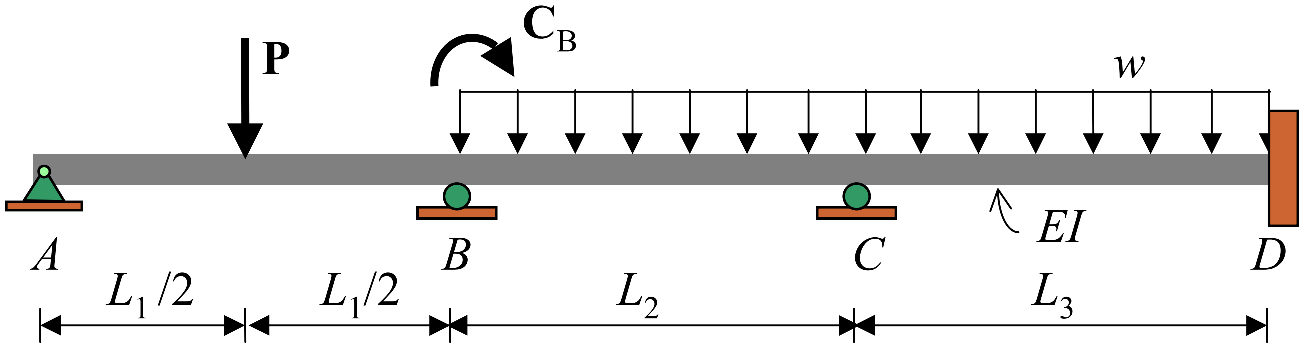
DISPLACEMENT MEDTHOD OF ANALYSIS: MOMENT DISTRIBUTION

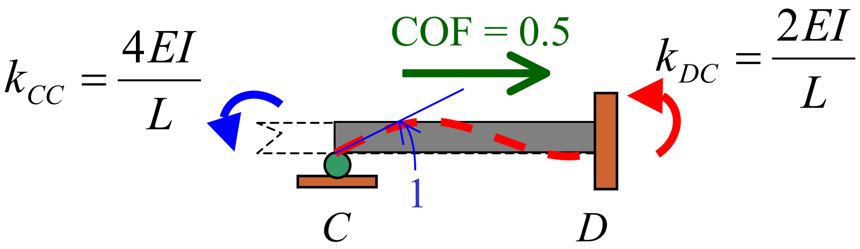
- Member Stiffness Factor (*K*)
- Distribution Factor (DF)
- Carry-Over Factor
- Distribution of Couple at Node
- Moment Distribution for Beams
  - General Beams
  - Symmetric Beams
- Moment Distribution for Frames: No Sidesway
- Moment Distribution for Frames: Sidesway

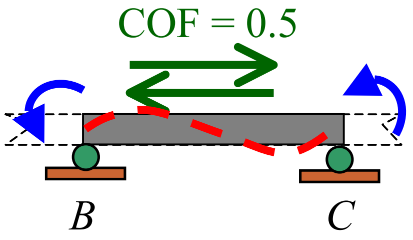
Member Stiffness Factor (*K*) & Carry-Over Factor (COF)



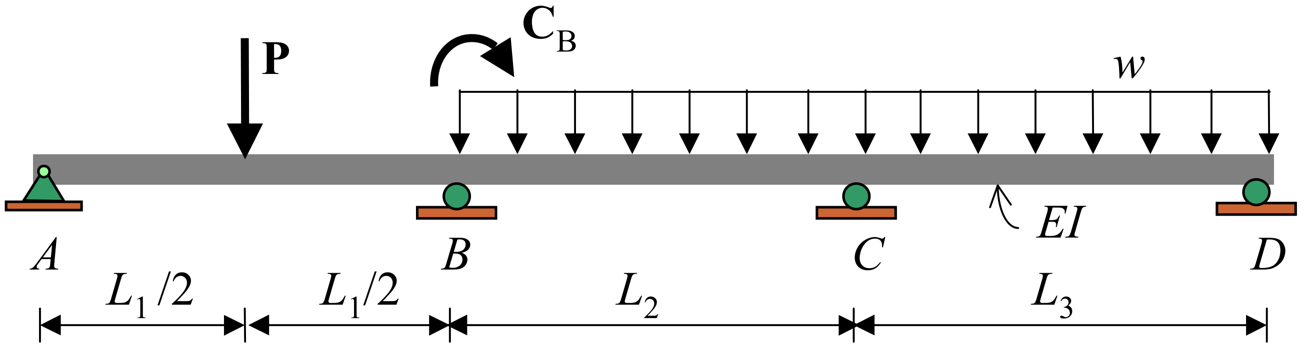
Internal members and **far-end member fixed** at **end support**:

$$\boxed{K = \frac{4EI}{L}}$$





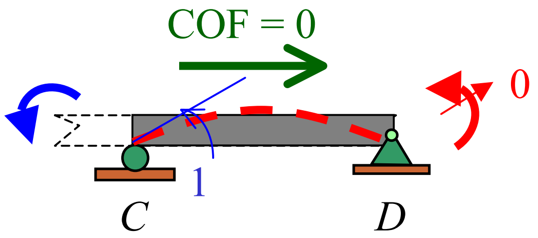
$K_{(BC)} = 4EI/L_2$, $K_{(CD)} = 4EI/L_3$



Far-end member pinned or roller end support:

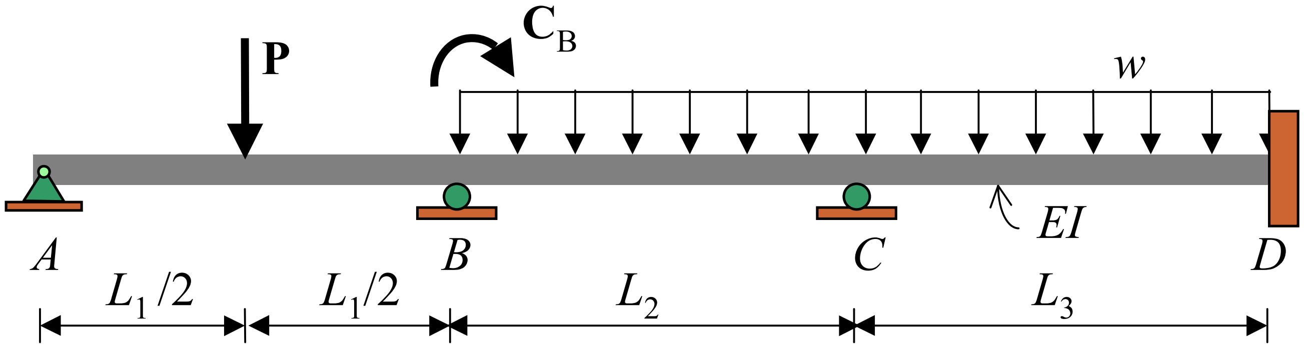
$$K = \frac{3EI}{L}$$

$$k_{AA} = \frac{3EI}{L}$$



$K_{(AB)} = 3EI/L_1$, $K_{(BC)} = 4EI/L_2$, $K_{(CD)} = 4EI/L_3$

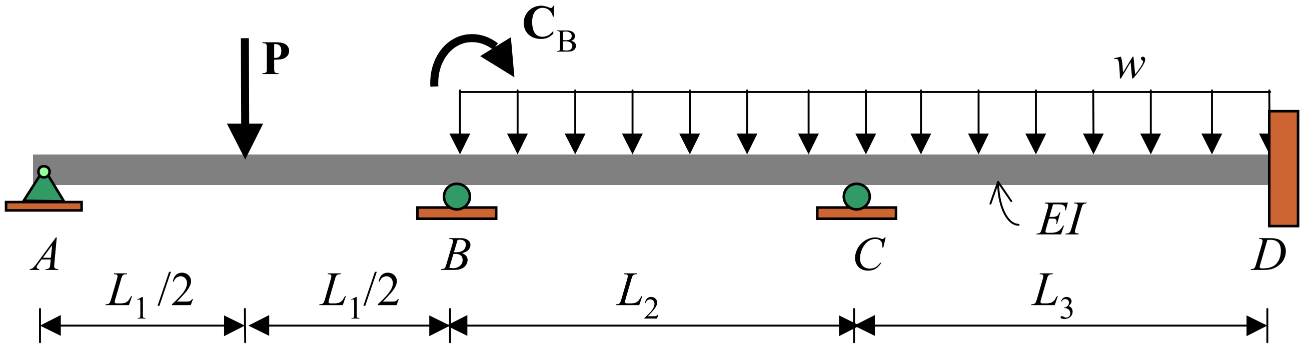
Joint Stiffness Factor (*K*)



$K_{(AB)} = 3EI/L_1$ $K_{(BC)} = 4EI/L_2,$ $K_{(CD)} = 4EI/L_3$

$$\boxed{K_{joint} = K_T = \Sigma K_{member}}$$

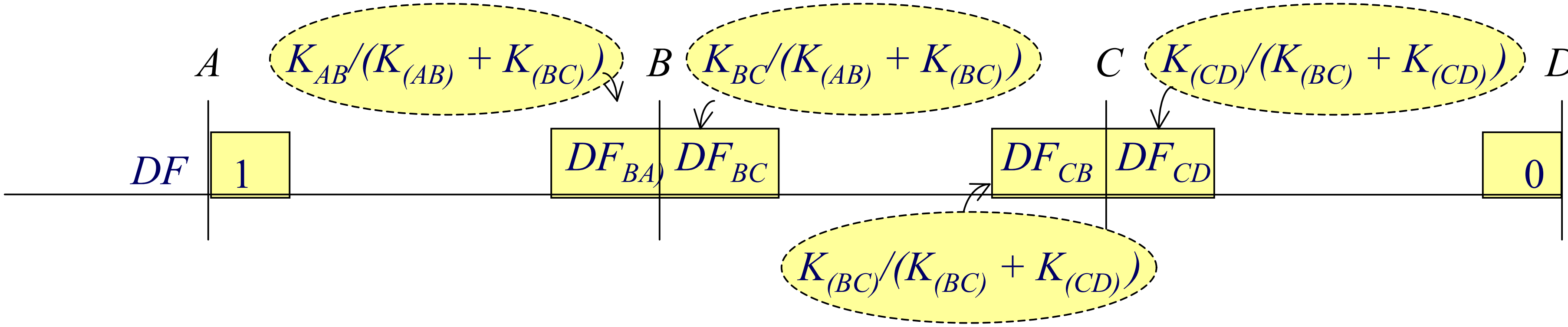
Distribution Factor (DF)



$$DF = \frac{K}{\Sigma K}$$

Notes:

- far-end pined (DF = 1)
- far-end fixed (DF = 0)



A B C D

DF

1

$K_{AB}/(K_{(AB)} + K_{(BC)})$ → $DF_{BA)}$

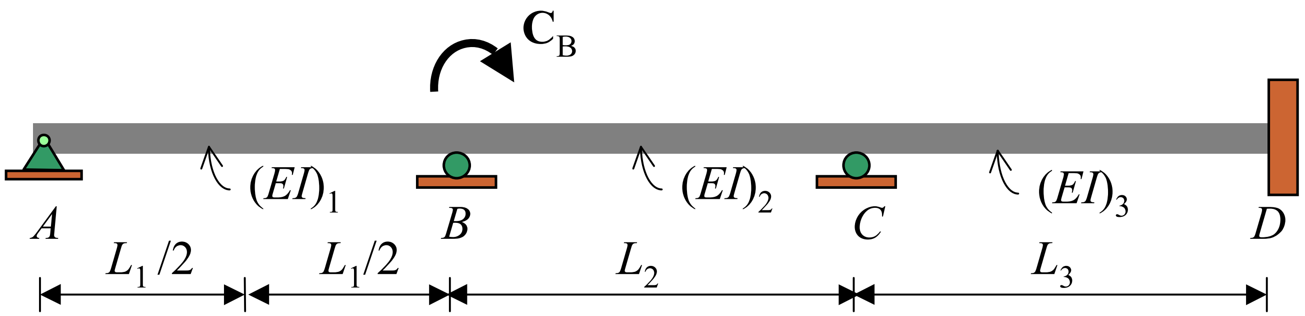
$K_{BC}/(K_{(AB)} + K_{(BC)})$ → DF_{BC}

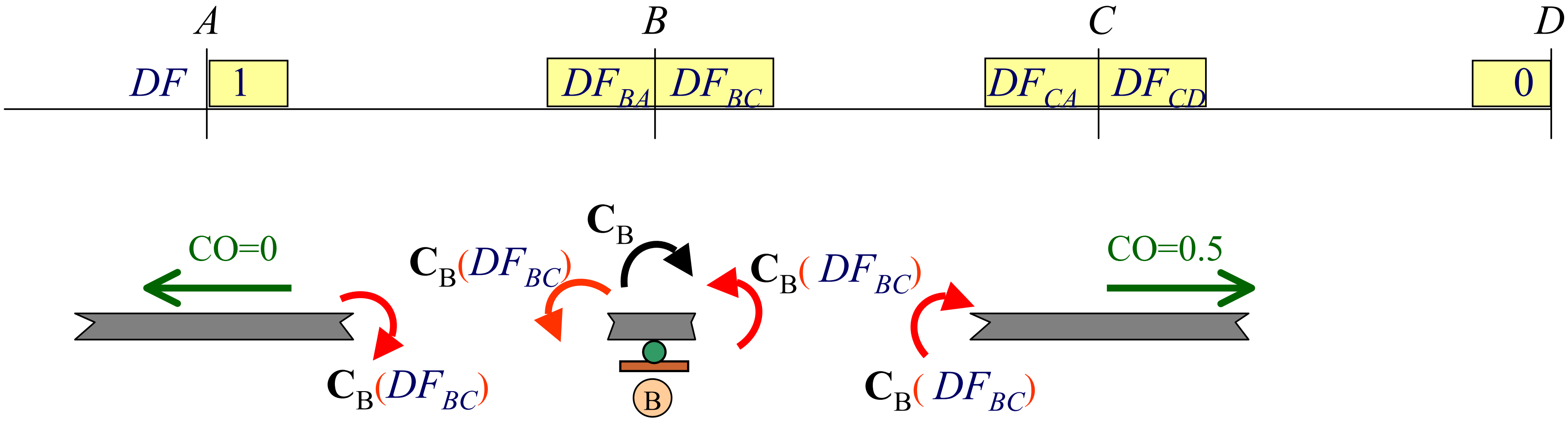
$K_{(BC)}/(K_{(BC)} + K_{(CD)})$ → DF_{CB}

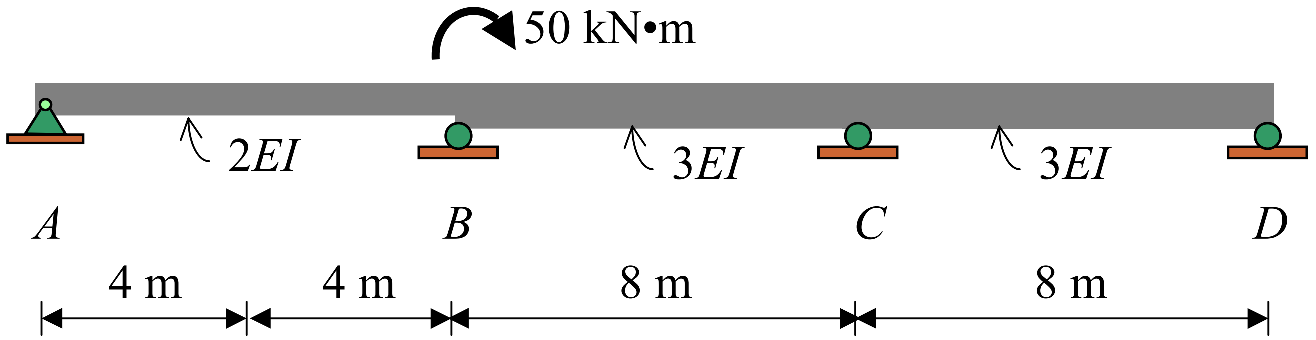
$K_{(CD)}/(K_{(BC)} + K_{(CD)})$ → DF_{CD}

0

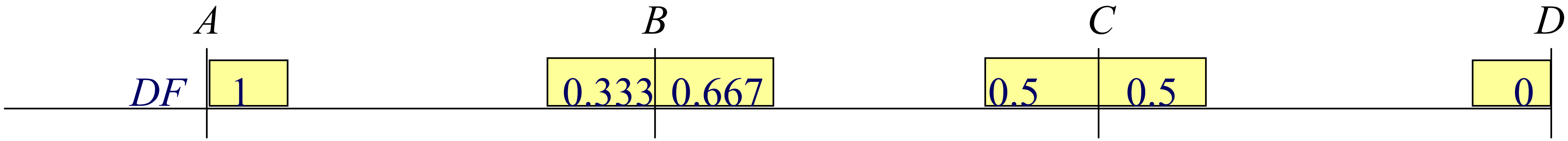
Distribution of Couple at Node

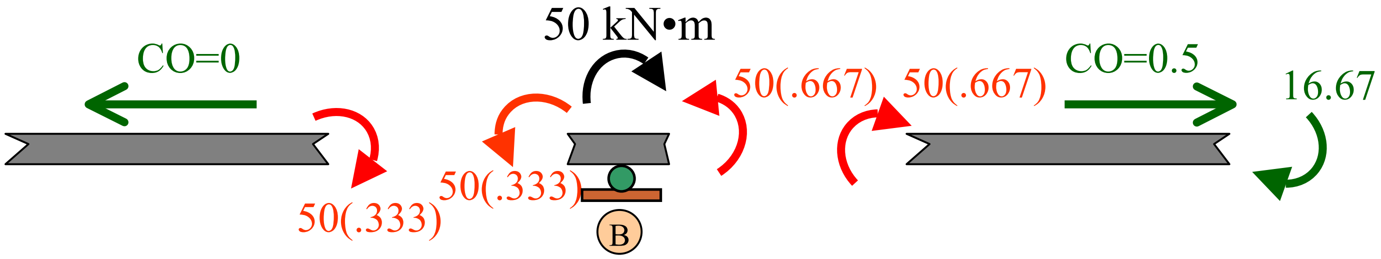




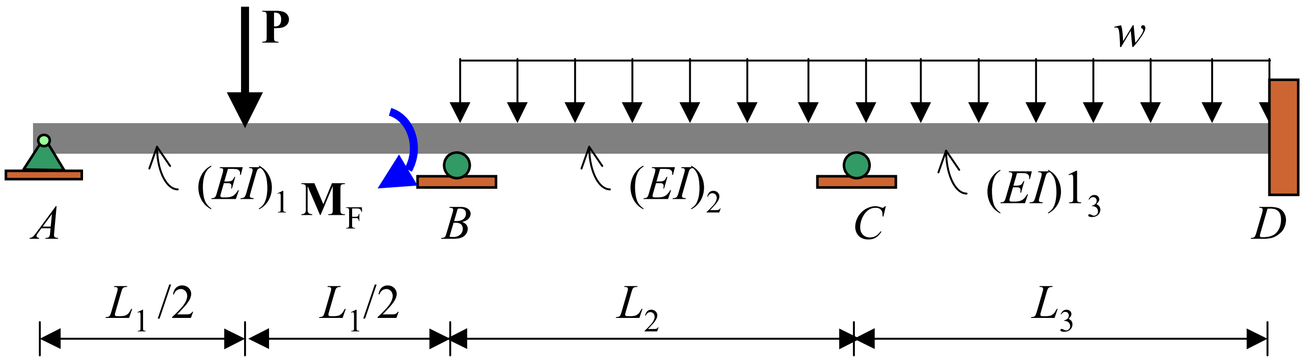

50 kN•m
2EI
3EI
3EI
A
B
C
D
4 m
4 m
8 m
8 m

$L_1 = L_2 = L_3$

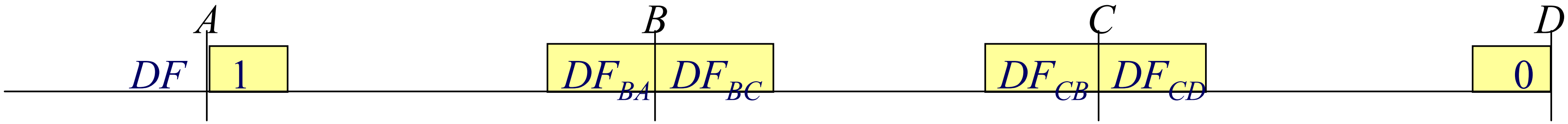

A
B
C
D
DF
1
0.333
0.667
0.5
0.5
0

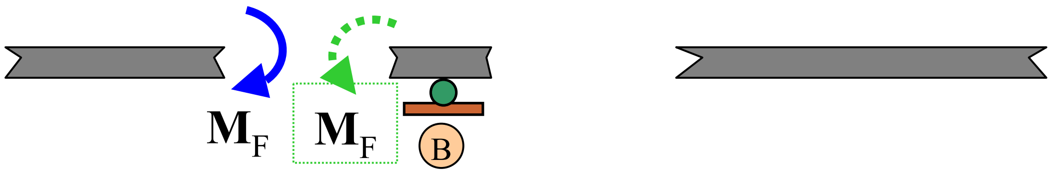

CO=0
50 kN•m
50(.667)
50(.667)
CO=0.5
16.67
50(.333)
50(.333)
B

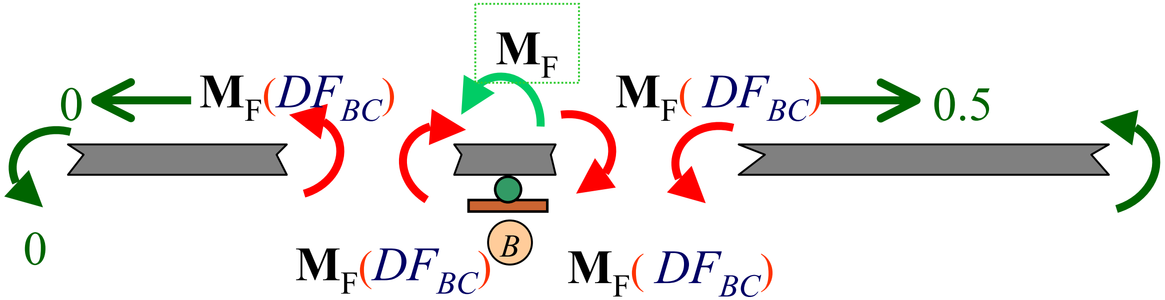
Distribution of Fixed-End Moments

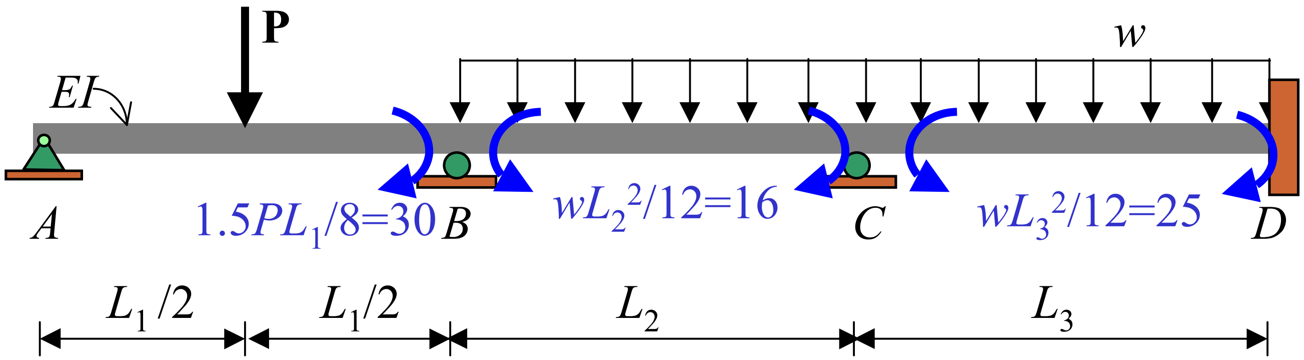


$L_1 = L_2 = 8$ m, $L_3 = 10$ m

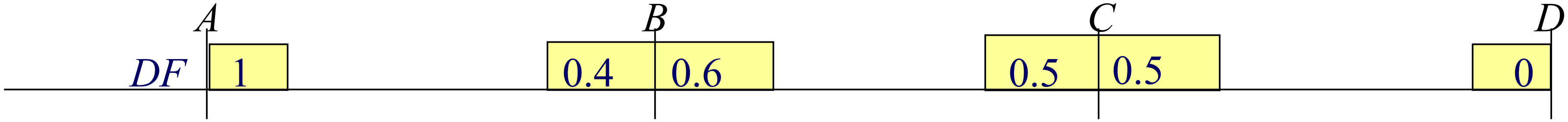


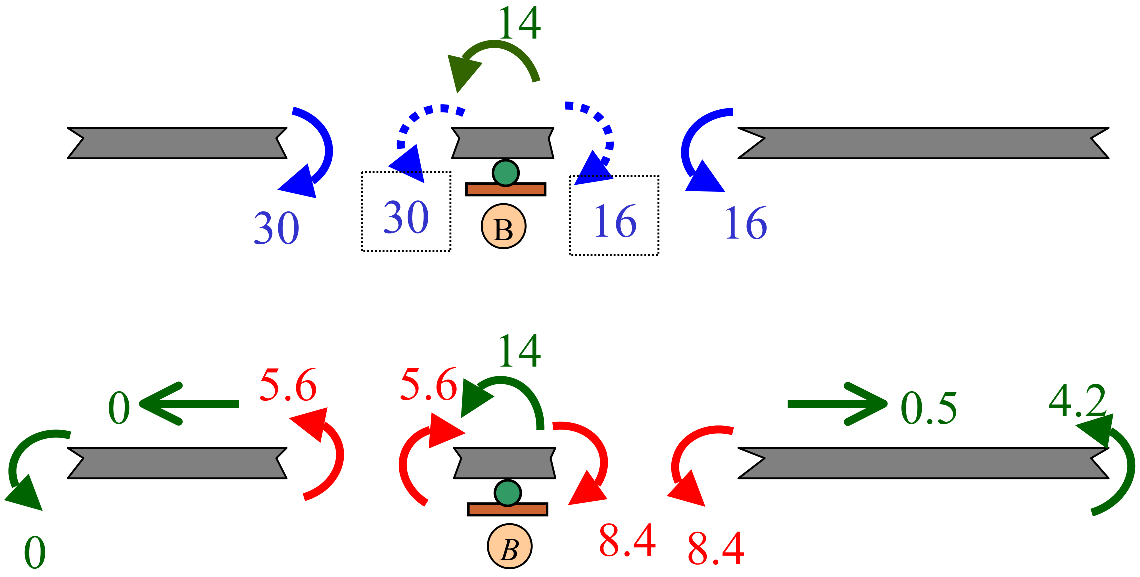






$L_1 = L_2 = 8$ m, $L_3 = 10$ m

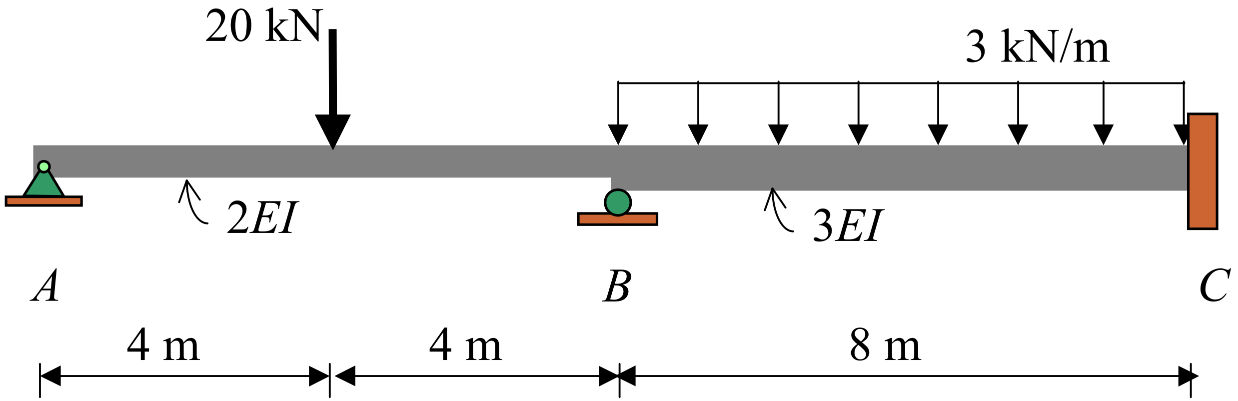


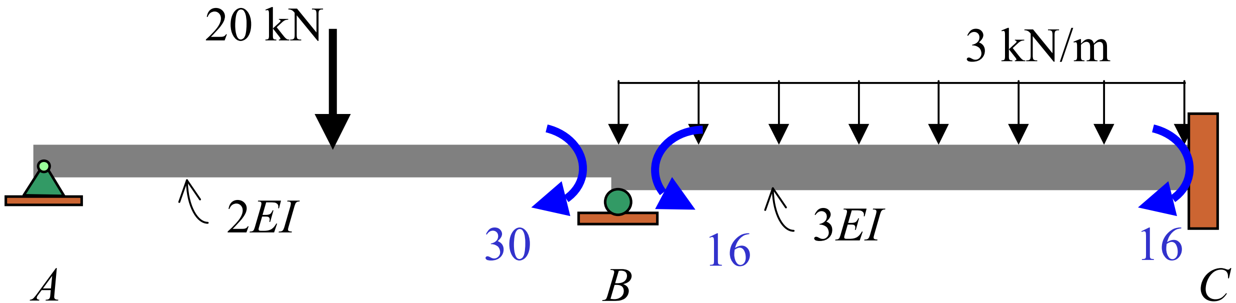


Moment Distribution for Beams

Example 1

The support *B* of the beam shown (E = 200 GPa, I = 50x10^6 mm^4).
Use the moment distribution method to:
(a) Determine all the reactions at supports, and also
(b) Draw its **quantitative shear** and **bending moment diagrams**,and **qualitative deflected shape.**





4 m | 4 m | 8 m

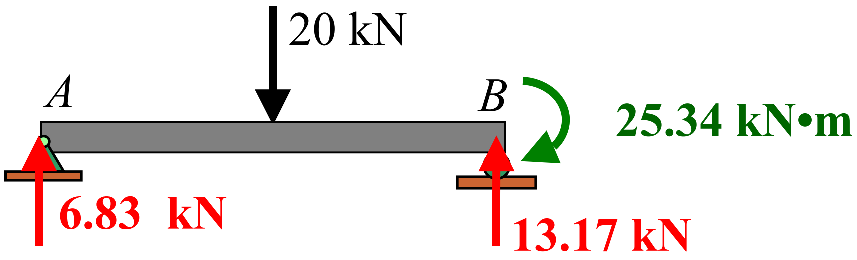
$K_1 = 3(2EI)/8$

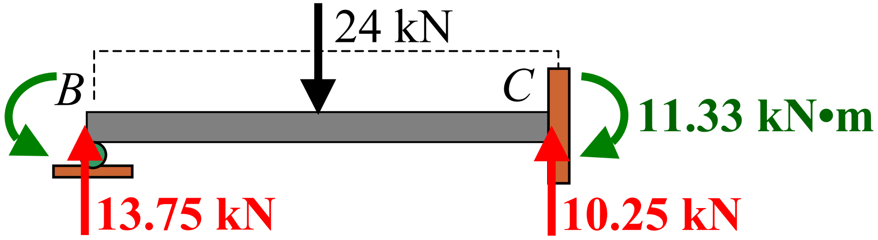
$K_2 = 4(3EI)/8$

$K_1/(K_1+K_2)$

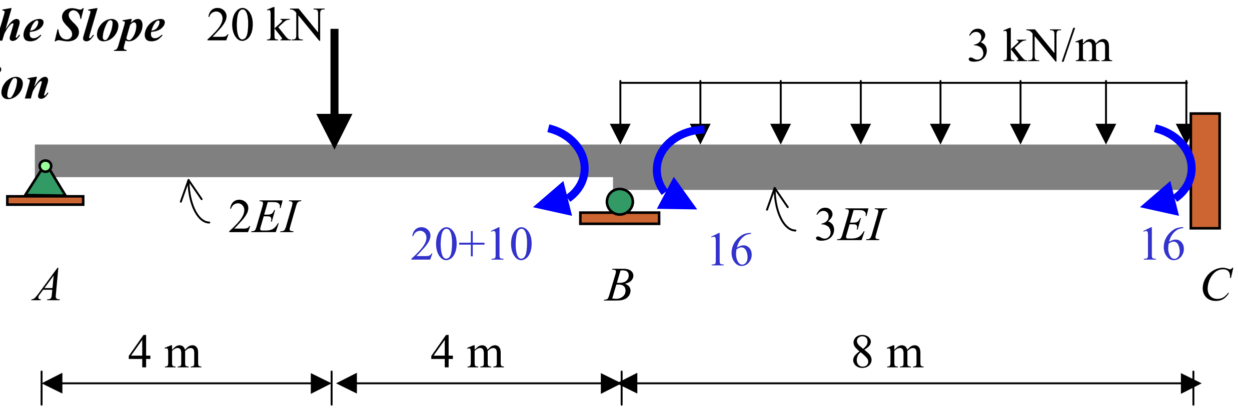
$K_2/(K_1+K_2)$

	A	B		C
CO	→ 0.5	← 0	→ 0.5	← 0
DF	1	0.333	0.667	0
$[\text{FEM}]_{\text{load}}$		-30	16	-16
Dist.		4.662	9.338	
CO				4.669
Σ		**-25.34**	**25.34**	**-11.33**



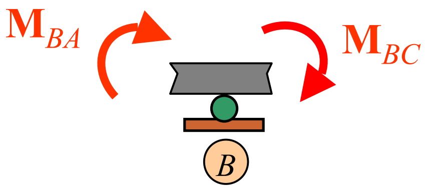


***Note :* Using the Slope Deflection**



$$M_{BA} = \frac{3(2EI)}{8}\theta_B - 30 \quad ---(1)$$

$$M_{BC} = \frac{4(3EI)}{8}\theta_B + 16 \quad ---(2)$$



$$\boxed{+\circlearrowright\ \Sigma \mathbf{M}_B = 0:\ -\mathbf{M}_{BA} - \mathbf{M}_{BC} = 0}$$

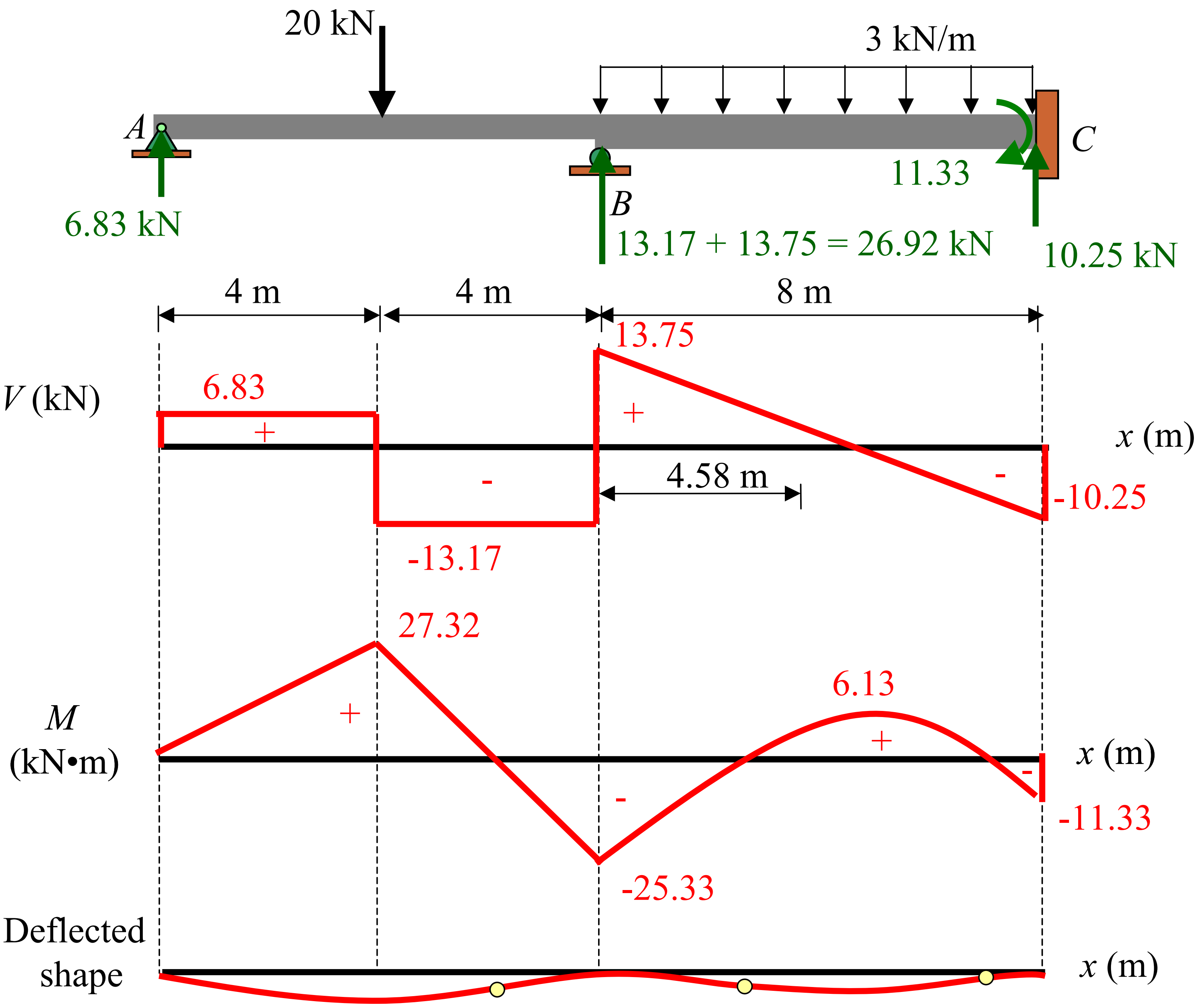
$$(0.75 + 1.5)EI\theta_B - 30 + 16 = 0$$

$$\theta_B = 6.22/EI$$

$$M_{BA} = -25.33 \text{ kN}\bullet\text{m},$$

$$M_{BC} = 25.33 \text{ kN}\bullet\text{m}$$

$$M_{CB} = \frac{2(3EI)}{8}\theta_B - 16 = -11.33\ kN \bullet m$$

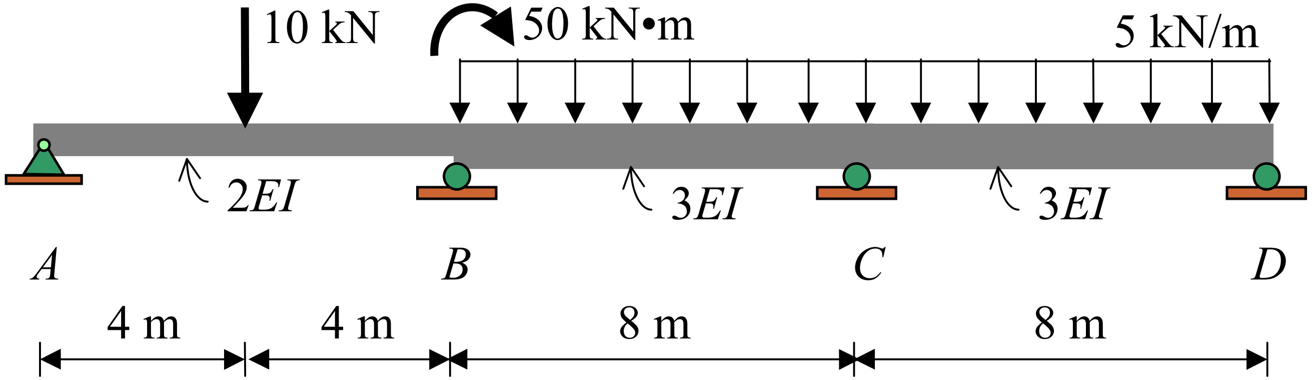

20 kN
3 kN/m
A
B
C
11.33
6.83 kN
13.17 + 13.75 = 26.92 kN
10.25 kN
4 m
4 m
8 m
V (kN)
13.75
6.83
+
x (m)
-
4.58 m
+
-
-10.25
-13.17
27.32
6.13
M
(kN•m)
+
+
x (m)
-
-
-11.33
-25.33
Deflected
shape
x (m)

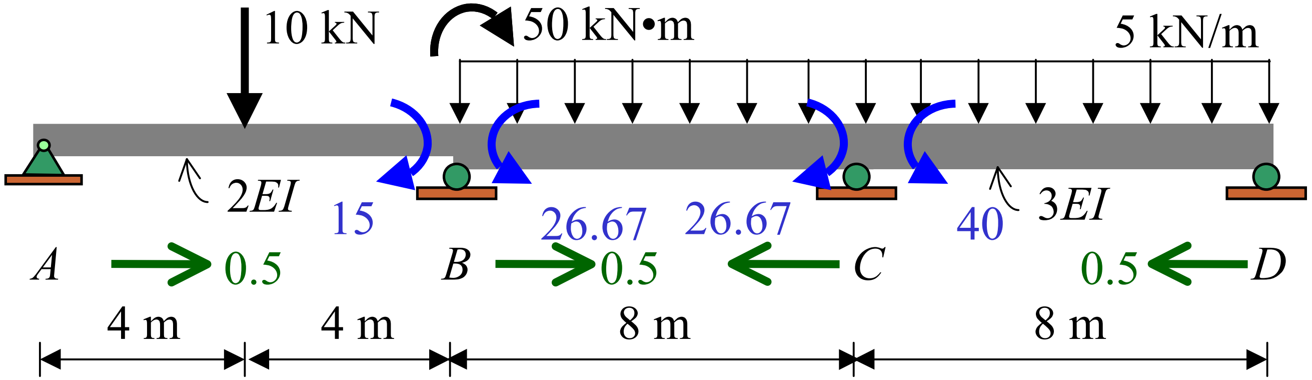
Example 2

From the beam shown use the moment distribution method to:

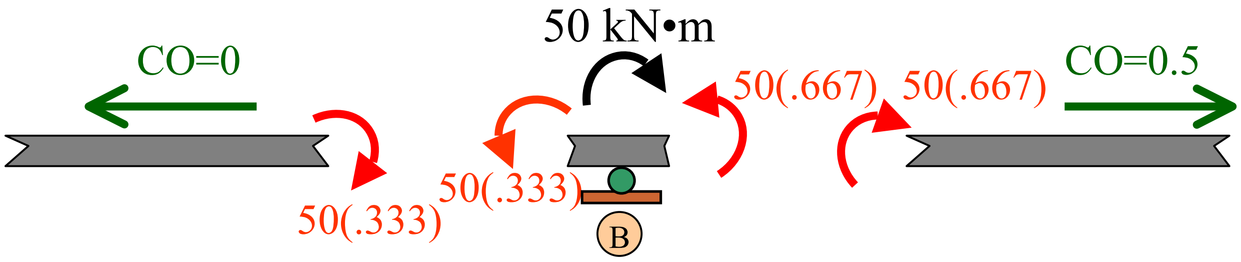
(a) Determine all the reactions at supports, and also

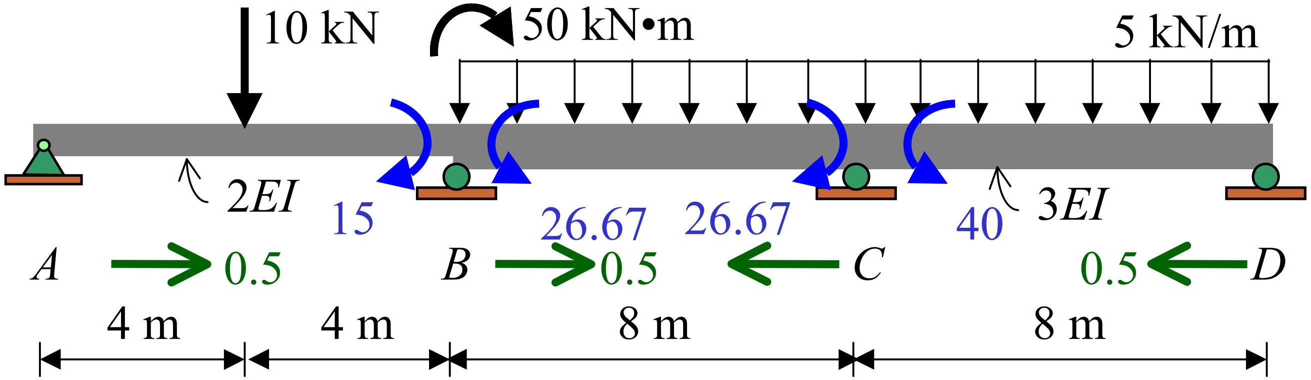
(b) Draw its **quantitative shear** and **bending moment diagrams**, and **qualitative deflected shape.**



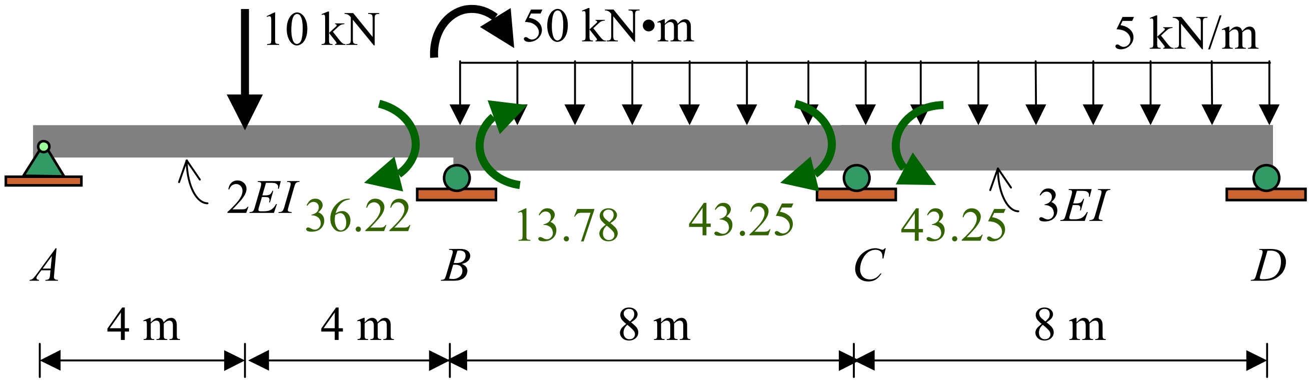


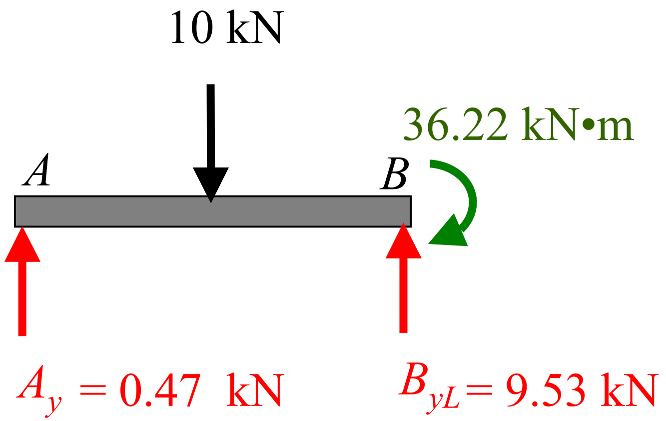
	$K_1 = 3(2EI)/8$		$K_2 = 4(3EI)/8$		$K_3 = 3(3EI)/8$	
		$K_1/(K_1+K_2)$	$K_2/(K_1+K_2)$	$K_2/(K_2+K_3)$	$K_3/(K_2+K_3)$	
DF	1	0.333	0.667	0.571	0.428	1
Joint couple		-16.65	-33.35			

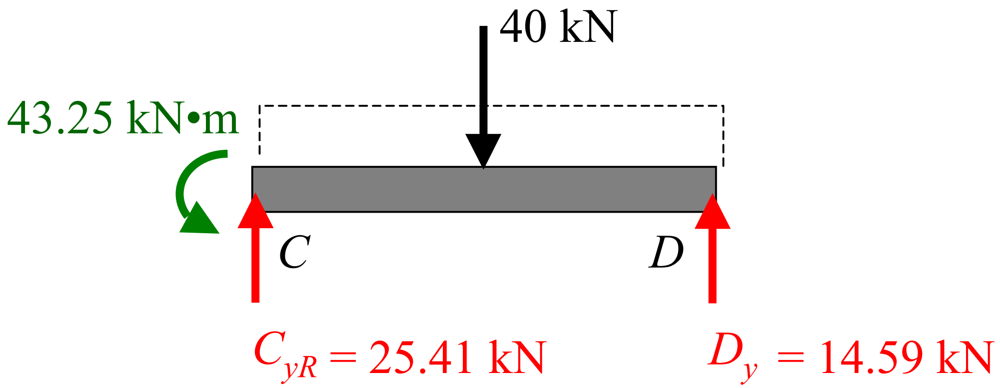


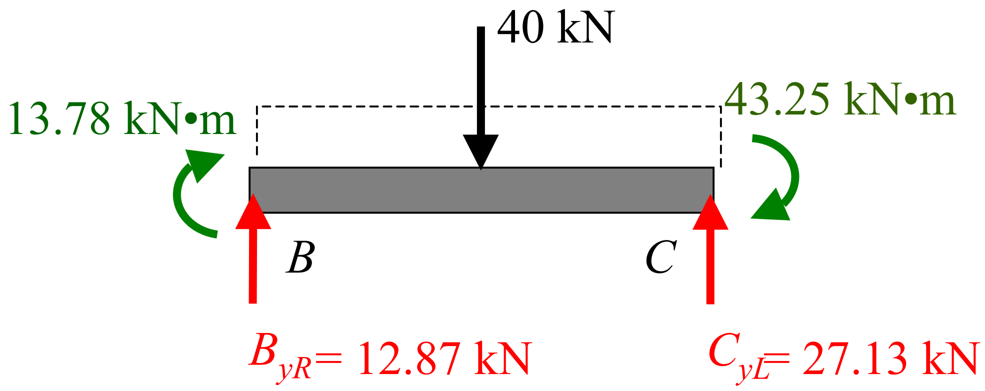


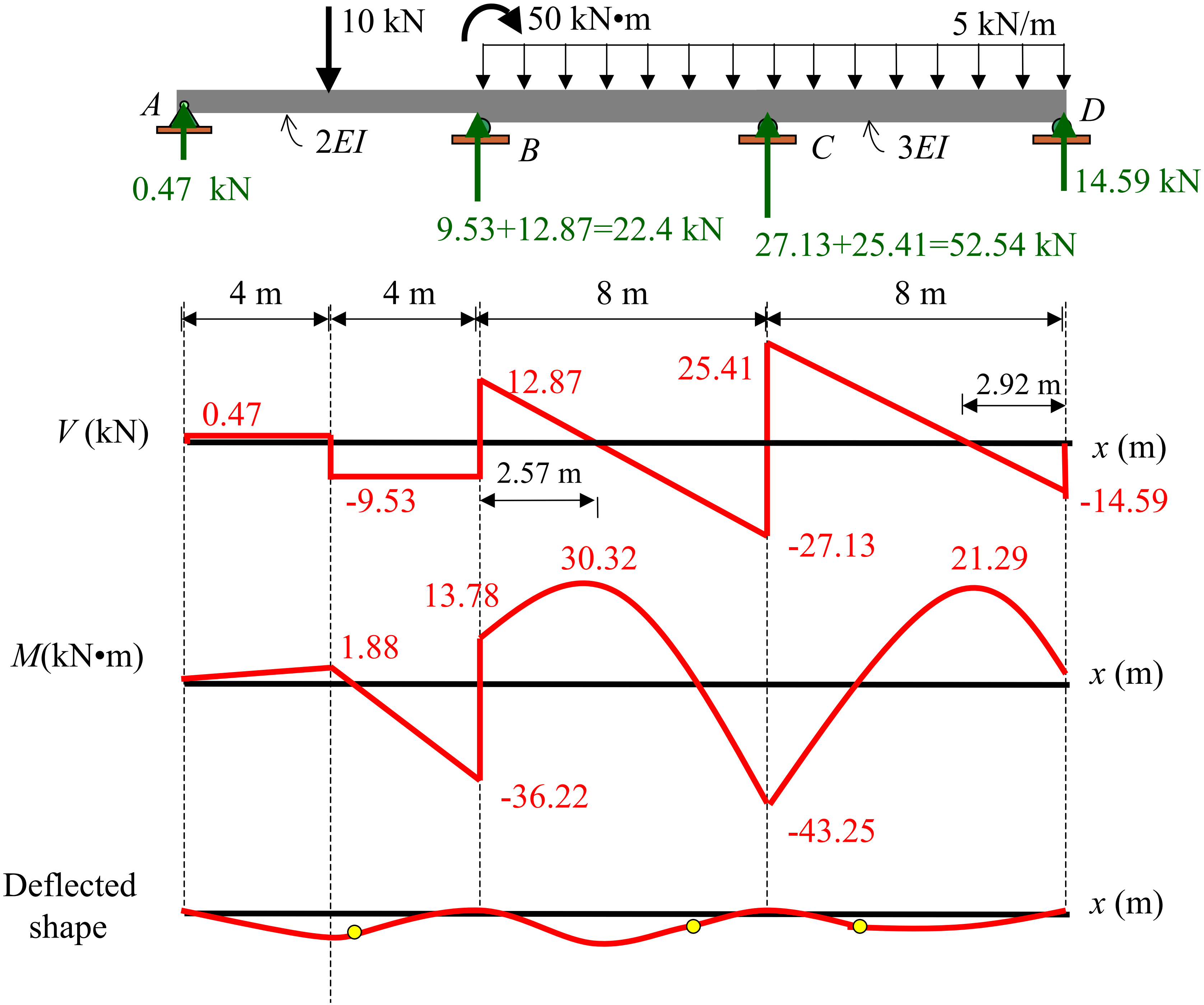
	$K_1 = 3(2EI)/8$		$K_2 = 4(3EI)/8$		$K_3 = 3(3EI)/8$	
		$K_1/(K_1+K_2)$	$K_2/(K_1+K_2)$	$K_2/(K_2+K_3)$	$K_3/(K_2+K_3)$	
DF	1	0.333	0.667	0.571	0.429	1
Joint couple		-16.65	-33.35			
CO				-16.675		
FEM		-15	26.667	-26.667	40	
Dist.		-3.885	-7.782	1.905	1.437	
CO			0.953	-3.891		
Dist.		-0.317	-0.636	2.218	1.673	
CO			1.109	-0.318		
Dist.		-0.369	-0.740	0.181	0.137	
Σ		**-36.22**	**-13.78**	**-43.28**	**43.25**	


10 kN
50 kN•m
5 kN/m
2EI
36.22
13.78
43.25
43.25
3EI
A
B
C
D
4 m
4 m
8 m
8 m


10 kN
36.22 kN•m
A
B
$A_y = 0.47$ kN
$B_{yL} = 9.53$ kN


40 kN
43.25 kN•m
C
D
$C_{yR} = 25.41$ kN
$D_y = 14.59$ kN


40 kN
13.78 kN•m
43.25 kN•m
B
C
$B_{yR} = 12.87$ kN
$C_{yL} = 27.13$ kN

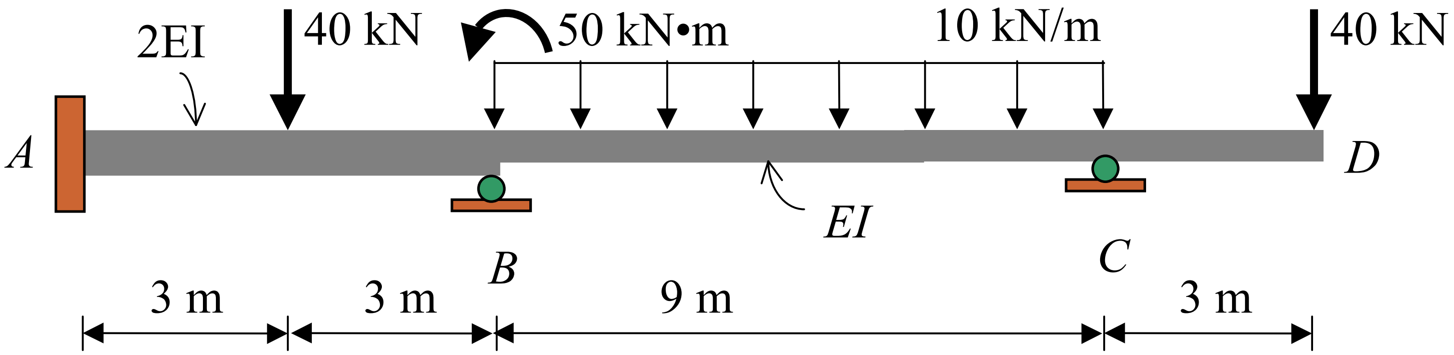

10 kN
50 kN•m
5 kN/m
A
2EI
B
C
3EI
D
0.47 kN
9.53+12.87=22.4 kN
27.13+25.41=52.54 kN
14.59 kN
4 m
4 m
8 m
8 m
V (kN)
0.47
12.87
25.41
2.92 m
x (m)
-9.53
2.57 m
-27.13
-14.59
M(kN•m)
30.32
13.78
21.29
1.88
x (m)
-36.22
-43.25
Deflected shape
x (m)

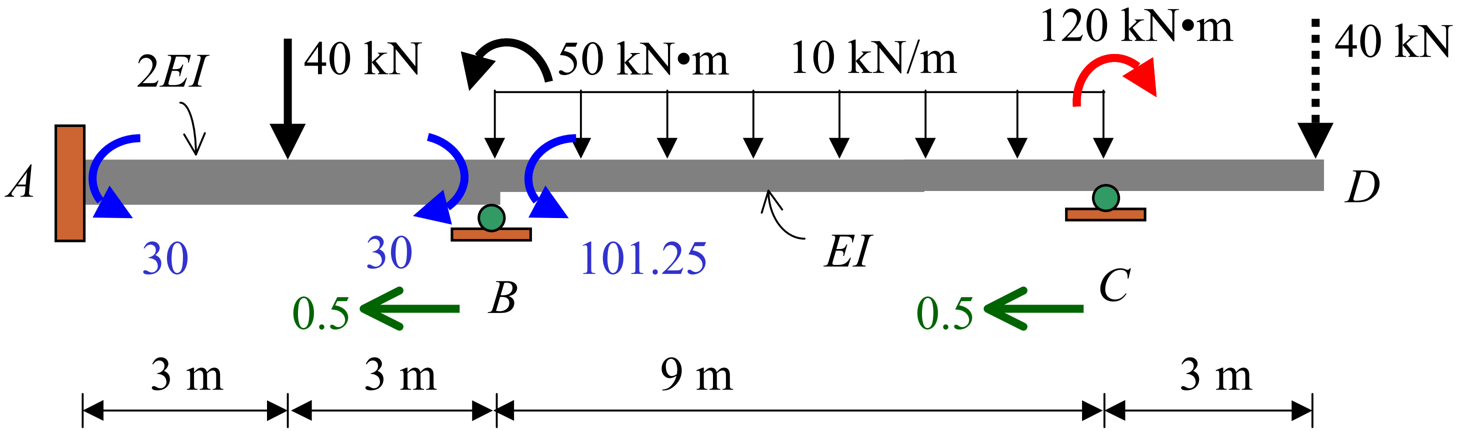
Example 3

From the beam shown use the moment distribution method to:

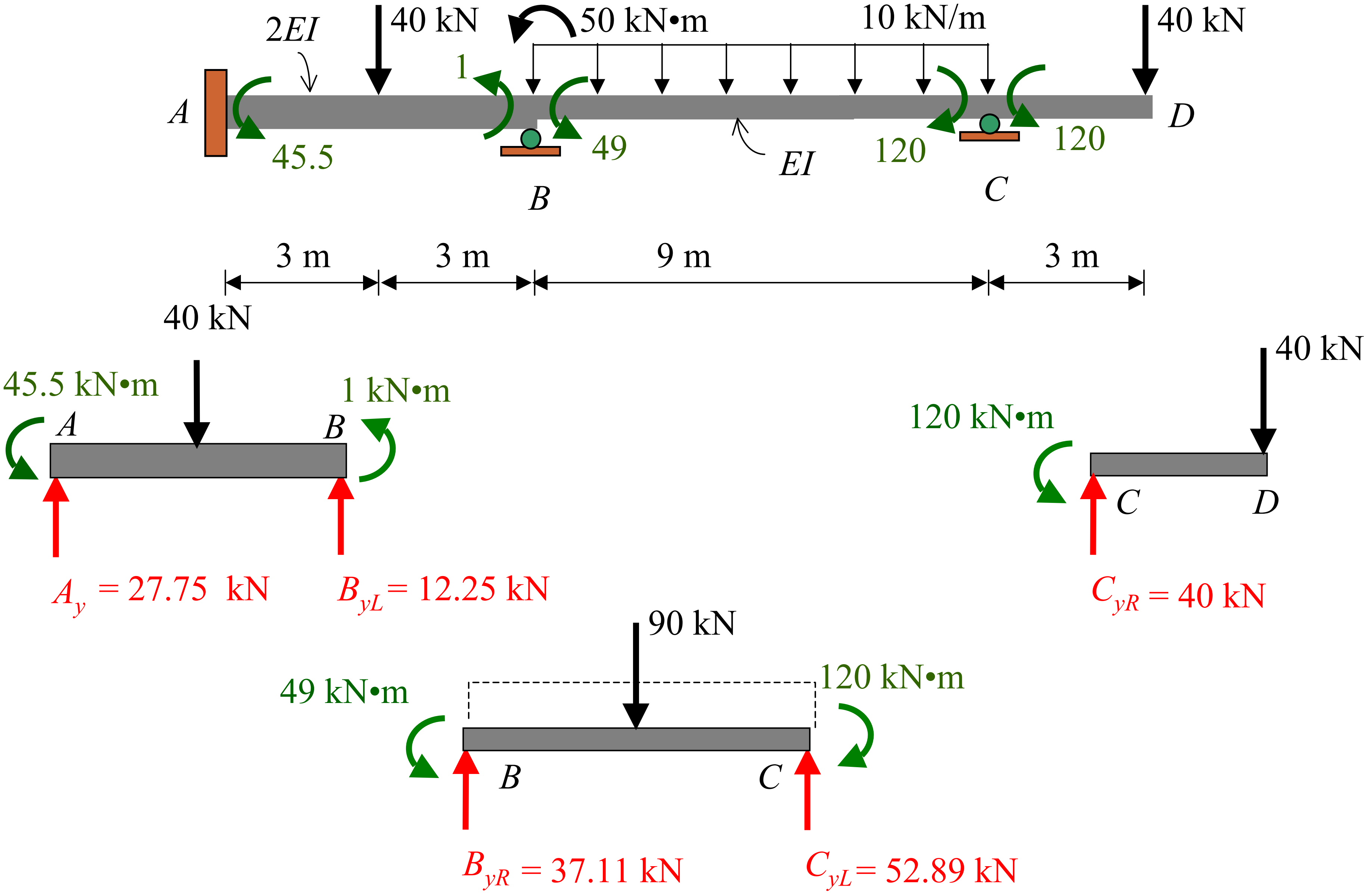
(a) Determine all the reactions at supports, and also

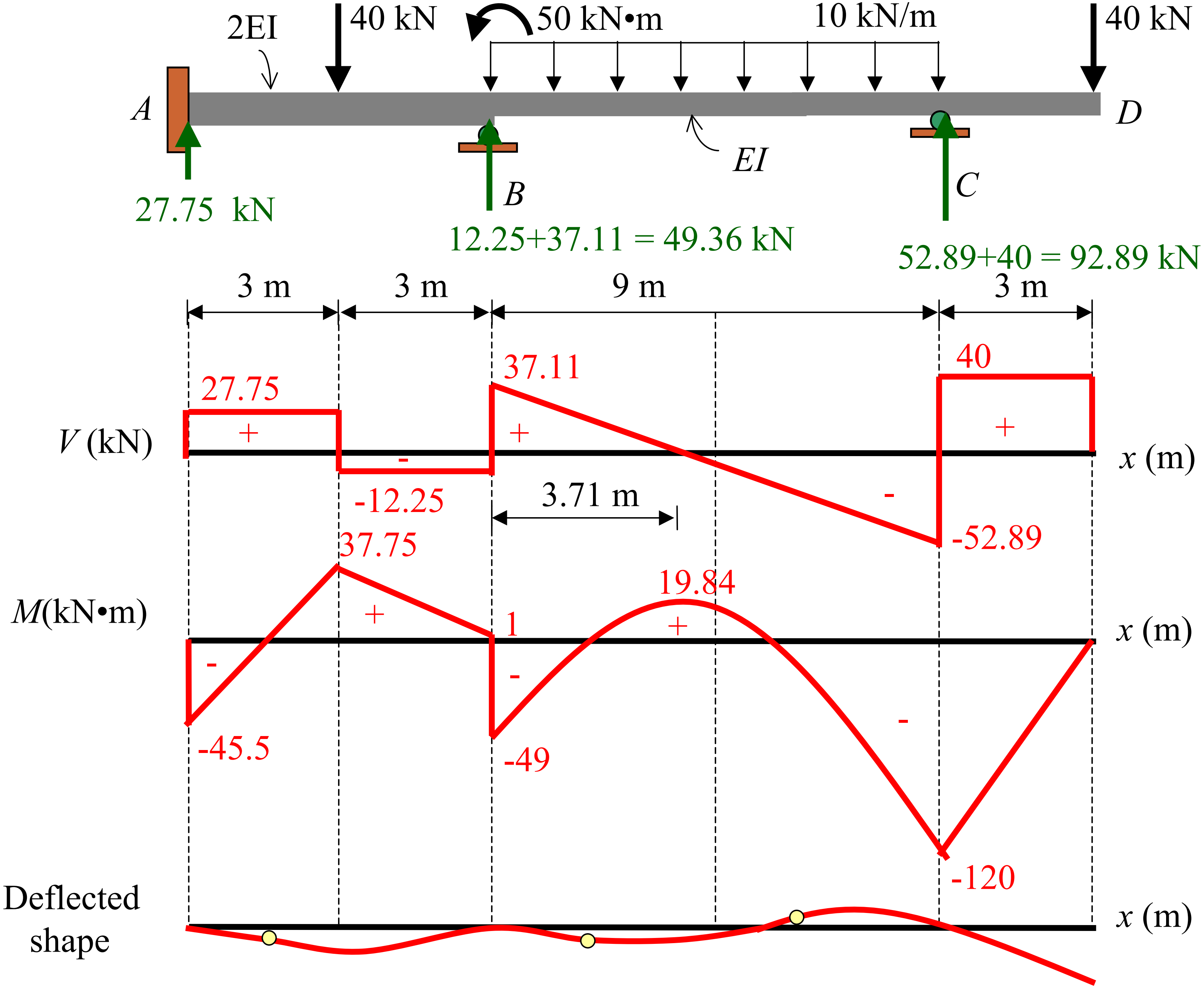
(b) Draw its **quantitative shear** and **bending moment diagrams**, and **qualitative deflected shape.**





	$K_1 = 4(2EI)/6$		$K_2 = 3(EI)/9$	
		$K_1/(K_1+K_2)$	$K_2/(K_1+K_2)$	
DF	0	0.80	0.20	1
Joint couple		40	10	-120
CO	20		-60	
FEM	30	-30	101.25	
Dist.				
Dist.		-9	-2.25	
CO	-4.5			
Σ	**45.5**	**1**	**49**	**-120**


40 kN
50 kN•m
10 kN/m
40 kN
2EI
1
A
45.5
49
120
120
D
B
EI
C
3 m
3 m
9 m
3 m
40 kN
45.5 kN•m
A
B
1 kN•m
A_y = 27.75 kN
B_{yL} = 12.25 kN
40 kN
120 kN•m
C
D
C_{yR} = 40 kN
90 kN
49 kN•m
120 kN•m
B
C
B_{yR} = 37.11 kN
C_{yL} = 52.89 kN

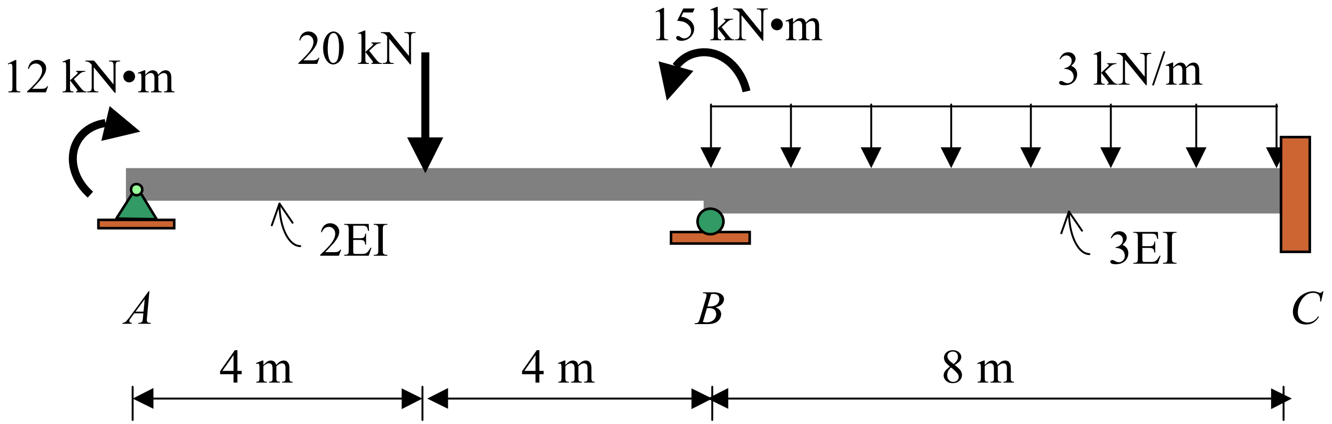

2EI
40 kN
50 kN•m
10 kN/m
40 kN
A
D
B
EI
C
27.75 kN
12.25+37.11 = 49.36 kN
52.89+40 = 92.89 kN
3 m
3 m
9 m
3 m
V (kN)
27.75
+
-
-12.25
37.11
+
3.71 m
-
-52.89
40
+
x (m)
M(kN•m)
37.75
+
1
19.84
+
-
-45.5
-
-49
-
-120
x (m)
Deflected shape
x (m)

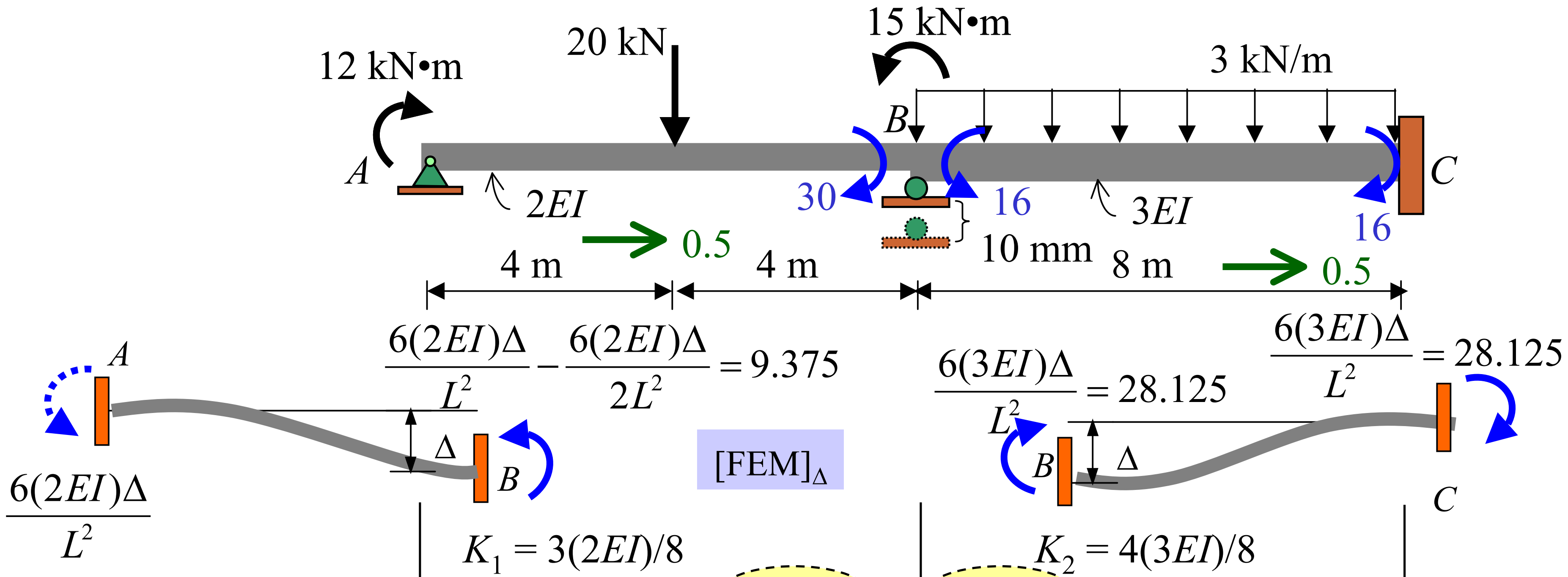
Example 4

The support *B* of the beam shown (E = 200 GPa, I = 50x10^6 mm^4) settles 10 mm. Use the moment distribution method to:

(a) Determine all the reactions at supports, and also

(b) Draw its **quantitative shear** and **bending moment diagrams**, and **qualitative deflected shape.**





$$\frac{6(2EI)\Delta}{L^2} - \frac{6(2EI)\Delta}{2L^2} = 9.375$$

$$\frac{6(3EI)\Delta}{L^2} = 28.125$$

$$\frac{6(3EI)\Delta}{L^2} = 28.125$$

$$\frac{6(2EI)\Delta}{L^2}$$

$K_1 = 3(2EI)/8$

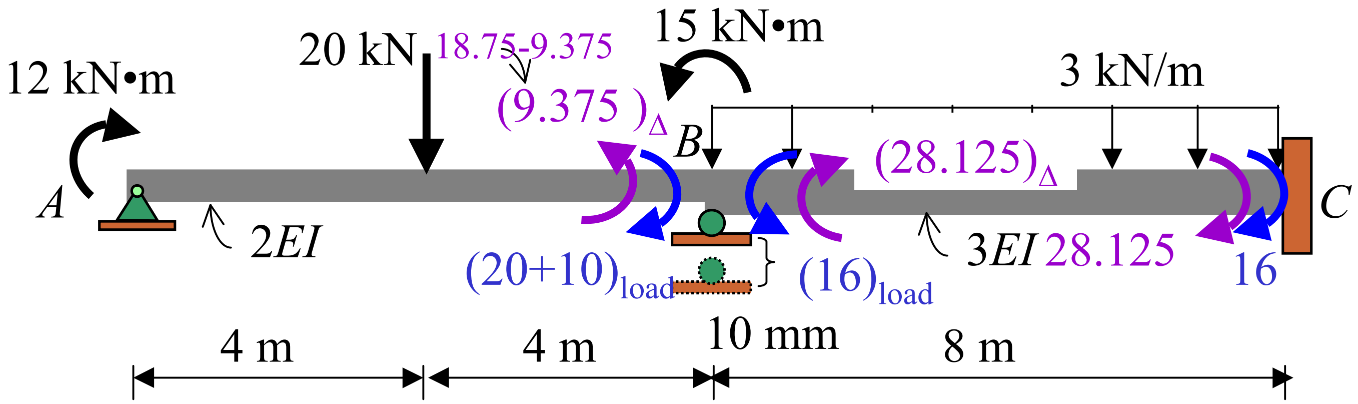
$K_2 = 4(3EI)/8$

$K_1/(K_1+K_2)$

$K_2/(K_1+K_2)$

DF	1	0.333	0.667	0
Joint couple	-12	5	10	
CO		-6		5
[FEM]$_{load}$		-30	16	-16
[FEM]$_\Delta$		9.375	-28.125	-28.125
Dist.		12.90	25.85	
CO				12.92
Σ	**-12**	**-8.72**	**23.72**	**-26.20**

***Note :* Using the slope deflection**



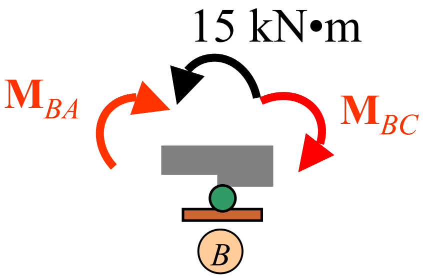
$$\overset{-12}{M_{AB}} = \frac{4(2EI)}{8}\theta_A + \frac{2(2EI)}{8}\theta_B + 20 - 18.75 \quad ---(1)$$

$$M_{BA} = \frac{2(2EI)}{8}\theta_A + \frac{4(2EI)}{8}\theta_B - 20 + 18.75 \quad ---(2)$$

$$\frac{(2)-(1)}{2} : M_{BA} = \frac{3(2EI)}{8}\theta_B - 30 + 9.375 - 12/2 \quad ---(2a)$$

$$M_{BC} = \frac{4(3EI)}{8}\theta_B + 16 - 28.125 \quad ---(3)$$

$$M_{CB} = \frac{2(3EI)}{8}\theta_B - 16 - 28.125 \quad ---(4)$$



$$+\circlearrowleft \ \Sigma M_B = 0: -\mathbf{M}_{BA} - \mathbf{M}_{BC} + 15 = 0$$

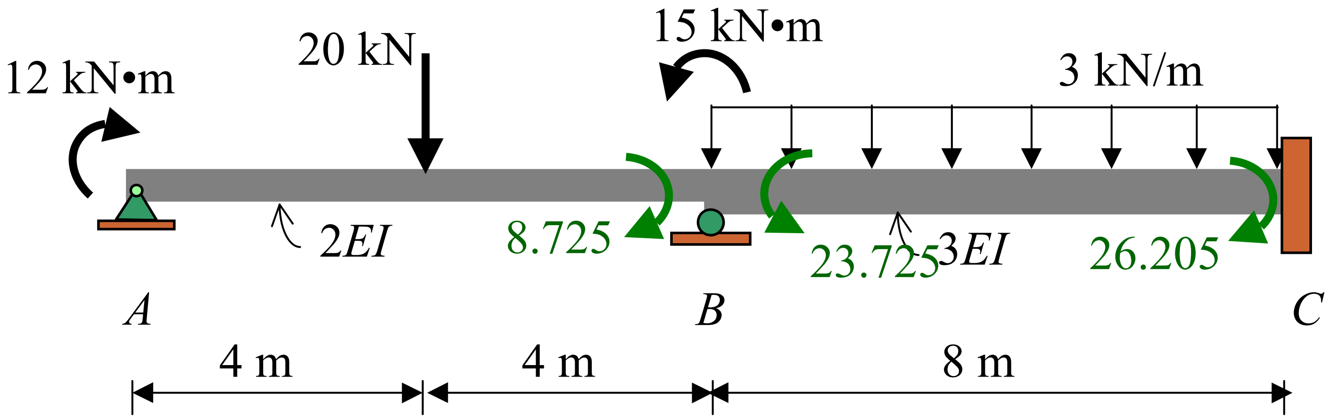
$$(0.75 + 1.5)EI\theta_B - 38.75 - 15 = 0$$

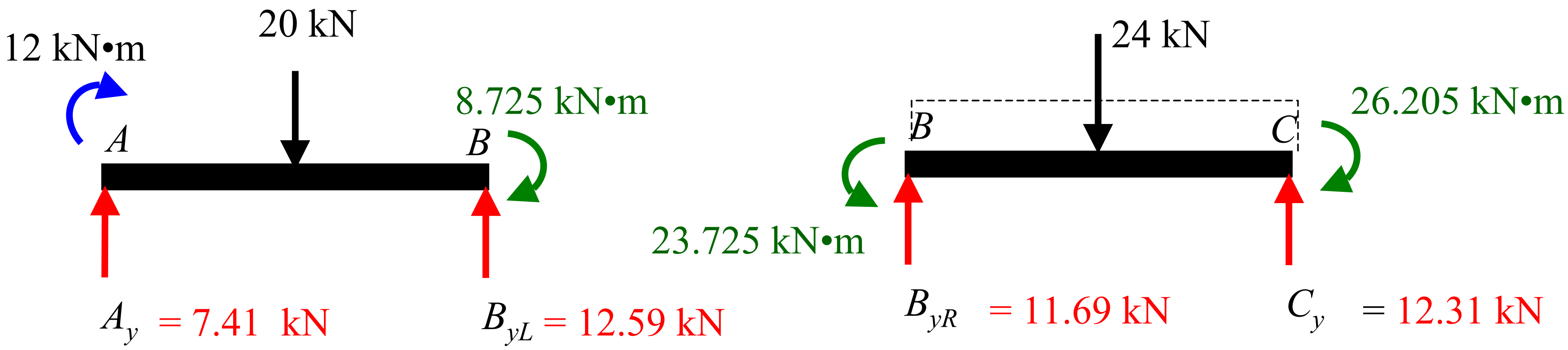
$$\theta_B = 23.9/EI$$

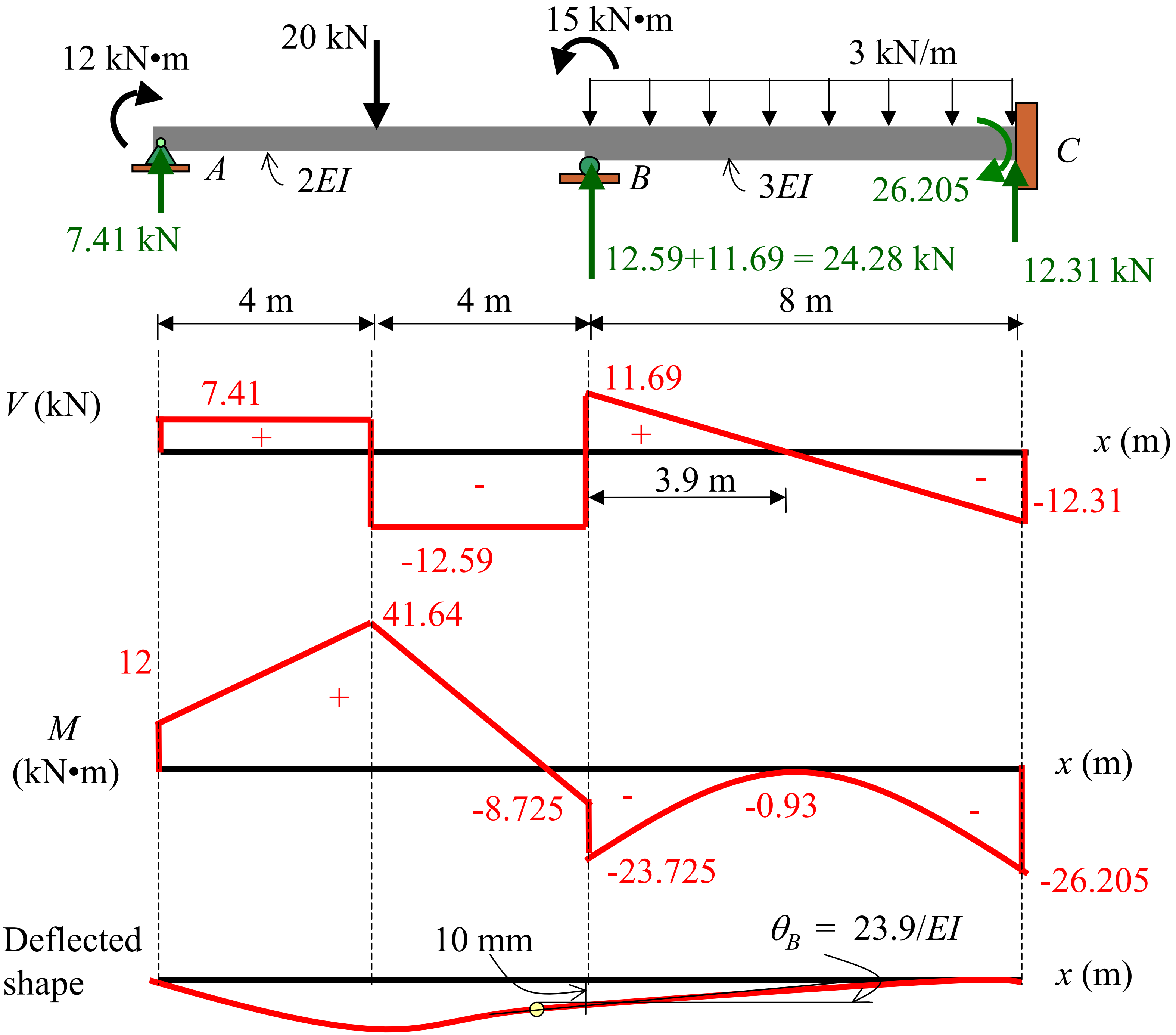
$$M_{BA} = -8.7 \text{ kN•m},$$

$$M_{BC} = 23.72 \text{ kN•m}$$

$$M_{CB} = \frac{2(3EI)}{8}\theta_B - 16 - 28.125$$
$$= -26.2\ kN \bullet m$$


12 kN•m
20 kN
15 kN•m
3 kN/m
2EI
8.725
23.725
3EI
26.205
A
B
C
4 m
4 m
8 m


12 kN•m
20 kN
A
B
8.725 kN•m
A_y = 7.41 kN
B_{yL} = 12.59 kN
24 kN
B
C
26.205 kN•m
23.725 kN•m
B_{yR} = 11.69 kN
C_y = 12.31 kN


12 kN•m
20 kN
15 kN•m
3 kN/m
A
2EI
B
3EI
C
26.205
7.41 kN
12.59+11.69 = 24.28 kN
12.31 kN
4 m
4 m
8 m
V (kN)
7.41
+
11.69
+
x (m)
-
3.9 m
-
-12.31
-12.59
41.64
12
+
M
(kN•m)
x (m)
-8.725
-
-0.93
-
-23.725
-26.205
Deflected
shape
10 mm
θ_B = 23.9/EI
x (m)

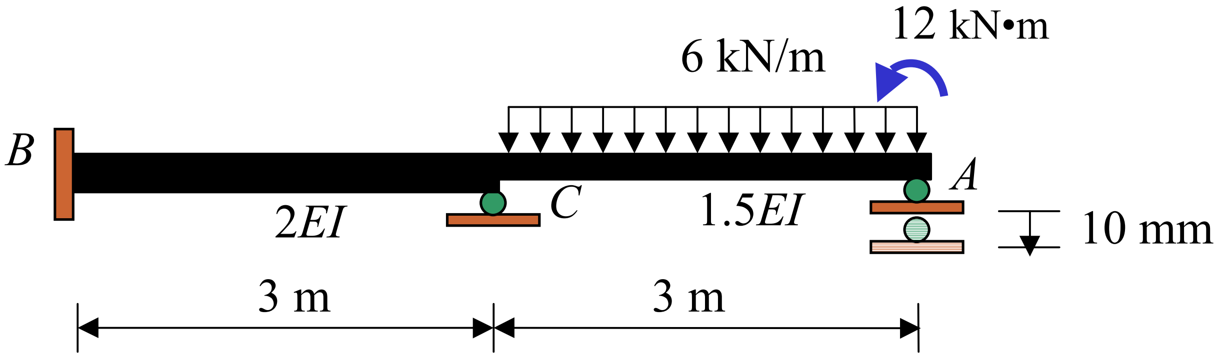
Example 5

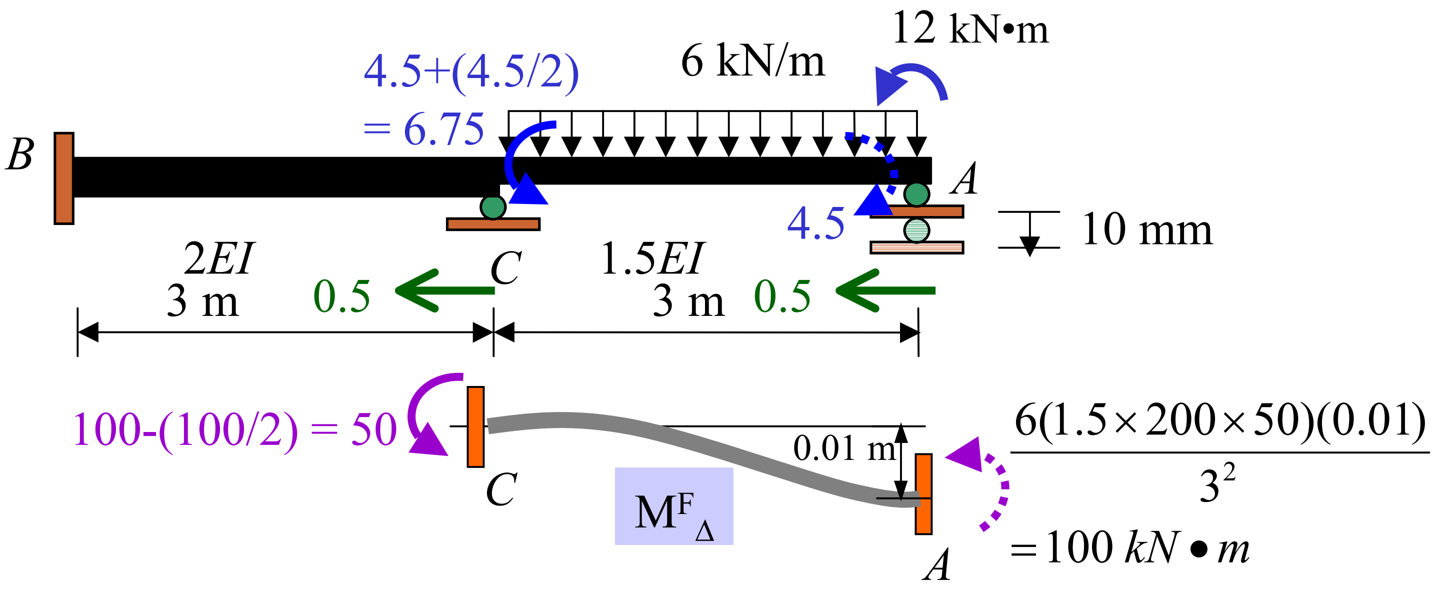
For the beam shown, support A settles 10 mm downward, use the moment distribution method to

(a)Determine all the **reactions** at supports

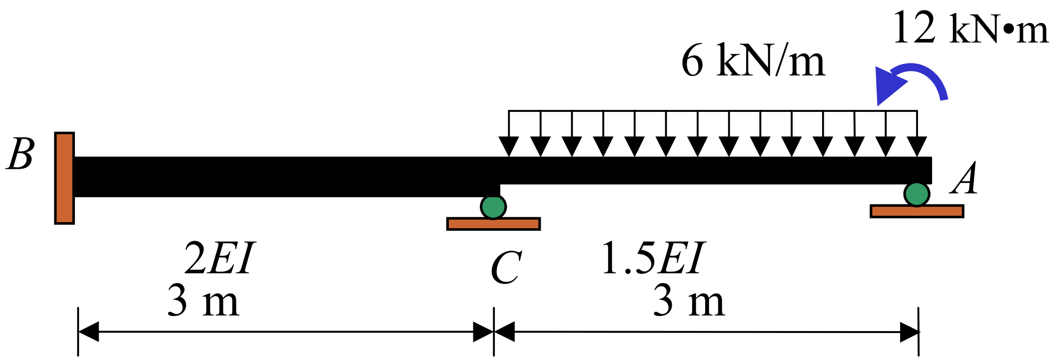
(b)Draw its **quantitative shear, bending moment diagrams**, and **qualitative deflected shape**.

Take E= 200 GPa, I = 50(10^6) mm^4.





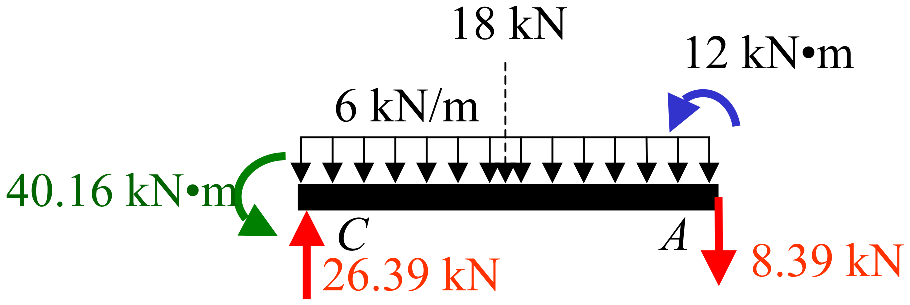
	$K_1 = 4(2EI)/3$		$K_2 = 3(1.5EI)/3$	
		$K_1/(K_1+K_2)$	$K_2/(K_1+K_2)$	
DF	0	0.64	0.36	1
Joint couple				12
CO			6	
[FEM]$_{\text{load}}$			6.75	
[FEM]$_\Delta$			50	
Dist.		-40.16	-22.59	
CO	-20.08			
Σ	**-20.08**	**-40.16**	**40.16**	**12**

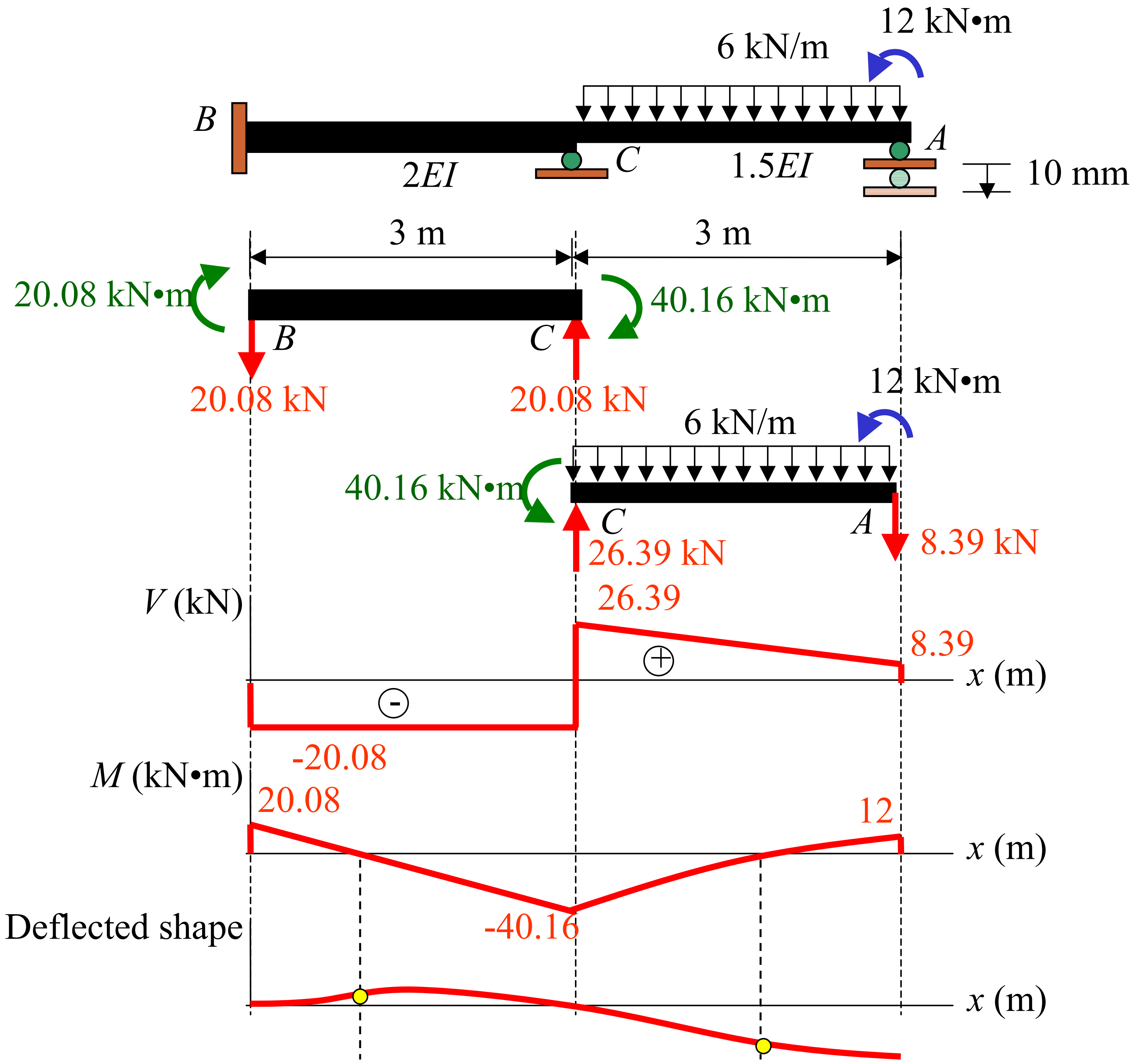


ΣM	**-20.08**	**-10.16**	**40.16**	**12**

20.08 kN

$$\frac{40.16+20.08}{3}=20.08\ kN$$




12 kN•m
6 kN/m
B
2EI
C
1.5EI
A
10 mm
3 m
3 m
20.08 kN•m
40.16 kN•m
B
C
20.08 kN
20.08 kN
12 kN•m
6 kN/m
40.16 kN•m
C
A
26.39 kN
8.39 kN
V (kN)
26.39
8.39
x (m)
-20.08
M (kN•m)
20.08
12
x (m)
-40.16
Deflected shape
x (m)

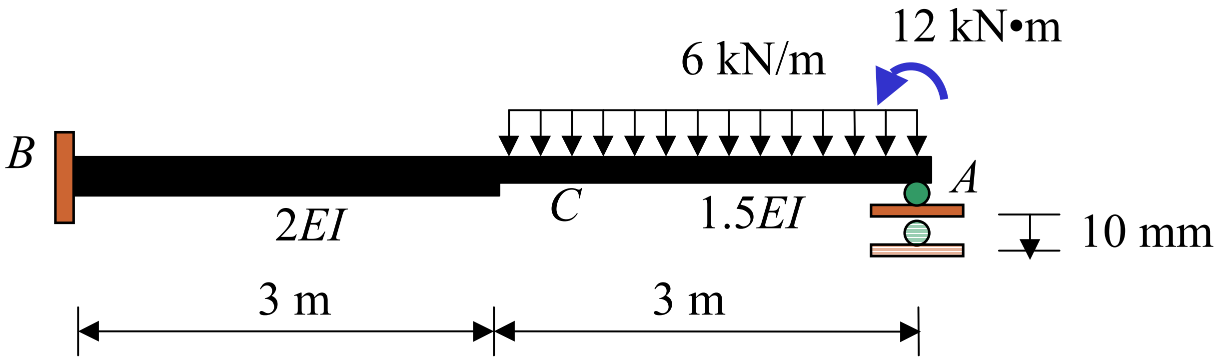
Example 6

For the beam shown, support A settles 10 mm downward, use the moment distribution method to

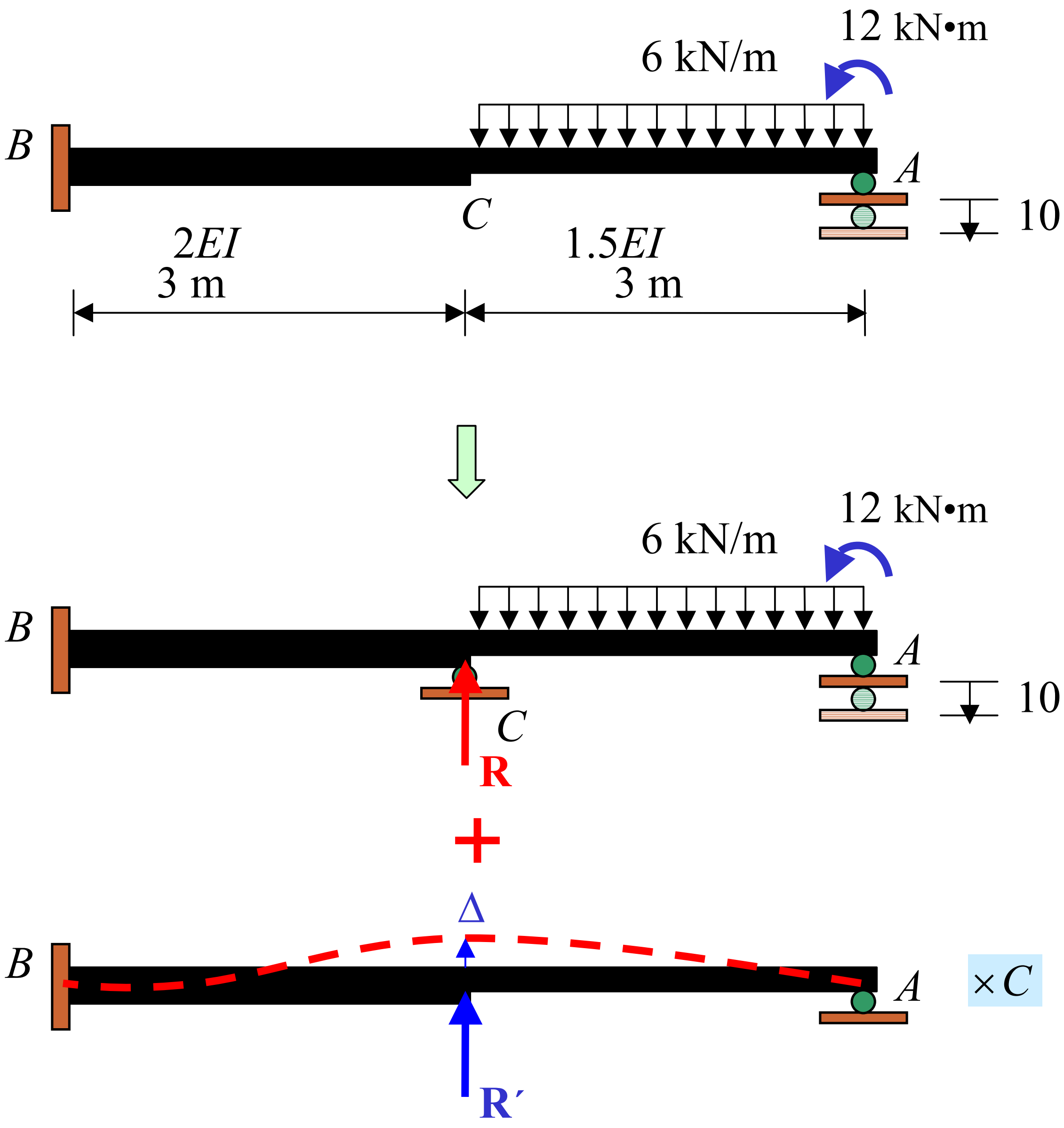
(a)Determine all the **reactions** at supports

(b)Draw its **quantitative shear, bending moment diagrams**, and **qualitative deflected shape**.

Take E= 200 GPa, I = 50(10^6) mm^4.

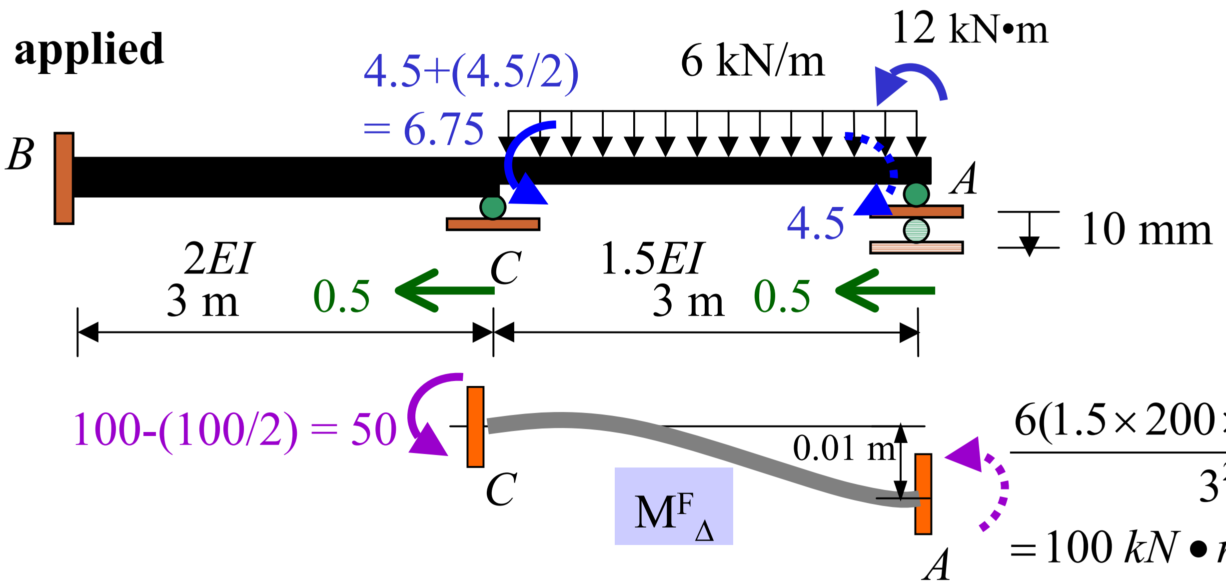


- **Overview**



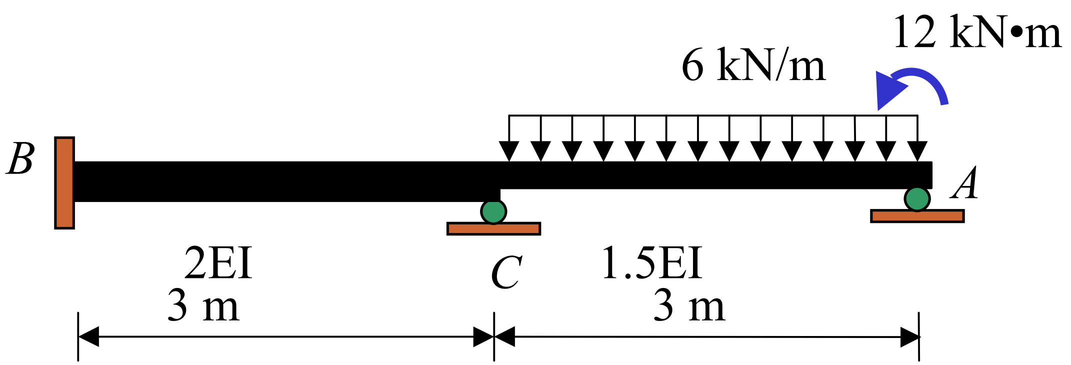
$$R + R'C = 0 \quad ---(1^*)$$

- **Artificial joint applied**



$$\frac{6(1.5\times200\times50)(0.01)}{3^2}$$
$$=100\ kN\bullet m$$

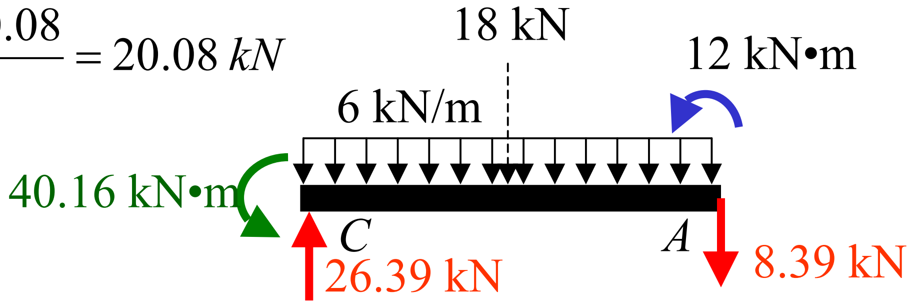
	$K_1 = 4(2EI)/3$		$K_2 = 3(1.5EI)/3$	
		$K_1/(K_1+K_2)$	$K_2/(K_1+K_2)$	
DF	0	0.64	0.36	1
Joint couple				12
CO			6	
$[\mathrm{FEM}]_{\mathrm{load}}$			6.75	
$[\mathrm{FEM}]_{\Delta}$			50	
Dist.		-40.16	-22.59	
CO	-20.08			
Σ	**-20.08**	**-40.16**	**40.16**	**12**

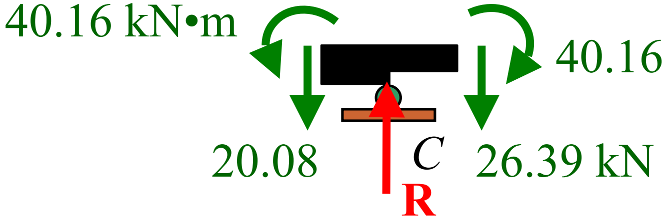


ΣM	**-20.08**	**-40.16**	**40.16**	**12**

$20.08\ kN$

$$\frac{40.16+20.08}{3}=20.08\ kN$$

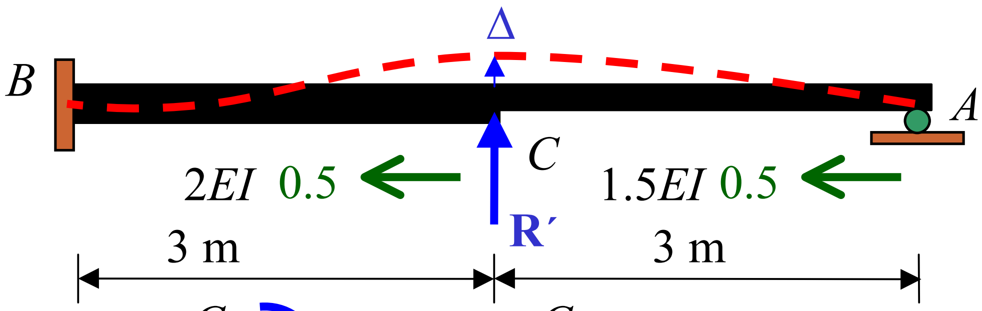




$$+\uparrow\quad \Sigma F_y=0:\quad -20.08-26.39+R=0$$

$$R=46.47\ kN$$

- **Artificial joint removed**

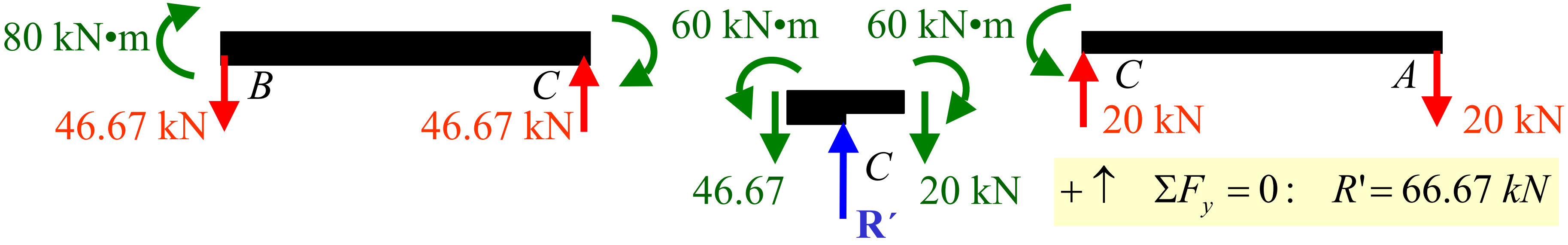


$$\frac{6(2EI)\Delta_C}{3^2} = 100 \rightarrow \Delta = \frac{75}{EI}$$

75-(75/2)
= 37.5

$$\frac{6(1.5EI)(\frac{75}{EI})}{3^2} = 75$$

DF	0	0.64	0.36	1
$[\mathbf{FEM}]_\Delta$	-100	-100	+37.5	
Dist.		40	22.5	
CO	20			
Σ	**-80**	**-60**	**60**	



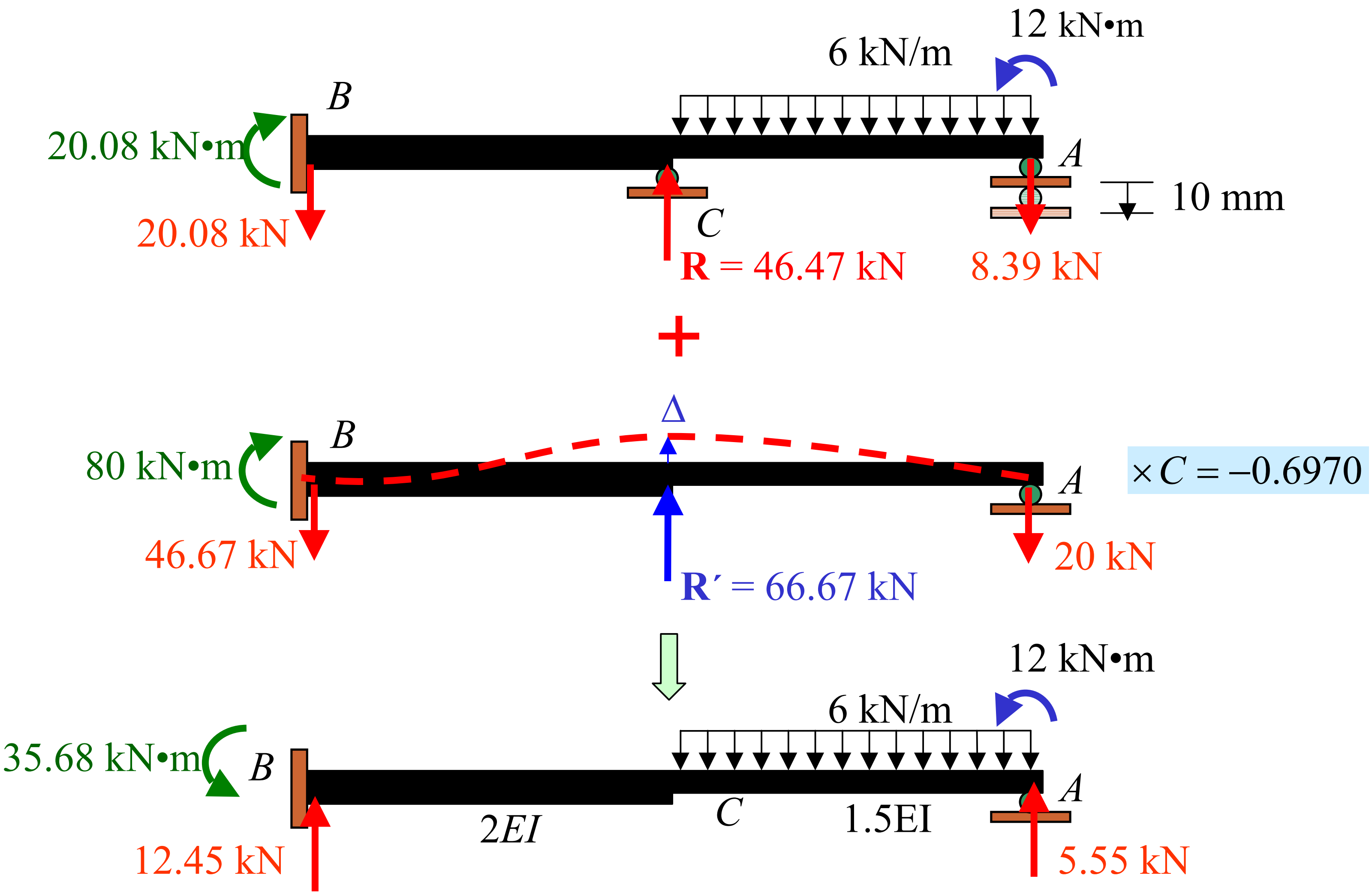
$$+\uparrow \quad \Sigma F_y = 0: \quad R' = 66.67\ kN$$

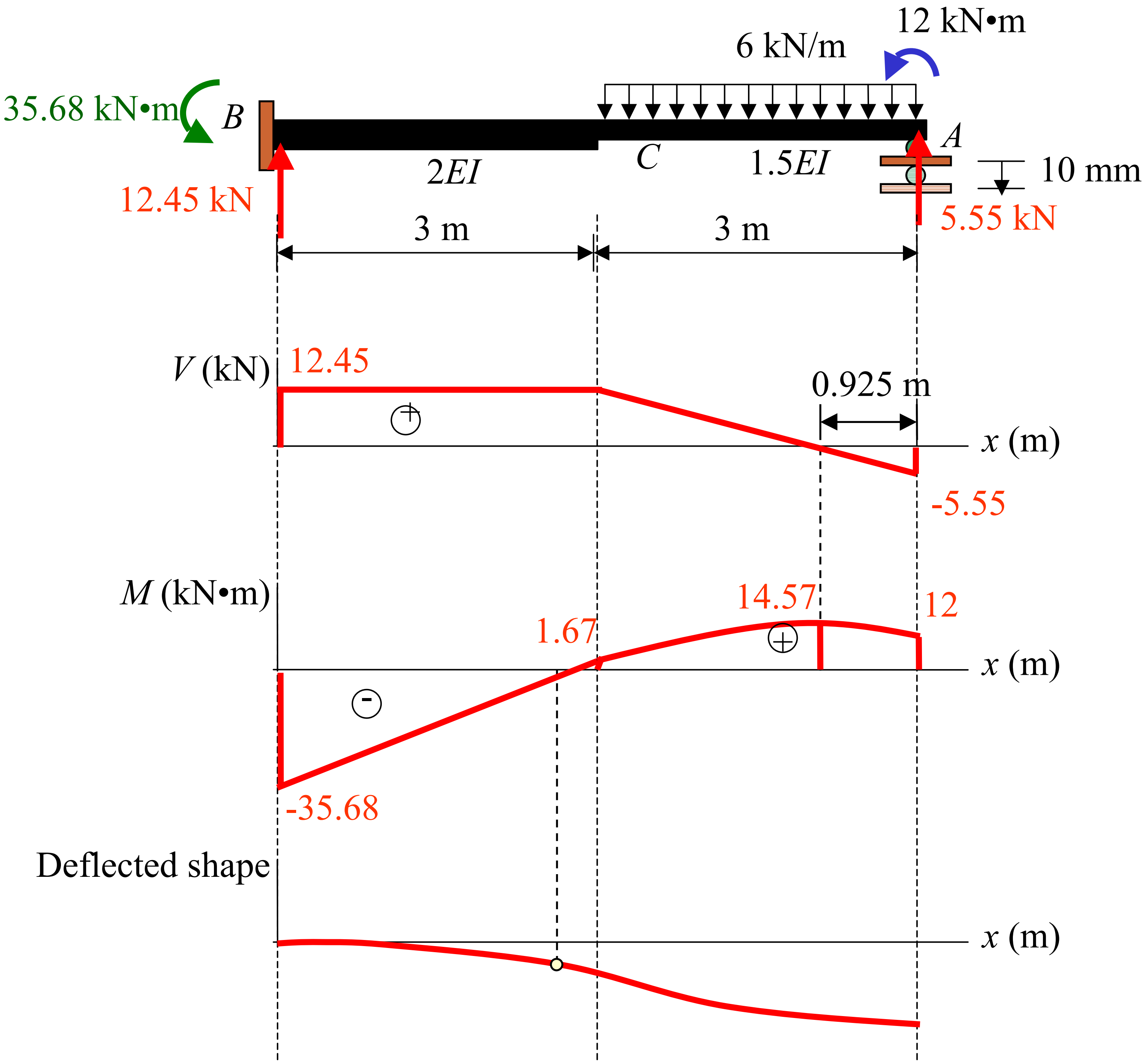
- **Solve equation**

$$Substitute\ R = 46.47\ kN\ and\ R' = 66.67\ kN\ in\ (1^*)$$

$$46.47 + 66.67C = 0$$

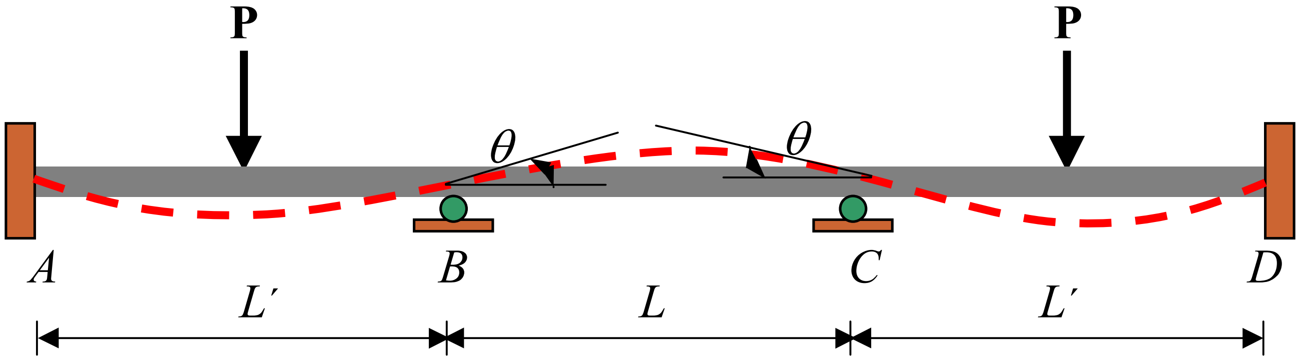
$$C = -0.6970$$



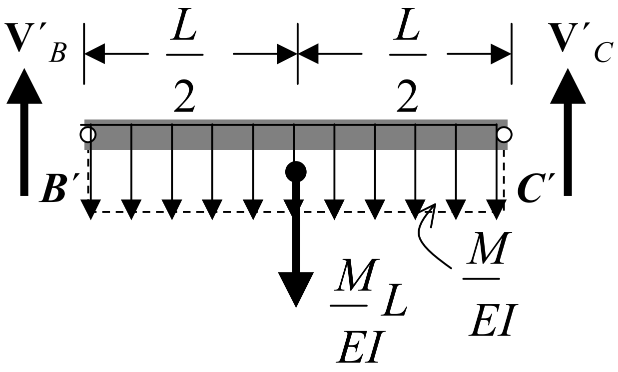

12 kN•m
6 kN/m
35.68 kN•m
B
A
C
2EI
1.5EI
10 mm
12.45 kN
5.55 kN
3 m
3 m
V (kN)
12.45
0.925 m
x (m)
+
-5.55
M (kN•m)
14.57
12
1.67
+
x (m)
-
-35.68
Deflected shape
x (m)

Symmetric Beam

- **Symmetric Beam and Loading**



real beam



conjugate beam

$$\overset{+}{\circlearrowright}\ \Sigma M_{C'} = 0: \qquad -V_{B'}(L) + \frac{M}{EI}(L)(\frac{L}{2}) = 0$$

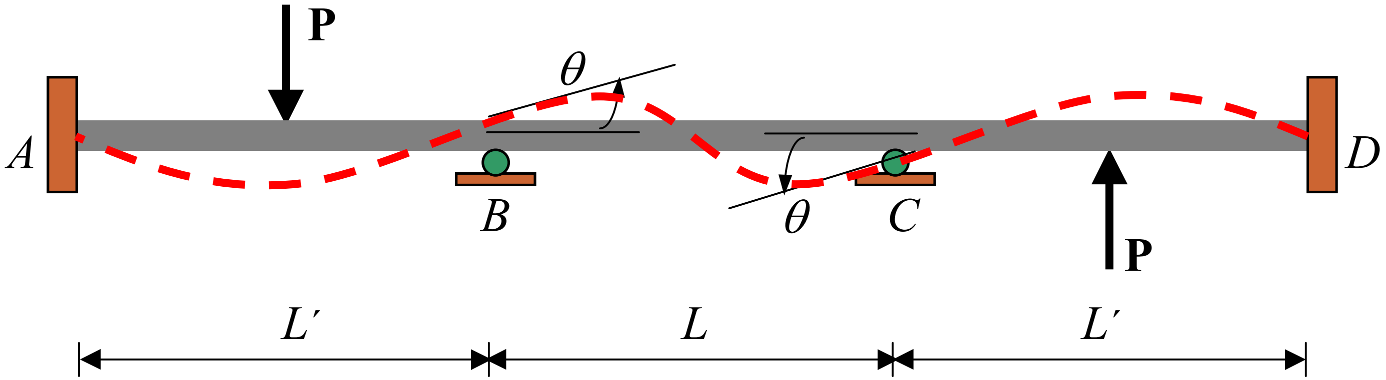
$$V_{B'} = \theta = \frac{ML}{2EI}$$

$$M = \frac{2EI}{L}\theta$$

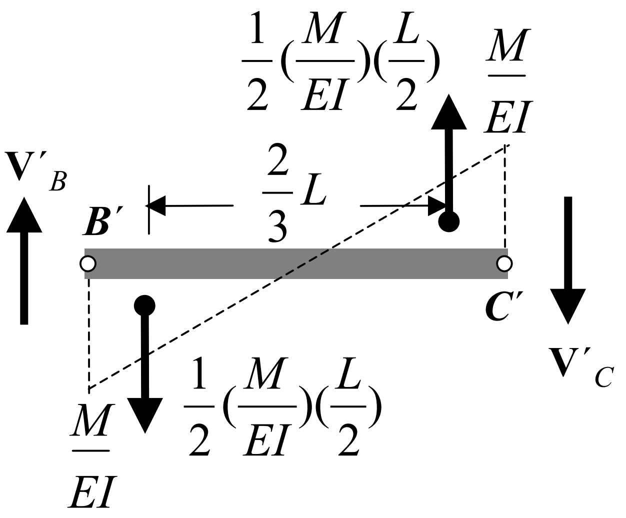
The stiffness factor for the center span is, therefore,

$$\boxed{K = \frac{2EI}{L}}$$

- **Symmetric Beam with Antisymmetric Loading**



real beam



conjugate beam

$$+\curvearrowright \Sigma M_{C'} = 0: \quad -V_{B'}(L) + \frac{1}{2}(\frac{M}{EI})(\frac{L}{2})(\frac{2L}{3}) = 0$$

$$V_{B'} = \theta = \frac{ML}{6EI}$$

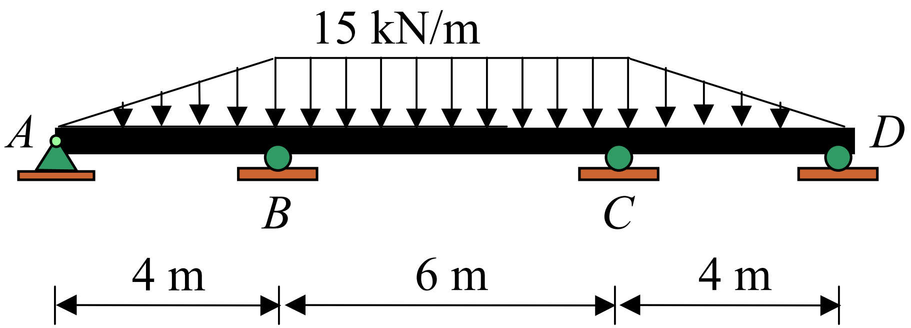
$$M = \frac{6EI}{L}\theta$$

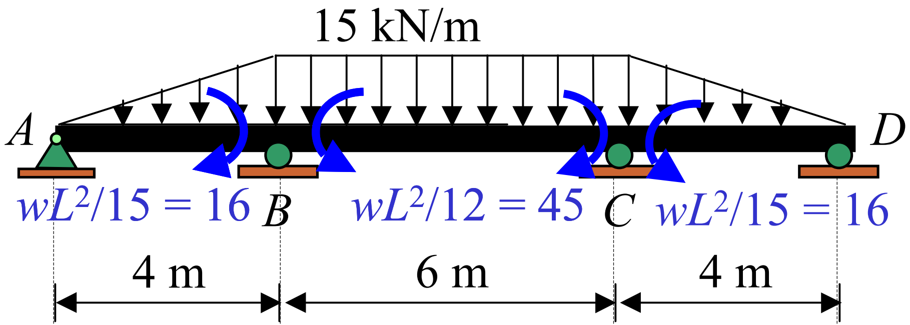
The stiffness factor for the center span is, therefore,

$$\boxed{K = \frac{6EI}{L}}$$

Example 5a

Determine all the reactions at supports for the beam below. *EI* is constant.



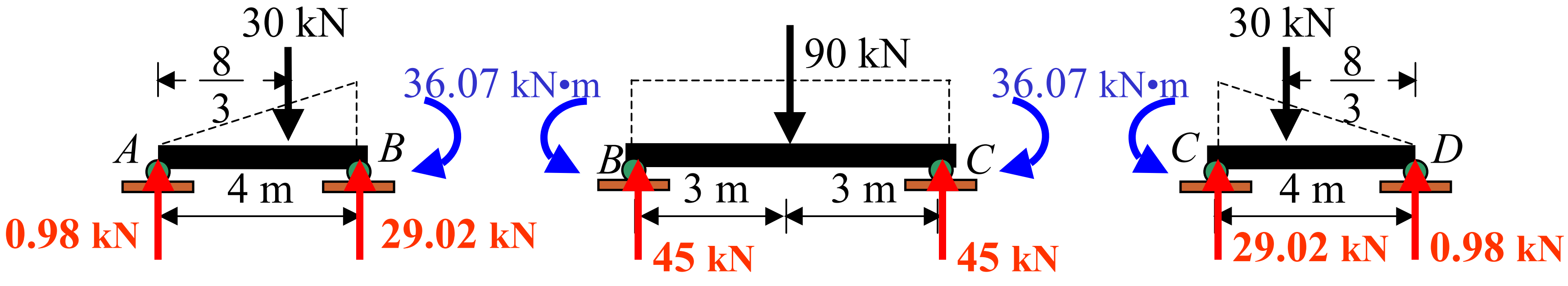


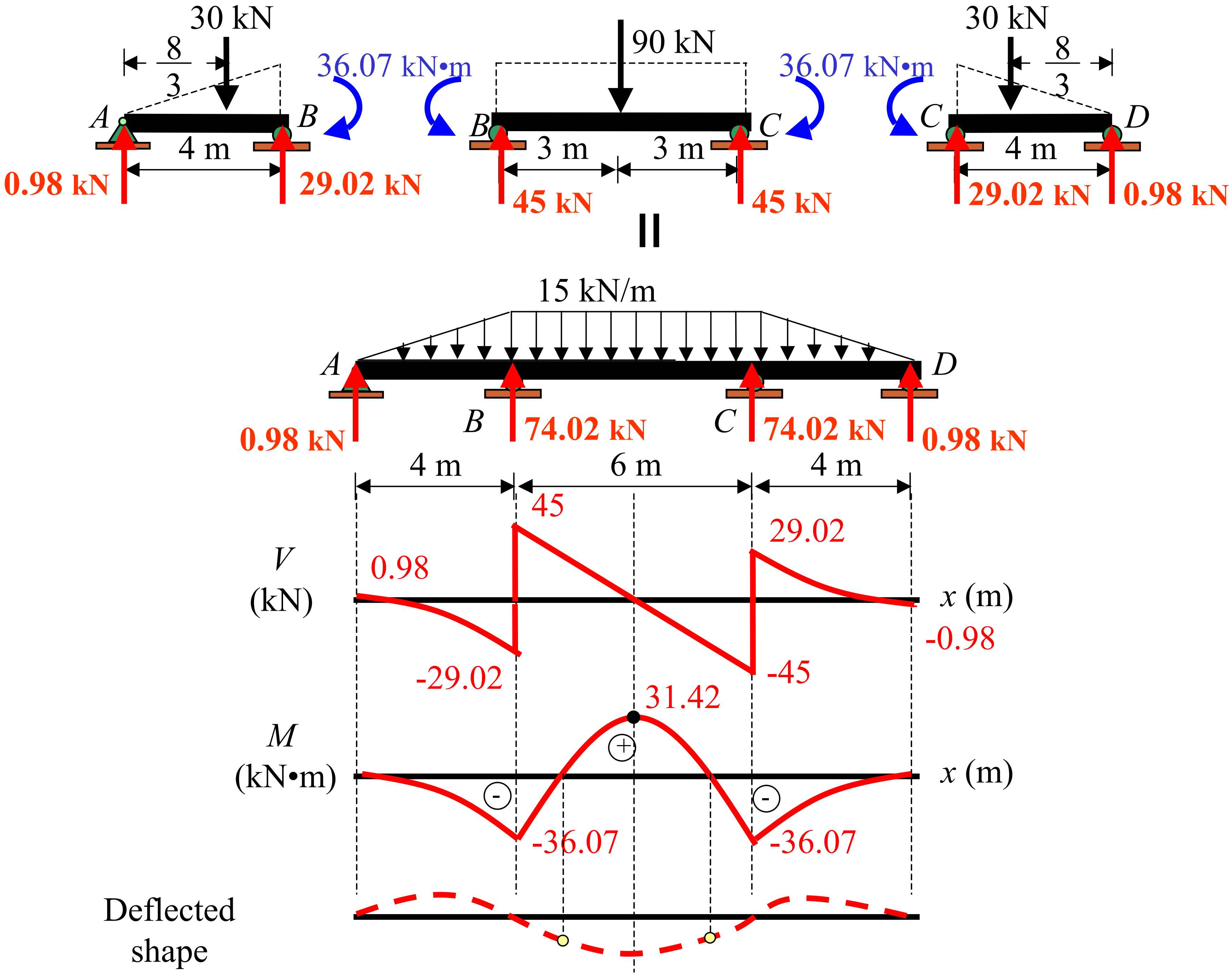
$$K_{(AB)} = \frac{3EI}{L} = \frac{3EI}{4}, \quad K_{(BC)} = \frac{2EI}{L} = \frac{2EI}{6}$$

$$(DF)_{AB} = \frac{K_{(AB)}}{K_{(AB)}} = 1, (DF)_{BA} = \frac{K_{(AB)}}{K_{(AB)} + K_{(BC)}} = \frac{(3EI/4)}{(3EI/4) + (2EI/6)} = 0.692,$$

$$(DF)_{BC} = \frac{K_{(BC)}}{K_{(AB)} + K_{(BC)}} = \frac{(2EI/6)}{(3EI/4) + (2EI/6)} = 0.308$$

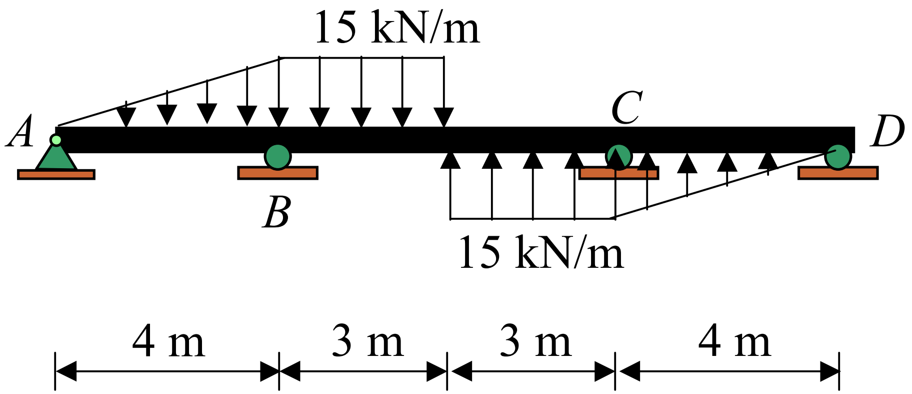
DF	1.0	0.692	0.308
[FEM]$_{\text{load}}$	0	-16	+45
Dist.		-20.07	-8.93
ΣM		-36.07	+36.07



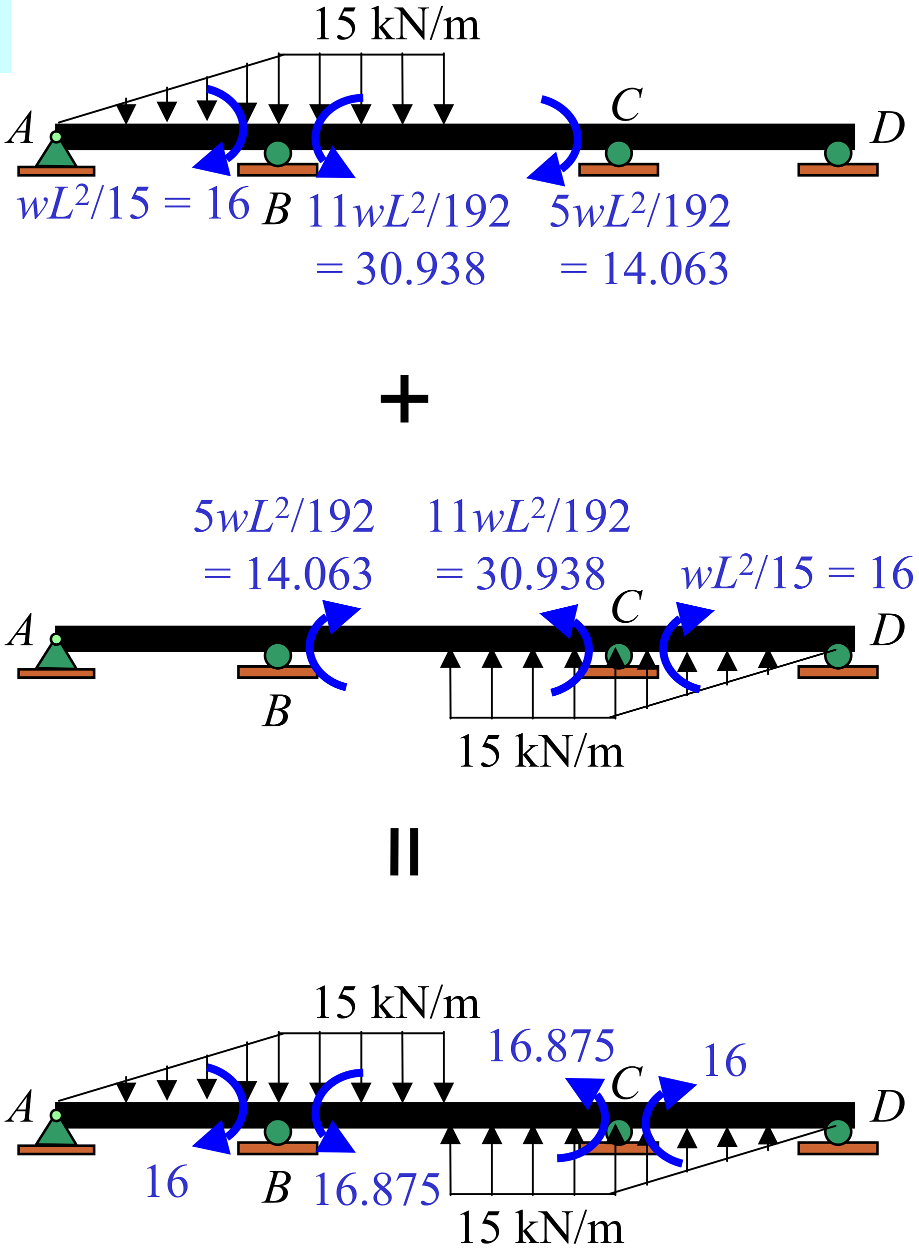

30 kN
8/3
36.07 kN•m
A
B
4 m
0.98 kN
29.02 kN
90 kN
36.07 kN•m
B
C
3 m
3 m
45 kN
45 kN
30 kN
8/3
C
D
4 m
29.02 kN
0.98 kN
=
15 kN/m
A
D
B
C
0.98 kN
74.02 kN
74.02 kN
0.98 kN
4 m
6 m
4 m
V
(kN)
45
0.98
29.02
x (m)
-0.98
-29.02
-45
M
(kN•m)
31.42
+
-
-
x (m)
-36.07
-36.07
Deflected
shape

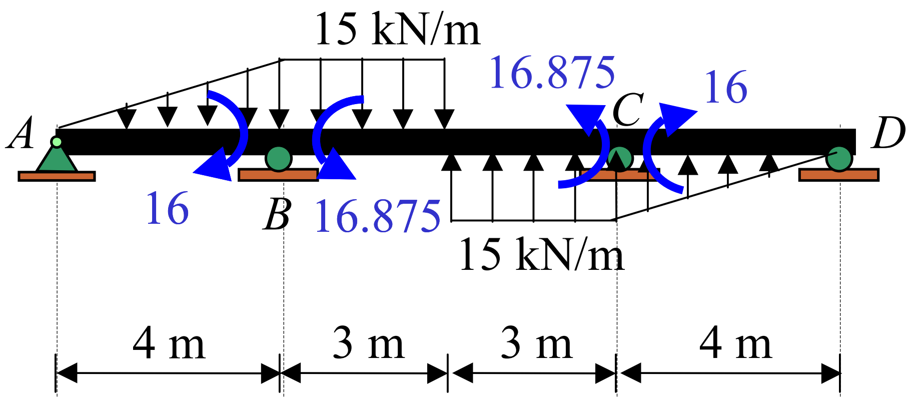
Example 5b

Determine all the reactions at supports for the beam below. *EI* is constant.



Fixed End Moment

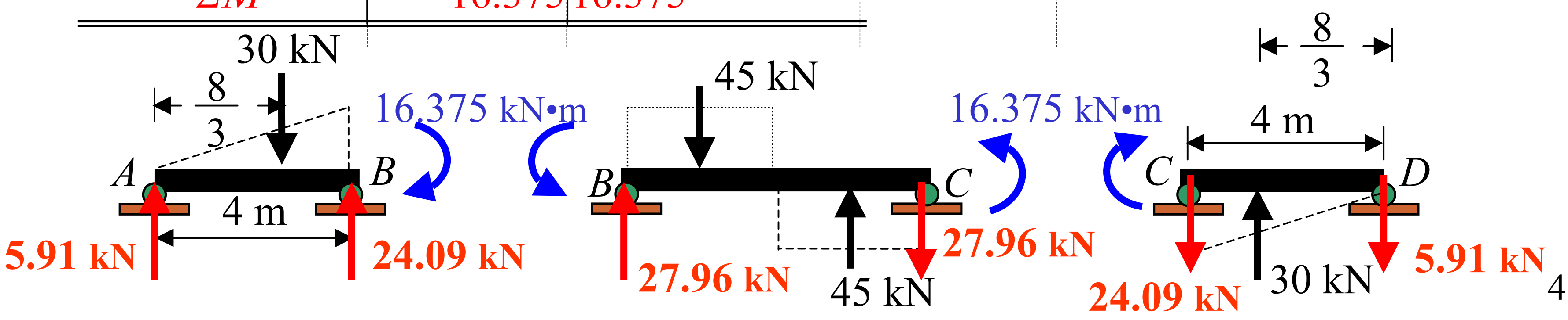


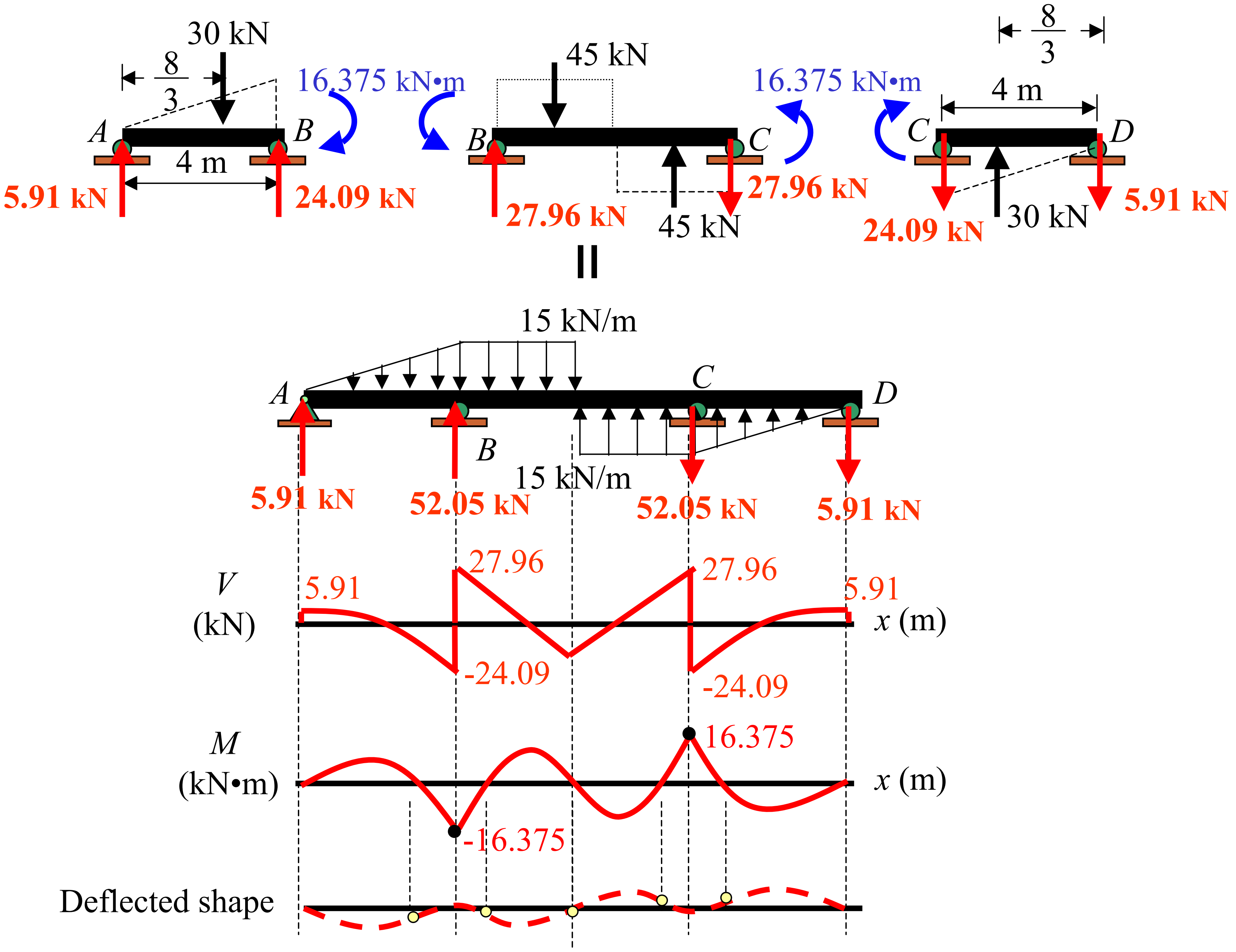


$$K_{(AB)}=\frac{3EI}{L}=\frac{3EI}{4}=0.75EI,\quad K_{(BC)}=\frac{6EI}{L}=\frac{6EI}{6}=EI$$

$$(DF)_{AB}=1,\qquad (DF)_{BA}=\frac{0.75}{0.75+1}=0.429,\qquad (DF)_{BC}=\frac{1}{0.75+1}=0.571$$

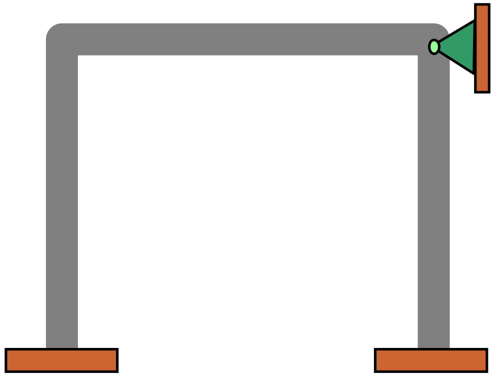
DF	1.0	0.429	0.571
[FEM]$_{\text{load}}$	0	-16	16.875
Dist.		-0.375	-0.50
ΣM		-16.375	16.375





30 kN
8
3
16.375 kN•m
A
B
4 m
5.91 kN
24.09 kN
45 kN
B
C
27.96 kN
45 kN
16.375 kN•m
27.96 kN
8
3
4 m
C
D
24.09 kN
30 kN
5.91 kN
=
15 kN/m
A
C
D
B
15 kN/m
5.91 kN
52.05 kN
52.05 kN
5.91 kN
V
(kN)
5.91
27.96
-24.09
27.96
-24.09
5.91
x (m)
M
(kN•m)
16.375
-16.375
x (m)
Deflected shape

Moment Distribution Frames: No Sidesway

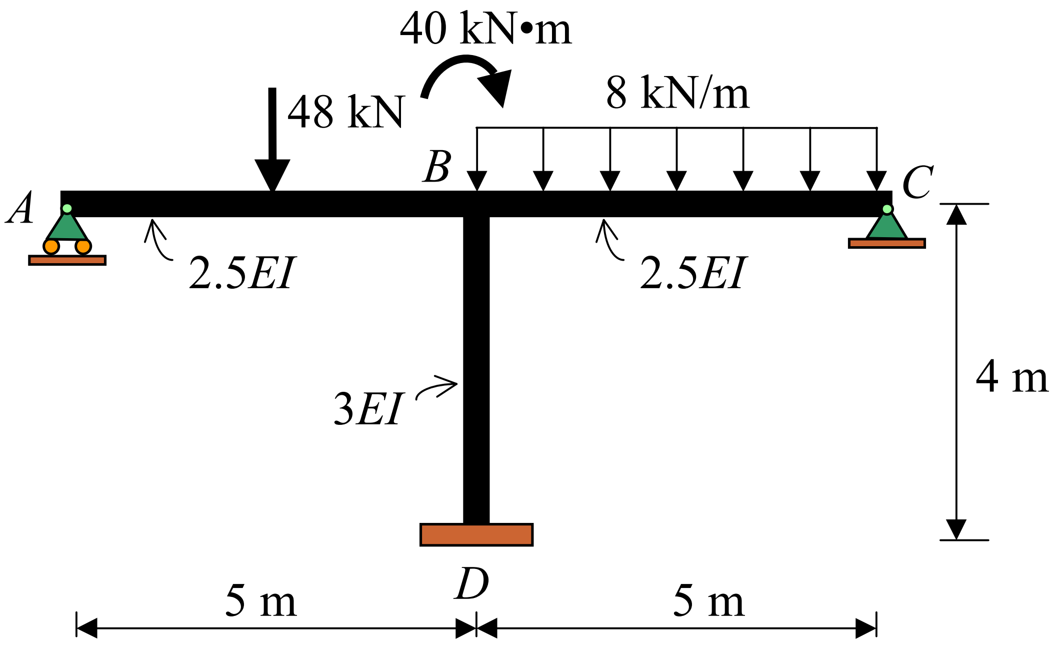


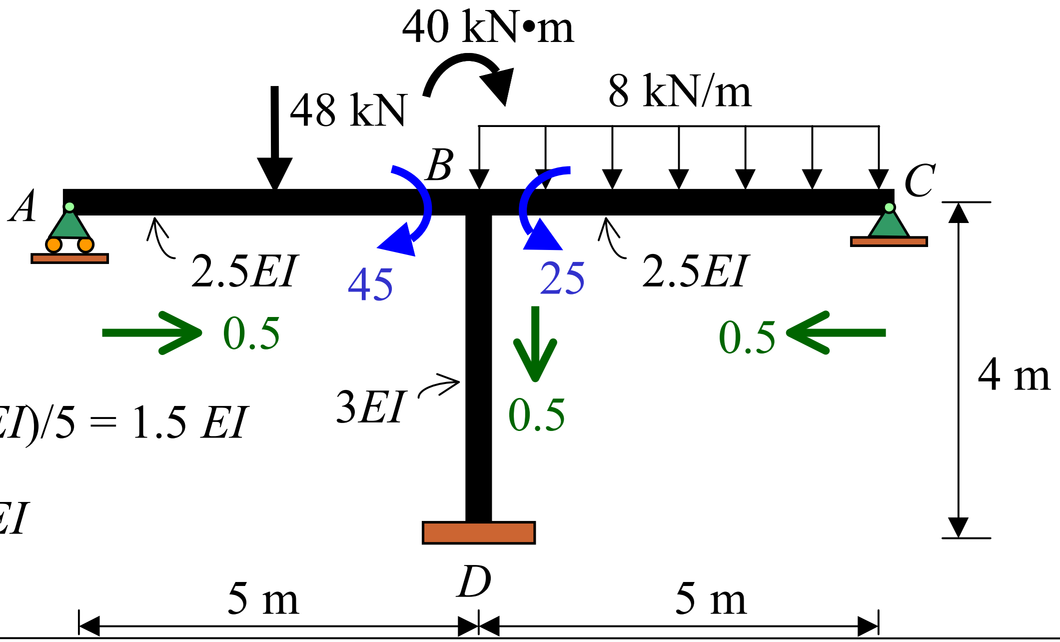
Example 6

From the frame shown use the moment distribution method to:

(a) Determine all the **reactions** at supports

(b) Draw its **quantitative shear** and **bending moment diagrams**, and **qualitative deflected shape.**

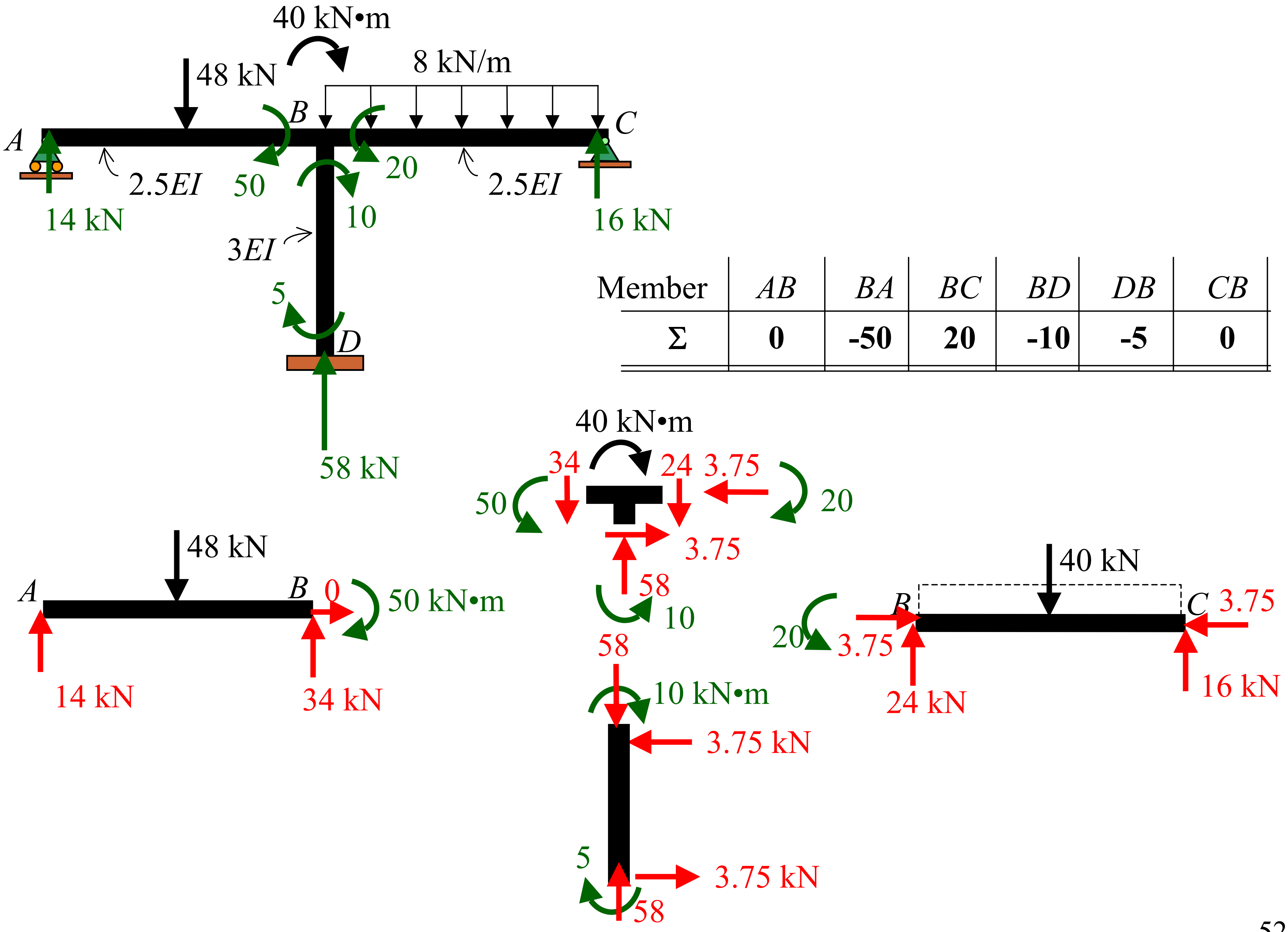




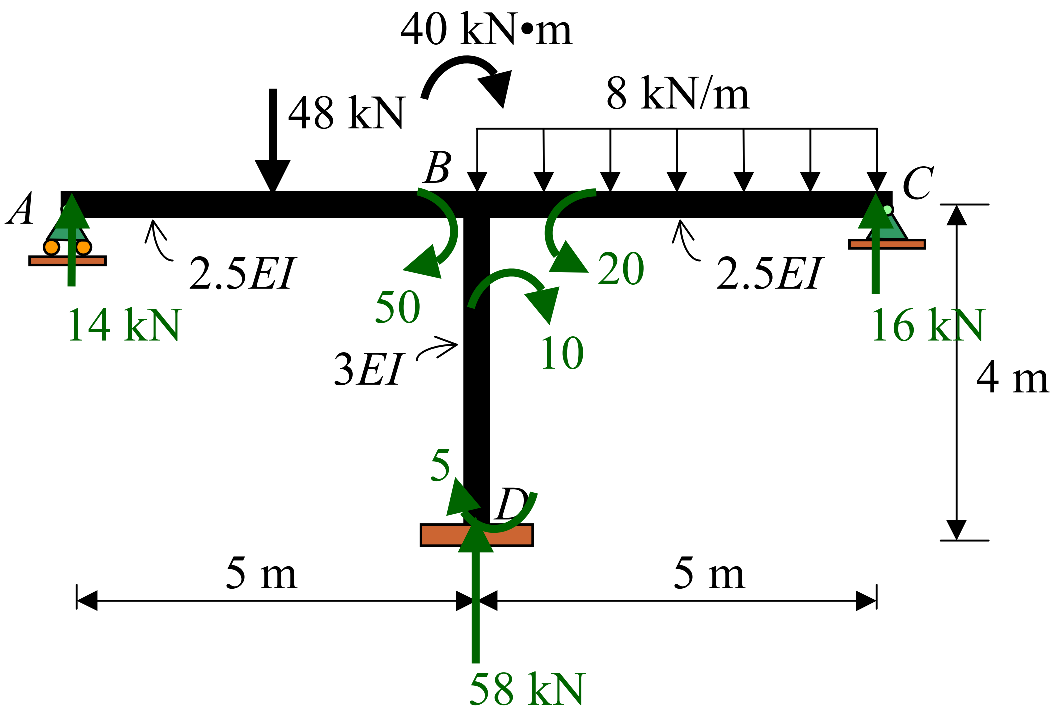
$K_{AB} = K_{BC} = 3(2.5EI)/5 = 1.5\ EI$

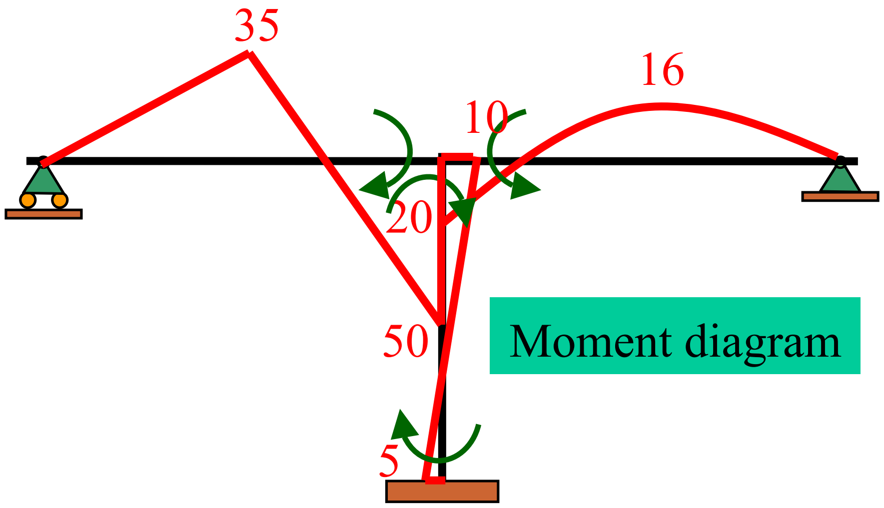
$K_{BD} = 4(3EI)/4 = 3EI$

	A	B			D	C
Member	AB	BA	BC	BD	DB	CB
DF	1	0.25	0.25	0.5	0	1
Joint load		-10	-10	-20		
CO					-10	
FEM		-45	25			
Dist.		5	5	10		
CO					5	
Σ	**0**	**-50**	**20**	**-10**	**-5**	**0**

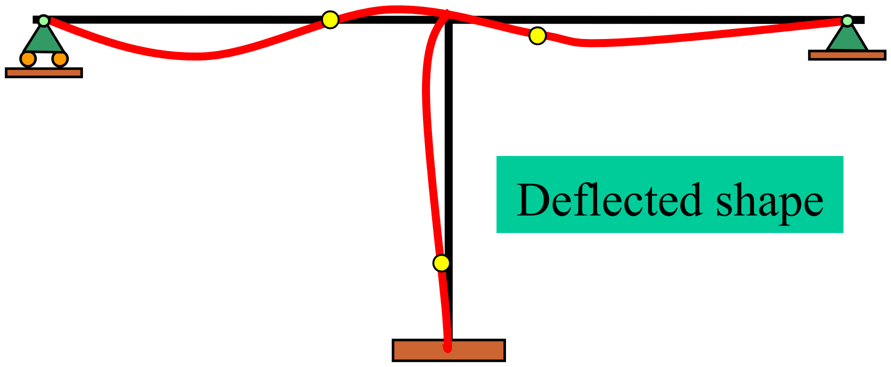


Member	*AB*	*BA*	*BC*	*BD*	*DB*	*CB*
Σ	**0**	**-50**	**20**	**-10**	**-5**	**0**



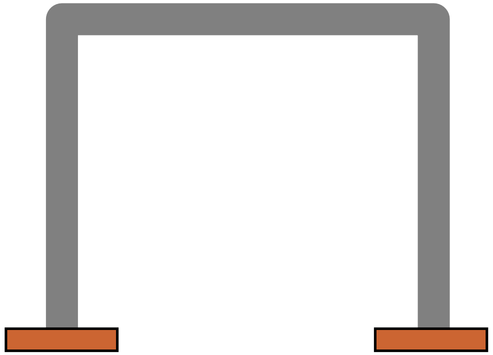


Moment diagram

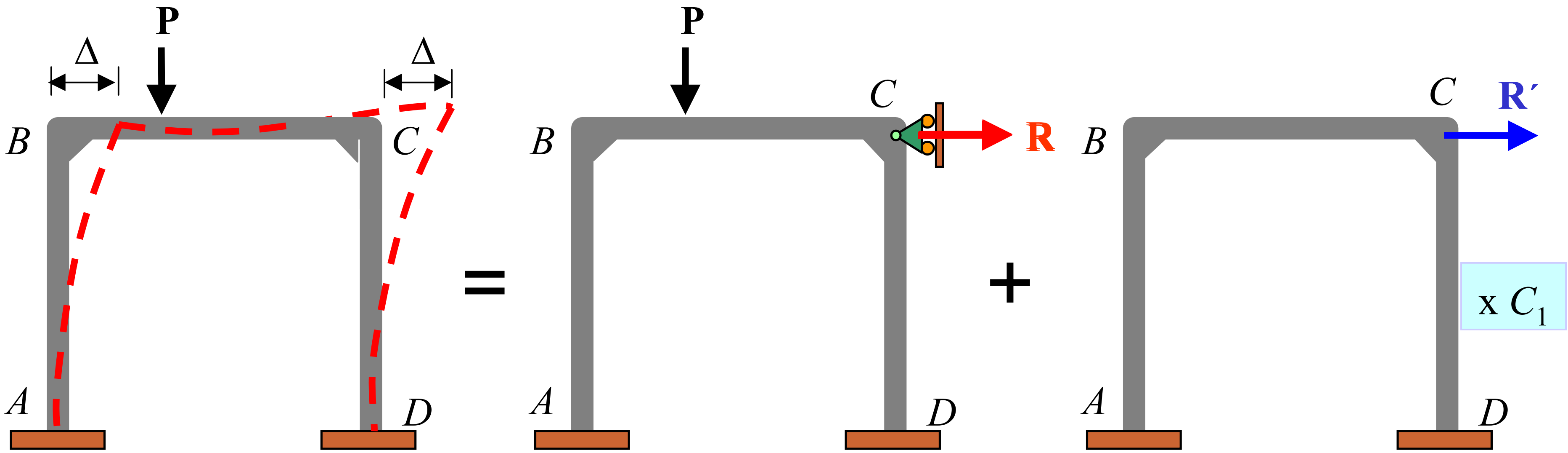


Deflected shape

Moment Distribution for Frames: Sidesway



Single Frames

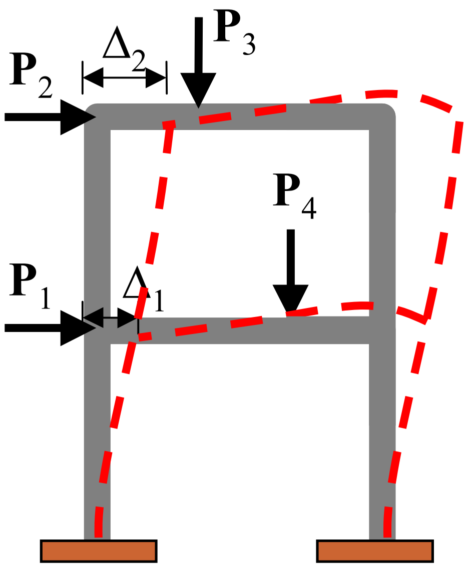


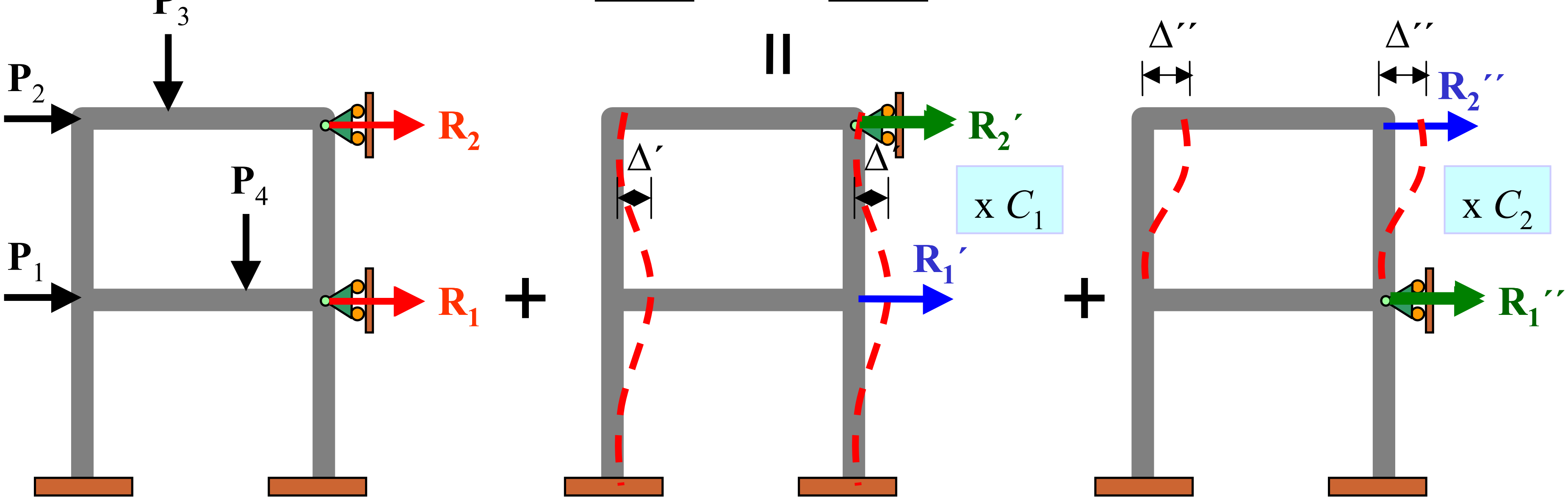
Artificial joint applied
(no sidesway)

Artificial joint removed
(sidesway)

$$\boxed{0 = R + C_1 R'}$$

Multistory Frames





$$0 = R_2 + C_1R_2' + C_2R_2''$$

$$0 = R_1 + C_1R_1' + C_2R_2''$$

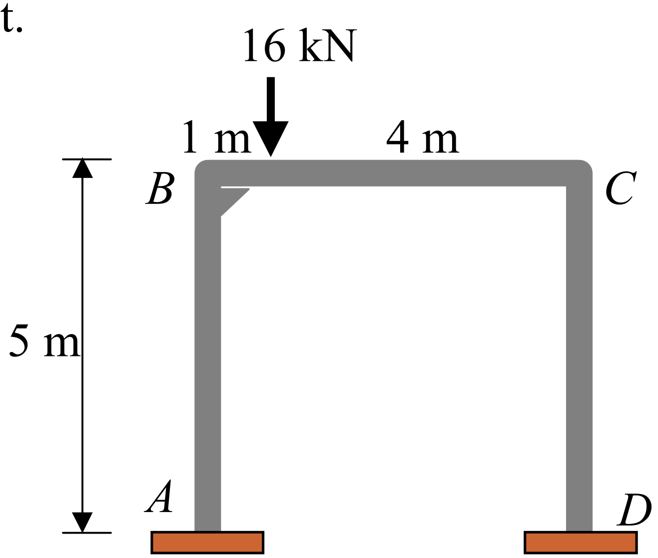
Example 7

From the frame shown use the moment distribution method to:

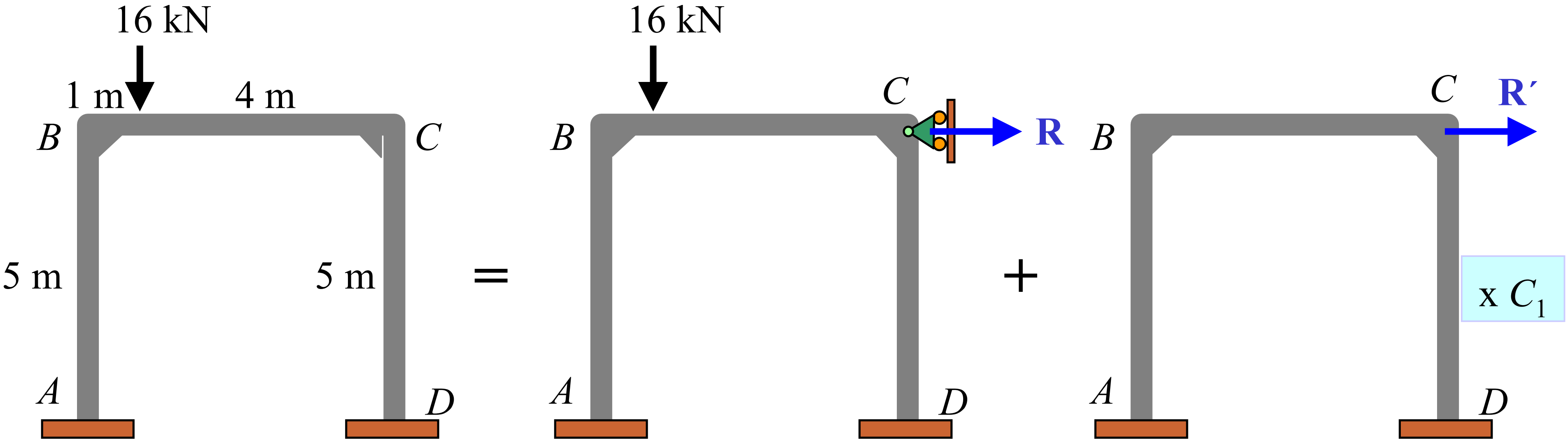
(a) Determine all the reactions at supports, and also

(b) Draw its **quantitative shear** and **bending moment diagrams**, and **qualitative deflected shape.**

EI is constant.



- **Overview**



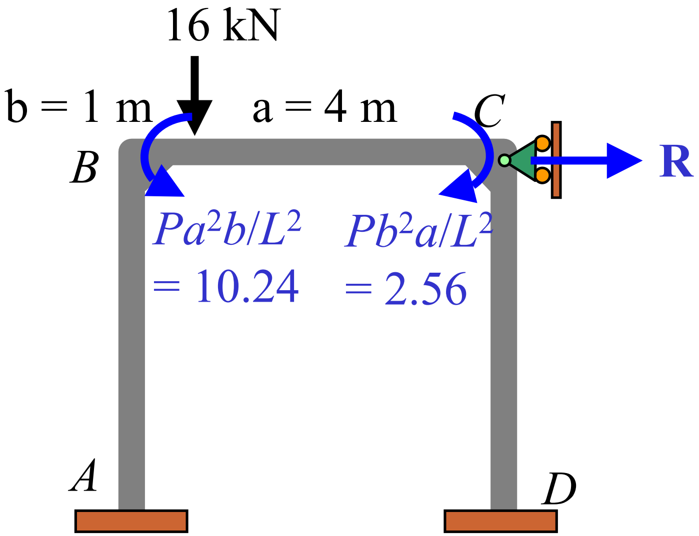
artificial joint applied
(no sidesway)

artificial joint removed
(sidesway)

$$\boxed{R + C_1R' = 0}$$ ---------(1)

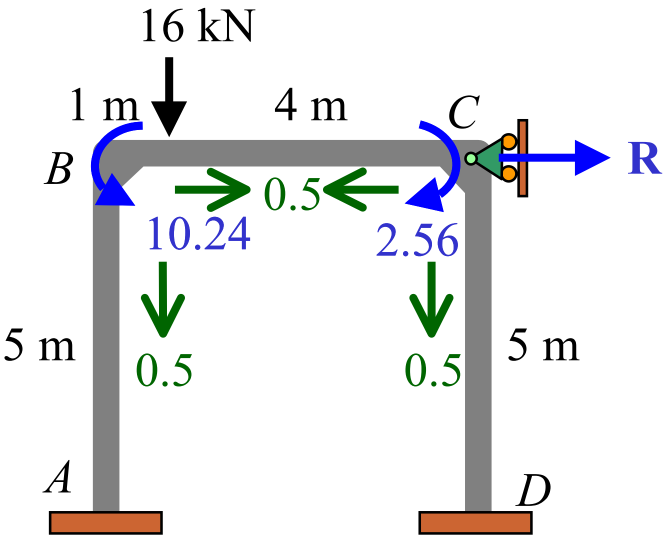
- **Artificial joint applied (no sidesway)**

Fixed end moment:

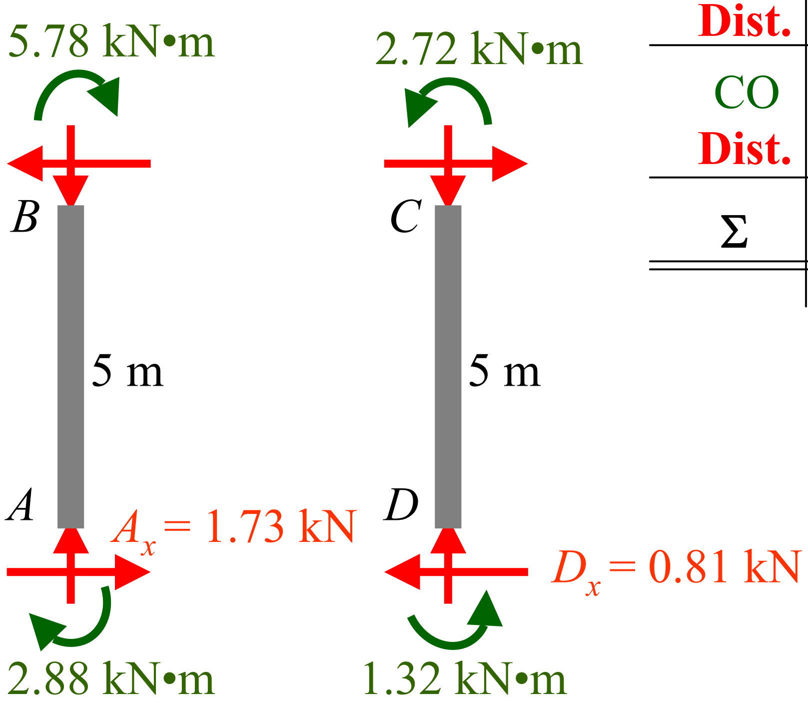


Equilibrium condition :

$$\overset{+}{\rightarrow} \Sigma F_x = 0: \quad A_x + D_x + R = 0$$



	A	B		C		D
DF	0	0.50	0.50	0.50	0.50	0
FEM			10.24	-2.56		
Dist.		-5.12	-5.12	1.28	1.28	
CO	-2.56		0.64	-2.56		0.64
Dist.		-0.32	-0.32	1.28	1.28	
CO	-0.16		0.64	-0.16		0.64
Dist.		-0.32	-0.32	0.08	0.08	
CO	-0.16		0.04	-0.16		0.04
Dist.		-0.02	-0.02	0.08	0.08	
Σ	**-2.88**	**-5.78**	**5.78**	**-2.72**	**2.72**	**1.32**



Equilibrium condition :

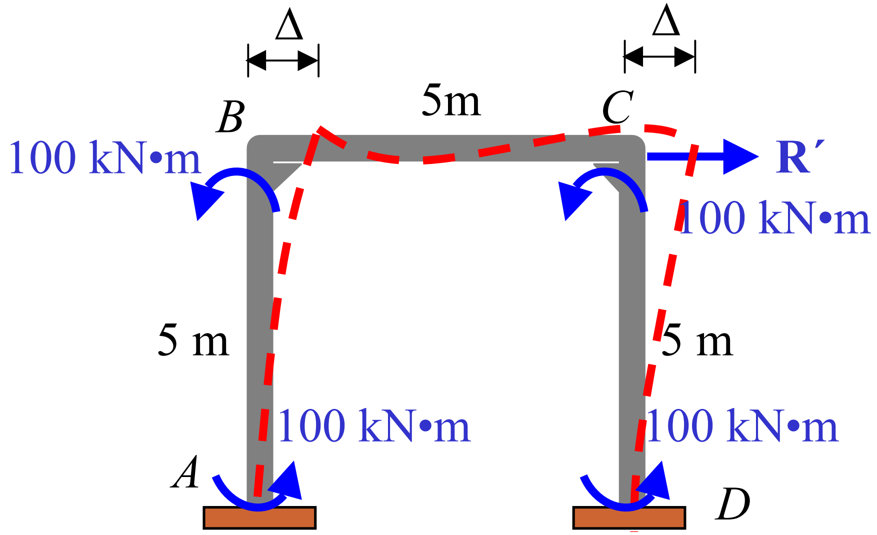
$$\xrightarrow{+} \Sigma F_x = 0: \quad 1.73 - 0.81 + R = 0$$

$$\boxed{R = -0.92 \text{ kN}} \leftarrow$$

- **Artificial joint removed (sidesway)**

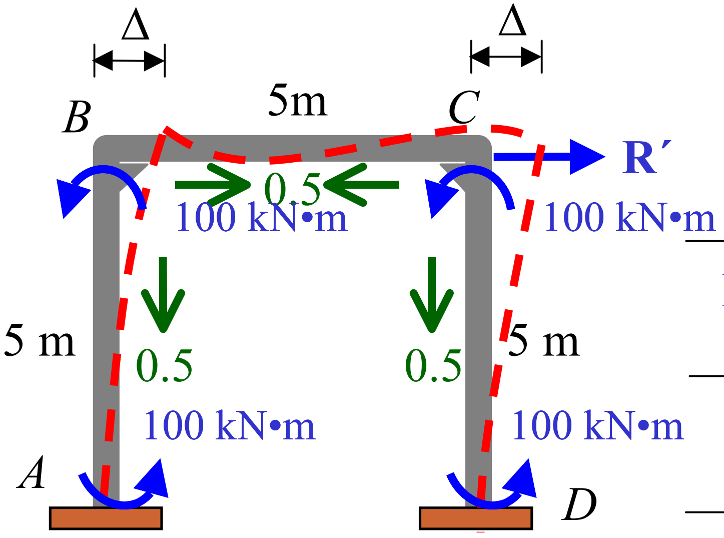
Fixed end moment:

Since both *B* and *C* happen to be displaced the same amount Δ, and *AB* and *DC* have the same *E*, *I*, and *L* so we will assume fixed-end moment to be 100 kN•m.

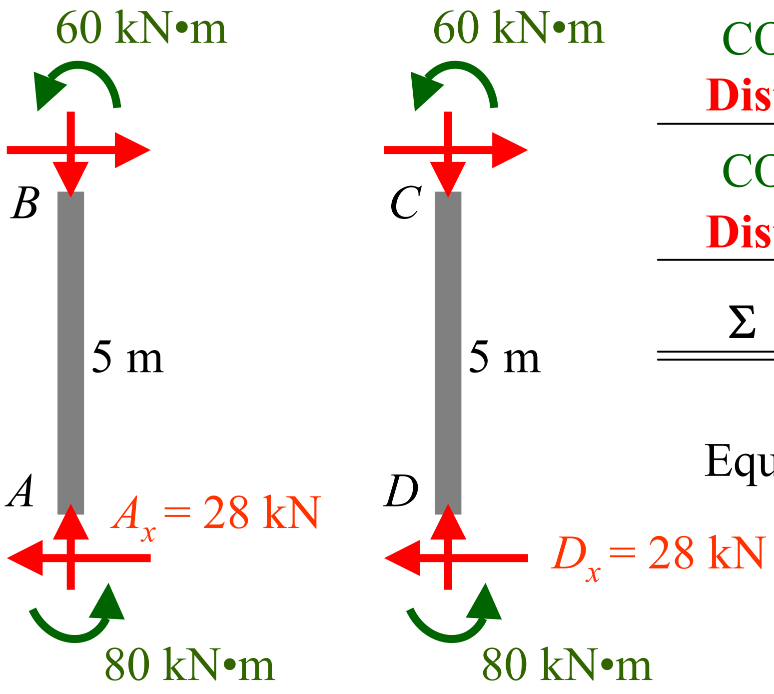


Equilibrium condition :

$$\overset{+}{\rightarrow} \Sigma F_x = 0: \quad A_x + D_x + R' = 0$$



	A	*B*		*C*		*D*
DF	0	0.50	0.50	0.50	0.50	0
FEM	100	100			100	100
Dist.		-50	-50	-50	-50	
CO	-25.0		-25.0	-25.0		-25.0
Dist.		12.5	12.5	12.5	12.5	
CO	6.5		6.5	6.5		6.5
Dist.		-3.125	-3.125	-3.125	-3.125	
CO	-1.56		-1.56	-1.56		-1.56
Dist.		0.78	0.78	0.78	0.78	
CO	0.39		0.39	0.39		0.39
Dist.		-0.195	-0.195	-0.195	-0.195	
Σ	**80**	**60**	**-60**	**-60**	**60**	**80**



Equilibrium condition: $\xrightarrow{+} \Sigma F_x = 0$:

$$-28 - 28 + R' = 0$$

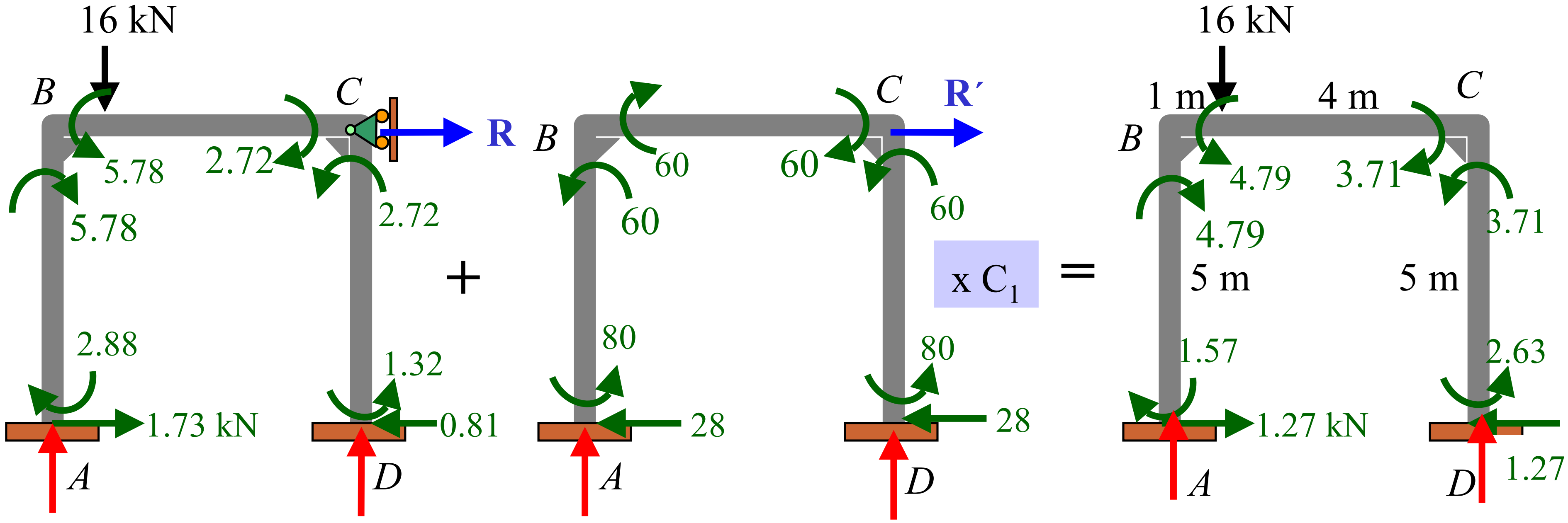
$$\boldsymbol{R'} = 56 \text{ kN}$$

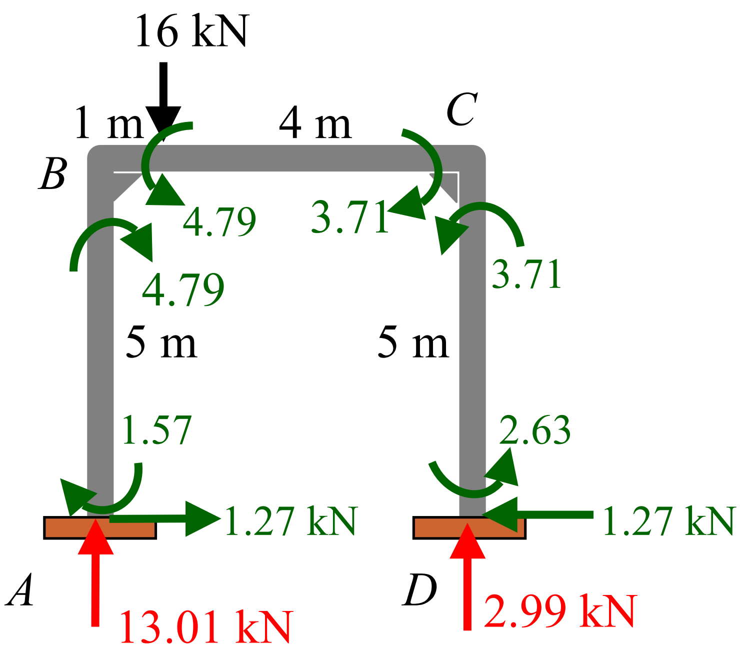
Substitute R = -0.92 and R'= 56 in (1) :

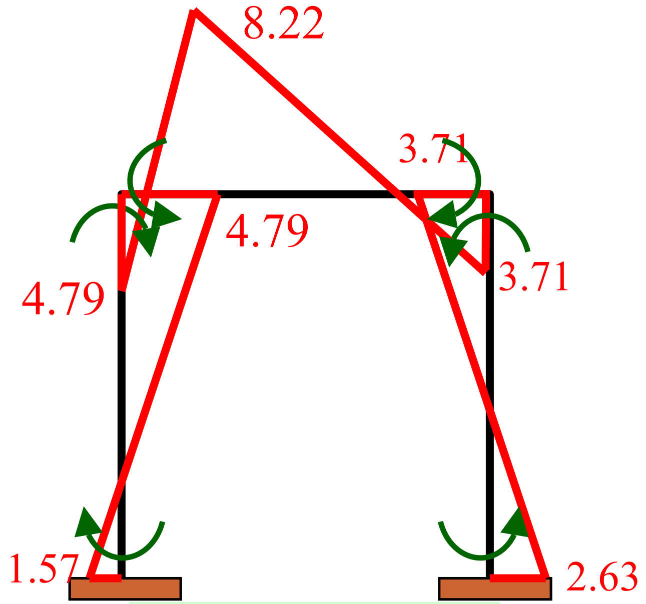
$$R + C_1 R' = 0$$

$$-0.92 + C_1(56) = 0$$

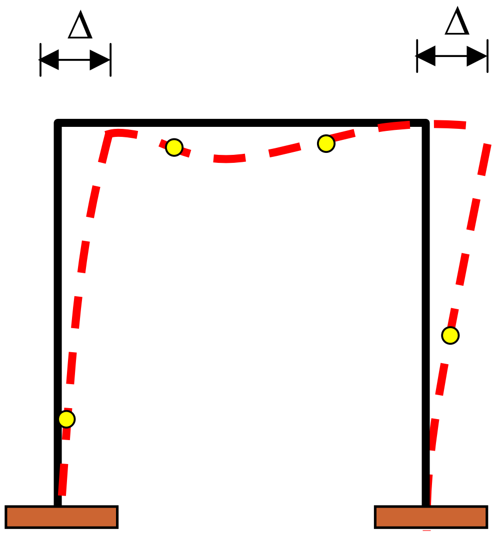
$$C_1 = \frac{0.92}{56}$$







Bending moment diagram (kN•m)



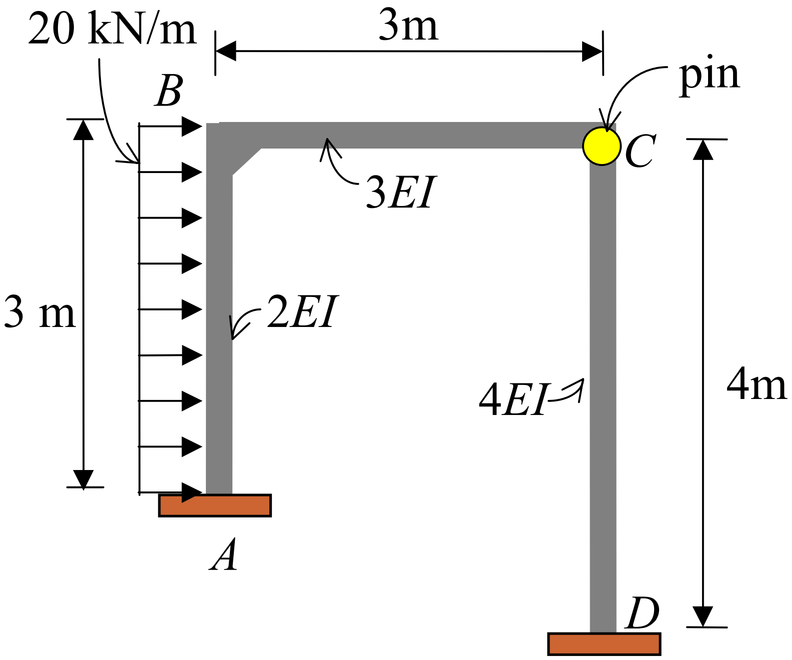
Deflected shape

Example 8

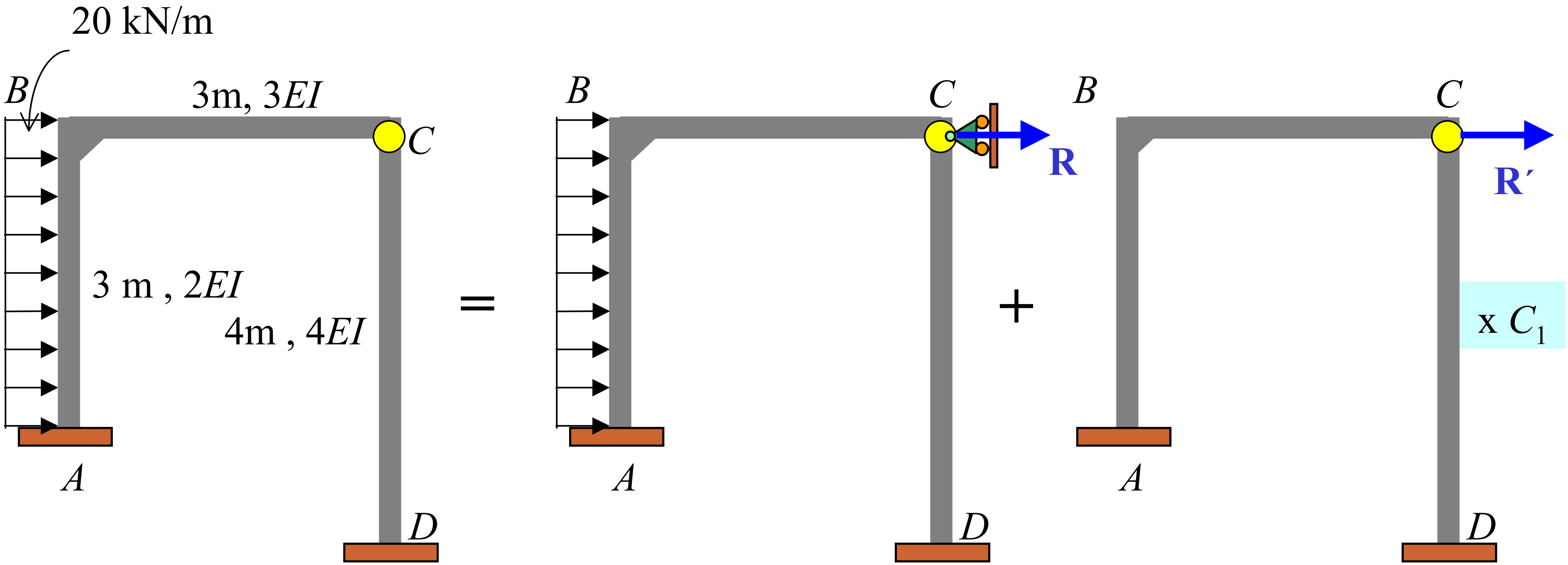
From the frame shown use the moment distribution method to:

(a) Determine all the reactions at supports, and also

(b) Draw its **quantitative shear** and **bending moment diagrams**, and **qualitative deflected shape.**



- **Overview**

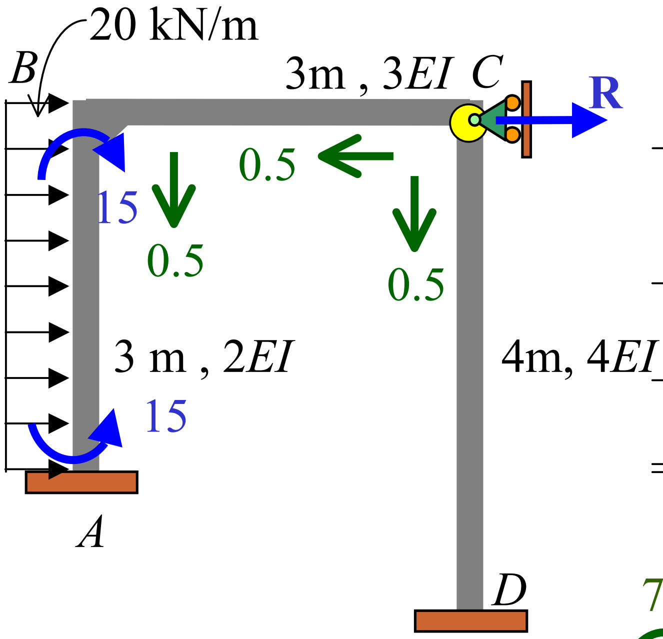


artificial joint applied
(no sidesway)

artificial joint removed
(sidesway)

$$\boxed{R + C_1 R' = 0} \qquad \text{----------(1)}$$

- **Artificial joint applied (no sidesway)**

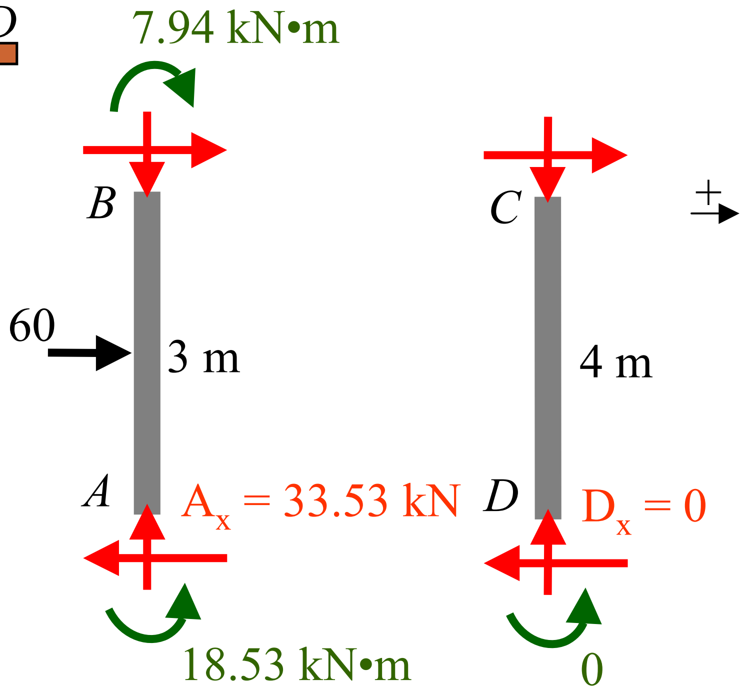


	A	B		C		D
DF	0	0.471	0.529	1.00	1.00	0
FEM	15.00	-15.00				
Dist.		7.065	7.935			
CO	3.533					
Σ	**18.53**	**-7.94**	**7.94**			

$K_{BA} = 4(2EI)/3 = 2.667EI$

$K_{BC} = 3(3EI)/3 = 3EI$

$K_{CD} = 3(4EI)/4 = 3EI$



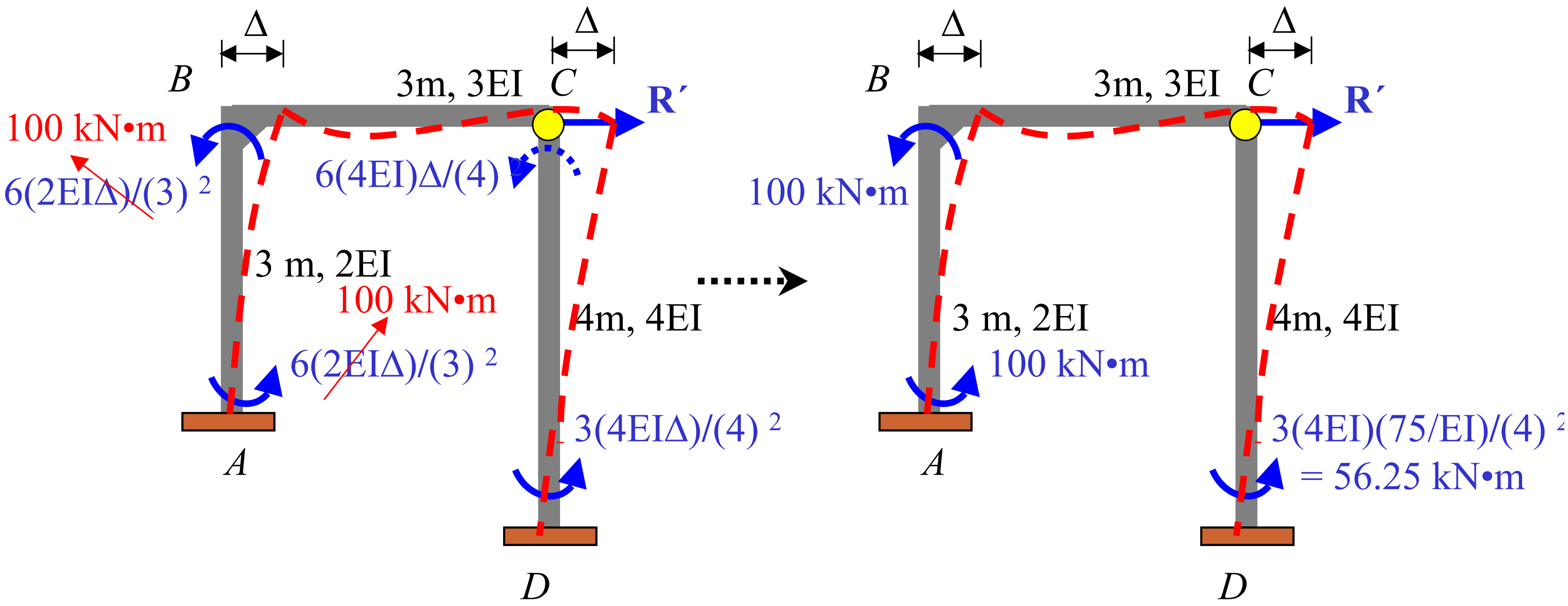
$\overset{+}{\rightarrow} \Sigma F_x = 0:$

$60 - 33.53 - 0 + R = 0$

$R = -26.47$ kN

←

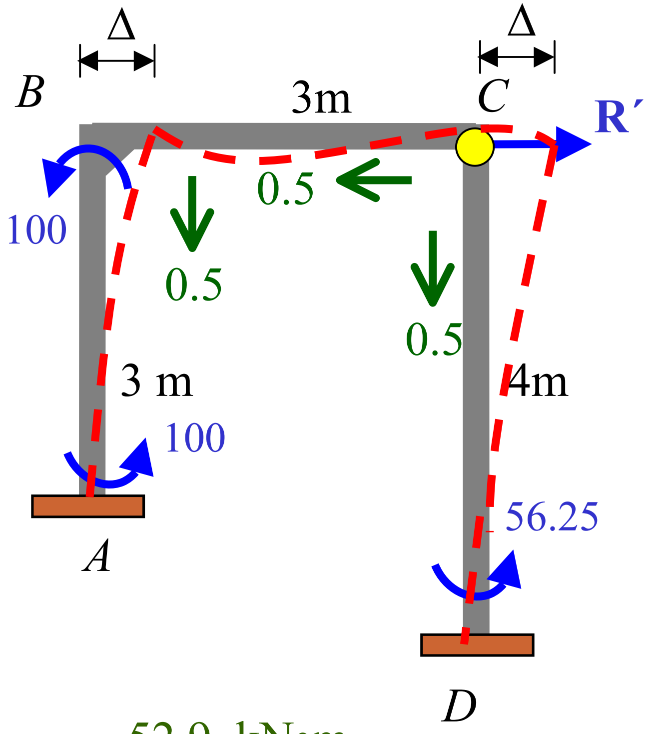
- **Artificial joint removed (sidesway)**
  - **Fixed end moment**



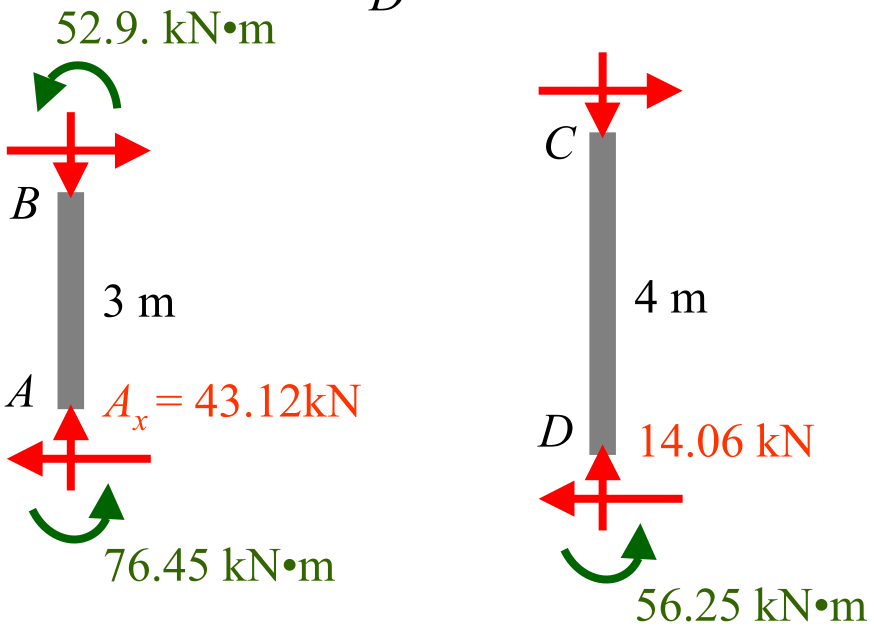
Assign a value of $(\text{FEM})_{AB} = (\text{FEM})_{BA} = 100$ kN•m

$$\frac{6(2EI)\Delta}{3^2} = 100$$

$$\Delta_{AB} = 75/EI$$



	A	B		C		D
DF	0	0.471	0.529	1.00	1.00	0
FEM	100	100				56.25
Dist.		-47.1	-52.9			0
CO	-28.55					
Σ	**76.45**	**52.9**	**-52.9**			**56.25**



$$\xrightarrow{+} \Sigma F_x = 0:$$

$$-43.12 - 14.06 + R' = 0$$

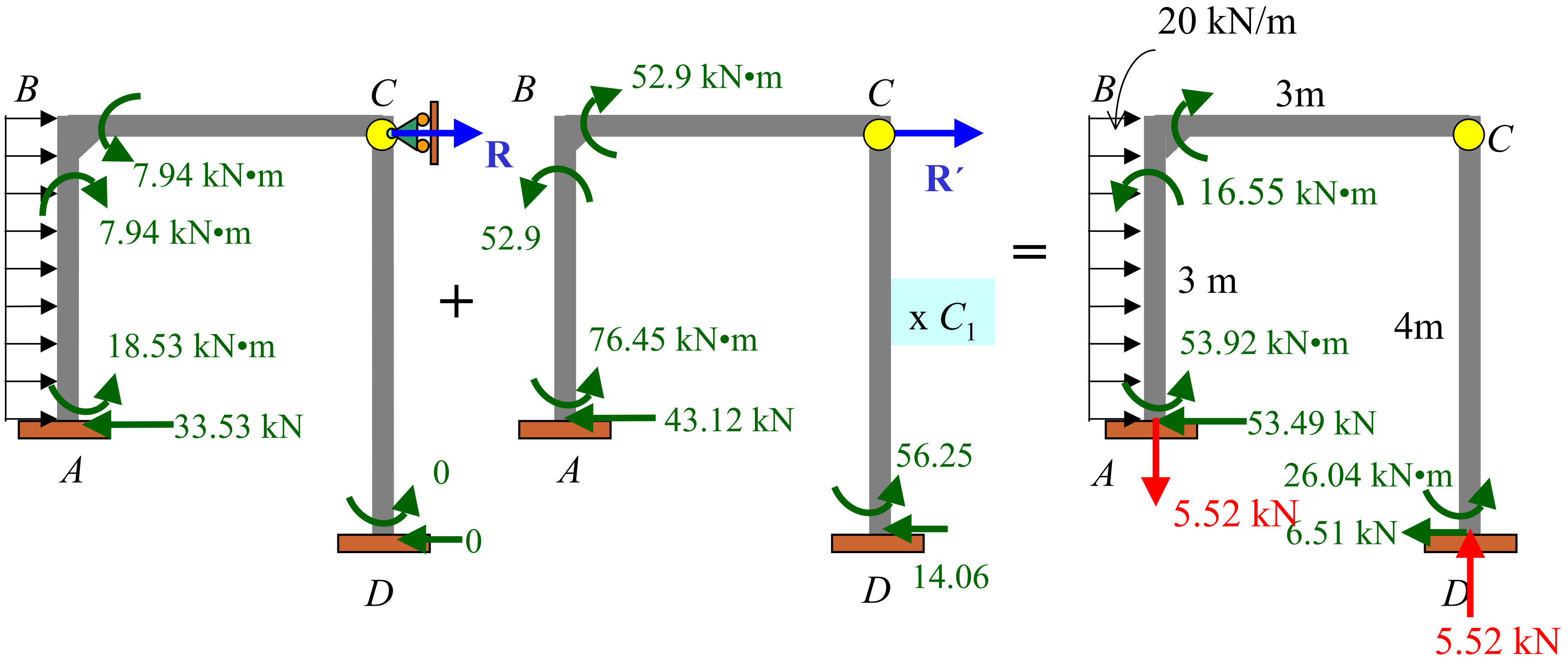
$$R' = 57.18 \text{ kN}$$

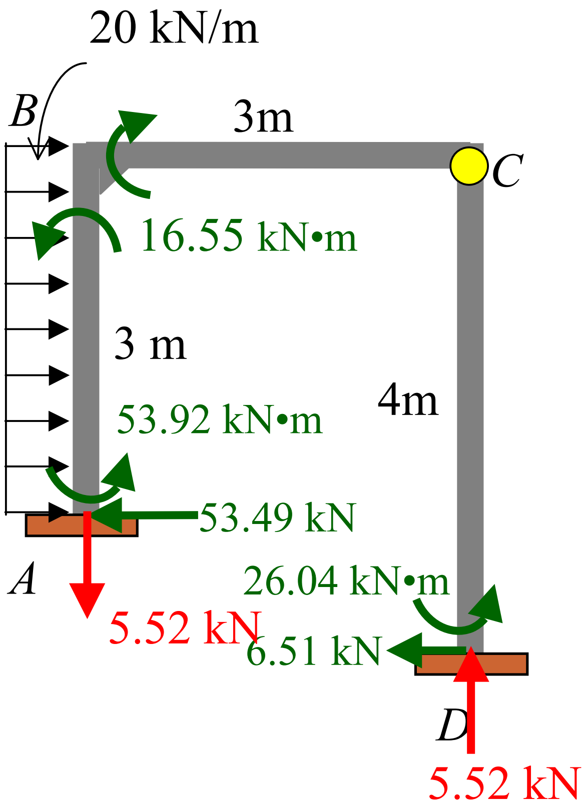
Substitute R = -26.37 and R′= 57.18 in (1) :

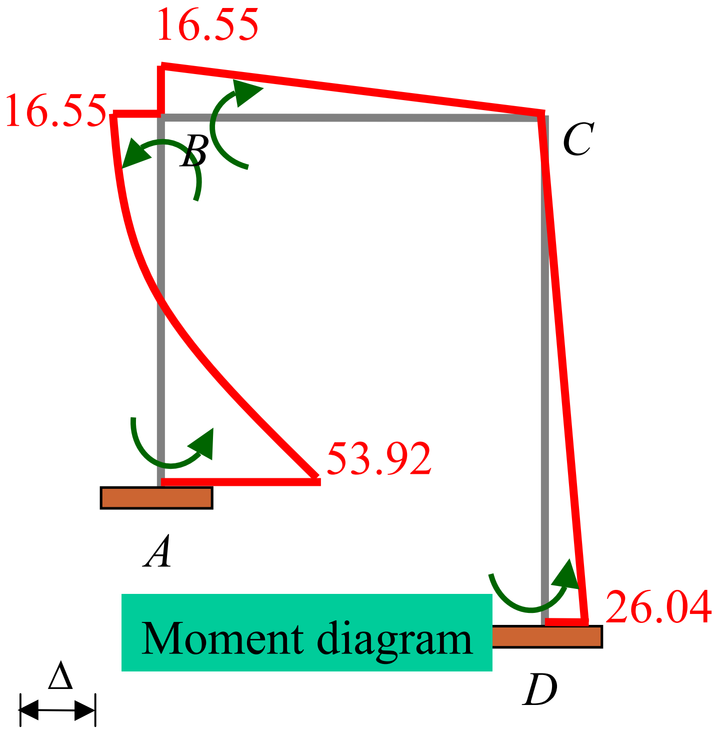
$$R + C_1R' = 0$$

$$-26.47 + C_1(57.18) = 0$$

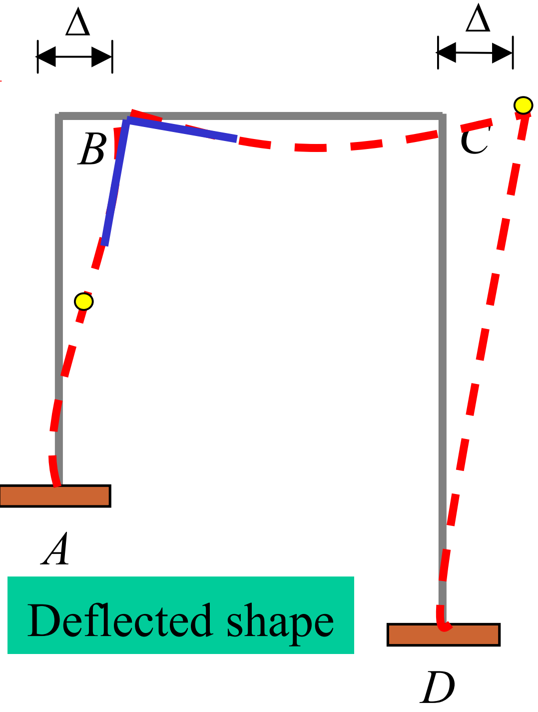
$$C_1 = \frac{26.47}{57.18}$$







Moment diagram



Deflected shape

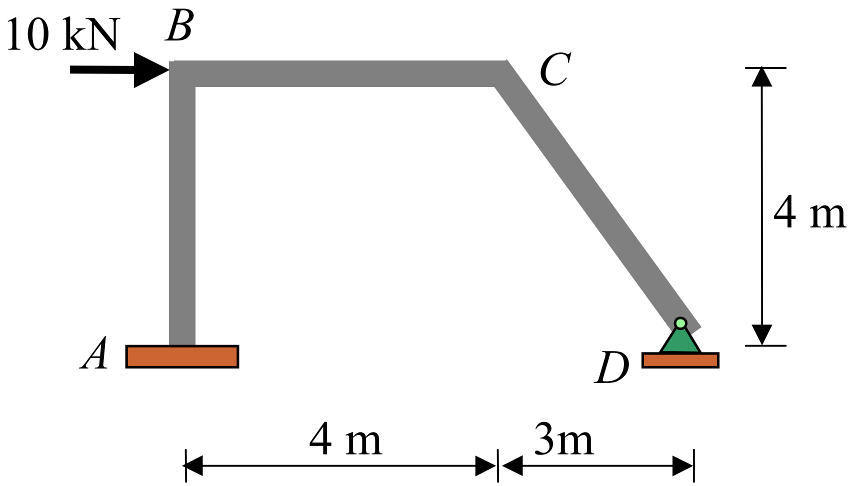
Example 8

From the frame shown use the moment distribution method to:

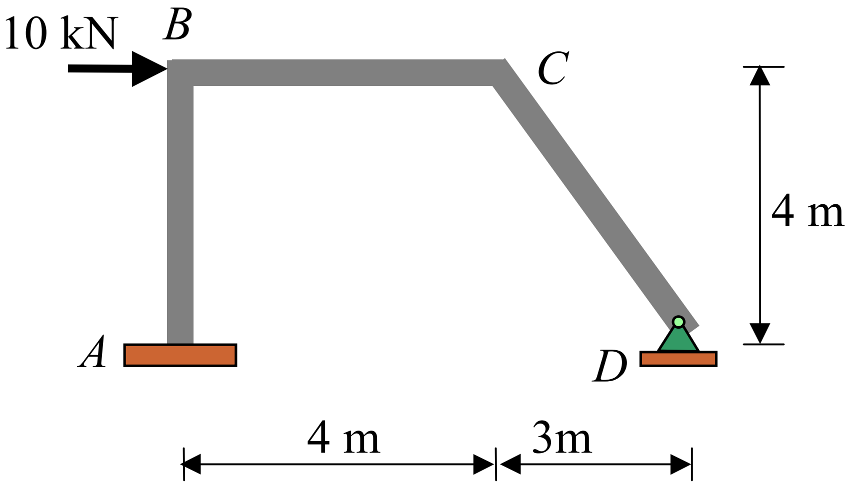
(a) Determine all the reactions at supports, and also

(b) Draw its **quantitative shear** and **bending moment diagrams**, and **qualitative deflected shape.**

EI is constant.

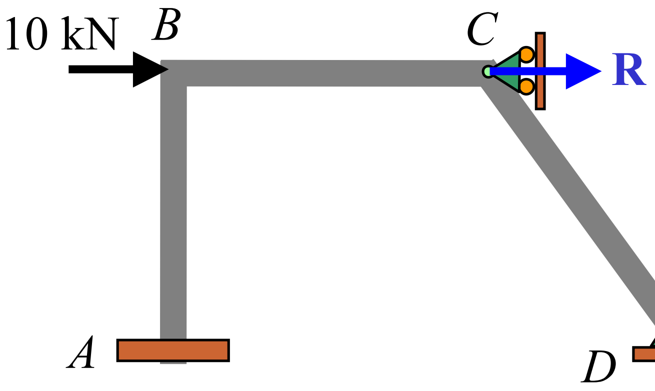


- **Overview**



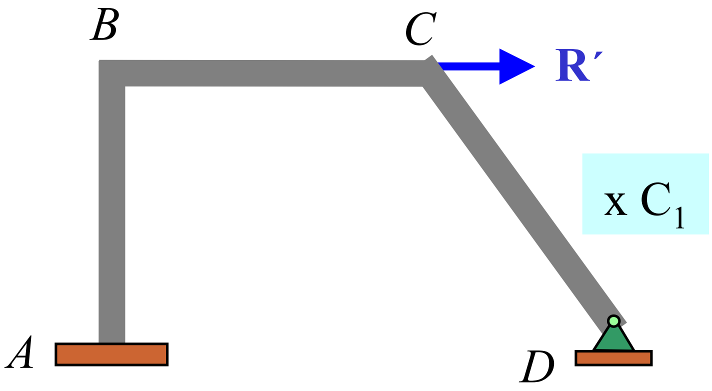
$$\boxed{R + C_1 R' = 0} \quad \text{---------(1)}$$

=



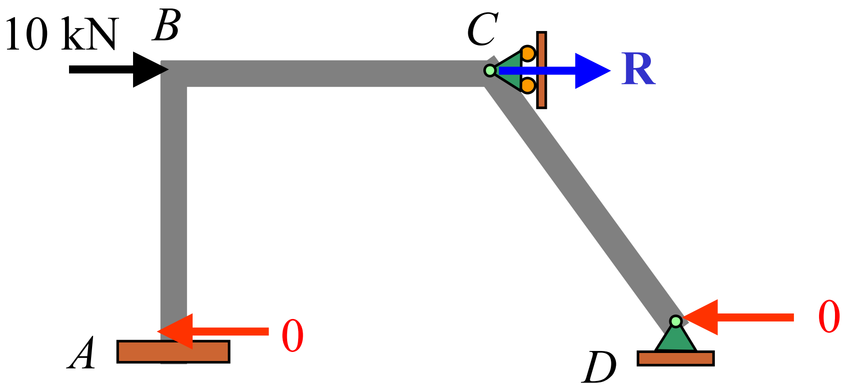
artificial joint applied
(no sidesway)

+



artificial joint removed
(sidesway)

- **Artificial joint applied (no sidesway)**



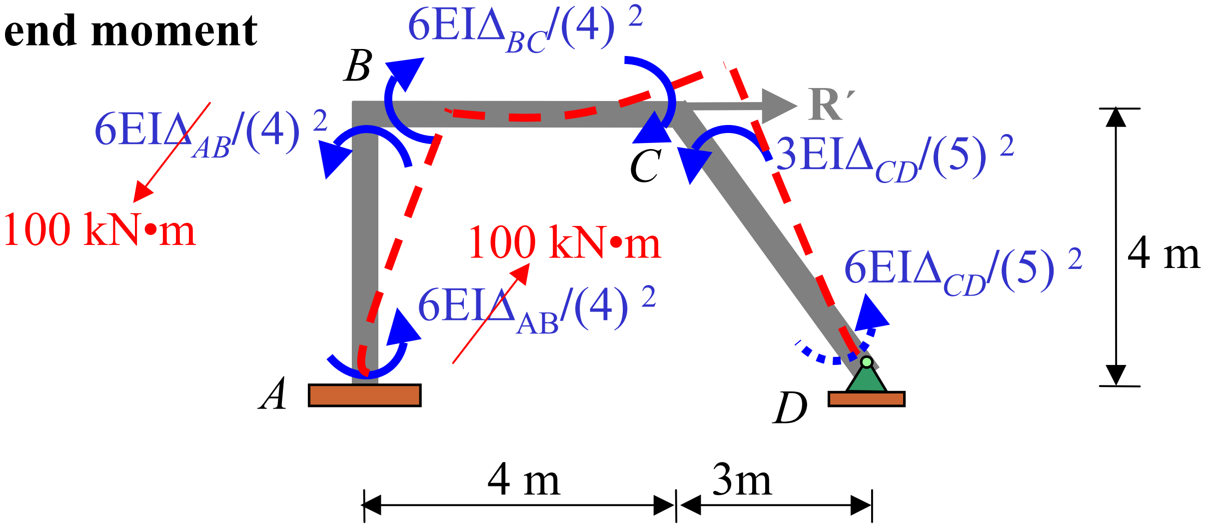
Equilibrium condition : $\overset{+}{\rightarrow} \Sigma F_x = 0$:

$$10 + R = 0$$

$$\boxed{R = -\ 10 \text{ kN}} \leftarrow$$

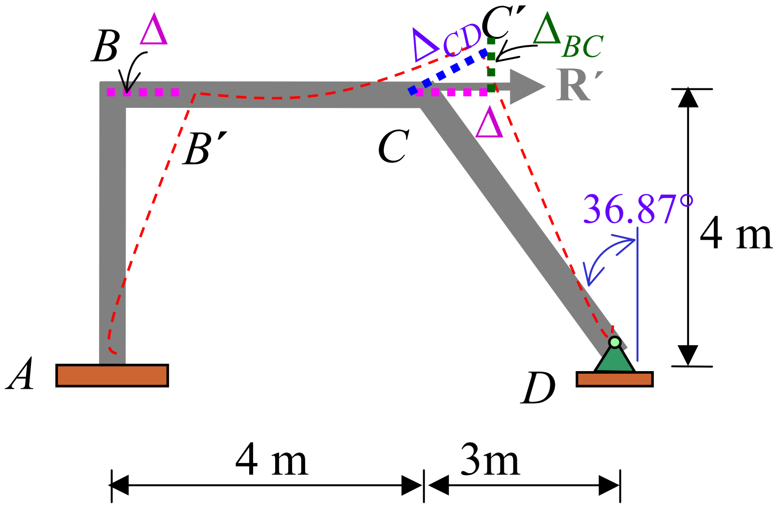
- **Artificial joint removed (sidesway)**

  - **Fixed end moment**

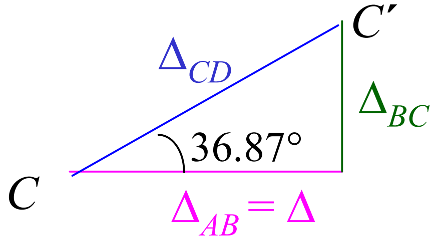


Assign a value of $(\text{FEM})_{AB} = (\text{FEM})_{BA} = 100$ kN•m : $\dfrac{6EI\Delta_{AB}}{4^2} = 100$

$$\Delta_{AB} = 266.667/EI$$

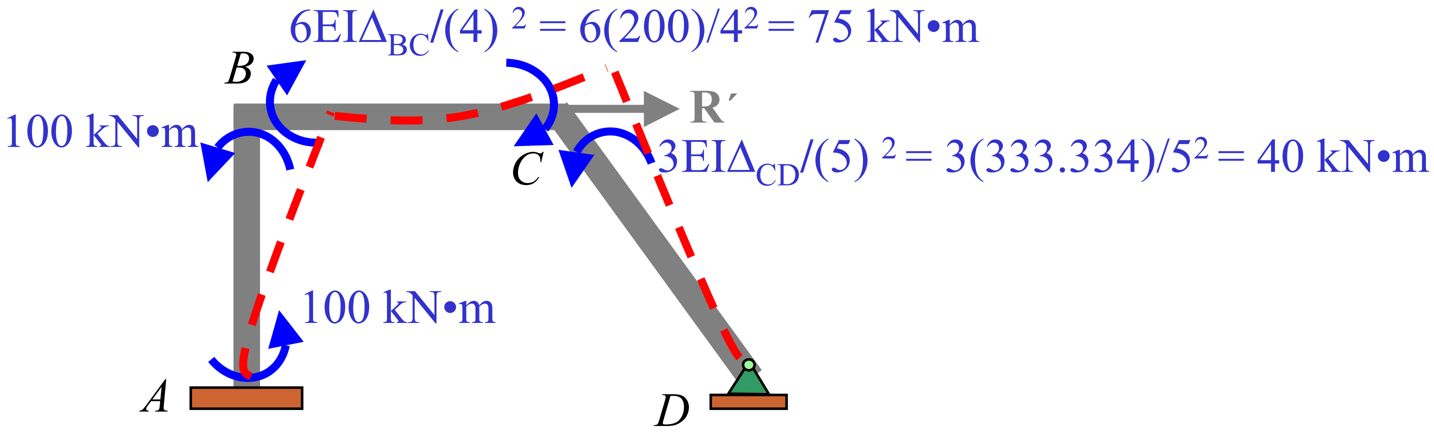


$$\Delta_{CD} = \Delta / \cos 36.87° = 1.25\,\Delta = 1.25(266.667/\text{EI})$$
$$= 333.334/\text{EI}$$



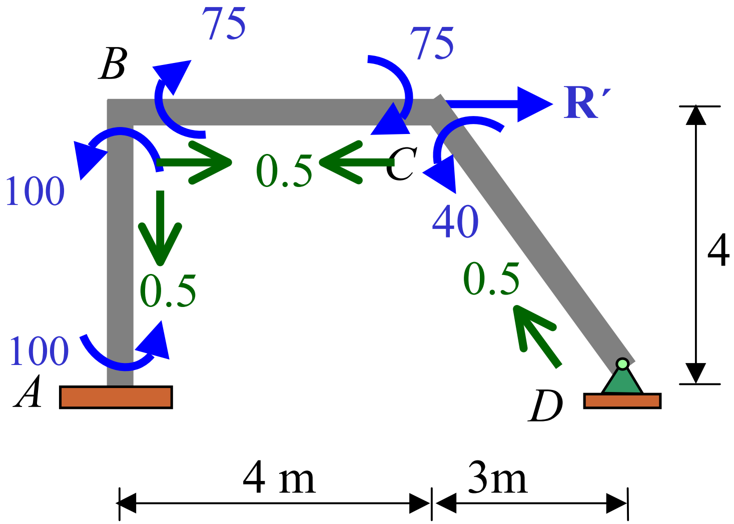
$$= \Delta \tan 36.87° = 0.75\,\Delta$$
$$= 0.75(266.667/EI)$$
$$= 200/EI$$

$\Delta_{BC} = 200/EI$, $\Delta_{CD} = 333.334/EI$



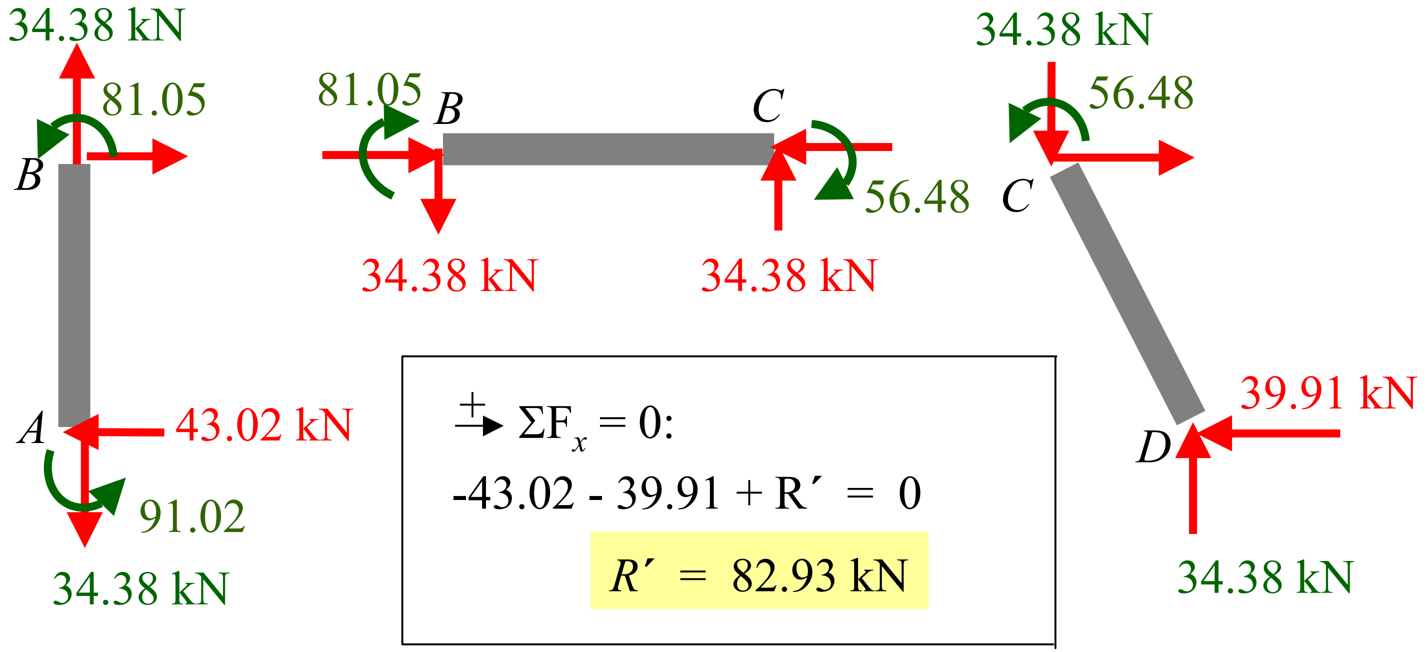
Equilibrium condition :

$$\overset{+}{\rightarrow} \Sigma F_x = 0: \quad A_x + D_x + R' = 0$$



$K_{BA} = 4EI/4 = EI$, $K_{BC} = 4EI/4 = EI$,
$K_{CD} = 3EI/5 = 0.6EI$

	A	*B*		*C*		*D*
DF	0	0.50	0.50	0.625	0.375	1
FEM	100	100	-75	-75	40	
Dist.		-12.5	-12.5	21.875	13.125	
CO	-6.25		10.938	-6.25		
Dist.		-5.469	-5.469	3.906	2.344	
CO	-2.735		1.953	-2.735		
Dist.		-0.977	-0.977	1.709	1.026	
Σ	**91.02**	**81.05**	**-81.05**	**-56.48**	**56.48**	



$\xrightarrow{+}$ $\Sigma F_x = 0$:

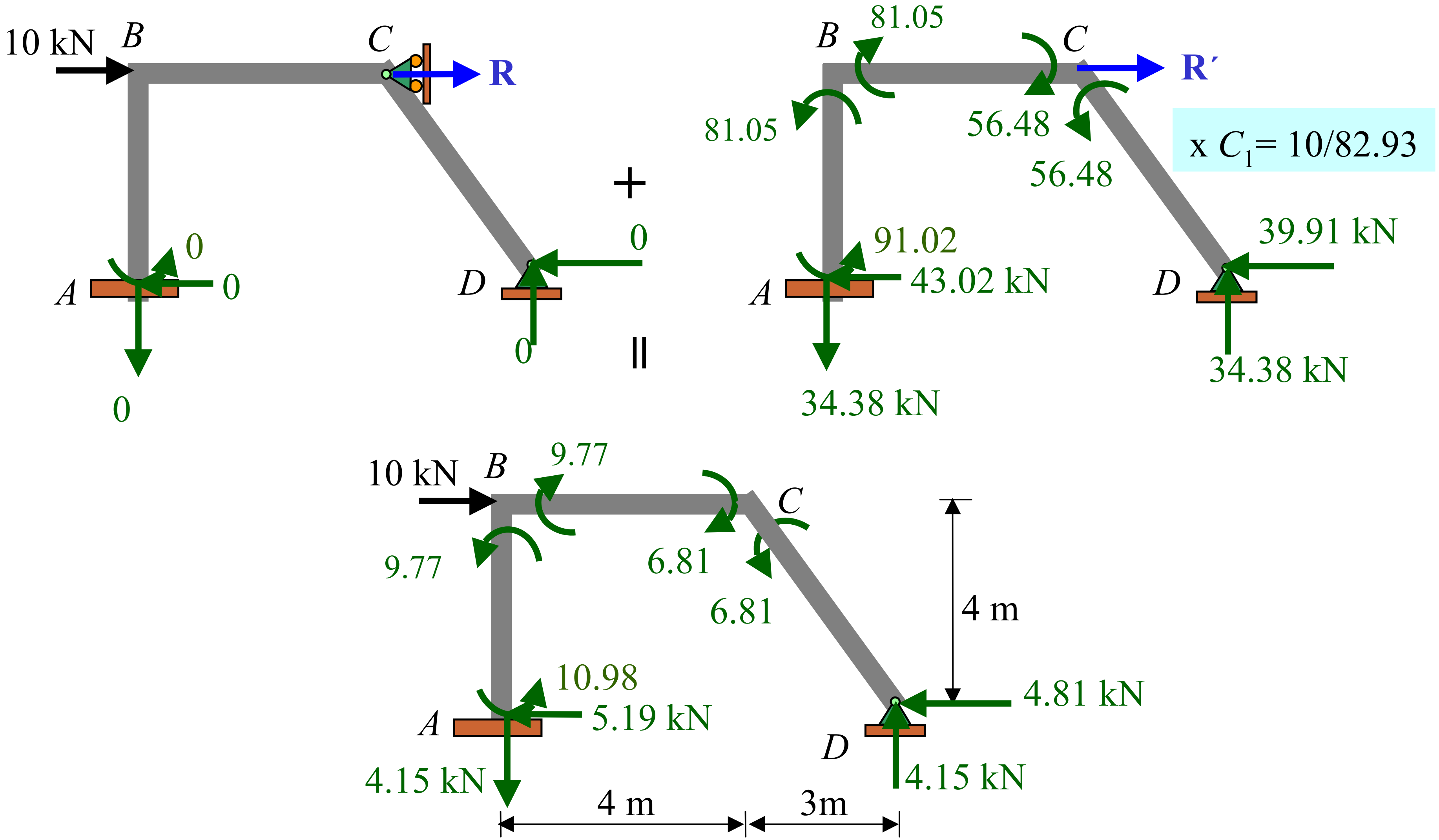
$-43.02 - 39.91 + R' = 0$

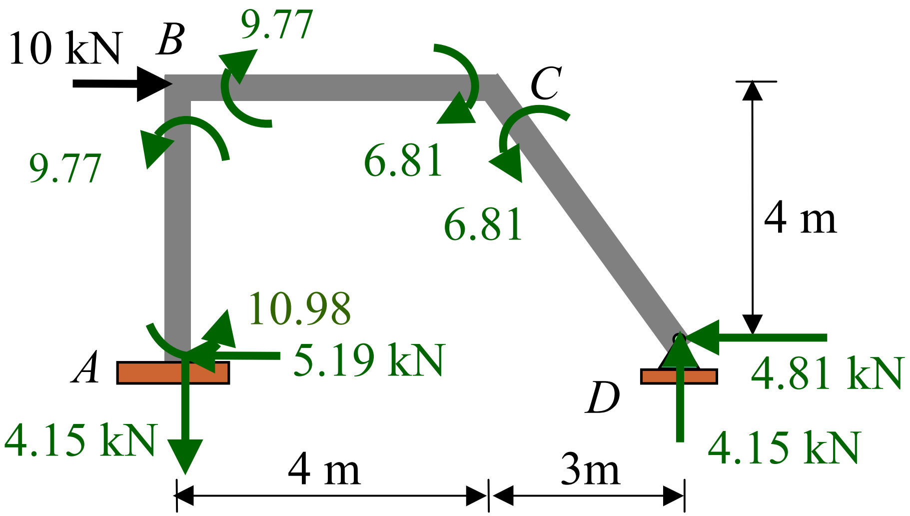
$R' = 82.93$ kN

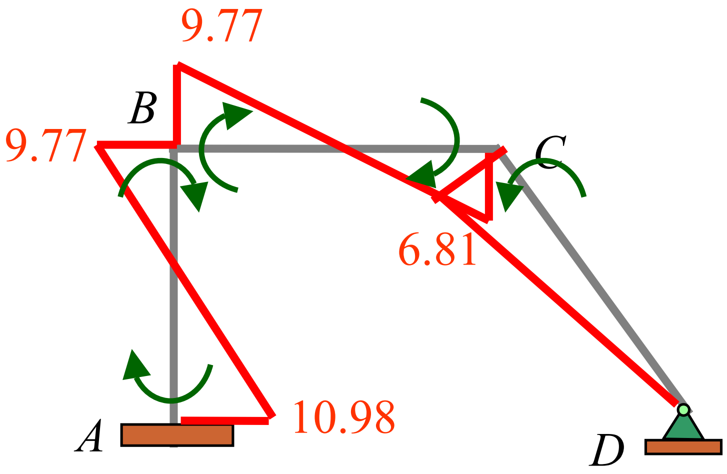
Substitute R = -10 kN and R′= 82.93 kN in (1) : $-10 + C_1(82.93) = 0$

$\boxed{R + C_1R' = 0}$ ---------(1)

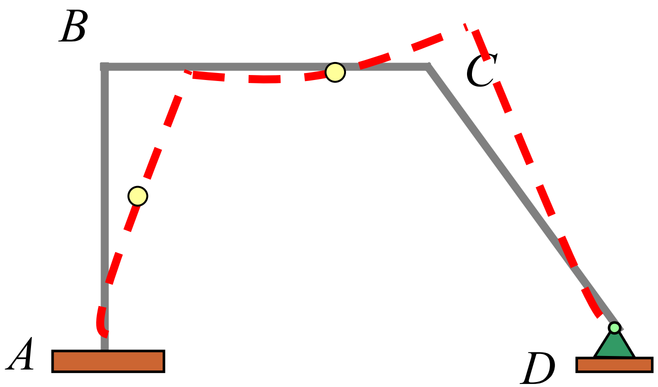
$C_1 = 10/82.93$







Bending moment diagram
(kN•m)



Deflected shape

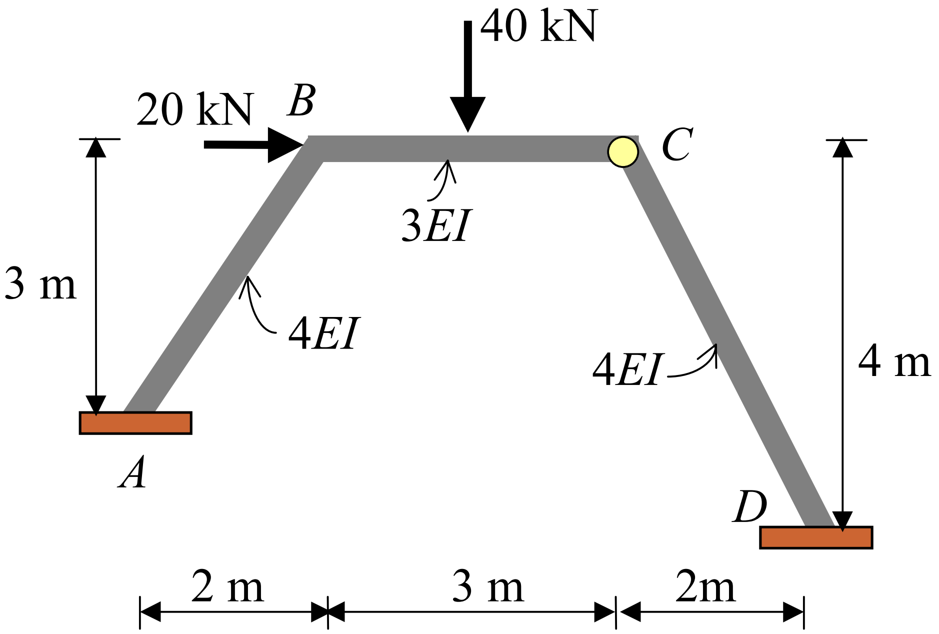
Example 9

From the frame shown use the moment distribution method to:

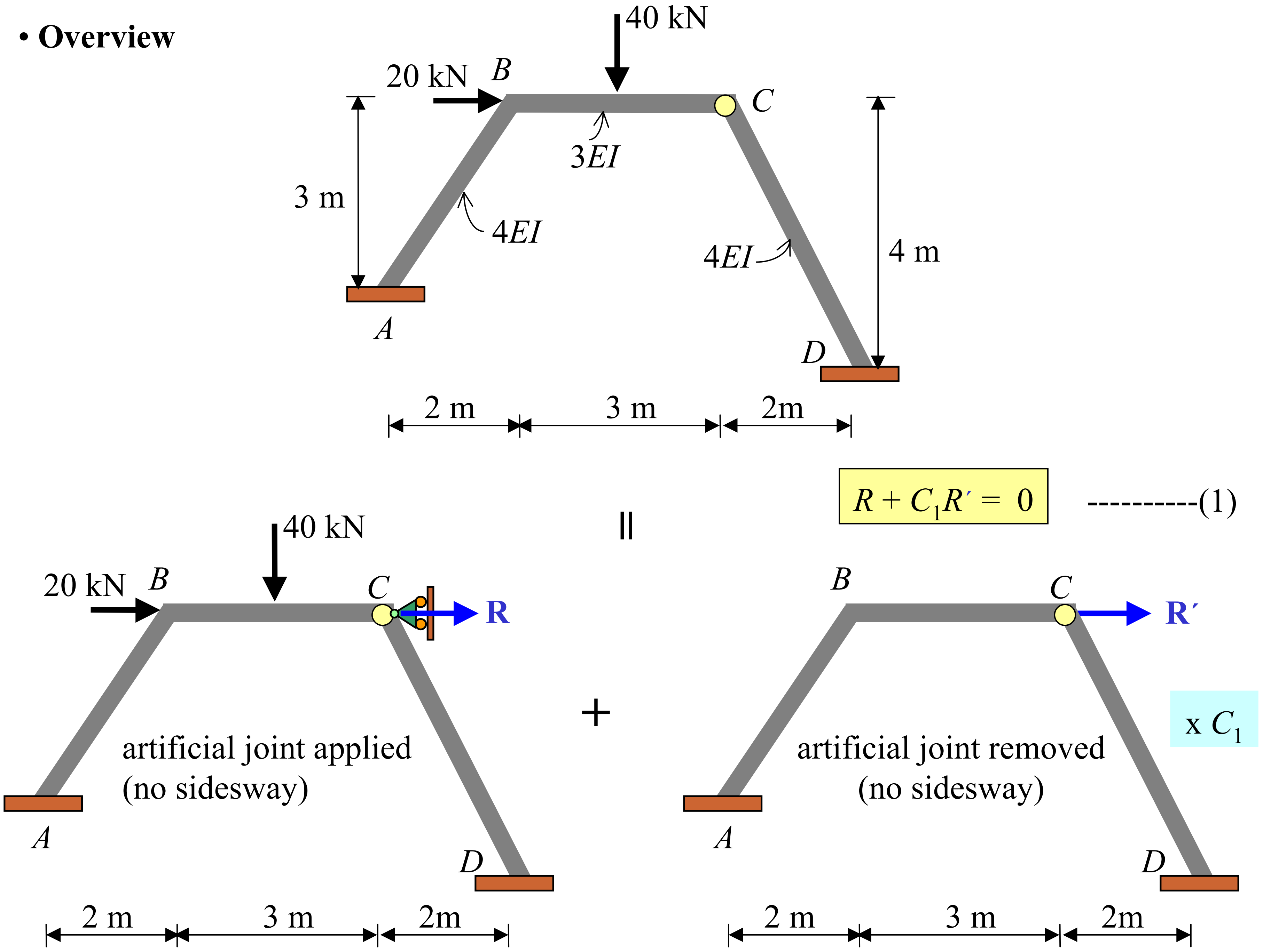
(a) Determine all the reactions at supports, and also

(b) Draw its **quantitative shear** and **bending moment diagrams**,and **qualitative deflected shape.**

EI is constant.



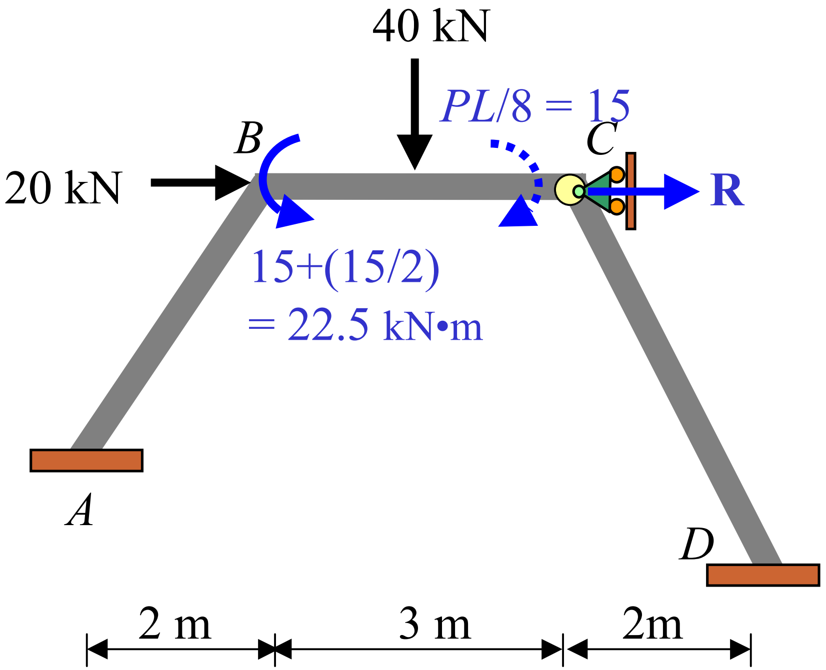
- **Overview**



$$\boxed{R + C_1 R' = 0} \quad \text{----------(1)}$$

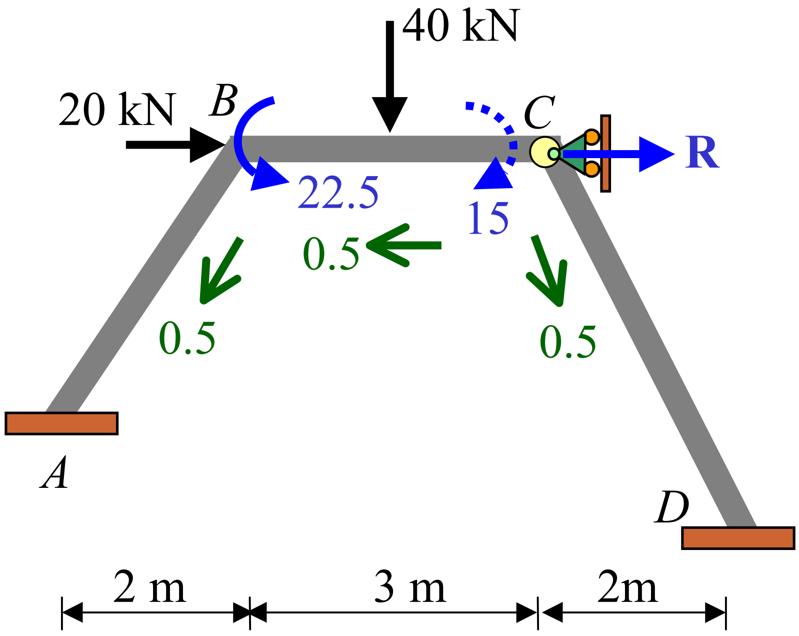
- **Artificial joint applied (no sidesway)**

Fixed end moments:



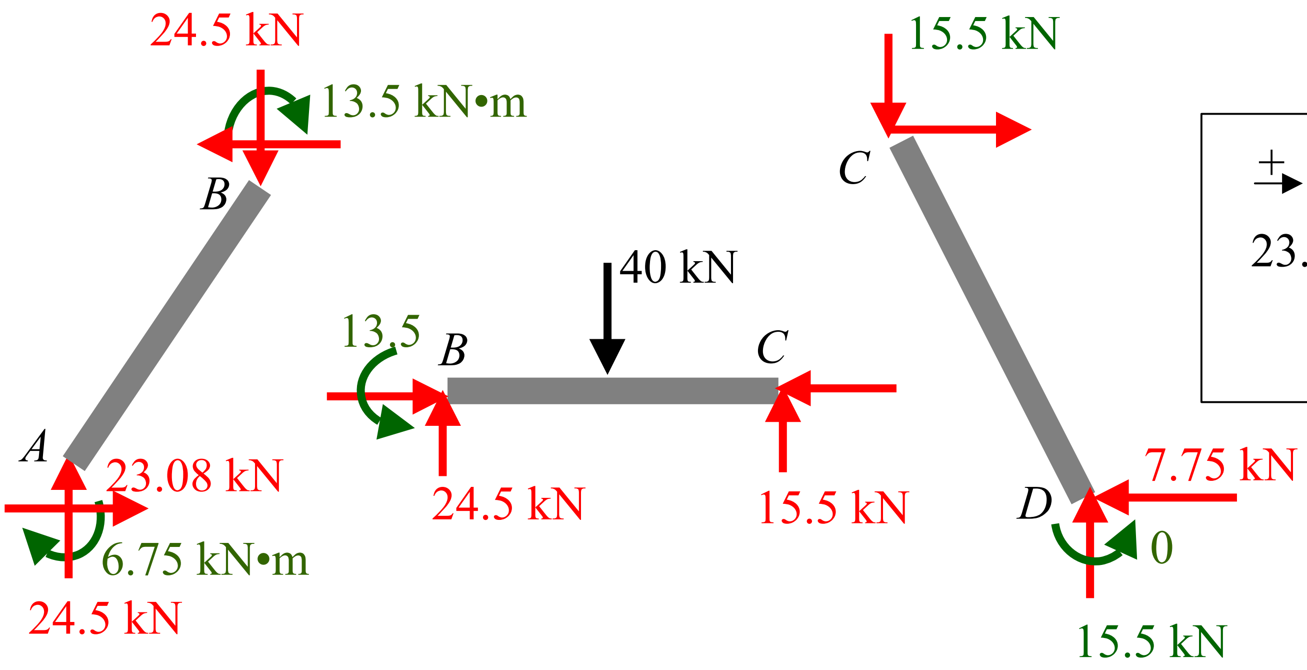
Equilibrium condition :

$$\overset{+}{\rightarrow} \Sigma F_x = 0: \quad A_x + D_x + R = 0$$



	A	B		C		D
DF	0	0.60	0.40	1.00	1.00	0
FEM			22.5			
Dist.		-13.5	-9.0			
CO	-6.75					
Σ	**-6.75**	**-13.5**	**13.5**			

$K_{BA} = 4(4EI)/3.6 = 4.444EI,\ K_{BC} = 3(3EI)/3 = 3EI,$



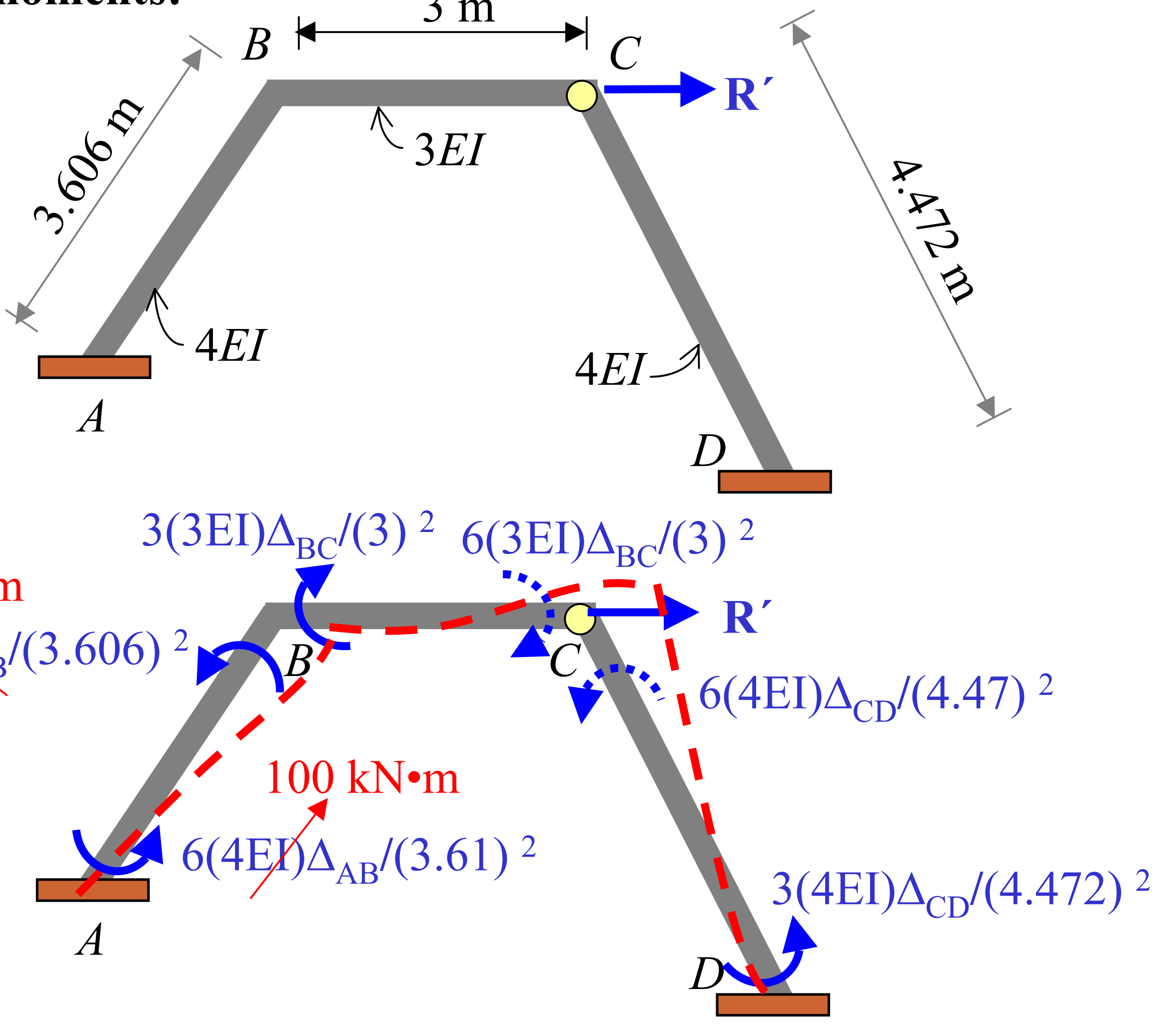
$\xrightarrow{+}\ \Sigma F_x = 0$:

$23.08 + 20 - 7.75 + R' = 0$

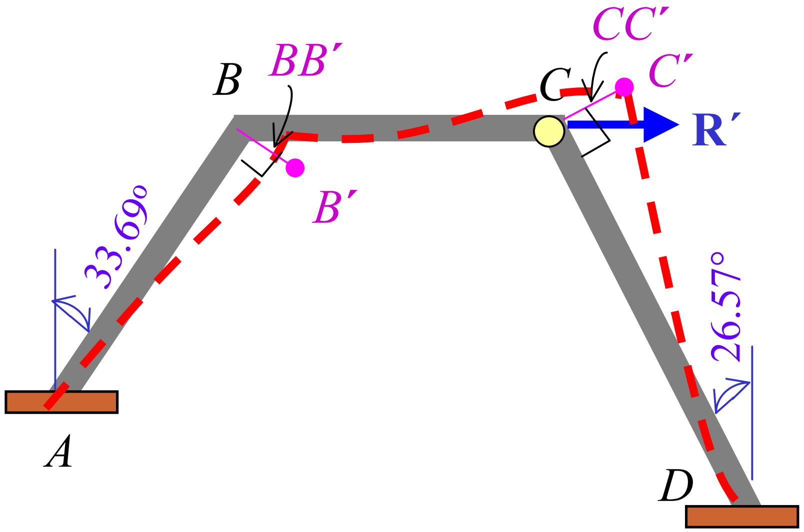
$R' = -35.33$ kN ←

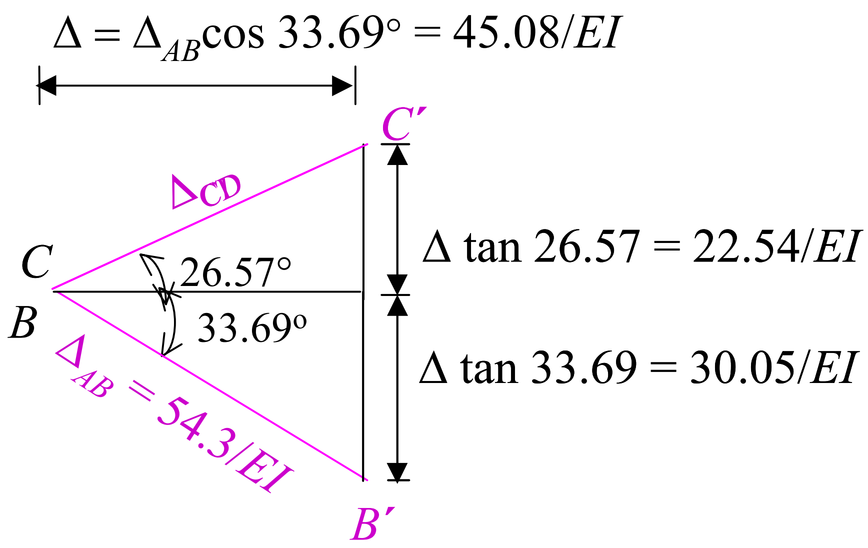
- **Artificial joint removed (sidesway)**

Fixed end moments:



Assign a value of $(\text{FEM})_{AB} = (\text{FEM})_{BA} = 100$ kN•m : $\frac{6(4EI)\Delta_{AB}}{3.61^2} = 100,$ $\Delta_{AB} = 54.18/\text{EI}$





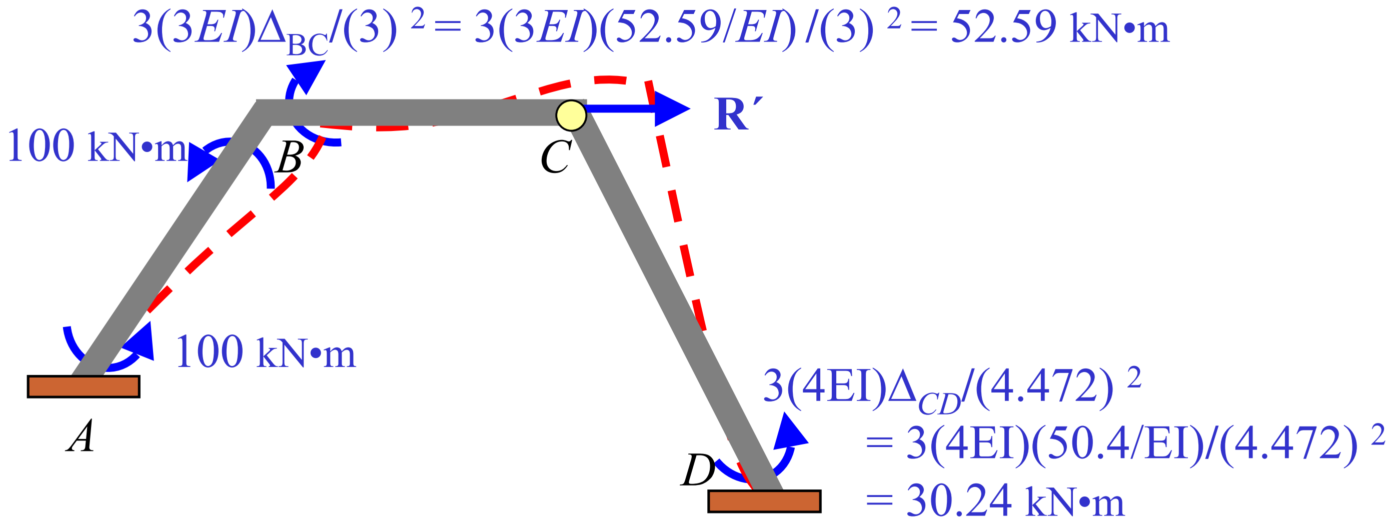
$\Delta = \Delta_{AB}\cos 33.69° = 45.08/EI$

$\Delta \tan 26.57 = 22.54/EI$

$\Delta \tan 33.69 = 30.05/EI$

$\Delta_{BC} = B'C' = 22.54/EI + 30.05/EI = 52.59/EI$

$\Delta_{CD} = \Delta/\cos 26.57° = 50.4/EI$

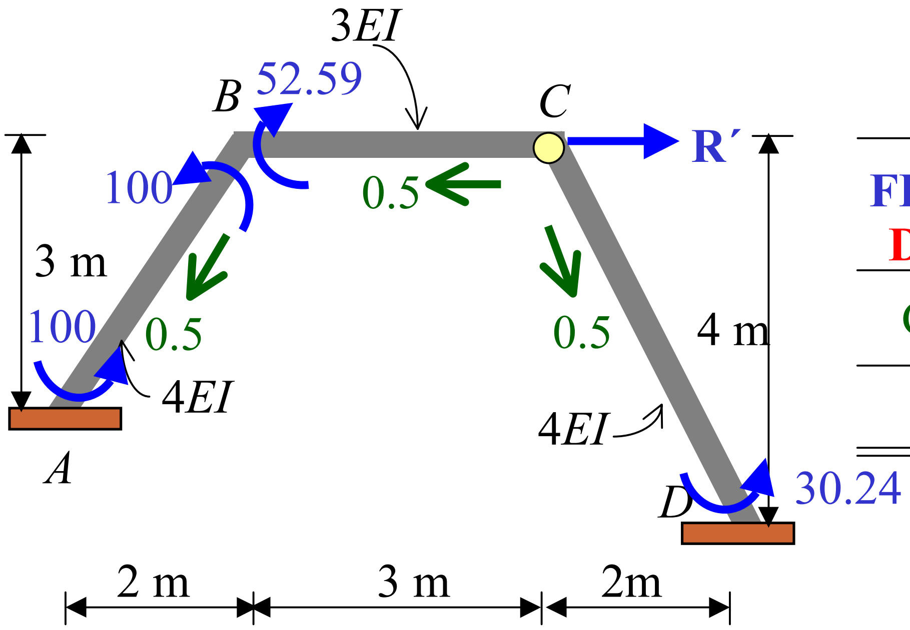


$3(3EI)\Delta_{BC}/(3)^2 = 3(3EI)(52.59/EI)/(3)^2 = 52.59$ kN•m

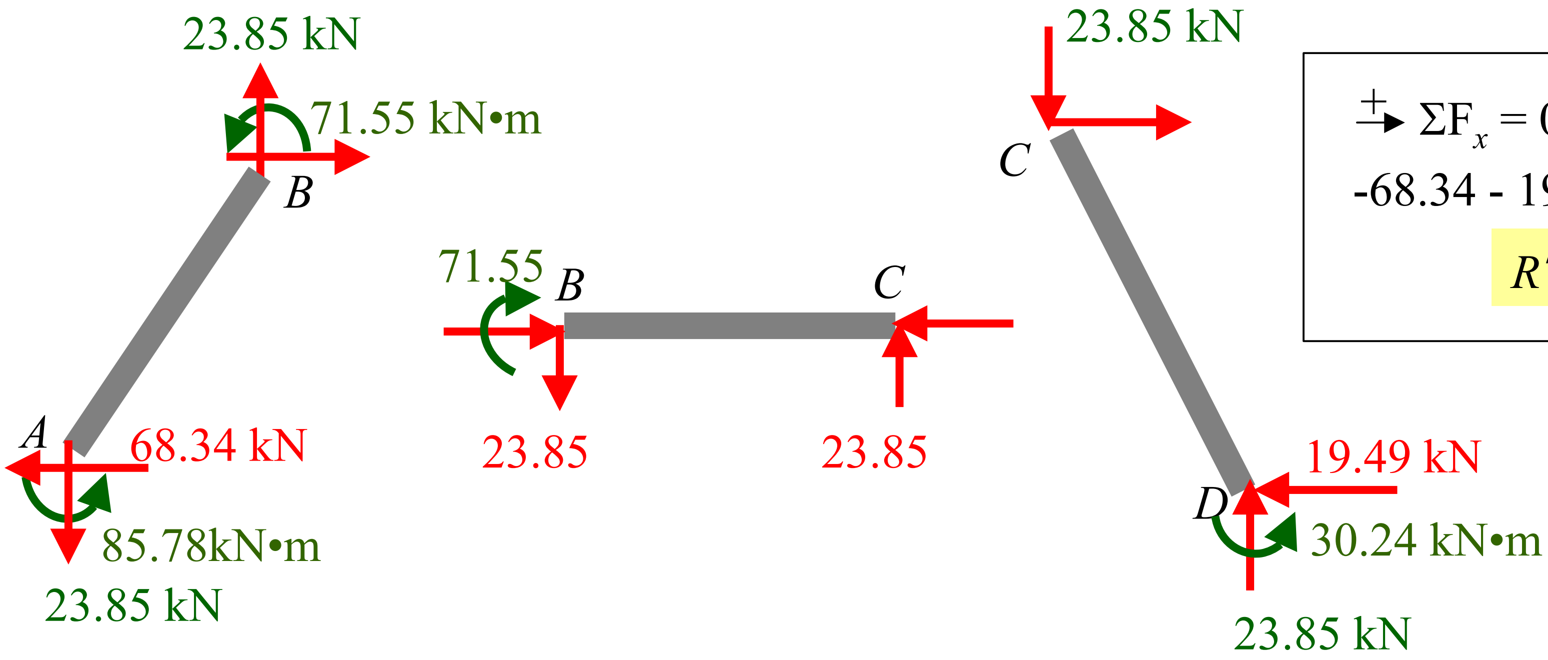
$3(4EI)\Delta_{CD}/(4.472)^2$

$= 3(4EI)(50.4/EI)/(4.472)^2$

$= 30.24$ kN•m



	A	B		C		D
DF	0	0.60	0.40	1.00	1.00	0
FEM	100	100	-52.59			30.24
Dist.		-28.45	-18.96			
CO	-14.223					
Σ	**85.78**	**71.55**	**-71.55**			**30.24**



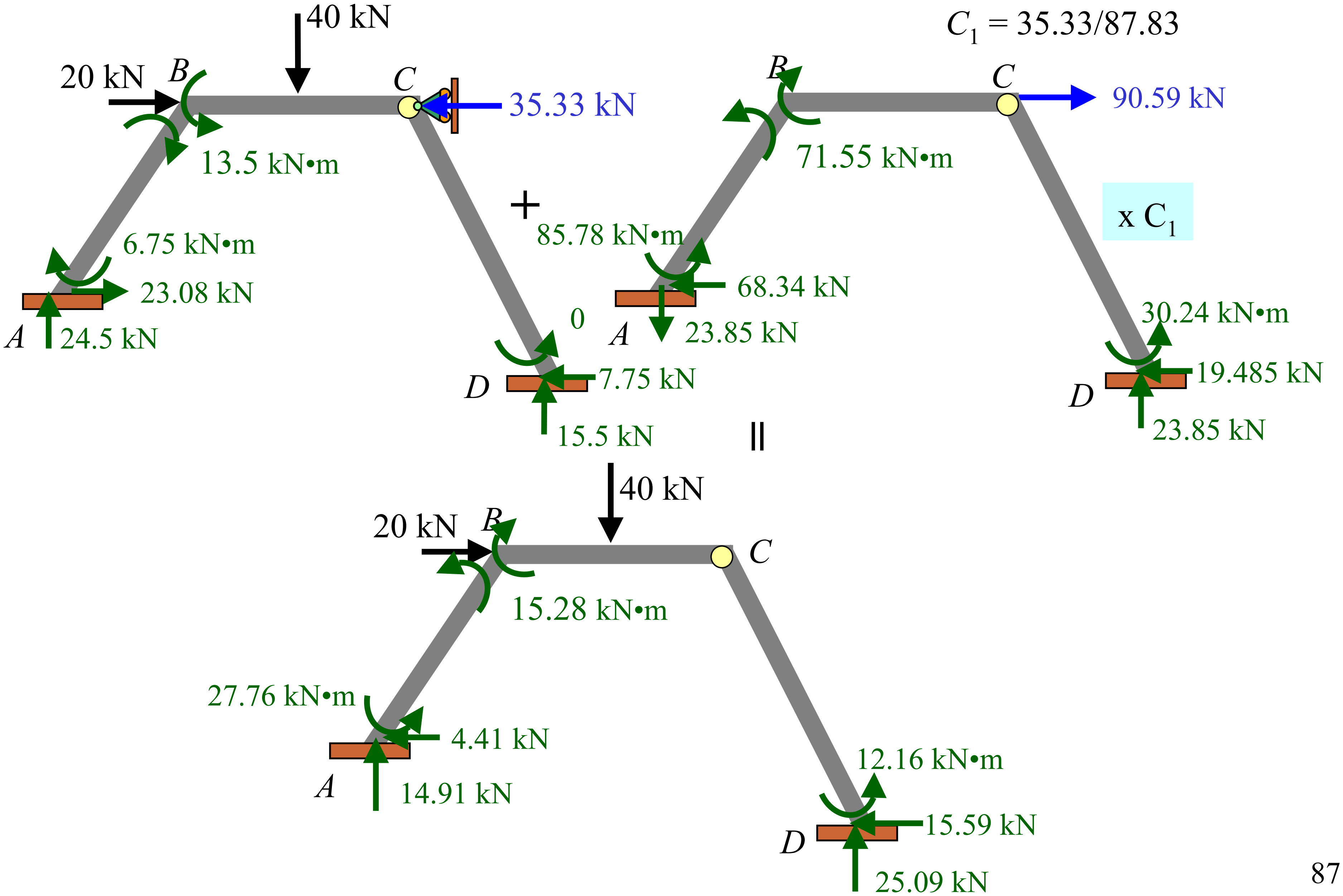
$\xrightarrow{+}$ $\Sigma F_x = 0$:

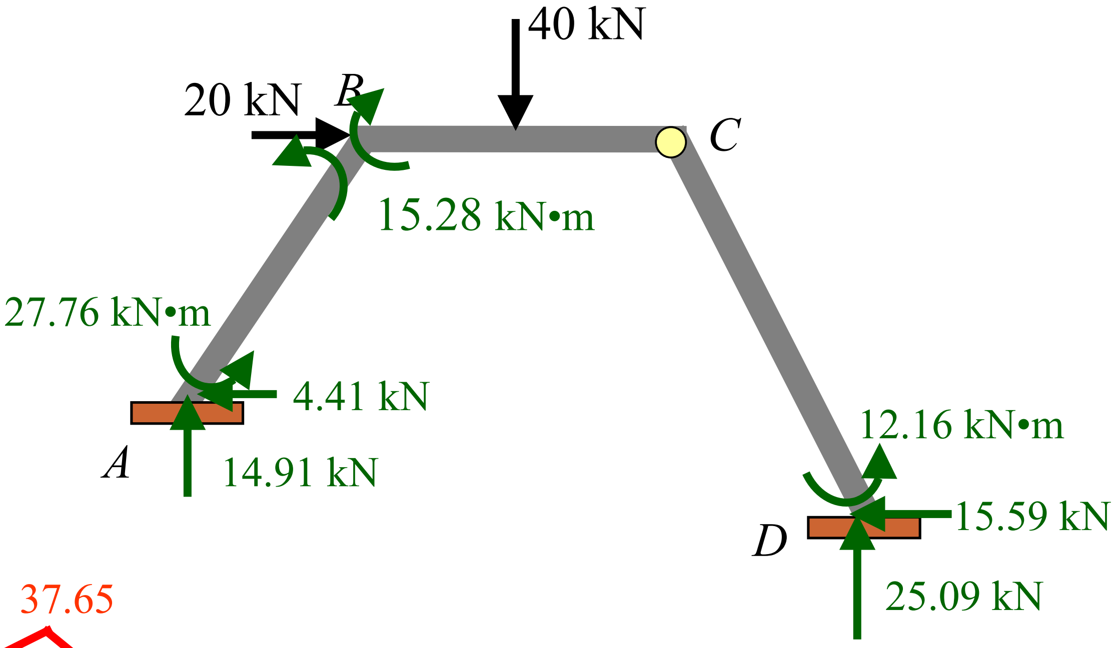
$-68.34 - 19.49 + R' = 0$

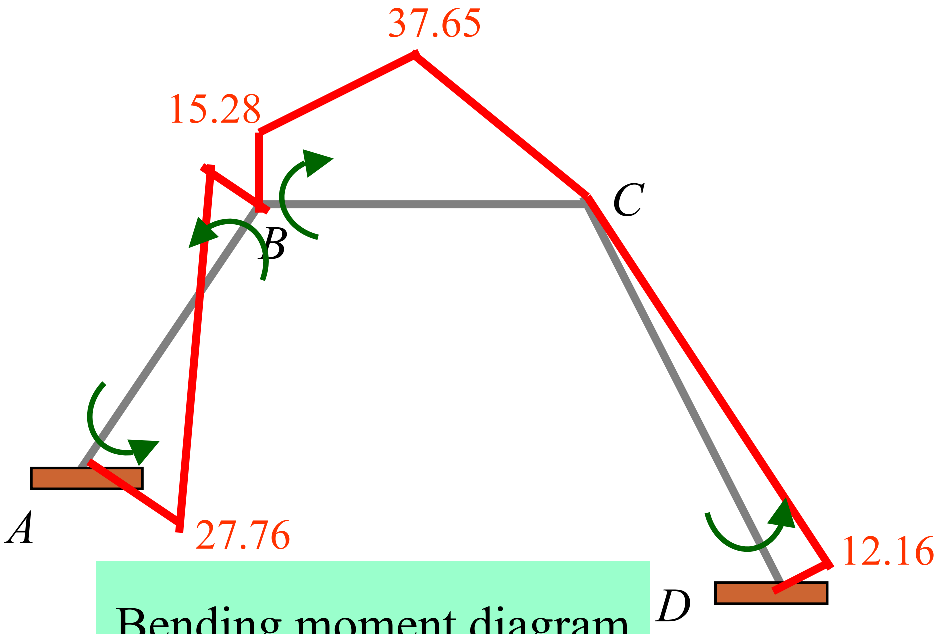
$R' = 87.83$ kN

Substitute R = -35.33 and R′= 87.83 in (1) : $-35.33 + C_1(87.83) = 0$

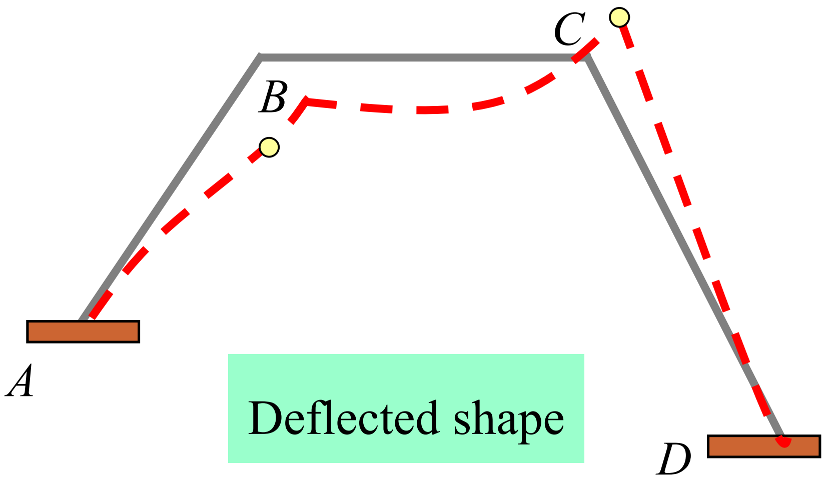
$C_1 = 35.33/87.83$







Bending moment diagram



Deflected shape