STEEL STRUCTURES

Design of Compression Members

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CHAPTER # 3

DESIGN OF COMPRESSION MEMBERS

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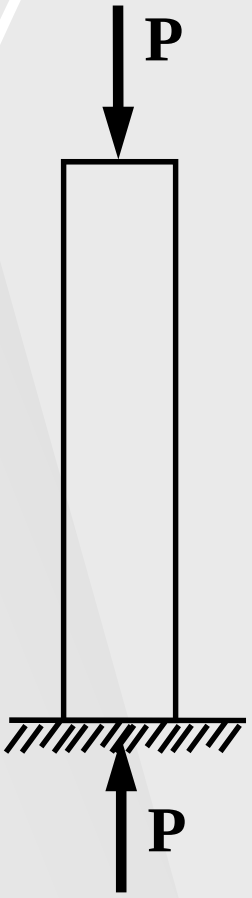
COMPRESSION MEMBERS

When a load tends to **squeeze or shorten** a member, the stresses produced are said to be compressive in nature and the member is called a compression member (Figure 3.1).



Examples are struts (short compression members without chances of buckling), eccentrically loaded columns, top chords of trusses, bracing members, compression flanges of beams and members that are subjected simultaneously to bending and compressive loads.

There are two significant differences between the behavior of tension and compression members, as under:

1.

Tensile Forces	Compression Forces
Tensile Forces will straighten a member which is initially not straight	Compressive Forces will bend/skew a member which is initially straight

2. The presence of rivet or bolt holes in tension members reduces the area available for resisting loads; but in compression members the rivets or bolts are assumed to fill the holes and the entire gross area is available for resisting load.

CONCENTRICALLY AND ECCENTRICALLY LOADED COLUMNS

The ideal type of load on a column is a concentric load and the member subjected to this type of load is called concentrically loaded column.

The load is distributed uniformly over the entire cross-section with the center of gravity of the loads coinciding with the center of gravity of the columns.

Due to load patterns, the live load on slabs and beams may not be concentrically transferred to interior columns.

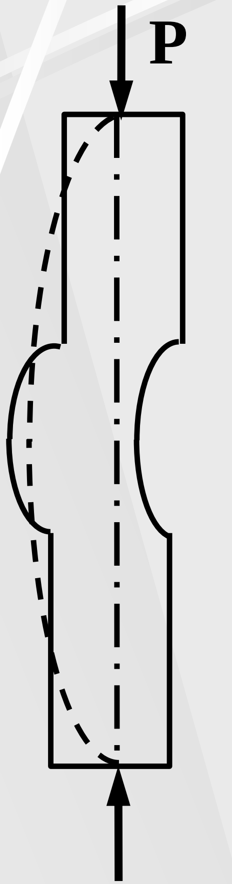
Similarly, the dead and live loads transferred to the exterior columns are, generally, having large eccentricities, as the center of gravity of the loads will usually fall well on the inner side of the column.

In practice, majority of the columns are eccentrically loaded compression members

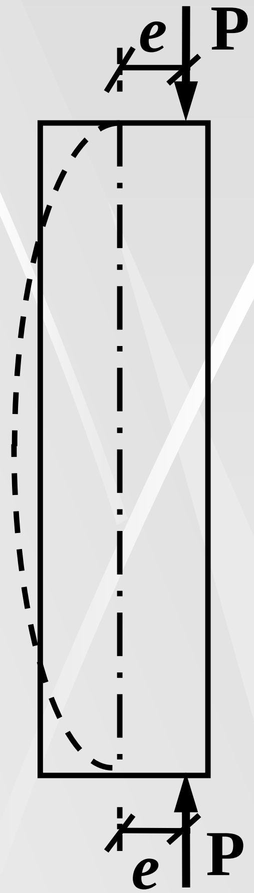
Slight initial crookedness, eccentricity of loads, and application of simultaneous transverse loads produce significant bending moments as the product of high axial loads (***P***) multiplied with the eccentricity, ***e***.

This moment, $\boldsymbol{P} \times \boldsymbol{e}$, facilitates buckling and reduces the load carrying capacity.

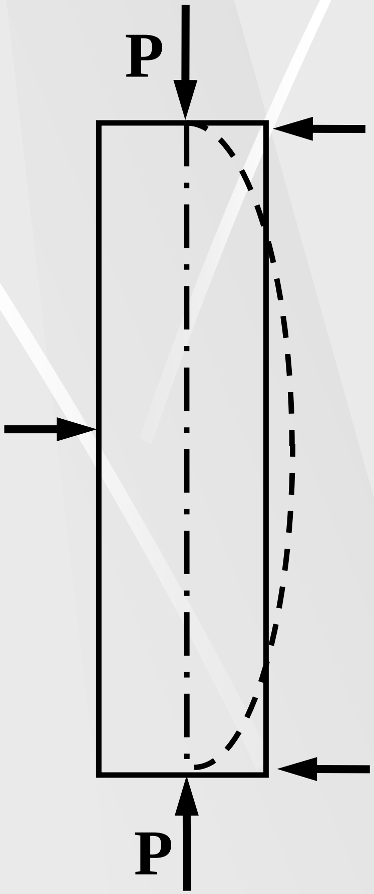
Eccentricity, $\boldsymbol{e}$, may be relatively smaller, but the product ($\boldsymbol{P} \times \boldsymbol{e}$) may be significantly larger.



a) Initial Crookedness



b) Eccentric Load



c)Simultaneous Transverse Load

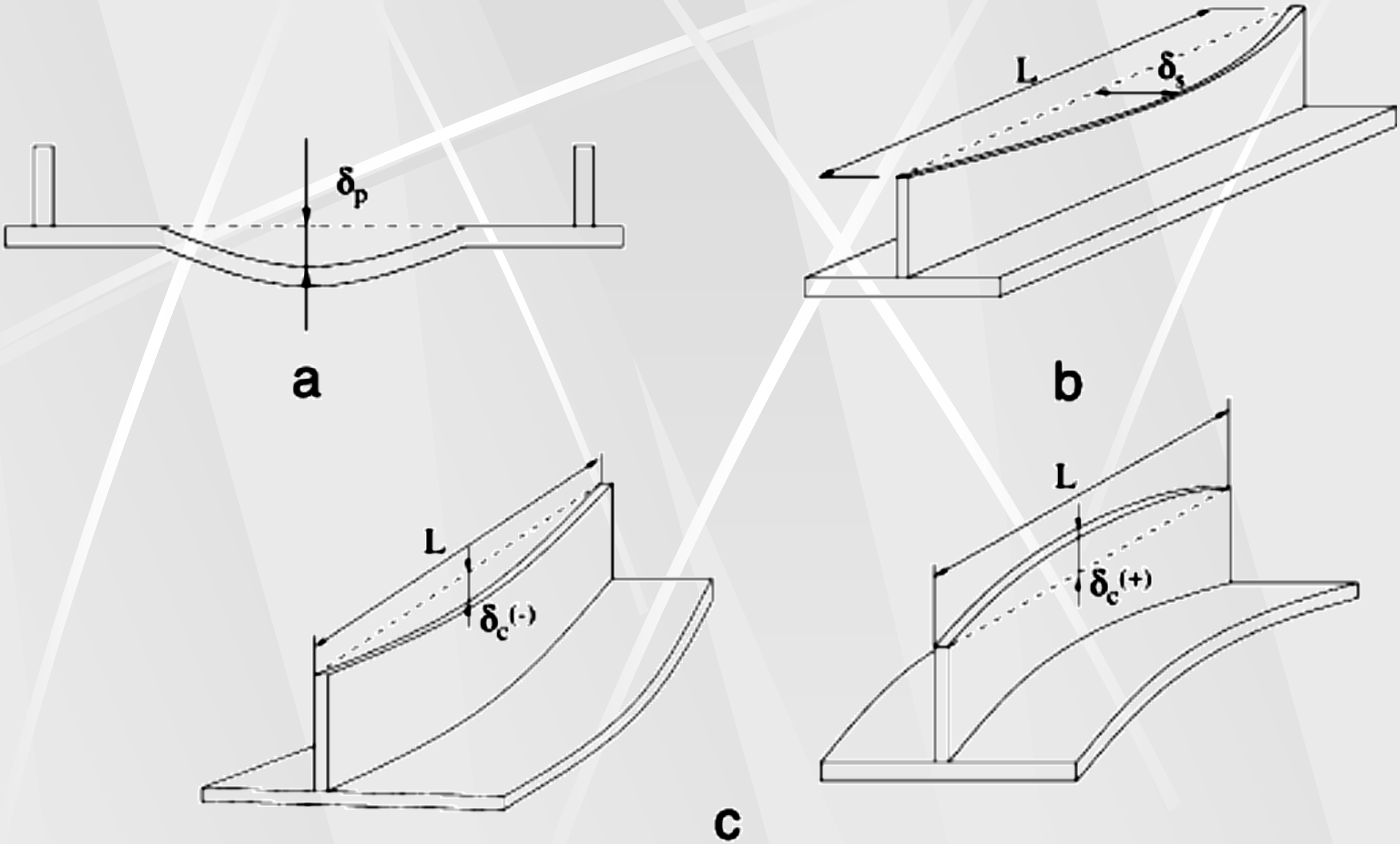


Figure: Typical initial deflections of stiffened panels: (a) out-of-flatness of a subpanel; (b) out-of-straightness of a stiffener; (c) stiffener camber deflection. (IDM, 1987)

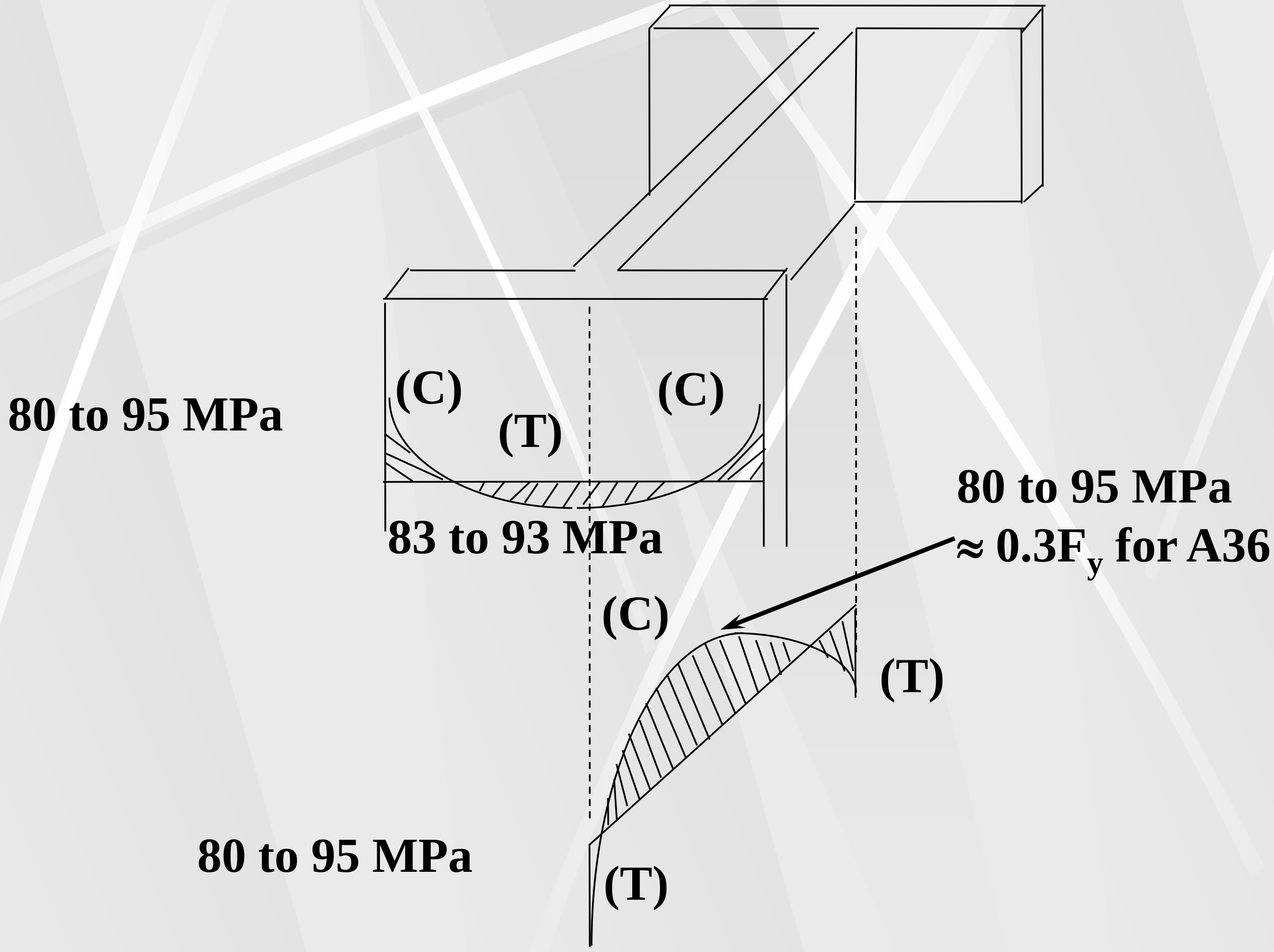
The AISC Code of Standard Practice specifies an acceptable upper limit on the out-of-plumbness and initial crookedness equal to the length of the member divided by 500.
(**equal to 0.002**, **AISC C2-2b-3**).

RESIDUAL STRESSES

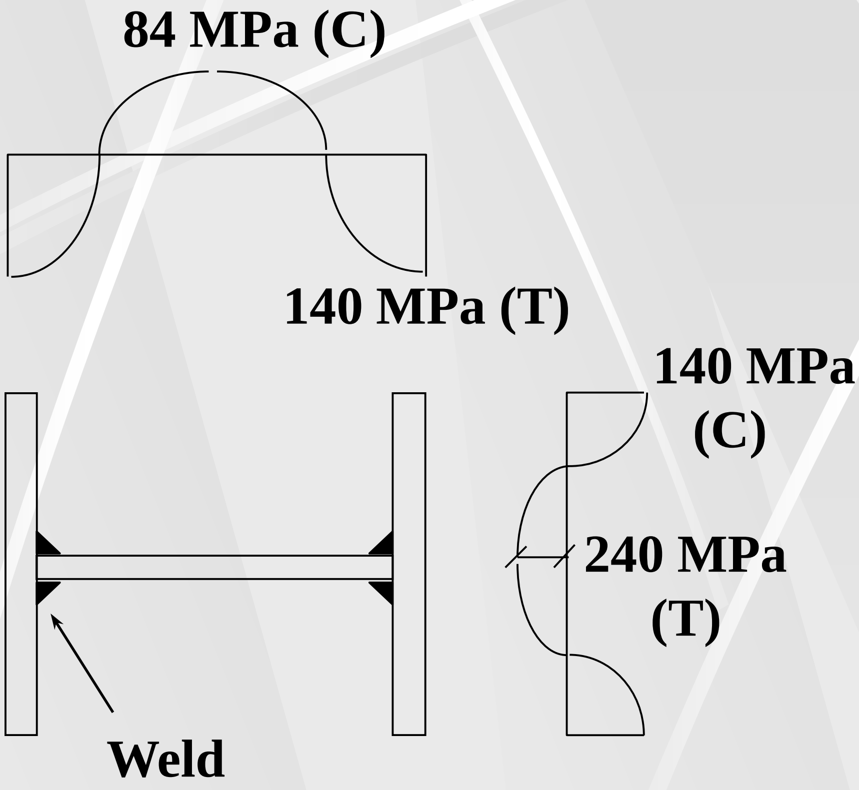
Residual stresses are stresses that remain in a member after it has been formed into a finished product.

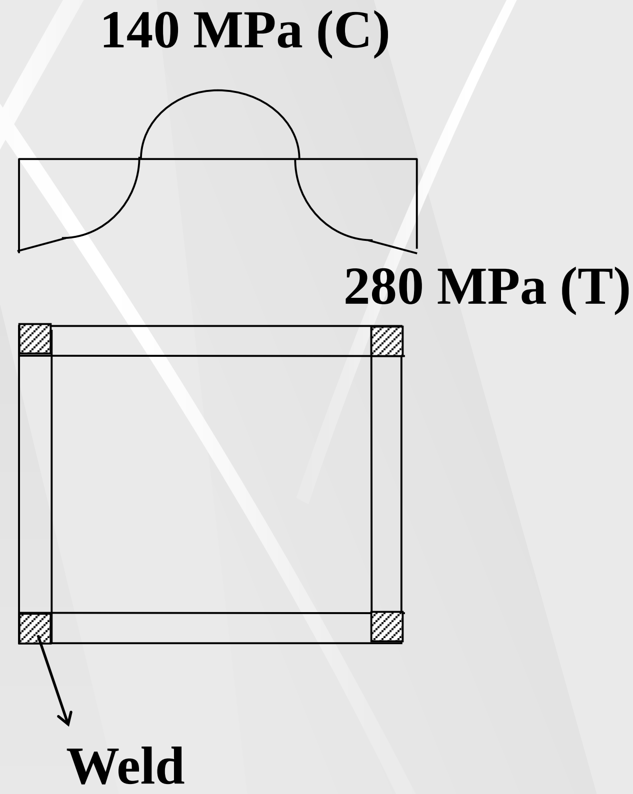
These are always present in a member even without the application of loads.

The magnitudes of these stresses are considerably high and, in some cases, are comparable to the yield stresses (refer to Figure 3.4).



a)Rolled Shapes





b)Welded Shapes

The causes of presence of residual stresses are as under:

1. Uneven cooling which occurs after hot rolling of structural shapes produces thermal stresses, which are permanently stored in members.

The thicker parts cool at the end, and try to shorten in length. While doing so they produce compressive stresses in the other parts of the section and tension in them.

Overall magnitude of this tension and compression remain equal for equilibrium.

In I-shape sections, after hot rolling, the thick junction of flange to web cools more slowly than the web and flange tips.

Consequently, compressive residual stress exists at flange tips and at mid-depth of the web (the regions that cool fastest), while tensile residual stress exists in the flange and the web at the regions where they join.

2. Cold bending of members beyond their elastic limit produce residual stresses and strains within the members.

Similarly, during fabrication, if some member having extra length is forced to fit between other members, stresses are produced in the associated members.

3. Punching of holes and cutting operations during fabrication also produce residual stresses.

4. Welding also produces the stresses due to uneven cooling after welding.

Welded part will cool at the end inviting other parts to contract with it.

This produces compressive stresses in parts away from welds and tensile stresses in parts closer to welds.

SECTIONS USED FOR COLUMNS

Single angle, double angle, tee, channel, W-section, pipe, square tubing, and rectangular tubing may be used as columns.

Different combinations of these structural shapes may also be employed for compression members to get built-up sections as shown in Figure 3.5.

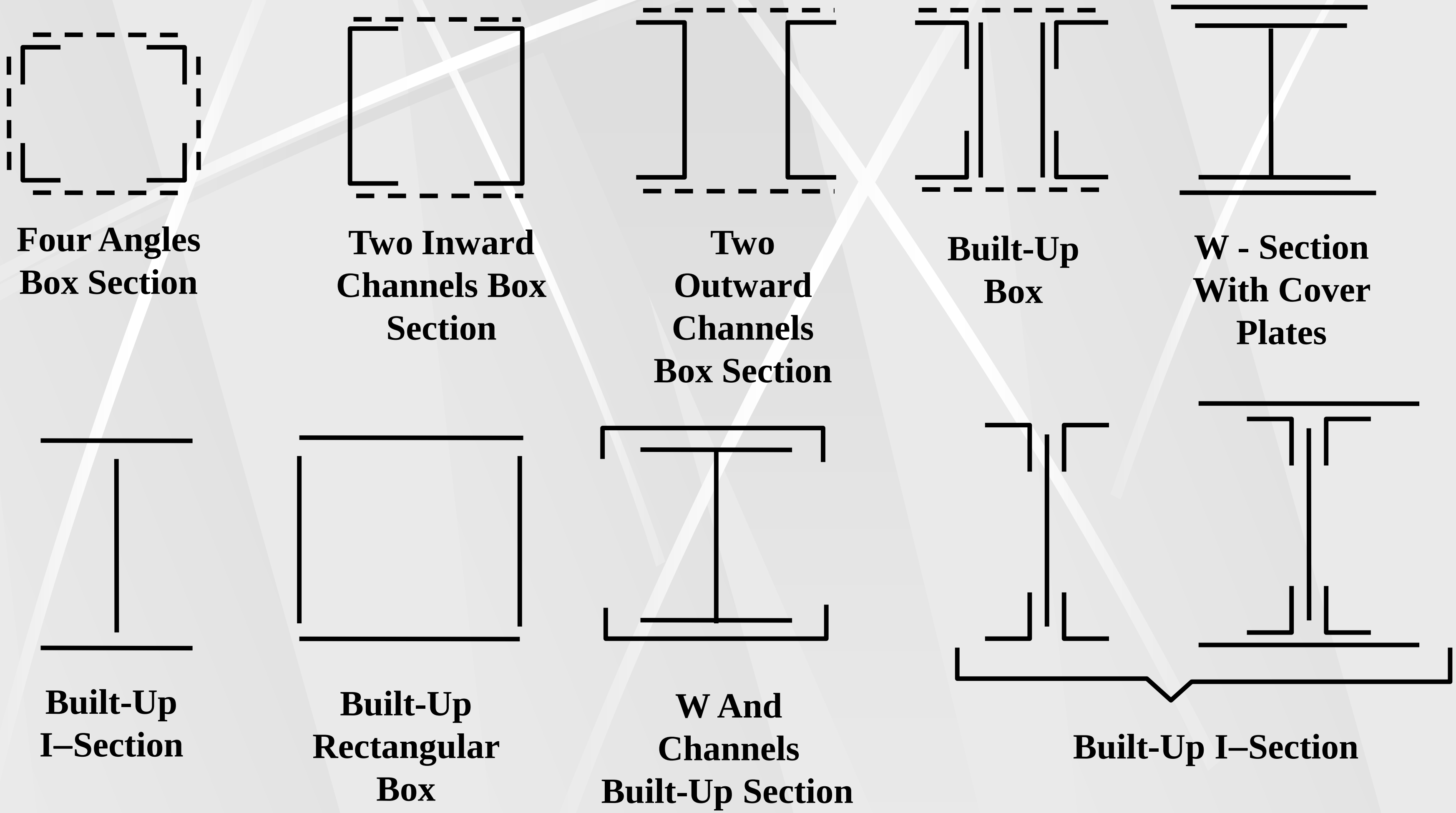


Figure 3.5: Built-up Section s for Compression Member

LIMITING SLENDERNESS RATIO

The slenderness ratio of compression members should preferably ***not exceed 200*** (AISC E2).

This means that in exceptional cases, the limit may be exceeded.

INSTABILITY OF COLUMNS

a) Local Instability

During local instability, the individual parts or plate elements of cross-section buckle without overall buckling of the column.

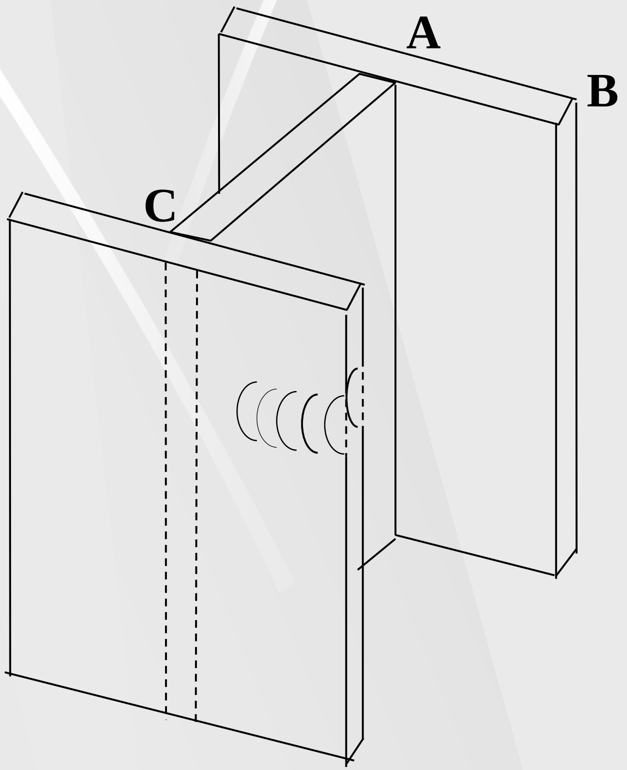


Figure 3.6. Local Flange Instability.

Width/thickness ratio of each part gives the slenderness ratio **($\lambda = b/t$)**, which controls the local buckling.

Local buckling should never be allowed to occur before the overall buckling of the member except in few cases like web of a plate girder.

An ***unstiffened element*** is a projecting piece with one free edge parallel to the direction of the compressive force.

The example is half flange AB in Figure 3.6.

A ***stiffened element*** is supported along the two edges parallel to the direction of the force.

The example is web AC in the same figure.

For unstiffened flange of figure, *b* is equal to half width of flange $(b_f / 2)$ and *t* is equal to t_f. Hence, $b_f / 2t_f$ ratio is used to find λ.

For ***stiffened web***, **h** *is the width of web and* t_w is the thickness of web and the corresponding value of λ or b/t ratio is h / t_w, which controls web local buckling.

Overall Instability

In case of overall instability, the column buckles as a whole between the supports or the braces about an axis whose corresponding slenderness ratio is bigger.

Single angle sections may buckle about their weak axis (z-axis, Figure 3.10).

Calculate $\boldsymbol{L_e / r_z}$ to check the slenderness ratio.

In general, all un-symmetric sections having non-zero product moment of inertia $\boldsymbol{(I_{xy})}$ have a weak axis different from the y-axis.

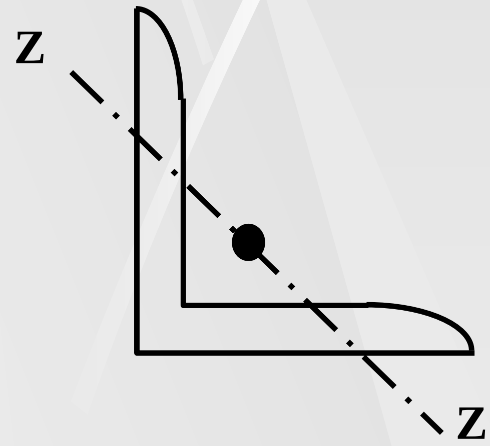
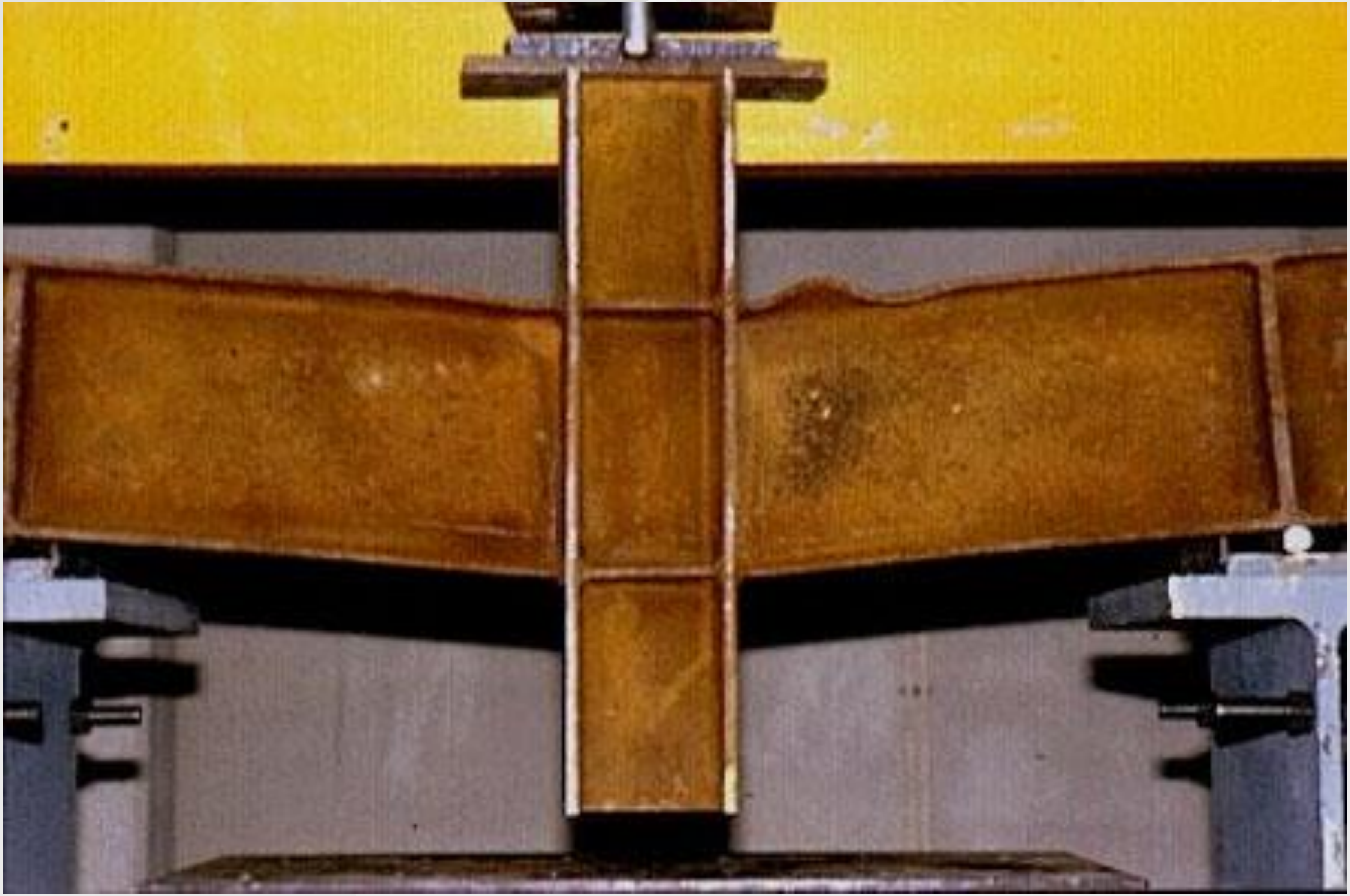
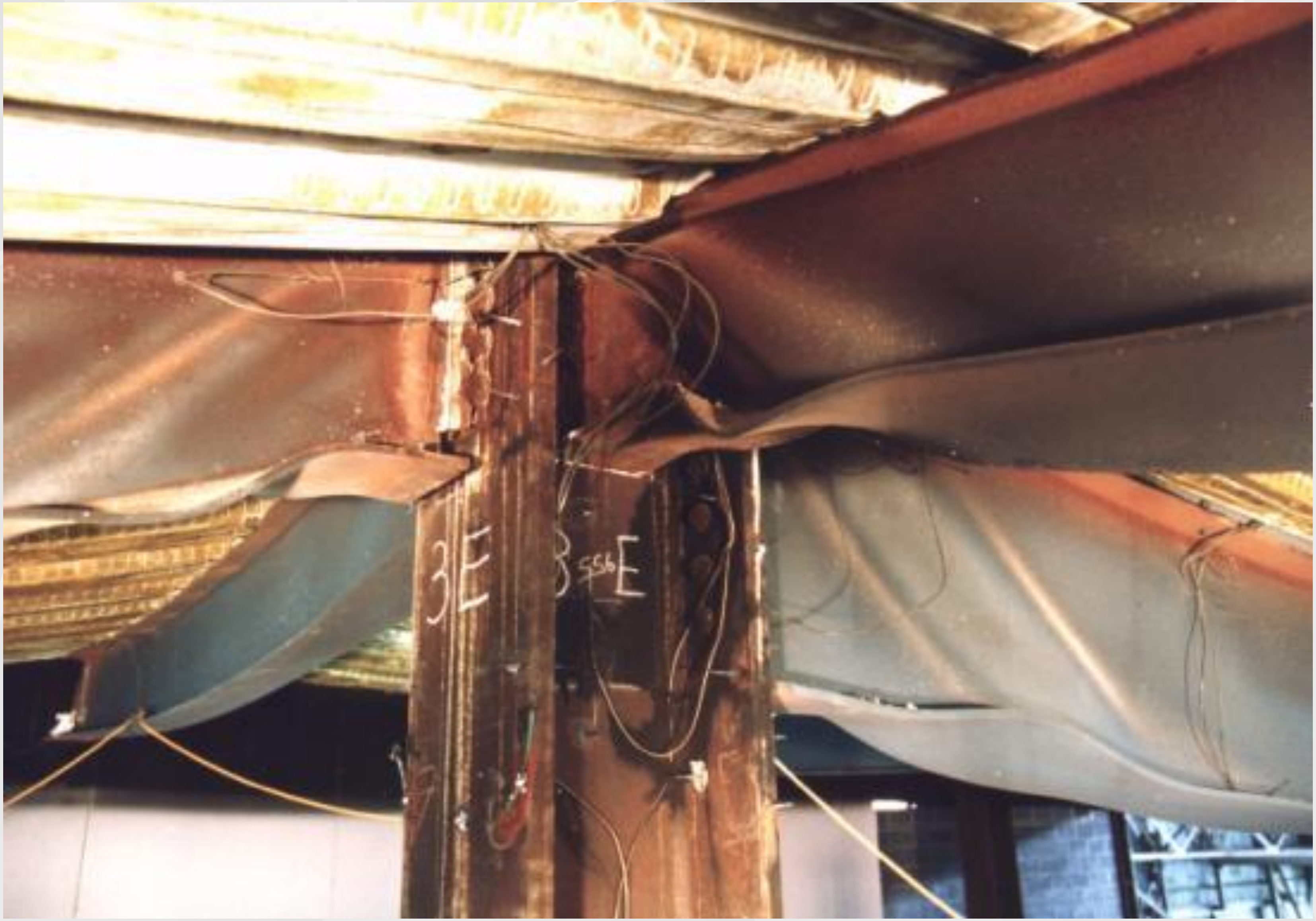
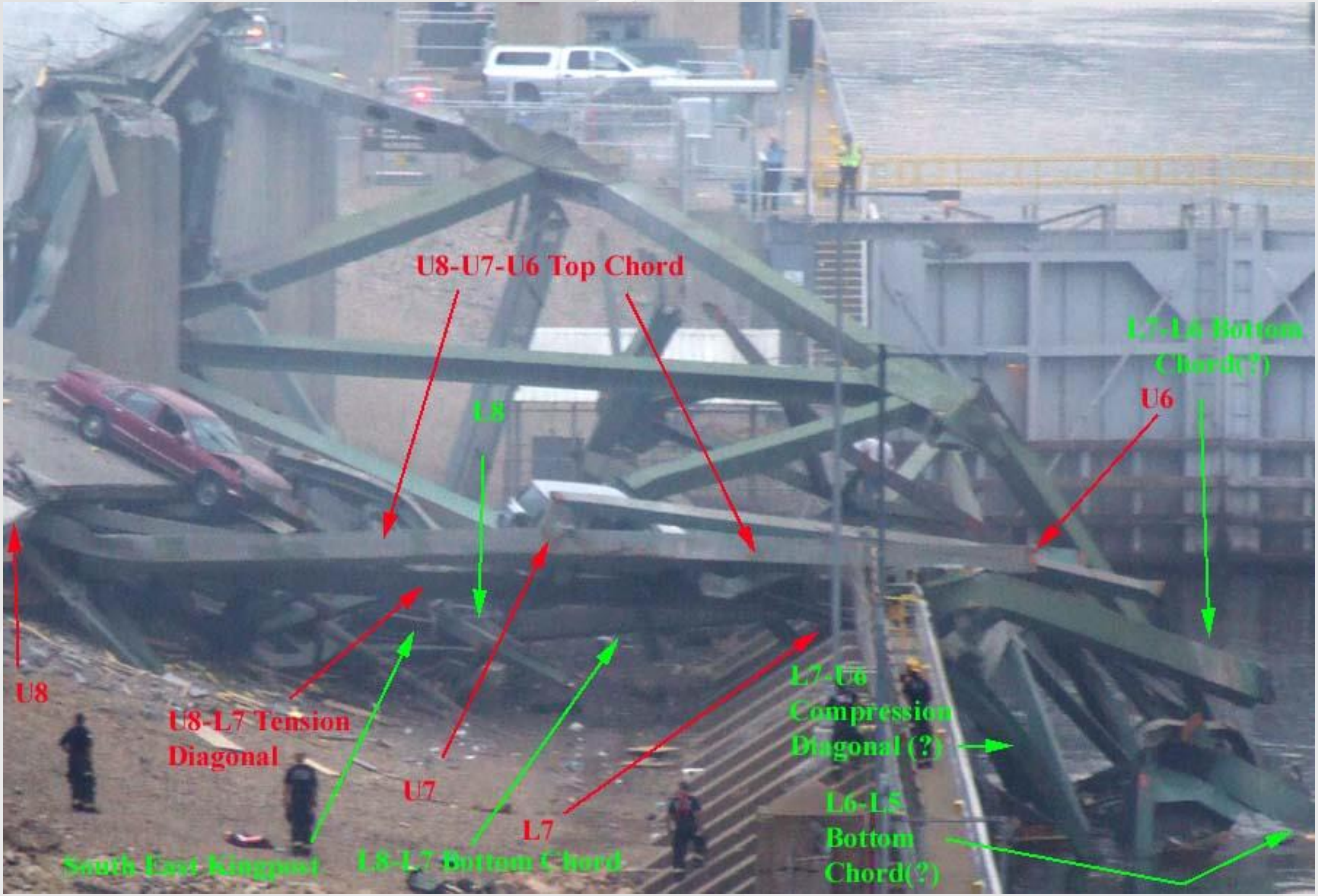


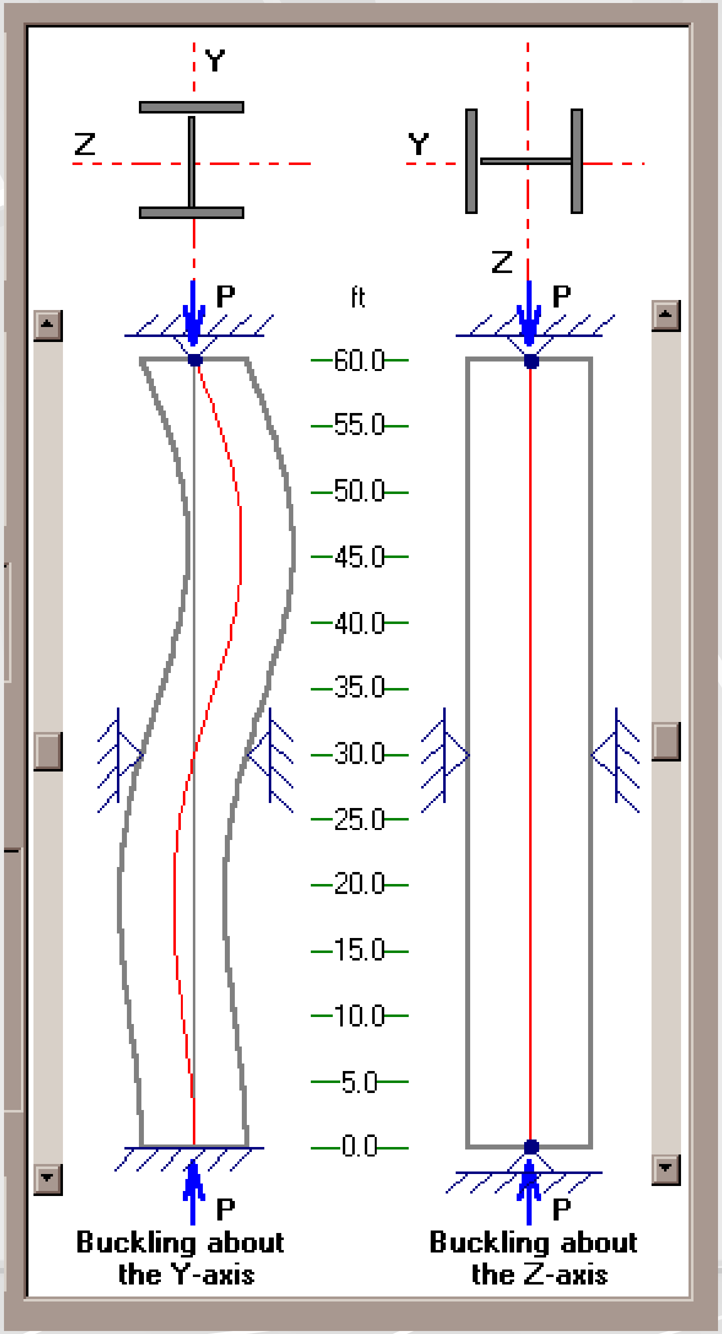
Figure 3.10. Axis of Buckling For Single Angle Section.




D2


3E


U8-U7-U6 Top Chord
L7-L6 Bottom Chord(?)
U6
L8
U8
U8-L7 Tension Diagonal
U7
L7
L7-U6 Compression Diagonal (?)
L6-L5 Bottom Chord(?)
South East Kingpost
L8-L7 Bottom Chord


Y
Z
Y
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60.0
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45.0
40.0
35.0
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25.0
20.0
15.0
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5.0
0.0
P
P
Buckling about the Y-axis
Buckling about the Z-axis

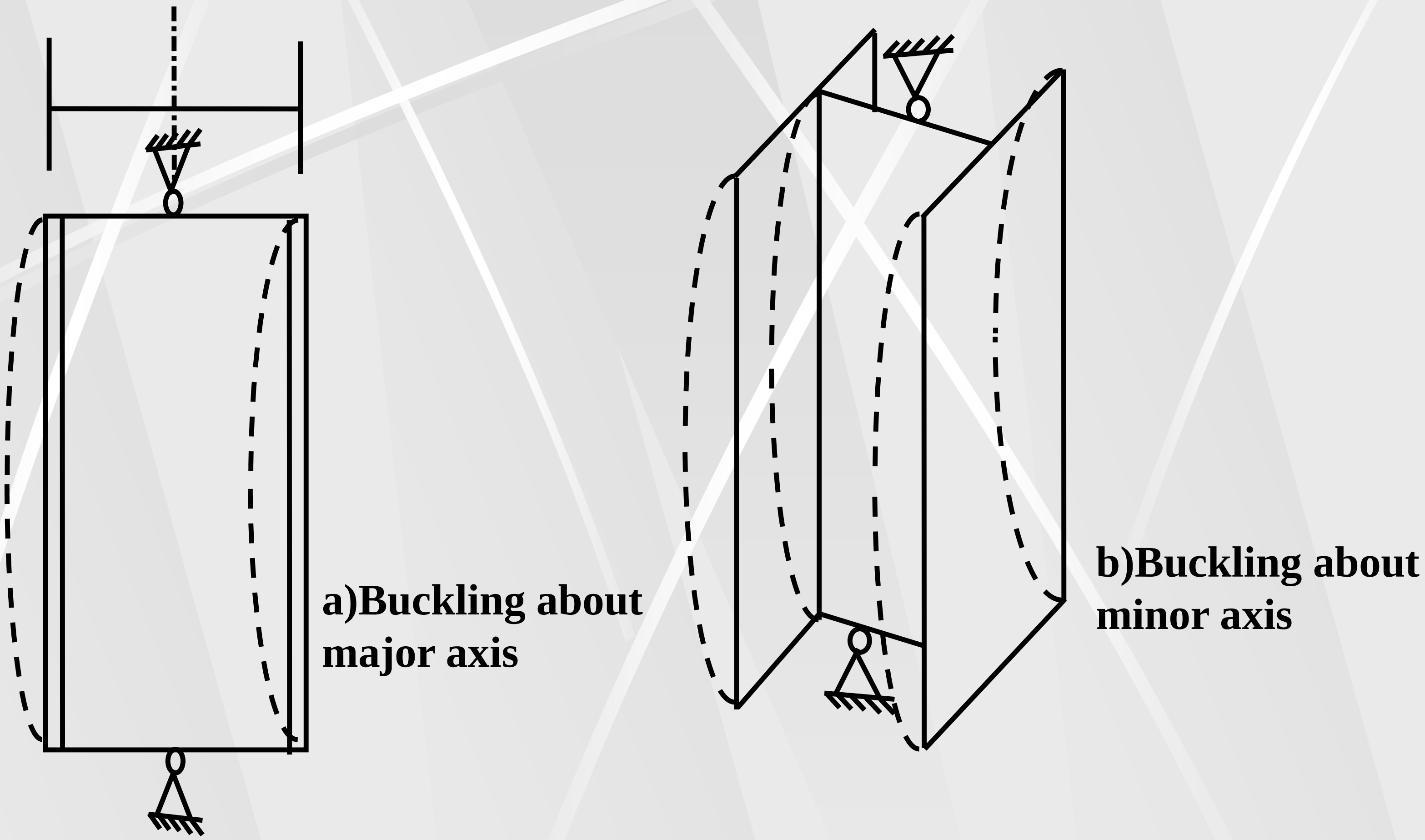
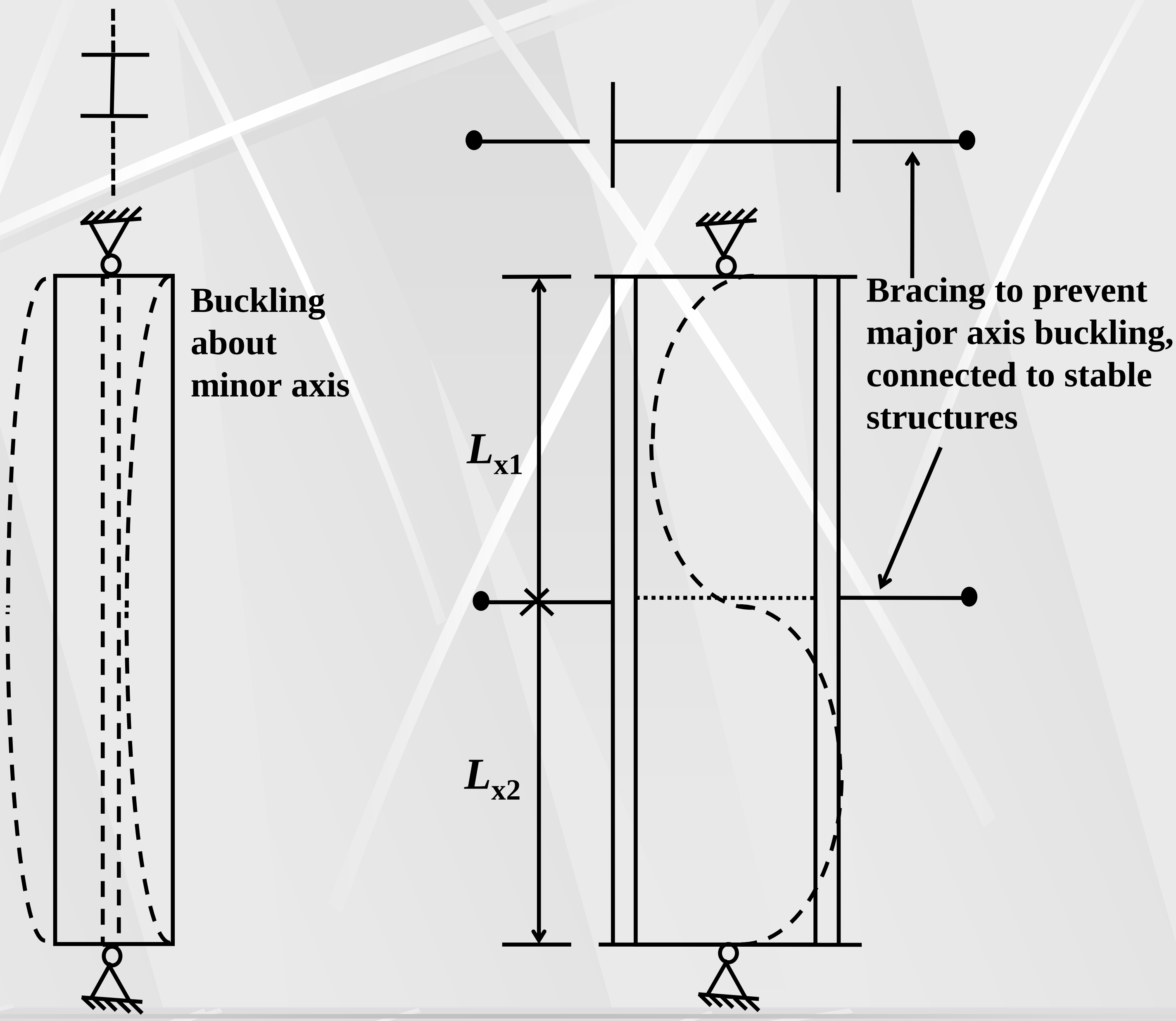
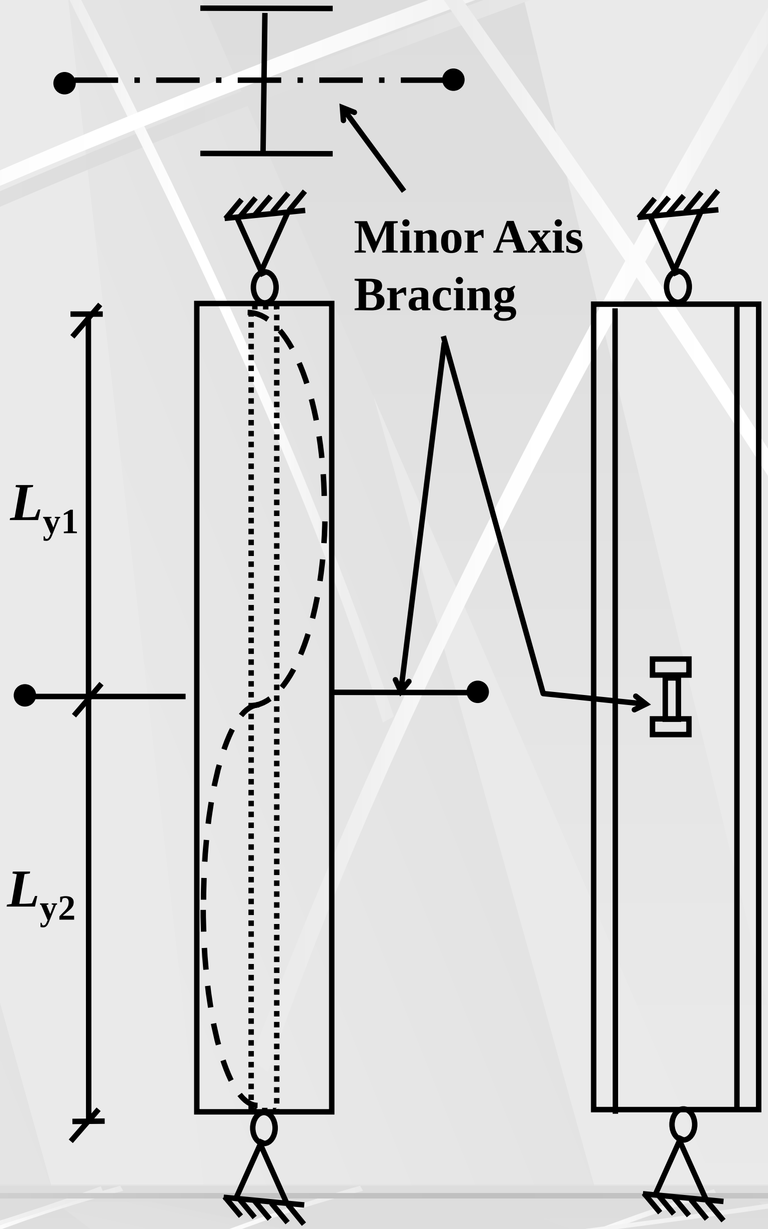


Figure 3.6- Buckling of a Column Without Intermediate Bracing


Buckling
about
minor axis
Bracing to prevent
major axis buckling,
connected to stable
structures
L_{x1}
L_{x2}


Minor Axis
Bracing
L_{y1}
L_{y2}

UNSUPPORTED LENGTH

It is the length of column between two consecutive supports or braces denoted by L_{ux} or L_{uy} in the x and y directions, respectively.

A different value of unsupported length may exist in different directions and must be used to calculate the corresponding slenderness ratios.

To calculate unsupported length of a column in a particular direction, only the corresponding supports and braces are to be considered neglecting the bracing preventing buckling in the other direction.

EFFECTIVE LENGTH OF COLUMN

The length of the column corresponding to one-half sine wave of the buckled shape or the length between two consecutive inflection points or supports after buckling is called the effective length.

BUCKLING OF STEEL COLUMNS

Buckling is the sudden lateral bending produced by axial loads due to initial imperfection, out-of-straightness, initial curvature, or bending produced by simultaneous bending moments.

Chances of buckling are directly related with the slenderness ratio ***KL/r*** and hence there are three parameters affecting buckling.

1. Effective length factor (***K***), which depends on the end conditions of the column.

2. Unbraced length of column (L_u), in strong direction or in weak direction, whichever gives more answer for ***KL/r***.

3. Radius of gyration (r), which may be r_x or r_y (strong and weak direction) for uniaxially or biaxially symmetrical cross-sections and least radius of gyration (r_z) for un-symmetrical cross-sections like angle sections.

The reason is that the rotation becomes free at this point and only the lateral movement is prevented.

EFFECTIVE LENGTH FACTOR (*K*)

This factor gives the ratio of length of half sine wave of deflected shape after buckling to full-unsupported length of column.

This depends upon the end conditions of the column and the fact that whether sidesway is permitted or not.

Greater the *K*-value, greater is the effective length and slenderness ratio and hence smaller is the buckling load.

K-value in case of no sidesway is between 0.5 and 1.0, whereas, in case of appreciable sidesway, it is greater than or equal to 1.0

$$\boldsymbol{L_e = K\,L_u}$$

SIDE SWAY

Any appreciable lateral or sideward movement of top of a vertical column relative to its bottom is called sidesway, sway or lateral drift.

If sidesway is possible, ***K*-value** increases by a greater degree and column buckles at a lesser load.

Sidesway in a frame takes place due to:-

a) Lengths of different columns are unequal.

b) When sections of columns have different cross-sectional properties.

c) Loads are un-symmetrical.

d) Lateral loads are acting.

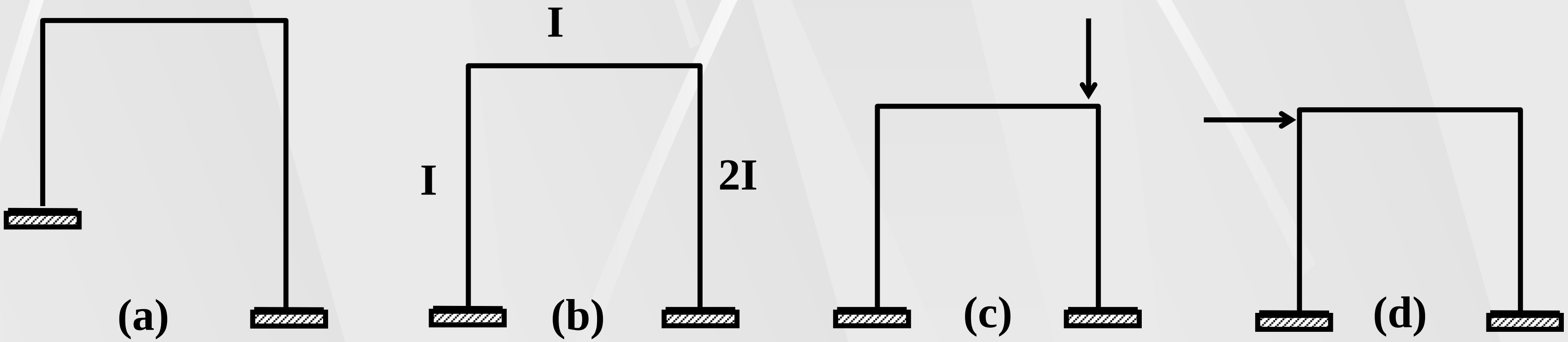


Figure 3.11. Causes of Sidesway in a Building Frame.

Sidesway may be prevented in a frame by:

a. Providing shear or partition walls.

b. Fixing the top of frame with adjoining rigid structures.

c. Provision of properly designed lift well or shear walls in a building, which may act like backbone of the structure reducing the lateral deflections.

Shear wall is a structural wall that resists shear forces resulting from the applied transverse loads in its own plane and it produces frame stability.

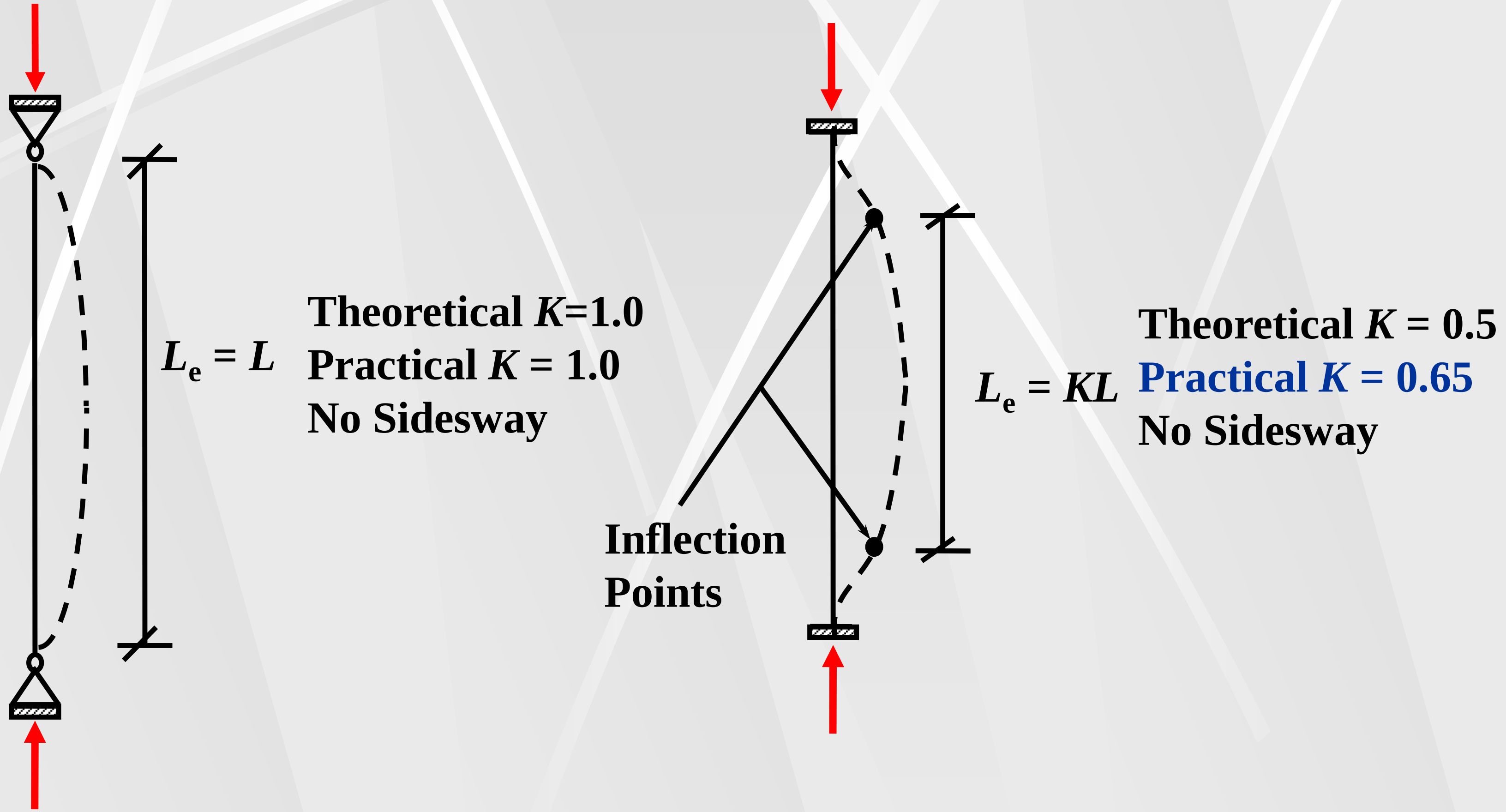
d. Provision of lateral bracing, which may be of following two types:

   i. Diagonal bracing, and

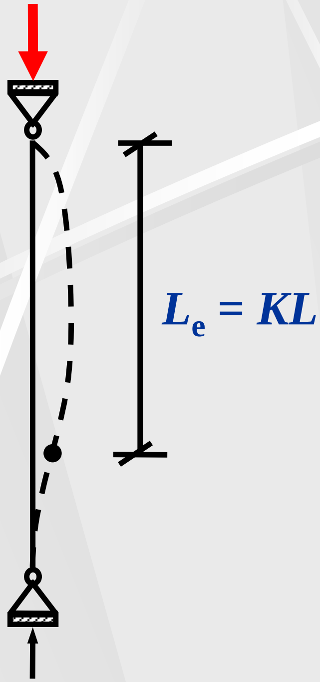
   ii. Longitudinal bracing.

Unbraced frame is defined as the one in which the resistance to lateral load is provided by the bending resistance of frame members and their connections without any additional bracing.

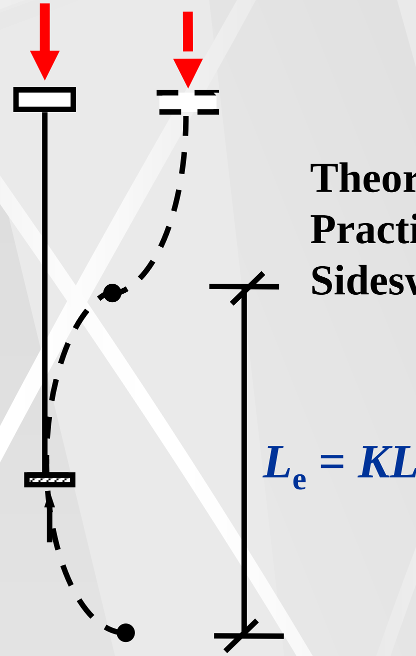
K-Factor for Columns Having Well Defined End Conditions



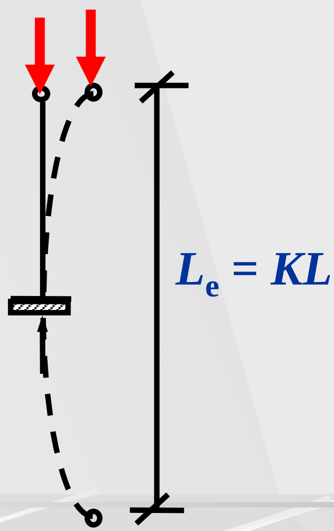
- Check the ***K*-valu**e for various end condition at **Page 103 on Refernce-1** (LRFD Steel Design Aids)



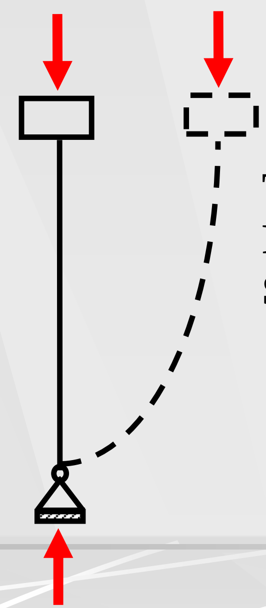
Theoretical K = 0.7
Practical K = 0.8
No Sidesway



Theoretical K=1.0
Practical K = 1.2
Sidesway Present



Theoretical K=2.0
Practical K = 2.10
Sidesway Present



Theoretical K=2.0
Practical K = 2.0
Sidesway Present

PARTIALLY RESTRAINED COLUMNS

Consider the example of column AB shown in Figure 3.13.

The ends are not free to rotate and are also not perfectly fixed. Instead these ends are partially fixed with the fixity determined by the ratio of relative flexural stiffness of columns meeting at a joint to the flexural stiffness of beams meeting at that joint.

This ratio is denoted by **G** or ψ and is determined for each end of the columns using following expression.

$$\psi \text{ or } G \text{ at each end} = \frac{\sum (EI/\ell) \text{ of columns}}{\sum (EI/\ell) \text{ of beams}}$$

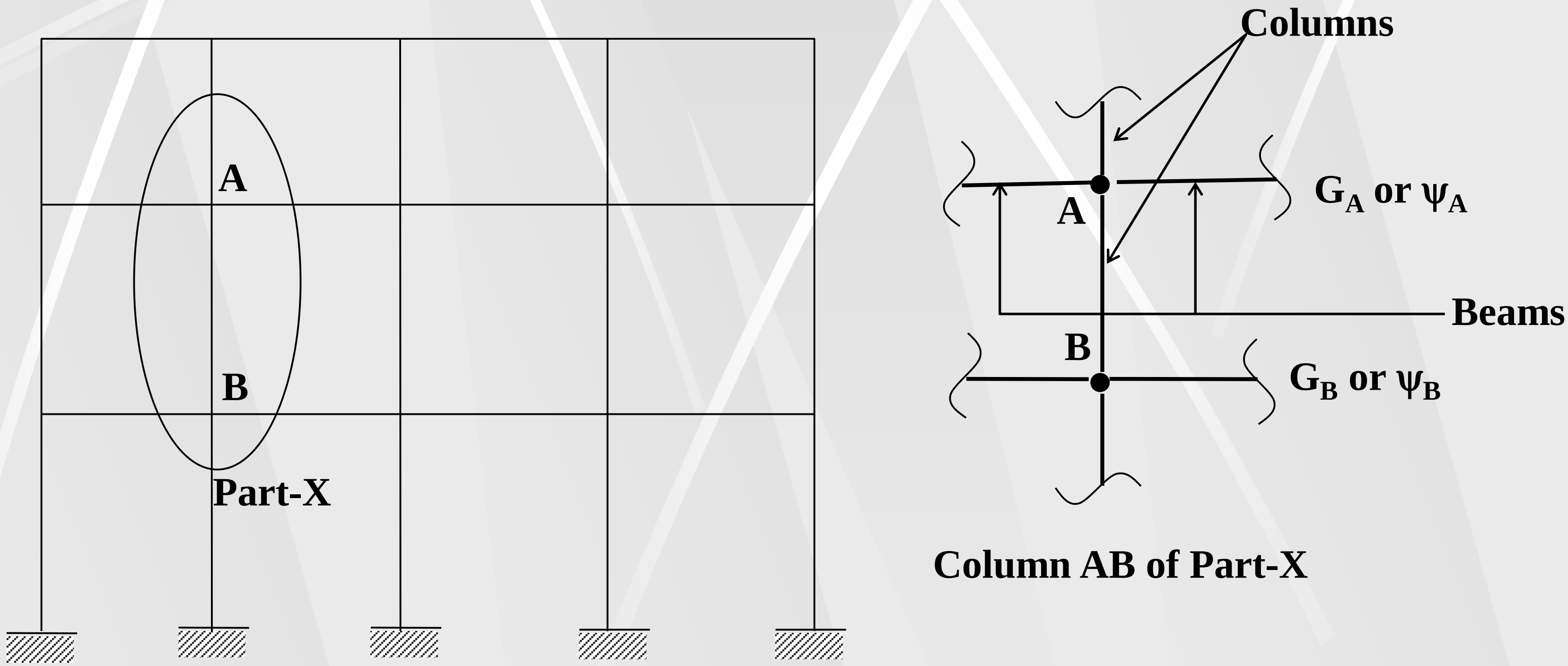


Figure 3.13. Partially restrained Columns

The **effective length factor** of column is determined using the charts given in the Figure 3.14 expression.
(check **Page- 104** on Reference - 1)

Note: *The charts does not give the actual values.*

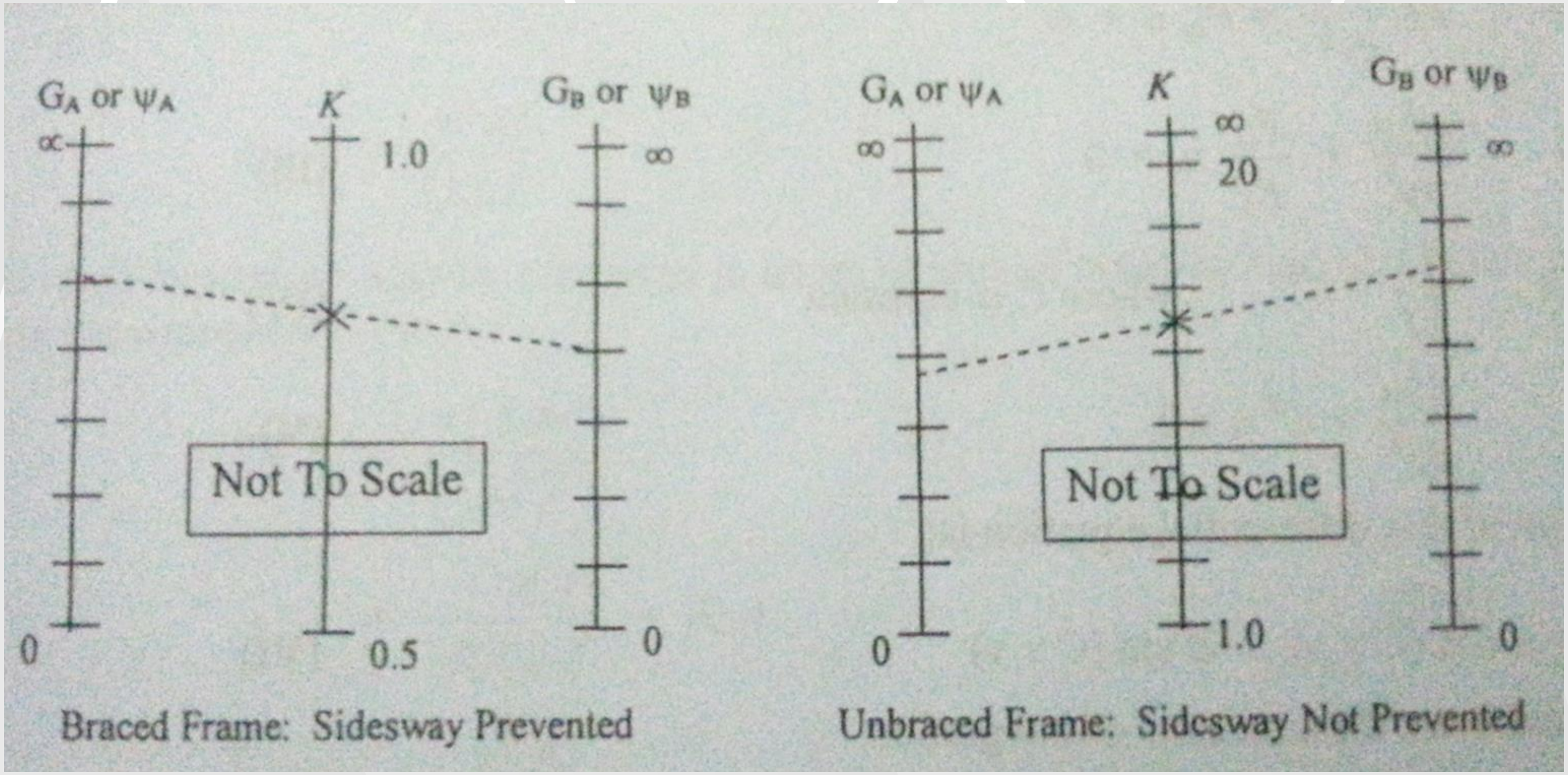


Figure 3.14. Determination of K-value for partially Restrained Columns

K-Values For Truss And Braced Frame Members

The effective length factor, ***K***, is considered equal to 1.0 for members of the trusses and braced frame columns.

In case the value is to be used less than one for frame columns, detailed buckling analysis is required to be carried out and bracing is to be designed accordingly.

ELASTIC BUCKLING LOAD FOR LONG COLUMNS

A column with pin connections on both ends is considered for the basic derivation, as shown in Figure 3.15.

The column has a length equal to L and is subjected to an axial compressive load, P.

Buckling of the column occurs at a critical compressive load, P_{cr}.

The lateral displacement for the buckled position at a height y from the base is u. The bending moment at this point D is:

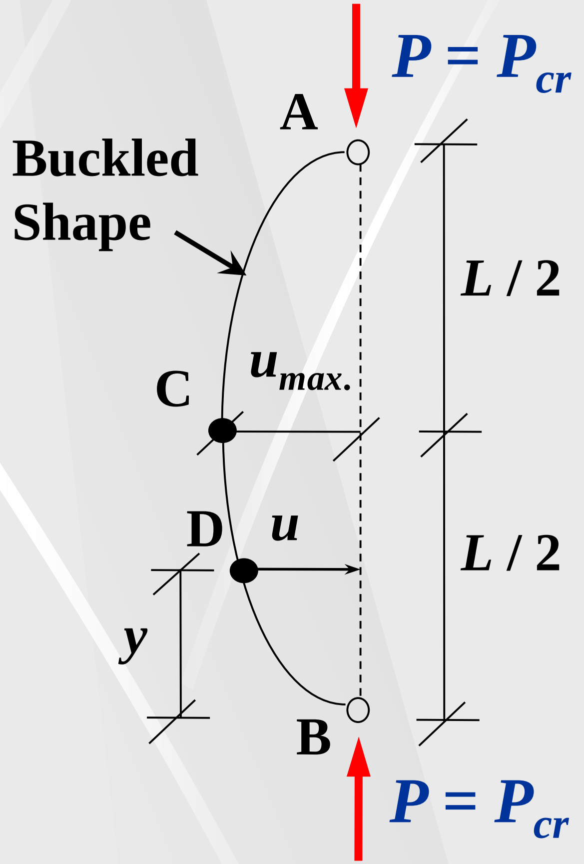


Figure 3.15. Buckled Elastic Curve for Long Columns

$$M = P_{cr} \times u \qquad \text{(I)}$$

This bending moment is function of the deflection unlike the double integration method of structural analysis where it is independent of deflection.

The equation of the elastic curve is given by the Euler-Bernoulli Equation, which is the same as that for a beam.

$$EI \frac{d^2u}{dy^2} = -M \qquad \text{(II)}$$

or $$EI \frac{d^2u}{dy^2} + P_{cr}\, u = 0$$

or $$\frac{d^2u}{dy^2} + \frac{P_{cr}}{EI}\, u = 0 \qquad \text{(III)}$$

Let $\dfrac{P_{cr}}{EI} = C^2$ where C is constant (IV)

$$\therefore \quad \frac{d^2u}{dy^2} + C^2 u = 0 \qquad \text{(V)}$$

The solution of this differential equation is:

$$u = A\cos(C \times y) + B\sin(C \times y) \qquad \text{(VI)}$$

where, A and B are the constants of integration.

Boundary Condition No. 1:

At $y = 0$, $u = 0$

$0 = A\cos(0°) + B\sin(0°) \quad \Rightarrow \quad A = 0$

$$\therefore \quad u = B \sin (C \times y) \qquad \text{(VII)}$$

Boundary Condition No. 2:

At $y = L, \quad u = 0$

From Eq. VII: $\quad 0 = B \sin (C\,L)$

$$\Rightarrow \quad \text{Either } B = 0 \text{ or } \sin (C\,L) = 0 \qquad \text{(VIII)}$$

If $B = 0$, the equation becomes $u = 0$, giving un-deflected condition. Only the second alternate is left for the buckled case.

$$\sin\ (C\,L) = \sin\left(\sqrt{\frac{P_{cr}}{EI}}\,L\right) = 0 \qquad \text{(IX)}$$

$\sin\theta = 0$ for $\theta = 0, \pi, 2\pi, 3\pi, \ldots$ (radians)

Or $\boldsymbol{n\pi}$ where $\boldsymbol{n} = 0, 1, 2, \ldots$ (X)

Hence, from Eq. IX: $\sqrt{\frac{P_{cr}}{EI}}L = n\pi$

$$P_{cr} = \frac{n^2 \pi^2 EI}{L^2} \qquad \text{(XI)}$$

The smallest value of P_{cr} is for $n = 1$, and is given below:

$$P_{cr} = \frac{\pi^2 EI}{L^2} \qquad \text{(XII)}$$

For other columns with different end conditions, we have to replace L by the effective length, $L_e = K L$.

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} \qquad \text{(XIII)}$$

$$P_{cr} = \frac{\pi^2 E A r^2}{(KL)^2}$$

$$P_{cr} = \frac{\pi^2 E A}{(K L/r)^2} = F_e A \qquad \text{(XIV)}$$

and $$\boldsymbol{F}_{\mathbf{e}} = \frac{\pi^2 E}{(K L/r)^2} \qquad \text{(XV)}$$

TYPES OF COLUMNS DEPENDING ON BUCKLING BEHAVIOUR

Elastic Critical Buckling Stress

The elastic critical buckling stress is defined as under:

F_e = Elastic critical buckling (Euler) stress

$$= \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2}$$

The critical slenderness ratio dividing the expected elastic and the inelastic buckling is denoted by R_c and is given below:

$$R_c = 4.71\sqrt{\frac{E}{F_y}} \approx \mathbf{133} \text{ for A36 steel}$$

Long Columns

In long columns, elastic buckling is produced and the deformations are recovered upon removal of the load.

Further, the stresses produced due to elastic buckling remains below the proportional limit.

The Euler formula is used to find strength of long columns.

Long columns are defined as those columns for which the slenderness ratio is greater than the critical slenderness ratio, R_c.

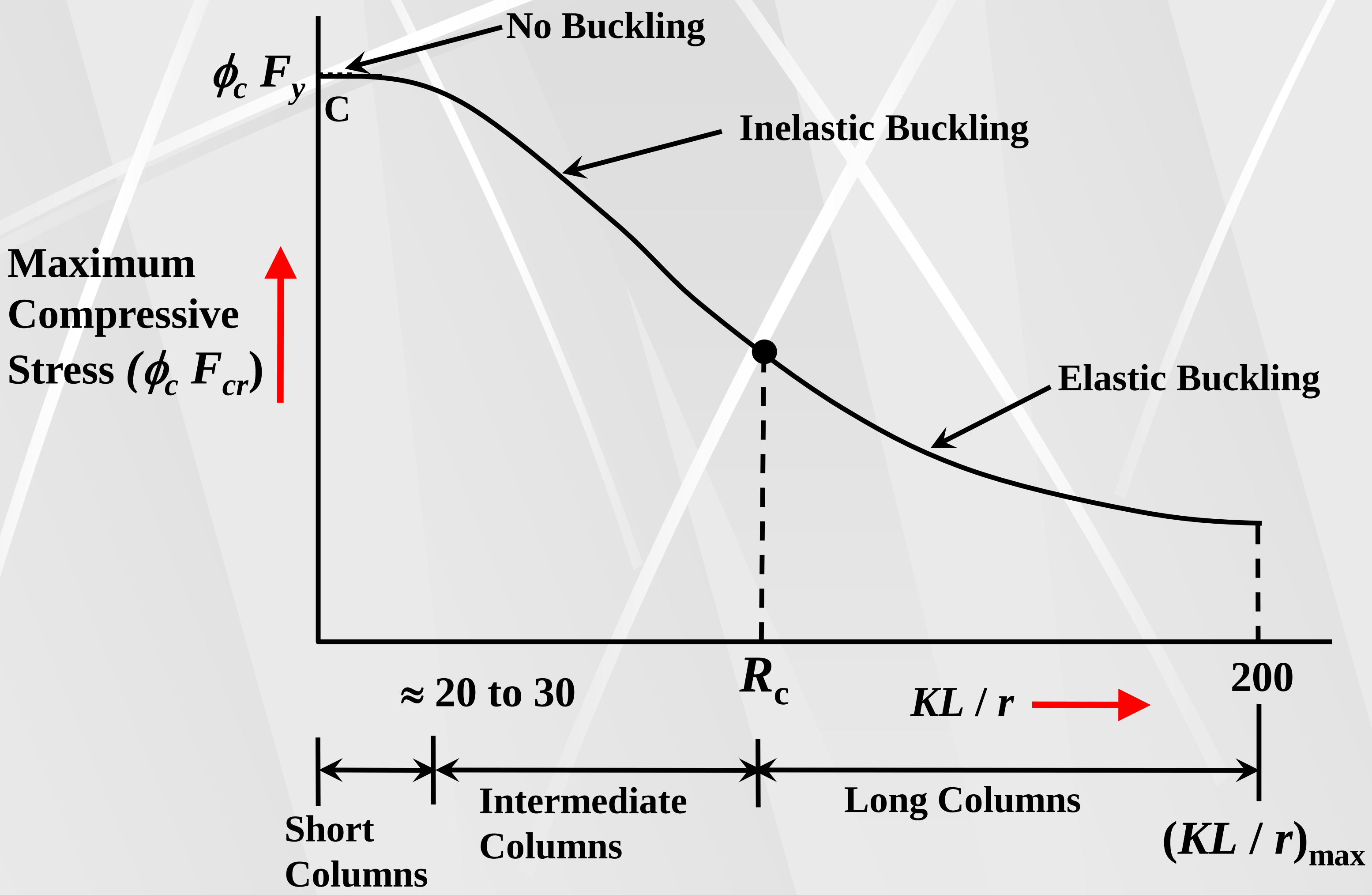
Short Columns

For very short columns, when the ***slenderness ratio is less than 20 to 30***, the failure stress will equal the yield stress and no buckling occurs. In practice, very few columns meet this condition.

For design, these are considered with the intermediate columns subjected to the condition that failure stress should not exceed the yield stress.

Intermediate Columns

Intermediate columns buckle at a relatively higher load (more strength) as compared with long columns.



No Buckling
$\phi_c\ F_y$
C
Inelastic Buckling
Maximum Compressive Stress ($\phi_c\ F_{cr}$)
Elastic Buckling
≈ 20 to 30
R_c
$KL\ /\ r$
200
Short Columns
Intermediate Columns
Long Columns
$(KL\ /\ r)_{max}$

The buckling is inelastic meaning that part of the section becomes inelastic after bending due to buckling.

The columns having slenderness ratio lesser than the critical slenderness ratio (R_c) are considered as intermediate columns, as shown in Figure 3.16.

COLUMN STRENGTH FORMULAS

The design compressive strength $(\phi_c P_n)$ and the allowable compressive strength (P_n / Ω_c) of compression members, whose elements do not exhibit elastic local instability (only compact and non-compact sections), are given below:

ϕ_c = 0.90 (LRFD) : $P_n = F_{cr} A_g$

Ω_c = 1.67 (ASD) : $P_n = F_{cr} A_g$

F_{cr} = critical or ultimate compressive strength based on the limit state of flexural buckling determined as under:

Elastic Buckling

When $KL / r > R_c$ or $F_e < 0.44F_y$

$$F_{cr} = 0.877\, F_e \qquad \text{(AISC Formula E3-2)}$$

where F_e is the Euler's buckling stress and **0.877** is a factor to estimate the ***effect of out-of-straightness*** of about **1/1500**.

Inelastic Buckling and No Buckling

When $KL/r \leq R_c$ or $F_e > 0.44F_y$

$$F_{cr} = \left(0.658^{\frac{F_y}{F_e}}\right)F_y \qquad \text{(AISC Formula E3-3)}$$

TYPES OF COLUMN SECTIONS FOR LOCAL STABILITY

Compact Sections

A compact section is one that has sufficiently thick elements so that it is capable of developing a fully plastic stress distribution before buckling.

The term plastic means stressed throughout to the yield stress.

For a compression member to be classified as compact, its flanges must be continuously connected to its web or webs and the width thickness ratios (***b/t***) of its compression elements may not be greater than the limiting ratios λ_p give in AISC Table B4.1 and reproduced in Table 3.1.

Element	λ_p	λ_p **For A36**
Un-stiffened:	Defined only for flexure	–
Stiffened: Flanges of hollow sections subjected to compression.	$1.12\sqrt{\dfrac{E}{F_y}}$	**31.8**

Non-Compact Sections

A non-compact section is one for which the yield stress can be reached in some but not all of its compression elements just at the buckling stage.

It is not capable of reaching a fully plastic stress distribution.

In AISC Table B4.1, the non-compact sections are defined as those sections which have width-thickness ratios (***b/t***) greater than λ_p but not greater than λ_r.

Values of limiting ***b/t*** ratios **(λ_r)** are given in Table 3.2.

Element	Width-Thickness Ratio	λ_r	λ_r For A36 Steel
<u>Unstiffened</u> **1. Flanges of I-shaped sections in pure compression, plates projecting from compression elements, outstanding legs of pairs of angles in continuous contact, and flanges of channels in pure compression.**	$\frac{b}{t}$	$0.56\sqrt{\frac{E}{F_y}}$	**15.9**
2. Legs of single angle struts, legs of double angle struts with separators and other un-stiffened elements supported along one edge.	$\frac{b}{t}$	$0.45\sqrt{\frac{E}{F_y}}$	**12.8**
3. Stems of tees.	$\frac{d}{t}$	$0.75\sqrt{\frac{E}{F_y}}$	**21.3**
4. Flanges of built-up I-sections with projecting plates or angles.	$\frac{b}{t}$	$0.64\sqrt{\frac{k_c E}{F_y}}$	$18.1\sqrt{k_c}$

Element	Width-Thickness Ratio	λ_r	λ_r For A36 Steel
stiffened **1. Flanges of rectangular hollow sections of uniform thickness used for uniform compression.**	$\frac{b}{t}$	$1.40\sqrt{\frac{E}{F_y}}$	**39.7**
2. Flexure in webs of doubly symmetric I-shaped sections and channels.	$\frac{h}{t_w}$	$5.70\sqrt{\frac{E}{F_y}}$	**161.8**
3. Uniform compression in webs of doubly symmetric I-shaped sections and uniform compression in all other stiffened elements.	$\frac{b}{t}$	$1.49\sqrt{\frac{E}{F_y}}$	**42.3**
4. Circular hollow sections in axial compression.	$d\,/\,t$	$0.11\,(E\,/\,F_y)$	**88.6**

Slender Compression Sections

These sections consist of elements having width-thickness ratios greater than λ_r and will buckle elastically before the yield stress is reached in any part of the section.

A special design procedure for slender compression sections is provided in Section E7 of the AISC Specification.

However, it will not be covered in detail here.

Width Of Un-stiffened Elements

For un-stiffened elements, which are supported along only one edge parallel to the direction of the compression force, the width shall be taken as follows:

a. For flanges of I-shaped members and tees, the width ***b*** is half the full nominal width **(b_f/2)**.

b. For legs of angles, the width *b* is the longer leg dimension.

c. For flanges of channels and zees, the width *b* is the full nominal dimension **(b_f)**.

d. For plates, the width ***b*** is the distance from the free edge to the first row of fasteners or line of welds.

e. For stems of tees, ***d*** is taken as the full nominal depth.

Width Of Stiffened Elements

a. For webs of rolled or formed sections, h is the clear distance between the flanges less the fillet or corner radius at each flange and h_c is twice the distance from the centroidal axis to the inside face of the compression flange less the fillet or corner radius.

b. For webs of built-up sections,

h is the clear distance between the inner lines of fasteners on the web or the clear distance between flanges when welds are used.

h_c is twice the distance from the centroidal axis to the nearest line of fasteners at the compression flange or the inside face of the compression flange when welds are used, and

h_p is twice the distance from the plastic neutral axis to the nearest line of fasteners at the compression flange or the inside face of the compression flange when welds are used.