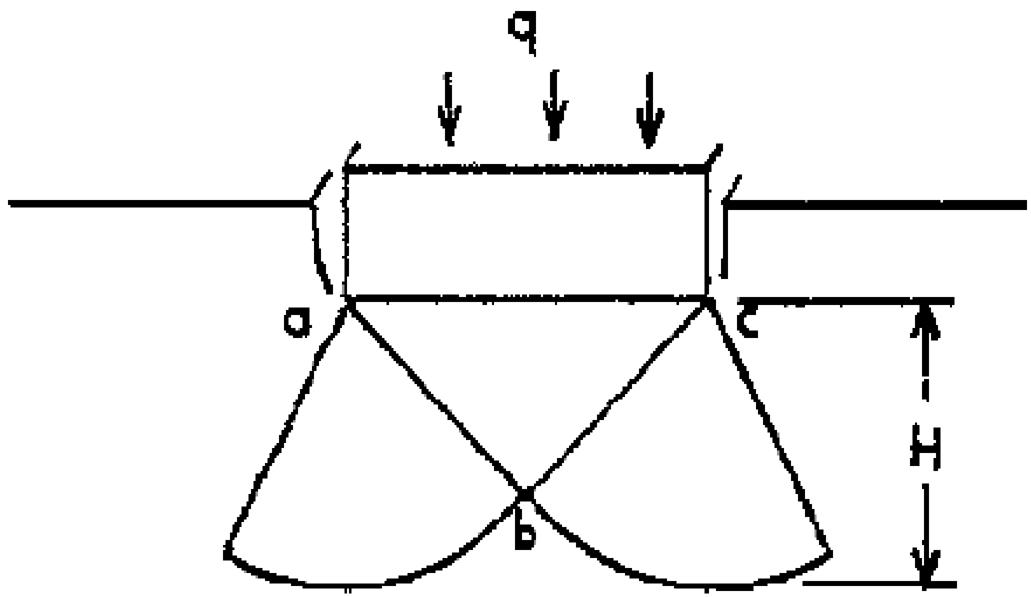
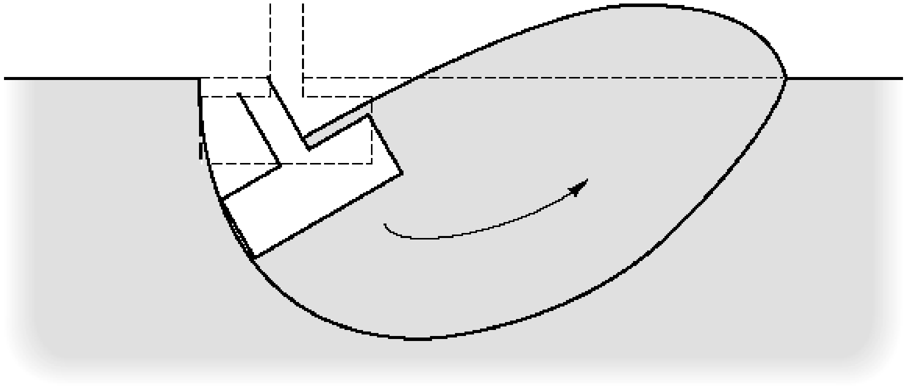
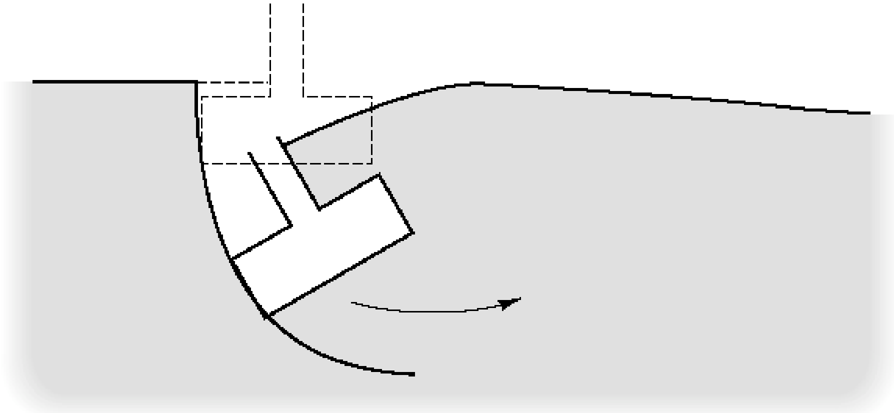
ENCE 461
Foundation Analysis and Design



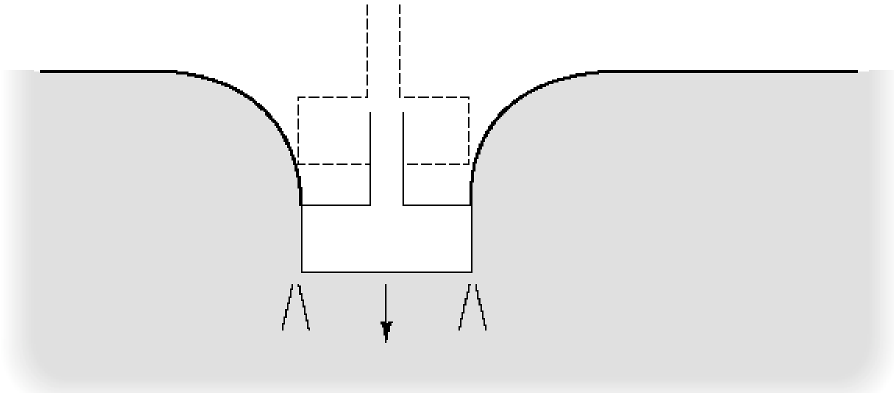
Bearing Capacity of Shallow Foundations



(a) General Shear Failure



(b)



(c)

Bearing Capacity Failure

a) General Shear Failure

Most common type of shear failure; occurs in strong soils and rocks

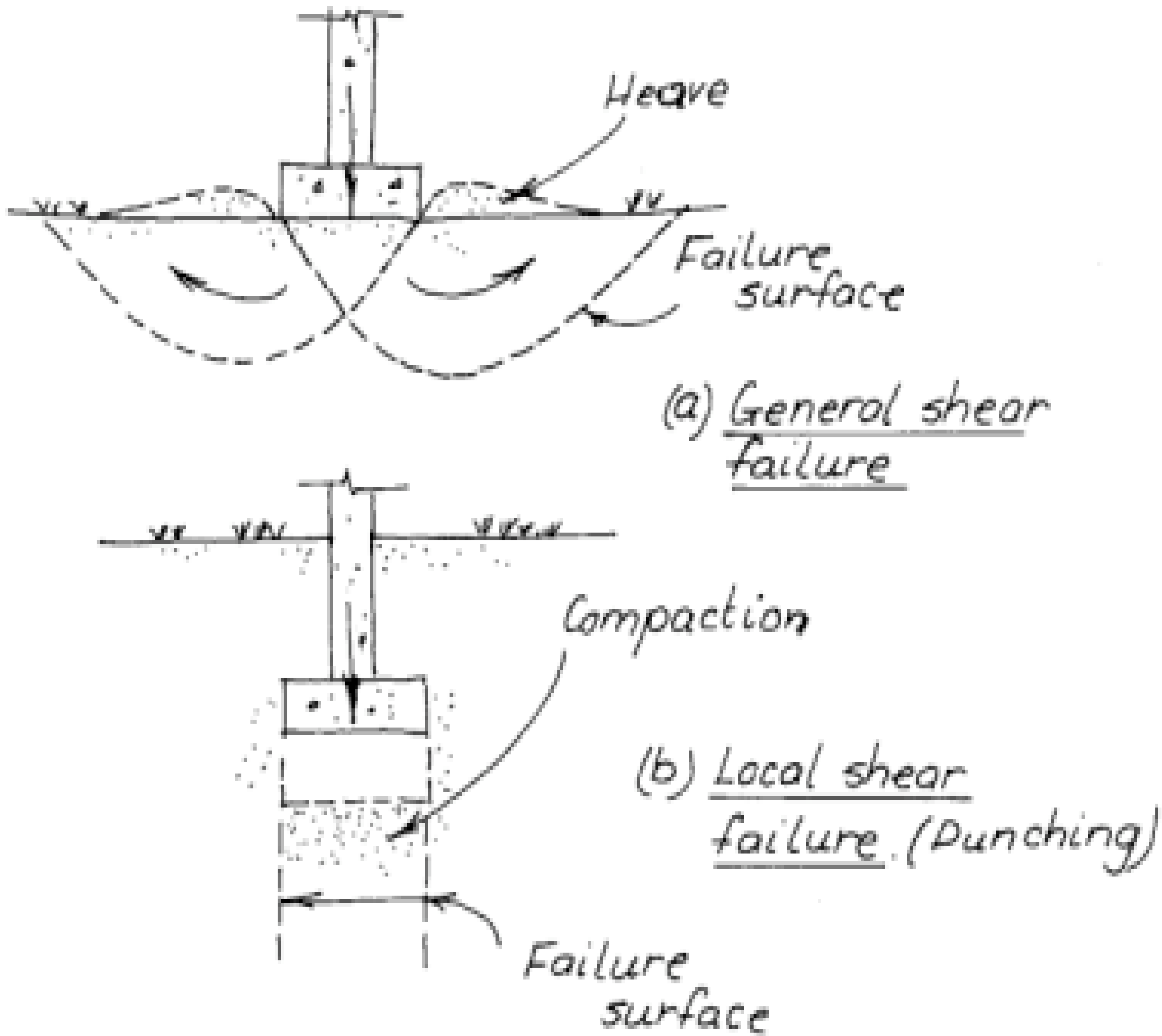
b) Local Shear Failure

Intermediate between general and punching shear failure

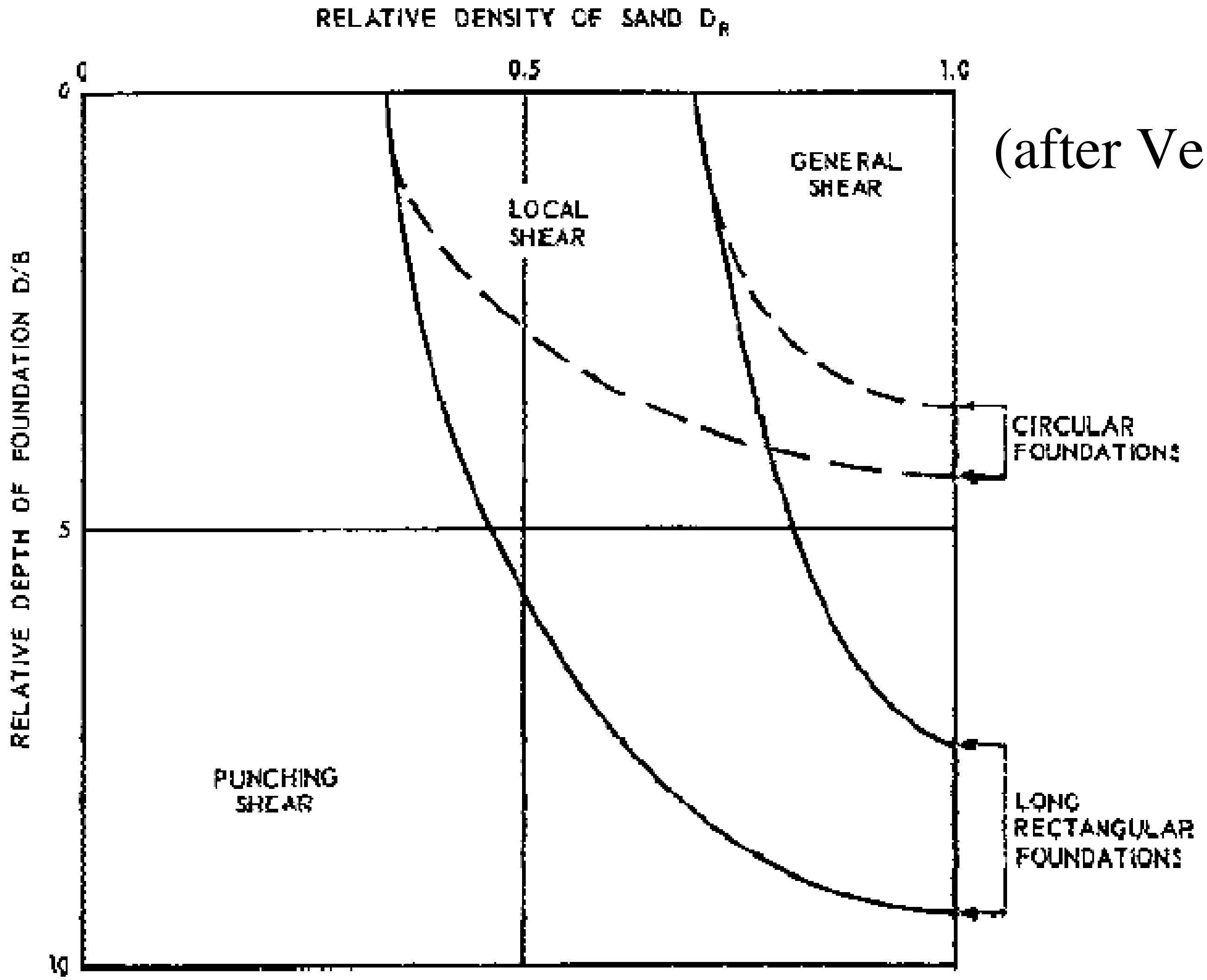
c) Punching Shear Failure

Occurs in very loose sands and weak clays

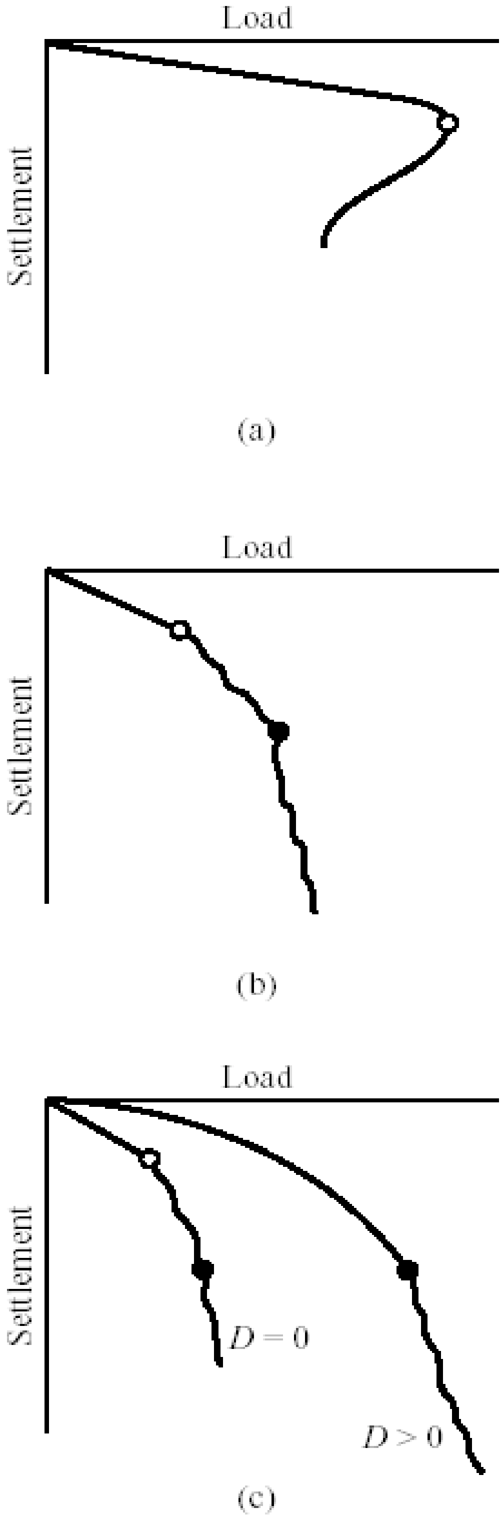
General and Punching Shear Failure



Soil Conditions and Bearing Capacity Failure



(after Vesić (1973))



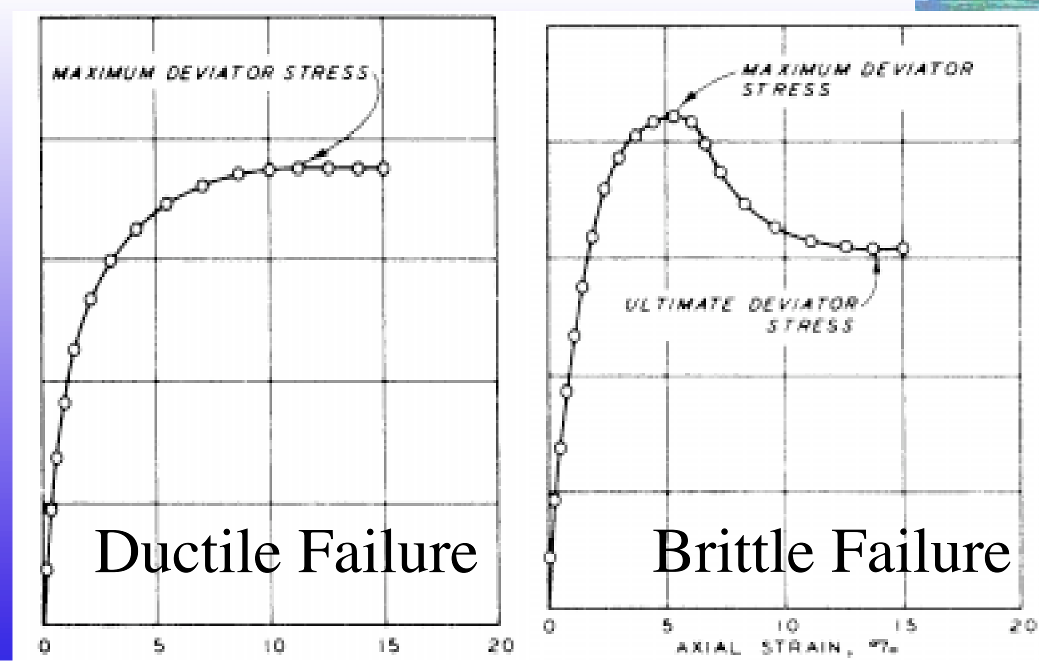
Load Displacement Curves

(after Vesić (1973)

a) General Shear Failure

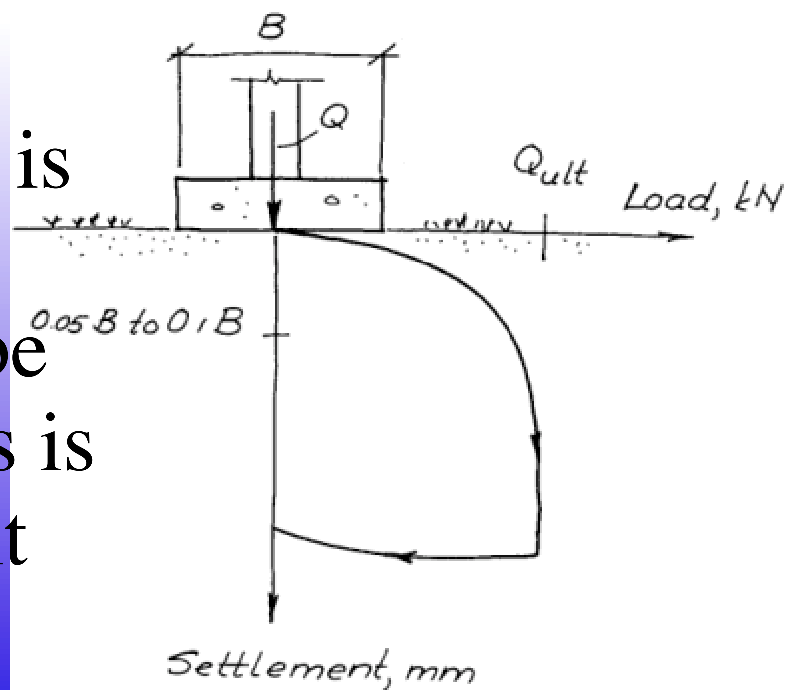
b) Local Shear Failure

c) Punching Shear Failure



Comments on Shear Failure

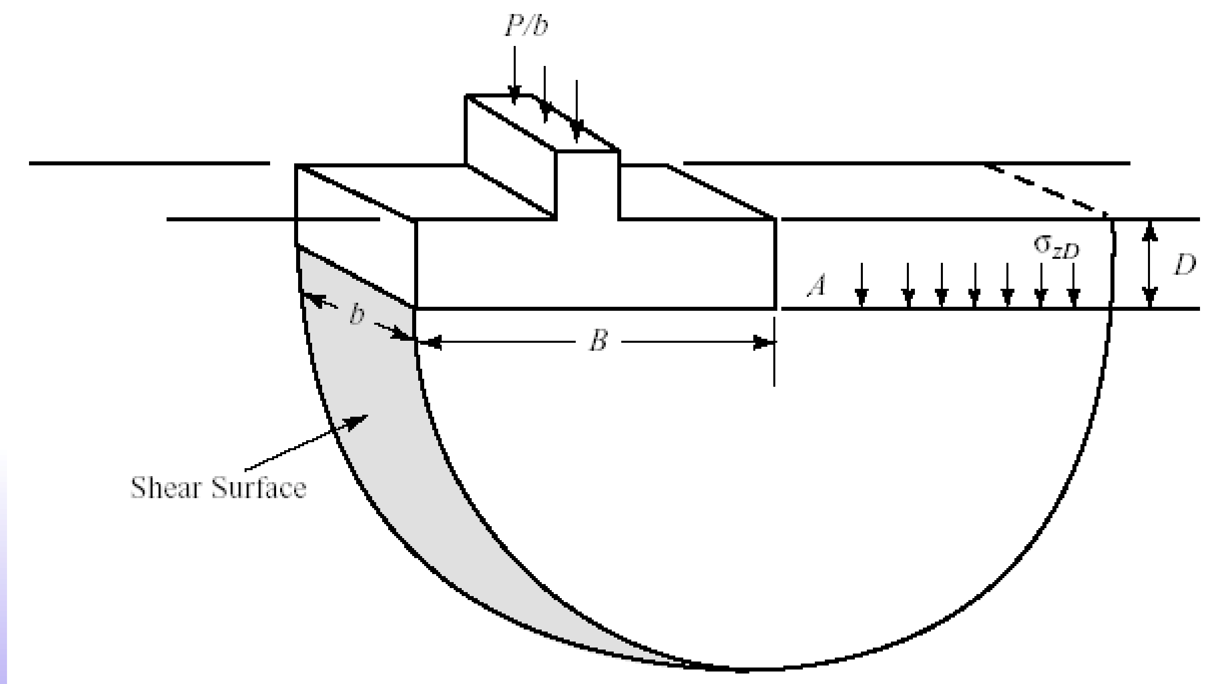
- Usually only necessary to analyse general shear failure
- Local and punching shear failure can usually be anticipated by settlement analysis
- Failure in shallow foundations is generally settlement failure; bearing capacity failure must be analysed, but in practical terms is usually secondary to settlement analysis



Development of Bearing Capacity Theory

- Application of limit equilibrium methods first done by Prandtl on the punching of thick masses of metal
- Prandtl's methods adapted by Terzaghi to bearing capacity failure of shallow foundations
- Vesić and others improved on Terzaghi's original theory and added other factors for a more complete analysis

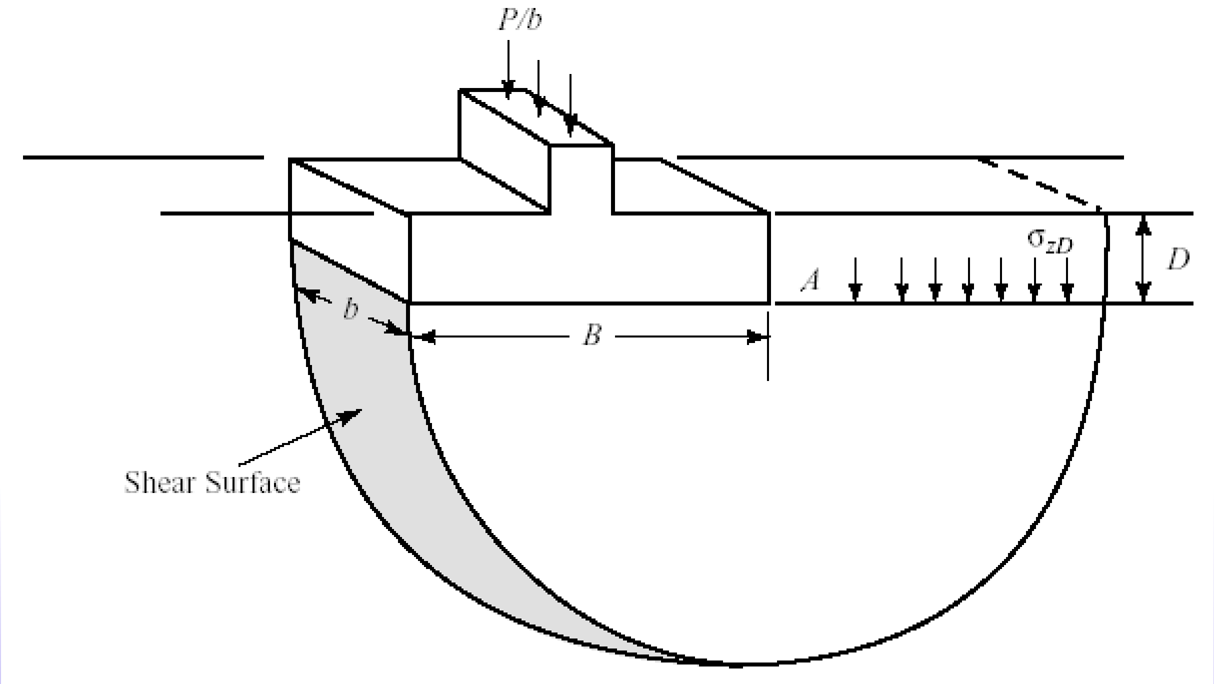
Sample Limit Equilibrium Method



$$\sum M_a = q_{ult} Bb \times \frac{B}{2} - s_{uBb} \times B - \sigma_{zD} Bb \times \frac{B}{2} = 0$$

Assume: No soil strength due to internal friction, shear strength above foundation base neglected

Sample Limit Equilibrium Method



$$q_{ult} = 2\pi s_u + \sigma_{zD}$$

$$N_c = 2\pi \approx 6.28$$

$$q_{ult} = N_c s_u + \sigma_{zD}$$

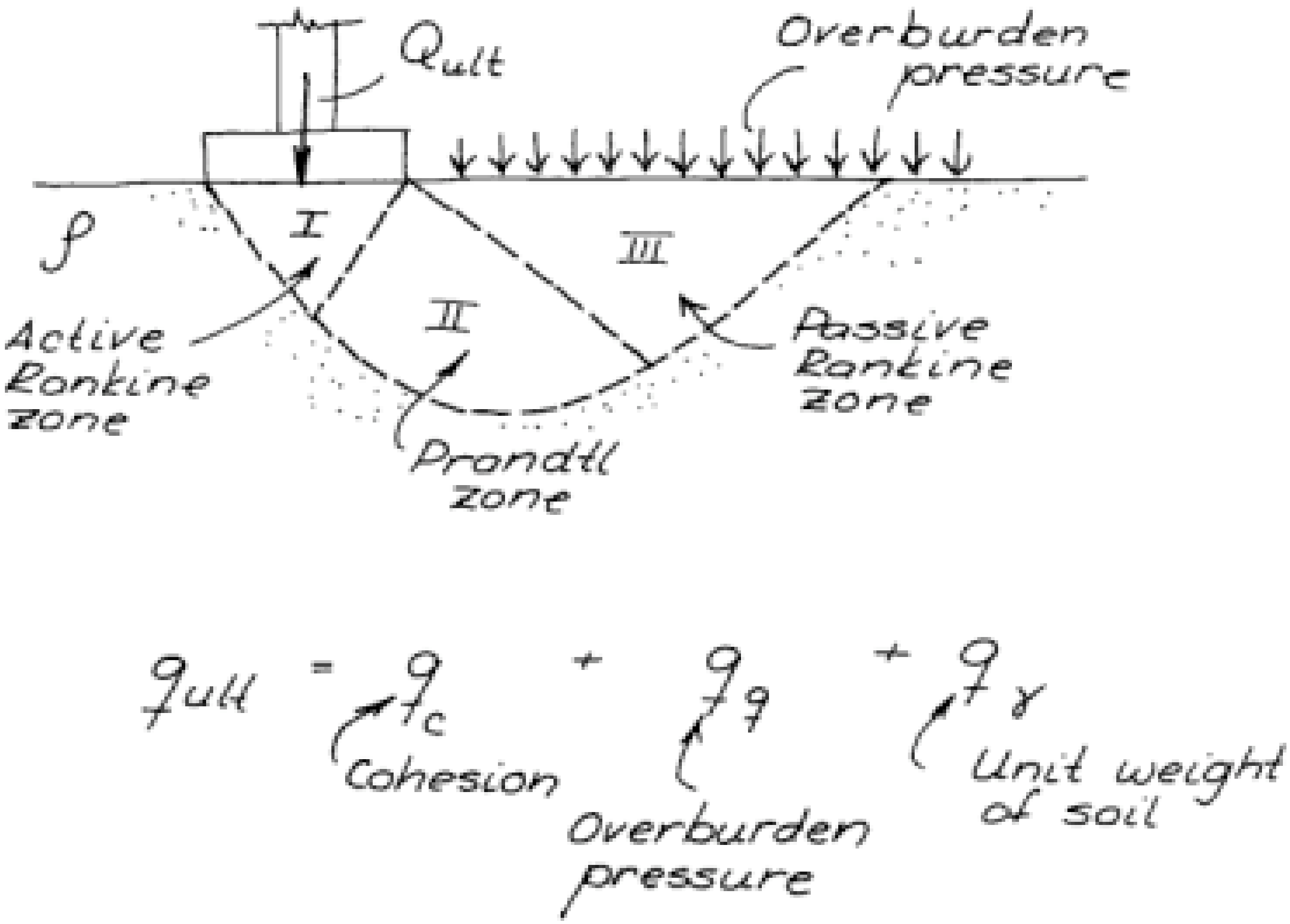
Assumptions for Terzaghi's Method

- Depth of foundation is less than or equal to its width
- No sliding occurs between foundation and soil (rough foundation)
- Soil beneath foundation is homogeneous semi-infinite mass
- Mohr-Coulomb model for soil
- General shear failure mode is the governing mode (but not the only mode)

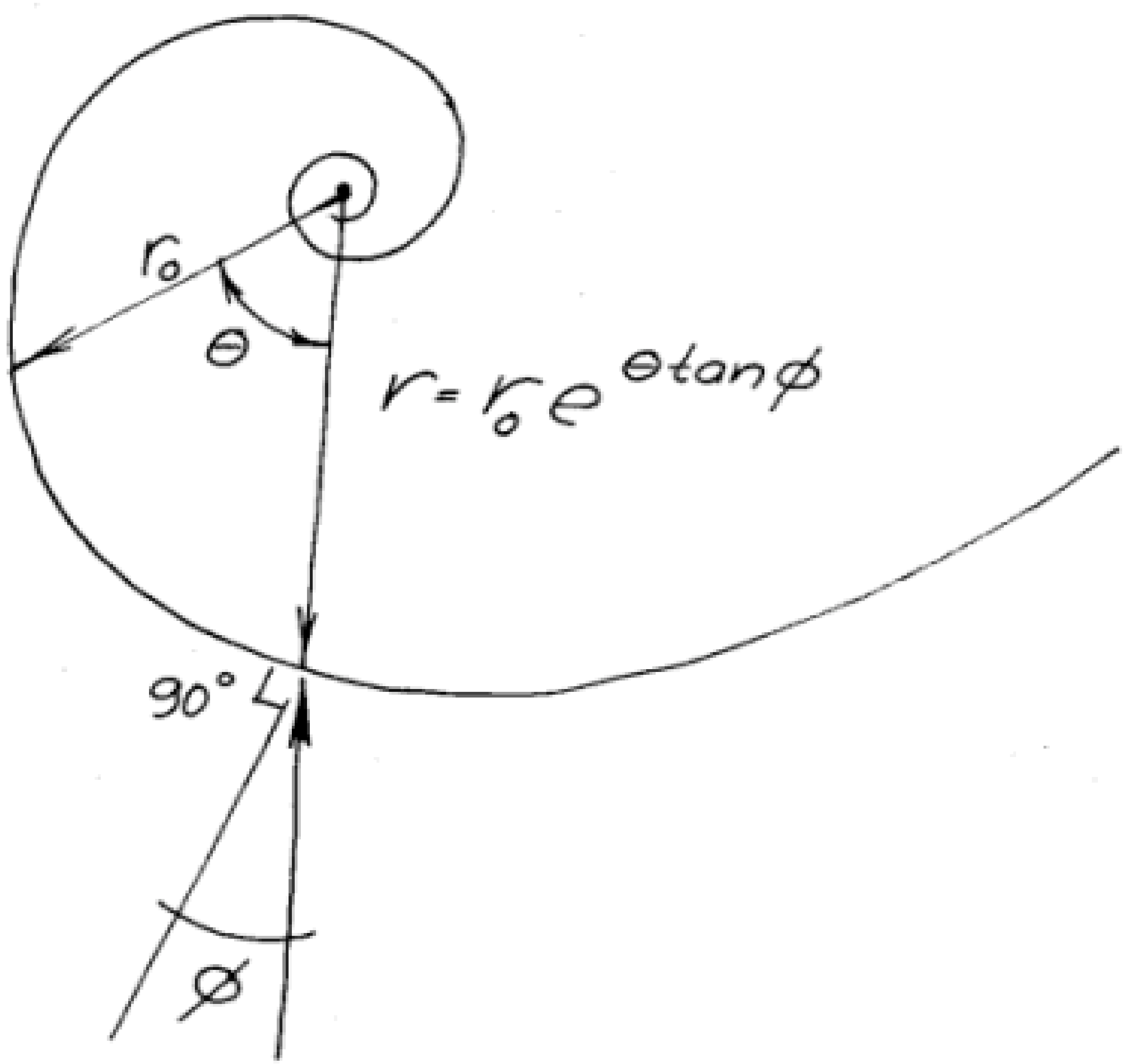
Assumptions for Terzaghi's Method

- No soil consolidation occurs
- Foundation is very rigid relative to the soil
- Soil above bottom of foundation has no shear strength; is only a surcharge load against the overturning load
- Applied load is compressive and applied vertically to the centroid of the foundation
- No applied moments present

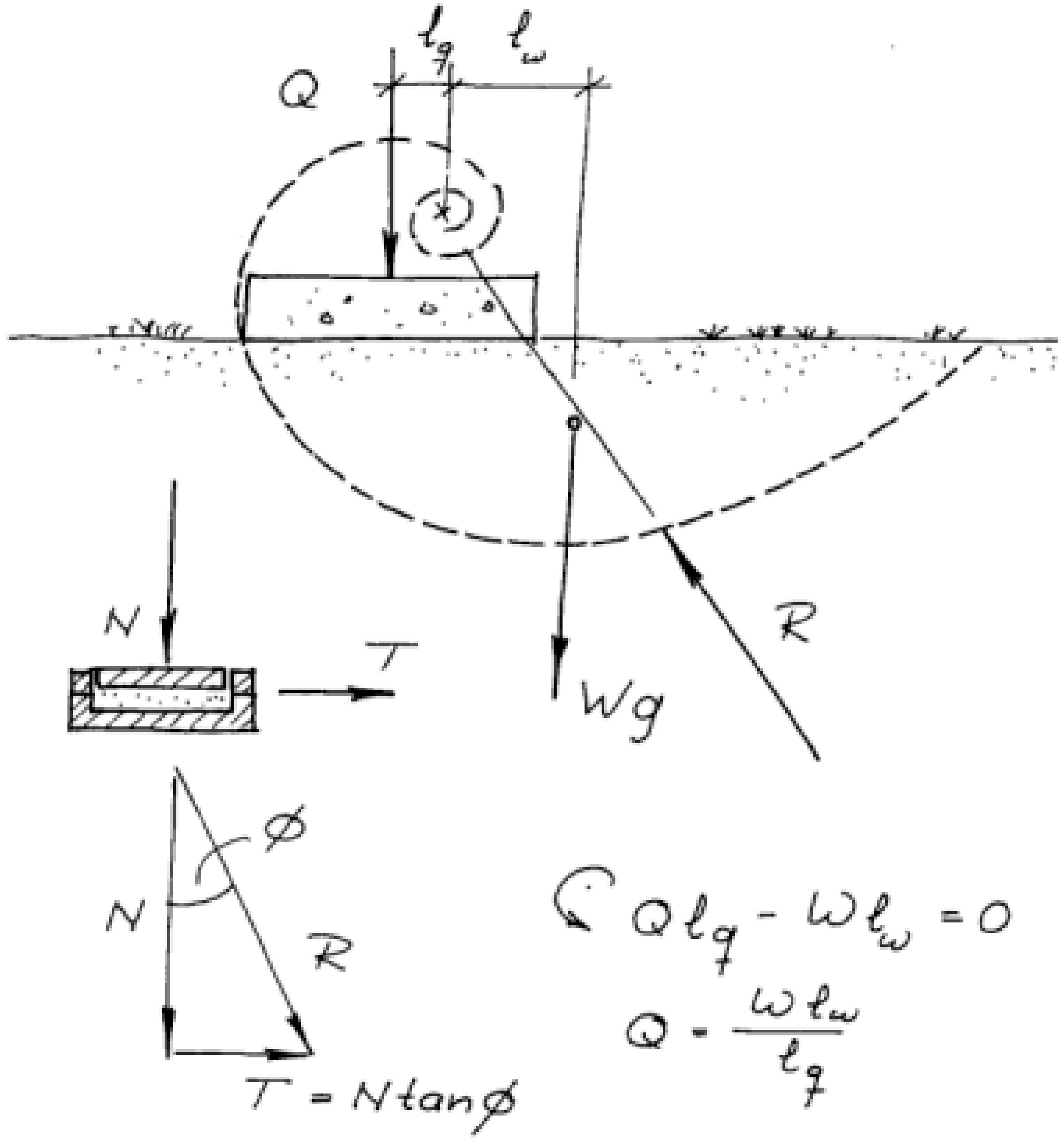
Failure Geometry for Terzaghi's Method



Log-Spiral Geometry



Application of Log-Spiral Geometry



Bearing Capacity Factors

$$q_{ult} = A\,N_c + B\,N_q + C\,N_\gamma$$

- N_c = factor of soil cohesion
- N_q = factor of overburden pressure
- N_γ = factor for unit weight of soil
- A, B, C = constants depending upon geometry, soil properties, etc.
- q_{ult} = ultimate bearing capacity of the soil

Terzaghi Equations and Factors

Loaded strip, width B

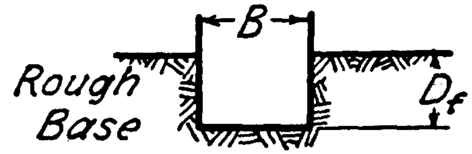
Total load per unit length of footing

General shear failure: $Q_d = B(cN_c + \gamma D_f N_q + \frac{1}{2}\gamma B N_\gamma)$

Local shear failure: $Q'_d = B(\frac{2}{3}cN'_c + \gamma D_f N'_q + \frac{1}{2}\gamma B N'_\gamma)$

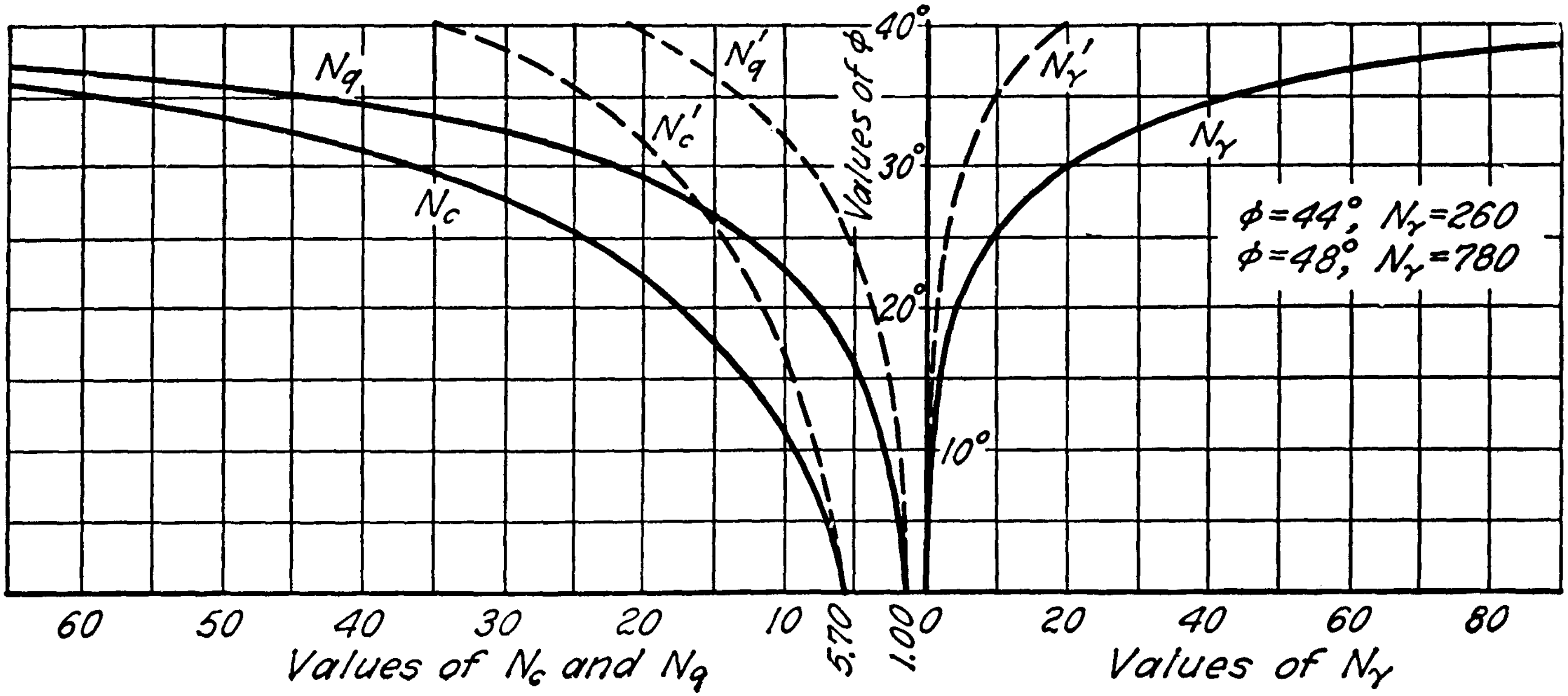
Square footing, width B

Total critical load: $Q_{d_s} = B^2(1.3cN_c + \gamma D_f N_q + 0.4\gamma B N_\gamma)$



Unit weight of earth $= \gamma$

Unit shear resistance, $s = c + \sigma \tan\phi$



Terzaghi Bearing Factors

$$N_c = (N_q - 1)\cot\phi'$$

$$N_q = \frac{e^{\frac{270-\phi'}{180}\pi\tan\phi'}}{2\cos^2(45+\phi'/2)}$$

- Equations for N_c, N_q shown at left
- Values for N_γ originally derived from Cullman's graphical method; can be approximated by equations
- All strictly a function of ϕ

Tabulated Terzaghi Bearing Factors

ϕ'	N_q	N_c	N_γ
28	17.81	31.61	15.7
30	22.46	37.16	19.7
32	28.52	44.04	27.9
34	36.50	52.64	36.0
35	41.44	57.75	42.4
36	47.16	63.53	52.0
38	61.55	77.50	80.0
40	81.27	95.66	100.4
42	108.75	119.67	180.0
44	147.74	151.95	257.0
45	173.29	172.29	297.5
46	204.19	196.22	420.0
48	287.85	258.29	780.1
50	415.15	347.51	1153.2

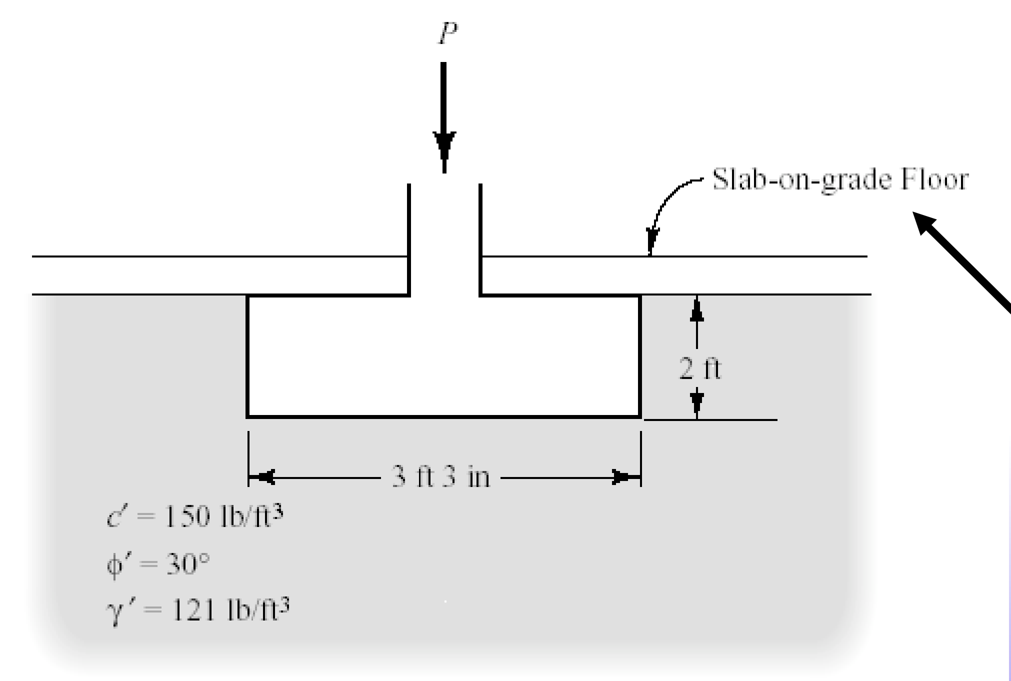
ϕ'	N_q	N_c	N_γ
0	1.00	5.70	0.0
2	1.22	6.30	0.2
4	1.49	6.97	0.4
6	1.81	7.73	0.6
8	2.21	8.60	0.9
10	2.69	9.60	1.2
12	3.29	10.76	1.7
14	4.02	12.11	2.3
16	4.92	13.68	3.0
18	6.04	15.52	3.9
20	7.44	17.69	4.9
22	9.19	20.27	5.8
24	11.40	23.36	7.8
26	14.21	27.09	11.7

Application to Square and Circular Foundations

$$q_{ult} = 1.3c' N_c + \sigma'_{zD} N_q + 0.4\gamma' B N_\gamma$$
$$(Square)$$

$$q_{ult} = 1.3c' N_c + \sigma'_{zD} N_q + 0.3\gamma' D N_\gamma$$
$$(Circular)$$

Example of Terzaghi's Method



Use Terzaghi's Method

- Given
  - Square Foundation as Shown
  - Grounwater table is 50' below surface
  - Ignore slab-on-grade flooring
- Find
  - Ultimate bearing capacity and column load to produce same

Example of Terzaghi's Method

$$q_{ult} = 1.3c'N_c + \sigma'_{zD}N_q + 0.4\gamma'BN_\gamma$$

$$(Square)$$

- Solve for q_{ult}
  - Obtain Bearing Capacity Factors

ϕ'	N_q	N_c	N_γ
0	1.00	5.70	0.0
2	1.22	6.30	0.2
4	1.49	6.97	0.4
6	1.81	7.73	0.6
8	2.21	8.60	0.9
10	2.69	9.60	1.2
12	3.29	10.76	1.7
14	4.02	12.11	2.3
16	4.92	13.68	3.0
18	6.04	15.52	3.9
20	7.44	17.69	4.9
22	9.19	20.27	5.8
24	11.40	23.36	7.8
26	14.21	27.09	11.7
28	17.81	31.61	15.7
30	22.46	37.16	19.7
32	28.52	44.04	27.9
34	36.50	52.64	36.0
35	41.44	57.75	42.4
36	47.16	63.53	52.0
38	61.55	77.50	80.0
40	81.27	95.66	100.4
42	108.75	119.67	180.0
44	147.74	151.95	257.0
45	173.29	172.29	297.5
46	204.19	196.22	420.0
48	287.85	258.29	780.1
50	415.15	347.51	1153.2

Example of Terzaghi's Method

$$q_{ult} = 1.3c' N_c + \sigma'_{zD} N_q + 0.4\gamma' B N_\gamma$$

$$q_{ult} = (1.3)(150)(37.16) + (121)(2)(22.46) + (0.4)(121)(3.25)(19.7)$$

$$q_{ult} = 7246 + 5435 + 3099$$

$$q_{ult} = 15{,}780\ psf$$

- Compute weight of foundation
  - $W_f = (3.25)^2(2)(150) = 3169$ lbs.
- Compute design load

$$q = \frac{P + W_f}{A} - u_D$$

  - $15780 = (P + 3169)/(3.25)^2 - 0$
  - P = 163,507 lbs. = 163.5 kips

Notes on Terzaghi's Method

- Since soil cohesion can be difficult to quantify, conservative values of c (cohesion) should be used
- Frictional strength is more reliable and does not need to be as conservative as cohesion
- Terzaghi's method is simple and and familiar to many geotechnical engineers; however, it does not take into account many factors, nor does it consider cases such as rectangular foundations

Vesić's Method

Bearing capacity factor $f_1(\phi)$ — Depth factor — Ground factor — Base factor

Cohesion — Shape factor ○, □, ▭ — Inclination factor

Bearing capacity factor $f_2(\phi)$ — Overburden pressure

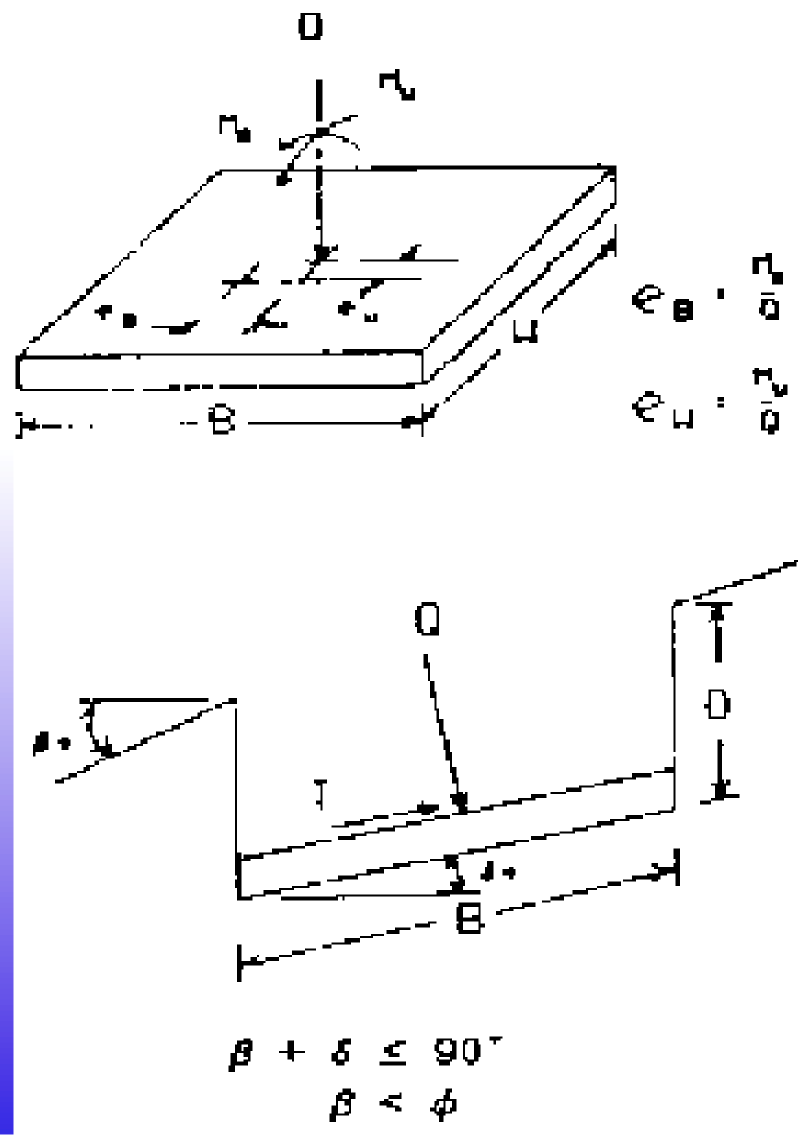
Bearing capacity factor $f_3(\phi)$ — Unit weight of soil — width

$$q_{ult} = \underbrace{c\, N_c\, s_c\, d_c\, i_c\, g_c\, b_c}_{q_c} + \underbrace{\bar{q}\, N_q\, s_q\, d_q\, i_q\, g_q\, b_q}_{q_q} + \underbrace{\tfrac{1}{2} \rho g\, B\, N_\gamma\, s_\gamma\, d_\gamma\, i_\gamma\, g_\gamma\, b_\gamma}_{q_\gamma}$$

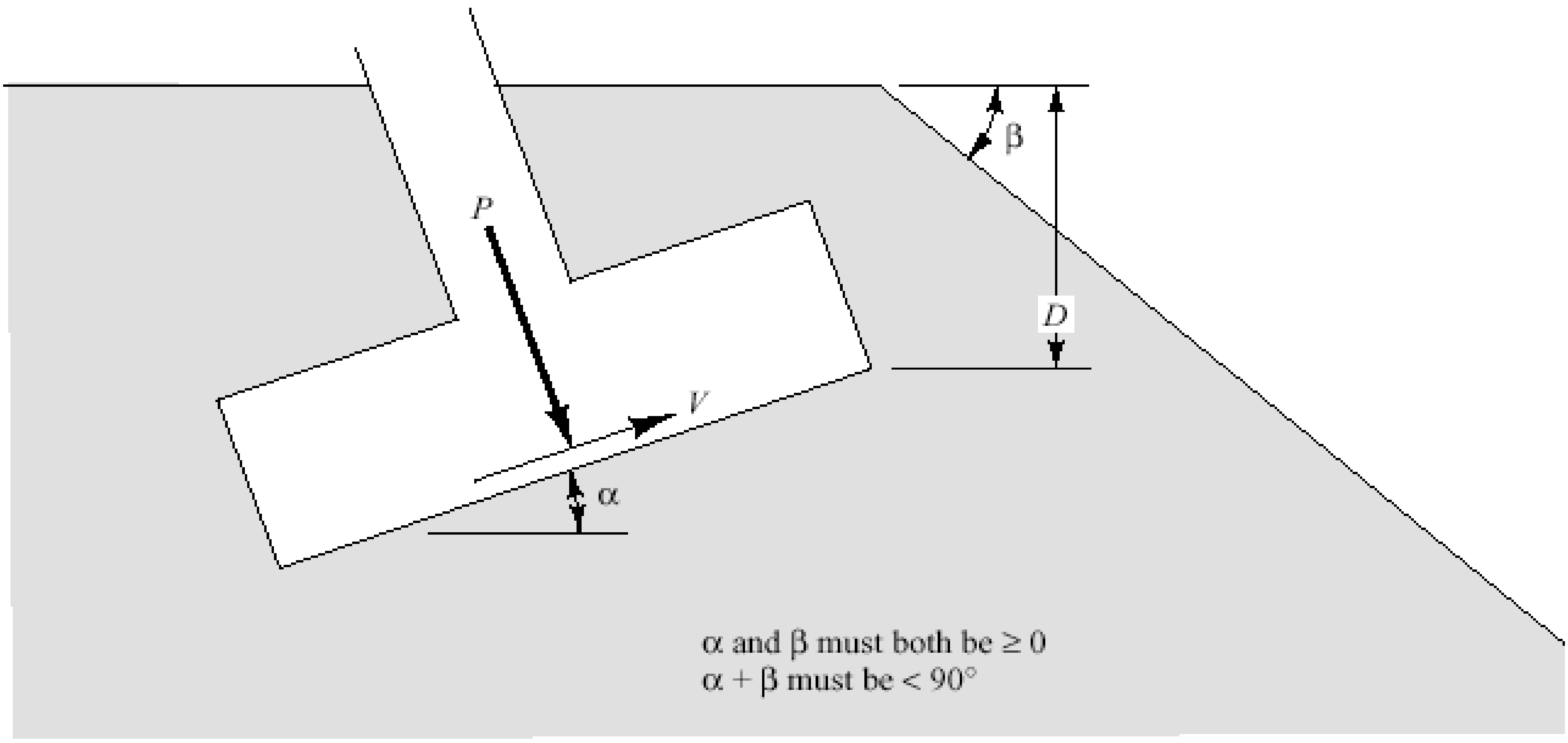
- Similar in basic format to Terzaghi's Method, but takes into account a large number of factors
- Some variations in the way it is implemented

Factors in Vesić Method

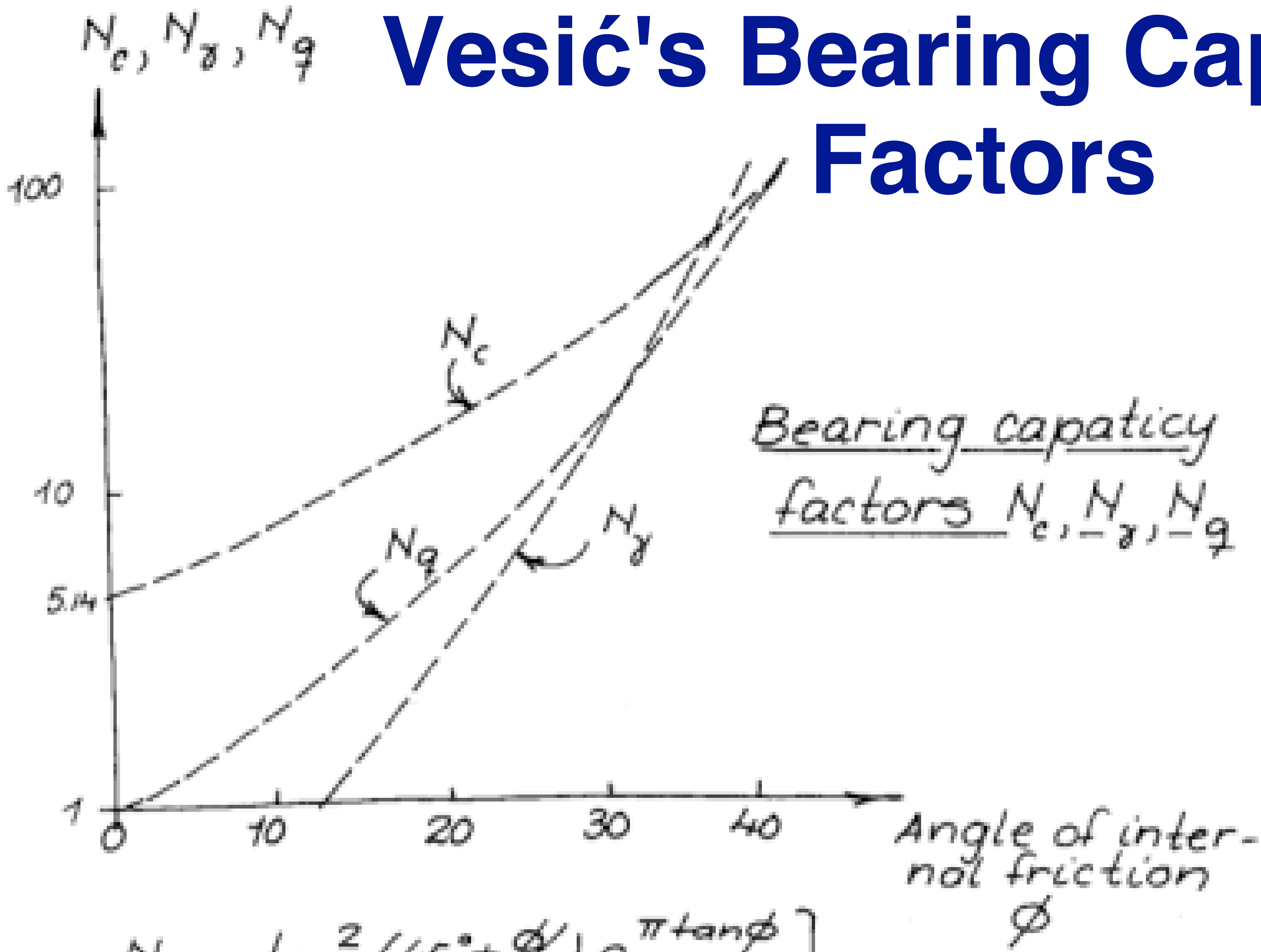
- Bearing Capacity Factor (N)
- Shape Factor (s)
- Depth Factor (d)
- Inclination Factor (i)
- Ground Factor (g)
- Base Factor (b)



Notations for Vesić's Method



Vesić's Bearing Capacity Factors



$$\left.\begin{aligned} N_q &= \tan^2(45° + \phi/2)\, e^{\pi \tan\phi} \\ N_c &= (N_q - 1)\cot\phi \\ N_\gamma &= \cancel{1.8}\ \underset{2.0}{}(N_q - 1)\tan\phi \end{aligned}\right\} f(\phi)$$

Tabulated Vesić "N" Factors

				N_γ		
ϕ	N_ϕ	N_c	N_q	Meyerhof	Hansen	Vesic
0	1.00	5.14	1.00	0.00	0.00	0.00
2	1.07	5.63	1.20	0.01	0.01	0.15
4	1.15	6.18	1.43	0.04	0.05	0.34
6	1.23	6.81	1.72	0.11	0.11	0.57
8	1.32	7.53	2.06	0.21	0.22	0.86
10	1.42	8.34	2.47	0.37	0.39	1.22
12	1.52	9.28	2.97	0.60	0.63	1.69
14	1.64	10.37	3.59	0.92	0.97	2.29
16	1.76	11.63	4.34	1.37	1.43	3.06
18	1.89	13.10	5.26	2.00	2.08	4.07
20	2.04	14.83	6.40	2.87	2.95	5.39
22	2.20	16.88	7.82	4.07	4.13	7.13
24	2.37	19.32	9.60	5.72	5.75	9.44
26	2.56	22.25	11.85	8.00	7.94	12.54
28	2.77	25.80	14.72	11.19	10.94	16.72
30	3.00	30.14	18.40	15.67	15.07	22.40
32	3.25	35.49	23.18	22.02	20.79	30.21
34	3.54	42.16	29.44	31.15	28.77	41.06
36	3.85	50.59	37.75	44.43	40.05	56.31
38	4.20	61.35	48.93	64.07	56.17	78.02
40	4.60	75.31	64.19	93.69	79.54	109.41
42	5.04	93.71	85.37	139.32	113.95	155.54
44	5.55	118.37	115.31	211.41	165.58	224.63
46	6.13	152.10	158.50	328.73	244.64	330.33
48	6.79	199.26	222.30	526.44	368.88	495.99
50	7.55	266.88	319.05	873.84	568.56	762.85

Shape Factor (s)

$$s_c = 1 + \frac{B}{L}\frac{N_q}{N_c}$$

$$s_q = 1 + \frac{B}{L}\tan\phi'$$

$$s_\gamma = 1 - 0.4\frac{B}{L}$$

For continuous footings, s = 1

Depth Factor (d)

$$d_c = 1 + 0.4k$$

$$d_q = 1 + 2k\tan\phi'(1 - \sin\phi')^2$$

$$d_\gamma = 1$$

- Values of k
  - D/B < 1, k = D/B
  - D/B > 1, k = arctan (D/B), result in radians
  - Discontinuity when D = B

Load Inclination Factor (i)

$$i_c = 1 - \frac{mV}{Ac'N_c} \geqslant 0$$

$$i_q = \left[1 - \frac{V}{P + \frac{Ac'}{\tan\phi'}}\right]^m \geqslant 0$$

$$i_\gamma = \left[1 - \frac{V}{P + \frac{Ac'}{\tan\phi'}}\right]^{m+1} \geqslant 0$$

Load Inclination Factor (i)

- Variables
  - V = applied shear load
  - P = applied normal load
  - A = base area of footing
  - c' = effective cohesion (use $c = s_u$ for undrained analyses)
  - φ' = effective friction angle (use $\varphi = 0$ for undrained analyses)
  - B = foundation width
  - L = foundation length

Load Inclination Factor (i)

- Values of m
  - Loads inclined in the B direction: $m = \dfrac{2 + B/L}{1 + B/L}$
  - Loads inclined in the L direction: $m = \dfrac{2 + L/B}{1 + L/B}$
- i = 1 if either loads act perpendicular to footing or soil is purely cohesive ($\varphi = 0$)
- Applies to loads that are not perpendicular to the base of the foundation; does not apply to eccentric loads

Ground Inclination Factor (g)

$$g_c = 1 - \frac{\beta}{147^o}$$

$$g_q = g_\gamma = [1 - \tan\beta]^2$$

- For level ground surface, g = 1

Base Factor (b)

$$b_c = 1 - \frac{\alpha}{147^o}$$

$$b_q = b_\gamma = [1 - \tan\beta]^2$$

- For footings with angled foundation bases
- When footing is level, b = 1

Allowable Bearing Capacity

Factors when considering selection of a factor of safety

Soil Type	Sand	Clay
Site Characterization Data	Extensive	Minimal
Soil Variability	Uniform	Erratic
Importance of Structure & Consequences of Failure	Low	High
Likehood of Design Load Occurring	Low	High
Design F — Extreme Values	2.0	4.0
Design F — Typical Range	2.5	3.5

- Most foundations designed by ASD for geotechnical strength

$$q_a = \frac{q_{ult}}{F}$$

- Foundation is then designed so that the allowable bearing pressure is not exceeded

General Bearing Capacity Example

- Given
  - Continuous Foundation
  - Width of foundation = 1100 mm (B)
  - Depth of Foundation = 1500 mm (D)
  - Soil cohesion c = 15 kPa
  - Soil internal friction angle $\varphi = 28^{o}$, $\gamma = 19\ kN/m^3$
  - Water table even with depth of foundation
- Find
  - Design loading, FS = 3
  - Use depth factor term but ignore foundation weight

Bearing Example

Bearing capacity factor $f_1(\phi)$
Depth factor
Ground factor
Base factor
Cohesion
Inclination factor
Shape factor ○, □, ▭

$$q_{ult} = \underbrace{c\, N_c\, s_c\, d_c\, i_c\, g_c\, b_c}_{q_c}$$

Bearing capacity factor $f_2(\phi)$
Overburden pressure

$$+ \underbrace{\bar{q}\, N_q\, s_q\, d_q\, i_q\, g_q\, b_q}_{q_q}$$

Bearing capacity factor $f_3(\phi)$
Unit weight of soil
width

$$+ \underbrace{\tfrac{1}{2}\, \rho g\, B\, N_\gamma\, s_\gamma\, d_\gamma\, i_\gamma\, g_\gamma\, b_\gamma}_{q_\gamma}$$

- For continuous footing, s = 1
- For perpendicular load, i = 1
- For level foundation, b =1
- For level ground, g = 1
- Need to compute factors N, d

"N" Factors for Example

				N_γ		
ϕ	N_ϕ	N_c	N_q	Meyerhof	Hansen	Vesic
0	1.00	5.14	1.00	0.00	0.00	0.00
2	1.07	5.63	1.20	0.01	0.01	0.15
4	1.15	6.18	1.43	0.04	0.05	0.34
6	1.23	6.81	1.72	0.11	0.11	0.57
8	1.32	7.53	2.06	0.21	0.22	0.86
10	1.42	8.34	2.47	0.37	0.39	1.22
12	1.52	9.28	2.97	0.60	0.63	1.69
14	1.64	10.37	3.59	0.92	0.97	2.29
16	1.76	11.63	4.34	1.37	1.43	3.06
18	1.89	13.10	5.26	2.00	2.08	4.07
20	2.04	14.83	6.40	2.87	2.95	5.39
22	2.20	16.88	7.82	4.07	4.13	7.13
24	2.37	19.32	9.60	5.72	5.75	9.44
26	2.56	22.25	11.85	8.00	7.94	12.54
28	2.77	25.80	14.72	11.19	10.94	16.72
30	3.00	30.14	18.40	15.67	15.07	22.40
32	3.25	35.49	23.18	22.02	20.79	30.21
34	3.54	42.16	29.44	31.15	28.77	41.06
36	3.85	50.59	37.75	44.43	40.05	56.31
38	4.20	61.35	48.93	64.07	56.17	78.02
40	4.60	75.31	64.19	93.69	79.54	109.41
42	5.04	93.71	85.37	139.32	113.95	155.54
44	5.55	118.37	115.31	211.41	165.58	224.63
46	6.13	152.10	158.50	328.73	244.64	330.33
48	6.79	199.26	222.30	526.44	368.88	495.99
50	7.55	266.88	319.05	873.84	568.56	762.85

Depth Factor (d)

- Values of k
  - D/B > 1.5/1.1 = 1.363
  - k = arctan (1.363) = 0.938 radians

$$d_c = 1 + 0.4k = 1.375$$

$$d_q = 1 + 2k \tan\phi'(1 - \sin\phi')^2 = 1.281$$

$$d_\gamma = 1$$

Solution of Bearing Capacity Equation

$$q_{ult} = c' N_c d_c + \sigma'_{zD} N_q d_q + 0.5 \gamma' B N_\gamma d_\gamma$$

$$(15)(25.8)(1.375) + (19)(1.5)(14.72)(1.281)$$

$$+ (0.5)(19 - 9.81)(1.1)(16.72)(1)$$

$$q_{ult} = 532.13 + 537.4 + 84.51 = 1154\, kPa$$

$$q_a = 1154/3 = 385\, kPa$$

- Allowable wall loading per lineal metre
  - (385 kPa) (1.1 m wide) (1 m long) = 423 kN/m

Example Using Square Foundation

- Given
  - Square foundation, load of 1500 kN
  - On Soil Surface
  - Soil Conditions
    - Sand, No Cohesion, $\gamma = 20\ kN/m^3$, $\varphi = 36^o$
- Find
  - Acceptable foundation size for square foundation
  - Neglect effect of foundation weight

Bearing Capacity Equation for Sand

<u>Sand.</u> Effective stress analysis
(ϕ-analysis)

$c = 0, \quad \phi' \approx \phi_d$

From drained triaxial or direct shear tests

$$\phi_{\text{plain strain}} \approx 1.1\,\phi_{\text{triaxial}}$$

Vertical load
Horizontal surface

$$q_{ult} = \cancel{c N_c s_c d_c i_c g_c b_c} \qquad c = 0$$

$$+ \bar{q}\, N_q\, \underbrace{s_q}_{1 + \frac{B}{L}\tan\phi}\, \underbrace{d_q}_{1.0\ (\text{if shear strength is neglected})}\, \underbrace{i_q}_{1.0}\, \underbrace{g_q}_{1.0}\, \underbrace{b_q}_{1.0}$$

$$+ 0.5\, N_\gamma\, \rho g\, \underbrace{s_\gamma}_{(1 - 0.4\, B/L)}\, \underbrace{d_\gamma}_{1.0}\, \underbrace{i_\gamma}_{1.0}\, \underbrace{g_\gamma}_{1.0}\, \underbrace{b_\gamma}_{1.0}$$

Surcharge Overburden Pressure

"q" factor zero if foundation is at the surface

$(1 - 0.4\, B/L)$ =1 – 0.4 = 0.6 in this case

Example Using Square Foundation

$$q_{ult} = \frac{1}{2}\gamma' B N_\gamma s_\gamma$$

- For $\varphi = 36^o$, $N_\gamma = 56.31$
- $s_\gamma = 0.6$
- $q_{ult} = (0.5)(20)(B)(56.31)(0.6) = 337.86B$ kPa
- $q_a = q_{ult}/FS = 337.86/3 = 112.62B$ kPa
- $q_a = Q/A = 1500$ kN/A $= 1500$ kN/B^2

Example Using Square Foundation

- $q_a = q_{ult}/FS = 337.86/3 = 112.62B$ kPa
- $q_a = Q/A = 1500$ kN/A $= 1500$ kN/B^2
- $1500/B^2 = 112.62B$
- $B^3 = 1500/112.62 = 13.32$
- $B = 2.37$ m

Bearing Capacity in Clay

Clay. Total stress analysis

($\phi = 0$-analysis, c-analysis)

c_u from field vane tests,
undrained triaxial tests
unconfined compression tests

Horizontal surface
Vertical load

$$q_{ult} = c \underset{5.14}{N_c} \overset{(1+0.2B/L)}{s_c} \underset{(1+0.4D/B)}{d_c} \overset{1.0}{i_c} \underset{1.0}{g_c} \overset{1.0}{b_c}$$

$$+ \underset{D\rho g}{\bar{q}} \overset{1.0}{N_q} \underset{\sim 1.0}{s_q} \overset{\sim 1.0}{d_q} \underset{1.0}{i_q} \overset{1.0}{b_q} \qquad N_q = 1.0 \; (\phi = 0)$$

$$+ \cancel{\tfrac{1}{2} N_\gamma \rho g s_\gamma d_\gamma i_\gamma b_\gamma} \qquad N_\gamma = 0 \; (\phi = 0)$$

Questions?