Technical Note

Development of Pakistan's New Area Weighted Rainfall Using Thiessen Polygon Method

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ABSTRACT

In this paper effort has been made to re-determine the area weighted rainfall data set of Pakistan, especially after the demarcation of more sub-regions. Monthly rainfall data of 56 climatological stations' rain gauges for the period of 50 years, from 1961-2010, have been used. The stations selected are almost evenly covering the mountainous and plain areas of the country. Based on this newly developed area weighted precipitation dataset of 56 stations in Pakistan, temporal and spatial variation of rainfall for Pakistan, as a whole country, and its six sub-regions, have been assessed for the period 1961–2010. The mean annual rainfall, for the fifty years period, of Pakistan comes out to be 297.6 mm with 63.4 mm standard deviation and 21.3 % coefficient of variability. Similarly the mean (1961-2010) annual rainfall for Azad Kashmir, Balochista,n Gilgit-Balitistan, Khyber-Pakhtunkhwa, Punjab & Balochistan are calculated as 418.7 mm, 160.6 mm, 207.0 mm, 734.6 mm, 386.3 mm & 172.4 mm respectively. Pakistan mean rainfall for winter (Jan-Mar) and Monsoon (Jul-Sep) seasons are 74.3 mm & 140.8 mm respectively, which contributes the 25 & 47 percentage of annual rainfall.

Introduction

In order to evaluate area rainfall quantity or area weighted rainfall (AWR), Thiessen Polygon is one of the widely used technique and important method. Its calculation method is fast, simple and some what accurate. With this method, the calculation of rainfall is simple, in a way that only the station's rainfall amount and the calculated station weight, area of the influence of each station (also called Theissen Constant or Area Factor) are required. Thiessen Polygon method is a standard method for computing mean areal precipitation (MAP) for the area having more or less homogeneous topographic as well as meteorological features (Majeed, 2003).

Station weights are scalar factors used to transform point precipitation observed at rainfall gauging stations into an associated mean precipitation over an area that the station data are assumed to represent (Fiedler 2003). There are two primary ways currently available to compute station weights: (1) the classical Thiessen methodology; and (2) inverse distance squared weighting (NWS 2002). However, station data are often sparse and do not represent areal processes well, particularly in mountainous regions. Gauges are commonly located at lower elevations in mountainous regions, causing consistent underestimation of MAP (NWS 2002). In these cases, users either subjectively modify (increase) computed station weights, or develop ''synthetic stations'' at high elevations; precipitation at these synthetic stations is also estimated with the inverse distance squared technique (NWS 2002).

The existing MAP and Area Factors being used in the Pakistan Meteorological Department (PMD) have been developed by Majeed (1988). The practice being adopted at PMD, based on 56 numbers of climate stations of country and covering the four geographical regions (provinces) namely; Sindh, Punjab, Balochistan and Khyber-Pakhtunkhwa (former; North Western Frontier Province). Later, Majeed (2002) recalculated the mean aerial precipitation procedure based on same four geographical regions, as mentioned above. In this study, 56 number of stations were used, which also covers the two remaining two sub-regions (Gilgit-Balitistan and Azad Kashmir).

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Data and Method

Estimation of Mean Areal Precipitation

There are several methods for converting a network of precipitation measurements into areal estimates. Some are:

Arithmetic Mean

In this method areal precipitation are calculated using the arithmetic mean of all the points of rainfall measurements considered in the analysis.

Isohyetal Analysis

In this graphical technique, lines of equal rainfall, called isohyetes, are drawn over an area of precipitation point measurements. The magnitude and extent of the resultant rainfall areas of coverage are used to estimate the areal precipitation value.

Thiessen Polygon

This is a method which calculates station weights based on the areas of each station. Each weights are then multiplied by the station precipitation to obtain the areal average precipitation.

Distance Weighting/Gridded

This is another station weighting technique. A grid of point estimates is made, based on a distance weighting scheme. Each observed point value is given a unique weight for each grid point based on the distance from the grid point in question. The grid point precipitation value is calculated based on the sum of the individual station weight multiplied by observed station value. Once the grid points have all been estimated, they are summed and the sum is divided by the number of grid points to obtain the areal average precipitation.

Data used

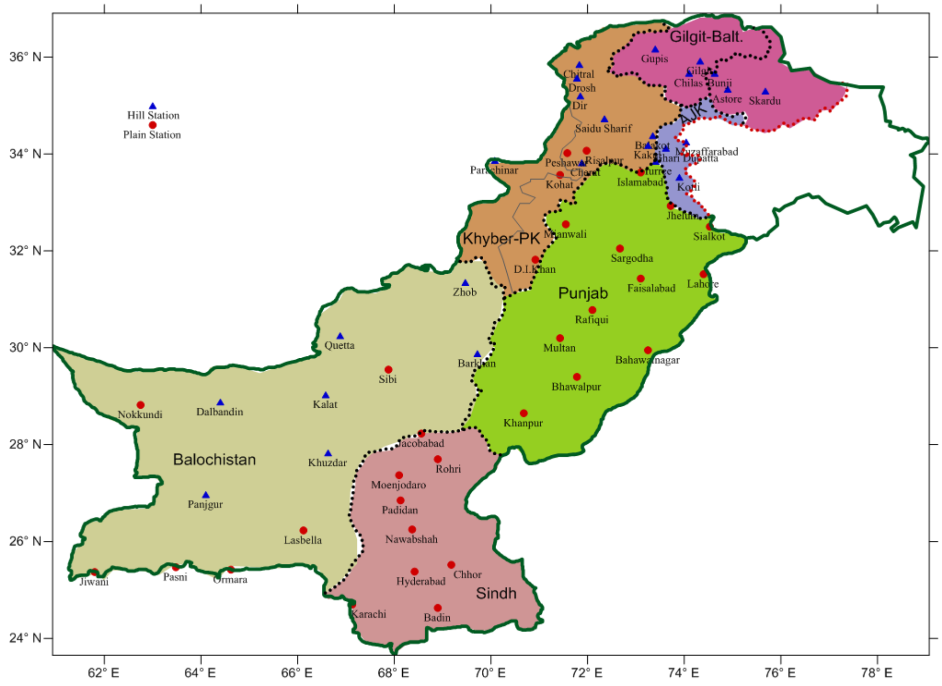

Figure 1: Location of the plain & hilly stations used in study

In this paper effort has been made to re-determine the area weighted rainfall data set of Pakistan, especially after the demarcation of more sub-regions. To prepare rainfall series, 56 rain-gauge stations have been selected for this study. The stations selected have the maximum available data for the period of fifty years, covering time span from 1961 to 2010. The network of selected stations (Figure-1) includes 25 numbers hilly precipitation measuring points while 31 are located on plain areas. Most of these selected stations have the data for the more than 90% of the years considered (1961-2010). However if any station rainfall data is found missing, it has been filled up by constructing isohytel map of the particular month, using the available rainfall data of the neighboring rain-gauge stations. Figure-2 shows the location of the stations being used by PMD. The numbers of the stations used in both cases are same, as 56 rain gauges. Some of the stations (plain area) used in PMD scheme were not considered in this syudy due to paucity of data, whereas equal number of hilly stations were incorporated, in order to get the representation of remaining two sub-regions.

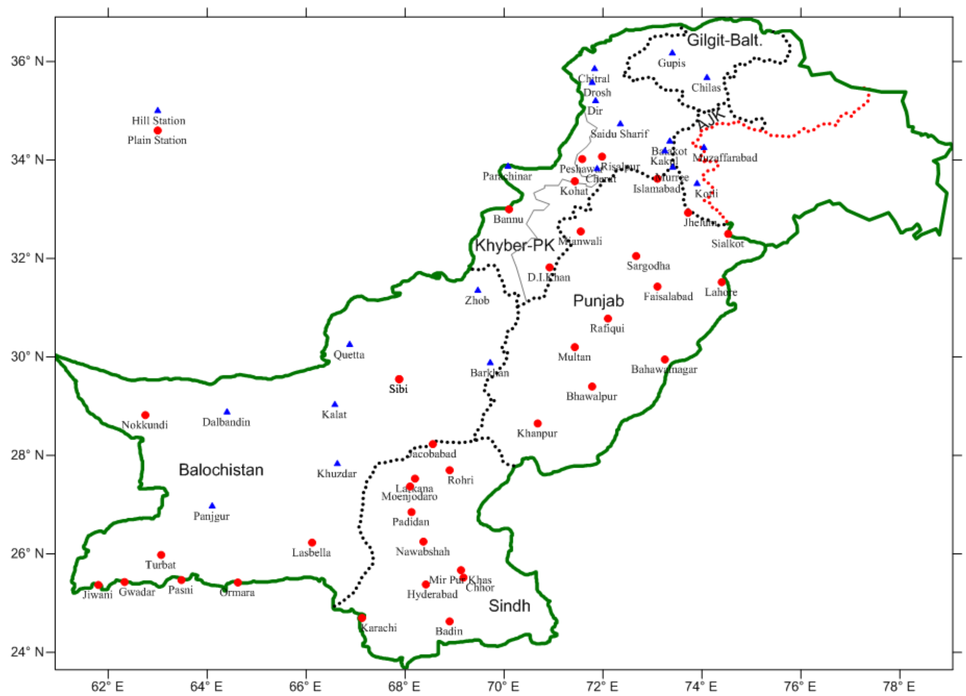

Figure 2: Location of the stations used by PMD

Analysis Technique

The whole country is gridded to a 0.20° x 0.20° grid (Figure-3). In order to determine area weighted rainfall for each station with the help of Theissen Polygon method, each station was connected by straight lines with the several nearest stations to form a series of triangles. Perpendicular bisectors were erected on each of these lines and extend them to intersect with other bisectors, thus forming a series of irregular polygons (Figure-4). Then estimate the area of each station polygon by counting grid squares (points) for the polygons formed by the bisect lines and multiplying them by cosine of latitude because on higher latitude each grid corresponds to lesser area.

There are 2025 grid points with in the boundary of Pakistan. Area of each station was calculated by summing the grids laying in that particular polygon and finally the station weights were calculated by dividing the station area by the total area of country or sub-region. The sum of all the station weights comes 1. Similar procedures were adopted for the calculation of station weights for each six sub-regions. The area weighted rainfall for each station determined by multiplying its area factor and the rainfall amount recorded at the station. All the stations' area weighted rainfall is sum up to calculate

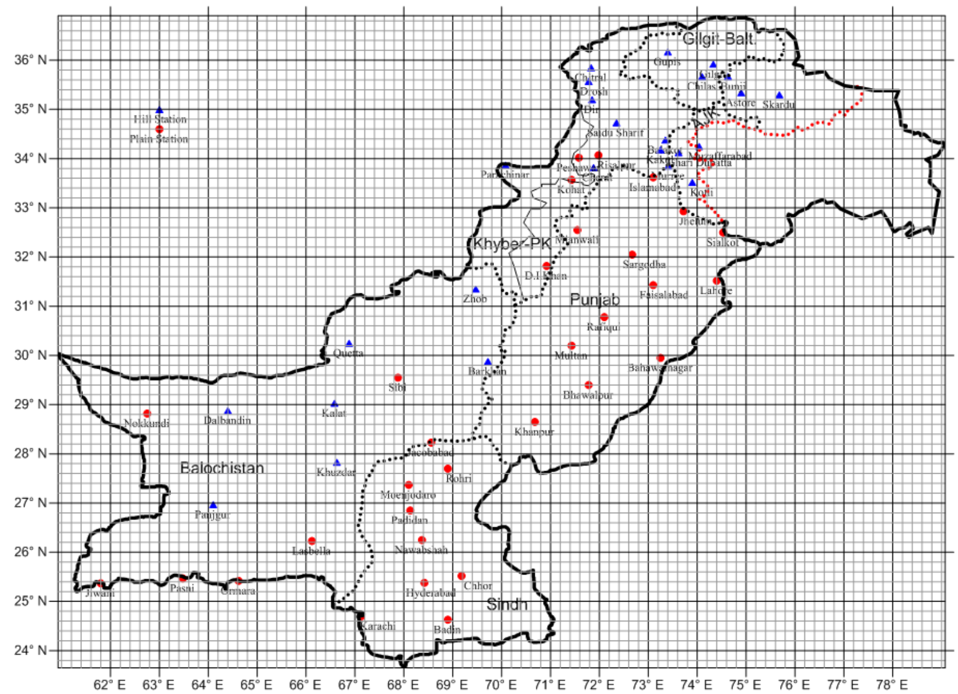

Figure 3: Gridded scheme of the area

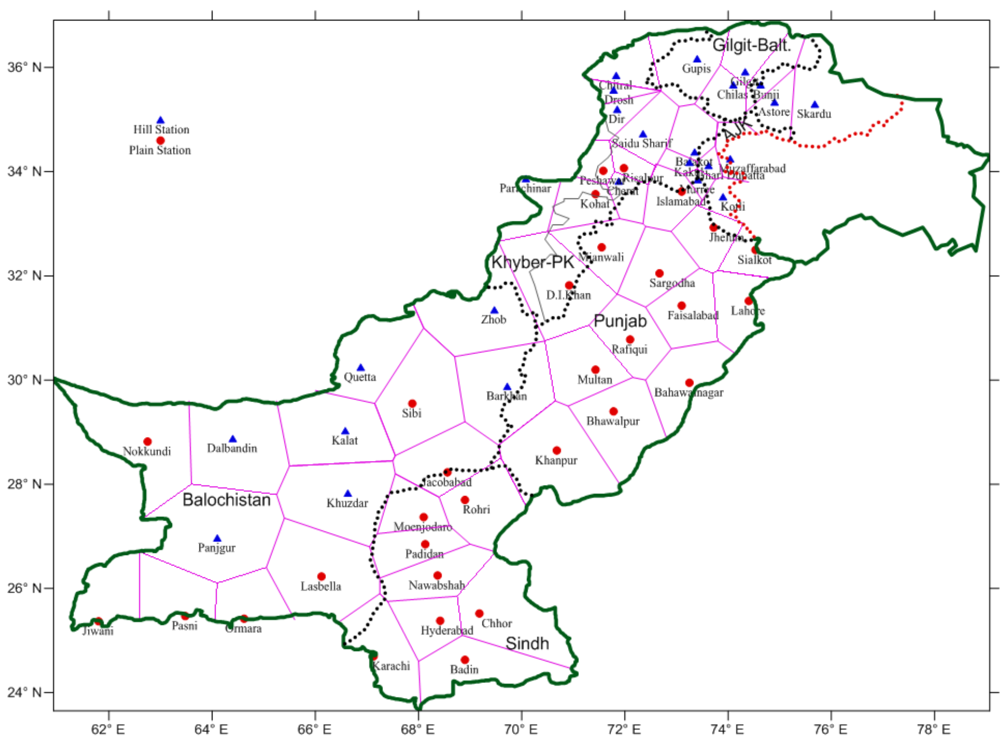

Figure 4: Map showing the area of each polygons

final area weighted rainfall of the country. Similar procedure is adopted for the final area weighted rainfall for six sub regions.

$$A_i = \sum_{j=1}^{k} n_k \cos \emptyset_k$$

Where:

A_i = Area of polygon

n = Number of grid points on same latitude

k = Number of latitude on which a polygon spread

$$AF = A_i / A_t$$

Where:

AF = Area Factor of polygon/station

A_t = Total area of region

$$R_{aw} = \sum_{i=1}^{m} AF_i \, r_i$$

Where:

R_{aw} = Area Weighted Rainfall of the region

r_i = Station's rainfall

Monthly, seasonal and annual series of area weighted rainfall for Pakistan, as a whole country, as well as its six provinces, is constructed by summing the area weighted rainfall of all 56 stations. The mean annual rainfall, for the fifty years period, of Pakistan comes out to be 297.6 mm (Table-1). Similarly the long period mean (1961-2010) annual rainfall for Azad Kashmir, Balochistan, Gilgit-Balitistan, Khyber-Pakhtunkhwa, Punjab & Sindh are calculated as 418.7 mm, 160.6 mm, 208.0 mm, 734.6 mm, 386.3 mm & 172.4 mm respectively. Figure-5 illustrates the distribution of long-term (1961-2010) monthly average rainfall over country and its six sub-regions. Brief trend analysis is conducted on country and six provinces, on annual basis, over the period of fifty years. Figure-6 depicts the increasing or decreasing trends, in mm, over fifty years of period. Annual rainfall is increasing on national and majority of the provinces except Azad Kashmir and Sindh, where it is almost unchanged over the period.

The contribution of Pakistan's monthly rainfall of July (21%) & August (19%) to the annual rainfall are highest, almost similar position is seen over the provinces too (Table-2). Pakistan mean rainfall for winter (Jan-Mar) and Monsoon (Jul-Sep) seasons are 74.3 mm & 140.8 mm, which contribute the 25 & 47 percentage of total annual rainfall respectively.

Figure-7 shows the comparison of Pakistan's PMD mean annual rainfall time series with the rainfall series developed in this study. The mean annual rainfall of PMD is 294.3 mm whereas the mean area weighted rainfall series developed by this study is 297.5 mm. The slight difference between the mean values of two series is because of the consideration of some stations located in Gilgit-Baltistan and Azad Kashmir regions, which mainly comprises of hilly area. The standard deviation and coefficient of variability for the PMD series are 75.7 mm and 25.7% and the same for the present time series are 63.4 mm, 21.3% (Table-3) respectively. Coefficient of variation of the present time series is smaller compared to PMD time series.

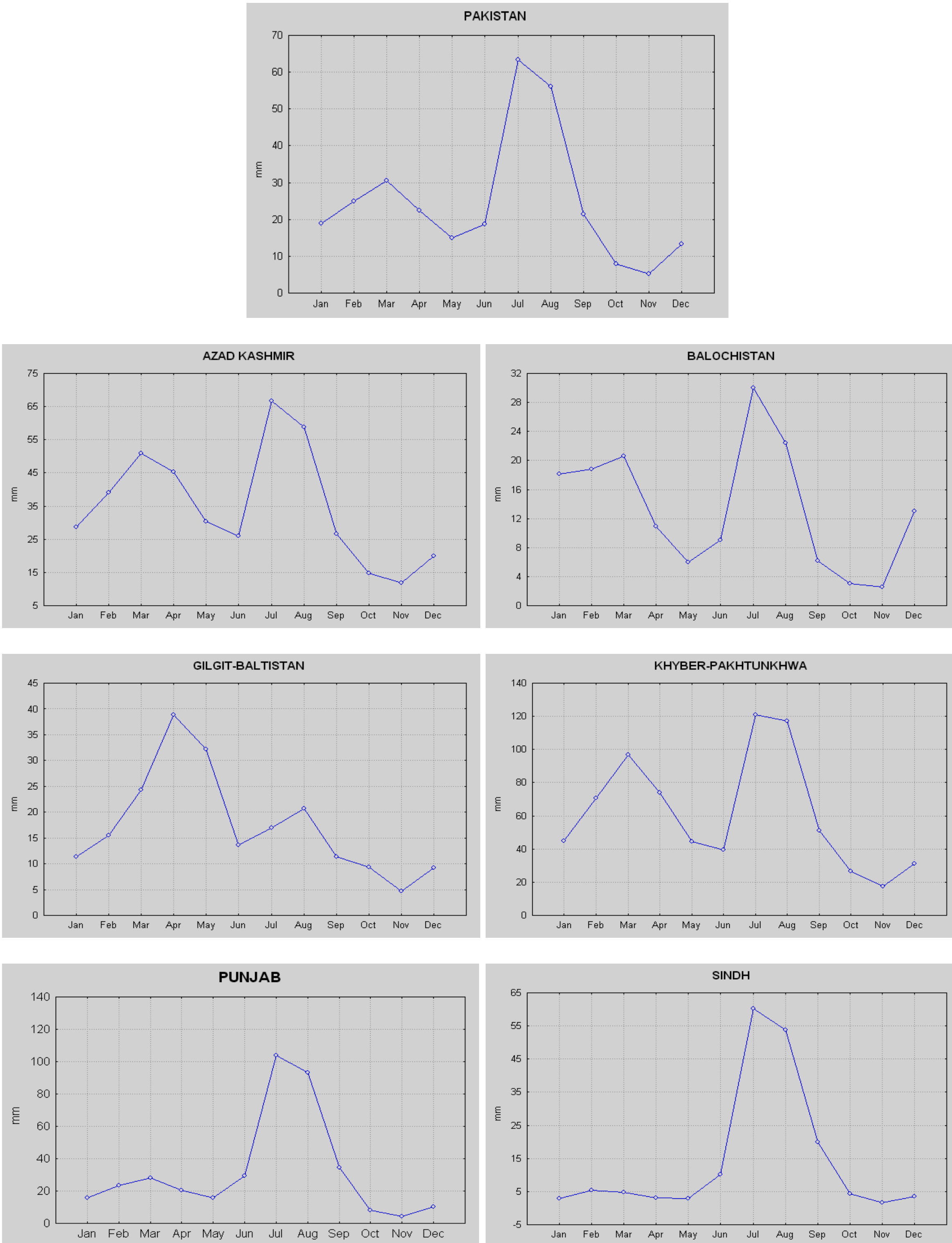

Figure 5: Distribution of monthly average rainfall

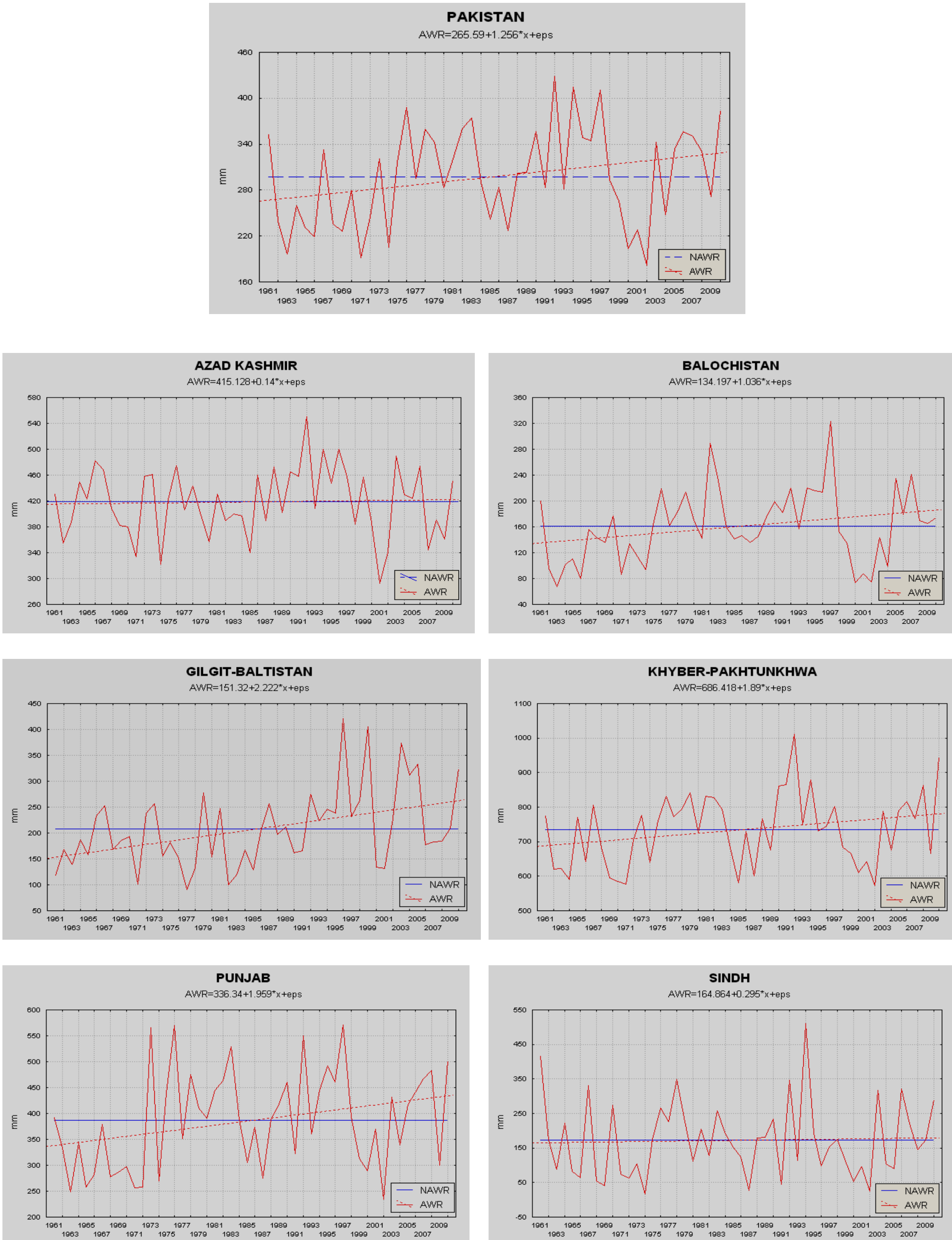

Figure 6: Area-Weighted time series of annul precipitation (red solid lines) along with trends and normalized annual precipitation (blue doted lines)

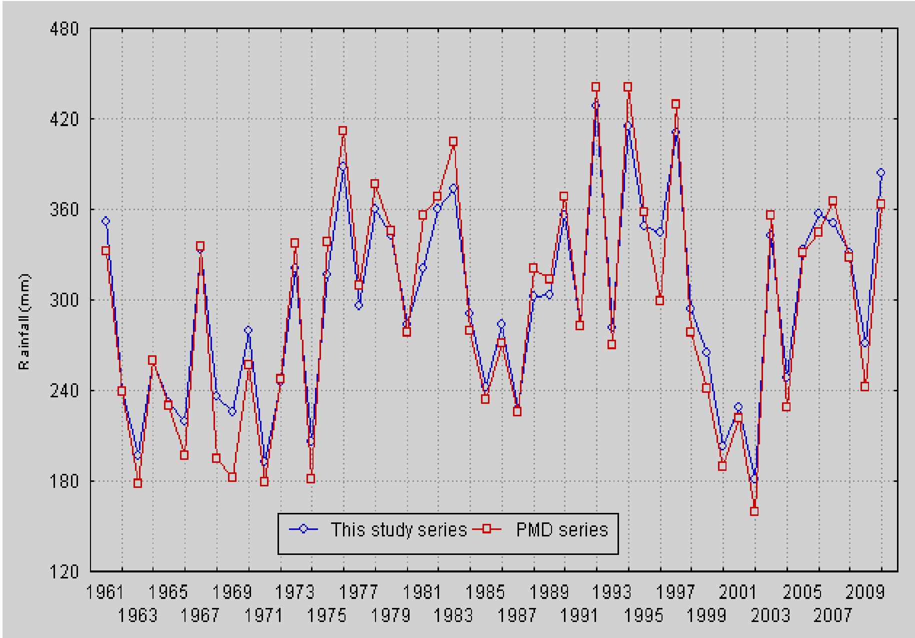

Figure 7: Comparison of Pakistan's PMD mean annual. series and series developed in this study.

Table 1: Descriptive Statistics of Annual Rainfall

	Valid N	Mean	Minimum	Maximum	Lower Quartile	Upper Quartile	Range	Variance	Std. Dev.	Standard Error
Pakistan	50	297.6	181.4	428.2	241.8	348.9	246.8	4016.7	63.4	9.0
Azad Kashmir	50	418.7	292.3	550.7	386.3	458.4	258.4	2830.7	53.2	7.5
Balochistan	50	160.6	67.7	323.1	133.6	198.8	255.4	3072.4	55.4	7.8
Gilgit-Baltistan	50	208.0	90.6	421.6	155.0	247.7	331.0	5755.7	75.9	10.7
Khyber-PK	50	734.6	572.6	1011.4	642.5	802.1	438.8	10559.7	102.8	14.5
Punjab	50	386.3	234.8	571.4	300.0	460.4	336.6	8793.6	93.8	13.3
Sindh	50	172.4	18.1	510.7	89.2	226.9	492.6	11868.1	108.9	15.4

Table 2: Normal (1961-2010) Area Weighted Rainfall (mm) of Pakistan & Provinces

Month/ Season	Pakistan	Azad Kashmir	Balochistan	Gilgit-Baltistan	Khyber-Pakhtunkhwa	Punjab	Sindh
January	18.9	28.6	18.1	11.4	45.0	15.6	2.8
February	24.9	39.0	18.8	15.5	70.5	23.2	5.3
March	30.6	51.0	20.6	24.3	96.7	28.0	4.7
April	22.5	45.2	10.9	38.8	73.9	20.6	3.1
May	14.9	30.4	6.0	32.2	44.3	15.6	2.9
June	18.6	25.9	9.0	13.6	39.6	29.4	10.2
July	63.3	66.7	30.0	16.9	120.7	103.8	60.3
August	56.1	58.9	22.4	20.7	117.3	93.1	53.7
September	21.4	26.6	6.2	11.4	51.3	34.6	19.9
October	8.0	14.8	3.0	9.4	26.6	8.1	4.3
November	5.2	11.8	2.6	4.7	17.5	4.2	1.6

December	13.4	20.0	13.0	9.2	31.2	10.2	3.5
Jan-Mar	74.3	118.6	57.5	51.2	212.2	66.8	12.8
Apr-Jun	55.9	101.5	25.9	84.5	157.8	65.6	16.3
Jul-Sep	140.8	152.1	58.7	49.0	289.3	231.4	133.9
Oct-Dec	26.5	46.5	18.5	23.3	75.3	22.5	9.4
Annual	297.6	418.7	160.6	208.0	734.6	386.3	172.4

Table 3: Annual Area Weighted Rainfall (mm)

Year	Pakistan	Azad Kashmir	Balochistan	Gilgit-Baltistan	Khyber-Pakhtunkhwa	Punjab	Sindh
1961	352.1	430.2	200.5	117.8	774.9	392.6	416.1
1962	240.4	354.8	95.6	168.3	619.8	332.7	173.9
1963	196.7	387.6	67.7	140.1	622.4	248.6	88.4
1964	259.2	449.6	100.7	186.5	591.4	342.9	222.1
1965	231.4	423.2	110.4	157.3	770.7	257.3	82.4
1966	219.5	482.4	79.5	232.8	642.5	281.5	63.7
1967	333.2	468.0	156.1	252.9	805.6	378.8	330.8
1968	235.7	408.4	143.0	169.1	683.4	277.8	55.3
1969	226.0	382.2	135.9	186.2	593.9	286.8	40.3
1970	279.5	380.3	177.1	193.6	584.7	297.9	274.4
1971	192.0	334.1	86.1	101.5	576.1	256.2	73.8
1972	245.1	457.5	133.6	237.6	705.6	257.3	62.7
1973	320.9	460.6	114.0	256.7	776.0	566.0	104.2
1974	205.5	321.9	93.5	155.0	639.9	269.5	18.1
1975	316.4	422.4	163.3	182.2	760.4	440.0	175.7
1976	388.4	474.6	219.2	151.1	832.5	570.2	265.5
1977	295.5	406.0	162.0	90.6	772.2	352.0	226.9
1978	359.8	442.8	182.8	132.8	793.4	475.1	349.3
1979	342.6	397.6	213.3	277.8	841.4	411.0	226.7
1980	283.4	357.2	170.3	154.5	726.2	391.2	112.6
1981	320.6	430.7	141.6	247.7	832.6	445.1	205.6
1982	359.9	389.4	289.0	100.1	827.6	462.7	127.7
1983	373.9	400.4	231.6	119.8	793.8	529.5	257.6
1984	290.3	396.4	159.2	167.5	681.4	388.5	190.6
1985	241.8	339.9	140.9	129.2	581.5	305.0	151.7
1986	283.4	460.4	146.1	203.5	728.5	373.3	123.3
1987	227.1	389.1	136.3	256.6	601.1	274.9	27.6
1988	302.0	472.6	144.8	197.2	765.8	388.1	178.6
1989	303.0	402.2	174.4	211.4	675.1	418.1	181.1
1990	356.0	465.7	198.8	162.4	861.9	460.8	233.5
1991	283.3	458.1	182.5	165.8	866.0	322.1	44.3
1992	428.2	550.7	220.2	274.8	1011.4	549.6	345.9
1993	281.8	408.4	156.8	223.7	748.3	361.1	113.3

1994	414.4	499.6	220.5	245.7	879.6	444.4	510.7
1995	348.9	447.5	215.9	237.6	730.8	491.9	193.4
1996	344.3	499.9	213.3	421.6	745.0	460.4	98.6
1997	410.4	458.4	323.1	231.3	802.1	571.4	152.7
1998	293.5	384.1	152.2	262.8	683.6	393.1	175.2
1999	265.3	456.5	135.1	405.6	666.5	313.8	112.2
2000	203.0	386.3	74.0	134.3	610.9	289.1	52.6
2001	228.3	292.3	87.9	132.2	641.2	370.0	95.3
2002	181.4	341.0	74.9	227.3	572.6	234.8	24.7
2003	342.5	489.0	143.7	373.3	787.7	431.5	317.1
2004	247.9	429.2	98.8	312.2	675.1	340.1	103.4
2005	333.4	424.6	234.4	333.3	791.1	417.4	89.2
2006	356.5	473.3	179.0	177.9	816.1	441.1	321.7
2007	350.3	343.9	241.6	182.7	766.8	467.3	221.2
2008	331.0	390.3	169.6	184.2	863.4	483.4	145.4
2009	271.2	361.2	165.3	208.9	665.6	300.0	173.2
2010	383.6	452.0	174.2	323.7	944.6	501.4	288.7
ave	297.6	418.7	160.6	208.0	734.6	386.3	172.4
SD	63.4	53.2	55.4	75.9	102.8	93.8	108.9
c.v.	21.3	12.7	34.5	36.5	14.0	24.3	63.2

Conclusion

After the creation of new province, there was need for development of a spatially and temporally area weighted rainfall series for all the six sub regions (provinces), in addition to, country as a whole. The new rainfall series constructed by this study is distributed uniformly all over the country and it represents all the existing provinces. In this study time series of area weighted rainfall have been developed using Theissen Polygen method, which is simple, fast and practical method. The newly time series represents an improvement over earlier, as it includes the rainfall data of hilly areas as well as plains and serves effectively for the requirement of existing needs.

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