CHAPTER 13

SHEAR STRENGTH OF SOIL

Shear Strength

- **The shear strength of the soil is the internal resistance per unit area that the soil mass can offer to resist failing/failure and sliding along any plane inside it.**
- **Mohr (1900) presented a theory saying that rupture (or failure) in material is because of a critical combination of normal stress and shearing stress and not from either max. normal or shear stress alone.**
- **For most soil mechanics problems, it is sufficient to approximate the shear stress on the failure plane as linear function of the normal stress (Coulomb, 1776). This linear function is :**

$$\tau_f = c + \sigma \tan\phi$$

$$or \;\; \tau_f = c' + \sigma' \tan\phi' \;\; (in\ terms\ of\ effective\ stress)$$

Mohr-Coulomb Failure Criterion

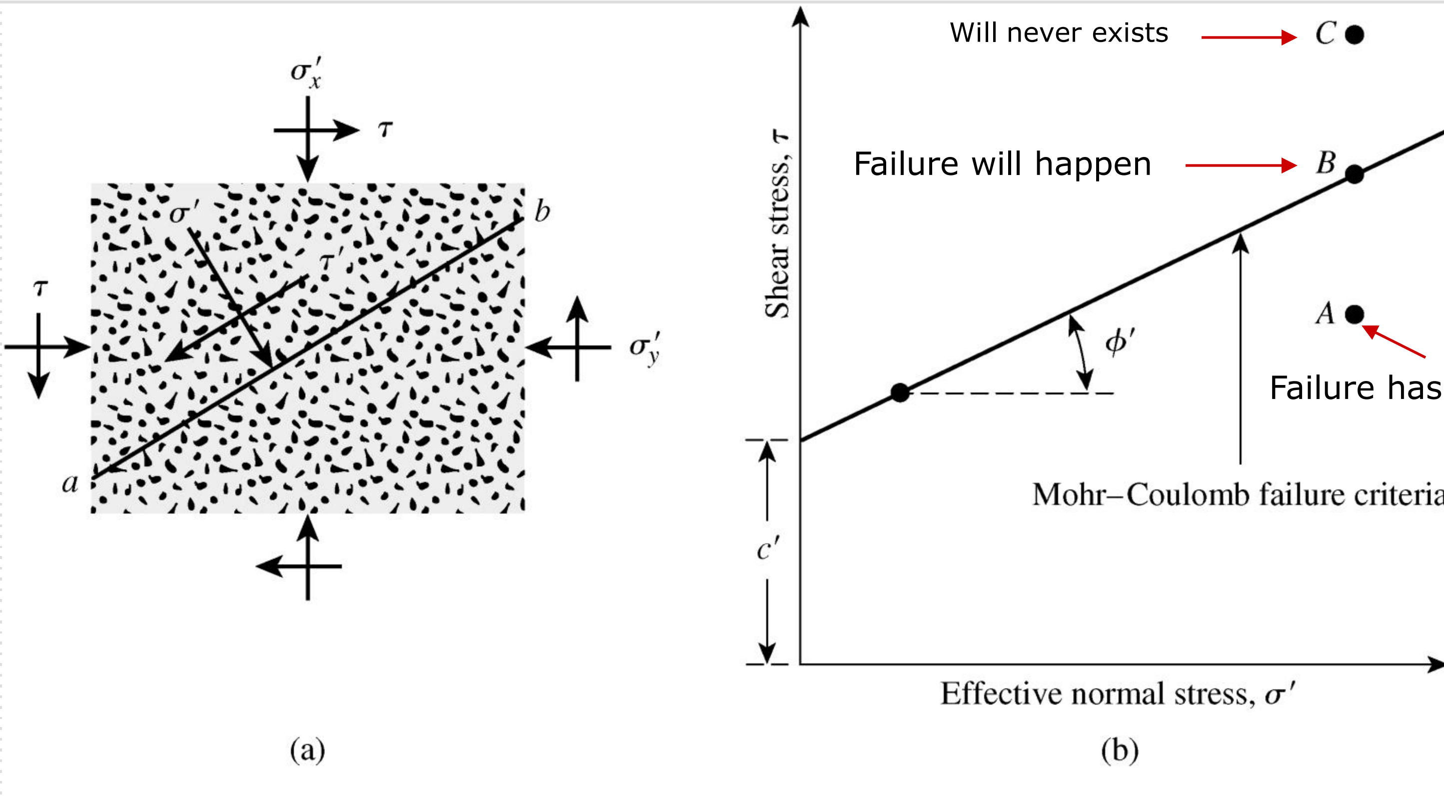


Figure 11.1 Mohr–Coulomb failure criterion

Factors That Affect The Shear Strength of Soil

- Cohesionless soil
  1. Soil type
  2. Soil density
  3. Grain size distribution
  4. Mineral type, angularity and particle size
  5. Deposit variability-because of variations in soil types, particle arrangements etc., the effective friction angle is rarely uniform with depth.
  6. Etc.

Factors That Affect The Shear Strength of Soil

- Cohesive soil

1. Soil type – the more plastic the soil (higher PI), the lower the values of c' and ϕ'.
2. Particle Bonding
3. Stress history
4. Peak and ultimate shear strength
5. Etc.-such as sample disturbance, strain rate, anisotropy etc.

Shear Strength

Table 11.1 Typical Values of Drained Angle of Friction for Sands and Silts

Soil type	**ϕ' (deg)**
Sand: Rounded grains	
Loose	27–30
Medium	30–35
Dense	35–38
Sand: Angular grains	
Loose	30–35
Medium	35–40
Dense	40–45
Gravel with some sand	34–48
Silts	26–35

Inclination of The Plane of Failure Caused by Shear

- As stated by the Mohr-Coulomb failure criterion, failure from shear will occur when the shear stress on a plane reaches a value given by:

$$\tau_f = c' + \sigma' \tan\phi'$$

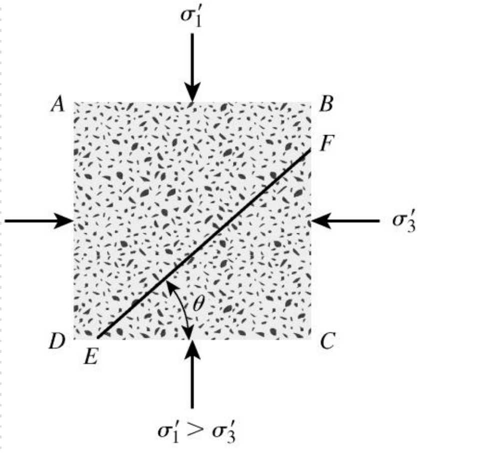


Figure 11.2 Inclination of failure plane in soil with major principal plane

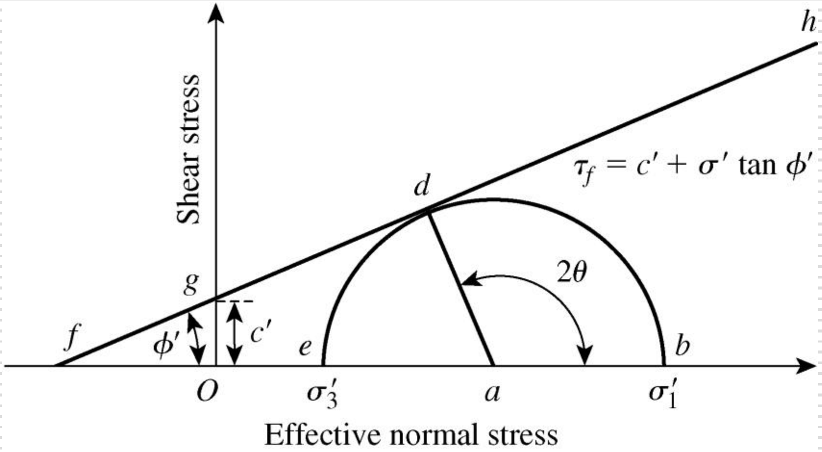


Figure 11.3 Mohr's circle and failure envelope

Inclination of The Plane of Failure Caused by Shear

- From Figures 11.2 and 11.3, angle bad = 2θ = 90°+ϕ

or

$$\theta = 45 + \frac{\phi'}{2}$$

and

$$\frac{1+\sin\phi'}{1-\sin\phi'} = \tan^2\left(45 + \frac{\phi'}{2}\right)$$

$$\sigma'_1 = \sigma'_3 \tan^2\left(45 + \frac{\phi'}{2}\right) + 2c'\tan\left(45 + \frac{\phi'}{2}\right)$$

← General formula

Drained and Undrained Conditions

- The drained condition exists when the rate of loading is slow compared to the rate of drainage. Thus, water is able to easily flow into or out of the voids and essentially no excess pore water pressures develop in the soil.
- The undrained condition exists when the rate of loading is rapid compared to the rate of drainage. Thus, water is not able to flow into or out of the voids quickly enough, and excess pore water pressures develop in the soil.

Laboratory Tests For Detemination of Shear Strength Parameters

- We do tests to get shear strength parameters such as c, ϕ, c′, ϕ′, etc. There are several tests conducted in the laboratory.

1. Direct shear test
2. Triaxial test
3. Unconfined compression test
4. Vane shear test
5. etc.

Direct Shear Test

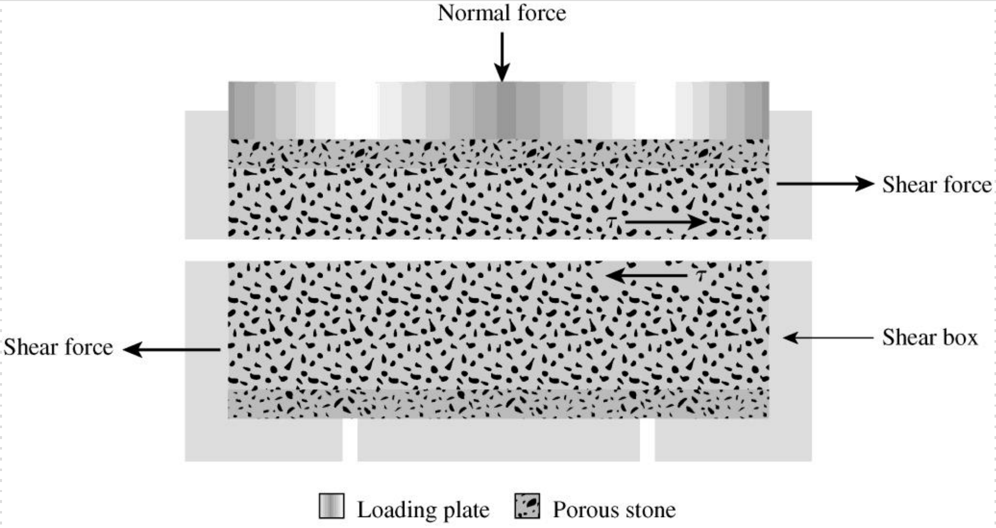


Figure 11.4 Diagram of direct shear test arrangement

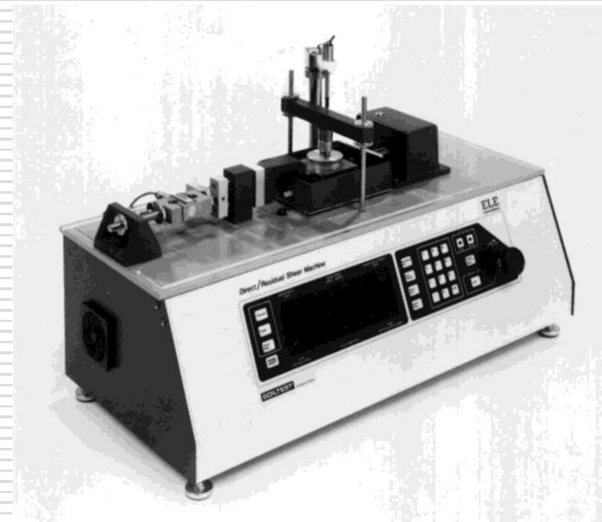


Figure 11.5 Strain-controlled direct shear test equipment (*Courtesy* of Soiltest, Inc., Lake Bluff, Illinois)

- Size of specimen is 100mm X 100mm
- Can be either stress or strain controlled
- Normal stress of specimen max. is approximately 1050 kN/m^2
- Resisting shear force of the soil corresponding to any shear displacement can be measured by a horizontal proving ring or load cell.

Direct Shear Test

- In the direct shear test arrangement, the shear box that contains the soil specimen is generally kept inside a container that can be filled with water to saturate the specimen.
- A drained test is made on a saturated soil specimen by keeping the rate of loading slow enough so that the excess pore water pressure generated in the soils is dissipated completely by drainage.
- Pore water from the specimen is drained through two porous stone.

Direct Shear Test

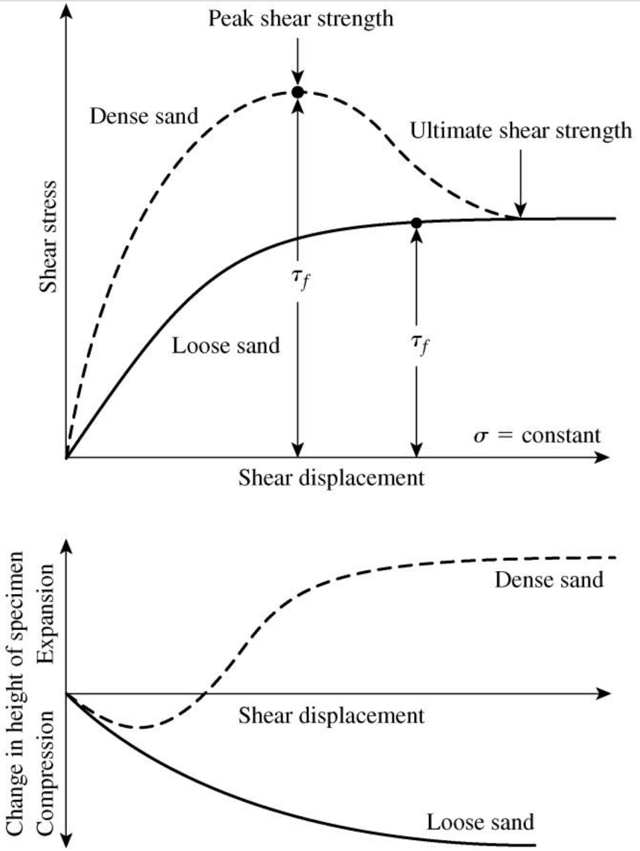


Figure 11.6 Plot of shear stress and change in height of specimen against shear displacement for loose and dense dry sand (direct shear test)

Direct Shear Test

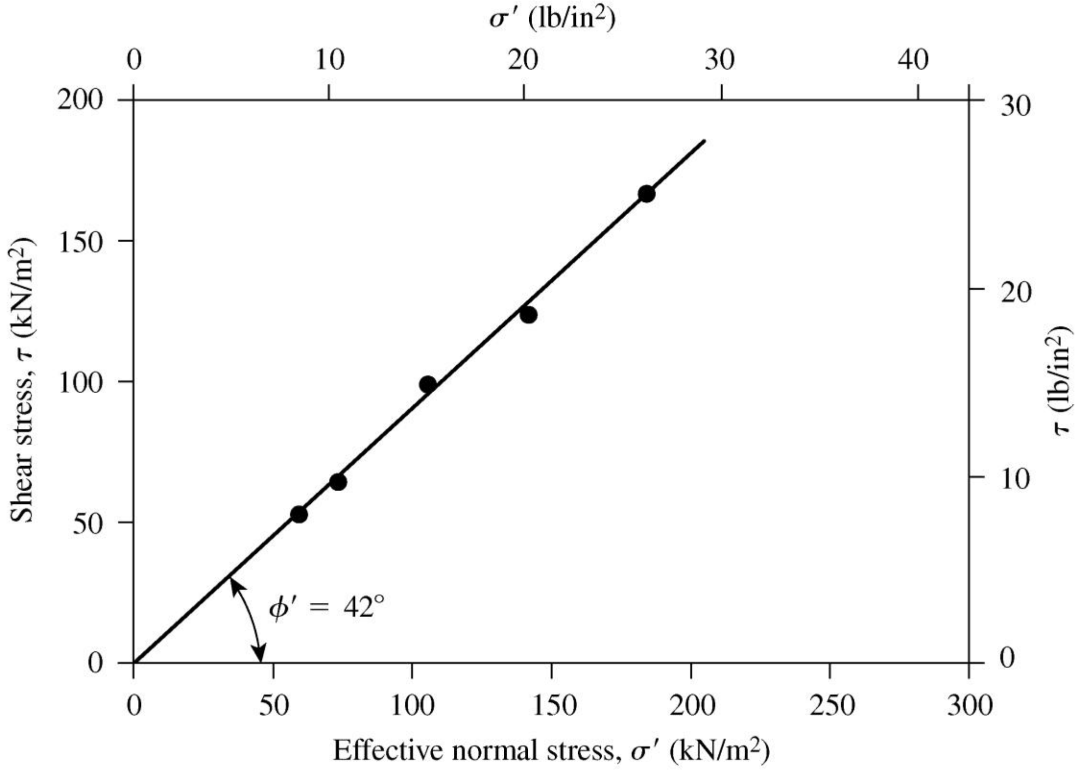


Figure 11.7 Determination of shear strength parameters for a dry sand using the results of direct shear tests

Direct Shear Test

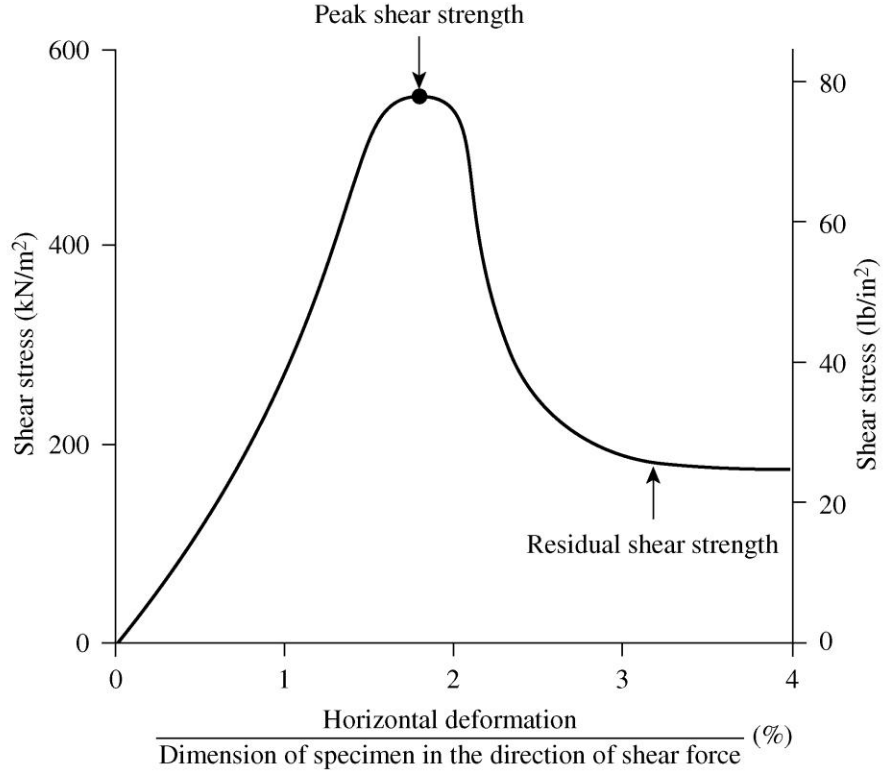


Figure 11.8 Results of a drained direct shear test on an overconsolidated clay. [*Note:* Residual shear strength in clay is similar to ultimate shear strength in sand (see Figure 11.6).]

Direct Shear Test

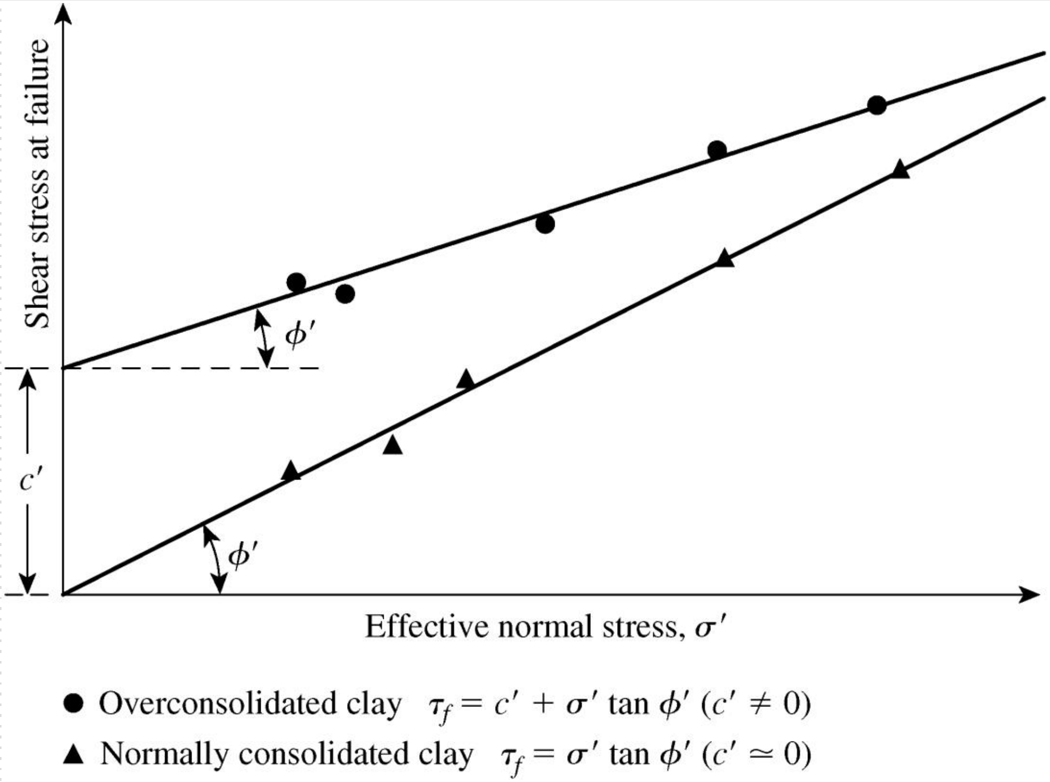


Figure 11.9
Failure envelope for clay obtained from drained direct shear tests

Triaxial Test

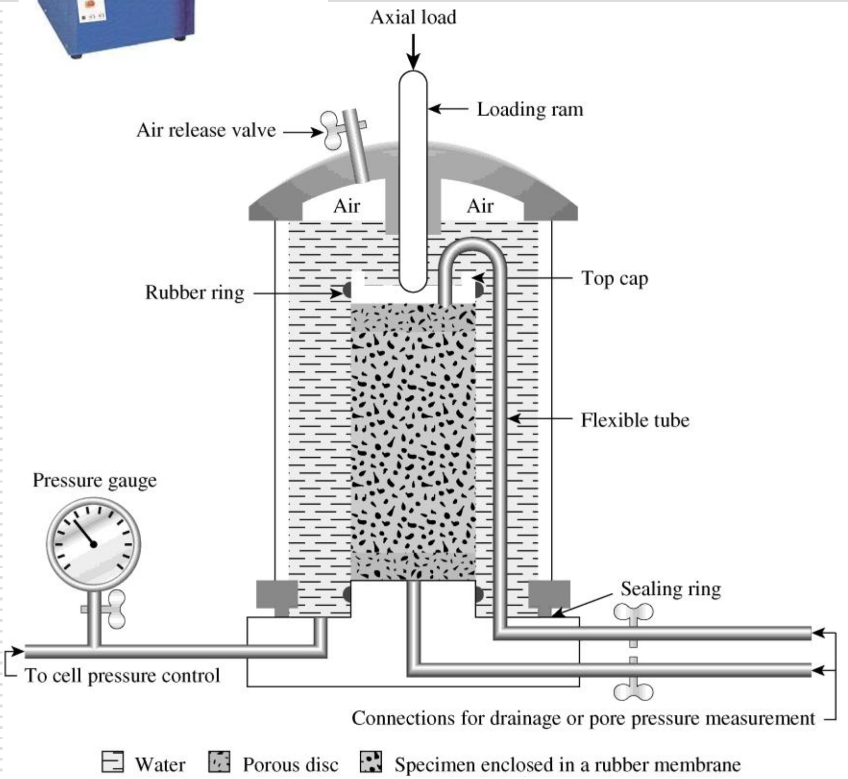


Figure 11.18 Diagram of triaxial test equipment (After Bishop and Bjerrum, 1960)

In triaxial test, specimen is subjected to a confining pressure (σ_3) by compression of the fluid in chamber.

To cause shear failure, axial stress is applied through a vertical loading ram. This axial stress is sometimes called deviator stress.

The axial load applied by the loading ram corresponding to a given axial deformation is measured by a proving ring or load cell attached to the ram.

Triaxial Test

- There are three standard types of Triaxial Test:

1. Consolidated-drained test or drained test (CD Test)
2. Consolidated-undrained test or undrained test (CU Test)
3. Unconsolidated Undrained test (UU Test)

Consolidated-Drained Test (CD)

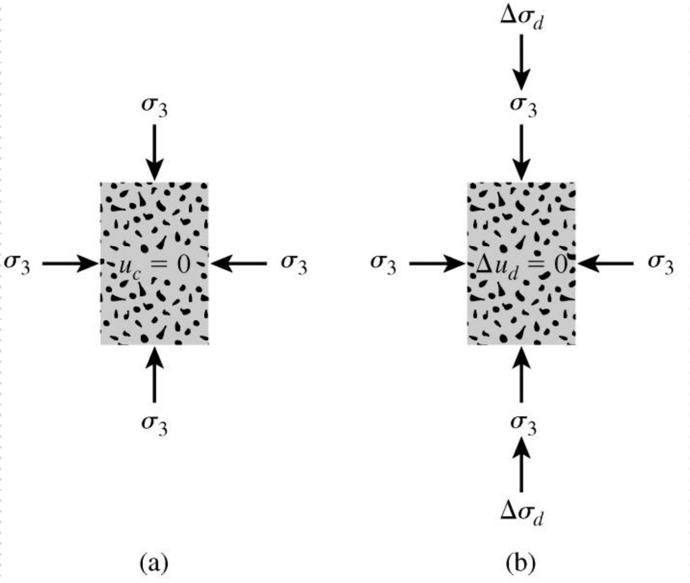


Figure 11.19
Consolidated-drained triaxial test: (a) specimen under chamber-confining pressure; (b) deviator stress application

Because the pore water pressure developed during the test is completely dissipated, therefore u=0.

From $\sigma_{3(\text{Total})} = u + \sigma'_3$

when u=0 $\sigma_3 = 0 + \sigma'_3$

$$\sigma_3 = \sigma'_3$$

Total and effective axial stress at failure

$$= \sigma_3 + (\Delta\, \sigma_d)_f = \sigma_1 = \sigma'_1$$

$$\tau_f = \sigma' \tan\phi'$$

Consolidated-drained triaxial test on clayey soil may take several days to complete. This time period is required because deviator stress must be applied very slowly to ensure full drainage. For this reason the CD type is uncommon.

Consolidated-Drained Test (CD)

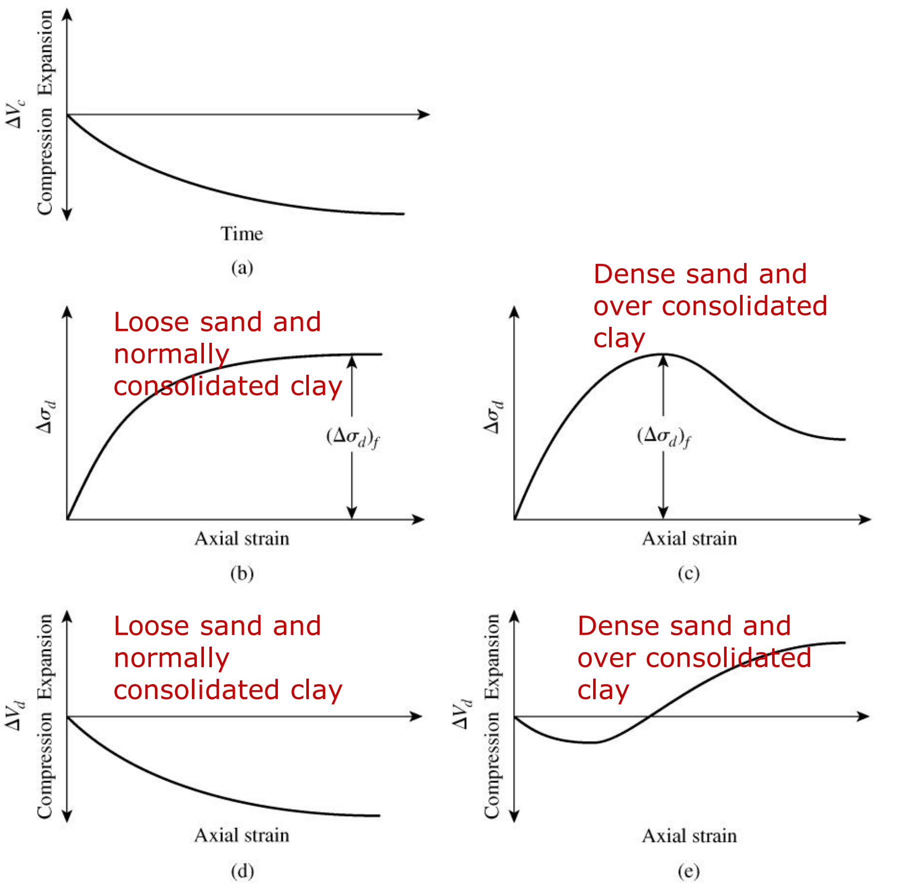


Figure 11.20 Consolidated-drained triaxial test: (a) volume change of specimen caused by chamber-confining pressure; (b) plot of deviator stress against strain in the vertical direction for loose sand and normally consolidated clay; (c) plot of deviator stress against strain in the vertical direction for dense sand and overconsolidated clay; (d) volume change in loose sand and normally consolidated clay during deviator stress application; (e) volume change in dense sand and overconsolidated clay during deviator stress application

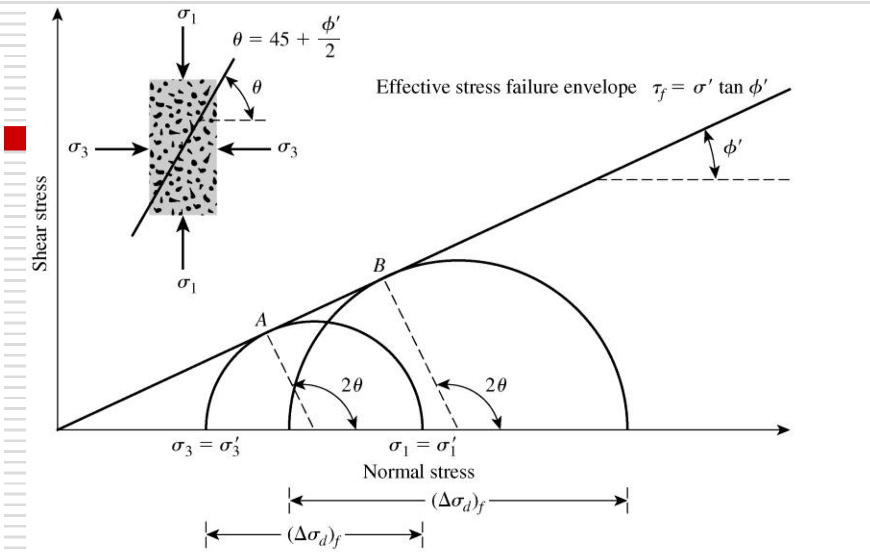


Figure 11.21 Effective stress failure envelope from drained tests on sand and normally consolidated clay

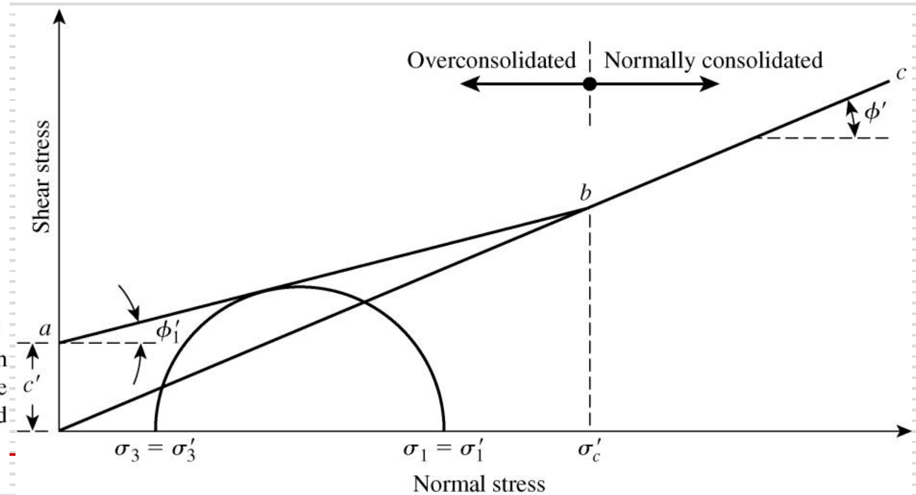
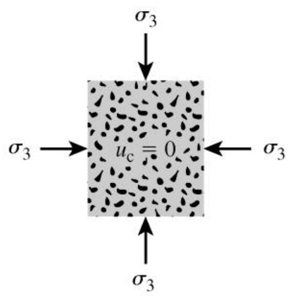


Figure 11.22 Effective stress failure envelope for overconsolidated clay

Consolidated-Undrained Test (CU)



(a)Specimen under chamber confining pressure

Major principal stress at failure (total): $\sigma_{3(t)}+(\Delta\sigma_d)_f=\sigma_{1(t)}$

Major principal stress at failure (effective): $\sigma_{1(t)}-(\Delta u_d)_f= \sigma'_1$

Minor principal stress at failure (total): $\sigma_{3(t)}$

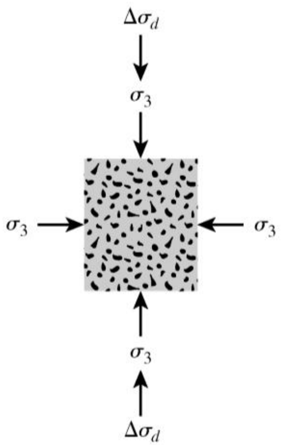
Minor principal stress at failure (effective): $\sigma_{3(t)} -(\Delta u_d)_f=\sigma'_3$

Where $(\Delta u_d)_f$ = pore water pressure at failure

$\sigma_{1(t)}- \sigma_{3(t)} = \sigma'_1- \sigma'_3$

$\tau_f = \sigma \tan\phi_{(cu)}$

Consolidated-undrained is the most common type of triaxial test. Because drainage is not allowed in these tests during the application of deviator stress, they can be performed quickly.



(b) Deviator stress application

Consolidated Undrained Test (CU)

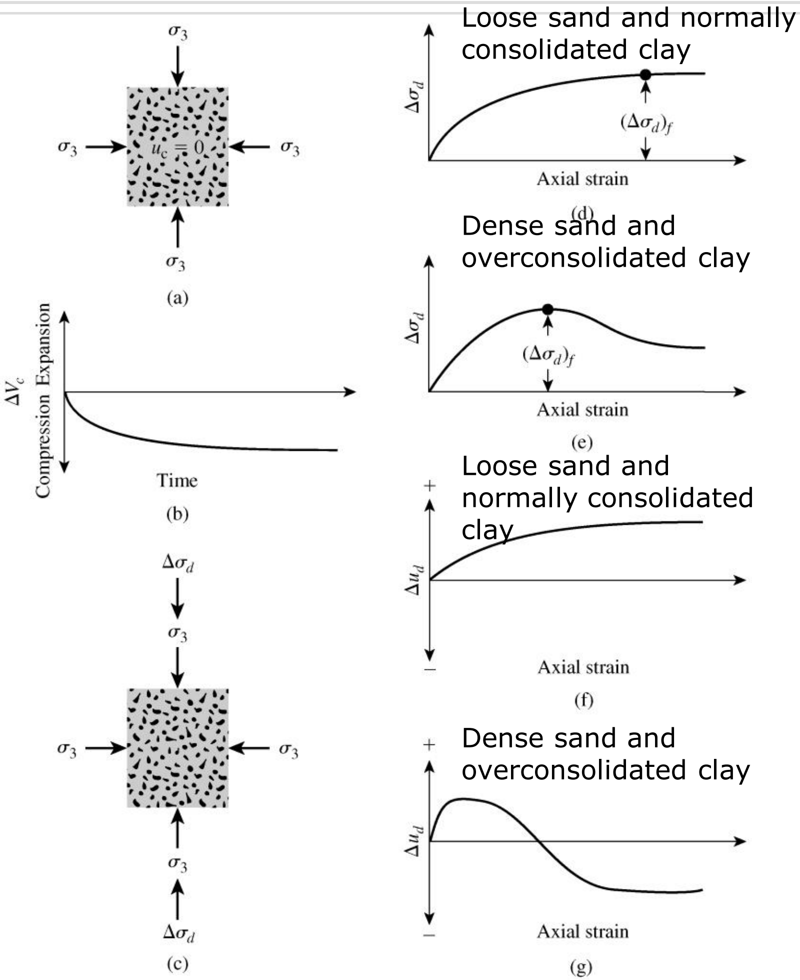


Figure 11.26 Consolidated undrained test: (a) specimen under chamber confining pressure; (b) volume change in specimen caused by confining pressure; (c) deviator stress application; (d) deviator stress against axial strain for loose sand and normally consolidated clay; (e) deviator stress against axial strain for dense sand and overconsolidated clay; (f) variation of pore water pressure with axial strain for loose sand and normally consolidated clay; (g) variation of pore water pressure with axial strain for dense sand and overconsolidated clay

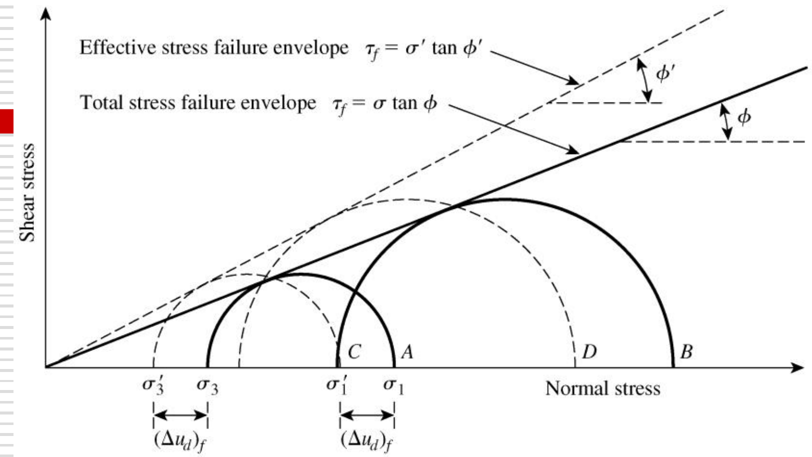


Figure 11.27 Total and effective stress failure envelopes for consolidated undrained triaxial tests. (*Note:* The figure assumes that no back pressure is applied.)

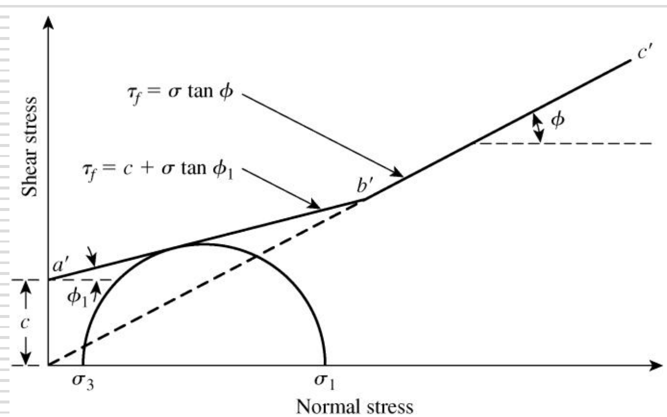


Figure 11.28 Total stress failure envelope obtained from consolidated-undrained tests in over-consolidated clay

Unconsolidated Undrained Test (UU)

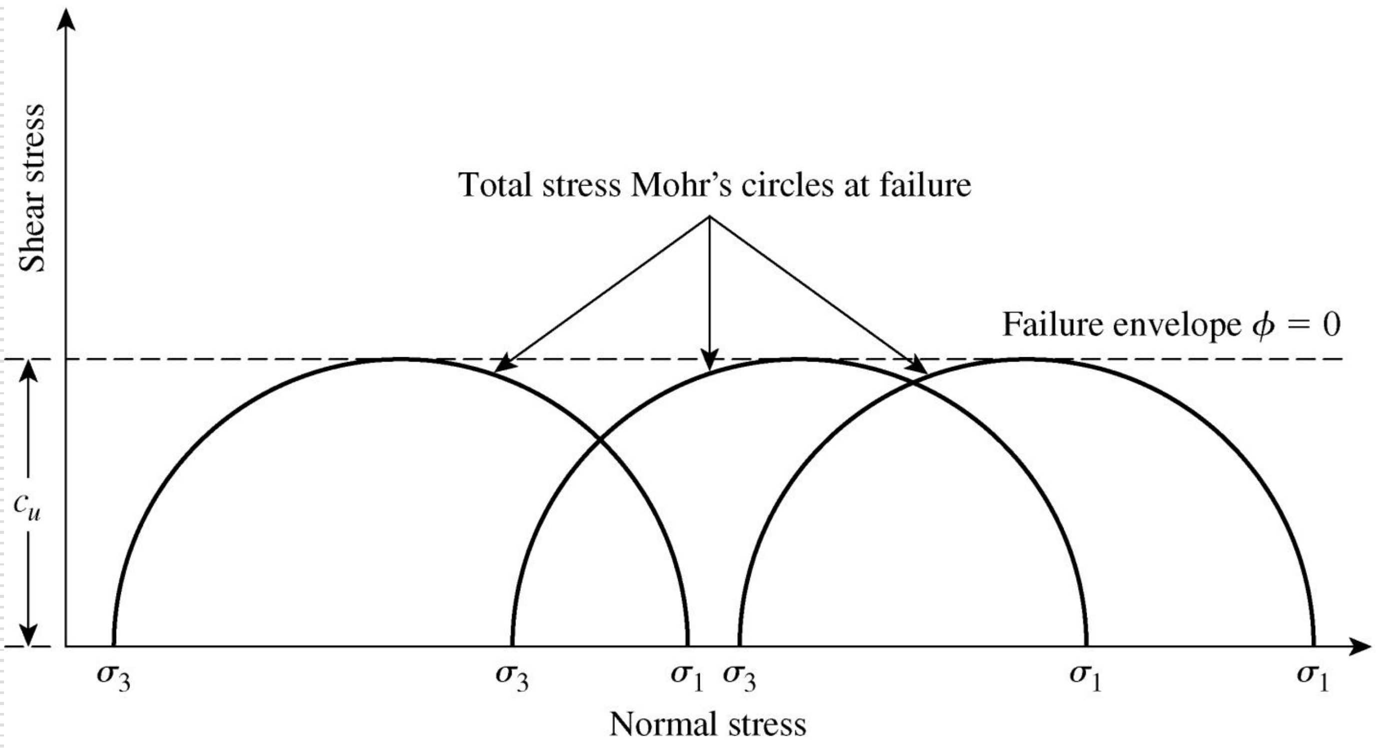


Figure 11.31 Total stress Mohr's circles and failure envelope ($\phi = 0$) obtained from unconsolidated-undrained triaxial tests on fully saturated cohesive soil

Unconsolidated Undrained Test (UU)

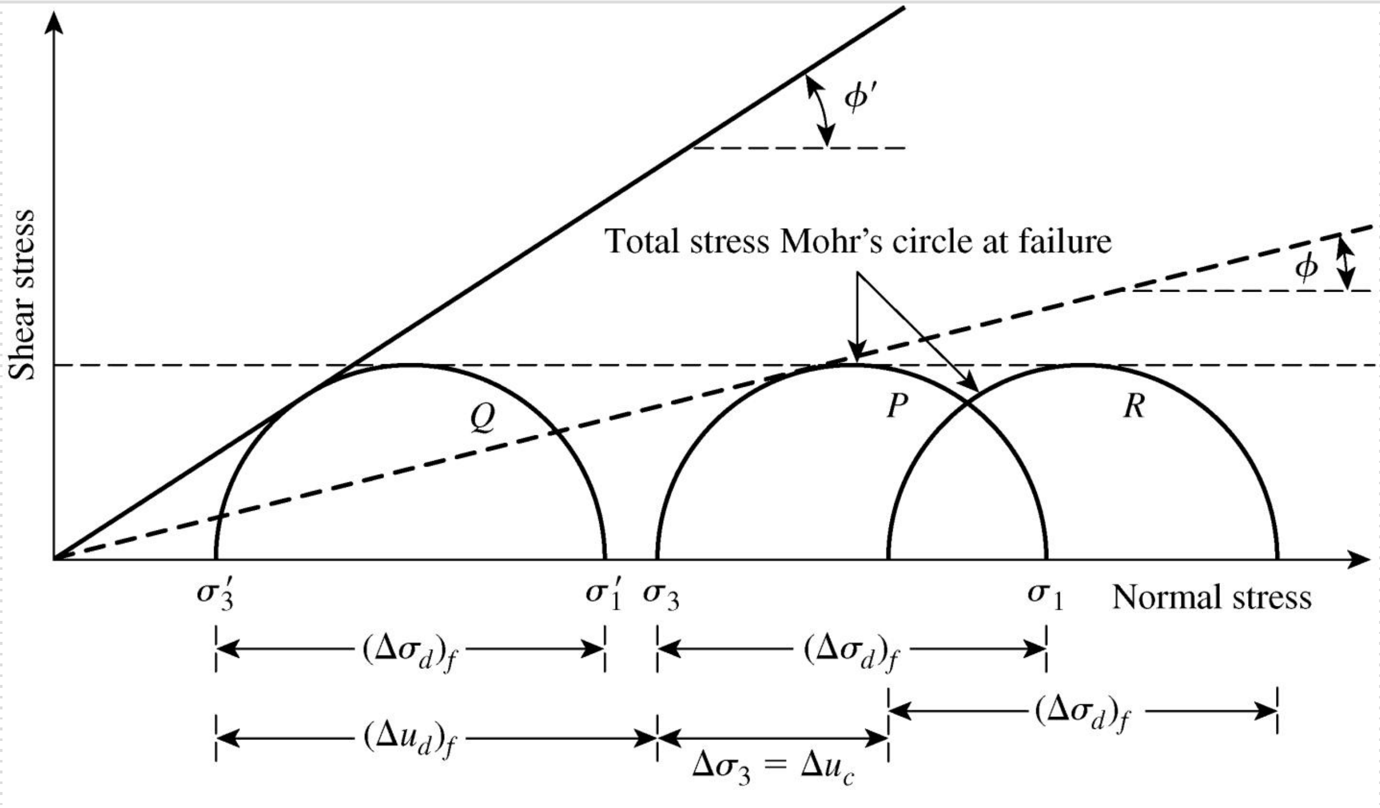


Figure 11.32 The $\phi = 0$ concept

Discussion on Triaxial Test

Laboratory test generally is divided into 2 categories.

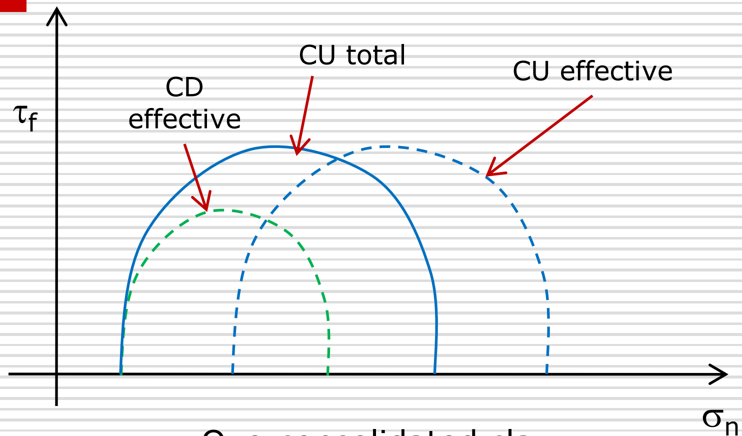
1. Shear strength tests based on total stress:
   - Also known as undrained shear strength test
   - To get undrained shear strength, S_u, c, ϕ - total stress
   - Test type – vane shear test, unconfined compression, unconsolidated undrained (UU), consolidated undrained (CU)
   - Performed exclusively on plastic (cohesive) soil
   - Normally for evaluation of foundation and embankment supported by cohesive soil
   - Analysis for rapid loading or unloading conditions
   - Conditions applicable to field situations where change in shear stress occurs quickly enough that soft cohesive soil does not have time to consolidate
   - Known as short term analysis
   - Will give a bigger value of strength carried out with the same cell pressure in triaxial test.

Discussion on Triaxial Test

2. Shear strength test based on effective stress.
   - To get effective strength – c′, ϕ′,etc.
   - Referred to as drained shear strength test.
   - Test include direct shear test, CD, CU, etc.
   - Performed on plastic (cohesive) soil and nonplastic (cohesionless) soil
   - Effective shear parameters used for long-term analysis where condition are relatively constant.
   - Eg. includes long term stability of slopes, embankments, earth supporting structure, foundation etc.
   - Effective shear stress analysis fundamentally models the shear strength of soil

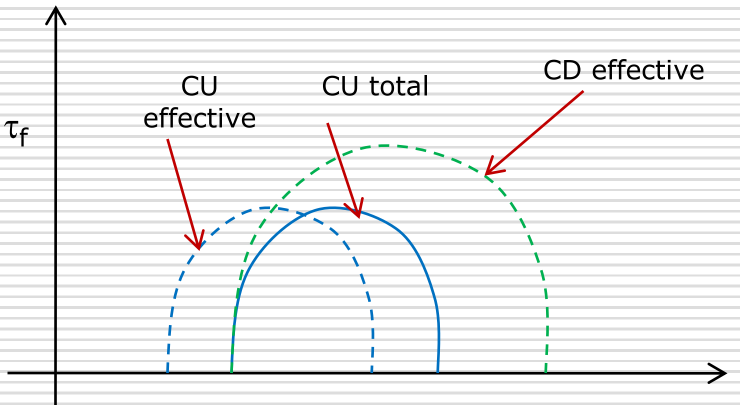
Discussion on Triaxial Test

Strength tests conducted on samples of a stiff overconsolidated clay gave lower strengths for CD tests than CU tests because since an overconsolidated specimen tends to expand during shear (in undrained condition), the pore water pressure decreases or even goes negative and thus the effective stress is increased.



Overconsolidated clay

Because $\sigma'_{3(f)} = \sigma_{3(f)}-(-\Delta u_f)$
or $\sigma'_{1(f)} = \sigma_{1(f)}-(-\Delta u_f)$
the effective stresses are greater than the total stress.
Thus the undrained strength is greater than the drained strength, which is opposite to the behavior of normally consolidated clay.



Normally consolidated clay

Therefore, in normally consolidated clay
Shear strength of CD>CU
In CD, soil becomes stiffer due to the reduction of volume because $(\sigma_1-\sigma_3)_{fcd}>(\sigma_1-\sigma_3)_{fcu}$

But in overconsolidated clay shear strength CU>CD
– In CD, soil dilate and pore pressure reduces

Use of CD Test in Engineering Practice

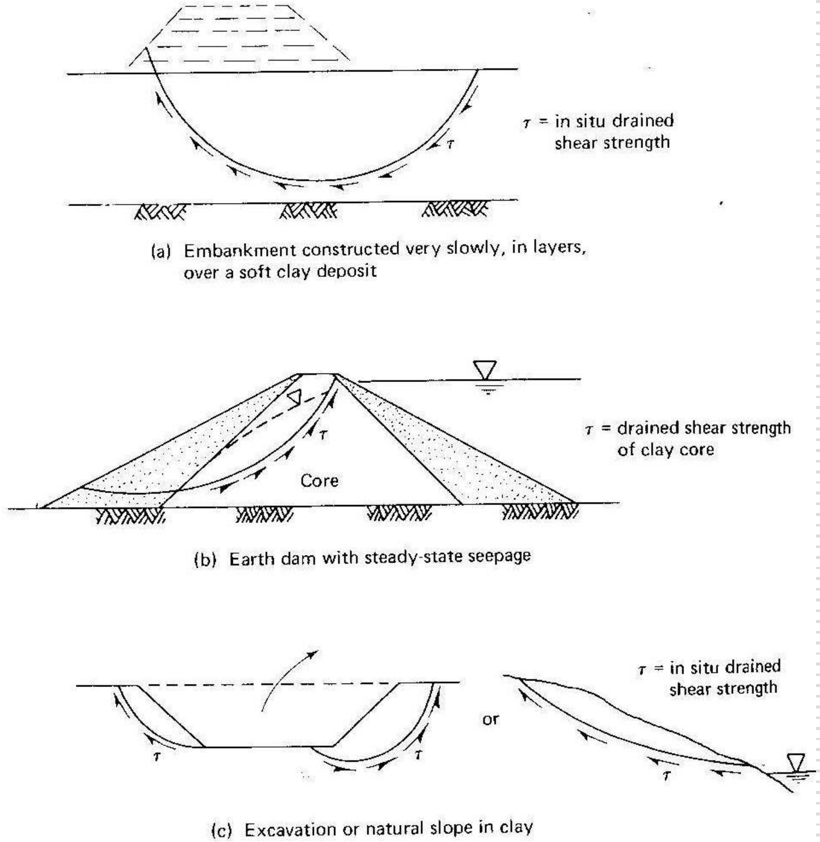


Fig. 11.28 Some examples of CD analyses for clays (after Ladd, 1971b).

CD conditions are the most critical for the long-term steady seepage case for embankment dams and the long-term stability of excavations or slopes in both soft and stiff clays.

Note: we should be aware that, practically speaking, it is not easy to actually conduct a CD test on a clay in the laboratory because of time factor. Therefore CD test is not very popular.

Use of CU Test in Engineering Practice

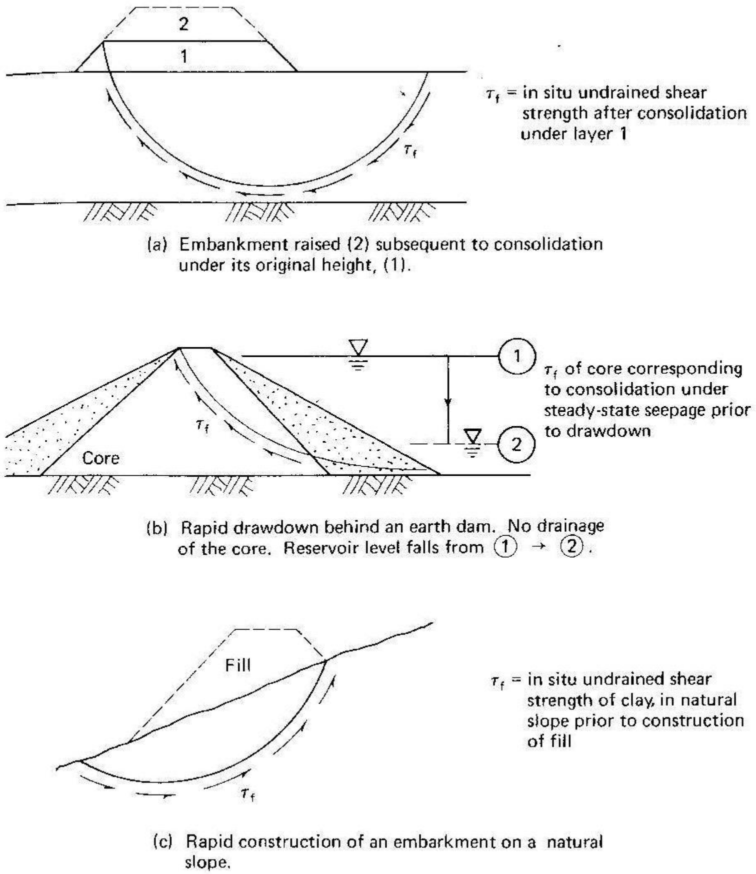


Fig. 11.37 Some examples of CU analyses for clays (after Ladd, 1971b).

CU strengths are used for stability problems where the soils have first become fully consolidated and are at equilibrium with the existing stress system. Then, for some reason, additional stresses are applied quickly, with no drainage occuring. Practical examples include rapid drawdown of embankment dams and the slopes of reservoirs and canals.

Since it is possible to measure the induced pore pressures in a CU test and thereby calculate the effective stresses in the specimen, CU tests are more practical for obtaining the effective stress strength parameters.

Use of UU Test in Engineering Practice

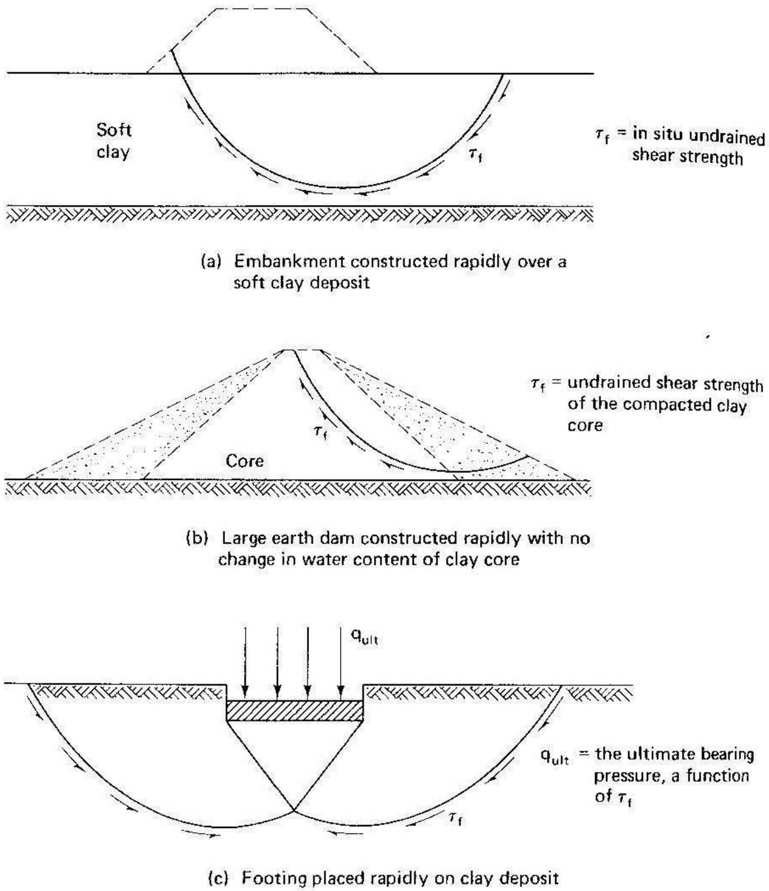


Fig. 11.57 Some examples of UU analyses for clay (after Ladd, 1971b).

Applicable in situations where the engineering loading is assumed to take place so rapidly that there is no time for the induced excess pore water pressure to dissipate or for consolidation to occur during the loading period. We also assume that the change in total stress during construction does not affect the in situ undrained shear strength. Examples include end of construction of embankment dams and foundations for embankment, piles, and footings on normally consolidated clays

For these cases, often the most critical design condition is immediately after the application of the load (at the end of construction) where the induced pore pressure is the greatest but before consolidation has had the time to take place. Once consolidation begins, the void ratio and the water content naturally decrease and the strength increases. So the embankment or foundation becomes increasing safer with time.

Unconfined Compression Test

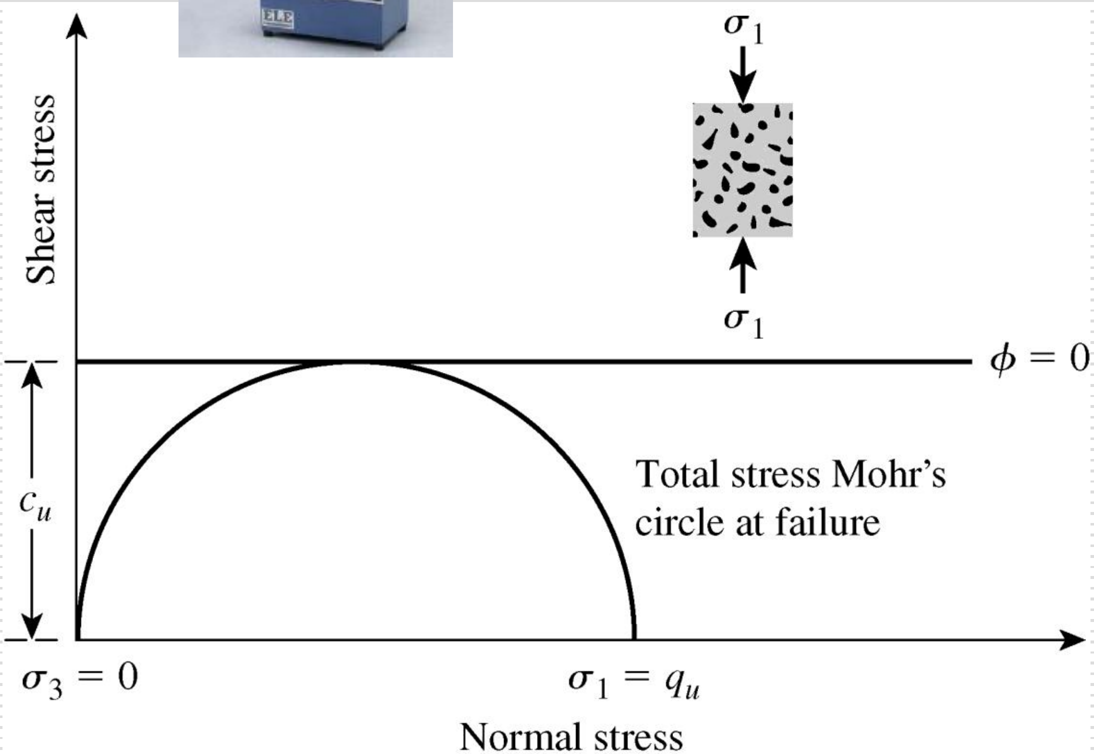


Figure 11.33 Unconfined compression test

The unconfined compression test is a special type of unconsolidated-undrained test that is commonly used for clay specimens.

In this test, the confining pressure σ_3 is 0.

At failure, the total minor principal stress is zero and the total major principal stress is $\sigma 1$ (Figure 11.33).

$$\tau_f = \frac{\sigma_1}{2} = \frac{q_1}{2} = c_u$$

Unconfined Compression Test

Table 11.4 General Relationship of Consistency and Unconfined Compression Strength of Clays

	q_u	
Consistency	**kN/m²**	**ton /ft²**
Very soft	0–25	0–0.25
Soft	25–50	0.25–0.5
Medium	50–100	0.5–1
Stiff	100–200	1–2
Very stiff	200–400	2–4
Hard	>400	>4

Unconfined Compression Test

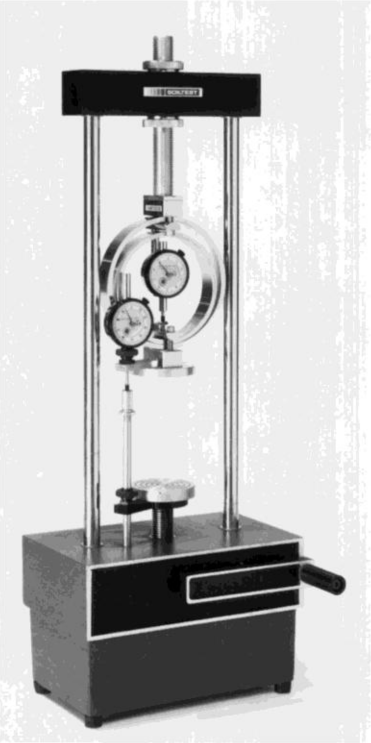


Figure 11.34 Unconfined compression test equipment (*Courtesy* of Soiltest, Inc., Lake Bluff, Illinois)

Table 11.5 Empirical Equations Related to c_u and σ'_O

Reference	Relationship	Remarks
Skempton (1957)	$\frac{c_{u(\mathrm{VST})}}{\sigma'_O} = 0.11 + 0.0037(PI)$ PI = plasticity index (%) $c_{u(\mathrm{VST})}$ = undrained shear strength from vane shear test	For normally consolidated clay
Chandler (1988)	$\frac{c_{u(\mathrm{VST})}}{\sigma'_c} = 0.11 + 0.0037(PI)$ σ'_c = preconsolidation pressure	Can be used in overconsolidated soil; accuracy ±25%; not valid for sensitive and fissured clays
Jamiolkowski, *et al.* (1985)	$\frac{c_u}{\sigma'_c} = 0.23 \pm 0.04$	For lightly overconsolidated clays
Mesri (1989)	$\frac{c_u}{\sigma'_O} = 0.22$	
Bjerrum and Simons (1960)	$\frac{c_u}{\sigma'_O} = 0.45\left(\frac{PI\%}{100}\right)^{0.5}$ for $PI > 0.5$	Normally consolidated clay
	$\frac{c_u}{\sigma'_O} = 0.18(LI)^{0.15}$ for LI = liquidity index > 0.5	Normally consolidated clay
Ladd, *et al.* (1977)	$\frac{\left(\frac{c_u}{\sigma'_O}\right)_{\text{overconsolidated}}}{\left(\frac{c_u}{\sigma'_O}\right)_{\text{normally consolidated}}} = (OCR)^{0.8}$ OCR = overconsolidation ratio	

Vane Shear Test

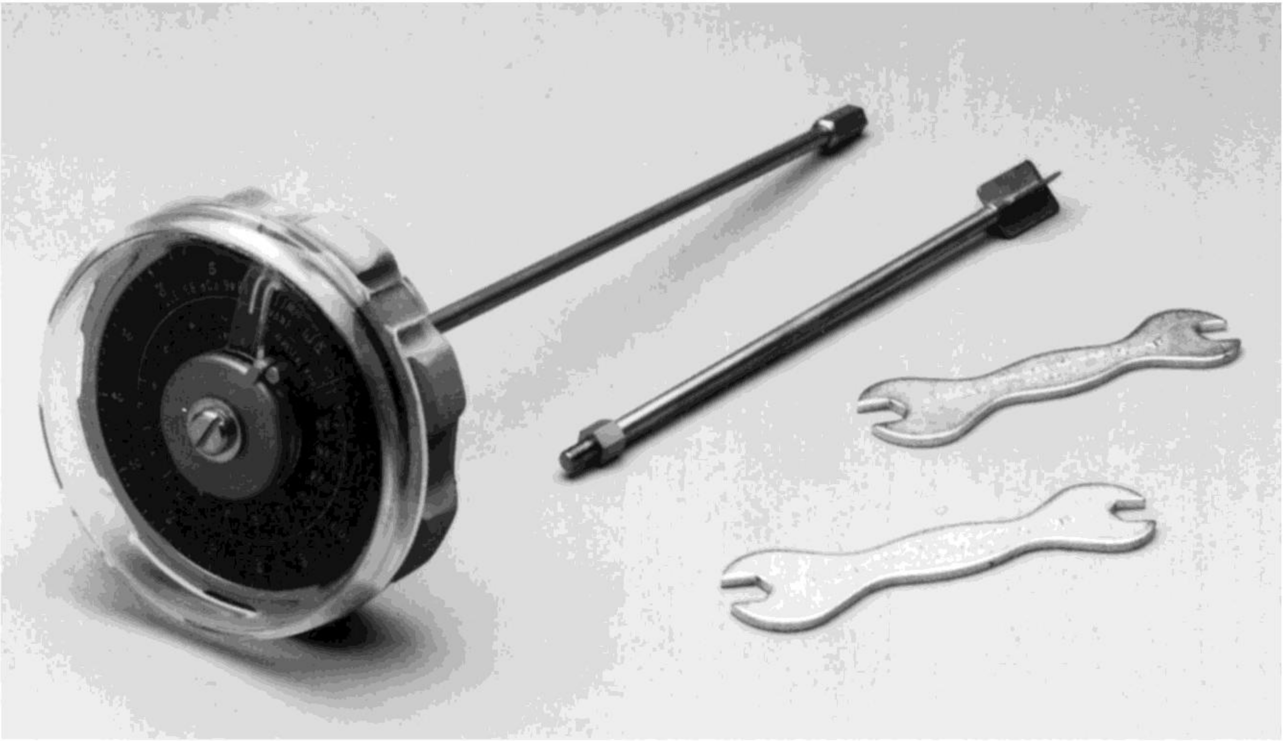


Figure 11.44 Laboratory vane shear test device (*Courtesy* of Soiltest, Inc., Lake Bluff, Illinois)

Vane Shear Test

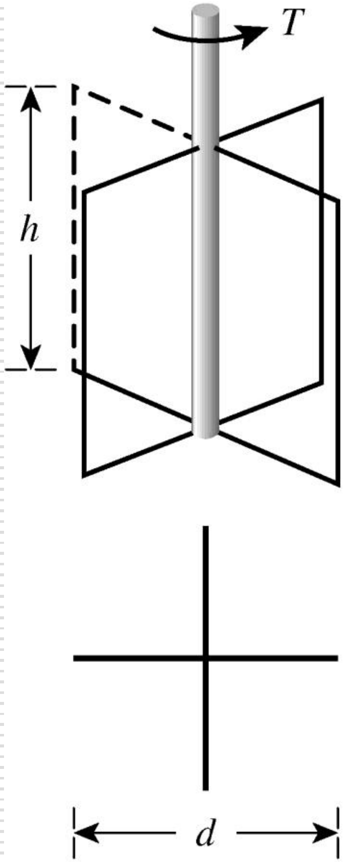


Figure 11.42
Diagram of vane shear test equipment

Fairly reliable results for the undrained shear strength, c_u (ϕ=0 concept), of very soft to medium cohesive soils may be obtained directly from vane shear tests.

If T is the maximum torque applied at the head of the torque rod to cause failure, it should be equal to the sum of the resisting moment of the shear force along the side surface of the soil cylinder (M_s) and the resisting moment of the shear force at each end (M_e) (Figure 11.43).

Vane Shear Test

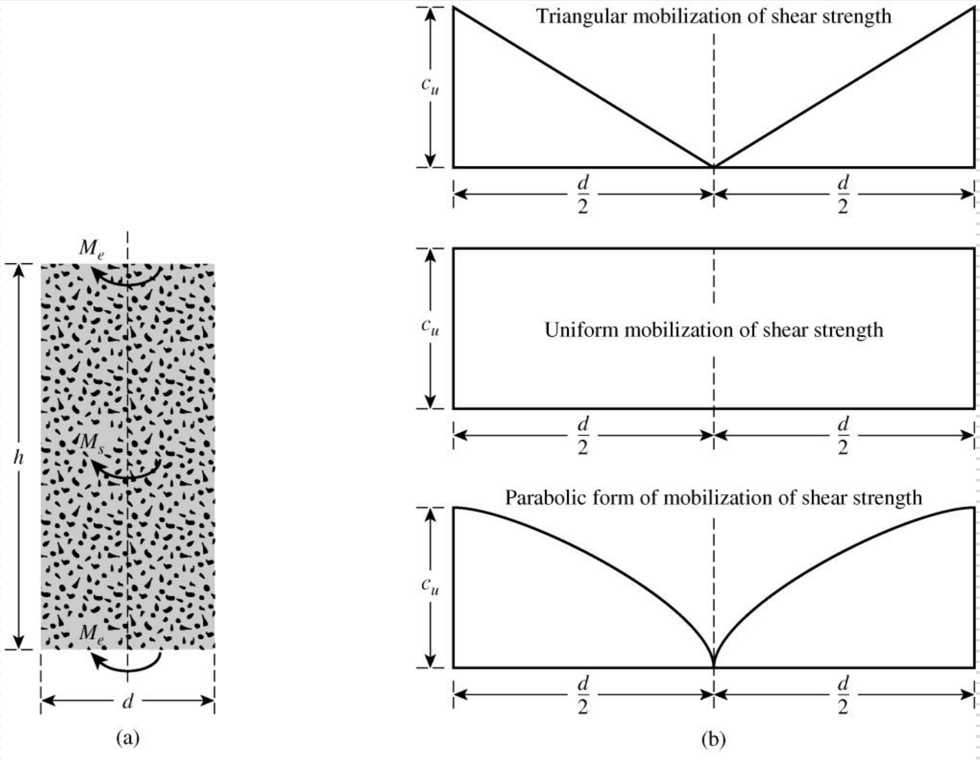


Figure 11.43 Derivation of Eq. (11.31): (a) resisting moment of shear force; (b) variations in shear strength-mobilization

$T = M_s + M_e + M_e$

Two ends

The resisting moment can be given as:

$M_s = (\pi dh)c_u \ (d/2)$

Surface area — Moment arm

$$T = \pi c_u \left[\frac{d^2 h}{2} + \beta \frac{d^3}{4} \right]$$

$$c_u = \frac{T}{\pi \left[\frac{d^2 h}{2} + \beta \frac{d^3}{4} \right]}$$

Example

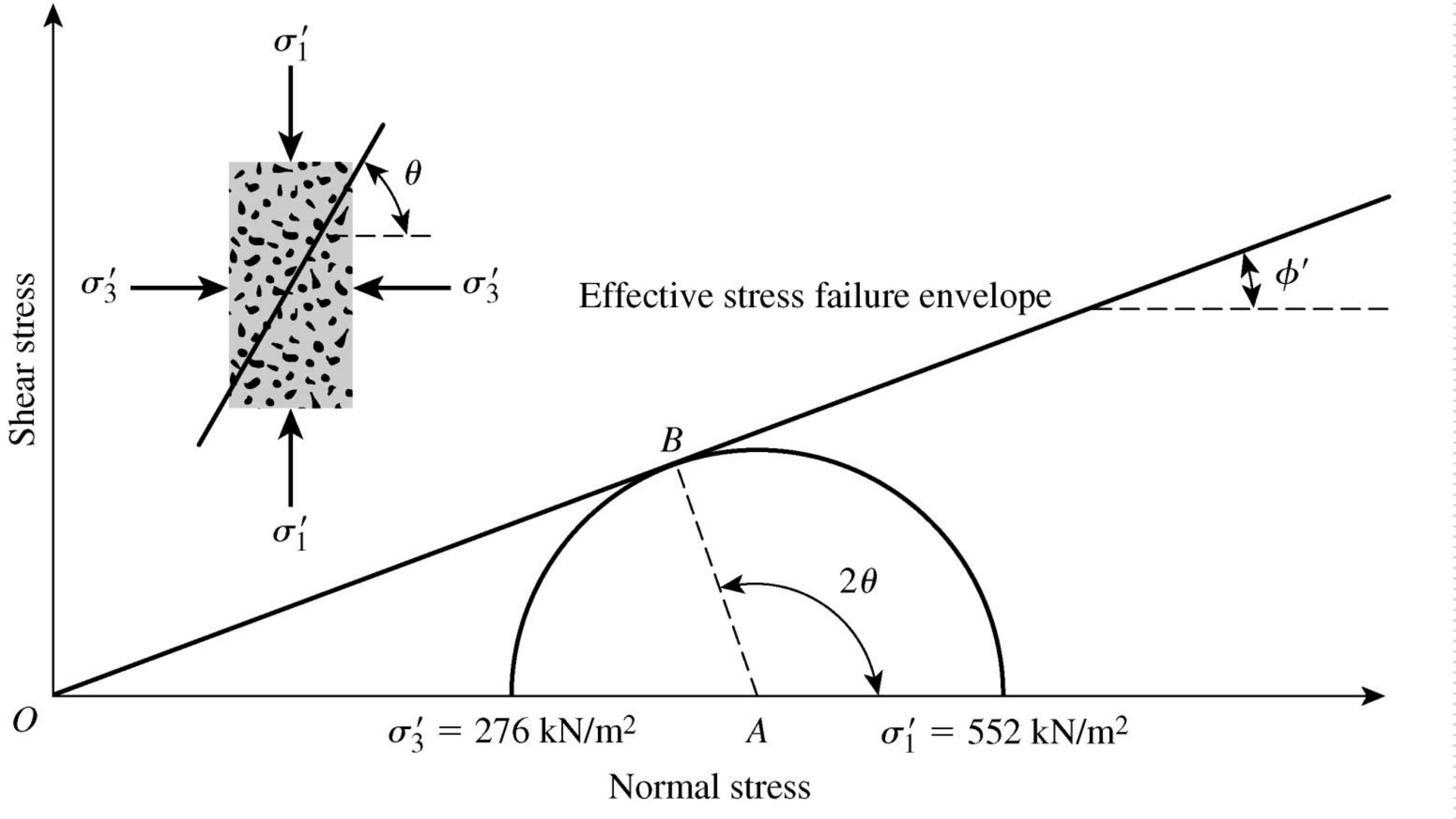


Figure 11.24 Mohr's circle and failure envelope for a normally consolidated clay