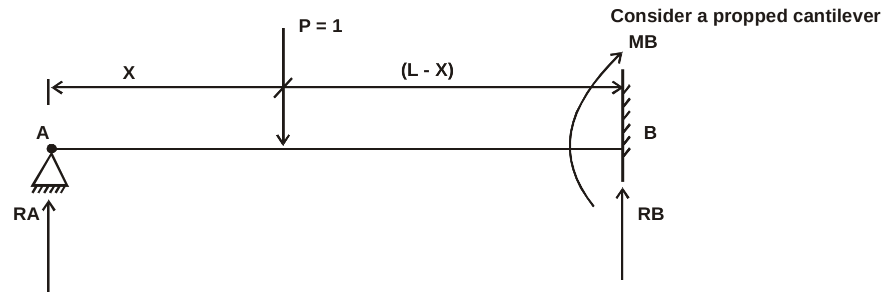
We have obtained ILD for B.M at X in a simple beam

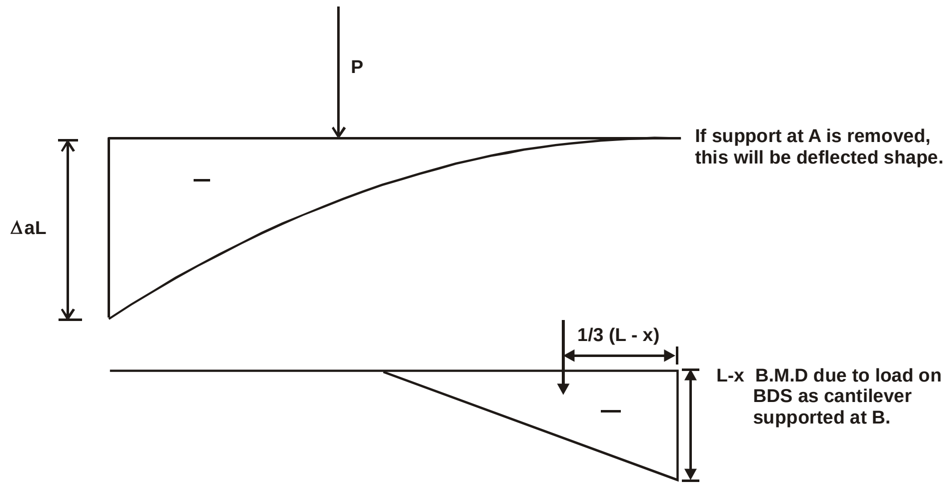
Let us now consider the shown conjugate beam.



Applying same concepts we get following ILD

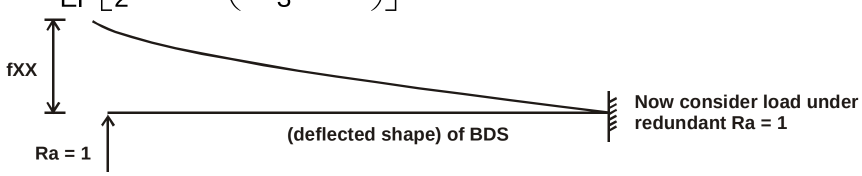


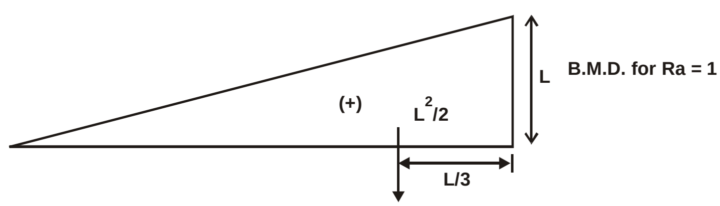




Applying moment area thereon, deflection at part A due to loads is

$$\Delta XL = \frac{1}{EI}\left[\frac{P}{2}(l-X)^2\left(l-\frac{1}{3}(l-X)\right)\right]$$





Applying moment area thereon, deflection at A due to Ra = 1

$$fXX = \frac{1}{EI}\left[\frac{1}{2}(l)^2\left(\frac{2l}{3}\right)\right] = \frac{l^3}{3EI}$$

Equation for compatibility

$\Delta al - fXX\ Ra = 0$ because A is a support. Net deflection should be zero.

$Ra = \dfrac{Sal}{fxx}$, $\boxed{Ra \; = \; \dfrac{P(l - X)^2\,(2l + X)}{2l^3}}$ after putting values of Sal and fxx

$Rb \; = \; 1 - Ra$ (equilibrium requirement)

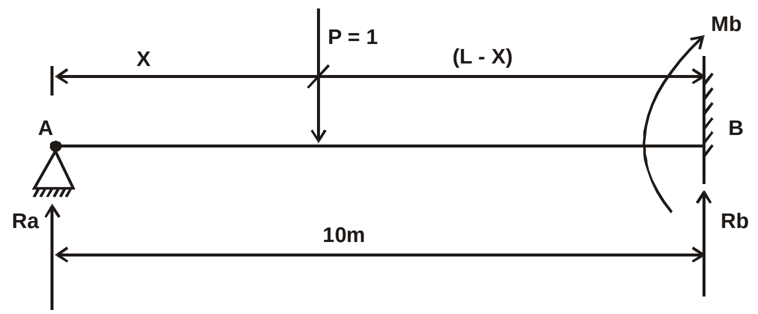
So we get $\boxed{Rb \; = \; \dfrac{X(3l^2 - X^2)}{2l^3}}$

We know

$Mb \; = \; Ra \times L - P\,(l - x)$. Put value of Ra and simplify

$\boxed{Mb \; = \; \dfrac{PX(l^2 - X^2)}{2l^2}}$ This expression will help in plotted ILD for Mb

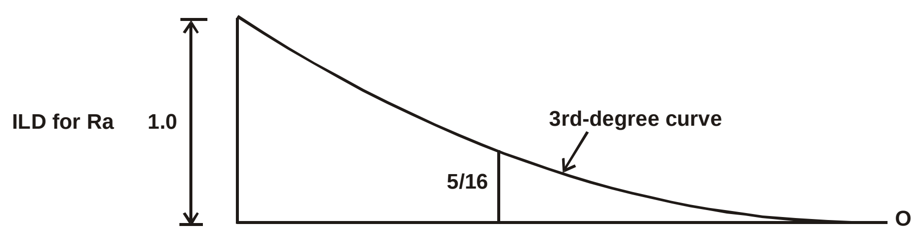
ILD for Ra



$$Ra \; = \; \frac{P(l - X)^2\,(2l + X)}{2\,l^3}$$

When $X = 0 \;\Rightarrow\; Ra = 1.0$ (put in above equation for Ra)

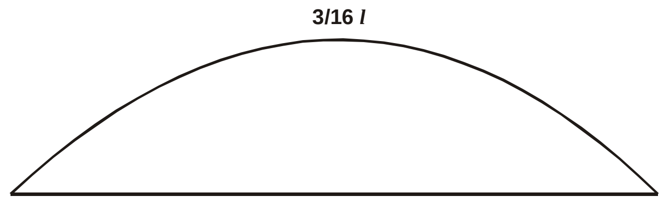
When $X = 5 \;\Rightarrow\; Ra = \dfrac{5}{16}$ (put in above equation for Ra)



Simplify ILD for Rb can be plotted as below:

Putting boundary conditions in the Mb expression ILD for Mb is obtained.

$$M_b = \frac{PX(l^2 - X^2)}{2l^2}$$

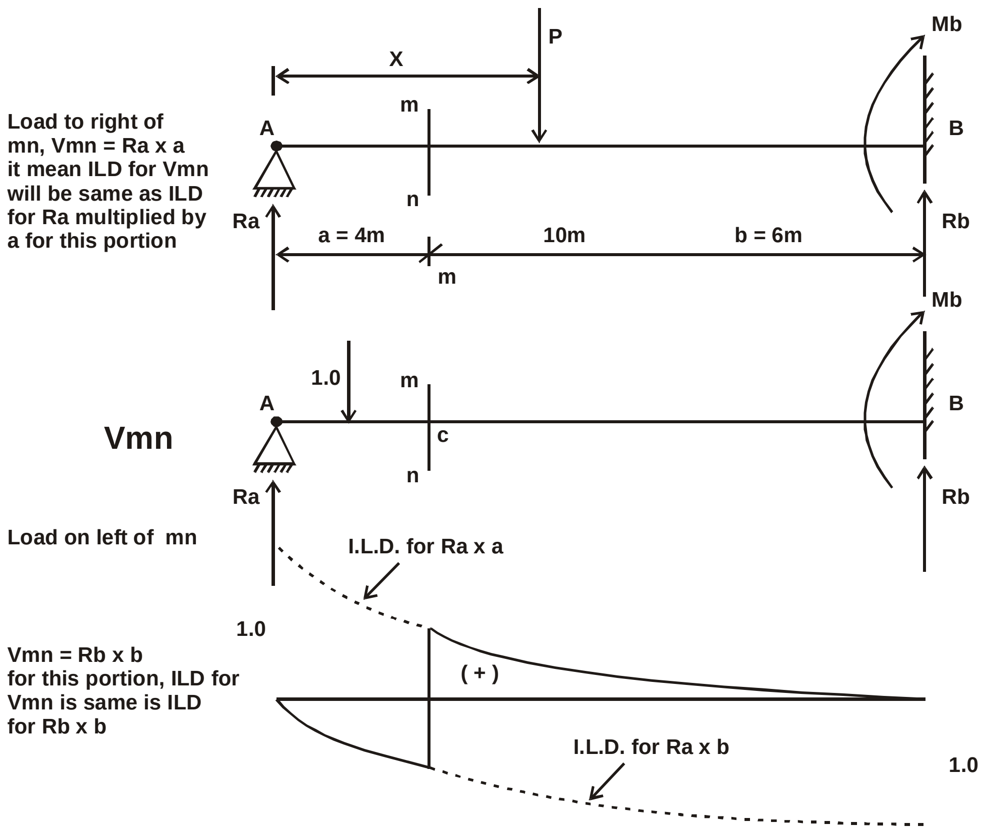


ILD for Mb

$$R_a l - P(l - X) + M_b = 0$$

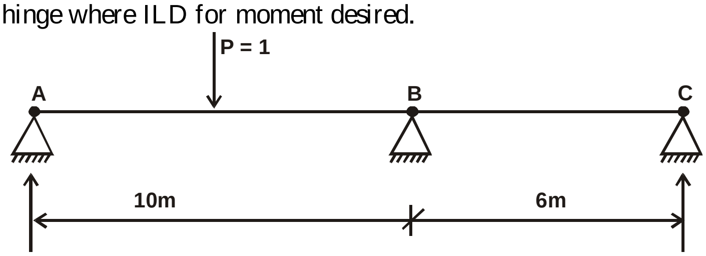
$$M_b = 1(l - X) - R_a l$$

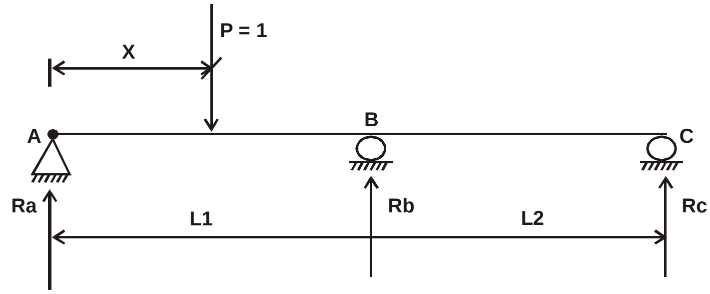
10.20. ILD for shear at Section mn:



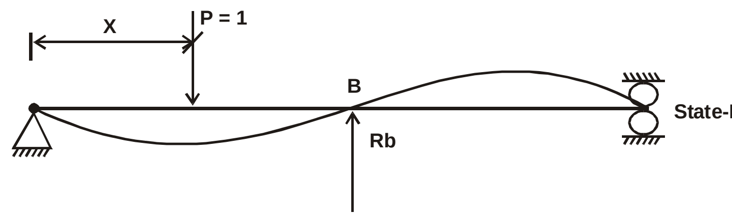
10.21. ILD for M_{mn}

Consider a hinge where ILD for moment desired.





Primary structure or BDS under load P = 1 and redundant Rb at B.

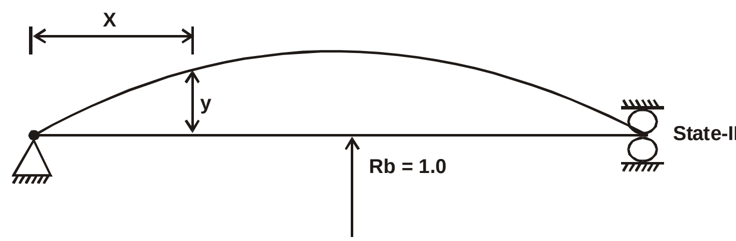


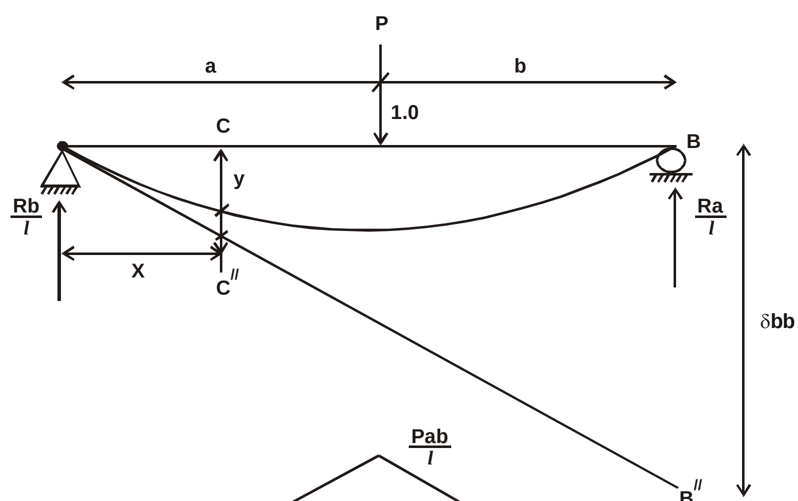
Compatibility equation at point B.

$$Rb\ \delta bb - Py = 0$$

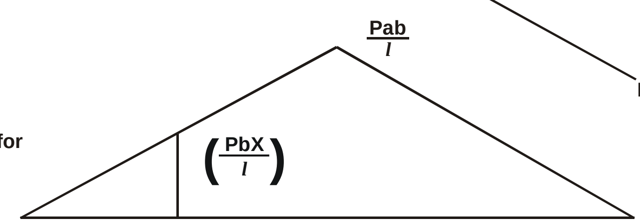
$$P = 1$$

$$Rb = \frac{y}{\delta bb}$$





We know this is ILD for moment at B in a simple beam.



$$y = \frac{PbX}{6EI\,l}\,(l^2 - b^2 - X^2) \qquad (X = 0 - a)$$

$$y = \frac{PaX}{6EI\,l}\,(l^2 - a^2 - X^2) \qquad (X = 0 - b)$$

$$y = \frac{l_2 X (l^2 - l_2^2 - X^2)}{6EI\,l}$$

$$\delta bb = \frac{l_2 X\,(l^2 - l_2^2 - l_1^2)}{6EI\,l}$$

$$\delta bb = \frac{l_1^2\, l_2^2}{3EI\,l}$$

and $$\left[Rb = \frac{X\,(l^2 - l_1^2 - X^2)}{2 l_1^2\, l_2} \right]$$

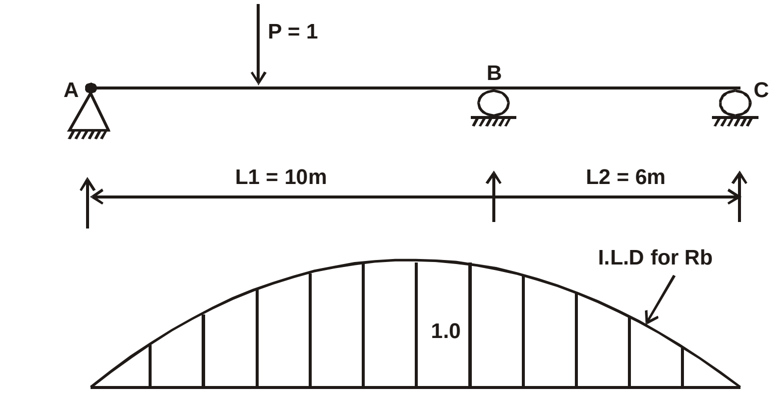
$X = 0 - l_1$ with Origin at A

$$Rb = \frac{X\,(l^2 - l_1^2 - X^2)}{(2 l_1^2\, l_2^2)} \qquad X = 0 \text{ to } l_2$$

Origin at C

Now assume same values of spans and re-calculate.

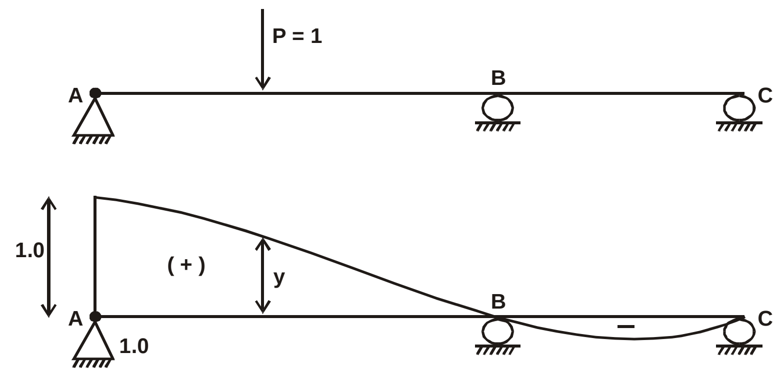
We know
$l_1 + l_2 = L$

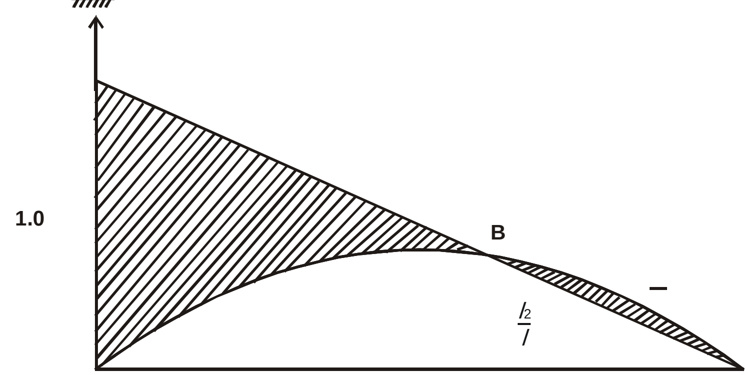


Compatibility at A

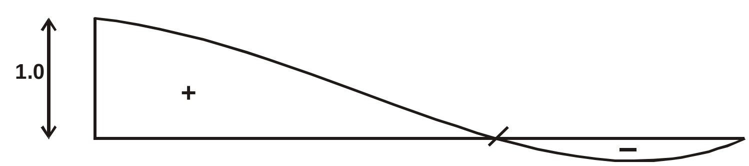
$$Ra\,\delta aa - Py = 0$$

$$Ra = \left(\frac{y}{\delta aa}\right)$$





I.L.D. for Ra



$$Rb = \frac{X\,(l^2 - l_2^2 - X^2)}{(2 \times l_1^2 \times l_2)} = \frac{X\,(16^2 - 6^2 - X^2)}{2 \times 10^2 \times 6}$$ by putting values of $L_1 l_1$ and l_2

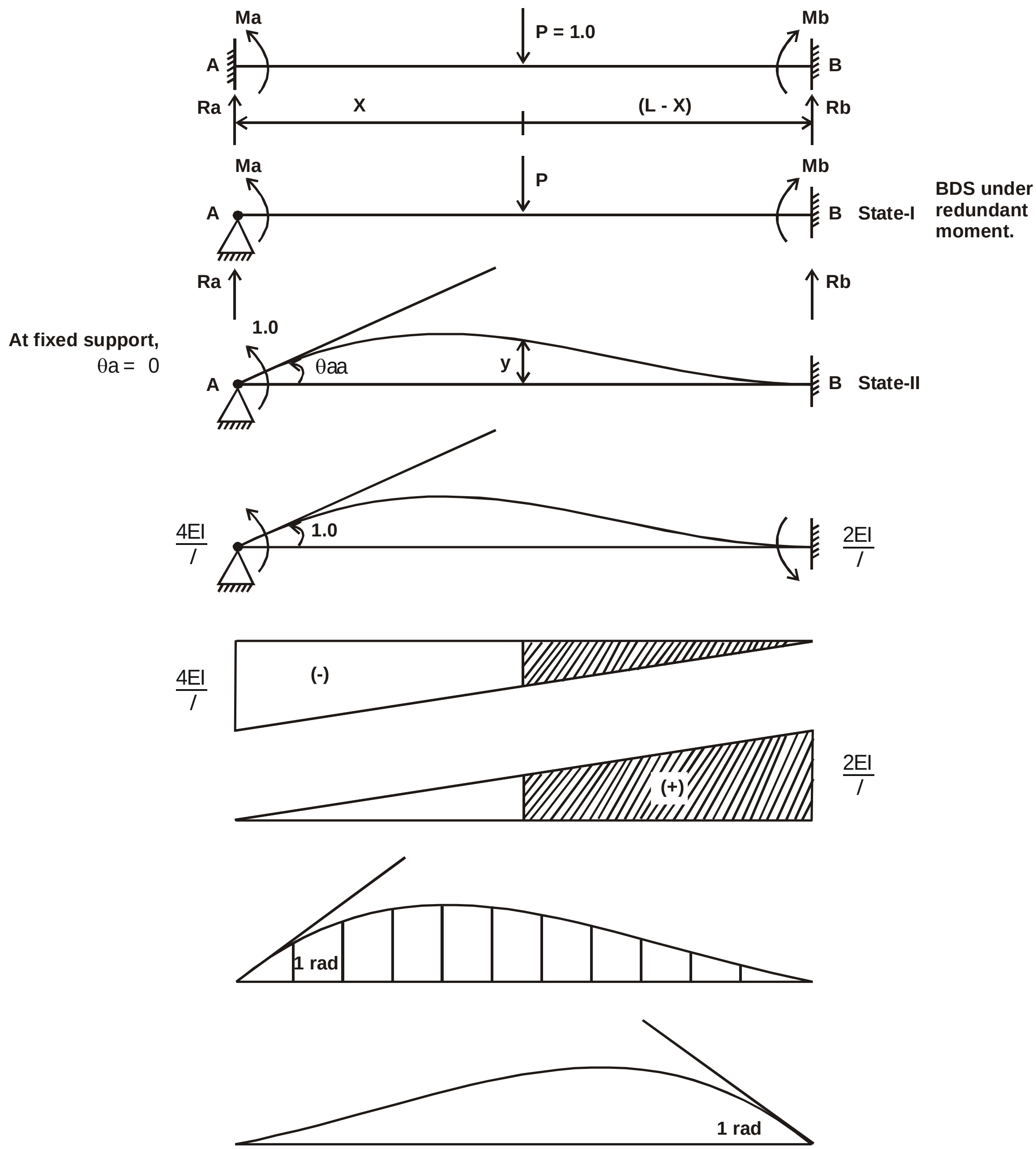
X	Rb	Ra	Rc
0	0		
1	0.1825		
2	0.36		
3	0.5275		
4	0.68		
5	0.8125		
6	0.92		
7	0.997	Calculate	Calculate
8	1.04	yourself	yourself
9			
10			
0	Calculate		
1	yourself		
2			
3			
4			

ILD for Ra can be obtained from ILD for Rb. Taking moments about C is equality to zero.

$$Ra\,l + Rb \times l_2 - P(l - X) = 0$$

So $$Ra = P\left(\frac{l - X}{l}\right) - \frac{Rb\, l_2}{l}$$

and $$Rb = \frac{(l_1 - X)}{2 l_1^2\, l}\,(2 l_1\, l - l_1 X - X^2)$$


Ma
P = 1.0
Mb
A
B
Ra
X
(L - X)
Rb
Ma
P
Mb
A
B State-I
BDS under redundant moment.
Ra
Rb
1.0
At fixed support, θa = 0
θaa
y
A
B State-II
4EI / l
1.0
2EI / l
4EI / l
(-)
2EI / l
(+)
1 rad
1 rad