CHAPTER # 4

ANALYSIS AND DESIGN OF BEAMS

2/2

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FLOW CHART FOR DESIGN OF BEAMS

Write Known Data

↓

Estimate self-weight of the member.

a) The self-weight may be taken as ***10 percent of the applied dead*** UDL or dead point load distributed over all the length.

b) If only live load is applied, self-weight may be taken equal to ***5 percent of its magnitude***.

c) In case only factored loads are given, self-wt. may be taken equal to ***3 % of the given loads***.

↓

Calculate Factored Loads

Draw B.M. and S.F. Diagrams

Calculate C_b For Each Unbraced Segment

Find $M_{u,max}$, $V_{u,max}$, L_b for each segment and guess which segment is the most critical.

Design this segment first and then check for others.

Assume the section to be compact without **LTB** in the start and calculate Z_x accordingly.

Assumed

$$Z_{x,req} = \frac{M_u \times 10^6}{\phi_b F_y}$$

$\phi_b = 0.9$

$F_y = 250\ MPa$ for A36 steel

Selection of Section

i. $Z_{sel} \geq Z_{req}$

ii. *Minimum* weight

iii. $d \geq d_{min}$

$$d_{\min} = \frac{F_y L}{5500}$$

For Δ_{max} = $L/360$

d_{min} **= L /22 for A36 steel and simply supported beams**

For Δ_{max} required to be lesser than L /360, like L /500 or L /800, find $(I_x)_{req}$ from the deflection formula, with only the live load acting, and select section such that $I_x \geq (I_x)_{req}$.

Method 1: *Use Of Selection Tables*

Refernce-1, Page 155

These tables are applicable only if $L_b < L_r$ and $C_b = 1$

1. Enter the column headed Z_x and find a value equal to or just greater than the plastic section modulus required.
2. The beam corresponding to this value in the shape column and all beams above it have sufficient flexural strength based on these parameters.
3. The first beam appearing in ***boldface type*** (top of a group) adjacent to or above the required Z_x is the lightest suitable section.

4. If the beam must have to satisfy a certain depth or minimum moment of inertia criterion, proceed up the column headed “Shape” until a beam fulfilling the requirements is reached.

5. If $C_b > 1.0$, use L_m in place of L_p for the approximate selection.

6. If L_b is larger than L_m of the selected section, use the unbraced design charts.

7. Apply moment capacity, shear capacity, deflection and all other checks.

8. The column headed $\phi_b M_p$ may also be used in place of the Z_x column in the above method.

Z_x	For shapes used as beams, $\phi_b = 0.90$											
		$F_y = 345$ MPa									Torsional Constant	Warping Constant
Z_x mm^3x10^3	SI Shape	$\phi_b M_p$ kN-m	$\phi_b M_r$ kN-m	L_p m	L_r m	BF kN	d mm	$b_f/2t_f$	h/t_w	Z_y mm^3x10^3	J mm^4x10^4	C_w mm^6x10^6
480	**W 310 x 32.7**	**149.0**	**90.4**	**0.91**	**2.78**	**34.82**	**312**	**4.74**	**41.8**	**60.0**	**12.2**	**44000**
464	S 250 x 37.8	144.1	87.6	1.02	3.81	22.47	254	4.75	25.6	81.8	25.1	40800
446	W 200 x 41.7	138.5	86.5	1.74	6.40	12.40	205	7.03	22.3	166	22.4	83800
426	**W 250 x 32.7**	**132.3**	**82.6**	**1.43**	**4.20**	**19.93**	**259**	**7.99**	**36.9**	**100**	**10.0**	**73800**
405	**W 310 x 28.3**	**125.8**	**75.9**	**0.89**	**2.62**	**31.90**	**310**	**5.72**	**46.2**	**48.8**	**7.49**	**35200**
379	W 200 x 35.9	117.7	74.3	1.73	5.80	11.85	201	8.12	25.9	140	14.4	69600
354	**W 250 x 28.4**	**109.9**	**66.9**	**0.94**	**2.96**	**23.65**	**259**	**5.09**	**35.4**	**54.9**	**9.70**	**27900**
334	W 200 x 31.3	103.7	64.8	1.36	4.51	13.72	210	6.59	27.5	93.2	11.7	40800
329	**W 310 x 23.8**	**102.2**	**60.9**	**0.83**	**2.45**	**28.42**	**305**	**7.53**	**49.4**	**37.0**	**4.29**	**26000**
315	S 200 x 34	97.81	57.60	0.86	4.09	13.81	203	4.91	14.1	60.1	22.9	16400
310	W 150 x 37.1	96.26	59.55	1.64	7.22	7.30	162	6.68	15.5	140	19.2	40300
306	W 250 x 25.3	95.01	57.60	0.91	2.79	22.14	257	6.08	36.9	45.9	6.49	22900
285	**W 310 x 21**	**88.49**	**53.03**	**0.81**	**2.35**	**25.50**	**302**	**8.82**	**54.3**	**31.1**	**2.93**	**21600**
279	W 200 x 26.6	86.63	54.12	1.32	4.11	12.96	207	7.95	29.9	76.4	7.16	32800
270	**M 318 x 18.5**	**83.84**	**50.64**	**0.80**	**2.24**	**25.58**	**318**	**8.22**	**74.8**	**27.5**	**2.05**	**20400**
270	S 200 x 27.4	83.84	51.29	0.89	3.48	13.96	203	4.71	22.9	52.1	13.9	14200
262	W 250 x 22.3	81.35	49.12	0.87	2.63	20.40	254	7.41	38.5	37.7	4.33	18300
246	**M 318 x 17.3**	**76.38**	**45.64**	**0.72**	**2.04**	**25.84**	**318**	**8.29**	**74.8**	**22.5**	**1.72**	**15300**
246	W 150 x 29.8	76.38	47.82	1.61	6.02	7.20	157	8.25	18.7	110	10.0	30300
234	M 310 x 17.6	72.66	42.82	0.60	1.78	28.14	305	6.81	62.5	18.8	2.08	10100
223	W 200 x 22.5	69.24	41.95	0.94	3.06	14.33	206	6.37	28.1	43.8	5.70	13900

Method 2: *Use Of Unbraced Design Charts*

This method is applicable in cases where the above method is not fully applicable and $L_b \geq L_p$.

The design charts are basically developed for uniform moment case with $C_b = 1.0$.

Following notation is used to separate full plastic, inelastic LTB, and elastic LTB ranges:

Solid Circle —●— represents L_p

Hollow Circle —○— represents L_r

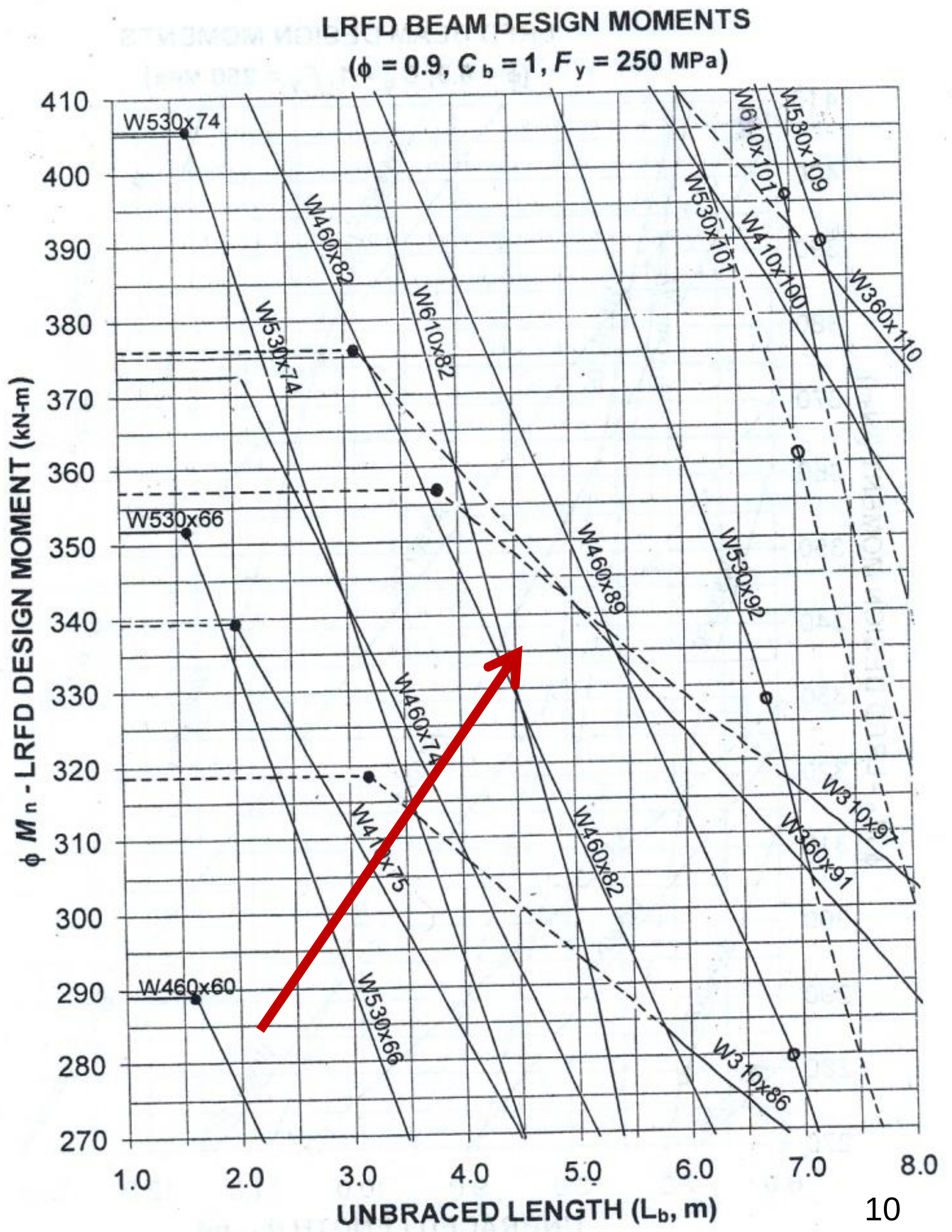
1. According to M_u in *kN-m* units and L_b in meters, enter into the charts.

1. Any section represented by a curve to the right and above (↗) the point selected in **No.1** will have a greater allowed unbraced length and a greater moment capacity than the required values of the two parameters.

3. A dashed line section is not an economical solution. If dashed section is encountered while moving in top-right direction, proceed further upwards and to the right till the first solid line section is obtained. Select the corresponding section as the trial section, and it will be the lightest available section for the requirements.

4. If $\boldsymbol{C_b > 1.0}$, use $M_{u,req} = M_u / C_b$ but check that the selected section has $\phi_b M_p > M_u$.

Move to the right and above in the arrow direction the point selected



Check the three conditions of compact section for internal stability, namely,

1. *web continuously connected with flange*,
2. *flange stability criterion, and*
3. *web stability* criterion.

If any one out of the above three is not satisfied, revise the section.

Either calculate $\boldsymbol{L_p}$, $\boldsymbol{L_r}$, and $\boldsymbol{L_m}$ or find their values from ***beam selection tables***.

$$L_p = 1.76\, r_y \sqrt{E/F_{yf}}\,, (mm)$$

$$L_p = 0.05\, r_y\, (m) \text{ for A36 steel}$$

$$L_r = 1.95\, r_{ts} \frac{E}{0.7F_y} \sqrt{\frac{J\,c}{S_x h_o}} \sqrt{1 + \sqrt{1 + 6.76 \left(\frac{0.7F_y}{E} \frac{S_x h_o}{J\,c} \right)^2}}$$

$$M_r = 0.7F_y\, S_x / 10^6 \qquad \textbf{(kN-m)}$$

$$r_{ts}^{\;2} = \frac{\sqrt{I_y C_w}}{S_x} = \frac{I_y h_0}{2S_x}$$

For doubly symmetric I-sections

$\boldsymbol{c}$ = 1.0 for a doubly symmetric I-shape

$\boldsymbol{h_o}$ = $d - t_f$

$$BF = \frac{M_p - M_r}{L_r - L_p}$$

$$L_m = L_p + \frac{M_p}{BF} \left(\frac{C_b - 1}{C_b} \right) \le L_r$$

Calculate design flexural strength:

1. If $\boldsymbol{L_b \leq L_m}$ $\quad M_n = M_p = Z_x F_y / 10^6 \ (kN-m)$
2. If $\boldsymbol{L_m < L_b \leq L_r}$ $\quad M_n = C_b\,[M_p - BF(L_b - L_p)] \leq M_p \ (kN-m)$
3. If $\boldsymbol{L_b > L_r}$ $\quad M_n = C_b F_{cr} S_x \leq M_p$

where

$$F_{cr} = \frac{C_b \pi^2 E}{\left(\frac{L_b}{r_{ts}}\right)^2}\sqrt{1 + 0.078\frac{J\,c}{S_x h_o}\left(\frac{L_b}{r_{ts}}\right)^2} \approx \frac{C_b \pi^2 E}{\left(L_b / r_{ts}\right)^2}$$

Design moment = $\phi_b M_n$, ***Where ϕ_b=0.9***

Bending strength check:

$$M_u \le \phi_b M_n \qquad \textbf{(OK)}$$

If not satisfied revise the trial selection.

Calculate design Shear Strength

For $\frac{h}{t_w} \le 2.24\sqrt{E / F_{yw}}$ **(= 63.4 for A36 steel)** C_v **= 1.0**

$$\phi_v V_n = \frac{0.9 \times 0.6}{1000} F_{yw} A_w C_v, (kN)$$

Shear check:

$$V_u \le \phi_v V_n$$

If not satisfied revise the trial selection

Deflection check:

Find Δ_{act} due to service live loads.

Note: If Live Loads are not directly known, service load may be taken equal to Factored load divided by 2.5

$\boldsymbol{\Delta_{act} \leq L/360}$ or other specified limit (***OK***)

Check self-weight:

Calculated self weight $\leq$ 1.2 $\times$ assumed self weight (***OK***)

Otherwise, revise the loads and repeat the calculations.

Write final selection using standard designation.

Example 4.1:

BEAMS WITH CONTINUOUS LATERAL SUPPORT

Design a **7m** long simply supported I-section beam subjected to ***service live load of* 5 *kN/m*** and ***imposed dead load of* 6 *kN/m***, as shown in Figure 4.19. The compression flange is continuously supported.

Use **(a)** A36 steel and **(b)** steel with f_y = 345 MPa and permissible live load deflection of span / 450.

Solution:

In beams with continuous lateral support, unbraced length is not applicable or it may be assumed equal to zero in calculations.

Assumed self weight = 10% of superimposed **DL**

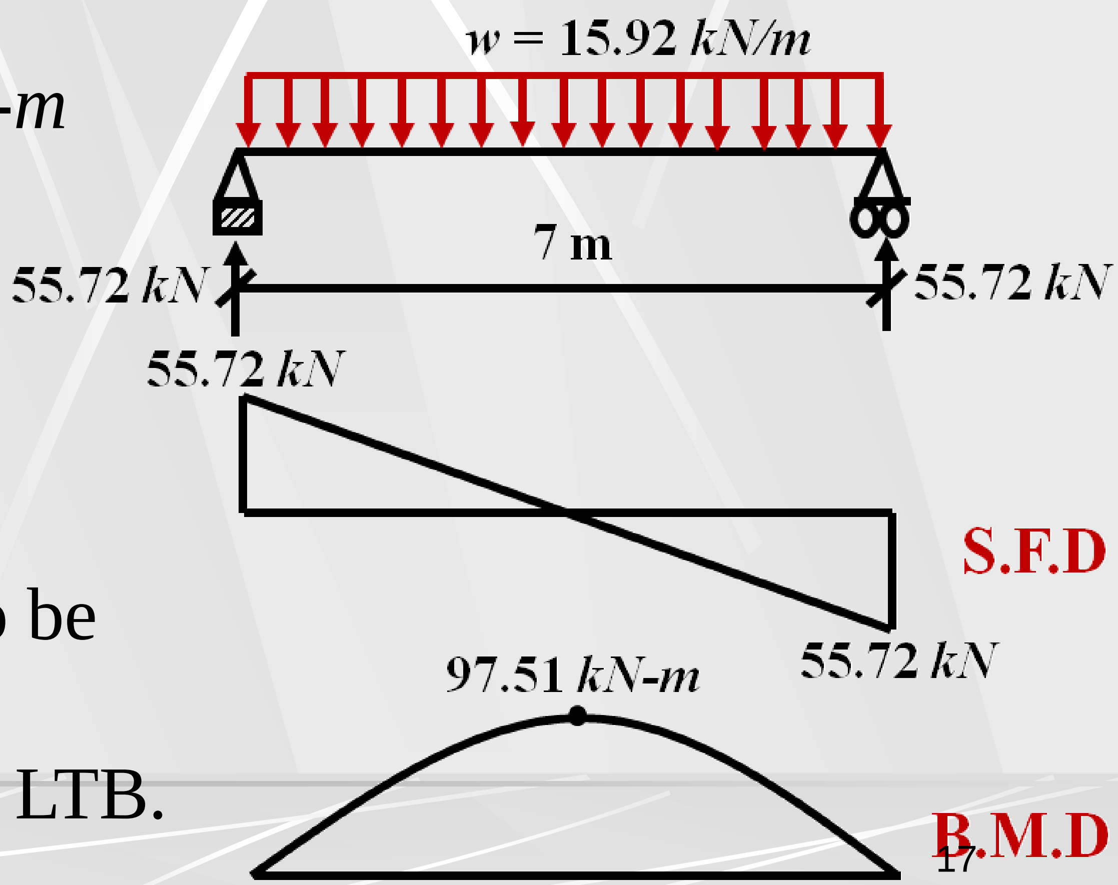
= 0.1x6 = 0.6 *kN/m*

$$w_u = 1.2D + 1.6L = 1.2 \times 6.6 + 1.6 \times 5$$
$$= 15.92 \text{ kN/m}$$

If the beam is continuously braced, C_b value is not applicable but may be considered equal to **1.0** in case it is required in the formulas.

$M_u = M_{max} = 97.51$ *kN-m*
$V_u = V_{max} = 55.72$ *kN*

Compression flange continuously braced.



(a) A36 Steel

Assuming the section to be internally compact and knowing that there is no LTB.

$$(Z_x)_{req} = \frac{M_u \times 10^6}{\phi_b F_y} = \frac{97.51 \times 10^6}{0.9 \times 250} = 433.4 \times 10^3, mm^3$$

$$d_{\min} = \frac{L}{22} = \frac{7000}{22} = 318mm$$

Selection of Section

i. $Z_{sel} \geq Z_{req} = 433.4\ mm^3$

ii. *Minimum* weight

iii. $d \geq d_{min} = 318\ mm$

- Consulting beam selection tables **(Reference-1, Page 164)**, following result is obtained.

Trial Section	Z_x (mm³)	Depth	Remark
W310 × 32.7	**480 × 10^3**	**313 < $d_{\min}$ = 318**	**Not Good**
W360 × 32.9	**544 × 10^3**	**349 > $d_{\min}$ =318**	**Check for Capacity**

Reference-1, Page 164

Z_x

For shapes used as beams

$\phi_b = 0.90$

Z_x mm^3x10^3	SI Shape	F_y = 250 MPa $\phi_b M_p$ kN-m	$\phi_b M_r$ kN-m	L_p m	L_r m	BF kN	d mm	$b_f/2t_f$	h/t_w	Z_y mm^3x10^3	Torsional Constant J mm^4x10^4	Warping Constant C_w mm^6x10^6	$\sqrt{\frac{EC_w}{GJ}}$ mm
895	W 360 x 51	201	125	1.93	5.87	21.5	355	7.4	43.1	174	23.7	287,333	1770
885	W 410 x 46.1	199	122	1.48	4.41	29.3	403	6.3	51.6	115	19.1	198,448	1638
839	W 310 x 52	189	118	1.95	6.43	17.6	317	6.3	36.2	188	30.8	236,043	1410
803	W 200 x 71	181	112	2.63	14.53	6.4	216	5.9	15.8	375	81.6	250,007	892
775	W 360 x 44	174	108	1.88	5.53	20.1	352	8.7	45.4	147	15.8	238,191	1974
767	W 250 x 58	173	109	2.50	9.69	9.9	252	7.5	25.0	282	40.8	266,388	1300
724	W 410 x 38.8	163	99	1.42	4.12	26.3	399	8.0	56.8	89.8	10.8	151,723	1905
706	W 310 x 44.5	159	100	1.92	5.91	16.5	313	7.4	41.8	157	19.1	193,346	1618
659	W 360 x 39.0	148	91	1.37	4.15	22.8	353	6.0	48.1	90.8	15.0	108,757	1372
652	W 200 x 59	147	91.7	2.58	12.22	6.3	210	7.2	17.6	303	46.6	194,957	1041
636	W 250 x 49.1	143	90	2.45	8.52	9.6	247	9.1	27.1	229	24.1	212,143	1509
610	W 310 x 38.7	137	86	1.91	5.57	15.5	310	8.5	47.2	134	12.5	163,001	1839
600	W 250 x 44.8	135	84	1.73	6.35	12.4	266	5.7	29.5	145	25.8	111,174	1057
569	W 200 x 52	128	80.5	2.57	10.96	6.3	206	8.1	20.4	264	32.0	166,224	1158
544	W 360 x 32.9	122	75	1.31	3.86	20.8	349	7.5	53.3	71.9	8.7	84,320	1580
513	W 250 x 38.5	115	72	1.72	5.77	11.9	262	6.6	34.0	123	16.6	92,645	1201
498	W 200 x 46.1	112	71.0	2.55	9.93	6.2	203	9.2	22.2	231	22.5	142,324	1280
480	W 310 x 32.7	108	66	1.07	3.46	19.8	313	4.7	41.8	60	12.1	44,040	973
446	W 200 x 41.7	100	62.7	2.05	8.48	6.5	205	7.0	22.2	166	22.5	83,783	983
426	W 250 x 32.7	96	60	1.68	5.25	11.2	258	8.0	36.9	100	10.0	73,847	1384
405	W 310 x 28.3	91	55	1.04	3.22	18.4	309	5.7	46.2	48.8	7.5	35,178	1102
380	W 200 x 35.9	85.5	53.9	2.04	7.57	6.4	201	8.1	25.8	140	14.6	69,551	1113

Trial Section 1: **W360 × 32.9 ;** Z_x **= 544 × 10^3 mm^3**

Check internal compactness of section as under:

1. Web is continuously connected **(*OK*)**

2. $\frac{b_f}{2t_f} = 7.5 < \lambda_p$ λ_p = **10.8** **(*OK*)**

3. $\frac{h}{t_w} = 53.3 < \lambda_p$ λ_p = **107** **(*OK*)**

∴ **The section is internally compact.**

4. Continuously braced conditions imply: $L_b < L_p$ and C_b = not included in formulas

Check for Moment Carrying Capacity:

$$\phi_b M_n = \phi_b M_p \quad \textit{for} \quad L_b < L_p$$

$$\phi_b M_n = \phi_b M_p = 0.9 \times 544 \times 10^3 \times 250 / 10^6$$

$$\cong 121\ kN\text{-}m$$

$$\boldsymbol{M_u} = 97.51\ kN\text{-}m < \boldsymbol{\phi_b M_n} = 121\ kN\text{-}m \qquad \textbf{(OK)}$$

Check for Shear Capacity:

$\boldsymbol{h/t_w} = 53.3 < 63.4 \Rightarrow$ shear yield formula is applicable

$$\phi_v V_n = \frac{0.9 \times 0.6}{1000} F_{yw} A_w C_w = \frac{0.9 \times 0.6}{1000} \times 250.(349 \times 5.8).1.0$$

$$\phi_v V_n = 273.27\, kN > \boldsymbol{V_u = 55.72\ kN} \qquad \textbf{(OK)}$$

Check for Deflections:

Check only for service live load

$$\Delta_{act} = \frac{5}{384} \times \frac{w_L\, L^4}{EI} = \frac{5}{384} \times \frac{5 \times (7000)^4}{2 \times 10^5 \times 8{,}300 \times 10^4} = 9.41 mm$$

$$\Delta_{all} = \frac{L}{360} = \frac{7000}{360} = 19.44$$

$$\Delta_{act} = 9.4 \quad < \quad \Delta_{all} = 19.44 \qquad (OK)$$

Check for Self-weight:

$$Selfweight = \frac{32.9 \times 9.81}{1000} = 0.323, kN/m$$

lesser than 1.2x(assumed self-weight of 0.6 *kN/m)*   *(OK)*

Final Section : W360x32.9

(b) Steel With F_y = 345 MPa

Assuming the section to be internally compact and knowing that there is no LTB,

$$(Z_x)_{req} = \frac{M_u \times 10^6}{\phi_b F_y} = \frac{97.51 \times 10^6}{0.9 \times 345} = 314.0 \times 10^3, mm^3$$

$$d_{\min} = \frac{F_y L}{5500} = \frac{L}{16} = \frac{7000}{16} = 437.5 mm$$

Selection of Section

i. $Z_{sel} \geq Z_{req} = 314.0\ mm^2$

ii. *Minimum* weight

iii. $d \geq d_{min} = 437.5\ mm$

This $d_{\min}$ is much larger, so the direct deflection criteria will be used.

Equating the, $\Delta_{act} = \Delta_{all}$

$$\frac{L}{450} = \frac{5}{384} \times \frac{w_L L^4}{EI_{min}}$$

$$I_{min} = \frac{5 \times 450}{384} \times \frac{w_L L^3}{E} = \frac{5 \times 450}{384} \times \frac{5 \times (7000)^3}{2 \times 10^5}$$

$$\mathbf{I_{min} = 5024 \times 10^4\ mm^4}$$

Selection of Section

i. $Z_{sel} \geq Z_{req} = 314.0\ mm^3$

ii. *Minimum* weight

iii. $I \geq I_{min} = 5024 \text{x} 10^4\ mm^4$

- Consulting beam selection tables for ***F_y =345 MPa*** **(Reference-1, Page 179)**, following result is obtained.

W310 × 28.3 provides sufficient strength and moment of inertia.

$\mathbf{I_x = 5410 \times 10^4\ mm^4}$; $\mathbf{Z_x = 405 \times 10^3\ mm^3}$

Reference-1, Page 179

Z_x	For shapes used as beams, ϕ_b = 0.90											
		F_y = 345 MPa									Torsional Constant	Warping Constant
Z_x mm^3x10^3	SI Shape	$\phi_b M_p$ kN-m	$\phi_b M_r$ kN-m	L_p m	L_r m	BF kN	d mm	$b_f/2t_f$	h/t_w	Z_y mm^3x10^3	J mm^4x10^4	C_w mm^6x10^6
480	W 310 x 32.7	149.0	90.4	0.91	2.78	34.82	312	4.74	41.8	60.0	12.2	44000
464	S 250 x 37.8	144.1	87.6	1.02	3.81	22.47	254	4.75	25.6	81.8	25.1	40800
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354	W 250 x 28.4	109.9	66.9	0.94	2.96	23.65	259	5.09	35.4	54.9	9.70	27900
334	W 200 x 31.3	103.7	64.8	1.36	4.51	13.72	210	6.59	27.5	93.2	11.7	40800
329	W 310 x 23.8	102.2	60.9	0.83	2.45	28.42	305	7.53	49.4	37.0	4.29	26000
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306	W 250 x 25.3	95.01	57.60	0.91	2.79	22.14	257	6.08	36.9	45.9	6.49	22900
285	W 310 x 21	88.49	53.03	0.81	2.35	25.50	302	8.82	54.3	31.1	2.93	21600
279	W 200 x 26.6	86.63	54.12	1.32	4.11	12.96	207	7.95	29.9	76.4	7.16	32800
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270	S 200 x 27.4	83.84	51.29	0.89	3.48	13.96	203	4.71	22.9	52.1	13.9	14200
262	W 250 x 22.3	81.35	49.12	0.87	2.63	20.40	254	7.41	38.5	37.7	4.33	18300
246	M 318 x 17.3	76.38	45.64	0.72	2.04	25.84	318	8.29	74.8	22.5	1.72	15300
246	W 150 x 29.8	76.38	47.82	1.61	6.02	7.20	157	8.25	18.7	110	10.0	30300
234	M 310 x 17.6	72.66	42.82	0.60	1.78	28.14	305	6.81	62.5	18.8	2.08	10100
223	W 200 x 22.5	69.24	41.95	0.94	3.06	14.33	206	6.37	28.1	43.8	5.70	13900

Check internal compactness of section as under:

1. Web is continuously connected **(*OK*)**

2. $\frac{b_f}{2t_f} = 5.7 < \lambda_p$ $\lambda_p = 9.1$ **(*OK*)**

3. $\frac{h}{t_w} = 46.2 < \lambda_p$ $\lambda_p = 90.5$ **(*OK*)**

$\therefore$ **The section is internally compact.**

4. Continuously braced conditions imply: $L_b < L_p$ and C_b = not included in formulas

Check for Moment Carrying Capacity:

$$\phi_b M_n = \phi_b M_p = 0.9 \times 405 \times 10^3 \times 345 / 10^6$$

$$\cong 125.8 \; kN\text{-}m$$

$$M_u = 97.51 \; kN\text{-}m < \phi_b M_n = 125.8 \; kN\text{-}m$$ **(*OK*)**

Check for Shear Capacity:

Reference-1, Page 318

$$\lambda_p = 2.24\sqrt{E / F_y} = 2.24\sqrt{2\times 10^5 / 345} = 53.93$$

$h/t_w = 46.2 < \lambda_p = 53.93 \Rightarrow$ shear yield formula is applicable

$$\phi_v V_n = \frac{0.9\times 0.6}{1000} F_{yw} A_w C_w = \frac{0.9\times 0.6}{1000}\times 345.(309\times 6.0).1.0$$

$$\phi_v V_n = 345.4 kN \quad > \quad V_u = 55.72\ kN \qquad (OK)$$

Check for Deflections:

Already Satisfied

$$\Delta_{act} = 9.4 \quad < \quad \Delta_{all} = 19.44 \qquad (OK)$$

Check for Self-weight:

$$Selfweight = \frac{28.3\times 9.81}{1000} = 0.28, kN / m$$

lesser than 1.2x(assumed self-weight of 0.6 *kN/m*)   (*OK*)

Final Section : W360x32.9

ALTERNATIVES TO THE SECTION REVISION

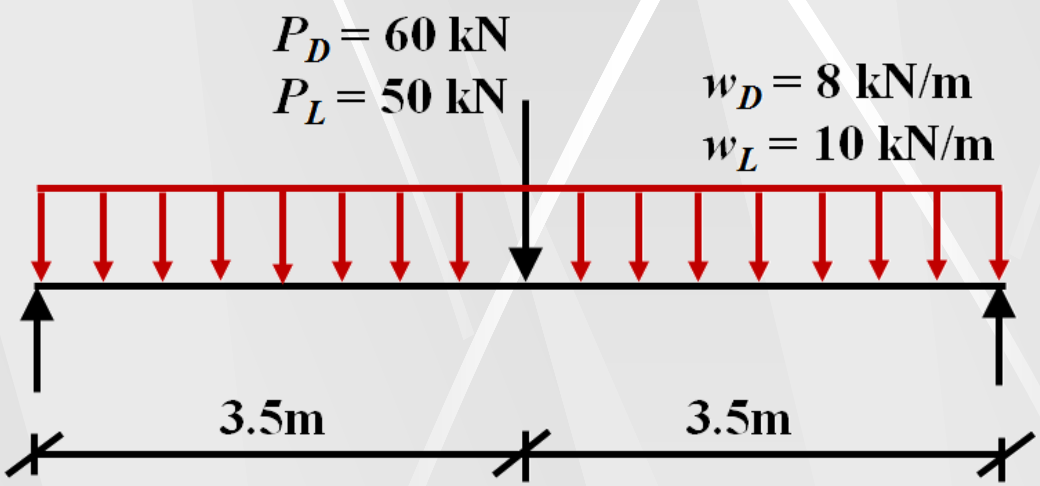
Sometime, in place of revising the section in case of less strength, following alternatives may be adopted.

1. *Flange cover plates can be provided to increase the moments capacity and reduce the deflections*
2. *Pre-stress tendons may be used to induce opposite resisting moment*
3. *The shear capacity may be increased by using the web-doublers or Web stiffener.*

Example 4.2:

BEAMS WITH COMPRESSION FLANGE LATERALY SUPPORTED (BRACED) AT REACTION POINTS ONLY

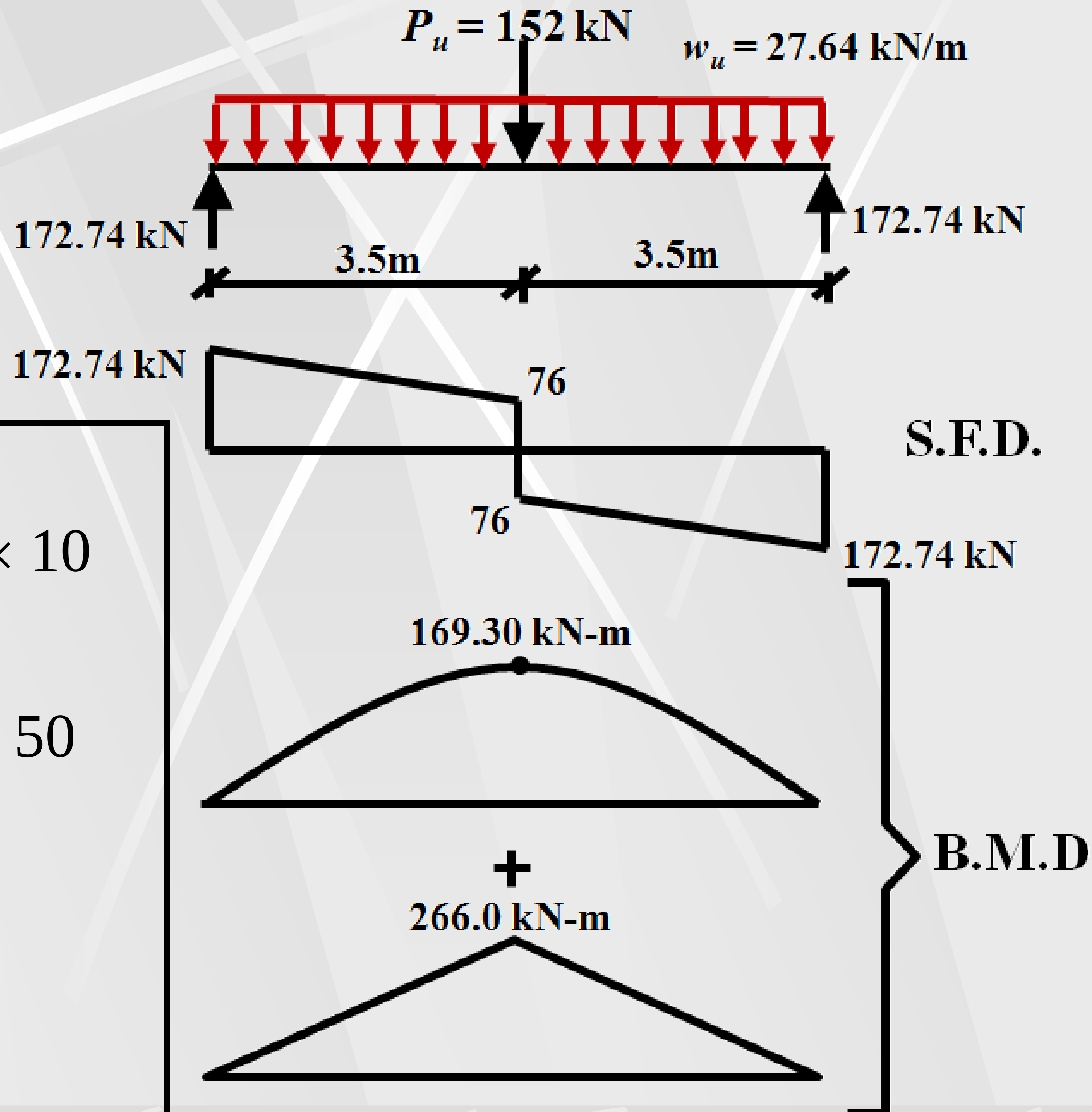
Design the beam of Figure 4.20 with the lateral bracing only provided at the reaction points. Use A36 steel.



Solution:

Assumed self weight:

$$(w_d)_{ass} = 0.1 \times \left(w_d + \frac{P_d}{L} \right) = 1 \times \left(8 + \frac{60}{7} \right) = 1.7, kN/m$$



Factored Loads:

$W_u = 1.2 \times 9.7 + 1.6 \times 10$

$= 27.64$ kN/m

$P_u = 1.2 \times 60 + 1.6 \times 50$

$= 152$ kN

$\mathbf{M_{max} = 435.3}$ **kN-m**

$\mathbf{V_{max} = 172.74}$ **kN**

$L_b = 7.0\ m$

Calculation of C_b Value:

$M_B = M_{max} = 435.3$ kN-m

$M_A = M_C = 172.4 \times 1.75 - 27.64 \times 1.75^2/2$

$= 259.97$ kN-m

$$C_b = \frac{12.5M\max}{2.5M_{\max} + 3M_A + 4M_B + 3M_C}$$

$$C_b = \frac{12.5 \times 435.30}{6.5 \times 435.30 + 6 \times 259.97} = 1.24$$

Assuming the section to be internally compact with no LTB

$$(Z_x)_{req} = \frac{M_u \times 10^6}{\phi_b F_y} = \frac{435.3 \times 10^6}{0.9 \times 250} = 1935 \times 10^3, mm^3$$

$$d_{\min} = \frac{L}{22} = \frac{7000}{22} = 318mm$$

Selection of Section

i. $Z_{sel} \geq Z_{req} = 1935\ mm^3$

ii. *Minimum weight*

iii. $d \geq d_{min} = 318\ mm$

- Consulting beam selection tables **(Reference-1, Page 162)**, following result is obtained.

Trial Section: **W610 × 82**

ϕM_p = 494.0 kN-m, ϕM_r = 295.0 kN-m,

BF = 64.99, L_p = 1.69 m , L_r = 5.10 m

$$L_m = L_p + \frac{M_p}{BF}\left(\frac{C_b - 1}{C_b}\right) \leq L_r$$

$$L_m = 1.69 + \frac{494/0.9}{65}\left(\frac{1.24-1}{1.24}\right) = 3.32m$$

L_b **= 7.0 >>** L_m **=3.32** ⇒	***Revise the section using beam selection charts of Reference-1***

Reference-1, Page 162

Z_x

For shapes used as beams

$\phi_b = 0.90$

Z_x	SI Shape				F_y = 250 MPa									Torsional Constant	Warping Constant	
Z_x mm³x10³					$\phi_b M_p$ kN-m	$\phi_b M_r$ kN-m	L_p m	L_r m	BF kN	d mm	$b_f/2t_f$	h/t_w	Z_y mm³x10³	J mm⁴x10⁴	C_w mm⁶x10⁶	$\sqrt{\frac{EC_w}{GJ}}$ mm
2622	**W**	**530**	**x**	**101**	**590**	**361**	**2.28**	**7.06**	**53.3**	**537**	**6.0**	**43.6**	**400**	**102**	**1,815,302**	**2146**
2573	W	360	x	134	579	369	4.68	16.86	19.2	356	10.2	25.9	1239	169	4,296,574	2565
2507	**W**	**610**	**x**	**92**	**564**	**339**	**1.74**	**5.33**	**69.9**	**603**	**6.0**	**50.1**	**257**	**71.2**	**1,240,636**	**2123**
2458	W	410	x	114	553	347	3.12	10.83	29.8	420	6.8	31.2	674	149	2,306,723	2004
2409	W	310	x	143	542	339	3.91	19.11	14.9	323	6.8	17.7	1106	286	2,526,923	1514
2409	W	250	x	167	542	324	3.39	26.97	10.3	289	4.2	10.4	1134	629	1,616,586	815
2376	W	460	x	106	535	328	2.15	7.61	42.1	469	4.7	32.4	405	145	1,262,119	1501
2360	**W**	**530**	**x**	**92**	**531**	**328**	**2.24**	**6.76**	**50.0**	**533**	**6.7**	**46.9**	**356**	**76.2**	**1,600,474**	**2332**
2278	W	360	x	122	513	318	3.14	13.36	21.1	363	5.9	22.4	734	211	1,801,876	1486
2196	**W**	**610**	**x**	**82**	**494**	**295**	**1.69**	**5.10**	**65.0**	**599**	**6.9**	**54.6**	**218**	**49.1**	**1,039,234**	**2342**
2179	W	460	x	97	490	302	2.14	7.22	41.1	466	5.1	35.7	369	114	1,138,592	1610
2163	W	310	x	129	487	304	3.88	17.56	14.8	318	7.5	18.9	990	212	2,220,792	1646
2130	W	410	x	100	479	302	3.11	10.00	28.5	415	7.7	35.9	582	99.5	1,960,312	2258
2130	W	250	x	149	479	290	3.35	24.10	10.1	282	4.6	11.6	1000	454	1,382,960	889
2114	W	530	x	85	476	287	1.71	5.35	57.6	535	5.0	46.3	243	73.7	856,629	1735
2065	W	360	x	110	465	290	3.14	12.41	20.9	360	6.4	25.3	665	161	1,608,530	1605
2016	W	460	x	89	454	279	2.14	6.94	40.4	463	5.4	38.7	338	90.3	1,033,863	1722
1950	W	310	x	117	439	276	3.86	16.18	14.7	314	8.2	20.7	890	160	1,968,368	1786
1885	W	360	x	101	424	266	3.11	11.59	20.7	357	7.0	27.5	605	126	1,444,723	1725
1852	W	250	x	131	417	254	3.33	21.37	10.0	275	5.2	13.0	870	313	1,162,760	980
1835	**W**	**460**	**x**	**82**	**413**	**254**	**2.11**	**6.63**	**39.2**	**460**	**6.0**	**41.2**	**303**	**69.1**	**921,078**	**1857**
1803	**W**	**530**	**x**	**74**	**406**	**244**	**1.64**	**5.03**	**53.0**	**529**	**6.1**	**49.4**	**200**	**47.5**	**690,137**	**1941**
1770	W	310	x	107	398	252	3.84	14.97	14.6	311	9.0	22.6	806	122	1,756,225	1930
1721	W	410	x	85	387	238	2.02	7.09	32.7	417	5.0	33.0	310	92.4	714,305	1415
1671	W	360	x	91	376	238	3.10	10.77	20.0	353	7.7	30.4	537	91.6	1,264,804	1892

For

L_b = 7m and

$M_{u,eq} = M_u / C_b$

$= 435.3 / 1.24$

$= 351.05$ *kN-m*

From design charts of

Reference-1, Page 211

Trial Section

W410 × 100

ϕM_p = 479 kN-m > M_u

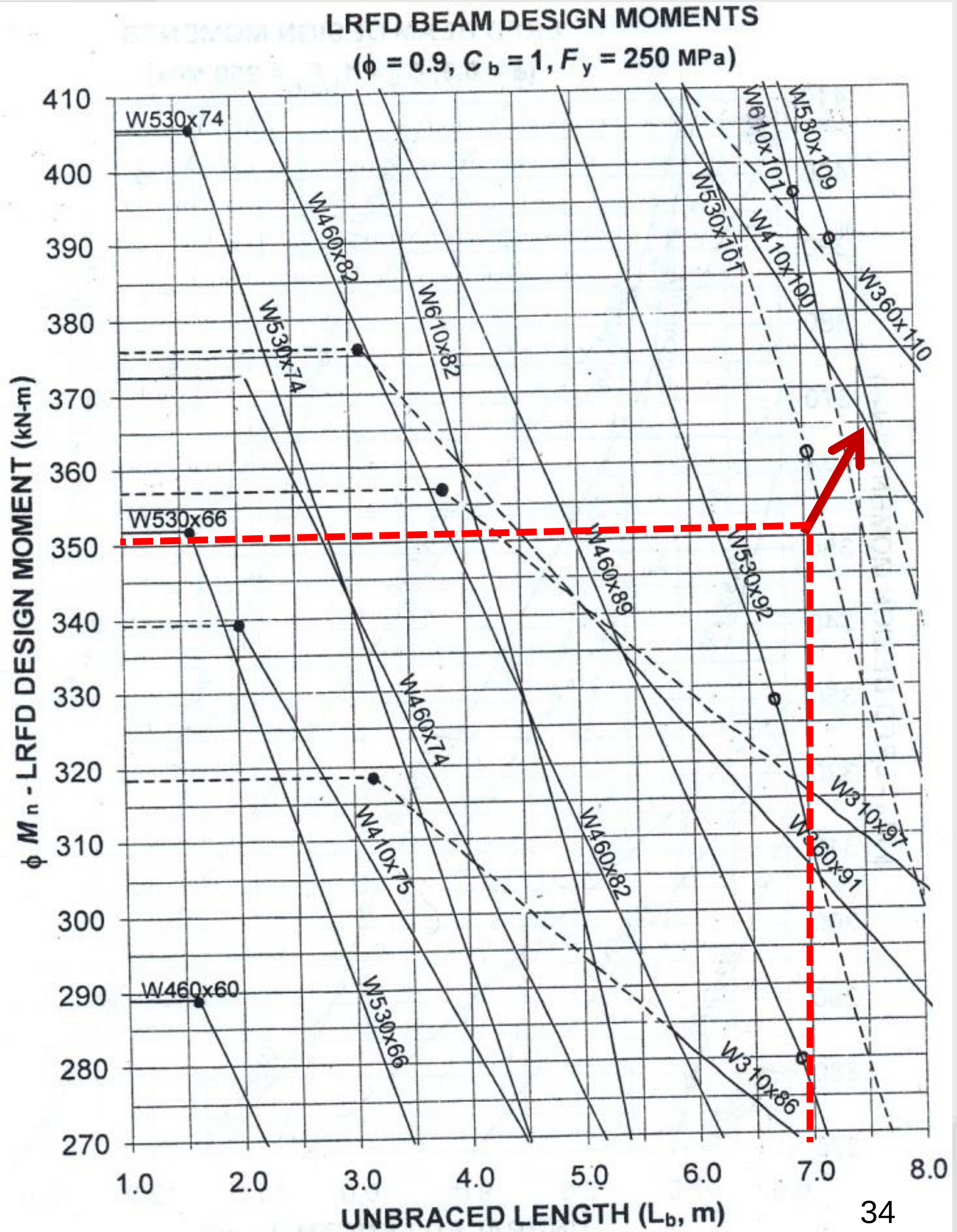
d = 410 > d_{min} ***(OK)***

ϕM_r = 295.0 kN-m

BF = 28.53

L_p = 3.11 m

L_r = 10.0 m



Check internal compactness of section as under:

1. Web is continuously connected **(*OK*)**

2. $\frac{b_f}{2t_f} = 7.5 < \lambda_p$ $\boldsymbol{\lambda_p = 10.8}$ **(*OK*)**

3. $\frac{h}{t_w} = 53.3 < \lambda_p$ $\boldsymbol{\lambda_p = 107}$ **(*OK*)**

$\therefore$ **The section is internally compact.**

$$L_m = L_p + \frac{M_p}{BF}\left(\frac{C_b - 1}{C_b}\right) \le L_r$$

$$L_m = 3.11 + \frac{494/0.9}{28.5}\left(\frac{1.24 - 1}{1.24}\right) = 6.72m$$

Check for Moment Carrying Capacity:

$L_m < L_b \le L_r \Rightarrow$ **Inelastic Buckling**

$$\phi_b M_n = C_b \times [\phi_b M_p - \phi_b BF(L_b - L_p)] \le \phi_b M_p$$

$$\phi_b M_n = 1.24 \times [479 - 0.9\text{x}28.53(7.00 - 3.11)]$$

$$= 470.44 \text{ kN-m}$$

$$M_u = 435.3 \ kN\text{-}m < \phi_b M_n = 470.44 \ kN\text{-}m \qquad \textbf{(OK)}$$

Check for Shear Capacity:

$h/t_w = 35.9 < 63.4 \Rightarrow$ shear yield formula is applicable

$$\phi_v V_n = \frac{0.9 \times 0.6}{1000} F_{yw} A_w C_w = \frac{0.9 \times 0.6}{1000} \times 250.(415 \times 10.0).1.0$$

$$\phi_v V_n = 560.25 kN > V_u = 172.74 \ kN \qquad \textbf{(OK)}$$

Check for Deflections:

Check only for service live load

$$\Delta_{act} = \frac{5}{384} \times \frac{w_L L^4}{EI} + \frac{Pa}{12EI}\left(0.75L^2 - a^2\right)$$

$$\Delta_{act} = \frac{5}{384} \times \frac{10 \times (7000)^4}{200000 \times 39700 \times 10^4} + \frac{50000 \times 3500}{12 \times 200000 \times 39700 \times 10^4}\left(0.75 \times (7000)^2 - (3500)^2\right)$$

$$\Delta_{act} = 3.94 + 4.50 = 8.44\ mm$$

$$\Delta_{all} = \frac{L}{360} = \frac{7000}{360} = 19.44$$

$$\Delta_{act} = 8.44 \quad < \quad \Delta_{all} = 19.44 \qquad (OK)$$

Check for Self-weight:

$$Selfweight = \frac{100 \times 9.81}{1000} = 0.981, kN/m$$

lesser than 1.2x(assumed self-weight of 1.7 *kN/m*) (*OK*)

Final Section : W410x100

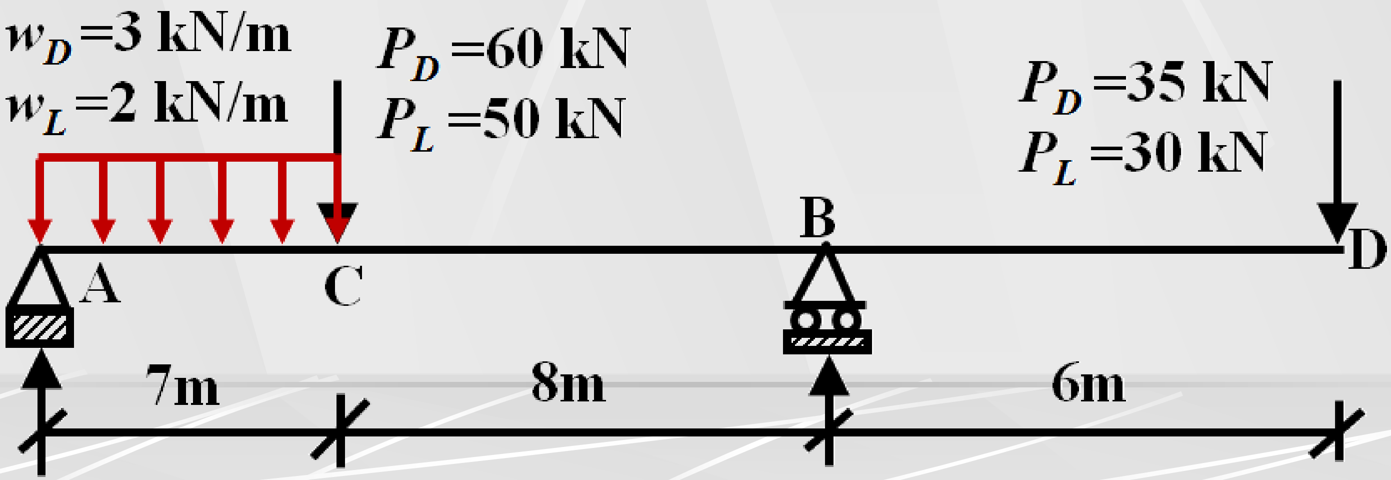
Example 4.3:

BEAMS WITH COMPRESSION FLANGE LATERALLY SUPPORTED AT INTERVALS

Design the beam of Figure 4.21, ignoring self-weight and deflection check. Compression flange is laterally supported at points A, B and C. Use A36 steel.

Solution:

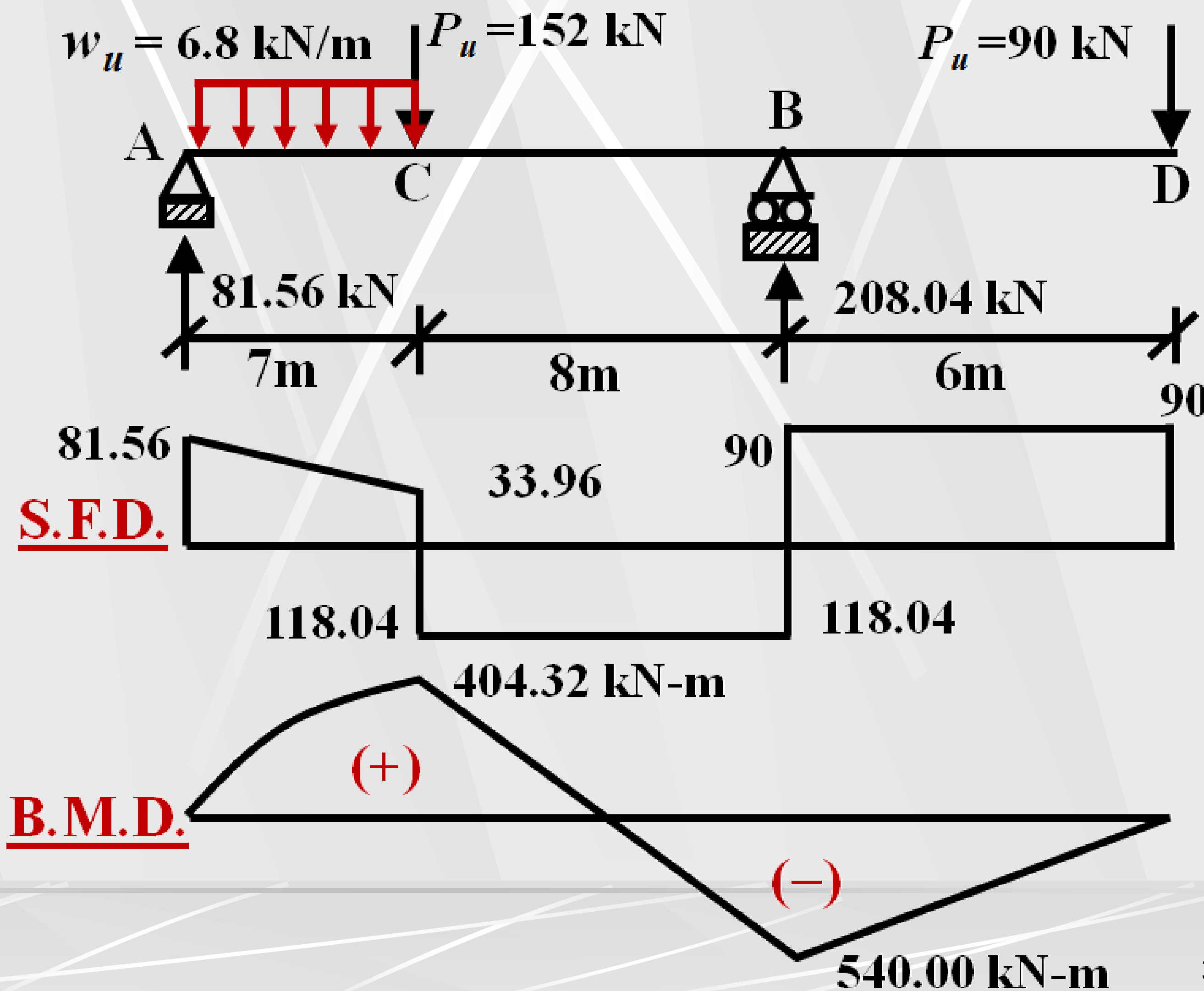
The factored loading and the shear force and bending moment diagrams are shown in Figure 4.22.



$M_u = M_{max}$ = 540 kN-m , $(M_u)_{AC}$ = 404.32 kN-m

$(M_u)_{CB}$ = 540 kN-m , $(M_u)_{BD}$ = 540.00 kN-m

$V_u = V_{max}$ = 118.04 kN



Portion AC

$(L_b)_{AC}$ = 7 m

$(L_b)_{CB}$ = 8 m

$(L_b)_{BD}$ = 6 m

Calculation of Cb:

i) Portion AC

$$M = 81.56\,x - 6.8\,x^2/2$$

M_{max} = 404.32 kN-m

M_A = 132.32 kN-m (x = 1.75 m)

M_B = 243.81 kN-m (x = 3.50 m)

M_C = 334.48 kN-m (x = 5.25 m)

$$C_b = \frac{12.5 \times 404.32}{2.5 \times 404.32 + 3 \times 132.32 + 4 \times 243.81 + 3 \times 334.48} = 1.49$$

ii) Portion CB

$$M = 404.32 - 118.04x$$ **(*x* starts from the end C)**

M_{max} = 540 kN-m

M_A = 168.24 kN-m (x = 2 m)

M_B = 67.84 kN-m (x = 4 m)

M_C = 303.92 kN-m (x = 6 m)

$$C_b = \frac{12.5 \times 540.00}{2.5 \times 540.00 + 3 \times 168.24 + 4 \times 67.84 + 3 \times 303.92} = 2.22$$

iii) Portion BD

C_b = **1.0** , *for overhanging portion*

Portion	$(M_u)_{max}$ (kN-m)	$(V_u)_{max}$ (kN)	L_b (m)	C_b
AC	404.32	81.56	7	1.49
CB	540	118.04	8	2.22
BD	540.0	90.0	6	1.0

Selection of Critical Segment:

- Greater value of M_u makes a segment more critical.
- Similarly, a longer unbraced length reduces the member capacity.
- However, smaller value of C_b is more critical for a particular segment.

The effect of these three factors together determines whether a selected segment is actually more critical than the other segments.

The decision about which part of the beam is more critical for design depends on experience but may not always be fully accurate.

If a wrong choice is made for this critical section, the procedure will correct itself. Only the calculations will be lengthy in such cases with the end result being always the same.

For this example, assume that portion **BD** is more critical and first design it. Later on, check for the other two portions.

$$d_{\min} \approx \frac{L}{22} = \frac{15000}{22} = 682$$

Depth for portion **AB** may be relaxed a little due to the presence of cantilever portion.

However, the portion **BD** may have its own larger depth requirements.

Portion BD

Assuming the section to be internally compact with no LTB

$$(Z_x)_{req} = \frac{M_u \times 10^6}{\phi_b F_y} = \frac{540 \times 10^6}{0.9 \times 250} = 2400 \times 10^3, mm^3$$

Selection of Section

i. $Z_{sel} \geq Z_{req} = 2400\ mm^3$

ii. *Minimum weight*

iii. $d \geq d_{min} = 682\ mm$

- Consulting beam selection tables **(Reference-1, Page 162)**, following result is obtained.

Trial Section: **W610 × 92** $C_b = 1$ **so** $L_p = L_m$

ϕM_p = 564.0 kN-m, ϕM_r = 339.0 kN-m,

BF = 69.9, L_p = 1.74 m , L_r = 5.33 m

$L_b = 6 \gg L_m = 1.74 \Rightarrow$ ***Revise the section using beam selection charts of Reference-1***

Reference-1, Page 162

Z_x					For shapes used as beams $\phi_b = 0.90$											
					F_y = 250 MPa									Torsional Constant	Warping Constant	
Z_x mm^{3}x10^3	SI Shape				$\phi_b M_p$ kN-m	$\phi_b M_r$ kN-m	L_p m	L_r m	BF kN	d mm	$b_f/2t_f$	h/t_w	Z_y mm^{3}x10^3	J mm^{4}x10^4	C_w mm^{6}x10^6	$\sqrt{\frac{EC_w}{GJ}}$ mm
2622	**W**	**530**	**x**	**101**	**590**	**361**	**2.28**	**7.06**	**53.3**	**537**	**6.0**	**43.6**	**400**	**102**	**1,815,302**	**2146**
2573	W	360	x	134	579	369	4.68	16.86	19.2	356	10.2	25.9	1239	169	4,296,574	2565
2507	**W**	**610**	**x**	**92**	**564**	**339**	**1.74**	**5.33**	**69.9**	**603**	**6.0**	**50.1**	**257**	**71.2**	**1,240,636**	**2123**
2458	W	410	x	114	553	347	3.12	10.83	29.8	420	6.8	31.2	674	149	2,306,723	2004
2409	W	310	x	143	542	339	3.91	19.11	14.9	323	6.8	17.7	1106	286	2,526,923	1514
2409	W	250	x	167	542	324	3.39	26.97	10.3	289	4.2	10.4	1134	629	1,616,586	815
2376	W	460	x	106	535	328	2.15	7.61	42.1	469	4.7	32.4	405	145	1,262,119	1501
2360	**W**	**530**	**x**	**92**	**531**	**328**	**2.24**	**6.76**	**50.0**	**533**	**6.7**	**46.9**	**356**	**76.2**	**1,600,474**	**2332**
2278	W	360	x	122	513	318	3.14	13.36	21.1	363	5.9	22.4	734	211	1,801,876	1486
2196	**W**	**610**	**x**	**82**	**494**	**295**	**1.69**	**5.10**	**65.0**	**599**	**6.9**	**54.6**	**218**	**49.1**	**1,039,234**	**2342**
2179	W	460	x	97	490	302	2.14	7.22	41.1	466	5.1	35.7	369	114	1,138,592	1610
2163	W	310	x	129	487	304	3.88	17.56	14.8	318	7.5	18.9	990	212	2,220,792	1646
2130	W	410	x	100	479	302	3.11	10.00	28.5	415	7.7	35.9	582	99.5	1,960,312	2258
2130	W	250	x	149	479	290	3.35	24.10	10.1	282	4.6	11.6	1000	454	1,382,960	889
2114	W	530	x	85	476	287	1.71	5.35	57.6	535	5.0	46.3	243	73.7	856,629	1735
2065	W	360	x	110	465	290	3.14	12.41	20.9	360	6.4	25.3	665	161	1,608,530	1605
2016	W	460	x	89	454	279	2.14	6.94	40.4	463	5.4	38.7	338	90.3	1,033,863	1722
1950	W	310	x	117	439	276	3.86	16.18	14.7	314	8.2	20.7	890	160	1,968,368	1786
1885	W	360	x	101	424	266	3.11	11.59	20.7	357	7.0	27.5	605	126	1,444,723	1725
1852	W	250	x	131	417	254	3.33	21.37	10.0	275	5.2	13.0	870	313	1,162,760	980
1835	**W**	**460**	**x**	**82**	**413**	**254**	**2.11**	**6.63**	**39.2**	**460**	**6.0**	**41.2**	**303**	**69.1**	**921,078**	**1857**
1803	**W**	**530**	**x**	**74**	**406**	**244**	**1.64**	**5.03**	**53.0**	**529**	**6.1**	**49.4**	**200**	**47.5**	**690,137**	**1941**
1770	W	310	x	107	398	252	3.84	14.97	14.6	311	9.0	22.6	806	122	1,756,225	1930
1721	W	410	x	85	387	238	2.02	7.09	32.7	417	5.0	33.0	310	92.4	714,305	1415
1671	W	360	x	91	376	238	3.10	10.77	20.0	353	7.7	30.4	537	91.6	1,264,804	1892

For

L_b = 6m and

M_u = 540 *kN-m*

From design charts of

Reference-1, Page 207

Trial Section

W690 × 125

ϕM_p = 900 kN-m > M_u

$d = 678 \approx d_{min} = 682$

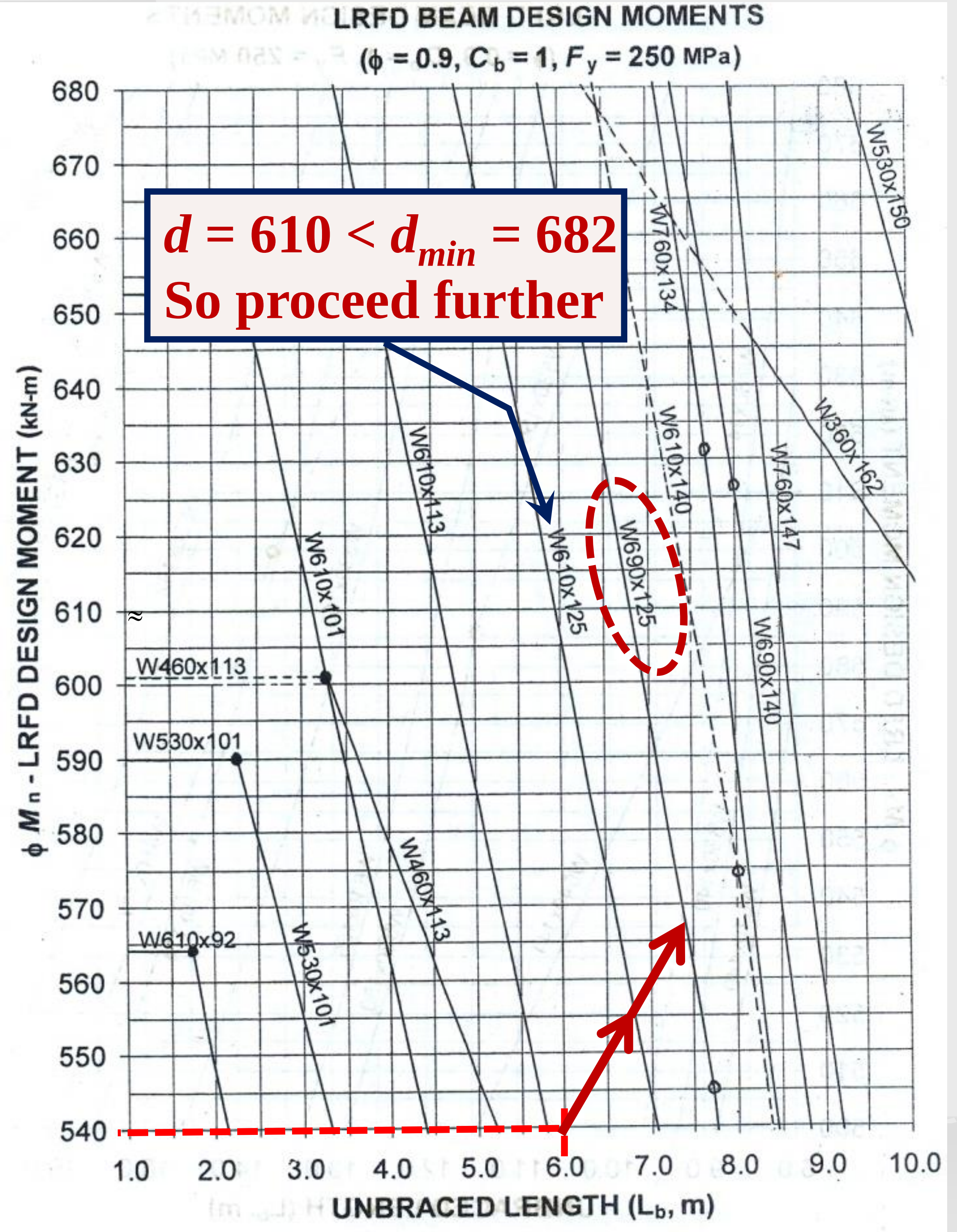
within the acceptable range (OK)

ϕM_r = 555.0 kN-m

BF = 77 kN

L_p = 2.62 m

L_r = 7.67m, $L_m = L_p$



Check internal compactness of section as under:

1. Web is continuously connected **(*OK*)**

2. $\frac{b_f}{2t_f} = 7.8 < \lambda_p$ $\qquad \lambda_p = 10.8$ **(*OK*)**

3. $\frac{h}{t_w} = 52.7 < \lambda_p$ $\qquad \lambda_p = 107$ **(*OK*)**

$\therefore$ **The section is internally compact.**

Check for Moment Carrying Capacity:

$L_m < L_b \leq L_r \Rightarrow$ **Inelastic Buckling**

$$\phi_b M_n = C_b \times [\phi_b M_p - \phi_b BF(L_b - L_p)] \leq \phi_b M_p$$

$$\phi_b M_n = 1.0 \times [900 - 0.9 \text{x} 77.0(6.00 - 2.62)]$$

$$= 665.8 \text{ kN-m}$$

M_u = 540 *kN-m* < $\phi_b M_n$ = 665.8 *kN-m* **(*OK*)**

Check for Shear Capacity:

$\boldsymbol{h/t_w}$ = 52.7 < 63.4 ⇒ shear yield formula is applicable

$$\phi_v V_n = \frac{0.9\times 0.6}{1000} F_{yw} A_w C_w = \frac{0.9\times 0.6}{1000} \times 250.(678\times 11.7).1.0$$

$$\phi_v V_n = 1070.9 kN \quad > \quad \boldsymbol{V_u = 118.04\ kN} \qquad \textbf{(OK)}$$

CHECK FOR PORTION AC

$$\boldsymbol{L_b} = 7.0 \text{ m}, \quad \boldsymbol{C_b = 1.49}$$

$\boldsymbol{L_p}$ **= 2.62 <** $\boldsymbol{L_b}$ **= 7 ≤** $\boldsymbol{L_r}$ **= 7.67 ⇒ Inelastic Buckling**

$$\boldsymbol{\phi_b M_n} = C_b \times [\phi_b M_p - \phi_b BF(L_b - L_p)] \leq \phi_b M_p$$

$$\boldsymbol{\phi_b M_n} = 1.49 \times [900 - 0.9\text{x}77.0(7.00 - 2.62)]$$

$$= 888.7 \text{ kN-m}$$

$$\boldsymbol{M_u} = 540\ kN\text{-}m < \boldsymbol{\phi_b M_n} = 888.7\ kN\text{-}m \qquad \textbf{(OK)}$$

CHECK FOR PORTION CB

L_b = 8.0 m, $C_b = 2.22$

$L_b = 8 > L_r = 7.67 \Rightarrow$ **Elastic Buckling**

For W690x125:

$S_x = 3490 \times 10^3$ mm^3, $J = 117 \times 10^4$ mm^4, $I_y = 4410 \times 10^4$ mm^4

$r_y = 52.6$, $d = 678$ mm, $t_f = 16.3$ mm, $\phi_b M_p = 900$ kN-m

$$\phi_b M_n = \phi_b C_b F_{cr} S_x \leq \phi_b M_p \,(kN\text{-}m)$$

$$F_{cr} = \frac{\pi^2 E}{\left(\dfrac{L_b}{r_{ts}}\right)^2}\sqrt{1 + 0.078\,\frac{J\,c}{S_x h_o}\left(\frac{L_b}{r_{ts}}\right)^2} \approx \frac{\pi^2 E}{\left(L_b / r_{ts}\right)^2} \qquad r_{ts} = \sqrt{\frac{I_y h_0}{2 S_x}}$$

$$\mathbf{h_o} = d - t_f = 678 - 16.3 = 661.7 \text{ mm}$$

$$r_{ts} = \sqrt{\frac{I_y h_0}{2S_x}} = \sqrt{\frac{4410 \times 10^4 \times 661.7}{2 \times 3490 \times 10^3}} = 64.66 mm$$

$$\frac{L_b}{r_{ts}} = \frac{8000}{64.59} = 123.72$$

$$F_{cr} = \frac{\pi^2 \times 2 \times 10^5}{(123.72)^2} \sqrt{1 + 0.078 \frac{117 \times 10^4 \times 1}{3490 \times 10^3 \times 661.7} (123.72)^2}$$

$$\mathbf{F_{cr}} = 128.96\text{x}(1.27) = 163.78 \text{ MPa}$$

$$\phi_b M_n = \phi_b C_b F_{cr} S_x = 0.9\text{x}(2.22\text{x}163.78\text{x}3490\text{x}10^3)/10^6$$

$$\phi_b M_n = 1142 \text{ kN-m} > \phi_b M_p = 900 \text{ kN-m}$$

Hence, $\phi_b M_n = \phi_b M_p = 900$ *kN-m* $> M_u = 540$ *kN-m*

and portion BD is most critical as expected earlier.

Deflection Check:

Requires detailed calculations for $\boldsymbol{\Delta_{actual}}$ that is not asked for in this example. Very approximate calculations are performed as under:

For cantilever Beam ; $$\Delta_{max} \approx \frac{Pa^2(L+a)}{3EI}$$

Where, ***a*** is the cantilever length and ***L*** is the span

Check only for service live load P_L = 30 kN at Free End

$$\Delta_{max} = \frac{30{,}000 \times 6{,}000^2 (15{,}000 + 6{,}000)}{3 \times 200{,}000 \times 119{,}000 \times 10^4} = 31.76mm$$

(Deflection will be reduced by loads on span '*L*')

Also $\boldsymbol{\Delta_{max}}$ considering one end of the overhang as fixed

$$\Delta_{max} = \frac{Pa^3}{3EI_x} = \frac{30{,}000 \times 6000^3}{3 \times 2 \times 10^5 \times 119{,}000 \times 10^4} = 9.08mm$$

$$\Delta_{all} = \frac{L}{360} = \frac{6000}{360} = 16.67\,mm$$

$\therefore$ Deflection may be critical, detailed calculations are recommended.

Check for Self-weight:

$$Selfweight = \frac{125 \times 9.81}{1000} = 1.23, kN/m$$

(asked to be ignored in the problem statement)

Final Section : W690x125

Example 4.4:

Select suitable trial section for a **8m** long simply supported W-section beam subjected to service live load **(P_L)** of **10 kN/m** and imposed dead load **(P_D)** of **4 kN/m**. The compression flange is continuously supported in the lateral direction and the deflection is limited to **span/1500**.

Solution:

Assumed self weight = 10% of superimposed **DL**

= 0.1x4 = 0.4 *kN/m*

say **0.8 kN/m** as live load is greater and deflection requirements are strict

w_u = $1.2D + 1.6L$ = $1.2 \times 4.8 + 1.6 \times 10$

= 21.76 kN/m

$$M_u = M_{\max} = \frac{wL^2}{8} = \frac{21.76 \times 8^2}{8} = 174.08, kN - m$$

$$V_u = V_{\max} = \frac{wL}{2} = \frac{21.76 \times 8}{2} = 87.04, kN$$

- Assuming the section to be internally compact and knowing that there is no LTB,

$$(Z_x)_{req} = \frac{M_u \times 10^6}{\phi_b F_y} = \frac{174.08 \times 10^6}{0.9 \times 345} = 773.7 \times 10^3, mm^3$$

$$d_{\min} = \frac{L}{22} = \frac{8000}{22} = 364 mm$$

To satisfy the deflection criterion equating the,

$$\Delta_{all} = \Delta_{act}$$

$$\frac{L}{1500} = \frac{5}{384} \times \frac{w_L L^4}{EI_{\min}}$$

$$I_{\min} = \frac{5\times1500}{384}\times\frac{w_L L^3}{E} = \frac{5\times1500}{384}\times\frac{10\times(8000)^3}{2\times10^5}$$

$$\mathbf{I_{min} = 50{,}000 \times 10^4\ mm^4}$$

Selection of Section

i. $Z_{sel} \geq Z_{req} = 314.0\ mm^3$

ii. *Minimum* weight

iii. $d \geq d_{min} = 364\ mm$

iv. $I \geq I_{min} = 50{,}000\text{x}10^4\ mm^4$

- Consulting beam selection tables

Trial section no.1: **W360 × 44**

$d = 352\ \text{mm} < d_{min}$

$I_x = 12{,}100 \times 10^4\ \text{mm}^4$

Skipping W410 × 60 and W460 × 60,

Trial section no.5: **W530 × 66**

$I_x = 35{,}100 \times 10^4\ mm^4$

Trial section no.6: **W530 × 74**

$I_x = 41{,}000 \times 10^4\ mm^4$

Trial section no.7: **W610 × 82**

$I_x = 56{,}200 \times 10^4\ mm^4$

Trial Section: W610 × 82

After selection of the trial section, all the checks are to be performed. This part of the exercise is left for the reader to be completed.

End of Chapter 4

ASSIGNMENTS